

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2017/2018

(SOLUTION)

PMT0101 – MATHEMATICS I

(Foundation in Information Technology)

OCTOBER 2017

(2 Hours)

Question 1 [10 marks]

a)

$$\begin{aligned}
 & \frac{\left(16x^2 y^{-\frac{1}{3}}\right)^{\frac{3}{4}}}{\left(xy^{\frac{3}{2}}\right)^{\frac{1}{2}}} \\
 &= \frac{(2^4)^{\frac{3}{4}} (x^2)^{\frac{3}{4}} \left(y^{-\frac{1}{3}}\right)^{\frac{3}{4}}}{x^{\frac{1}{2}} \left(y^{\frac{3}{2}}\right)^{\frac{1}{2}}} \\
 &= \frac{8x^{\frac{3}{2}} y^{-\frac{1}{4}}}{x^{\frac{1}{2}} y^{\frac{3}{4}}} \\
 &= \frac{8x}{y} \quad [1+0.5+0.5]
 \end{aligned}$$

b)

$$\begin{aligned}
 & \frac{1+\sqrt{5}}{\sqrt{5}-1} \\
 &= \frac{(1+\sqrt{5})(\sqrt{5}+1)}{(\sqrt{5}-1)(\sqrt{5}+1)} \\
 &= \frac{\sqrt{5}+1+5+\sqrt{5}}{5-1} \\
 &= \frac{2\sqrt{5}+6}{4} \\
 &= \frac{\sqrt{5}+3}{2} \quad [0.5+0.5+0.5+0.5]
 \end{aligned}$$

c)

$$\begin{aligned}
 & 5x\sqrt{\frac{3y^2}{2}} - 3y\sqrt{\frac{27x^2}{2}} + 2\sqrt{\frac{3x^2y^2}{2}} \\
 &= 5xy\sqrt{\frac{3}{2}} - 3xy\sqrt{\frac{3(9)}{2}} + 2xy\sqrt{\frac{3}{2}} \\
 &= 5xy\sqrt{\frac{3}{2}} - 9xy\sqrt{\frac{3}{2}} + 2xy\sqrt{\frac{3}{2}} \\
 &= -2xy\sqrt{\frac{3}{2}} \quad [1 + 0.5 + 0.5]
 \end{aligned}$$

d)

$$\begin{aligned}
 & 3(x^2 + 10x + 25) - (4x + 20) \\
 &= 3(x + 5)(x + 5) - 4(x + 5) \\
 &= (x + 5)(3x + 15 - 4) \\
 &= (x + 5)(3x + 11) \quad [1 + 0.5 + 0.5]
 \end{aligned}$$

e)

$$\begin{aligned}
 & i^7 + 4i^3 - 2i^2 + 1 \\
 &= i^4 i^2 i + 4i^2 i - 2i^2 + 1 \\
 &= 1(-1)i + 4(-1)i - 2(-1) + 1 \\
 &= -i - 4i + 2 + 1 \\
 &= 3 - 5i \quad [0.5 + 0.5 + 0.5 + 0.5]
 \end{aligned}$$

Question 2 (10 marks)

a)

$$kx^2 + x + 4 = 0$$

$$b^2 - 4ac < 0$$

$$1 - 4(k)(4) < 0$$

$$1 - 16k < 0$$

$$-16k < -1$$

$$k > \frac{1}{16} \quad [0.5 + 0.5 + 0.5 + 0.5]$$

b)

$$x^4 = 4x^2$$

$$x^4 - 4x^2 = 0$$

$$x^2(x^2 - 4) = 0 \quad [0.5 + 0.5]$$

$$x^2 = 0 \quad \text{or} \quad x^2 - 4 = 0$$

$$x = 0 \quad \text{or} \quad x = \pm 2 \quad [0.5 + 0.5]$$

c)

$$\frac{3}{4}|x+1| = 9$$

$$|x+1| = 12 \quad [0.5]$$

$$x+1 = 12 \quad \text{or} \quad x+1 = -12$$

$$x = 11 \quad [0.5] \quad \text{or} \quad x = -13 \quad [0.5]$$

d) i) The domain is $(-\infty, -3) \cup (-3, \infty)$ [0.5]

ii)

	-3	-1	2	
Sign of $(x-2)$	---	---	---	+++
Sign of $(x+1)$	---	---	+++	+++
Sign of $(x+3)$	---	+++	+++	+++
Sign of $\frac{(x-2)(x+1)}{x+3}$	---	+++	---	+++

[0.5]

[0.5]

[0.5]

[0.5]

The solution set is $(-\infty, -3) \cup [-1, 2]$ [0.5+0.5]

$$\text{iii)} \quad \frac{(x-2)(x+1)}{x+3} \geq 0$$

The domain of $h(x)$ is $(-3, -1] \cup [2, \infty)$ [0.5 + 0.5]

Question 3 (10 marks)

a)i)

Domain of f is $[0, \infty)$

Domain of g is $(-\infty, \infty)$ [0.5 + 0.5]

ii)

$$\begin{aligned} (g \circ f)(x) &= g(f(x)) \\ &= g(3x-5) \\ &= \sqrt{3x-5} \end{aligned} \quad [0.5 + 0.5]$$

Domain of $g \circ f$ is $\left[\frac{5}{3}, \infty\right)$ [0.5]

iii) Let

$$y = 3x - 5$$

$$x = \frac{y+5}{3}$$

$$\therefore g^{-1}(x) = \frac{x+5}{3} \quad [0.5 + 0.5]$$

b)

i) Domain of f is $(-2, \infty)$ [0.5]

ii)

$$f(5) = 5 + 1 = 6 \quad [0.5]$$

$$f(0) = 3(0) = 0 \quad [0.5]$$

c)

i) Degree = 6 [0.5]

ii) The zeros of f are 4, -3 , and 1

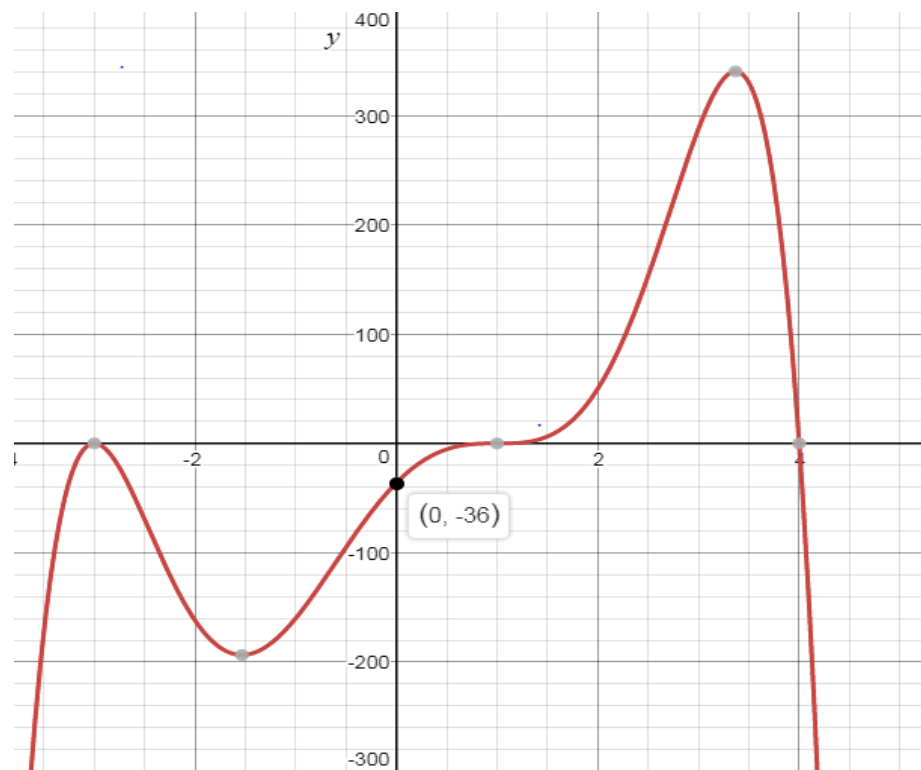
Zeros	Multiplicities	Crosses or Touches the x -axis	
4	1	Crosses	[0.5]
-3	2	Touches	[0.5]
1	3	Crosses	[0.5]

iii) y -intercept, $f(0) = -36$ [0.5]

iv) As $x \rightarrow -\infty$, $y \rightarrow -\infty$ [0.5]

As $x \rightarrow \infty$, $y \rightarrow -\infty$ [0.5]

v) Sketch the graph:



Shows all intercepts [0.5]

Proper end behaviour [0.5]

Shows correct crossing or touching at x -intercepts [0.5]

Question 4 (10 marks)

a)

$$\begin{aligned}
 f(-3) &= 2(-3)^3 - (-3)^2 + 2(-3) - 3 \\
 &= 2(-27) - 9 - 6 - 3 \\
 &= -54 - 9 - 6 - 3 \\
 &= -72 \neq 0 \quad [0.5 + 0.5]
 \end{aligned}$$

$\therefore (x+3)$ is NOT a factor of f . [0.5]

b)

$$\begin{aligned}
 &\log_a 12 \\
 &= \log_a (2^2 \times 3) \\
 &= 2\log_a 2 + \log_a 3 \\
 &= 2m + n \quad [0.5 + 0.5 + 0.5]
 \end{aligned}$$

c)

i)

$$\begin{aligned}
 3\log_2(k-1) &= 9 \\
 \log_2(k-1) &= 3 \\
 k-1 &= 2^3 \\
 k &= 9 \quad [0.5 + 0.5 + 0.5]
 \end{aligned}$$

ii)

$$\begin{aligned}
 3\log_2(x-1) &= 0 \\
 \log_2(x-1) &= 0 \\
 x-1 &= 1 \\
 x &= 2 \quad [0.5 + 0.5 + 0.5]
 \end{aligned}$$

d)

i)

$$\begin{aligned}
 5^{2x+3} &= \frac{1}{125} \\
 5^{2x+3} &= 5^{-3} \\
 2x+3 &= -3 \\
 x &= -3 \quad [0.5 + 0.5 + 0.5]
 \end{aligned}$$

ii)

$$\begin{aligned}
 7^{x+3} &= e^x \\
 \ln(7^{x+3}) &= \ln e^x \\
 (x+3)\ln 7 &= x\ln e \\
 x\ln 7 + 3\ln 7 &= x \\
 x(1 - \ln 7) &= 3\ln 7 \\
 x &= \frac{3\ln 7}{1 - \ln 7} \quad [0.5 + 0.5 + 0.5 + 0.5 + 0.5]
 \end{aligned}$$

Question 5 (10 marks)

a)

$$3x + ky + 2 = 0$$

$$y = -\frac{3}{k}x - \frac{2}{k}$$

$$m_1 = -\frac{3}{k} \quad [0.5]$$

$$4x + y - 1 = 0$$

$$y = -4x + 1$$

$$m_2 = -4 \quad [0.5]$$

<p>i) Parallel</p> $-\frac{3}{k} = -4$ $k = \frac{3}{4} \quad [0.5]$	<p>ii) Perpendicular</p> $-\frac{3}{k} \times -4 = -1$ $k = -12 \quad [0.5]$
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b)

$$x^2 + y^2 + 8x - 10y + 16 = 0$$

$$x^2 + 8x + \left(\frac{8}{2}\right)^2 + y^2 - 10y + \left(-\frac{10}{2}\right)^2 = -16 + \left(\frac{8}{2}\right)^2 + \left(-\frac{10}{2}\right)^2$$

$$(x+4)^2 + (y-5)^2 = 25 \quad [1+1]$$

$$(x-h)^2 + (y-k) = r^2$$

$$\text{Center} = (-4, 5) \quad [0.5]$$

$$\text{Radius} = 5 \quad [0.5]$$

c)

$$PS = PT$$

$$\sqrt{(x+3)^2 + (y-2)^2} = \sqrt{(x-13)^2 + (y-6)^2}$$

$$x^2 + 6x + 9 + y^2 - 4y + 4 = x^2 - 26x + 169 + y^2 - 12y + 36$$

$$32x + 8y - 192 = 0$$

$$4x + y - 24 = 0 \quad [1+1+0.5+0.5]$$

d)

$$m = \frac{3-1}{1-2} = -2 \quad [0.5]$$

$$Y = mX + c$$

$$3 = -2(1) + c$$

$$c = 5 \quad [0.5]$$

$$\frac{1}{y} = -2\left(\frac{1}{x}\right) + 5$$

$$\frac{1}{y} = \frac{-2 + 5x}{x}$$

$$y = \frac{x}{-2 + 5x} \quad [0.5 + 0.5]$$

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