TUTORIAL CHAPTER 5 – WAVES

1. A fisherman notices that wave crests pass the bow of his anchored boat every 3.0 s. He measures the distance between two crests to be 8.5 m. How fast are the waves traveling?

2. A sound wave in air has a frequency of 262 Hz and travels with a speed of 330 m/s. How far apart are the wave crests (compressions)?

3. AM radio signal shave frequencies between 550 kHz and 1600 kHz and travel with a speed of 3.0×10^8 m/s. What are the wavelengths of these signals? On FM, the frequencies range from 88.0 MHz to 108 MHz and travel at the same speed; what are their wavelengths?

4. The velocity of waves on a string is 92 m/s. If the frequency of standing waves is 475 Hz, how far apart are two adjacent nodes?

5. Determine the speed and direction of propagation of each of the following sinusoidal waves, assuming that x is measured in meters and t is in seconds.

   (a) $y(x,t) = 0.60 \cos (3.0x - 15t)$

   (b) $y(x,t) = 0.40 \cos (3.0x - 15t)$

   (c) $y(x,t) = 1.2 \sin (15t + 2.0x)$

   (d) $y(x,t) = 0.20 \sin (12t + x/2)$

6. Two sinusoidal waves in a string are defined by the functions

   $y_1 = (2.0 \text{ cm}) \sin (20x - 30t)$

   $y_2 = (2.0 \text{ cm}) \sin (25x - 40t)$

   where y and x are in centimeters and t is in seconds. What is the phase difference between these two waves at the point $x = 5.0$ cm at $t = 2.0$ s?

7. Two waves in a long string are given by:

   $y(x,t) = (3.0) \cos (4.0x - 5.0t)$

   $y(x,t) = (4.0) \sin (5.0x - 2.0t)$

   where the y(x,t) and x are in centimeters and t is in seconds. Find the superposition of the waves at the point,

   (a) x = 1.0 cm, t = 1.0 s

   (b) x = 1.0 cm, t = 0.5 s

   (c) x = 0.5 cm, t = 0.0 s

8. A standing wave has nodes at $x = 0$ cm, $x = 6.0$ cm, $x = 12.0$ cm, and $x = 18$ cm.

   (a) What is the wavelength of the waves that are interfering to produce this standing wave?

   (b) At what positions are the antinodes?

9.

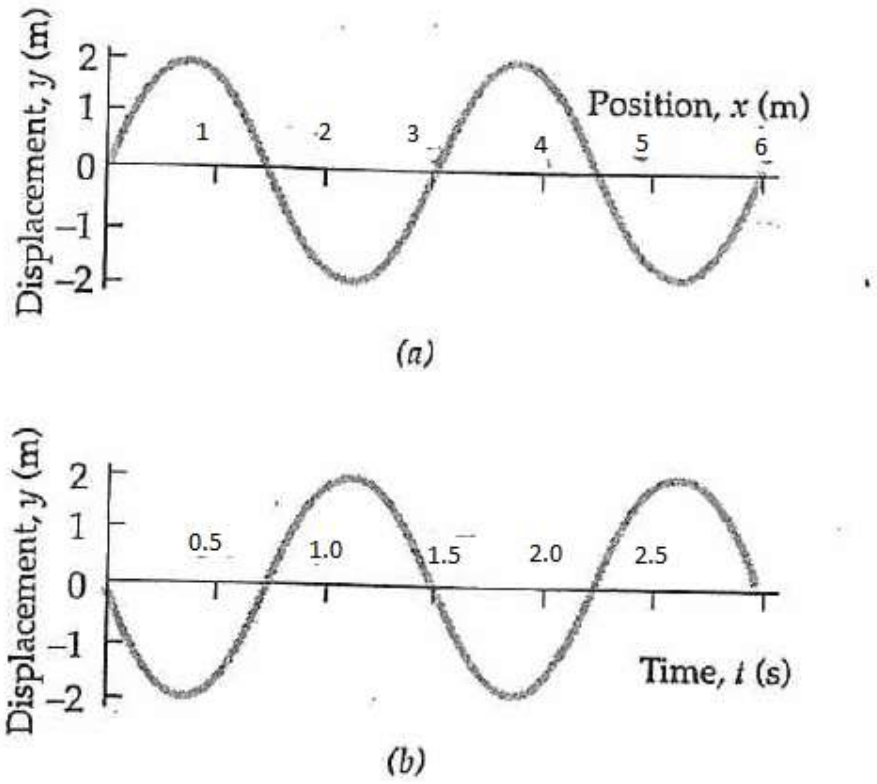

Figure 5.1

Figure 5.1(a) shows a wave plotted as a function of position at time t = 0, while Figure 5.1(b) shows the same wave plotted as a function of time at position x = 0. Find:

(a) the wavelength,

(b) the period,

(c) the wave speed,

(d) the direction if propagation, and

(e) the mathematical description of the wave.

10. (a) Water in a tank C is disturbed and the waves generated can be represented by the equation

$y = 0.50 \sin (16t - 6.0x)$

with x and y in centimeters and t in seconds. Calculate

(i) the wavelength,

(ii) the frequency,

(iii) the speed of the wave.

(b) As compared to water waves at tank C in part (a), water waves at tank D have:

The same frequency; but

Half the amplitude;

Twice the speed of propagation; and

Travel in the opposite direction.

Write down an equation to represent water waves at tank D.

11. Two sine waves, $A \sin(kx - \omega t)$ and $A \sin(kx - \omega t - \varphi)$, combine to form a sine wave with the same amplitude A.

(a) For what values of φ of the second wave is this possible?

(b) Write an expression for the resultant wave.

(c) State the type of interference of the resultant wave.

12. The displacements, y_1 and y_2, of two harmonic waves are given by

$$y_1 = (0.025m)\sin\left(\left(\frac{3}{2}m^{-1}\right)x - (2s^{-1})t\right) \text{ and}$$

$$y_2 = (0.025m)\sin\left(\left(\frac{3}{2}m^{-1}\right)x - (2s^{-1})t + \frac{1}{4}\right)$$

(a) What is the phase difference (in radians) between the waves?

(b) What is the amplitude of the wave that is the superposition, $y_1 + y_2$, of these two waves, and what type of interference produced?

13. The waves emitted from a source A is given by

$$y_1 = 6 \sin \pi(20t - 0.5x)$$

where y_1, x and t are expressed in units of meter and seconds.

(a) Determine the wavelength and the frequency of the radio waves.

At the same time, Source B generates waves y_2 that is identical to y_1 except that there is a phase difference of $\frac{\pi}{3}$.

(b) Write down the wave equation for y_2.

If y_2 were superimposed to y_1,

(c) what is the phase constant and the amplitude of the resultant wave?

(d) write down the equation of the resultant waves.

14. The displacements, y_1 and y_2, of two waves are described by

$$y_1 = (0.04\text{ m}) \sin \pi[(5\text{ m}^{-1})x - (800\text{ s}^{-1})t] \text{ and}$$

$$y_2 = (0.04\text{ m}) \sin \pi[(5\text{ m}^{-1})x + (800\text{ s}^{-1})t]$$

(a) Determine the amplitude of the resulting standing wave.

(b) Find the position of the nodes.

15. Two traveling waves that move in opposite directions along the x-axis interfere to produce a standing wave of the form, $\sin kx \cos \omega t$, where $k = 3\pi\text{ m}^{-1}$ and $\omega = 24\pi\text{ s}^{-1}$. What are the amplitudes, frequencies and speeds of the traveling waves?

Answers:

1. 2.8 m/s. 2. $\lambda =$ 1.26 m.
3. For AM : 545 m; 188 m.
   For FM : 3.41 m; 2.78 m.
4. 0.097 m.
5. a) $v = f\lambda = 2.1 \times 2.4 = 5.04$ m/s (direction is positive x axis)
   b) $v = f\lambda = 2.1 \times 2.4 = 5.04$ m/s (direction is negative x axis)
   c) $v = f\lambda = 3.14 \times 2.4 = 7.54$ m/s (direction is negative x axis)
   d) $v = f\lambda = 12.56 \times 1.9 = 23.86$ m/s (direction is positive x axis)
6. a) $\Delta\phi = 5 radians = 287^\circ = 74^\circ$
7. a) $y = y_1 + y_2 = 1.62 + 0.58 = 2.18 cm$
   b) $y = y_1 + y_2 = 0.22 + (-3.01) = -2.82 cm$
   c) $y = y_1 + y_2 = -1.24 + 2.41 = 1.14 cm$
8. a) 12 cm
   b) The antinodes are in the middle between the nodes. So they are at 3.0 cm, 9.0 cm 15.0 cm .....
9. a) $\lambda = 3$ m b) $T = 1.5$ s
   c) 2 m/s d) to the right
   e) k = 2.09 m^{-1}, 4.19 s^{-1}
   $y = (2\text{ m}) \sin[(2.09\ m^{-1})\, x - (4.19\ s^{-1})\, t]$
10. a) i) 1.05 cm ii) 2.55 Hz iii) 0.0268 m/s
    b) $y = 0.25 \sin(16t - 3.0x)$
11. a) $\phi = \pm 2\pi/3$
    b) $y = 2A\cos(\frac{\phi}{2}) \sin(kx - \omega t - \frac{\phi}{2}) = A \sin(kx - \omega t - \frac{\pi}{3})$
    c) intermediate interference
12. a) $\pi/4$
    b) amplitude = 0.046 m
    intermediate interference
13. a) $\lambda = \frac{2\pi}{0.5} = 4\pi$ m, $f = \frac{\omega}{2\pi} = \frac{10}{2\pi} = \frac{5}{\pi}$ Hz
    b) $y_1 = 6 \sin \pi(20\, t - 0.5\, x - \frac{\pi}{3})$
    c) phase constant = $\frac{\pi/3}{2} = \frac{\pi}{6}$, amplitude = $2A\cos(\frac{\phi}{2}) = 2(6)\cos(\frac{\pi}{6}) = 1.851$ m
    d) $y = 2A\cos(\frac{\phi}{2}) \sin(kx - \omega t - \frac{\phi}{2}) = 1.851 \sin(20t - 0.5x - \frac{\pi}{6})$
14. a) amplitude = $0.08 \sin 5\pi x$
    b) $x = n / 5$, where n = integer
15. $A = ½$ m , $f = \frac{\omega}{2\pi} = \frac{24\pi}{2\pi} = 12$ Hz, $v = \frac{\omega}{k} = \frac{24\pi}{3\pi} = 8$ m/s

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