$z_1 = 2-3i, \quad z_2 = 1+2i$

$\boxed{1}$

$\boxed{a}$ $\cancel{z\bar{z}}$ $z_1\bar{z}_1$

$$(2-3i)(2+3i) = 4-9i^2 = 4+9 = \boxed{13}$$

$\boxed{b}$ $z_1 z_2$

$$= (2-3i)(1+2i) = 2+4i-3i-6i^2$$

$$\boxed{= 8+i}$$

$\boxed{c}$ $(z_1+3z_2)^2$

$$\cancel{= z_1^2 + 6\,z_1 z_2 +} = \left[(2-3i)+3(1+2i)\right]^2$$

$(2-3i)(2-3i)$ $\quad = \left[(2-3i)+(3+6i)\right]^2$

$(3+6i)(3+6i)$

$$= (2-3i)^2 + 2(2-3i)(3+6i) + (3+6i)^2 \qquad = 9+18i+18i+36i^2$$

$$= -27+36i$$

$$= 4-6i-6i+9i^2 + \cancel{4-6i}\; 2\left(6+12i-9i-18i^2\right) \quad \cancel{-}27+36i$$

$$= -5 - 12i + 48 + 6i - 27 + 36i \qquad = 2(24+3i)$$

$$\Rightarrow \boxed{16+30i} \;\#$$

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d) $\left[Z_1+(1+Z_2)\right]^2$

$= \cancel{Z_1^2 + 2Z_1(1+Z_2) + (1+Z_2)^2}$ $\quad (4-6i)(4+4i)$

$= \cancel{4-6i-6i+9i^2 + (4-6i)(2+2+4i) + 1 + 2Z_2 + Z_2^2}$

$= \cancel{-5-12i + 16 + 16i - 24i - 24i^2 + 1 + 2 + 4i + (1+2i)^2}$

$\hookrightarrow \cancel{(1+2i)(1+2i)}$

$= \cancel{1+2i+2i+4i^2}$

$= \cancel{-3+4i}$

$= \cancel{38-16i-3+4i}$ $\quad \cancel{= \boxed{35-12i}}$ #

$= \left[(2-3i) + (1+(1+2i))\right]^2$ $\quad 2\left[(2-3i)(2+2i)\right]$

$= \left[(2-3i) + (2+2i)\right]^2$ $\quad 2\left[4+4i-6i-6i^2\right]$

$(2-3i)(2-3i)$ $\quad 2(10-2i) =$

$20-4i$

$= (2-3i)^2 + 2(2-3i)(2+2i) + (2+2i)^2$

$\hookrightarrow (2+2i)(2+2i)$

$= 4-6i-6i+9i^2+20-4i+4+4i+4i+4i^2$

$\boxed{= 15-8i}$ #

2

a) $\frac{1+4i}{5-12i} \cdot \frac{5+12i}{5+12i}$

$= \frac{5+12i+20i+48i^2}{25-144i^2} = \frac{-43+32i}{169}$

$\boxed{= \frac{-43}{169} + \frac{32}{169}i}$ #

b) $(2+i)^3$

$= 2^3 + 3(2)^2(i) + 3(2)(i)^2 + i^3$

1

1 1

1 2 1

$\boxed{1 \quad 3 \quad 3 \quad 1}$

$= 8 + 12i - 6 - i$

$\boxed{= 2+11i}$ #

$\boxed{c}$ $3\sqrt{-50}+\sqrt{-72}$

$3\sqrt{-25\times 2}+\sqrt{-2\times 36}$

$=3(5)i\sqrt{2}+6i\sqrt{2}$

$=15i\sqrt{2}+6i\sqrt{2}$ $\boxed{=21\sqrt{2}\,i}$ #

$\cancel{\boxed{=21i\sqrt{2}}}$ ~~#~~

$\boxed{d}$ $\dfrac{1}{5-3i}-\dfrac{1}{5+3i}$ $\cancel{=\dfrac{1}{5-3i}-\dfrac{1}{5-3i}}$

$=\dfrac{\cancel{5}+3i-\cancel{5}+3i}{25+\cancel{15i}-\cancel{15i}-9i^2}=\dfrac{6i}{34}=\boxed{\dfrac{3}{17}i}$ #

$\boxed{3}$

$\boxed{a}$ $e^{i\theta}=\cos\theta+i\sin\theta$

$\boxed{b}$

(i) $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$e^{i\alpha}\cdot e^{i\beta}=(\cos\alpha+i\sin\alpha)(\cos\beta+i\sin\beta)$

$=\boxed{e^{i(\alpha+\beta)}=\cos\alpha\cos\beta+i(\cos\alpha\sin\beta+\cancel{i}\sin\alpha\cos\beta)+i^2\sin\alpha\sin\beta}$

$=\boxed{\cos\alpha\cos\beta-\sin\alpha\sin\beta+i(\cos\alpha\sin\beta+\cancel{i}\sin\alpha\cos\beta)}$

$e^{i(\alpha+\beta)}=\cos(\alpha+\beta)+i\sin(\alpha+\beta)$

$\boxed{\therefore\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta}$

(ii) $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$\therefore\sin(\alpha+\beta)=\sin\beta\cos\alpha+\sin\alpha\cos\beta$

$=\boxed{\sin\alpha\cos\beta+\cos\alpha\sin\beta}$ #

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$\boxed{4}$

(i) $\sin 2\theta = 2\sin\theta\cos\theta$

$\boxed{* \; e^{i\theta} = \cos\theta + i\sin\theta}$

$e^{i\theta} \cdot e^{i\theta} = (\cos\theta + i\sin\theta)(\cos\theta + i\sin\theta)$

$\boxed{e^{i(2\theta)} = \cos^2\theta + i\cos\theta\sin\theta + i^2\sin^2\theta + i\sin\theta\cos\theta}$ $\boxed{1}$

$\boxed{e^{i(2\theta)} = \cos(2\theta) + i\sin(2\theta)}$ $\boxed{2}$

$= \cos^2\theta - \sin^2\theta + i(2\cos\theta\sin$

$\boxed{\therefore \sin 2\theta = 2\sin\theta\cos\theta}$

(ii) ~~$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$~~

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\boxed{\therefore \cos 2\theta = \cos^2\theta - \sin^2\theta}$

$\boxed{a}$ $\sin(\theta+\theta) = \sin\theta\cos\theta + \cos\theta\sin\theta \Rightarrow$ ~~$\sin\theta\cos\theta$~~

$\boxed{\sin 2\theta = 2\sin\theta\cos\theta}$

$\boxed{b}$ $e^{i(2\theta)} = (e^{i\theta})^2 = e^{i\theta} \cdot e^{i\theta}$

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$\boxed{5}$ $\sin 3\theta$

$$e^{i(3\theta)} = (\cos\theta + i\sin\theta)^3$$

1

1 1

1 2 1

$\boxed{1 \quad 3 \quad 3 \quad 1}$

$$= \cos^3\theta + 3\cos^2\theta \cdot (i\sin\theta) + 3\cos\theta \cdot (i\sin\theta)^2 + (i\sin\theta)^3$$

$$= \cos^3\theta + \boxed{3i\cos^2\theta\sin\theta} - 3\sin^2\theta\cos\theta \boxed{- i\sin^3\theta}$$

$$\boxed{= \cos^3\theta - 3\sin^2\theta\cos\theta + i(3\cos^2\theta\sin\theta - \sin^3\theta)} \quad \boxed{1}$$

$$\boxed{e^{i(3\theta)} = \cos 3\theta + i\sin 3\theta} \quad \boxed{2}$$

$$\boxed{\therefore \sin 3\theta = 3\cos^2\sin\theta - \sin^3\theta} \quad \#$$

$\boxed{6}$

$$\boxed{\therefore \cos 3\theta = \cos^3\theta - 3\sin^2\theta\cos\theta} \quad \#$$

$\boxed{7}$ $\sin 4x \sin 5x$

$e^{i\theta} = \cos\theta + i\sin\theta$

$e^{-i\theta} = \cos\theta - i\sin\theta$

$\therefore \sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$

$\therefore \cos\theta = \dfrac{e^{i\theta} + e^{-i\theta}}{2}$

$$\left(\frac{e^{i4x} - e^{-i4x}}{2i}\right)\left(\frac{e^{i5x} - e^{-i5x}}{2i}\right)$$

$$= \frac{e^{i9x} - e^{-ix} - e^{ix} + e^{-i9x}}{4i^2}$$

$$= \frac{1}{2}\left(\frac{-e^{i9x} - e^{-i9x} + e^{-ix} + e^{ix}}{2}\right) = \frac{1}{2}\left(\frac{e^{ix} + e^{-ix}}{2} - \frac{e^{i9x} + e^{-i9x}}{2}\right)$$

$$\boxed{= \frac{1}{2}(\cos x - \cos 9x)} \quad \#$$

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$\boxed{8}$ $\cos 5x \cos 2x$

$$\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$$

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$= \left(\frac{e^{i5x} + e^{-i5x}}{2}\right)\left(\frac{e^{i2x} + e^{-i2x}}{2}\right)$$

$$= \frac{e^{i7x} + e^{i3x} + e^{-i3x} + e^{-i7x}}{4}$$

$$= \frac{1}{2}\left(\frac{e^{i3x} + e^{-i3x}}{2} + \frac{e^{i7x} + e^{-i7x}}{2}\right)$$

$$\boxed{= \frac{1}{2}\left(\cos 3x + \cos 7x\right)} \quad \#$$

$\boxed{9}$ $\sin 5x \cos 2x$

$$\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$$

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$= \left(\frac{e^{i5x} - e^{-i5x}}{2i}\right)\left(\frac{e^{i2x} + e^{-i2x}}{2}\right)$$

$$= \frac{e^{i7x} + e^{i3x} - e^{-i3x} - e^{-i7x}}{4i}$$

$$= \frac{1}{2}\left(\frac{e^{i3x} - e^{-i3x}}{2i} + \frac{e^{i7x} - e^{-i7x}}{2i}\right) = \boxed{\frac{1}{2}\left(\sin 3x + \sin 7x\right)} \quad \#$$

$\boxed{10}$ $\cos 5x \sin 3x$

$$\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$$

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

$$= \left(\frac{e^{i5x} + e^{-i5x}}{2}\right)\left(\frac{e^{i3x} - e^{-i3x}}{2i}\right)$$

$$= \frac{e^{i8x} - e^{i2x} + e^{-i2x} - e^{-i8x}}{4i}$$

$$= \frac{1}{2}\left(\frac{e^{i8x} - e^{-i8x}}{2i} - \frac{e^{i2x} - e^{-i2x}}{2i}\right) = \boxed{\frac{1}{2}\left(\sin 8x - \sin 2x\right)} \quad \#$$

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$\boxed{\text{III}}$

(i) $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\boxed{b}$ $\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$

$$e^{i\alpha}\cdot e^{-i\beta} = (\cos\alpha + i\sin\alpha)(\cos\beta - i\sin\beta)$$

$$= e^{i(\alpha-\beta)} \quad = \cos\alpha\cos\beta - i\cos\alpha\sin\beta + i\sin\alpha\cos\beta - i^2\sin\alpha\sin\beta$$

$i(\sin\alpha\cos\beta - \cos\alpha\sin\beta)$

$$\boxed{= \cos\alpha\cos\beta + \sin\alpha\sin\beta + i(-\cos\alpha\sin\beta + \sin\alpha\cos\beta)} \quad \boxed{1}$$

$$\boxed{e^{i(\alpha-\beta)} = \cos(\alpha-\beta) + i\sin(\alpha-\beta)} \quad \boxed{2}$$

$$\boxed{\therefore \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta} \quad \#$$

(ii) $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$$\boxed{\therefore \sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta}$$

$\boxed{a}$ $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

12 $\tan(\alpha+\beta) = ?$ $\Rightarrow$ $\boxed{\tan(\alpha+\beta) = \dfrac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}}$

$\tan(\alpha-\beta) = ?$ $\Rightarrow$ $\boxed{\tan(\alpha-\beta) = \dfrac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}}$

$$\boxed{\begin{aligned} \sin(\alpha+\beta) &= \sin\alpha\cos\beta + \cos\alpha\sin\beta \\ \cos(\alpha+\beta) &= \cos\alpha\cos\beta - \sin\alpha\sin\beta \\ \sin(\alpha-\beta) &= \sin\alpha\cos\beta - \cos\alpha\sin\beta \\ \cos(\alpha-\beta) &= \cos\alpha\cos\beta + \sin\alpha\sin\beta \end{aligned}}$$

$\Rightarrow$ * From previous Questions

$$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta - \sin\alpha\sin\beta}$$

$$= \frac{\dfrac{\sin\alpha\cancel{\cos\beta}}{\cos\alpha\cancel{\cos\beta}} + \dfrac{\cancel{\cos\alpha}\sin\beta}{\cancel{\cos\alpha}\cos\beta}}{\dfrac{\cancel{\cos\alpha\cos\beta}}{\cancel{\cos\alpha\cos\beta}} - \dfrac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}} \boxed{= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}}$$

#

$$\tan(\alpha-\beta) = \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)} = \frac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\alpha\cos\beta + \sin\alpha\sin\beta}$$

$$= \cancel{\frac{\dfrac{\sin\alpha\cos\beta}{\sin\alpha\cos\beta} - \dfrac{\cos\alpha\sin\alpha}{\sin\alpha\cos\beta}}{\dfrac{\cos\alpha\cos\beta}{\sin\alpha\cos\beta} + \dfrac{\sin\alpha\sin\beta}{\sin\alpha\cos\beta}}} \quad \cancel{无 1}$$

$$= \frac{\dfrac{\sin\alpha\cancel{\cos\beta}}{\cos\alpha\cancel{\cos\beta}} - \dfrac{\cancel{\cos\alpha}\sin\beta}{\cancel{\cos\alpha}\cos\beta}}{\dfrac{\cancel{\cos\alpha\cos\beta}}{\cancel{\cos\alpha\cos\beta}} + \dfrac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}} \boxed{= \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}}$$

#