

PMT0201 Tutorial 2 (Part 2)

Q1 Simplify the trigonometric expression.

a) $\sin u + \cot u \cos u$

b) $\cos^2 \theta (1 + \tan^2 \theta)$

c) $\frac{1 + \cos y}{1 + \sec y}$

d) $\frac{1 + \sin u}{\cos u} + \frac{\cos u}{1 + \sin u}$

Q2 Verify the identity

a) $(\sin x + \cos x)^2 = 1 + 2 \sin x \cos x$

b) $\tan \theta + \cot \theta = \sec \theta \csc \theta$

c) $(\sin x + \cos x)^4 = (1 + 2 \sin x \cos x)^2$

d) $\frac{\sec x + \csc x}{\tan x + \cot x} = \sin x + \cos x$

Q3 Use Addition or Subtraction Formula to find the exact value of the expressions below

a) $\sin 75^\circ$

b) $\sin 15^\circ$

c) $\cos 105^\circ$

d) $\tan 165^\circ$

e) $\sin \frac{19\pi}{12}$

f) $\cos \frac{17\pi}{12}$

g) $\sin\left(-\frac{5\pi}{12}\right)$

h) $\tan\left(-\frac{\pi}{12}\right)$

i) $\cos \frac{11\pi}{12}$

Q4 Prove the identity (Addition and Subtraction Formulas may be helpful)

a) $\tan\left(\frac{\pi}{2} - u\right) = \cot u$

b) $\sin\left(x - \frac{\pi}{2}\right) = -\cos x$

c) $\cos\left(x - \frac{\pi}{2}\right) = \sin x$

d) $\sin(x + y) - \cos(x - y) = 2 \cos x \sin y$

e) $\cos(x + y) + \cos(x - y) = 2 \cos x \cos y$

f) $\cos(x - \pi) = -\cos x$

g) $\cos(x + y) \cos(x - y) = \cos^2 x - \sin^2 y$

Q5 Evaluate each expression under the given conditions

a) $\sin(\theta - \phi)$; $\tan \theta = \frac{4}{3}$, θ in Quadrant III, $\sin \phi = -\frac{\sqrt{10}}{\sqrt{10}}$, ϕ in Quadrant IV.

b) $\cos(\theta - \phi)$; $\cos \theta = \frac{3}{5}$, θ in Quadrant IV, $\tan \phi = -\sqrt{3}$, ϕ in Quadrant II.

c) $\sin(\theta + \phi)$; $\sin \theta = \frac{5}{13}$, θ in Quadrant I, $\cos \phi = -\frac{2\sqrt{5}}{5}$, ϕ in Quadrant II.

Q6 Find $\sin 2x$, $\cos 2x$, and $\tan 2x$ from the given information

a) $\sin x = \frac{5}{13}$, x in Quadrant I

b) $\tan x = -\frac{4}{3}$, x in Quadrant II

c) $\csc x = 4$, $\tan x < 0$

d) $\cos x = \frac{4}{5}$, $\csc x < 0$

Q7 Use an appropriate Half-Angle Formula to find the exact value of the expression

a) $\sin 15^\circ$

b) $\tan 15^\circ$

c) $\cos 165^\circ$

d) $\cos \frac{3\pi}{8}$

e) $\tan \frac{\pi}{8}$

f) $\sin \frac{9\pi}{8}$

Q8 Find $\sin \frac{x}{2}$, $\cos \frac{x}{2}$, and $\tan \frac{x}{2}$ from the given information

a) $\sin x = \frac{3}{5}$, $0^\circ < x < 90^\circ$

b) $\cos x = -\frac{4}{5}$, $180^\circ < x < 270^\circ$

c) $\tan x = 1$, $0^\circ < x < 90^\circ$

d) $\cot x = 5$, $180^\circ < x < 270^\circ$

Q9 Find the exact value of the given expression

a) $\sin\left(2\cos^{-1}\frac{7}{25}\right)$ b) $\cos\left(2\tan^{-1}\frac{12}{5}\right)$ c) $\tan\left(\frac{1}{2}\cos^{-1}\frac{2}{3}\right)$

Q10 Find the value of the product or sum

a) $3\cos 37.5^\circ \cos 7.5^\circ$ b) $2\sin 52.5^\circ \sin 97.5^\circ$ c) $\cos 37.5^\circ \sin 7.5^\circ$
d) $\cos \frac{\pi}{12} + \cos \frac{\pi}{12}$ e) $\sin 75^\circ + \sin 15^\circ$ f) $\cos 225^\circ - \cos 195^\circ$

Q11 Prove the identity

a) $\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$ b) $\frac{\sin 4x}{\sin x} = 4\cos x \cos 2x$
c) $\frac{\sin 3x + \sin 7x}{\cos 3x - \cos 7x} = \cot 2x$ d) $\cos^4 x - \sin^4 x = \cos 2x$
e) $\frac{\sin 10x}{\sin 9x + \sin x} = \frac{\cos 5x}{\cos 4x}$

Q12 Show that

a) $\cos 100^\circ - \cos 200^\circ = \sin 50^\circ$ b) $\sin 130^\circ - \sin 110^\circ = -\sin 10^\circ$
c) $\cos 87^\circ + \cos 33^\circ = \sin 63^\circ$

Q13 Solve the equations below

a) $2\cos^2 \theta + \sin \theta = 1$ b) $2\sin 2\theta - 3\sin \theta = 0$
c) $2\sin^2 \theta - \cos \theta = 1$ d) $\tan \theta - 3\cot \theta = 3$

Q14 Find the solutions of equations below in the interval $[0, 2\pi)$

a) $2\cos 3\theta = 1$ b) $3\csc^2 = 4$ c) $\sqrt{3}\tan 3\theta + 1 = 0$
d) $\cos \frac{\theta}{2} - 1 = 0$ e) $\tan \frac{\theta}{4} + \sqrt{3} = 0$ f) $\sec \theta - \tan \theta = \cos \theta$
g) $\cos \theta \cos 2\theta + \sin \theta \sin 2\theta = \frac{1}{2}$ h) $\sin 3\theta \cos \theta - \cos 3\theta \sin \theta = 0$

Q15 Use a Double- or Half-angle Formula to solve the equation in the interval $[0, 2\pi)$

a) $\sin 2\theta + \cos \theta = 0$

b) $\cos 2\theta - \cos^2 \theta = 0$

c) $\tan \frac{\theta}{2} - \sin \theta = 0$

d) $\sin \theta - \cos \theta = \frac{1}{2}$

Q16 Solve the equation by first using a Sum-to-Product Formula

a) $\cos 5\theta - \cos 7\theta = 0$

b) $\sin 5\theta - \sin 3\theta = \cos 4\theta$