STUDENT ID NO

SEAT NO

VENUE:____________________

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2018/2019

PMT0301 – MATHEMATICS III

(All sections/ Groups)

OCT 2018
00.00 a.m. – 00.00 a.m.
(2 Hours)

Question	Marks
1	/10
2	/10
3	/10
4	/10
Total	/40

INSTRUCTIONS TO STUDENTS

1. This question paper consists of **EIGHT** printed pages excluding cover page and statistical table.
2. Answer **ALL** FOUR questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write all your answers in the **QUESTION BOOKLET**. All necessary working steps **MUST** be shown.

Question 1

(a) Find the symmetric equations of the line passing through the point $(1,0,6)$ and perpendicular to the plane $x+3y+z=5$. [1.5 marks]

Given a point $(1,0,6)$ and direction $\langle 1,3,1\rangle$

Symmetric Equations

$$x-1=\frac{y}{3}=z-6$$

(b) Find an equation of the plane that passes through the point (2,-2,0) and contain the line with parametric equations $x=1+4t$, $y=-4+3t$, $z=-t$. Give your final answer in the form of $ax+by+cz=d$. [3.5 marks]

Point on the plane: $P_0=(2,-2,0)$

Point on the line : $P_1=(1,-4,0)$

Vector on plane : $u=\overrightarrow{P_0P_1}=\langle -1,-2,0\rangle$

Vector from line: $v=\langle 4,3,-1\rangle$

Normal to plane = $uxv=\begin{vmatrix} i & j & k \\ -1 & -2 & 0 \\ 4 & 3 & -1 \end{vmatrix}=\langle 2,-1,5\rangle$

Equation of plane with point $P_0=(2,-2,0)$ and normal vector $\langle 2,-1,5\rangle$ is

$$\langle 2,-1,5\rangle\cdot\langle x-2,y-(-2),z-0\rangle=0$$

$$2(x-2)-(y+2)+5(z)=0$$

$$2x-4-y-2+5z=0$$

$$2x-y+5z=6$$

Continue…

(c) Given the following system of linear equations:

$$x-2y+z=1$$
$$y+2z=5$$
$$x+y+3z=8$$

Find the inverse matrix by using its adjoint, and hence solve the system of linear equations by using inverse method.

[5 marks]

$$A=\begin{bmatrix}1 & -2 & 1\\ 0 & 1 & 2\\ 1 & 1 & 3\end{bmatrix} \qquad B=\begin{bmatrix}1\\5\\8\end{bmatrix}$$

$$\text{Cofactor}=\begin{bmatrix} +\begin{vmatrix}1 & 2\\1 & 3\end{vmatrix} & -\begin{vmatrix}0 & 2\\1 & 3\end{vmatrix} & +\begin{vmatrix}0 & 1\\1 & 1\end{vmatrix}\\ -\begin{vmatrix}-2 & 1\\1 & 3\end{vmatrix} & +\begin{vmatrix}1 & 1\\1 & 3\end{vmatrix} & -\begin{vmatrix}1 & -2\\1 & 1\end{vmatrix}\\ +\begin{vmatrix}-2 & 1\\1 & 2\end{vmatrix} & -\begin{vmatrix}1 & 1\\0 & 2\end{vmatrix} & +\begin{vmatrix}1 & -2\\0 & 1\end{vmatrix}\end{bmatrix}=\begin{bmatrix}1 & 2 & -1\\ 7 & 2 & -3\\ -5 & -2 & 1\end{bmatrix}$$

1st column

$$|A|=1(1)-(-2)(-2)+1(-1)=-4$$

$$AdjA=\begin{bmatrix}1 & 2 & -1\\ 7 & 2 & -3\\ -5 & -2 & 1\end{bmatrix}^T \implies A^{-1}=\frac{1}{|A|}AdjA=\frac{1}{-4}\begin{bmatrix}1 & 7 & -5\\ 2 & 2 & -2\\ -1 & -3 & 1\end{bmatrix}$$

$$x=\begin{bmatrix}x\\y\\x\end{bmatrix}=A^{-1}B \qquad =\frac{1}{-4}\begin{bmatrix}1 & 7 & -5\\ 2 & 2 & -2\\ -1 & -3 & 1\end{bmatrix}\begin{bmatrix}1\\5\\8\end{bmatrix}$$

$$=\frac{1}{-4}\begin{bmatrix}1+35-40\\ 2+10-16\\ -1-15+8\end{bmatrix}$$

$$=\frac{1}{-4}\begin{bmatrix}-4\\-4\\-8\end{bmatrix}=\begin{bmatrix}1\\1\\2\end{bmatrix}$$

Continue…

Question 2

(a) An arithmetic sequence has first term $a_1 = 5$ and fourth term $a_4 = 11$. How many terms of this sequence must be added to get 2700. [3 marks]

$a_1 = 5$

$a_4 = 11 \Rightarrow a_1 + 3d = 11 \Longrightarrow d = 2$

$$S_n = 2700$$

$$\frac{n}{2}\left[2a + (n-1)d\right] = 2700$$

$$\frac{n}{2}\left[2(5) + (n-1)2\right] = 2700$$

$$n(2n+8) = 5400$$

$$n^2 + 4n - 2700 = 0$$

$$(n-50)(n+54) = 0$$

$$n = 50,\ n = -54$$

But n>0 $\quad\therefore n = 50$

(b) Find the coefficient of the term that contains y^3 in the expansion of $\left(\sqrt{2} - 3y\right)^7$.

[2 marks]

$$\binom{7}{3}\left(\sqrt{2}\right)^4(-3y)^3$$

$$= 35(4)\left(-27y^3\right) = -3780y^3$$

The coefficient is -3780

Continue…

(c) (i) The following data shows the weight (in kilogram) of 12 randomly selected bags of rice which are supposed to have a weight of 5 kilograms each.

5.01	4.98	5.02	5.00	4.99	5.00
4.97	5.02	5.00	5.01	5.01	5.00

Calculate the mean, median and mode. [2.5 marks]

Mean, $\bar{x} = \frac{\sum x}{n} = \frac{60.01}{12} = 5$

Ranked data 4.97, 4.98, 4.99, 5.00, 5.00, 5.00, 5.00, 5.01, 5.01, 5.01, 5.02, 5.02

$\therefore \text{Median} = \frac{5+5}{2} = 5$

Mode, $\hat{x} = 5$

(ii) Given below is the number of sales made last month for 10 sales staffs of a company:

5 3 $10+a$ $10-a$ 4 11 7 9 4 2

It is known that the sample variance is 11.8333. Find the possible value(s) of a **in nearest integer**. [2.5 marks]

$\sum x = 65$

$\sum x^2 = 521 + 2a^2$

Sample Variance, $s^2 = 11.8333$

$$\frac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1} = 11.8333$$

$$\frac{521 + 2a^2 - \frac{(65)^2}{10}}{9} = 11.8333$$

$$\frac{98.5 + 2a^2}{9} = 11.8333$$

$$98.5 + 2a^2 = 106.4997$$

$$2a^2 = 7.9997$$

$$a^2 = 3.99985$$

$$a = \pm\sqrt{3.99985}$$

$$a = \pm 2$$

Continue…

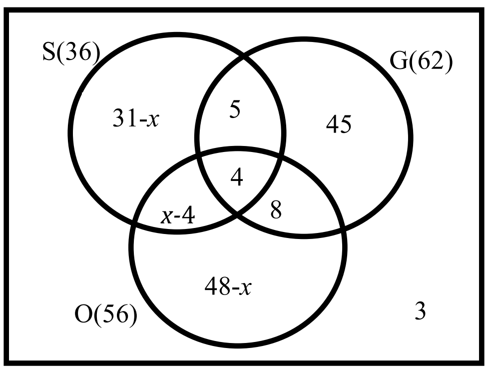
Question 3

(a) A group of 130 MMU students were surveyed, and it was found that each of the students surveyed liked at least one of the following three fruits: strawberries, grapes, and oranges.

3 students do not like fruits.
36 students like strawberries.
62 students like grapes.
56 students like oranges.
9 students like strawberries and grapes.
12 students like grapes and oranges.
x students like strawberries and oranges.
4 students like all three fruits.

(i) Draw a Venn diagram to visualize the above information. [3 marks]

Let S – Strawberries, G – Grapes, O – Oranges



(ii) Find the value of x from the Venn diagram obtained in part (i). [1 mark]

$$(31-x)+5+4+8+45+(x-4)+(48-x)+3=130$$
$$140-x=130$$
$$x=10$$

(iii) What is the probability of students who like grapes and oranges but not strawberries? [1 mark]

Let A be the event of students who like grapes and oranges but not strawberries.

$$P(A)=\frac{8}{130}=\frac{4}{65}$$

Continue…

(b) **[GIVE YOUR ANSWERS IN THE SIMPLEST FRACTION FORM.]**

Given event A and event B are two non-mutually exclusive events.

If $P(A)=\frac{1}{3}$, $P(A\cup B)=\frac{8}{15}$ and $P(A\cap B)=\frac{1}{5}$, find

(i) $P(B)$. [1.5 marks]

(ii) $P(A\mid B)$. [1.5 marks]

(iii) Are the events A and B independent? Justify your answer. [2 marks]

(i) $P(A\cap B)=P(A)+P(B)-P(A\cup B)$

$\therefore P(B)=P(A\cap B)+P(A\cup B)-P(A)$

$=\frac{1}{5}+\frac{8}{15}-\frac{1}{3}=\frac{2}{5}$

(ii)

$$P(A|B)=\frac{P(A\cap B)}{P(B)}$$

$$=\frac{1/5}{2/5}$$

$$=\frac{1}{2}$$

(iii) $P(A\cap B)=\frac{1}{5}$

$P(A)\times P(B)=\frac{1}{3}\times\frac{2}{5}=\frac{2}{15}$

Since $P(A\cap B)\neq P(A)P(B)$, A and B are not independent.

Continue…

Question 4

(a) Ahmad sells brownies to increase his personal income and the probability of making a sale is 0.04. Given that he randomly approaches 30 people, find the probability that he will make at least 2 sales. [3 marks]

(i) n=30 p=0.04 q=0.96

$$P(X \geq 2) = 1 - \left[P(X=0) + P(X=1)\right]$$
$$= 1 - \left[{}^{30}C_0 (0.04)^0 (0.96)^{30} + {}^{30}C_1 (0.04)^1 (0.96)^{29} \right]$$
$$= 1 - [0.2939 + 0.3673]$$
$$= 1 - 0.6612$$
$$= 0.3388$$

(b) The number of traffic accidents that occur on a particular stretch of road during a month (30 days) follows a Poisson distribution with a mean of 4. Find the probability that

(i) less than two accidents will occur next month on this stretch of road. [2 marks]

(ii) at least one accident will occur in the next 15 days on this stretch of road. [2 marks]

(i) $X \sim POI(4)$

$$P(X < 2) = P(X=0) + P(X=1)$$
$$= \frac{\left(4^0 e^{-4}\right)}{0!} + \frac{\left(4^1 e^{-4}\right)}{1!}$$
$$= 0.0183 + 0.0733$$
$$= 0.0916$$

(ii) $Y \sim POI(2)$

$$P(\mathrm{Y} \geq 1) = 1 - P(Y=0)$$
$$= 1 - \frac{\left(2^0 e^{-2}\right)}{0!}$$
$$= 1 - 0.1353$$
$$= 0.8647$$

Continue…

(c) A personnel test is designed to test a job applicant's cognitive and physical abilities. It is known that for all the tests administrated last year, the distribution of scores was approximately normal with a mean of 75 and a standard deviation of 7.5. If Marina is planning to take this test soon, what should be her score in the test so that only 10% of all applicants' scores will be higher than hers? Round up your answer to an integer.

[3 marks]

Let the score for Marina as h

$P\left(X < h\right) = 0.9$

$\left(Z < \dfrac{h-75}{7.5}\right) = 0.9$

$\dfrac{h-75}{7.5} = 1.28$

$h = 84.6 \approx 85$

End of Page