STUDENT ID NO

SEAT NO

VENUE:____________________

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2017/2018

PMT0301 – MATHEMATICS III

(All sections/ Groups)

16 OCTOBER 2017
2.30 p.m. – 4.30 p.m.
(2 Hours)

Question	Marks
1	/10
2	/10
3	/10
4	/10
Total	/40

INSTRUCTIONS TO STUDENTS

1. This question paper consists of **TEN** printed pages excluding cover page, formulae list and statistical table.

2. Answer **ALL** FOUR questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the **QUESTION BOOKLET**. All necessary working steps **MUST** be shown.

Question 1

a) Find the parametric equations of the line passing through the point $(2,-1,3)$ and parallel to the line $\frac{1}{5}(x+4)=y-3=-\frac{1}{2}z$. (2 marks)

A point on the line, $(x_0, y_0, z_0)=(2,-1,3)$
A parallel vector with the line, $\langle a,b,c\rangle=\langle 5,1,-2\rangle$

The line in parametric equations
$x=x_0+at \Leftrightarrow x=2+5t$
$y=y_0+bt \Leftrightarrow y=-1+t$
$z=z_0+ct \Leftrightarrow z=3-2t$

b) Find an equation of the plane which passes through $(1,-1,2)$ and parallel to $2x-5y+z=3$. (2.5 marks)

A point on the plane $=(1,-1,2)$
A vector that perpendicular with the plane, $=\langle 2,-5,1\rangle$

The plane equation
$\mathbf{n}.(\mathbf{r}-\mathbf{r_0})=0$
$\langle 2,-5,1\rangle\cdot\langle x-1, \mathrm{y}-(-1), \mathrm{z}-2\rangle=0$
$2(x-1)-5(y+1)+(z-2)=0$
$2x-2-5y-5+z-2=0$
$2x-5y+z=9$

Continued…

c) Express $0.00\overline{25}$ as a fraction. Simplify your answer. (2.5 marks)

$$0.00\overline{25} = 0.0025 + 0.000025 + \ldots$$
$$a = 0.0025$$
$$r = \frac{0.000025}{0.0025} = 0.01$$
$$S_\infty = \frac{a}{1-r}$$
$$= \frac{0.0025}{1-0.01}$$
$$= \frac{0.0025}{0.99}$$
$$= \frac{25}{9900} = \frac{1}{396}$$

d) Find the coefficient that contains x^3 in the expansion of $\left(\frac{1}{x} + 2x^2\right)^9$. (3 marks)

$$\binom{9}{r}\left(\frac{1}{x}\right)^{9-r}\left(2x^2\right)^r$$
$$= \binom{9}{r}\left(2^r\right)\left(x^{-1}\right)^{9-r}\left(x^{2r}\right)$$
$$= \binom{9}{r}\left(2^r\right)\left(x^{-9+r+2r}\right)$$

Comparing the variable

$$x^{-9+r+2r} = x^3$$

Comparing the index

$$-9 + r + 2r = 3$$
$$3r = 12$$
$$r = 4$$

$$\binom{9}{4}\left(\frac{1}{x}\right)^5\left(2x^2\right)^4$$
$$= 126\left(\frac{1}{x^5}\right)\left(16x^8\right)$$
$$= 2016x^3$$

Continued…

Question 2

a) Given the following system of linear equations:

$$x-2y+z \;=-4$$
$$-\mathrm{y}+3z \;=-7$$
$$x+2y=2$$

Find the inverse matrix by using its adjoint, and hence solve the system of linear equations by using inverse method. (5 marks)

$$A=\begin{bmatrix}1 & -2 & 1\\ 0 & -1 & 3\\ 1 & 2 & 0\end{bmatrix} \qquad X=\begin{bmatrix}x\\ y\\ z\end{bmatrix} \qquad B=\begin{bmatrix}-4\\ -7\\ 2\end{bmatrix}$$

$$\text{Cofactor}=\begin{bmatrix}+\begin{vmatrix}-1 & 3\\ 2 & 0\end{vmatrix} & -\begin{vmatrix}0 & 3\\ 1 & 0\end{vmatrix} & +\begin{vmatrix}0 & -1\\ 1 & 2\end{vmatrix}\\ -\begin{vmatrix}-2 & 1\\ 2 & 0\end{vmatrix} & +\begin{vmatrix}1 & 1\\ 1 & 0\end{vmatrix} & -\begin{vmatrix}1 & -2\\ 1 & 2\end{vmatrix}\\ +\begin{vmatrix}-2 & 1\\ -1 & 3\end{vmatrix} & -\begin{vmatrix}1 & 1\\ 0 & 3\end{vmatrix} & +\begin{vmatrix}1 & -2\\ 0 & -1\end{vmatrix}\end{bmatrix}=\begin{bmatrix}-6 & 3 & 1\\ 2 & -1 & -4\\ -5 & -3 & -1\end{bmatrix}$$

1st column

$$|A|=(1)(-6)+(0)(2)+(1)(-5)=-11$$

$$A^{-1}=\frac{1}{|A|}AdjA=-\frac{1}{11}\begin{bmatrix}-6 & 2 & -5\\ 3 & -1 & -3\\ 1 & -4 & -1\end{bmatrix}$$

$$X=\begin{bmatrix}x\\ y\\ x\end{bmatrix}=A^{-1}B$$

$$=-\frac{1}{11}\begin{bmatrix}-6 & 2 & -5\\ 3 & -1 & -3\\ 1 & -4 & -1\end{bmatrix}\begin{bmatrix}-4\\ -7\\ 2\end{bmatrix}$$

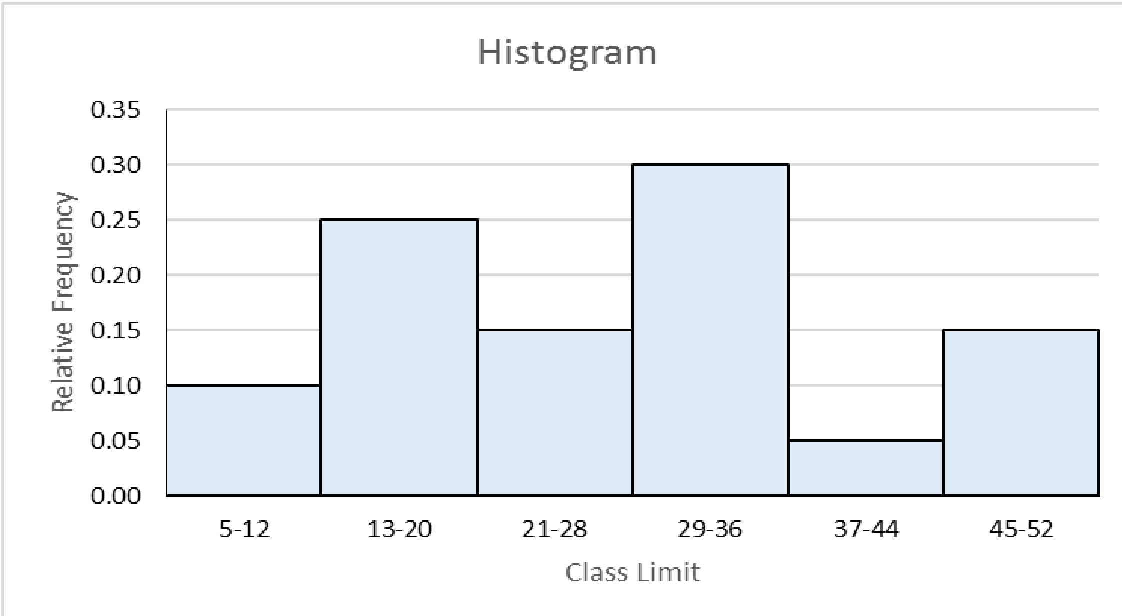
$$=-\frac{1}{11}\begin{bmatrix}24-14-10\\ -12+7-6\\ -4+28-2\end{bmatrix}$$

$$=-\frac{1}{11}\begin{bmatrix}0\\ -11\\ 22\end{bmatrix}=\begin{bmatrix}0\\ 1\\ -2\end{bmatrix}$$

$\therefore\; x=0,\quad y=1,\quad z=-2$

Continued…

b) Below is the histogram for the time spent (in minutes) by a random sample of 40 students in MMU library.



i) Based on the histogram, construct a frequency distribution table. (1 marks)

Class Limit	Frequency
5 - 12	$0.10 \times 40 = 4$
13 - 20	$0.25 \times 40 = 10$
21 - 28	$0.15 \times 40 = 6$
29 - 36	$0.30 \times 40 = 12$
37 - 44	$0.05 \times 40 = 2$
45 - 52	$0.15 \times 40 = 6$

ii) Calculate the mode. Give your answer correct to 1 decimal place. (1.5 marks)

Mode Class is 29- 36

$$\text{Mode} = L + \left(\frac{f_m - f_B}{(f_m - f_A) + (f_m - f_B)} \right) c$$

$$= 28.5 + \left(\frac{12 - 6}{(12 - 2) + (12 - 6)} \right) 8$$

$$= 28.5 + 3$$

$$= 31.5$$

Continued…

iii) Calculate the standard deviation. Give your answer correct to 2 decimal places.
(2.5 marks)

Midpoint, m	**Frequency, f**	mf	$m^2 f$
8.5	4	34	289
16.5	10	165	2722.5
24.5	6	147	3601.5
32.5	12	390	12675
40.5	2	81	3280.5
48.5	6	291	14113.5
		$\sum mf = 1108$	$\sum m^2 f = 36682$

Standard deviation

$$= \sqrt{\frac{\sum m^2 f - \frac{\left(\sum mf\right)^2}{\sum f}}{\sum f - 1}}$$

$$= \sqrt{\frac{36682 - \frac{1108^2}{40}}{39}}$$

$$= \sqrt{153.6}$$

$$= 12.39$$

Continued…

Question 3

a) An English lecturer wants to split a class of 10 students into three discussion groups. One group will have four students while the other two groups consist of three students each.

i) In how many ways can the lecturer form the groups? (1.5 marks)

$$\frac{10!}{4!3!3!2!}$$
$$= 2100$$

ii) The lecturer assigns one student as the group leader and another student as the secretary for each group. In how ways can the class be split? (1.5 marks)

$$2100 \times {}^4P_2 \times {}^3P_2 \times {}^3P_2$$
$$= 907200$$

b) Given 12 bottles of beverages which contain 4 coffees and 8 teas. If five bottles are selected at random, find the probability that at least three are coffees. (2 marks)

$$P(\text{at least three are coffees})$$
$$= P(\text{exactly three coffees}) + P(\text{exactly four coffees})$$
$$= \frac{{}^4C_3 \times {}^8C_2}{{}^{12}C_5} \times \frac{{}^4C_4 \times {}^8C_1}{{}^{12}C_5}$$
$$= \frac{112}{792} + \frac{8}{792}$$
$$= \frac{120}{792} = \frac{5}{33} / 0.1515$$

Continued…

c) Table below shows a study between smoking and dementia among 1000 senior citizens:

	No Dementia	Dementia
Smoker	95	375
Non-smoker	480	50

i) Suppose a person is selected at random from the study, find the probability that the person is a non-smoker or he/she has dementia. (1.5 marks)

Assume A=Non-smoker and B=Dementia

$$P(A \cup B) = \frac{n(A \cup B)}{n(S)}$$
$$= \frac{480 + 50 + 375}{1000}$$
$$= \frac{905}{1000} = \frac{181}{200} / 0.905$$

ii) Determine whether the events "Dementia" and "Smoking" are independent. (3.5 marks)

Assume A=Dementia and B=Smoking

By using, $P(A|B) = P(A)$

RHS:

$$P(A|B) = \frac{n(A \cap B)}{n(B)}$$
$$= \frac{375}{95 + 375}$$
$$= \frac{375}{470} = \frac{75}{94} /0.7979$$

LHS:

$$P(A) = \frac{n(A)}{n(S)}$$
$$= \frac{375 + 50}{1000}$$
$$= \frac{425}{1000} = \frac{17}{40} / 0.425$$

Therefore, $P(A|B) \neq P(A)$. As a conclusion, events 'A' and 'B' are not independent.

Continued…

or

$P(B\|A)=P(B)$ LHS : $P(B\|A)=\dfrac{n(B\cap A)}{n(A)}$ $=\dfrac{375}{425}=\dfrac{15}{17}$ /0.8824 RHS : $P(B)=\dfrac{470}{1000}=\dfrac{47}{100}$ / 0.47 $\therefore P(B\|A)\neq P(B)$	$P(A)\cdot P(B)=P(A\cap B)$ LHS : $P(A)\cdot P(B)$ $=\dfrac{17}{40}\times\dfrac{47}{100}=\dfrac{799}{4000}$ / 0.1998 RHS : $P(A\cap B)=\dfrac{375}{1000}=\dfrac{3}{8}$ / 0.375 $\therefore P(A)\cdot P(B)\neq P(A\cap B)$

Continued…

Question 4

a) You are offered to play a game in either one of the methods below:

Method A: 6 dice were thrown simultaneously. You will win if you get at least one dice with the number on the uppermost face is '6'.
Method B: 18 dice were thrown simultaneously. You will win if you get at least three dice with the number on the uppermost face is '6'.

Assume all the dice are a fair dice. Which method would you choose to play? Explain your answer. (5 marks)

Let X be the number of dice with the number on the uppermost face is '6'

Method A:

$n=6, p=\frac{1}{6}, q=\frac{5}{6}$

$P(win)=P(x\geq 1)$
$=1-P(x=0)$
$=1-\binom{6}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^6$
$=1-0.3349$
$=0.6651$

Method B:

$n=18, p=\frac{1}{6}, q=\frac{5}{6}$

$P(win)=P(x\geq 3)$
$=1-\left[P(x=0)+P(x=1)+P(x=2)\right]$
$=1-\left[\binom{18}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{18}+\binom{18}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^{17}+\binom{18}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{16}\right]$
$=1-(0.0376+0.1352+0.2299)$
$=1-0.4027$
$=0.5973$

Method A is favored due to the probability to win is higher.

Continued…

b) The average number of landed properties sold by a real estate agent is 2 properties per week. Assume that the sales follow a Poisson distribution, find the probability that an agent will sell exactly three properties on two consecutive weeks. (2 marks)

Average landed properties in two weeks= $\lambda = 2 \times 2 = 4$ properties per 2 weeks

$P(x = 3)$

$= \dfrac{4^3 e^{-4}}{3!}$

$= 0.1954$

c) Suppose the number of games in which badminton players play is normally distributed with the mean of 150 games and variance of 900 games. How many percent of the players will play in more than 200 games? (3 marks)

$P(X > 200)$

$= P\left(Z > \dfrac{200 - 150}{\sqrt{900}}\right)$

$= P(Z > 1.67)$

$= 1 - P(Z < 1.67)$

$= 1 - 0.9525$

$= 0.0475$

Percent of players= 0.0475 x 100% = 4.75%

End of Page