TUTORIAL 8

8.1 FUNCTIONS OF SEVERAL VARIABLES

QUESTION 1

(a) Let $f(x, y) = x^2 e^{3xy}$.
   Evaluate $f(2, 0)$.

(b) Let $f(x, y, z) = e^{\sqrt{z - x^2 - y^2}}$.
   Evaluate $f(2, -1, 6)$.

(c) Let $g(x, y, z) = \ln(25 - x^2 - y^2 - z^2)$.
   Evaluate $g(2, -2, 4)$.

(d) Let $f(x, y) = \ln(x + y - 1)$.
   (a) Evaluate $f(1, 1)$.
   (b) Evaluate $f(e, 1)$.

8.2 PARTIAL DERIVATIVES

QUESTION 2

Find the first partial derivatives of the function.

$$f(x, y) = 3x - 2y^4$$

$$f(x, y) = x^5 + 3x^3 y^2 + 3xy^4$$

$$z = xe^{3y}$$

$$f(x, y) = \frac{x - y}{x + y}$$

$$w = \sin \alpha \, \cos \beta$$

$$f(r, s) = r \ln(r^2 + s^2)$$

$$f(x, y) = xe^{xy} - e^x \ln(x + y)$$

$$g(x, y) = e^{-x} \sin(x + y)$$

QUESTION 3

Find the indicated partial derivatives.

$$f(x, y) = \sqrt{x^2 + y^2}; \quad f_x(3, 4)$$

$$f(x, y) = \sin(2x + 3y); \quad f_y(-6, 4)$$

8.3 CHAIN RULE

QUESTION 4

Use the chain rule to find $\dfrac{dz}{dt}$.

(i) $\quad z = x^2y + xy^2, \quad x = 2 + t^4, \quad y = 1 - t^3$

(ii) $\quad z = \sqrt{x^2 + y^2}, \quad x = e^{2t}, \quad y = e^{-2t}$

(iii) $\quad z = \sin x \cos y, \quad x = \pi t, \quad y = \sqrt{t}$

(iv) $\quad z = x \ln(x + 2y), \quad x = \sin t, \quad y = \cos t$

QUESTION 5

Use the chain rule to find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$.

(i) $\quad z = x^2 + xy + y^2, \quad x = s + t, \quad y = st$

(ii) $\quad z = x/y, \quad x = se^t, \quad y = 1 + se^{-t}$