

# PDS0101 Introduction to Digital Systems

## Tutorial 2 SAMPLE

## SOLUTIONS

### Tutorial outcomes

By the end of today's lab, you should be able to

- apply arithmetic operations to binary, hex and octal numbers
- convert between binary, hex and octal numbers
- expressed signed binary numbers in in signed- magnitude, 1's complement and 2's complement

### Theory based questions

#### **BINARY NUMBERS**

Perform the conversions from binary to decimal number or vice versa

- |                   |                  |
|-------------------|------------------|
| a) $1011_2$       | g) $24_{10}$     |
| b) $110101101_2$  | h) $15_{10}$     |
| c) $0.1101_2$     | i) $0.246_{10}$  |
| d) $0.00111_2$    | j) $0.0981_{10}$ |
| e) $101101.101_2$ | k) $56.625_{10}$ |
| f) $10111.1101_2$ | l) $110.75_{10}$ |

a)  $1011_2 = 8 + 2 + 1 = 11$

b)  $110101101_2 = 256 + 128 + 32 + 8 + 4 + 1 = 429$

c)  $0.1101_2 = 0.5 + 0.25 + 0.0625 = 0.8125$

d)  $0.00111_2 = 0.125 + 0.0625 + 0.03125 = 0.21875$

e)  $101101.101_2 = 32 + 8 + 4 + 1 + 0.5 + 0.125 = 45.625$

f)  $10111.1101_2 = 16 + 4 + 2 + 1 + 0.5 + 0.25 + 0.0625 = 23.8125$

g)  $24 = 16 + 8 = 2^4 + 2^3 = 11000_2$  (show alternative using repeated div too)

h)  $15 = 1111_2$

i)  $0.246 = 0.0011111$  (rounded to 7 binary points ... actual value

$0.001111101111100111011011001000101101000011100101011)$

j)  $0.0981 = 0.00011001$

k)  $56.625_{10} = 111000.101_2$

$0.625 \times 2 = 1.250$

$0.250 \times 2 = 0.500$

$0.500 \times 2 = 1.000$

l)  $110.75_{10} = 1101110.11_2$

What is the highest decimal number that can be represented by each of the following number of bits

- a) three → seven
- b) four → fifteen
- c) seven → one hundred twenty seven
- d) eight → two hundred fifty five

What is the minimum number of bits required to represent the following decimal numbers?

- a) 17 → 5
- b) 35 → 6
- c) 205 → 8
- d) 132 → 8

Perform the following arithmetic operations on binary numbers (unsigned)

- a)  $101 + 11 = 1000$
- b)  $1001 + 101 = 1110$
- c)  $1100 - 1001 = 11$
- d)  $110 - 101 = 1$
- e)  $11 \times 11 = 1001$
- f)  $1001 \times 110 = 110110$
- g)  $111 \times 101 = 100011$
- h)  $1001 \div 11 = 11$
- i)  $1100 \div 100 = 11$

Express each decimal number below in binary as an 8-bit sign-magnitude number as well as 1's and 2's complement form for use in arithmetic operations

a) -80	11010000	10101111	10110000
b) +80	01010000	01010000	01010000
c) -123	11111011	10000100	10000101
d) -34	10100010	11011101	11011110
e) +101	01100101	01100101	01100101
f) -125	11111101	10000010	10000011
g) +60	00111100	00111100	00111100

*Note the assumption that the numbers will be used in binary arithmetic operations, thus positive numbers remain as such in 1s and 2s complement*

**NUMBERS WILL BE USED IN NEXT QUESTION TO SPEED UP WORK!!!**

Perform the following arithmetic functions using signed 8-bit 2's complement form of each decimal number

- a)  $101 - 34 = 01100101 + 11011110 = 01000011 (= +67)$
- b)  $60 - 125 = 00111100 + 10000011 = 10111111 (= -63)$
- c)  $-125 + 80 = 10000011 + 01010000 = 11010011$  (-ve value, therefore rev 2sc = -45)
- d)  $-34 - 123 = 11011110 + 10000101 = 01100011 (= -99)$  ← two -ve num result in +ve, overflow occurred
- e)  $60 + 80 = 00111100 + 01010000 = 10001100 (= 140)$  ← overflow again
- f)  $80 - 80 = 01010000 + 10110000 = 00000000$
- g)  $-80 - 34 = 10110000 + 11011110 = 10001110 (= -114)$

Perform the arithmetic functions of the unsigned binary numbers below in 2's complement form (Note: with unsigned binary numbers, MSB represents a number value)

- a)  $10001100 + 00111001 = 11000101$
- b)  $11011001 + 11100111 = 11000000$  (overflow ignored)

- c)  $00110011-00010000 = 00110011 + 11110000 = 00100011$   
d)  $01100101-11101000 = 01100101 + 00011000 = 01111101$

## OCTAL NUMBERS

Convert the octal numbers below to decimal and decimal to octal

- a)  $12_8 = 8^1 \times 1 + 8^0 \times 2 = 8 + 2 = 10$       f)  $85 = 125_8$   
b)  $73_8 = 59$       g)  $103 = 147_8$   
c)  $56_8 = 46$       h)  $1024 = 2000_8$   
d)  $163_8 = 115$       i)  $98 = 142_8$   
e)  $1024_8 = 532$       j)  $999 = 1747_8$

Division method example for (f) and (h)

8	85	remainder
8	10	5
8	1	2
	0	1

8	1024	remainder
8	128	0
8	16	0
8	2	0
	0	2

Convert the following decimal fractions to its octal fraction equivalents and vice-versa

- a)  $28.175 = \text{see below}$   
b)  $59.080 = 73.05075_8$   
c)  $88.888 = 130.70651_8$   
d)  $110.01_8 = 72.015625$   
e)  $407.304_8 = 263.3828125$   
f)  $345.135_8 = 229.181641625$

E.g. for (a)

Split conversion into before and after decimal point

$28 = 34_8$  (using weighted or division method)

perform repeated multiplication on fractional

0.175
$\times 8$
1.4
$\times 8$
3.2
$\times 8$
1.6
$\times 8$
4.8
$\times 8$
6.4
$\times 8$

etc.

Therefore  $28.175 = 34.1346_8$

Convert each octal below to binary and each binary into octal

- a)  $13_8 = 1011$   
b)  $13271_8 = 1011010111001$

- c)  $1100_2 = 14_8$   
d)  $111100010111_2 = 7427_8$

Perform the calculations for the following octal values as shown below

- a)  $555_8 + 574_8 = 1351_8$   
b)  $711_8 - 45_8 = 644_8$   
c)  $456_8 + 123_8 = 601_8$   
d)  $77714_8 + 76_8 = 100012_8$   
e)  $765_8 - 444_8 = 321_8$   
f)  $44_8 - 6_8 = 36_8$

## HEXADECIMAL NUMBERS

Perform the following conversions from hexadecimal number to binary, octal and decimal numbers

	Covert each hex number into a quad-binary digit group	regroup binary values into triplets and obtain octal value	weighted position to obtain decimal value
a) A034B <sub>16</sub>	1010 0000 0011 0100 1011 10 100 000 001 101 001 011	2401513 <sub>8</sub>	656203
b) 666FA <sub>16</sub>	0110 0110 0110 1111 1010	1463372 <sub>8</sub>	419578
c) 66 <sub>16</sub>	0110 0110	146 <sub>8</sub>	102
d) 191 <sub>16</sub>	0001 1001 0001	621 <sub>8</sub>	401

Perform the calculation for the following data as shown below:

- $15h + 32h = 47h$   
 $12h + EBh = FDh$   
 $AAA_{16} + 111_{16} = BBB_{16}$   
 $DDF_{16} + 11_{16} = DF0h$   
 $16Fh + 4A2h = 611h$   
 $9EFh + 9EFh = 13Deh$   
 $C_{16} - 2_{16} = Ah$   
 $BB_{16} - C1_{16} = -6h$   
 $1586h - 243h = 1343h$   
 $576A_{16} - AB_{16} = 56BFh$   
 $1234_{16} - 4321_{16} = -30EDh$   
 $FD19_{16} - AC_{16} = FC6Dh$

## Applied knowledge questions

Digital systems represent characters of the roman/english alphabet using 7-bits of a byte. Using the ASCII conversion table below, translate the numbers below the table into its corresponding decimal value and decode the words below. You may have to first determine what numerical base is being used before attempting to decode. Note that only characters are used and there are no symbols in the encoded words.

# Regular ASCII Chart (character codes 0 - 127)

000	(nul)	016 ▶ (dle)	032 sp	048 0	064 @	080 P	096 `	112 p
001 ☉ (soh)	017 ◀ (dc1)	033 !	049 1	065 A	081 Q	097 a	113 q	
002 ☉ (stx)	018 † (dc2)	034 "	050 2	066 B	082 R	098 b	114 r	
003 ▼ (etx)	019 !! (dc3)	035 #	051 3	067 C	083 S	099 c	115 s	
004 + (eot)	020 ¶ (dc4)	036 \$	052 4	068 D	084 T	100 d	116 t	
005 ♣ (enq)	021 \$ (nak)	037 %	053 5	069 E	085 U	101 e	117 u	
006 ♣ (ack)	022 − (syn)	038 &	054 6	070 F	086 V	102 f	118 v	
007 ▪ (bel)	023 † (etb)	039 '	055 7	071 G	087 W	103 g	119 w	
008 ▣ (bs)	024 † (can)	040 (	056 8	072 H	088 X	104 h	120 x	
009 (tab)	025 † (em)	041 )	057 9	073 I	089 Y	105 i	121 y	
010 (lf)	026 (eof)	042 *	058 :	074 J	090 Z	106 j	122 z	
011 ♂ (vt)	027 ← (esc)	043 +	059 ;	075 K	091 [	107 k	123 {	
012 * (np)	028 L (fs)	044 ,	060 <	076 L	092 \	108 l	124	
013 (cr)	029 ↔ (gs)	045 −	061 =	077 M	093 ]	109 m	125 }	
014 ♂ (so)	030 ▲ (rs)	046 .	062 >	078 N	094 ^	110 n	126 ~	
015 ♂ (si)	031 ▼ (us)	047 /	063 ?	079 O	095 _	111 o	127 ò	

- 010000100110000101101110011000010110111001100001  
binary encoding - banana
  - 6910810111210497110116  
decimal encoding - elephant
  - 104151147151164141154  
octal - digital
  - 0x4d0x750x6c0x740x690x6d0x650x640x690x61  
hex encoding - multimedia
-