

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2017/2018

(SOLUTION)

PMT0101 – MATHEMATICS I
(Foundation in Information Technology)

02 MARCH 2018

(2 Hours)

QUESTION 1 [10 marks]

a)

$$\begin{aligned}
 & \frac{m^7 \times (8e^3)^{\frac{1}{3}}}{(2m^{-2}e^3)^2} \\
 &= \frac{m^7 (2^3)^{\frac{1}{3}} (e^3)^{\frac{1}{3}}}{2^2 (m^{-2})^2 (e^3)^2} \\
 &= \frac{2 m^7 e}{2^2 m^{-4} e^6} \\
 &= \frac{m^{11}}{2e^5} \quad [1 + 0.5 + 0.5]
 \end{aligned}$$

b)

$$\begin{aligned}
 & \frac{4\sqrt{3} + 2}{2\sqrt{3} - 1} \\
 &= \frac{(4\sqrt{3} + 2)(2\sqrt{3} + 1)}{(2\sqrt{3} - 1)(2\sqrt{3} + 1)} \\
 &= \frac{8(3) + 4\sqrt{3} + 4\sqrt{3} + 2}{4(3) - 1} \\
 &= \frac{26 + 8\sqrt{3}}{11} \quad [0.5 + 1 + 0.5]
 \end{aligned}$$

c)

$$\begin{aligned}
 & b^2 \left(\sqrt{\frac{32a^3b}{c^4}} \right) - \frac{ac^2}{b} \sqrt{\frac{98ab^7}{c^8}} \\
 &= b^2 \sqrt{\frac{16 \times 2a^2ab}{c^4}} - \frac{ac^2}{b} \sqrt{\frac{49 \times 2ab^6b}{c^8}} \\
 &= \frac{4ab^2 \sqrt{2ab}}{c^2} - \frac{ac^2(7b^3) \sqrt{2ab}}{bc^4} \\
 &= \frac{4ab^2 \sqrt{2ab}}{c^2} - \frac{7ab^2 \sqrt{2ab}}{c^2} \\
 &= \frac{-3ab^2 \sqrt{2ab}}{c^2} \quad [0.5 + 0.5 + 0.5 + 0.5]
 \end{aligned}$$

d)

$$\begin{aligned} & 27 - 8x^3 \\ &= 3^3 - (2x)^3 \\ &= (3 - 2x)(3^2 + 3(2x) + (2x)^2) \\ &= (3 - 2x)(9 + 6x + 4x^2) \quad [0.5 + 1 + 0.5] \end{aligned}$$

e)

$$\begin{aligned} & \left(\frac{\sqrt{3}}{2} - \frac{1}{2}i \right)^2 \\ &= \left(\frac{\sqrt{3}}{2} \right)^2 - 2 \left(\frac{\sqrt{3}}{2} \right) \left(\frac{i}{2} \right) + \left(\frac{i}{2} \right)^2 \\ &= \frac{3}{4} - \frac{\sqrt{3}}{2}i + \frac{1}{4}i^2 \\ &= \frac{1}{2} - \frac{\sqrt{3}}{2}i \quad [1 + 0.5 + 0.5] \end{aligned}$$

QUESTION 2 [10 marks]

a)i)

$$\begin{aligned}
 &6x^3 - 5x^2 - 6x \\
 &= x(6x^2 - 5x - 6) \quad [0.5] \\
 &= x(2x - 3)(3x + 2) \quad [1]
 \end{aligned}$$

$$x(2x - 3)(3x + 2) = 0$$

$$x = 0 \quad \text{or} \quad x = \frac{3}{2} \quad \text{or} \quad x = -\frac{2}{3} \quad [1.5]$$

ii)

	$-\frac{2}{3}$	0	$\frac{3}{2}$	
Sign of x	---	---	+++	+++
Sign of $(2x - 3)$	---	---	---	+++
Sign of $(3x + 2)$	---	+++	+++	+++
Sign of $x(2x - 3)(3x + 2)$	---	+++	---	+++

[0.5]

[0.5]

[0.5]

[0.5]

$$\text{The solution set is } \left[-\infty, -\frac{2}{3}\right] \cup \left[0, \frac{3}{2}\right] \quad [0.5+0.5]$$

b)

$$\begin{aligned}
 &\sqrt{12x + 13} = 2x + 3 \\
 &12x + 13 = (2x + 3)^2 \\
 &4x^2 + 12x + 9 - 12x - 13 = 0 \\
 &4x^2 - 4 = 0 \\
 &x^2 - 1 = 0 \\
 &(x + 1)(x - 1) = 0 \\
 &x = -1 \quad \text{or} \quad x = 1 \quad [0.5 + 0.5 + 0.5 + 0.5]
 \end{aligned}$$

Check the solution:

When $x = -1$, $LHS = \sqrt{12(-1) + 13} = 1$ $RHS = 2(-1) + 3 = 1$ $LHS = RHS \quad [0.5]$	When $x = 1$, $LHS = \sqrt{12(1) + 13} = 5$ $RHS = 2(1) + 3 = 5$ $LHS = RHS \quad [0.5]$
$\therefore x = -1 \text{ and } 1 \quad [0.5+0.5]$	

QUESTION 3 [10 marks]

a)

i) Let $y = 2x - 3$

$$x = \frac{y+3}{2}$$

$$f^{-1}(x) = \frac{x+3}{2} \quad [0.5 + 0.5]$$

ii)

$$(f^{-1} \circ g)(3)$$

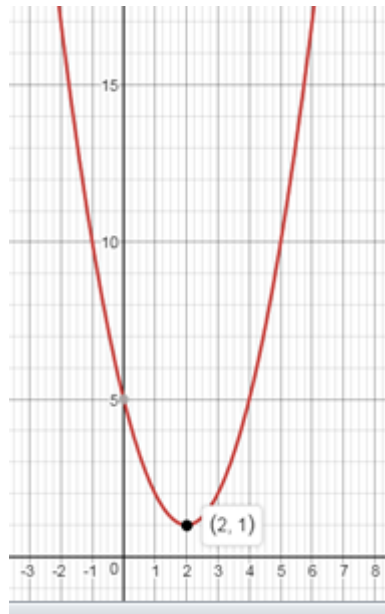
$$= f^{-1}(g(3))$$

$$= f^{-1}[(3-2)^2 + 1]$$

$$= f^{-1}(2)$$

$$= \frac{5}{2} \quad [0.5 + 0.5 + 0.5]$$

iii)

Minimum point is $(2, 1)$ [0.5]y-intercept, $f(0) = 5$ [0.5]

Shows correct shape, minimum point and y-intercept

[1]

Range of g is $[1, \infty)$

[0.5]

b)

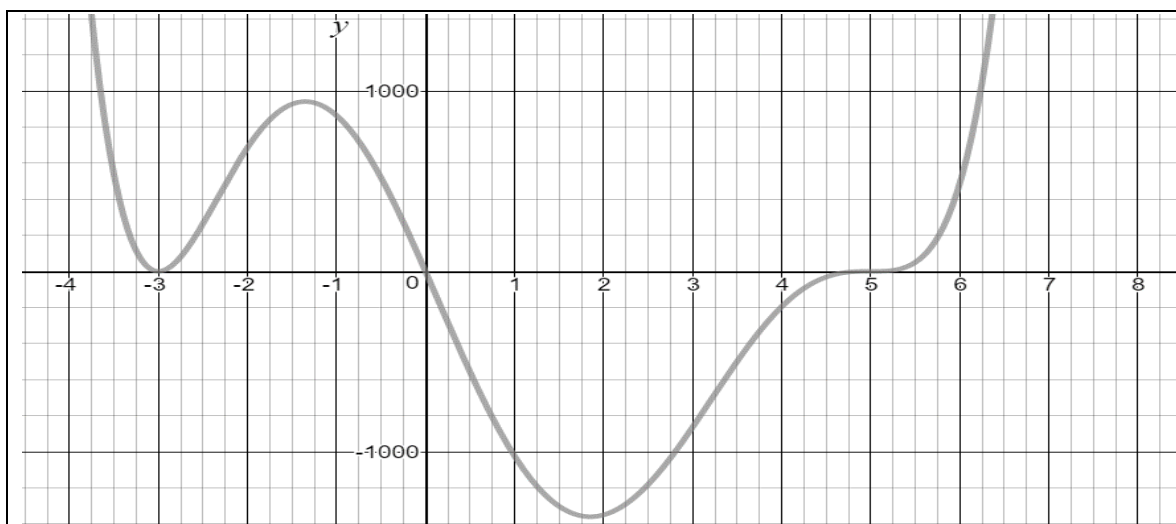
i) Degree = 6 [0.5]

ii) The zeros of f , let $f(x) = 0$.

Zeros	Multiplicities	Crosses/Touches x -axis	
0	1	Crosses	[0.5]
-3	2	Touches	[0.5]
5	3	Crosses	[0.5]

iii) y -intercept, $f(0) = 0$ [0.5]iv) As $x \rightarrow -\infty$, $y \rightarrow \infty$ [0.5]As $x \rightarrow \infty$, $y \rightarrow \infty$ [0.5]

v) Sketch the graph:



Shows all intercepts [0.5]

Proper end behaviour [0.5]

Shows correct crossing or touching at x -intercepts [0.5]

QUESTION 4 [10 marks]

a)

$$\begin{aligned}
 f(26) &= -1 \\
 2 - \log_3(26 + a) &= -1 \\
 \log_3(26 + a) &= 3 \\
 26 + a &= 27 \\
 a &= 1 \quad [0.5 + 0.5 + 0.5 + 0.5]
 \end{aligned}$$

b)

$$\begin{aligned}
 9^{-x+15} &= 27^{x+3} \\
 3^{2(-x+15)} &= 3^{3(x+3)} \\
 -2x + 30 &= 3x + 9 \\
 5x &= 21 \\
 x &= \frac{21}{5} \quad [0.5 + 0.5 + 0.5 + 0.5]
 \end{aligned}$$

c)

$$\begin{aligned}
 \ln(x-1) - \ln(x+6) &= \ln(x-2) - \ln(x+3) \\
 \ln\left[\frac{x-1}{x+6}\right] &= \ln\left[\frac{x-2}{x+3}\right] \\
 \frac{x-1}{x+6} &= \frac{x-2}{x+3} \\
 (x-1)(x+3) &= (x-2)(x+6) \\
 x^2 + 2x - 3 &= x^2 + 4x - 12 \\
 x &= \frac{9}{2} \quad [0.5 + 0.5 + 0.5 + 0.5]
 \end{aligned}$$

d)

$$\begin{aligned}
 P(2) &= 0 \\
 2(2)^3 + p(2)^2 - 4 &= 0 \\
 16 + 4p - 4 &= 0 \\
 4p &= -12 \\
 p &= -3 \quad [0.5 + 0.5 + 0.5 + 0.5]
 \end{aligned}$$

$$\begin{aligned}
 P(x) &= 2x^3 - 3x^2 - 4 \quad [0.5] \\
 P(-1) &= 2(-1)^3 - 3(-1)^2 - 4 \\
 &= -2 - 3 - 4 \\
 &= -9 \quad [0.5 + 0.5 + 0.5]
 \end{aligned}$$

QUESTION 5 [10 marks]

a)

$3x + ky + 6 = 0$ $y = -\frac{3}{k}x - \frac{6}{k}$ [0.5]	$5x - y - 1 = 0$ $y = 5x + 1$ [0.5]
$-\frac{3}{k} \times 5 = -1$ $k = 15$ [0.5 + 0.5]	

b)

$$x^2 - 12x + y^2 + 12y + 36 = 0$$

$$x^2 - 12x + (-6)^2 + y^2 + 12y + (6)^2 = -36 + (-6)^2 + (6)^2$$

$$(x-6)^2 + (y+6)^2 = 36 \quad [1+1]$$

$$\text{Center} = (6, -6)$$

$$\text{Radius} = 6 \quad [0.5+0.5]$$

c)

$$\text{i) } m = \frac{7-1}{2-4} = -3 \quad [0.5 + 0.5]$$

$$Y = mX + c$$

$$7 = -3(2) + c$$

$$c = 13$$

$$Y\text{-intercept} = 13 \quad [0.5+0.5]$$

$$\text{ii) } x\sqrt{y} = -3x^2 + 13 \quad [0.5]$$

iii) When $x = 2$,

$$2\sqrt{y} = -3(2)^2 + 13$$

$$\sqrt{y} = \frac{1}{2}$$

$$y = \frac{1}{4} \quad [0.5 + 0.5 + 0.5]$$

iv)

$$\text{At } (q, 6), \quad X = q, \quad Y = 6.$$

$$6 = -3q + 13$$

$$3q = 7$$

$$q = \frac{7}{3} \quad [0.5 + 0.5]$$