# Topic 6.1 Introduction to Graph

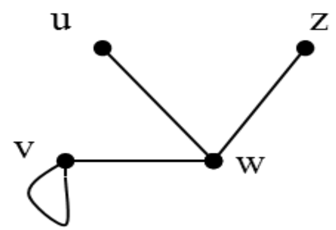
TMA1201 Discrete Structures & Probability
Faculty of Computing & Informatics
Multimedia University

# What you will learn in this lecture:

- Structure of graph
- Some terminologies and definitions
- Some special graphs
- Subgraphs
- Adjacency matrix
- The concept of a degree in a graph
- The Handshaking Lemma

# Graph

- A graph is a very simple (and thus very general) data structure




- Definition: A graph G = (V, E) consists of a set V of vertices (or nodes), together with a set E of edges which join two vertices.

- For the above graph we have
  - Vertex set, V = {u, v, w, z}
  - Edge set, E = {uw, vw, zw, vv}
  - Thus G = ({u, v, w, z}, {uw, vw, zw, vv})

Note 1:
uw has the same meaning as wu

Note 2: If the edges are labelled, then the edges is written according to the label instead of the vertices endpoints.

# Why are graphs important?

**Where are graph used?**

Networks are graphs:

- Computer networks (computing grids, the internet)
- Telephone networks
- Social networks (who's friends with whom on Facebook), family trees, etceteras.
- Transportation, shipping routes, roads, etceteras.

Usage in Computer science:

- Dependency trees for software, pathfinding, optimisation, etceteras.

# Some terminologies and definitions

Let G = (V, E)

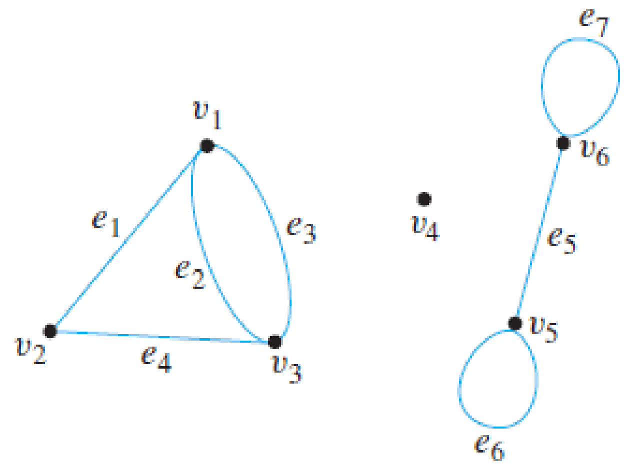
- An edge that connects a vertex back to itself is called a loop.
- Two distinct edges with same set of vertices endpoints are said to be parallel.
- Let *v* and *w* be vertices of G. If *v* and *w* are joined by an edge *e*, then *v* and *w* are said to be **adjacent** and *e* is said to be **incident** on *v* and *w*.
- Two edges incident on the same endpoint are called **adjacent**.
- A vertex that is an endpoint to a loop is called adjacent to itself.
- A vertex with no edges are incident is called isolated.

# Some terminologies and definitions

**Example:**

Consider the following graph:

a) Write the vertex set and the edge set, and give a table showing the edge-endpoint function.




**Solution:**

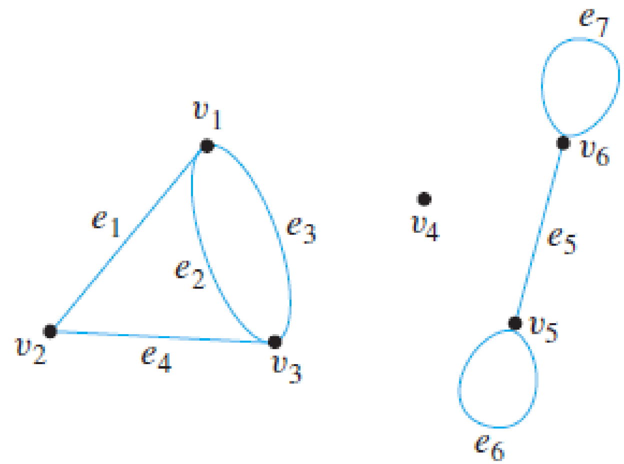
vertex set = {$v1$, $v2$, $v3$, $v4$, $v5$, $v6$}, edge set = {$e1$, $e2$, $e3$, $e4$, $e5$, $e6$, $e7$}

| Edge | Endpoints |
|---|---|
| $e_1$ | $\{v_1, v_2\}$ |
| $e_2$ | $\{v_1, v_3\}$ |
| $e_3$ | $\{v_1, v_3\}$ |
| $e_4$ | $\{v_2, v_3\}$ |
| $e_5$ | $\{v_5, v_6\}$ |
| $e_6$ | $\{v_5\}$ |
| $e_7$ | $\{v_6\}$ |

# Some terminologies and definitions

**Example (cont.):**

b) Find all edges that are incident on *v*1, all vertices that are adjacent to *v*1, all edges that are adjacent to *e*1, all loops, all parallel edges, all vertices that are adjacent to themselves, and all isolated vertices.

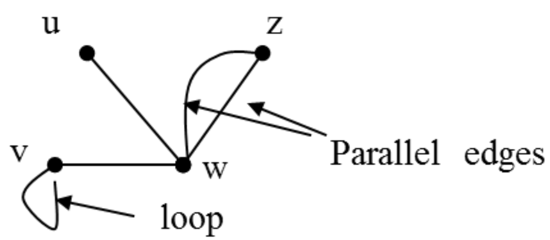



**Solution:**

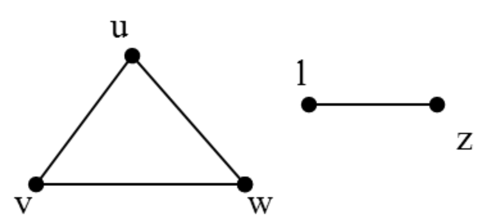
- *e*1, *e*2, and *e*3 are incident on *v*1.
- *v*2 and *v*3 are adjacent to *v*1.
- *e*2, *e*3, and *e*4 are adjacent to *e*1.
- *e*6 and *e*7 are loops.
- *e*2 and *e*3 are parallel.
- *v*5 and *v*6 are adjacent to themselves.
- *v*4 is an isolated vertex.

# Some terminologies and definitions

- A **simple graph** is a graph that does not have any loops or parallel edges
- A graph is called **connected** if there is a walk from any vertex to any other vertex, otherwise, the graph is **disconnected**.
- Example: a connected non-simple graph:



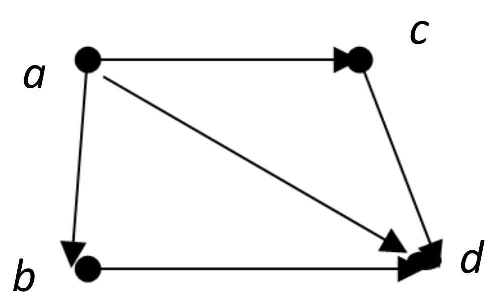

- Example: a disconnected simple graph:



# Directed graph

- In a graph, an edge is joining 2 vertices but does not provide any information on how the vertices are connected (no direction). Therefore, there is a need for having directed graphs.

- A directed graph (digraph) G = (V, E) consists of a set of vertices (or nodes) and a set of edges (or arcs) such that each edge $e \in E$ is associated with an ordered pair of vertices.



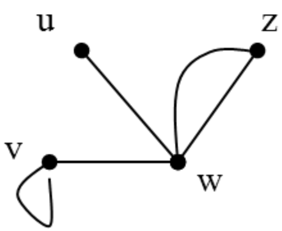

V = {*a*, *b*, *c*, *d*}, E = {*ab*, *ac*, *ad*, *bd*, *cd*}

Note: here we cannot write *ab* as *ba* because this is a digraph

# Subgraph

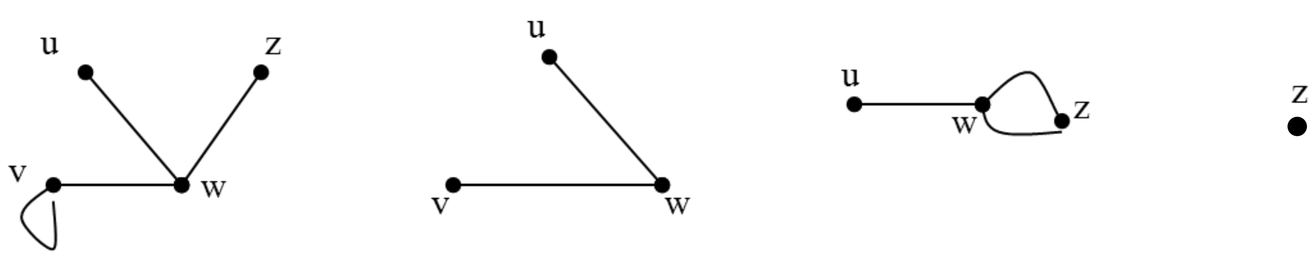
- Let G be a graph with vertex set V and edge set E, a graph H is said to be a subgraph of G ↔ every vertex in H is also a vertex in G, every edge in H is also an edge in G and every edge in H has the same endpoints as it has in G.

- Graph G




G

- Example for subgraphs of G:



# The concept of a degree in a graph

- Let $v$ be a vertex of a graph G, the degree of $v$, deg($v$), equals to the number of edges that are incident on $v$.
- An edge that is a loop will be counted twice towards the degree.
- The total degree of G is the sum of the degree of all the vertices of G.
- **The Handshaking Lemma said that**:

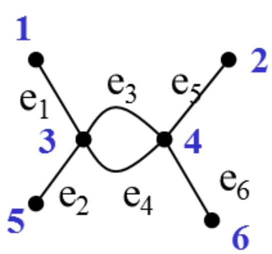
The sum of the degrees of all vertices of G equals twice the number of edges of G.

# Adjacency matrix

- Let G be a graph with n vertices labeled 1,2,3,…,n. The adjacency matrix $A_G$ is the n x n matrix in which the entry in row i and column j is the number of edges joining the vertices i and j.

Example:

M

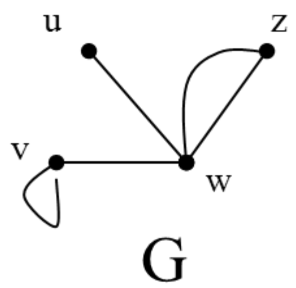



Adjacency matrix of M

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

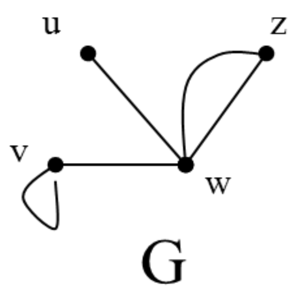
# Exercise

- Find the adjacency matrix for G



# Example

Find the degree of each of the vertex in G and also the degree of G




Verify if the handshaking lemma holds for G.

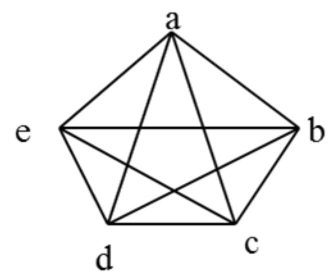
# Summary

Materials covered in this lecture?

- The structure of graph
- Some important terminologies related to graph
- Some special graphs
- The concept of degree
- The handshaking lemma
- The adjacency matrix

# Exercise 1

What is the adjacency matrix for this graph?



# Exercise 2

1) How many edges are there in a graph with 10 vertices each of degree 6?

2) Is there a graph with the degrees of vertices 1, 1, 1, 4?