CMA6134 - Tutorial 1

1. Given $x_1 = 1.333$, $x_1^* = 1.334$ and $x_2 = 0.001$, $x_2^* = 0.002$, which approximated value is better? State your reason.

2. Write the following in a way that avoids loss of significance
   (a) $\ln x - 1$
   (b) $\log x - \log \frac{1}{x}$
   (c) $f(x) = \frac{\sin x}{x - \sqrt{x^2 - 1}}$
   (d) $f(x) = \frac{x^2}{\sqrt{1 + x^2} - 1}$

3. Consider a function, $f(x) = \frac{\sqrt{x+4} - 2}{x}$.
   (a) Find $f(0.01)$ by using three-digit arithmetic and rounding.
   (b) Convert the $f(x)$ to the function that will avoid loss of significance, $g(x)$.
   (c) Find $g(0.01)$ by using three-digit arithmetic and rounding.
   (d) Given the actual value is 0.249844. Find the relative errors for $f(x)$ and $g(x)$.

4. Let $f(x) = x - \sin x$, $g(x) = 1 - \cos x$ and $h(x) = \frac{\sqrt{x+9} - 3}{x}$.
   (a) Write each function $f(x), g(x)$ and $h(x)$ in a way that avoids loss of significance. [HINT: Use three nonzero terms of Taylor series expansion]
   (b) Using results in (a), find $r(x) = h(x) - \frac{f(x)}{g(x)}$.
   (c) Approximate the value for $r(0)$.

5. Consider a function, $f(x) = \frac{\sqrt{x^2+1} - 1}{x^2} - \frac{x^2 \sin x}{x - \tan x}$.
   (a) Write the above in a way that avoids loss of significance in the vicinity of $x = 0$.
   (b) Approximate the value for $f(0)$ by using three nonzero terms of Taylor series expansion.

6. Consider the following polynomial function with the given value *x*.
   $$f(x) = x^3 - 3x^2 + 3x - 1 \text{ and } x = 2.19$$
   (a) Approximate the value of $f(2.19)$ by using 3 digits and use rounding.
   (b) Turn the given function into nested form.
   (c) Approximate the value of $f(2.19)$ for (b) by using 3 digits and rounding.

Find the absolute errors for (a) and (c). The actual value is 1.685159.