1. Let $\underset{\sim}{p} = \langle 2, k \rangle$ and $\underset{\sim}{q} = \langle 3, 5 \rangle$. Find the positive value of k such that

   a) $\underset{\sim}{p}$ and $\underset{\sim}{q}$ are parallel

   b) $\underset{\sim}{p}$ and $\underset{\sim}{q}$ are orthogonal

   c) the angle between $\underset{\sim}{p}$ and $\underset{\sim}{q}$ is $\frac{\pi}{4}$

2. Find a unit vector that is orthogonal to both $\underset{\sim}{u} = \langle -2, 1, 5 \rangle$ and $\underset{\sim}{v} = \langle 3, 0, -3 \rangle$.

3. Find the cross product for the following vectors:

   a) $\underset{\sim}{\mathrm{u}} = 3\mathbf{i} + \mathbf{j} - \mathbf{k}$, $\underset{\sim}{\mathrm{v}} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$

   b) $\underset{\sim}{\mathrm{u}} = 3\mathbf{i} + \mathbf{j} - \mathbf{k}$, $\underset{\sim}{\mathrm{v}} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$

4. Find the angle between vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$.
   (Please try both methods; Dot Product and Cross Product)

   a) $\underset{\sim}{a} = \langle 2, 2, -1 \rangle$ and $\underset{\sim}{b} = \langle 5, -3, 2 \rangle$

   b) $\underset{\sim}{a} = \langle 1, 1, 1 \rangle$ and $\underset{\sim}{b} = \langle 1, -2, 3 \rangle$

5. Find a **symmetric equation of a line** that passes through a point $A(1,2,3)$ and is parallel to the line joining points $B(-2,2,0)$ and $C(4,-1,7)$

6. Find a **parametric equation of a line** joining points $E(1,-1,8)$ and $F(10,-1,11)$.

7. Find a **parametric equation of a line** that passes through a point $A(1,-2,3)$ and is perpendicular to both the vector $\underset{\sim}{u} = \langle 1, 0, 0 \rangle$ and the line with symmetric equation

   $$\frac{x-4}{2} = \frac{y-3}{-1} = \frac{z}{5}$$

   (Hint: Vector perpendicular to both $\underset{\sim}{u}$ and $\underset{\sim}{v}$ $\Rightarrow$ parallel to $\underset{\sim}{u} \times \underset{\sim}{v}$)

8. Find an equation of a plane that passes through points:

   a) $P(-4,-1,-1)$, $Q(-2,0,1)$ and $R(-1,-2,-3)$.

   b) $P(5,4,3)$, $Q(4,3,1)$ and $R(1,5,4)$.

9. Find an equation of a plane that passes through point $P(-2,1,7)$ and is perpendicular to the planes $4x - 2y + 2z = -1$ and $3x + 3y - 6z = 5$.

10. Find an equation of a plane that passes through point $P(-3,1,6)$ and is perpendicular to the planes $3x - y + z = -1$ and $2x + 3y - z = 5$.

11. Find the angle between the planes $x + 4y + 7z = 3$ and $5x + 3y + z = 0$.

Answer:

1. a) $\frac{10}{3}$ b) $-\frac{6}{5}$ c) $\frac{1}{2}$

2. unit vector $= \left\langle -\frac{1}{\sqrt{11}}, \frac{3}{\sqrt{11}}, -\frac{1}{\sqrt{11}} \right\rangle$

3. a) $\underset{\sim}{u} \times \underset{\sim}{v} = \mathbf{i} - 10\mathbf{j} - 7\mathbf{k}$
   b) $\underset{\sim}{u} \times \underset{\sim}{v} = -3\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}$

4. a) $\theta = 83.79^0$ b) $\theta = 72.02^0$

5. $\frac{x-1}{6} = \frac{y-2}{-3} = \frac{z-3}{7}$

6. $x = 1 + 9t, \quad y = -1, \quad z = 8 + 3t$

7. $x = 1, \ \frac{y+2}{-5} = \frac{z-3}{-1}$

8. a) $2y - z + 1 = 0$
   b) $x + 9y - 5z = 26$

9. $x + 5y + 3z = 24$

10. $2x - 5y - 11z = -77$

11. $\theta = 60^0$