TMA1201 Tutorial 05 -
T3 Relation and function

1 Suppose $A = \{1, 2, 3\}$ and $B = \{0, 2, 6, 9\}$ and R is relation on sets A and B that $R = \{(x, y) \in A \times B \mid x \text{ divides } y\}$

i) Find the Cartesian products of $A \times B$

ii) List all the elements of R.

iii) Draw the corresponding arrow diagrams of $A \times B$ and R.

2 Given

$P = \{(1,1), (1,2), (2,5), (3,6), (4,2), (5,0)\}$ and $Q = \{(0,1), (2,1), (3,5), (6,2)\}$

Find

a) P^2 b) Q^2

c) $P \circ Q$ d) $Q \circ P$

3. Given two relations R = {(1,1), (2,1), (3,2), (4,3)} and T = {(1,3), (2,2), (2,3), (3,3), (3,4), (4,4)} on the set S = {1,2,3,4}.

a) Is R reflexive? Is R symmetric? Is R antisymmetric? Is R transitive? Justify each of your answer.

b) Is R an equivalence relation? Is R a partial order relation? Justify each of your answer.

c) Is R a function? Is T a function? Justify each of your answer.

d) Find R^{-1} and T^{-1}.

e) Find R∘T and T∘R.

4. Determine whether the following relations, R_1 and R_2 on the set of integers is reflexive, symmetric, antisymmetric, and/or transitive. Justify your answer.

a) $R_1 = \{(x,y) \mid x + y = 0\}$

b) $R_2 = \{(u,v) \mid uv \geq 0\}$

5. Suppose $C = \{0, 1, 2\}$, $D = \{3, 5, 7, 14\}$ and the relation $R = \{(x, y) \in C \times D \mid 2x + 5 = y\}$ and $S = \{(x, y) \in C \times D \mid x + y \text{ divisible by } 5\}$.

a) List all the elements of R. Determine whether R is a function? Explain your answer.

b) List all the elements of S. Determine whether S is a function? Explain your answer.

6. Let *f*, *g* and *h* be the following functions.

f: Z $\rightarrow$ {-1, 1} defined as $f(x) = \begin{cases} 1, & \text{if } x \text{ is even.} \\ -1, & \text{if } x \text{ is odd.} \end{cases}$

$g : \text{Z} \rightarrow \text{R}$ defined as $g(x) = x^2 - \frac{1}{2}$.

$h : \{x \mid x \in R \wedge x \geq 0\} \rightarrow \text{R}$ defined as $h(x) = \sqrt{x} + 2$.

a) Determine the range of *f*, *g* and *h*.
b) Is *f* one to one? Is *g* one to one? Is *h* one to one?
c) Is *f* onto? Is *g* onto? Is *h* onto?

7. Suppose

$f : R^+ \rightarrow R^+$ where $f(x) = \sqrt{2x}$ and

$g : Z^+ \rightarrow Z$ and $g(x) = x^4 + 2$.

a) Determine the range of *f* and *g*.
b) Is *f* one to one? Is *g* one to one? Explain your answers.
c) Is *f* onto? Is *g* onto? Explain your answer.