MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2016/2017

(*SOLUTION*)

PMT0101 – MATHEMATICS I
(All sections / Groups)

JUNE 2017

(2 Hours)

No calculators are allowed.
You are required to write proper steps.

QUESTION 1   [10 marks]

a) [2 marks]

$$\frac{x^3(xy)^{-4}z^{-3}}{x^{-3}yz^{-2}} = \frac{x^3x^{-4}y^{-4}z^{-3}}{x^{-3}yz^{-2}} = x^{3-4+3}y^{-4-1}z^{-3+2} = x^2y^{-5}z^{-1} = \frac{x^2}{y^5z}$$

   or equivalent

[0.5+0.5+0.5+0.5]

b) [2 marks]

$$\sqrt{75} - \sqrt{972} = \sqrt{3\cdot5^2} - \sqrt{2^2\cdot3^5} = 5\sqrt{3} - 18\sqrt{3} = -13\sqrt{3}$$

[0.5+(0.5+0.5)+0.5]

c) [3 marks]

$$\frac{x^2-4}{3x^2-9x} \div \frac{x^2+x-6}{x^2-9} = \frac{(x+2)(x-2)}{3x(x-3)} \times \frac{x^2-9}{x^2+x-6}$$
$$= \frac{(x+2)(x-2)}{3x(x-3)} \times \frac{(x+3)(x-3)}{(x+3)(x-2)}$$
$$= \frac{x+2}{3x}$$

[0.5+0.5+0.5+0.5+1]

d) [3 marks]

$$\frac{1}{2-i} + \frac{1}{1+2i} = \frac{1}{2-i}\cdot\frac{2+i}{2+i} + \frac{1}{1+2i}\cdot\frac{1-2i}{1-2i} = \frac{2+i}{5} + \frac{1-2i}{5} = \frac{3}{5} + \left(-\frac{1}{5}\right)i$$
[0.5+0.5+0.5+0.5+0.5+0.5]

Continued …….

QUESTION 2   [10 marks]

a) [2 marks]
   $5x + 11 = 41$ or $5x + 11 = -41$ [0.5+0.5]
   $5x = 30$ or $5x = -52$
   $x = 6$ or $x = -\dfrac{52}{5}$ [0.5+0.5]

b) [3 marks]
   Squaring, $15 - 3x = (1 + x)^2$
   $15 - 3x = 1 + 2x + x^2$
   $x^2 + 5x - 14 = 0$
   $(x + 7)(x - 2) = 0$
   $x + 7 = 0$ or $x - 2 = 0$
   $x = -7$ or 2 [0.5+0.5+0.5]

   Checking:

When $x = -7$,	When $x = 2$,
LHS $= \sqrt{15 - 3(-7)} = \sqrt{36} = 6$	LHS $= \sqrt{15 - 3(2)} = \sqrt{9} = 3$
RHS = 1-7=-6	RHS = 1+2=3
LHS $\neq$ RHS x	LHS = RHS $\sqrt{}$

   [0.5+0.5]

   Conclusion: $x = 2$ [0.5]

c) [5 marks]
   (i) $x^2 + 4x - 5 = 0$.
        $(x + 5)(x - 1) = 0$.
        $x + 5 = 0$ or $x - 1 = 0$
        $x = -5$ or 1 [0.5+0.5]

   (ii) $\dfrac{x^2+4x-5}{x+3} > 0$, i.e., $\dfrac{(x+5)(x-1)}{x+3} > 0$ [0.5]

		-5		-3		1	
Sign of $(x + 5)$	--		+		+		+	[0.5]
Sign of $(x - 1)$	--		--		--		+	[0.5]
Sign of $(x + 3)$	--		--		+		+	[0.5]
$\dfrac{(x + 5)(x - 1)}{x + 3}$	--		+		--		+	[0.5]

   OR equivalent

   Hence the solution set is $(-5, -3) \cup (1, \infty)$ [0.5]

   (iii) For the domain of $f(x) = \sqrt{\dfrac{x^2+4x-5}{x+3}}$, we want $\dfrac{x^2+4x-5}{x+3} \geq 0$

   Based on the answer from (ii),
         the domain of f is $[-5, -3) \cup [1, \infty)$ [0.5+0.5]

                                    Continued …….

QUESTION 3   [10 marks]

a) [2 marks]
   (i) Domain: $[-2,\ 2]$ Range: $[-4,-2) \cup [0,\ 4]$ [0.5+(0.5+0.5)]
   (ii) f is one-to-one.   [0.5]

b) [3 marks]
   (i)

$$(f \circ g)(1) = f\big(g(1)\big) = f\left(\frac{3}{1-5}\right) = f\left(-\frac{3}{4}\right) = \sqrt{-\frac{3}{4}+3} = \sqrt{\frac{9}{4}} = \frac{3}{2}$$

                                                                          [0.5+0.5+0.5]

   (ii) Let $y = \sqrt{x+3}$.   Need to solve for x in terms of y.

$$y^2 = x+3 \ , \quad x = y^2 - 3 \qquad\qquad [0.5+0.5]$$

$$f^{-1}(x) = x^2 - 3 \qquad\qquad [0.5] \qquad\qquad \text{or equivalent}$$

c)   [5 marks] $f(x) = (x-3)(x+3)^2(x-1)^4$
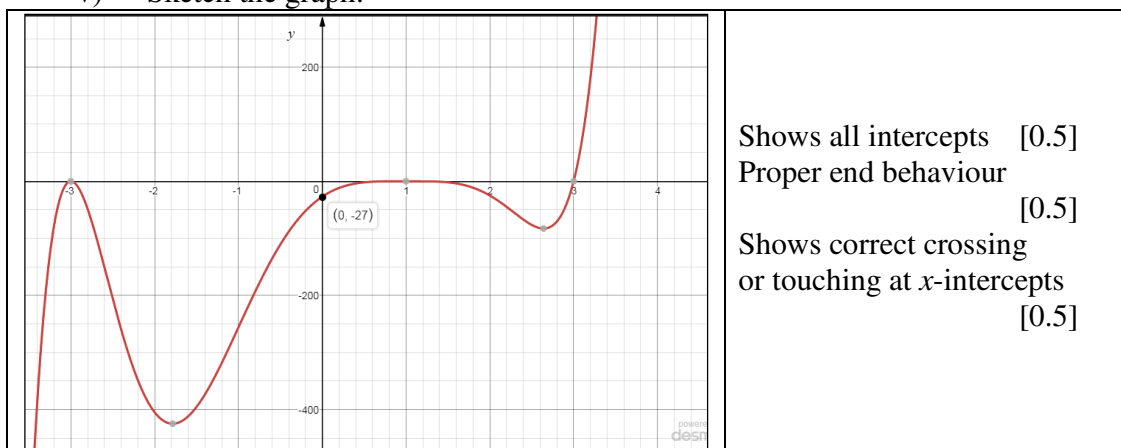
   (i) Degree = 7   [0.5]

   (ii) The zeros of f are $3, -3$ and 1 .

Zeros	Multiplicities	Crosses/Touches x-axis	
3	1	Crosses	[0.5]
-3	2	Touches	[0.5]
1	4	Touches	[0.5]

   (iii) y-intercept, $f(0) = -27$ [0.5]

   iv)   As $x \to -\infty,\ y \to -\infty$ [0.5]

          As $x \to \infty,\ y \to \infty$ [0.5]

   v)   Sketch the graph:

Shows all intercepts   [0.5]
Proper end behaviour
                                 [0.5]
Shows correct crossing
or touching at x-intercepts
                                 [0.5]

Continued …….

QUESTION 4 **[10 marks]**

a) [3 marks]

$$\begin{array}{r} 2x^3 + x \\ x^2-1 \overline{)2x^5 + 0x^4 - x^3 + 0x^2 + 0x + 2} \\ \underline{2x^5 \qquad -2x^3} \\ x^3 \qquad + 0x \\ \underline{x^3 \qquad -x} \\ x \quad +2 \end{array}$$

[0.5]
[0.5]

[0.5]

[0.5]

Quotient : $2x^3 + x$ [0.5]

Remainder : $x + 2$ [0.5]

b) [**3** marks]

(i)	(ii)
$H(-2) = 3\left(\dfrac{1}{2}\right)^{-2} - 2$ $= 3(2)^2 - 2$ $= 10 \qquad [0.5 + 0.5]$	$H(x) = -\dfrac{13}{8}$ $3\left(\dfrac{1}{2}\right)^x - 2 = -\dfrac{13}{8}$ $3\left(\dfrac{1}{2}\right)^x = \dfrac{3}{8}$ $\left(\dfrac{1}{2}\right)^x = \dfrac{1}{8}$ $x = 3 \qquad [0.5 + 0.5 + 0.5 + 0.5]$

c) [3 marks]

$y = a\ln(x + b)$

$a\ln b = 0$

$\ln b = 0$

$b = e^0$

$b = 1 \qquad [0.5 + 0.5 + 0.5]$

$y = a\ln(x + b)$

$-2\ln 3 = a\ln(2 + 1)$

$a = \dfrac{-2\ln 3}{\ln 3}$

$a = -2 \qquad [0.5 + 0.5 + 0.5]$

d) [1 mark]

$1 + \ln e^{x+1} = 5$

$(x + 1)\ln e = 4$

$x = 3 \qquad [0.5 + 0.5]$

Continued …….

QUESTION 5   [10 marks]

a) [2.5 marks]

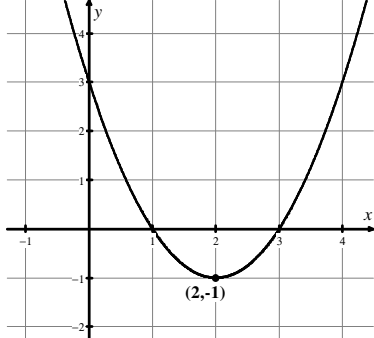$$\text{Center} = \left(\frac{-1+3}{2}, \frac{6-(-4)}{2}\right) = (1,5) \qquad [0.5]$$

Radius^2 = $(1-(-1))^2 + (5-6)^2 = 5$ [0.5+0.5]

Equation: $(x-1)^2 + (y-5)^2 = 5$ [0.5+0.5]

b) [2.5 marks]

$2x - 3y + 5 = 0$ $3y = 2x + 5$ $y = \dfrac{2}{3}x + \dfrac{5}{3} \quad \Rightarrow \quad m_1 = \dfrac{2}{3}$ $\hspace{4cm}$ [0.5+0.5] $m_1 m_2 = -1 \Rightarrow m_2 = -\dfrac{3}{2}$ $\hspace{3cm}$ [0.5]	Equation: $y - (-3) = -\dfrac{3}{2}(x-4)$ [0.5] $y + 3 = -\dfrac{3}{2}x + 6$ $y = -\dfrac{3}{2}x + 3$ $\qquad$ [0.5]

c) [3 marks]

Vertex: $(2, -1)$ $\hspace{2cm}$ [0.5] $f(0) = 4 - 1 = 3$, the y-intercept. [0.5] Solving $f(x) = 0, i.e., (x-2)^2 - 1 = 0$ $(x-2)^2 = 1$ $x - 2 = \pm 1$ $x = 3, 1$ $\quad$ the x-intercepts. $\quad$ [0.5+0.5]	 $\hspace{4cm}$ [0.5+0.5]

d) [2 marks]

$$y = ax^3 + bx \qquad \frac{y}{x} = ax^2 + b \qquad [0.5]$$

$$Y = aX + b$$

$$a = \text{slope}$$
$$b = Y\text{- intercept}$$

$$a = \frac{9-4}{1-6} = -1 \qquad [0.5]$$

$$9 = -1(1) + b$$

$$b = 10 \qquad\qquad [0.5 + 0.5]$$

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