

**TMA1201 Tutorial 02 -
T1.2 Predicate logic**

1. Given that $P(x, y)$ means " $x + 2y = xy$ ", where x and y are integers. Determine the truth value of the statement. Explain each of your answer.
 - a) $P(1, -1)$
 - b) $P(0, 0)$
 - c) $\exists y P(3, y)$
 - d) $\forall x \exists y P(x, y)$
 - e) $\exists x \forall y P(x, y)$
 - f) $\forall y \exists x P(x, y)$
 - g) $\exists y \forall x P(x, y)$
 - h) $\neg \forall x \exists y \neg P(x, y)$
 - i) $\forall x \exists y P(x, y) \rightarrow \exists x \forall y P(x, y)$

2. Express the negation of the following statements so that no negation symbols appear outside a quantifier or an expression involving logical connectives.
 - a) $\forall x \forall y [\neg F(x, y) \rightarrow (Q(x) \wedge \neg S(y))]$
 - b) $\forall x \neg \exists y [(Q(x) \wedge F(x, y)) \rightarrow S(y)]$
 - c) $\exists x \exists y [Q(x) \wedge S(y) \wedge F(x, y) \wedge P(x)]$

3. Given that the variable x represents students and y represents subject, and:

$U(y)$: y is a Delta level subject
 $M(y)$: y is a mathematics subject
 $F(x)$: x is a first-year student
 $B(x)$: x is a full-time student
 $T(x, y)$: student x is taking subject y

 - a) Translate each of the following into logic statements using the given predicates, suitable quantifiers, and logical connectives.
 - i) Eric is taking TMA1201.
 - ii) All students are first year student.
 - iii) Every first-year student is a full-time student.
 - iv) No mathematics subject is a Delta level subject.

 - b) Translate each of the following logic statements into English.
 - i) $\forall x \exists y T(x, y)$
 - ii) $\exists x \forall y T(x, y)$
 - iii) $\forall x \exists y [(B(x) \wedge F(x)) \rightarrow (M(y) \wedge T(x, y))]$

- 4) Establish these logical equivalences, where x does not occur as a free variable in A .

Assume that the domain is nonempty.

a) $\exists x(A \rightarrow P(x)) \equiv A \rightarrow \exists xP(x)$

b) $\exists x(P(x) \rightarrow A) \equiv \forall xP(x) \rightarrow A$