STUDENT ID NO

SEAT NO

VENUE:____________________

MULTIMEDIA UNIVERSITY

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2015/2016

PMT0301 – MATHEMATICS III

(All sections/ Groups)

7 FEBRUARY 2016
2.30 p.m. – 4.30 p.m.
(2 Hours)

Question	Marks
1	/10
2	/10
3	/10
4	/10
5	/10
Total	/50

INSTRUCTIONS TO STUDENTS

1. This question paper consists of **TEN** printed pages excluding cover page, formulae list and statistical table.

2. Answer **ALL** FIVE questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the **QUESTION BOOKLET**. All necessary working steps **MUST** be shown.

Question 1

a) Find the parametric equations of the line passing through the point $(-1, 2, 4)$ that is parallel to $3\bar{i} - 4\bar{j} + \bar{k}$. (2 marks)

$$\bar{r} = \bar{r}_o + t\bar{v}$$
$$\langle x, y, x\rangle = \langle -1,2,4\rangle + t\langle 3,-4,1\rangle$$
$$= \langle -1+3t, 2-4t, 4+t\rangle$$
$$\therefore x = -1+3t$$
$$y = 2-4t$$
$$z = 4+t$$

b) Find an equation of the plane that contains the line $x = -2+3t$, $y = 4+2t$, $z = 3-t$ and is ~~perpendicular~~ parallel to the plane $x - 2y + z = 5$. Give your final answer in the form of $ax + by + cz = d$. (2 marks)

Point on the plane: $(-2, 4, 3)$
Normal vector: $\langle 1, -2, 1\rangle$

$$1(x-(-2)) + (-2)(y-4) + 1(z-3) = 0$$
$$x + 2 - 2y + 8 + z - 3 = 0$$
$$x - 2y + z = -7$$

c) Find the point at which the line $x = 3-t$, $y = -2t$, $z = 1-2t$ intersects the plane $x + y - 2z = 2$. (2 marks)

$$(3-t) + (-2t) - 2(1-2t) = 2$$
$$3 - t - 2t - 2 + 4t = 2$$
$$t = 1$$

$$\therefore x = 3-1 = 2$$
$$y = -2(1) = -2$$
$$z = 1-2(1) = -1$$

The intersection point is $(2,-2,-1)$

Continued…

d) Find the sum of the geometric series $1+3+9+...+2187$. (2.5 marks)

$$T_n = ar^{n-1}$$
$$2187 = (1)(3)^{n-1}$$
$$(n-1)\log_{10} 3 = \log_{10} 2187$$
$$n-1 = \frac{\log_{10} 2187}{\log_{10} 3} = 7$$
$$n = 8$$

$$\therefore S_8 = \frac{a\left(1-r^n\right)}{1-r}$$
$$= \frac{1\left(1-3^8\right)}{1-3}$$
$$= 3280$$

e) Find the term that contains x^5 in the expansion of $(x+2y)^{15}$. (1.5 marks)

$$\binom{15}{10} x^5 (2y)^{10}$$
$$= 3003x^5\left(1024y^{10}\right)$$
$$= 3075072x^5 y^{10}$$

Continued…

Question 2

a) Solve the system of linear equations with Gauss-Jordan Elimination method. (6 marks)

$$2x-3y+5z=14$$
$$4x-y-2z=-17$$
$$-x-y+z=3$$

$$\left[\begin{array}{ccc|c}-1&-1&1&3\\2&-3&5&14\\4&-1&-2&-17\end{array}\right]\xrightarrow{(-1)R_1\to R_1}\left[\begin{array}{ccc|c}1&1&-1&-3\\2&-3&5&14\\4&-1&-2&-17\end{array}\right]\xrightarrow[(-4)R_1+R_3\to R_3]{(-2)R_1+R_2\to +R_2}\left[\begin{array}{ccc|c}1&1&-1&-3\\0&-5&7&20\\0&-5&2&-5\end{array}\right]$$

$$\xrightarrow{\frac{R_2}{(-5)}\to R_2}\left[\begin{array}{ccc|c}1&1&-1&-3\\0&1&-\frac{7}{5}&-4\\0&-5&2&-5\end{array}\right]\xrightarrow{(5)R_2+R_3\to R_3}\left[\begin{array}{ccc|c}1&1&-1&-3\\0&1&-\frac{7}{5}&-4\\0&0&-5&-25\end{array}\right]$$

$$\xrightarrow{\frac{R_3}{(-5)}\to R_3}\left[\begin{array}{ccc|c}1&1&-1&-3\\0&1&-\frac{7}{5}&-4\\0&0&1&5\end{array}\right]\xrightarrow[\left(\frac{7}{5}\right)R_3+R_2\to R_2]{R_3+R_1\to R_1}\left[\begin{array}{ccc|c}1&1&0&2\\0&1&0&3\\0&0&1&5\end{array}\right]\xrightarrow{R_2+R_1\to R_1}\left[\begin{array}{ccc|c}1&0&0&-1\\0&1&0&3\\0&0&1&5\end{array}\right]$$

$\therefore x=-1, y=3, z=5$

Continued…

b) Let $A=\begin{bmatrix}1 & -3 & 4\\ 2 & 0 & -2\\ -1 & -2 & 5\end{bmatrix}$, $B=\begin{bmatrix}-2 & 4 & -3\\ 1 & 5 & \frac{3}{2}\end{bmatrix}$, $C=\begin{bmatrix}0 & 1 & -3\\ 2 & 4 & \frac{1}{2}\end{bmatrix}$ and $D=\begin{bmatrix}1 & 5 & 4\\ 3 & -1 & 2\\ 2 & 0 & 1\end{bmatrix}$

i) Solve the matrix equation $\frac{1}{3}(X+C)=B$ for the unknown matrix *X*. (2 marks)

$$\frac{1}{3}(X+C)=B$$

$$\begin{aligned}X &= 3B-C\\ &= 3\begin{bmatrix}-2 & 4 & -3\\ 1 & 5 & \frac{3}{2}\end{bmatrix}-\begin{bmatrix}0 & 1 & -3\\ 2 & 4 & \frac{1}{2}\end{bmatrix}\\ &= \begin{bmatrix}-6 & 12 & -9\\ 3 & 15 & \frac{9}{2}\end{bmatrix}-\begin{bmatrix}0 & 1 & -3\\ 2 & 4 & \frac{1}{2}\end{bmatrix}\\ &= \begin{bmatrix}-6 & 11 & -6\\ 1 & 11 & 4\end{bmatrix}\end{aligned}$$

ii) Calculate AD. (2 marks)

$$\begin{aligned}AD &=\\ &= \begin{bmatrix}1 & -3 & 4\\ 2 & 0 & -2\\ -1 & -2 & 5\end{bmatrix}\begin{bmatrix}1 & 5 & 4\\ 3 & -1 & 2\\ 2 & 0 & 1\end{bmatrix}\\ &= \begin{bmatrix}1-9+8 & 5+3+0 & 4-6+4\\ 2+0-4 & 10+0+0 & 8+0-2\\ -1-6+10 & -5+2+0 & -4-4+5\end{bmatrix}\\ &= \begin{bmatrix}0 & 8 & 2\\ -2 & 10 & 6\\ 3 & -3 & -3\end{bmatrix}\end{aligned}$$

Continued…

Question 3

The following table shows the money spent for entertainment on weekends by a sample of 34 students.

Money (in RM)	Number of students
20 – 29	4
30 – 39	6
40 – 49	14
50 – 59	7
60 – 69	3
TOTAL	**34**

a) Compute the midpoint, class boundaries, $\sum mf$ and $\sum m^2 f$. (3 marks)

Money (in RM)	f	Midpoint, m	Class boundaries	mf	$m^2 f$
20 – 29	4	24.5	19.5 – 29.5	98	2401
30 – 39	6	34.5	29.5 – 39.5	207	7141.5
40 – 49	14	44.5	39.5 – 49.5	623	27723.5
50 – 59	7	54.5	49.5 – 59.5	381.5	20791.75
60 – 69	3	64.5	59.5 – 69.5	193.5	12480.75
TOTAL	**34**			$\sum mf = 1503$	$\sum m^2 f = 70538.5$

b) Calculate the mean. Give your answer correct to 2 decimal places. (1.5 marks)

$$Mean = \frac{\sum mf}{n} = \frac{1503}{34}$$
$$= 44.21$$

Continued…

c) Calculate the median. Give your answer correct to 2 decimal places. (2 marks)

Position of median = 17th
Median Class = 40 – 49
Median

$$= L + \left(\frac{\frac{\sum f}{2} - f_L}{f_m} \right) c$$

$$= 39.5 + \left(\frac{\frac{34}{2} - 10}{14} \right) 10$$

$$= 39.5 + 5 = 44.50$$

d) Calculate the mode. Give your answer correct to 2 decimal places. (2 marks)

Mode Class = 40 – 49

Mode

$$= L + \left(\frac{f_m - f_B}{(f_m - f_B) + (f_m - f_A)} \right) c$$

$$= 39.5 + \left(\frac{14 - 6}{(14 - 6) + (14 - 7)} \right) 10$$

$$= 39.5 + 5.33 = 44.83$$

e) Calculate the standard deviation. Give your answer correct to 2 decimal places. (1.5 marks)

Standard deviation

$$= \sqrt{\frac{\sum m^2 f - \frac{(\sum mf)^2}{\sum f}}{\sum f - 1}}$$

$$= \sqrt{\frac{70538.5 - \frac{1503^2}{34}}{33}}$$

$$= \sqrt{124.1533}$$

$$= 11.14$$

Continued…

Question 4

a) How many ways can a committee of five people be formed consisting of at least two boys and at most one girl to be selected from a group of fifteen boys and eight girls?

(2 marks)

$$^{15}C_4 \, ^8C_1 + ^{15}C_5$$
$$= 10920 + 3003$$
$$= 13923$$

b) Kevin is a Foundation student in a certain institution. According to the examination rules of the institution, the eligibility of a student to sit for a final examination is that he/she must have a good attendance rate and high coursework mark. Kevin shows a good attendance in Mathematics subject with a probability of 0.75. If his attendance is good, he will have a probability of 0.83 scoring high in his coursework assessment. If his attendance is not good, his probability of scoring high in his coursework assessment will be 0.35.

Let A be the event of having good attendance, $\overline{A}$ for **not** having good attendance, C for scoring high in coursework and $\overline{C}$ for **not** scoring high in coursework.

i) Find the probability that he is eligible to sit for the final exam of the Mathematics subject. (1 mark)

$$P(A \cap C) = P(A) \times P(C|A)$$
$$= 0.75 \times 0.83$$
$$= 0.6225$$

ii) Let the probability that Kevin has high coursework is 0.71, find the chance that he will have good attendance rate given that he does not score high in coursework assessment. Give your answer correct to 4 decimal places. (2 marks)

$$P(A|\overline{C}) = \frac{P(A \cap \overline{C})}{P(\overline{C})}$$
$$= \frac{0.75 \times (1 - 0.83)}{1 - 0.71}$$
$$= \frac{0.1275}{0.29}$$
$$= 0.4397$$

Continued…

iii) Are events 'good attendance' and 'high coursework' statistically independent? Explain your answer. (3 marks)

By using, $P(A|C) = P(A)$

RHS:

$$P(A|C) = \frac{0.75 \times 0.83}{0.71}$$
$$= 0.8768$$

LHS: $P(A) = 0.75$

Therefore, $P(A|C) \neq P(A)$

As a conclusion, events 'good attendance' and 'high coursework' are not independent.

or

$P(C\|A) = P(C)$ $\text{LHS}: P(C\|A) = \frac{0.75 \times 0.83}{0.75} = 0.83$ $\text{RHS}: P(C) = 0.71$ $\therefore P(C\|A) \neq P(C)$	$P(C) \cdot P(A) = P(C \cap A)$ $\text{LHS}: P(C) \cdot P(A) = 0.71 \times 0.75 = 0.5325$ $\text{RHS}: P(C \cap A) = 0.75 \times 0.83 = 0.6225$ $\therefore P(C) \cdot P(A) \neq P(C \cap A)$

c) There are 5 children, 4 men and 6 women in a gathering. Find the probability of arranging 6 persons from the gathering in a row if the second person must be a woman, follow by a child and the last person must be a man. Give your answer correct to 4 decimal places. (2 marks)

$$\frac{12 \times 6 \times 5 \times 11 \times 10 \times 4}{15 \times 14 \times 13 \times 12 \times 11 \times 10} \; or \; \frac{{}^{5}P_{1} \times {}^{4}P_{1} \times {}^{6}P_{1} \times {}^{12}P_{3}}{{}^{15}P_{6}}$$
$$= 0.0440$$

Continued…

Question 5

a) Given the probability for a car to have life expectancy of at least 25 years is 0.67. Among 20 such cars,

i) find the probability between 8 and 11 cars will have life expectancy of at least 25 years. Give your answer correct to 4 decimal places. (2 marks)

Let X be the number of cars with life expectancy of at least 25 years
n= 20, $p = 0.67$, $q = 0.33$

$$P(8 < x < 11)$$
$$= P(x = 9) + P(x = 10)$$
$$= \binom{20}{9} 0.67^9 0.33^{11} + \binom{20}{10} 0.67^{10} 0.33^{10}$$
$$= 0.0231 + 0.0516$$
$$= 0.0747$$

ii) find the mean and standard deviation for the number of cars with life expectancy of at least 25 years. (2 marks)

$$\mu = np = 20 \times 0.67$$
$$= 13.4$$

$$\sigma = \sqrt{npq} \quad = \sqrt{20 \times 0.67 \times 0.33}$$
$$= 2.1$$

b) The number of marriages per day in a district follows a Poisson distribution with a standard deviation of 3. Find the

i) mean of the number of marriages per day. (1 mark)

$$\sigma = 3$$
$$\sqrt{\lambda} = 3$$
$$\lambda = 9 = \mu$$

Continued…

ii) probability that exactly 21 marriages will occur on two given days. Give your answer correct to 4 decimal places. (2 marks)

2 day period of $\mu = 2 \times 9 = 18$

$P(x = 21)$

$= \dfrac{18^{21} e^{-18}}{21!}$

$= 0.0684$

c) Given that the weight of the adult chicken is normally distributed with a mean of $2.1kg$ and a standard deviation of $0.5kg$. What is the probability that the adult chicken weight is within 0.45 kg from the mean weight? (3 marks)

$P(x = 2.1 \pm 0.45)$

$= P(1.65 \le x \le 2.55)$

$= P\left(\dfrac{1.65 - 2.1}{0.5} \le z \le \dfrac{2.55 - 2.1}{0.5}\right)$

$= P(-0.9 \le z \le 0.9)$

$= 0.8159 - (1 - 0.8159)$

$= 0.6318$

End of Page