# Topic 5

## Events and Probability

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## 5.1.1 Introduction

- Our primary goal in studying sets is to use them in explaining and understanding probability.

# 5.1.2 Sets And Sets Operation

## 5.1.2.1 What is **SET**?

- A **set** is a collection whose members are specified by a list or a rule.
- The items in the collection are the **elements** in the sets.

# 5.1.2.2 Method to specify a SET

A) Specified by a list:

S = {Kelantan, Terengganu, Pahang, Johor}

B) Specify with a rule:

S = {$x$: $x$ is a state in Malaysia with shoreline on the South China Ocean}

"S is the set of all $x$ such that…}."

# Example 1

The set $A$ of even positive integers less than 15.

Method 1:

$A$ = {n : n is an even positive less than 15}

Method 2:

$A = \{2, 4, 6, 8, 10, 12, 14\}$

# Example 2

Set $B$ consists of prime numbers between 13 and 31

Method 1:

$B = \{a: a$ is a prime numbers between 13 and 31$\}$

Method 2:

$B = \{17, 19, 23, 29\}$

# 5.1.2.3 Symbols

1) $x \in X$

   $x$ is an element of a set $X$

2) $x \notin X$

   $x$ is **not** an element of $X$

<u>Example:</u>

$A = \{2, 4, 6, 8, 10, 12, 14\}$

$10 \in A$ and $13 \notin A$

# 5.1.2.3 Symbols

3) $\boldsymbol{A \subset B}$ set $A$ is a **subset** of a set $B$

Example 3:

A = {2, 4, 6, 8, 10}

B = {1, 2, ,3, 4, 5, 6, 7, 8, 9,10}

If $A \subset B$ and $B \subset A$,

then $A$ *and* $B$ have exactly the same elements,

Thus $A$ *and* $B$ are **equal**, we write $A = B$.

Example 4:

A = {2, 4, 6, 8, 10}

B = {2, 4, 6, 8, 10}

# 5.1.2.3 SYMBOLS

4) $\boldsymbol{A \cup B}$

***Set*** $A$ **union** with *set* $B$

Meaning = consists of all elements which are in $A$ **or** in $B$ or in both.

$$A \cup B = \{x : x \in A \quad or \quad x \in B\}$$

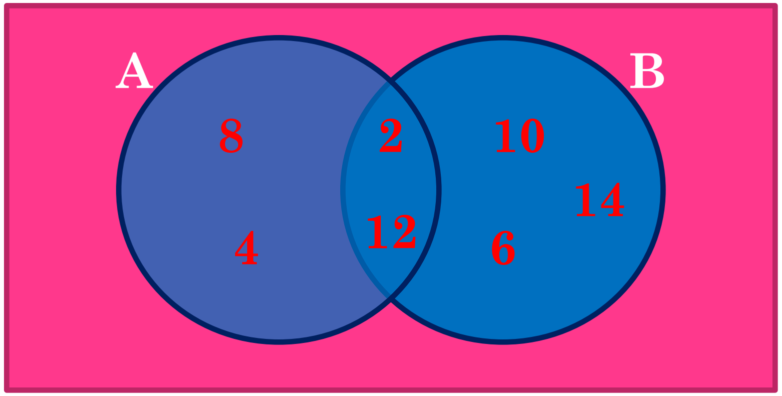
# Example 5

A = {2, 4, 8, 12}

B = {2, 6, 10, 12, 14}

Thus,

$\boldsymbol{A \cup B}$ = {2, 4, 6, 8, 10, 12, 14}




The operation which combines sets in this way is known as the **union**

# 5.1.2.3 Symbols

5) $A \cap B$

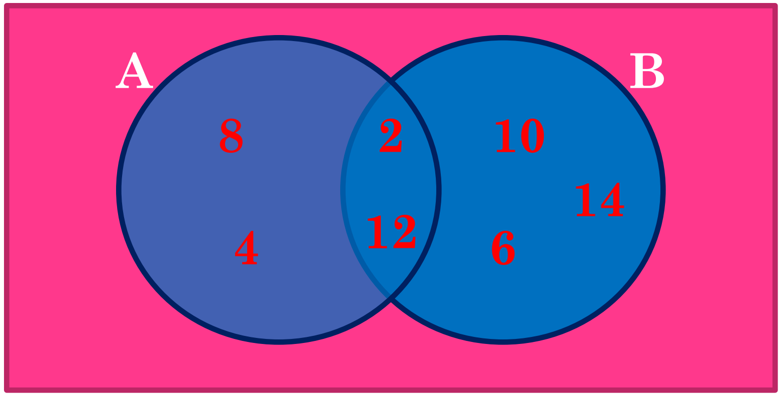
Set $A$ **intersect** with set $B$

Meaning = consists of all elements which are in both $A$ **and** $B$.

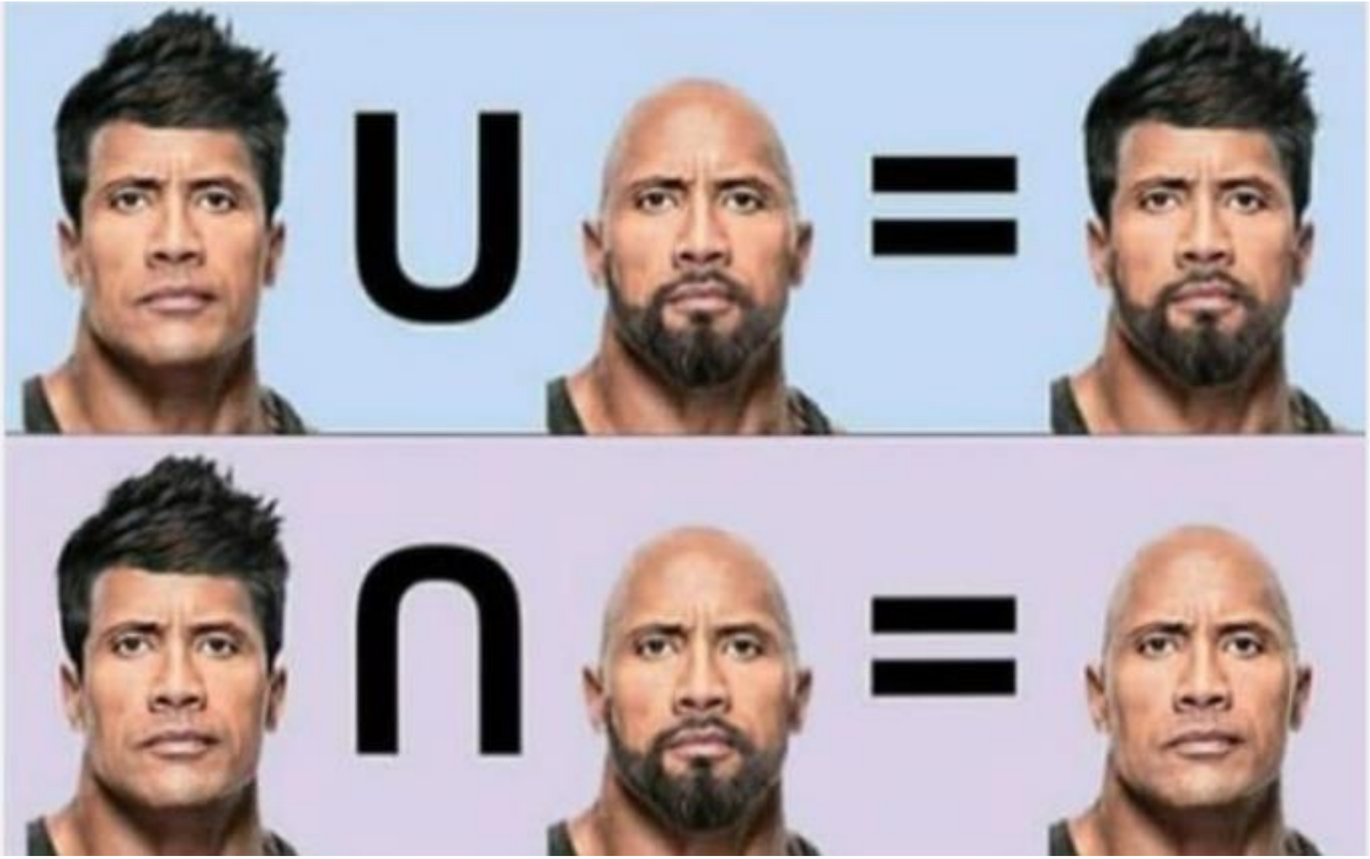
$$A \cap B = \{x : x \in A \quad and \quad x \in B\}$$

# Example 6

A = {2, 4, 8, 12}
B = {2, 6, 10, 12, 14}
Thus,
$\boldsymbol{A \cap B}$ = {2, 12}




The operation which combines sets in this way is known as the **intersection**.



# Example 7

Let the sets $S$, $E$, $O$, and $F$ be defined as follows:

$S = \{1, 2, 3, 4, 5, 6\}$ $E = \{2, 4, 6\}$ $O = \{1, 3, 5\}$ $F = \{1, 2, 3\}$

Find

i) $E \cap F$
$= \{2\}$

ii) $O \cap F$
$= \{1,3\}$

iii) $E \cup F$
$= \{1, 2, 3, 4, 6\}$

iv) $E \cup O$
$= \{1, 2, 3, 4, 5, 6\} = S$

v) $O \cup F$
$= \{1, 2, 3, 5\}$

# EXAMPLE 8

Let $A = \{a, b, c\}$, $B = \{a, c, e\}$, and $C = \{a, d\}$.
Find

i) $A \cap B \cap C$
   $= \{a\}$

ii) $A \cup B \cup C$
   $= \{a, b, c, d, e\}$

iii) $(A \cap B) \cup C$
   $= \{a, c\} \cup \{a, d\}$
   $= \{a, c, d\}$

iv) $A \cap (B \cup C)$
   $= \{a, b, c\} \cap \{a, c, d, e\}$
   $= \{a, c\}$

**Note:**

**1) $A \cap B \cap C = A \cap (B \cap C) = (A \cap B) \cap C$**

2) **$(A \cap B) \cup C \neq A \cap (B \cup C)$**

The parentheses are clearly crucial to the meaning of the expressions.

# 5.1.2.3 Symbols

6) $\varnothing$ is **empty set**, which contains no elements

Since the empty set has no elements, we see that for every set A,

$A \cap \varnothing = \varnothing$ and $A \cup \varnothing = A$

Example:

$A = \{2, 4, 6, 8, 10\}$

$B = \{\ \}$

Find

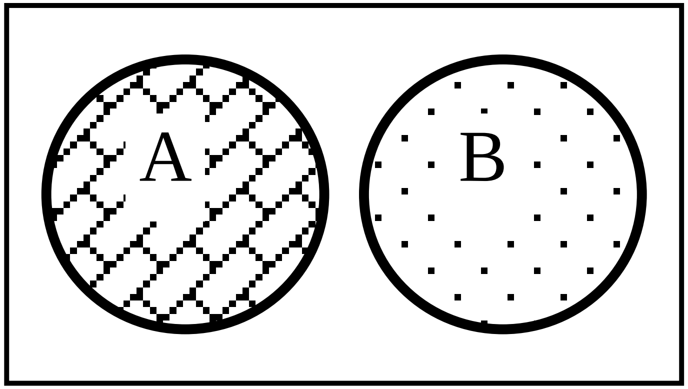
i) $A \cap B$

$= \varnothing$

ii) $A \cup B$

$= \{2, 4, 6, 8, 10\} = A$

## 5.1.3 Mutually Exclusive




Example:

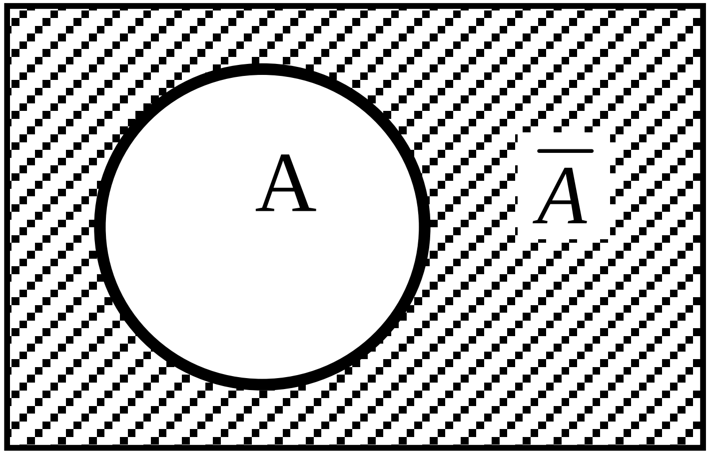
$A = \{1, 3, 5, 7, 9\}$

$B = \{2, 4, 6, 8, 10\}$

$A \cap B = \{\,\}$ / $\varnothing$

Two sets $A$ and $B$ are **disjoint** if $A \cap B = \varnothing$.

# 5.1.4 THE COMPLEMENT OF A (A' OR $\bar{A}$)




**Complement** of a subset $A$ of universal set is the set of all elements in U which are not in $A$

# Example 9

Let $U = \{a, b, c, d, e\}$, $A = \{a, c, e\}$, $B = \{b, d, e\}$, and $C = \{b, d\}$.

Find

i) $A'$

$= \{b, d\} = C$

ii) $B'$

$= \{a, c\}$

iii) $C'$

$= \{a, c, e\} = A$

iv) $B \cap C'$

$= \{e\}$

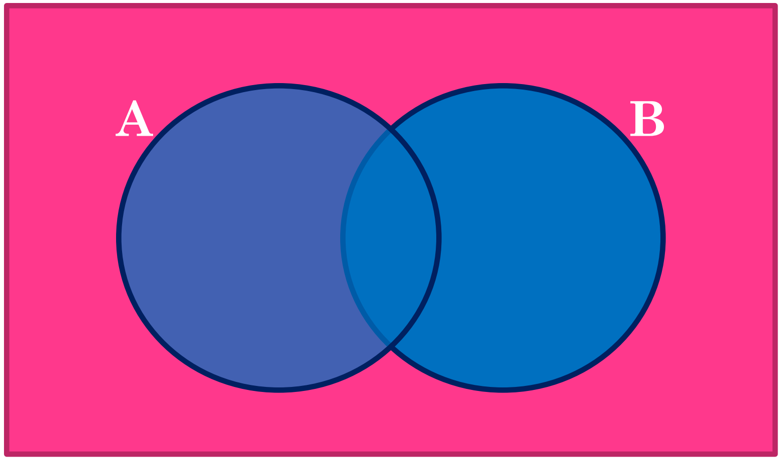
v) $A \cap C'$

$= \{a, c, e\} = A$

vi) $C \cap B'$

$= \varnothing$

# 5.1.5 Venn Diagrams




A **Venn diagram** which the universal set is usually represented by a rectangle, and the subsets such as sets A and B are usually circles inside the rectangle.

# TRY THIS

A total of 150 MMU students registered for foreign language classes: Japanese, French and Spanish. It is found that

96 students learn Japanese
20 students learn Japanese and French
17 students learn French and Spanish
12 students learn Japanese and Spanish
12 students learn all three languages
$x$ students learn only French
$y$ students learn only Spanish

The number of students who learn Japanese is three times bigger than the number of students who learn French.

a) Represent the above information in Venn diagram and calculate the value of x and y.

b) Find the number of student who learn exactly one of the three languages.

Solution: a) $x = 7$, $y = 42$ b) 125

# TRY THIS

100 students attended at least one of three projects (Business, Music and Art projects) during orientation week.

48 attended Business project
36 attended Music project
60 attended Art project
12 attended Business and Music projects
20 attended Music and Art projects
16 attended Business and Art projects
X attended all three projects

a. Find the value of X
b. How many students attended exactly one project?
c. How many students attended only Business training project?
d. How many students attended at least two projects?

Answer : (a) 4 (b) 60 (c) 24 (d) 40

# TRY THIS

Given

$n(A \cup B \cup C) = 85$

$n(A) = 50$

$n(B) = 40$

$n(C) = 35$

$n(A \cap B) = 18$

$n(B \cap C) = 12$

$n(A \cap B \cap C) = 5$

Find $n(A \cap C)$.

**Solution:**

$$n(A \cup B \cup C) = 85 = 5 + 7 + 13 + x + 15 + (23 - x) + (32 - x)$$
$$= 95 - x$$

Thus, $85 = 95 - x$ and, consequently, $x = n(A \cap B' \cap C) = 10$.

Finally, since $n(A \cap B \cap C) = 5$, we have $n(A \cap C) = 10 + 5 = 15$.