FINAL EXAM
MARKING SCHEME
TRIMESTER II, 2015/2016
PMT0101 – MATHEMATICS I

Question 1 [10 marks]
Solution:

a) $$\left(5x^2 y^{-\frac{3}{2}} z^{\frac{1}{4}}\right)\left(4x^4 y^{\frac{1}{2}} z^{\frac{3}{4}}\right)$$
$$= 20x^{2+4} y^{-\frac{3}{2}+\frac{1}{2}} z^{\frac{1}{4}+\frac{3}{4}}$$
$$= 20x^6 y^{-1} z^1$$
$$= \frac{20x^6 z}{y} \qquad [1 + 0.5 + 0.5]$$

b)
$$\frac{1-3\sqrt{2}}{3\sqrt{2}+2}$$
$$= \frac{1-3\sqrt{2}}{3\sqrt{2}+2} \cdot \frac{3\sqrt{2}-2}{3\sqrt{2}-2}$$
$$= \frac{3\sqrt{2}-2-9(2)+6\sqrt{2}}{9(2)-4}$$
$$= \frac{-2-18+9\sqrt{2}}{14}$$
$$= \frac{-20+9\sqrt{2}}{14} \qquad [0.5 + 0.5 + 0.5 + 0.5]$$

c) $$\sqrt{80} - 10\sqrt{20} + 4\sqrt{5}$$
$$= \sqrt{16(5)} - 10\sqrt{4(5)} + 4\sqrt{5}$$
$$= 4\sqrt{5} - 10(2)\sqrt{5} + 4\sqrt{5}$$
$$= -12\sqrt{5} \qquad [0.5+1+0.5]$$

d)

$$\frac{1}{2\sqrt{7}-3}-\frac{1}{2\sqrt{7}+3}$$
$$=\frac{2\sqrt{7}+3-\left(2\sqrt{7}-3\right)}{\left(2\sqrt{7}-3\right)\left(2\sqrt{7}+3\right)}$$
$$=\frac{2\sqrt{7}+3-2\sqrt{7}+3}{4(7)-9}$$
$$=\frac{6}{19} \qquad [1+0.5+0.5]$$

e)i) $z=3(7+7i)+(5+6i)i$
$$=21+21i+5i+6i^2$$
$$=21-6+26i$$
$$=15+26i \qquad [0.5+0.5+0.5]$$

ii) $\overline{z}=15-26i \qquad [0.5]$

Question 2 [10 marks]
Solution:

a) $x^2 - 4x + 8 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(8)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{-16}}{2}$$

$$x = \frac{4 \pm 4i}{2}$$

$x = 2 \pm 2i$ $\quad [0.5 + 0.5 + 0.5 + 0.5]$

b) $(2x+1)(x+2)(x-3) \geq 0$

Let $(2x+1)(x+2)(x-3) = 0$

$x = -\frac{1}{2}, \quad x = -2, \quad x = 3$ $\quad [0.5]$

$-\infty \qquad -2 \qquad -\frac{1}{2} \qquad 3 \qquad \infty$

Test value	**-3**	**-1**	**0**	**4**
Sign of $(2x+1)$	---	---	+++	+++
Sign of $(x+2)$	---	+++	+++	+++
Sign of $(x-3)$	---	---	---	+++
Sign of $(2x+1)(x+2)(x-3)$	---	+++	---	+++

Or equivalent [0.5+0.5+0.5]

The solution set is $\left[-2, -\frac{1}{2}\right] \cup [3, \infty)$ [0.5+0.5]

c) $|3x - 15| > 3$

$3x - 15 > 3$ $\qquad$ $3x - 15 < -3$

$3x > 18$ $\qquad$ or $\qquad$ $3x < 12$

$x > 6$ $\qquad$ $x < 4$ $\quad [0.5 + 0.5 + 0.5]$

The solutions are $(-\infty, 4) \cup (6, \infty)$ [0.5]

d) $\sqrt{x+1} = 3 - \sqrt{x}$

$$\left(\sqrt{x+1}\right)^2 = \left(3 - \sqrt{x}\right)^2$$

$$x + 1 = 9 - 6\sqrt{x} + x$$

$$6\sqrt{x} = 8$$

$$\sqrt{x} = \frac{4}{3}$$

$$x = \frac{16}{9} \qquad [0.5 + 0.5 + 0.5 + 0.5]$$

Check the solution:

From LHS:

$$\sqrt{\frac{16}{9} + 1}$$

$$= \sqrt{\frac{25}{9}}$$

$$= \frac{5}{3} \quad [0.25]$$

From RHS:

$$3 - \sqrt{\frac{16}{9}}$$

$$= 3 - \frac{4}{3}$$

$$= \frac{5}{3} \quad [0.25]$$

LHS=RHS

Therefore, the solution is $x = \frac{16}{9}$ [0.5]

Question 3 [10 marks]
Solution:

a) i) Given $h(x) = 2x + 3$ and $g(x) = \dfrac{1}{x-1}$,

$D_h = (-\infty,\ \infty)$ [0.5]	$x - 1 \neq 0$ $x \neq 1$ $D_g = (-\infty,\ 1) \cup (1,\ \infty)$ [0.5]

ii)

$(g \circ h)(x) = g(2x+3)$ $= \dfrac{1}{2x+3-1}$ $= \dfrac{1}{2x+2}$ [0.5 + 0.5]	$2x + 2 \neq 0$ $x \neq -1$ [0.5]

$D_{g \circ h} = (-\infty,\ -1) \cup (-1,\ \infty)$ [0.5]

iii)

$h(x) = 2x + 3$

Let $y = 2x + 3$

$\dfrac{y-3}{2} = x$

$h^{-1}(x) = \dfrac{x-3}{2}$ [0.5 + 0.5]

b)

i) Let $x - 2 = 0$, $x = 2$ ==> $c = 2$ [0.5]

By Remainder Theorem;

$P(2) = 2^3 - 19(2) + 30 = 0 \quad \Rightarrow \quad (x-2)$ is a factor of $P(x)$. [0.5 + 0.5] ii)

$$\begin{array}{r l}
 & x^2 + 2x - 15 \\
x-2 \big) & x^3 + 0x^2 - 19x + 30 \\
- & \underline{x^2 - 2x^2} \\
 & 2x^2 - 19x \\
- & \underline{2x^2 - 4x} \\
 & -15x + 30 \\
- & -\underline{15x + 30} \\
 & ..0
\end{array}$$

[0.5+0.5+0.5+0.5]

Quotient = $x^2 + 2x - 15$ [0.5]

iii)

$f(x) = x^3 - 19x + 30$

$f(x) = (x-2)(x^2 + 2x + -15)$ [0.5]

$f(x) = (x-2)(x-3)(x+5)$ [0.5]

vi)

$(x-2)(x-3)(x+5) = 0$

$x = 2, \quad 3, \quad -5$ [1]

Question 4 [10 marks]
Solution:

a) $f(x) = 2(x+4)^2(x+1)^2(x-4)$

i) Degree = 5 [0.5]

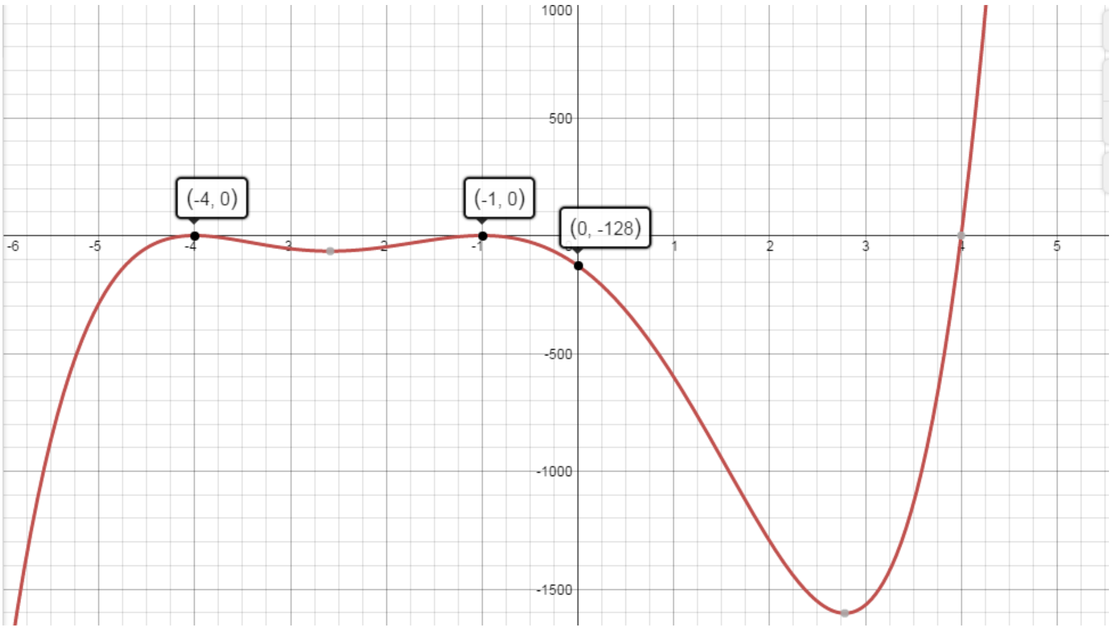
ii) **The zeros** of f are $-4,\ -1$ and 4

Zeros	Multiplicities	Cross/Touch
-4	2	Touch
-1	2	Touch
4	1	Cross
[0.5]	[0.5]	[0.5]

iii) y-intercept, $f(0) = -4.$ [0.5]

iv) As $x \to -\infty,\ y \to -\infty$ [0.5]
As $x \to \infty,\ y \to \infty$ [0.5]

v) Sketch the graph:



Shows all intercepts [0.5]
Proper end behaviour [0.5]
Shows correct crossing or touching at x-intercepts [0.5]

b)

i) When $x = -1.5, \quad y = 0$

$$\log_{10}\left(k(-1.5) + 4\right) = 0$$
$$-1.5k + 4 = 10^0$$
$$-1.5k = -3$$
$$k = 2 \qquad [0.5 + 0.5 + 0.5]$$

ii) Yes, f is a one-to-one function. [0.5]

c) $\log_2\left(x^2 + 2\right) = 1 + \log_2\left(x + 5\right)$

$$\log_2\left(x^2 + 2\right) = \log_2 2 + \log_2\left(x + 5\right)$$
$$\log_2\left(x^2 + 2\right) = \log_2 2\left(x + 5\right)$$
$$x^2 + 2 = 2x + 10$$
$$x^2 - 2x - 8 = 0$$
$$(x - 4)(x + 2) = 0$$
$$x = 4 \quad \textit{and} \quad x = -2 \qquad [0.5 + 0.5 + 0.5 + 0.5 + 0.5 + 0.5]$$

Question 5 [10 marks]
Solution:

a)i)

$$m_{PR} = \frac{3-(-9)}{16-0} = \frac{12}{16} = \frac{3}{4} \qquad [0.5+0.5]$$

$$m_{PR} \times m_{QT} = -1$$

$$m_{QT} = -\frac{4}{3} \qquad [0.5]$$

ii)

$$y - 8 = -\frac{4}{3}(x-6)$$

$$y = -\frac{4}{3}x + 16 \qquad [1+0.5]$$

iii) At T, $y = 0$,

$$\frac{4}{3}x = 16$$

$$x = 16 \times \frac{3}{4} = 12$$

$$\therefore \quad T(12,\ 0) \qquad [0.5+0.5]$$

b)

Let $S = (x,\ y)$

$$(12,\ 0) = \left(\frac{2x+3(6)}{3+2},\ \frac{2y+3(8)}{3+2}\right) \qquad [0.5]$$

$$\frac{2x+3(6)}{5} = 12 \qquad \frac{2y+3(8)}{5} = 0 \qquad [1+1]$$

$$x = 21 \qquad y = -12$$

$$\therefore \quad S(21,\ -12) \qquad [0.5]$$

c)

$$KQ = 3$$

$$\sqrt{(x-6)^2 + (y-8)^2} = 3$$

$$x^2 - 12x + 36 + y^2 - 16y + 64 = 9$$

$$x^2 + y^2 - 12x - 16y + 91 = 0 \qquad [1+1+1]$$