# MULTIMEDIA UNIVERSITY

# FINAL EXAMINATION

**TRIMESTER 3, 2016/2017**

# (*SOLUTION*)

## PMT0101 – MATHEMATICS I
(All sections / Groups)

MAY 2017

(2 Hours)

**No calculators are allowed.**
**You are required to write proper steps.**

**QUESTION 1    [10 marks]**

(a) [2 marks]

$$\left(\frac{-2xy^{-2}}{y^3}\right)^2 \cdot \left(\frac{x^{-4}}{4y^2}\right) = \frac{4x^2y^{-4}}{y^6} \cdot \frac{x^{-4}}{4y^2} = \frac{x^{-2}y^{-4}}{y^8} = x^{-2}y^{-12} = \frac{1}{x^2y^{12}} \text{ or equivalent}$$

[0.5+0.5+0.5+0.5]

(b) [2 marks]

$$\frac{1-\sqrt{5}}{1+2\sqrt{5}} = \frac{1-\sqrt{5}}{1+2\sqrt{5}} \cdot \frac{1-2\sqrt{5}}{1-2\sqrt{5}} = \frac{1-2\sqrt{5}-\sqrt{5}+2(5)}{1-4(5)} = \frac{11-3\sqrt{5}}{-19} \text{ or } \frac{3\sqrt{5}-11}{19}$$

[0.5+(0.5+0.5)+0.5]

(c) [2 marks]

$$-3y\sqrt{\frac{4x^2}{3}} + 2\sqrt{3x^2y^2} = -3y \cdot \frac{2x}{\sqrt{3}} + 2xy\sqrt{3} = \frac{-6xy}{\sqrt{3}} + 2xy\sqrt{3}$$

$$= \frac{-6xy + 2xy(3)}{\sqrt{3}} = 0$$

[0.5+0.5+0.5+0.5]

(d) [2 marks]

$$5xy^2 - 7xy - 6x = x(5y^2 - 7y - 6) = x(5y+3)(y-2)$$

[1+1]

(e) [2 marks]

$$\frac{3+2i}{2-3i} = \frac{3+2i}{2-3i} \cdot \frac{2+3i}{2+3i} = \frac{6+4i+9i+6i^2}{4+9}$$

$$= \frac{13i}{13} = 0+i \qquad \text{or} \quad i$$

[0.5+0.5+0.5+0.5]

**Continued …….**

**QUESTION 2   [10 marks]**

(a) [2.5 marks]

$x^3 - x^2 - 4x + 4$
$= x^2(x-1) - 4(x-1) = (x-1)(x^2-4) = (x-1)(x-2)(x+2)$        [0.5+0.5+0.5]

$x^3 - x^2 - 4x + 4 = 0$
$(x-1)(x-2)(x+2) = 0$    [0.5]
$x = 1, 2, \text{ or } -2$          [0.5]

(b) [3 marks]

$(x-4)^2 = 3x - 8$     [0.5]
$x^2 - 11x + 24 = 0$    [0.5]
$(x-3)(x-8) = 0$
$x = 3 \quad \text{or} \quad x = 8$    [0.5]
      Checking:

              When $x = 3$, LHS $= 3 - 4 = -1$, RHS $= \sqrt{3(5)+1} = \sqrt{16} = 4$  ✗

              When $x = 8$ , LHS $= 8 - 4 = 4$, RHS $= \sqrt{3(8)-8} = \sqrt{16} = 4$  ✓
                                              [0.5+0.5]
      Conclusion:   $x = 8$                          [0.5]

(c) [2 marks]

$1 \le \dfrac{2x-1}{-3} \le 5$
$-3 \ge 2x - 1 \ge -15$
$-2 \ge 2x \ge -14$
$-1 \ge x \ge -7$    Solution set : $[-7, -1]$                [0.5+0.5+0.5+0.5]

(d) [2.5 marks]

| [0.5+0.5] | [0.5] *for one of these* $4x - 3 = 13 \text{ or } 4x - 3 = -13$ | [0.5+0.5] |
|---|---|---|
| $4x - 3 = 13$ | | $4x - 3 = -13$ |
| $4x = 16$ | $4x - 3 = \pm 13$ | $4x = -10$ |
| $x = 4$ | | $x = -\dfrac{5}{2}$ |
| | or | |

                                                         **Continued …….**

**QUESTION 3    [10 marks]**

(a)  [2 marks]
   The domain is $[-3,0) \cup (0,4]$;  the range is  $[0,2)$.                    [(0.5+0.5)+0.5]
   The function is NOT one-to-one.                              [0.5]

(b) [3 marks]

   (i) $(f \circ g)(6) = f(g(6)) = f\left(\dfrac{6}{4(6)+1}\right) = f\left(\dfrac{6}{25}\right)$          [0.5+0.5]

   $$= \sqrt{3 + \frac{6}{25}} = \sqrt{\frac{81}{25}} = \frac{9}{5}$$          [0.5]

   (ii) Let    $y = \sqrt{3+x}$.        Need to solve for $x$ in terms of $y$.

   $$y^2 = 3 + x \ , \qquad x = y^2 - 3$$          [0.5+0.5]

   $$f^{-1}(x) = x^2 - 3$$          [0.5] or equivalent

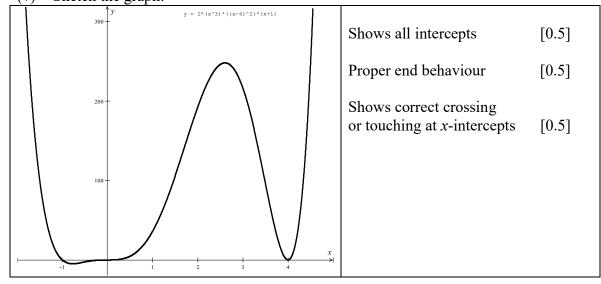(c)    [5 marks]          $f(x) = 2x^3(x-4)^2(x+1)$
(i)    Degree $= 6$          [0.5]
(ii)    **The zeros** of $f$ are $0, \ 4$ and $-1$

| Zeros | Multiplicities | Crosses or Touches |
|:-----:|:--------------:|:------------------:|
| 0 | 3 | Crosses |
| 4 | 2 | Touches |
| $-1$ | 1 | Crosses |
| [0.5] | [0.5] | [0.5] |

(iii)  $y$-intercept, $f(0) = 0$     [0.5]

(iv)  As $x \to -\infty, \ y \to \infty$     [0.5]          As $x \to \infty, \ y \to \infty$     [0.5]

(v)    Sketch the graph:



Shows all intercepts          [0.5]

Proper end behaviour          [0.5]

Shows correct crossing
or touching at $x$-intercepts     [0.5]

**Continued …….**

## QUESTION 4 [10 marks]

(a) [3 marks]

$$\begin{array}{r} 3x - 2 \qquad\qquad [0.5] \\ x^2 + 3 \enclose{longdiv}{3x^3 - 2x^2 + 10x - 7} \\ \underline{3x^3 \qquad + 9x} \qquad [0.5] \\ -2x^2 + x - 7 \qquad [0.5] \\ \underline{-2x^2 \qquad -6} \qquad [0.5] \\ x - 1 \end{array}$$

Quotient $= 3x - 2$
Remainder $= x - 1$ \qquad [0.5+0.5]

(b) [2 marks]

$$(2^3)^{3x-1} = (2^2)^{2x+3} \qquad\qquad [0.5]$$
$$2^{3(3x-1)} = 2^{2(2x+3)}, \; 3(3x-1) = 2(2x+3) \qquad [0.5+0.5]$$
$$9x - 3 = 4x + 6, \; x = \tfrac{9}{5} \qquad\qquad [0.5]$$

(c) [1 mark]

$$-2\left(\frac{1}{3}\right)^m + 1 = -53 \quad \frac{1}{3^m} = 27 \qquad [0.5]$$
$$3^m = \frac{1}{27} \quad 3^m = 3^{-3} \quad m = 3 \qquad [0.5]$$

(d) [2 marks]

$$2\log_{10} 5 + \log_{10} 12 - \log_{10} 3$$
$$= \log_{10} 5^2 + \log_{10} 12 - \log_{10} 3$$
$$= \log_{10}\left(\frac{25 \times 12}{3}\right)$$
$$= \log_{10} 100$$
$$= \log_{10} 10^2 = 2\log_{10} 10 = 2 \qquad [0.5+0.5+0.5+0.5]$$

(e) [2 marks]

$$2 = a + k\log_{10} 1 \quad a = 2 \qquad [0.5+0.5]$$
$$5 = a + k\log_{10} 10 \quad 5 = 2 + k \quad k = 3 \qquad [0.5+0.5]$$

**Continued …….**

## QUESTION 5    [10 marks]

(a) [2.5 marks]

(i) $x^2 - 4x + \left(\dfrac{-4}{2}\right)^2 + y^2 + 8y + \left(\dfrac{8}{2}\right)^2 = 5 + 4 + 16$        [0.5+0.5]

$(x-2)^2 + (y+4)^2 = 5^2$                    [0.5]

(ii)

Centre = $(2, \ -4)$          [0.5]

Radius = 5                 [0.5]

(b) [2.5 marks]

| $\begin{aligned} x + 3y &= 6 \\ 3y &= -x + 6 \end{aligned}$ | $y = -\dfrac{1}{3}x + 2$   [0.5]<br><br>$m_1 = -\dfrac{1}{3}$     [0.5] | $-\dfrac{1}{3} \times m_2 = -1$<br><br>$m_2 = 3$      [0.5] |
|---|---|---|

Equation of perpendicular line:

$y - 2 = 3(x+1)$         [0.5]

$y = 3x + 5$            [0.5]

(c) [2 marks]    $PA = PB$

$\sqrt{(x-(-2))^2 + (y-3)^2} = \sqrt{(x-4)^2 + (y-(-1))^2}$      [0.5]

$\left(\sqrt{(x+2)^2 + (y-3)^2}\right)^2 = \left(\sqrt{(x-4)^2 + (y+1)^2}\right)^2$      [0.5]

$x^2 + 4x + 4 + y^2 - 6y + 9 = x^2 - 8x + 16 + y^2 + 2y + 1$      [0.5]

$10x - 8y - 4 = 0 \ \text{ or } \ 5x - 4y - 2 = 0$      [0.5]

(d) [3 marks]

(i) $\dfrac{y}{x} = p + q\sqrt{x}$           [1]

(ii) $q = \dfrac{10-4}{2-5} = \dfrac{6}{-3} = -2$    [0.5 + 0.5]

$10 = -2(2) + p$

$p = 14$      [0.5 + 0.5]

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