Topic 3.2
Function

TMA1201 Discrete Structures & Probability
Faculty of Computing & Informatics
Multimedia University

What you will learn in this lecture:

- Definition of a function
- Domain, codomain, range
- Properties of functions

Definition of a Function

A special type of relation is known as function.

Definition:

A relation denoted as f from a set X to a set Y is a **function** from X to Y satisfies the condition that every element in X is related to exactly ONE element in Y.

Mathematical notation of function f is written as $\boldsymbol{f}\mathbf{: X \rightarrow Y}$

Set X is known as the **domain for function *f***

Set Y is known as the **codomain for function *f***

Informal Definition of a Function

Let X and Y be sets.

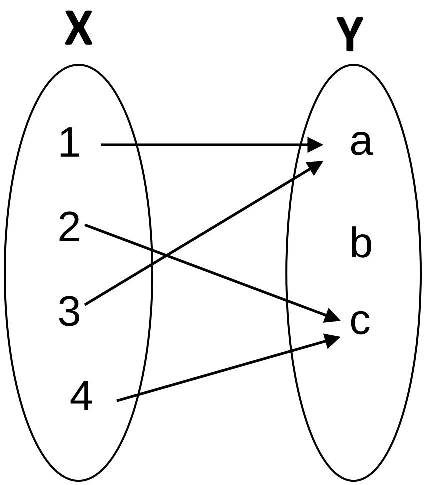
A function with domain X and codomain Y is a "box" which accepts elements of X as inputs and, for each element of X, outputs exactly one element of Y .

Note:

The domain and codomain are part of the function and must always be defined.

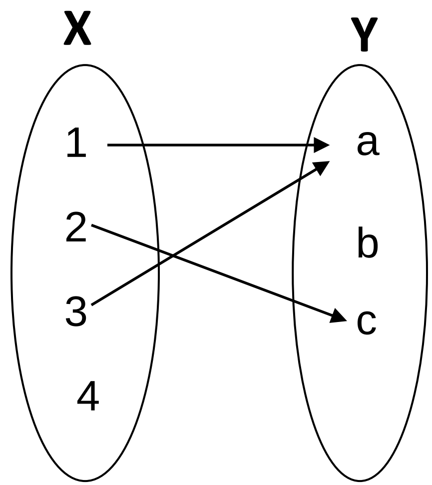
Function versus Relation

In an arrow diagram,



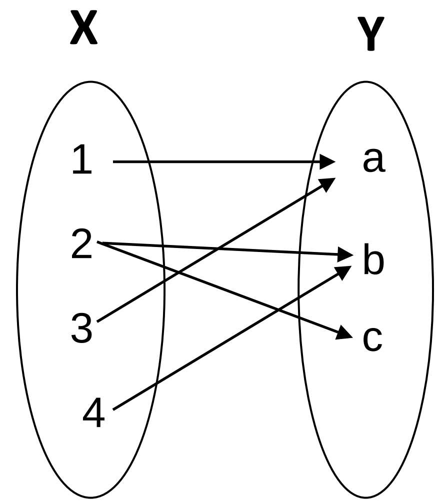
A relation

A function



A relation

Not a function



A relation

Not a function

Example 1

Which of the following rules define functions?

1) For each non-empty set S of natural numbers, let *f* (S) be the least member of S.

Yes. (But why? Can you justify your answer)

2) For each set X of real numbers between 0 and 1, let *g*(X) be the least member of X.

No - $g(\{x \in R \mid 0.5 < x < 1\})$ is not defined.

3) For each circle C in the (x, y) plane, let *h*(C) be the minimum distance from the origin of C to the x-axis.

Yes. (But why? Can you justify your answer)

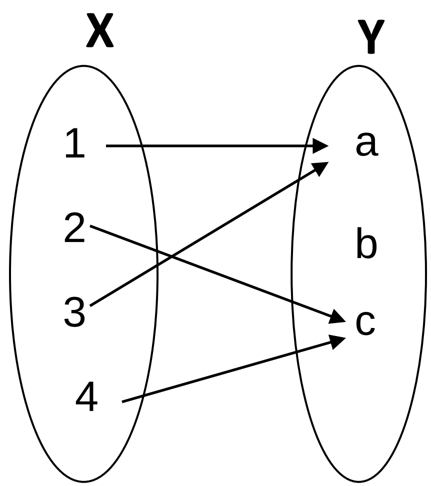
Domain, Codomain, Range

From the definition of a function and its association to relation, we see that a function consists of a domain X, a codomain Y, and a set of ordered pairs from $X \rightarrow Y$ which has exactly one ordered pair (x, y) for each $x \in X$.

The set of y values in set Y occurring in these ordered pairs is called the **range** of the function.

The range is always a subset of the codomain but they may not be equal.
If they are equal we say the function is onto.

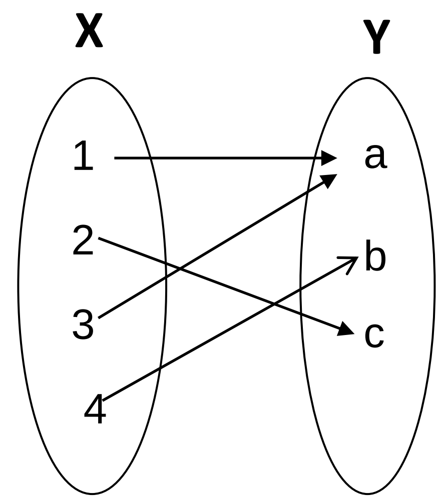
Example 2



Domain = _______

Codomain = ________

Range = ________



Domain = _______

Codomain = ________

Range = ________

Properties of Functions

A function **f: X $\rightarrow$ Y** is said to be

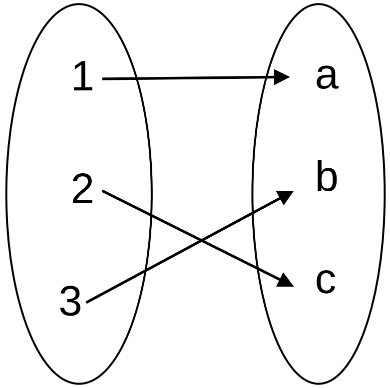
one-to-one (or **injective):** for all x and y if f(x) = f(y) then x = y
(A function is one-to-one (injective) when exactly one element in the domain is assigned to each of the element in the codomain.)

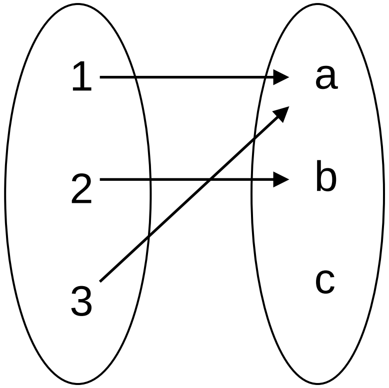
onto (or **surjective)** : for all *y* in Y there exist x in X such that f(x) = y.
(OR simply the range of f is equal to Y)

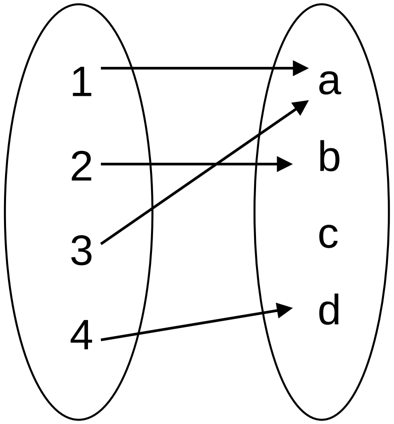
bijective : if it is both one-to-one and onto.

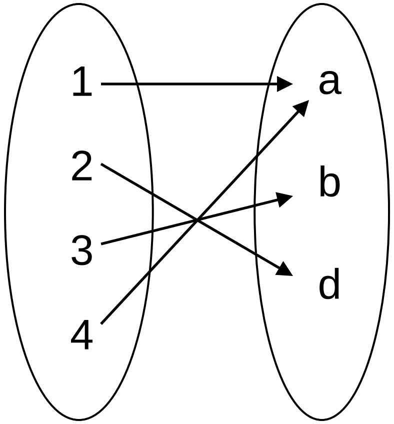
Example 3

Which of the following functions are injective, surjective, and/or bijective? Explain why.









Example 4

Determine whether the following functions are injective, surjective, and/or bijective. Explain why.

1) $f_1, f_2 : Z \rightarrow Z$

a) $f_1(x) = x^2$

b) $f_2(x) = x + 1$

2) f_3 : Country → City

$f_3(a)$ = capital city of a

Summary

We have learnt the following concepts related to functions:

- A function is a special type of Relations.
- A function has to be defined with its domain and codomain.
- A function’s range is a set consisting only of the values defined by the function at its codomain.
- If the range = codomain, the function is onto (surjective).
- A function is one-to-one (injective) when exactly one element in the domain is assigned each of the element in the codomain.

Exercise 1

Given,

$R_1 = \{(x, y) \mid x$ and y are human beings and x is taller than $y\}$

$R_2 = \{(x, y) \mid x$ and y are human beings and both have the same height$\}$

$R_3 = \{(a, b) \mid |a - b| \leq 4\}$, where $a, b \in Z$

$R_4 = \{(a, b) \mid (a - b)$ is a multiple of $7\}$, where $a, b \in Z$

Determine if each of these are functions. Justify your answer.

Exercise 2

Let the domain and codomain be the sets of real numbers. Which of the following is a function? If it is not a function, modify the domain so that it becomes a function. Then determine whether those functions are one to one and/or onto.

1) $f(x)=e^x$

2) $g(x)=1/x$

3) $h(x)=\log(x)$