STUDENT ID NO

SEAT NO

VENUE:____________________

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2017/2018

PMT0301 – MATHEMATICS III

(All sections/ Groups)

2 MARCH 2018
9.00 a.m. – 11.00 a.m.
(2 Hours)

Question	Marks
1	/10
2	/10
3	/10
4	/10
Total	/40

INSTRUCTIONS TO STUDENTS

1. This question paper consists of **NINE** printed pages excluding cover page and statistical table.

2. Answer **ALL FOUR** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the **QUESTION BOOKLET**. All necessary working steps **MUST** be shown to obtain maximum marks.

Question 1

a) Find the angle between the planes $x+y+z=8$ and $x-y+z=-3$. Correct your answer to the nearest degree. (3 marks)

Let $\mathbf{u}=\langle 1,1,1\rangle$ and $\mathbf{v}=\langle 1,-1,1\rangle$

$|\mathbf{u}|=\sqrt{1^2+1^2+1^2}=\sqrt{3}$

$|\mathbf{v}|=\sqrt{1^2+(-1)^2+1^2}=\sqrt{3}$

$\mathbf{u}\cdot\mathbf{v}=(1)(1)+(1)(-1)+(1)(1)=1$

$\therefore \mathbf{u}\cdot\mathbf{v}=|\mathbf{u}|\cdot|\mathbf{v}|\cos\theta$

$1=\sqrt{3}\cdot\sqrt{3}\cos\theta$

$\cos\theta=\dfrac{1}{\sqrt{3}\cdot\sqrt{3}}=\dfrac{1}{3}$

$\theta=70.53^\circ=71^\circ$

b) Find an equation of the plane that contains the line $x=1+3t$, $y=-1+t$, $z=3+8t$ and is perpendicular to the plane $4x-2y-3z=8$. Give your final answer in the form of $ax+by+cz=d$. (3 marks)

A known point on the plane: $(1,-1,3)$

A vector on the plane: $\langle 3,1,8\rangle$

The normal vector from $4x-2y-3z=8:\langle 4,-2,-3\rangle$

Let the cross product of $\langle 3,1,8\rangle$ and $\langle 4,-2,-3\rangle$ to be $\vec{n}$

$$=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 3 & 1 & 8\\ 4 & -2 & -3\end{vmatrix}$$

$=\mathbf{i}((-3)-(-16))-\mathbf{j}((-9)-32)+\mathbf{k}(-6-4)$

$=13\mathbf{i}+41\mathbf{j}-10\mathbf{k}$

$\mathbf{n}\cdot(\mathbf{r}-\mathbf{r_0})=0$

$\langle 13,41,-10\rangle\cdot(\langle x,y,z\rangle-\langle 1,-1,3\rangle)=0$

$13(x-1)+(41)(y-(-1))+(-10)(z-3)=0$

$13x-13+41y+41-10z+30=0$

$13x+41y-10z=-58$

Continued…

c) Adam purchases a machine for RM250,000. Assume the annual depreciation is by 10% of its value in the previous year. In what year will the value of the machine be less than RM75,000? Let $n = 1$ be the year the machine is purchased. (2.5 marks)

$a = \text{RM}250000$

$r = 1 - 0.1 = 0.9$

$ar^{n-1} < 75000$

$(250000)(0.9)^{n-1} < 75000$

$0.9^{n-1} < 0.3$

$$n - 1 > \frac{\log_{10} 0.3}{\log_{10} 0.9}$$

$n - 1 > 11.43$

$n > 12.43$

$\therefore n = 13$

d) Find the coefficient of the term that contains x^5 in the expansion of $(-2x-1)^6$.
(1.5 marks)

$\begin{pmatrix} 6 \\ 1 \end{pmatrix}(-2x)^5(-1)^1$

$= 6(-32x^5)(-1)$

$= 192x^5$

The coefficient is 192

Continued…

Question 2

a) Solve the system of linear equations with Gauss-Jordan Elimination method.

$$x+y+z=3$$
$$y+5z=-6$$
$$2y-3z=14$$

(4.5 marks)

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 5 & -6 \\ 0 & 2 & -3 & 14 \end{array}\right] \xrightarrow{R_3-2R_2\to R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 5 & -6 \\ 0 & 0 & -13 & 26 \end{array}\right] \xrightarrow{\frac{R_3}{(-13)}\to R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 5 & -6 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$\begin{array}{c} R_1-R_3\to R_1 \\ \to \\ R_2-5R_3\to R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 5 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -2 \end{array}\right] \xrightarrow{R_1-R_2\to R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$\therefore x=1, y=4, z=-2$

Continued…

b) The data below shows the age of 24 patients who are diagnosed with cancer in a certain district:

65	75	59	32
67	56	66	45
53	57	51	69
54	59	60	51
55	45	57	59
40	51	58	60

i) Based on the data, construct a frequency distribution table with the lower limit of the first class is 31 and the width of each class is at 10. (2 marks)

Class Limit	**Frequency**
31-40	2
41-50	2
51-60	15
61-70	4
71-80	1

ii) Based on part (bi), calculate the mode. Give your answer correct to 1 decimal place. (2 marks)

Mode Class is 51- 60

$$\text{Mode} = L + \left(\frac{f_m - f_B}{(f_m - f_A) + (f_m - f_B)} \right) c$$

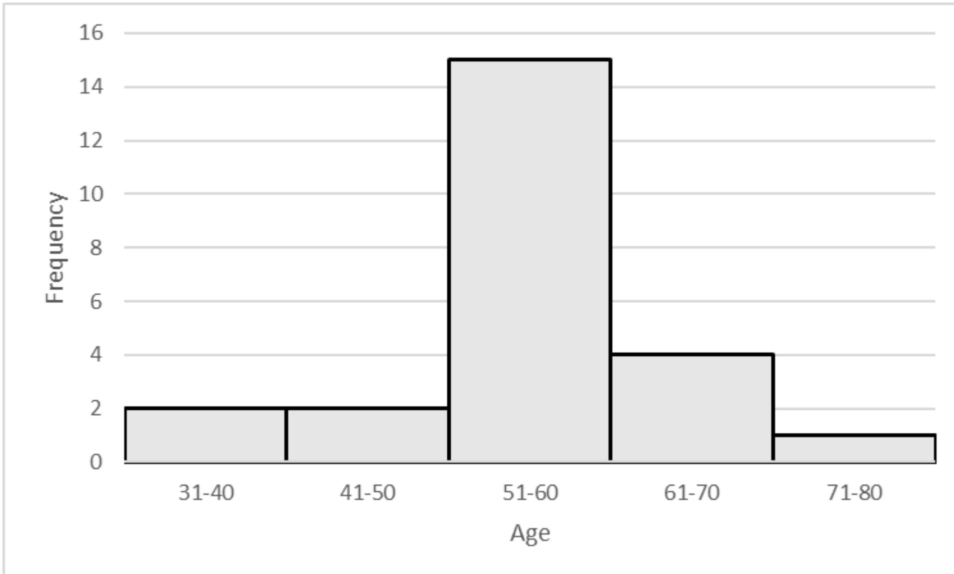
$$= 50.5 + \left(\frac{15 - 2}{(15 - 4) + (15 - 2)} \right) 10$$

$$= 50.5 + 5.4$$

$$= 55.9$$

Continued…

iii) Construct a histogram to represent the data. (1.5 marks)



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Question 3

a) A license plate consists of one letter followed by four digits. How many different plates are possible if the letters 'Y' and 'Z' are not allowed and the digit cannot have the zero as the lead position. (1.5 marks)

$$24 \times 9 \times 10 \times 10 \times 10$$
$$= 216000$$

b) How many ways are possible to sit ten students in a row if four specific students refuse to sit together? (2.5 marks)

$$6! \times {}^7P_4 = 720 \times 840 = 604800$$

c) A dice is rolled 3 times successively. What is the probability that the numbers on the uppermost surface add up will be exactly 16? (2 marks)

Set points with total of the three dice is $16 = \{466, 646, 664, 556, 565, 655\}$

$P(\text{total of the three dice is } 16)$

$$= \frac{6}{6 \times 6 \times 6}$$
$$= \frac{6}{216}$$
$$= \frac{1}{36} \text{ or } 0.0278$$

Continued…

d) The events A and B are such that $P(A) = 0.5$, $P(A \cup B) = 0.8$ and $P(B|A) = 0.3$.

i) Find $P(B)$. (2 marks)

$P(B|A) = 0.3$

$\frac{P(B \cap A)}{P(A)} = 0.3$

$P(B \cap A) = 0.3 \times 0.5 = 0.15$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.8 = 0.5 + P(B) - 0.15$

$P(B) = 0.45$

ii) Are events A and B statistically independent? Explain your answer. (2 marks)

By using, $P(A|B) = P(A)$

RHS:

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

$= \frac{0.15}{0.45}$

$= 0.3333$

LHS:

$P(A) = 0.5$

Therefore, $P(A|B) \neq P(A)$. As a conclusion, events 'A' and 'B' are not independent.

or

$P(B\|A) = P(B)$ $\text{LHS}: P(B\|A) = 0.3$ $\text{RHS}: P(B) = 0.45$ $\therefore P(B\|A) \neq P(B)$	$P(A) \cdot P(B) = P(A \cap B)$ $\text{LHS}: P(A) \cdot P(B)$ $= 0.5 \times 0.45 = 0.225$ $\text{RHS}: P(A \cap B) = 0.15$ $\therefore P(A) \cdot P(B) \neq P(A \cap B)$

Continued…

Question 4

a) The statistics show that 25% of engineers prefer to work in a local company. Based on a randomly selected 12 engineers,

i) evaluate the probability that at least 2 engineers prefer to work in a local company. (2 marks)

X: No. of engineers prefer to work in a local company

$n = 12, p = 0.25, q = 0.75$

$$P(X \geq 2)$$
$$= 1 - [P(x = 0) + P(x = 1)]$$
$$= 1 - \left[\binom{12}{0}(0.25)^0(0.75)^{12} + \binom{12}{1}(0.25)^1(0.75)^{11}\right]$$
$$= 1 - (0.0317 + 0.1267)$$
$$= 1 - 0.1584$$
$$= 0.8416$$

ii) find the expected value and standard deviation of the number of engineers prefer to work in a local company. (1 mark)

$$\mu = np = 12 \times 0.25 = 3$$

$$\sigma = \sqrt{npq} = \sqrt{12 \times 0.25 \times 0.75} = \sqrt{2.25} = 1.5$$

Continued…

b) According to Boeing's website, the total fuel consumption per miles of a Boeing plane follows a Poisson distribution with a standard deviation of 2.25 gallons.

i) Calculate the expected value of the fuel consumption in 8 miles. (1.5 marks)

$$\sigma = \sqrt{\lambda t} = 2.25$$
$$\mu = \lambda t = 5.0625 \text{ gallons per mile}$$
$$\therefore \mu = \lambda t = 5.0625 \times 8$$
$$= 40.5 \text{ gallons in 8 miles}$$

ii) What is the probability for the next 8 miles, the plane will consume between 40 and 43 gallons of fuel? (2.5 marks)

$$P(40 < x < 43)$$
$$= P(x = 41) + P(x = 42)$$
$$= \frac{40.5^{41} e^{-40.5}}{41!} + \frac{40.5^{42} e^{-40.5}}{42!}$$
$$= 0.06199 + 0.05977$$
$$= 0.12176$$

c) A survey conducted among a group of teenagers on the time spent in the usage of computer per day is found to be normally distributed with a mean of 7 hours and variance of 4 hours. Determine the probability that a teenager will use the computer from 4 to 6 hours per day. (3 marks)

$$P(4 \le X \le 6)$$
$$= P\left(\frac{4-7}{\sqrt{4}} \le Z \le \frac{6-7}{\sqrt{4}}\right)$$
$$= P(-1.5 \le Z \le -0.5)$$
$$= P(Z < 1.5) - P(Z < 0.5)$$
$$= 0.9332 - 0.6915$$
$$= 0.2417$$

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