CMA6134 - Tutorial 3

1. Find the actual value for the following integrals:

   (a) $\int_1^2 \frac{1-x}{x^2}\,dx$ (b) $\int_3^4 \frac{1}{x^2+x-6}\,dx$ (c) $e^{\int_1^2 \frac{4}{x}dx}$

2. By using the following rules, find an approximate value of the integral given in Q1(a) with five points. Then compare with the actual value obtained in Q1(a) and find its absolute error.
   (a) Composite Trapezoidal Rule
   (b) Simpson's Rule
3. By using the following rules, find an approximate value of the integral given in Q1(b) by dividing the range into 4 equal parts. Then compare with the actual value obtained in Q1(b) and find its absolute error.
   (a) Composite Trapezoidal Rule
   (b) Simpson's Rule
4. How many points should be considered if the Composite Trapezoidal Rule is used to approximate the integral in Q1(a) and Q1(c) with an error of at most $\frac{1}{2}\times 10^{-4}$?
5. Consider the integral $\int_2^7 \frac{1}{x}\,dx$. Consider your answers in FOUR decimal places.
   (a) Find the number of subintervals if Composite Simpson's Rule is used to approximate the integral with an error of at most 2×10^{-2}.
   (b) With the number of subintervals obtained in (a), find the approximation of the integral using the Composite Simpson's Rule. Consider each point *x* with TWO decimal places.
   (c) Find an upper bound for the error in your approximation.
6. What can you say about the absolute error in Q2 and Q3 that are obtained from the Composite Trapezoidal Rule and Simpson's Rule?
7. By using the Romberg algorithm, approximate $\int_0^2 \frac{4}{(1+x^2)}\,dx$ by evaluating $R(2,2)$.
8. What is $R(5,3)$ if $R(5,2)=12$ and $R(4,2)=-51$ in the Romberg algorithm?
9. By using the Romberg algorithm, approximate the integral in Q1(a) by evaluating $R(3,3)$. Then compare with the actual value obtained in Q1(a) and find its absolute error.