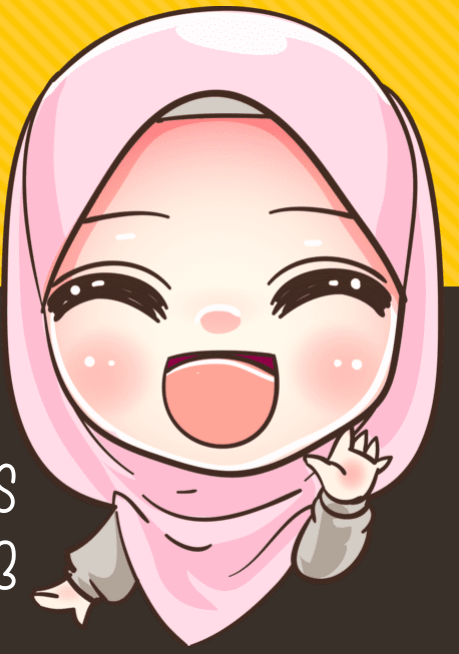
MMU
MULTIMEDIA UNIVERSITY

TUTORIAL 5
COMBINATIONAL LOGIC ANALYSIS 1



PDS0101: INTRODUCTION TO DIGITAL SYSTEMS
TRI 2, 2022-2023

Domain ABC

Boolean Expression (BE)

- standard BE
  - Sum of product (SOP)
    - standard SOP: $X = A\bar{B}\bar{C} + ABC$
    - non-standard SOP: $X = A\bar{B} + C + AB$
  - Product of sum (POS)
    - standard POS: $X = (A+B+C)(A+\bar{B}+\bar{C})$
    - non-standard POS: $X = (A+B)(\bar{A}+\bar{C})$
- non-standard BE: $X = (A+B)AB + A\bar{C}$, $X = (A+\bar{A}B)\bar{C}D$
  - not
    - not OR (NOR)
    - not and (NAND)

Domain

x y z

POS

(product of Sum)

product → and gate

Sum → sum term

Standard pos

$$A = (x + y + \bar{z})\ (x + \bar{y} + z)\ (\bar{x} + y + z)$$

non standard pos

$z + \boxed{0} = z$

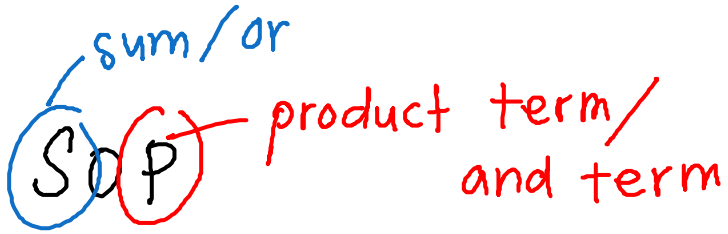
$z + z = z$

$$A = (x + y)(\bar{x} + \bar{z})(z)$$

$z + 0$

$z + z$

domain xyz



non standard SOP

$$A = x\bar{y} + yz + x\bar{z}$$

missing z; missing x; missing y

Standard SOP

$$A = \underbrace{xyz}_{1} + \underbrace{\bar{x}y\bar{z}}_{2} + \underbrace{x\bar{y}z}_{3}$$

product term / and term

THEORY BASED QUESTIONS

QUESTION 1

Identify which of the following expressions are in **proper SOP** and **POS** forms

a. $AB + CD\overline{E}$ **SOP** not and (NAND)

b. $AB + CD + \overline{BF}$ overbar extending multiple variables

c. $\overline{A}B + \overline{C}DE + CA$ **SOP**

d. $A(B + \overline{C}D)$

e. $\overline{(A + B)(C + D + E)}$ not AND; overbar extending multiple variables

f. $(\overline{A} + B)(A + \overline{B} + C)$ **POS**

g. $(W + \overline{X})(Y + \overline{Z})$ **POS**

h. $A(B + C)(E + \overline{D} + F)$ **POS**

i. $(H + I + J)(K + \overline{L})$ **POS**

Expression d is neither in POS or SOP form as shown but can be made into SOP

QUESTION 2

For all the expressions in (1) determine the **DOMAIN** of each expression

a. $AB + CD\overline{E}$

b. $AB + CD + \overline{BF}$

c. $\overline{A}B + \overline{C}DE + CA$

d. $A(B + CD)$

e. $\overline{(A + B)(C + D + E)}$

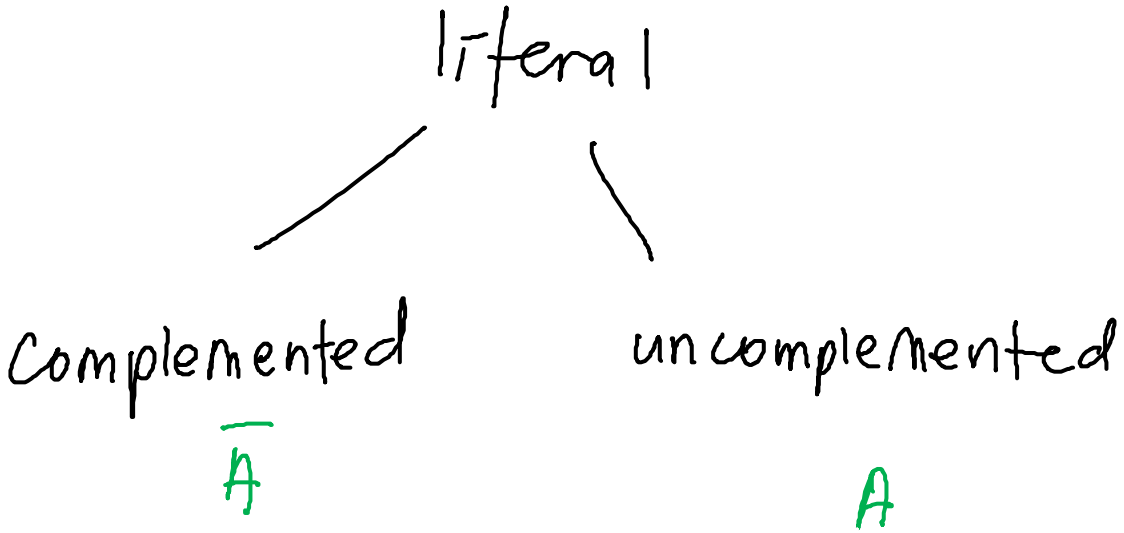
f. $(\overline{A} + B)(A + \overline{B} + C)$

g. $(W + \overline{X})(Y + \overline{Z})$

h. $A(B + C)(E + \overline{D} + F)$

i. $(H + I + J)(K + \overline{L})$

DOMAIN of a Boolean expression is the SET OF VARIABLES contained in either COMPLEMENTED or UNCOMPLEMENTED form


literal
Complemented
$\overline{A}$
uncomplemented
A

THEORY BASED QUESTIONS

QUESTION 2

For all the expressions in (1) determine the **DOMAIN** of each expression

a. $AB + CD\overline{E}$ **Domain : ABCDE**

b. $AB + CD + \overline{BF}$ **Domain : ABCDF**

c. $\overline{A}B + \overline{C}DE + CA$ **Domain : ABCDE**

d. $A(B + CD)$ **Domain : ABCD**

e. $\overline{(A + B)(C + D + E)}$ **Domain : ABCDE**

f. $(\overline{A} + B)(A + \overline{B} + C)$ **Domain : ABC**

g. $(W + \overline{X})(Y + \overline{Z})$ **Domain : WXYZ**

h. $A(B + C)(E + \overline{D} + F)$ **Domain : ABCDEF**

i. $(H + I + J)(K + \overline{L})$ **Domain : HIJKL**

DOMAIN of a Boolean expression is the SET OF VARIABLES contained in either COMPLEMENTED or UNCOMPLEMENTED form

QUESTION 3

Convert the following general expressions to SOP form

$b\bar{b} = 0$

$cc = c$

a. $(a + b)(c + \bar{b})$

$= ac + a\bar{b} + bc + b\bar{b}$ 0

$= ac + a\bar{b} + bc$ ✱

b. $(a + \bar{b}c)c$

$= ac + \bar{b}cc$

$= ac + \bar{b}c$ ✱

$= c(a + \bar{b})$

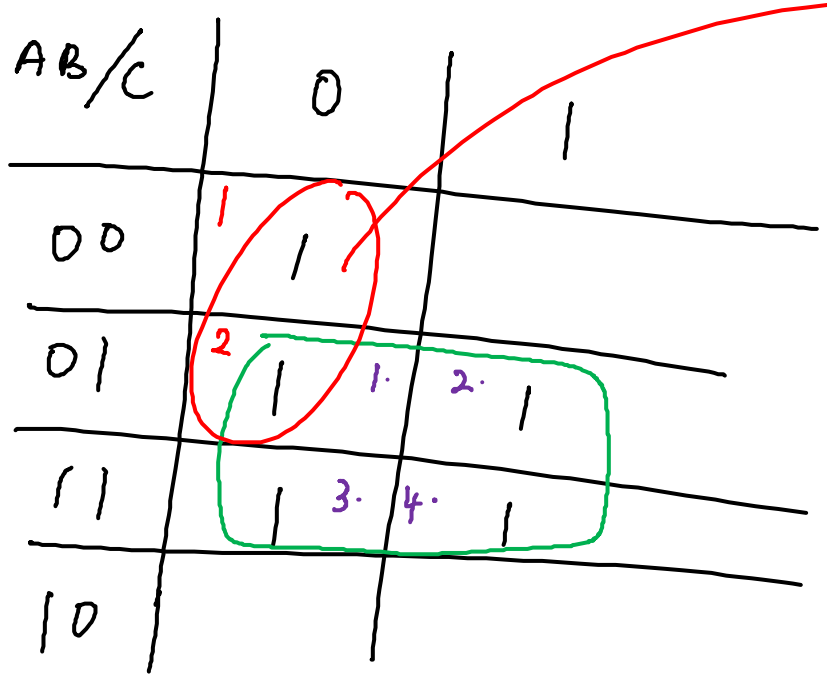
SOP

A = 1

$\bar{A}$ = 0

Output for SOP = 1

SOP K-MAP



A C

0 ⊗ 0

0 * 0

= $\bar{A}\bar{C}$

~~sum~~ and term

A B C

⊗ 1 ⊗

⊗ 1 *

= B

* 1 ⊗

* 1 *

and term

$X = \bar{A}\bar{C} + B$

4 SOP K-MAP

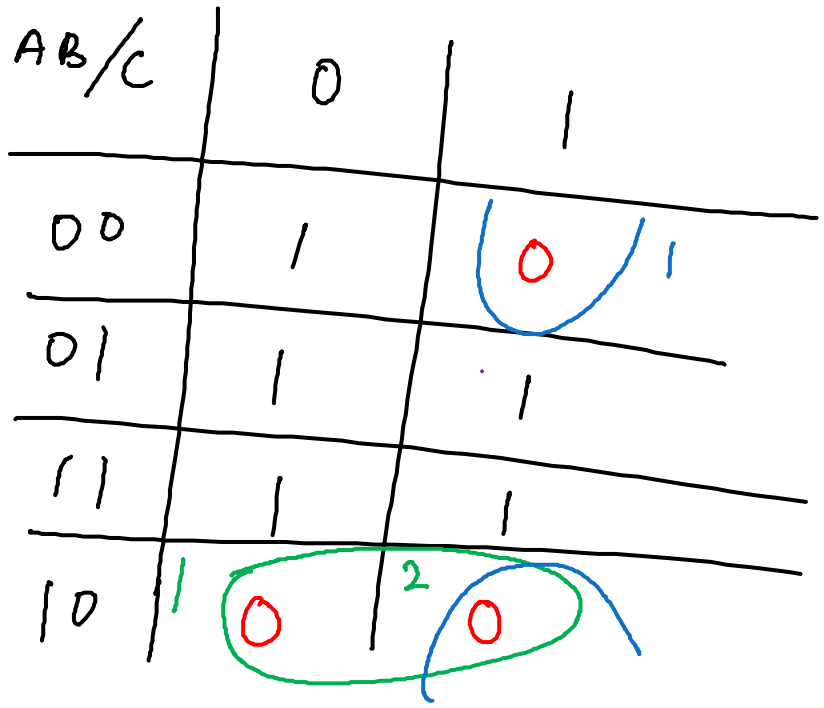
POS

A = 0

$\bar{A}$ = 1

Output for POS = 0

Standard POS



simplification

A B C

1 0 ⊗

1 0 *

$\bar{A}+B$

A B C

⊗ 0 1

* 0 1

$B+\bar{C}$

$X = (0+0+1)(1+0+0)(1+0+1)$

Simplification

$X = (\bar{A}+B)(B+\bar{C})$

$= (A+B+\bar{C})(\bar{A}+B+C)(\bar{A}+B+\bar{C})$

QUESTION 3

Convert the following general expressions to **SOP form**

Rule 10

$ac + acb = ac$

c. $(a + c)(ab + ac)$

$= aab + aac + abc + acc$

$= ab + ac + abc + ac$

$= ab + ac(1 + b)$

$= ab + ac$ ✱

d. $ab + c\,d(a\bar{b} + cd)$

$= ab + a\bar{b}cd + ccdd$

$= ab + cd(a\bar{b} + 1 + 1)$

$= ab + cd$ ✱

Convert the following general expressions to **SOP form**

e. $ab(\bar{b}\bar{c} + bd)$

$= ab\bar{b}\bar{c} + abbd$

$= abd$ ✱

f. $a + b[ac + (b + \bar{c})d]$

$= a + b(ac + bd + \bar{c}d)$

$= a + abc + bbd + b\bar{c}d$

$= a(1 + bc) + bd(1 + \bar{c})$

$= a + bd$ ✱

Convert the SOP expressions into **standard SOP form**

Domain : abc

2' = 2 b c 2' = 2 a 2' = 2

a. $ac + a\bar{b} + bc$

$= ac(b + \bar{b}) + a\bar{b}(c + \bar{c}) + bc(a + \bar{a})$

$= abc + a\bar{b}c + a\bar{b}c + a\bar{b}\bar{c} + abc + \bar{a}bc$

$= abc + a\bar{b}c + a\bar{b}\bar{c} + \bar{a}bc$

Domain ABC

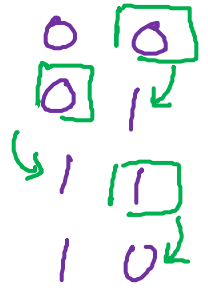
$2^3 = 8$ possible input combination

4X2

POS	SOP			
			0	1
$A + B$	$\bar{A}\bar{B}$	00	000	001
$A + \bar{B}$	$\bar{A}B$	01	010	011
$\bar{A} + \bar{B}$	AB	11	110	111
$\bar{A} + B$	$A\bar{B}$	10	100	101

K-MAP

Gray code (✓)



Binary code

0 0
0 1
1 0
1 1

Convert the SOP expressions into **standard SOP form**

b. $ac + \bar{b}c$

$= ac(b + \bar{b}) + \bar{b}c(a + \bar{a})$

$= abc + a\bar{b}c + a\bar{b}c + \bar{a}\bar{b}c$

$= abc + a\bar{b}c + \bar{a}\bar{b}c$

Convert the SOP expressions into **standard SOP form**

c. $ab + ac$

$= ab(c + \bar{c}) + ac(b + \bar{b})$

$= abc + ab\bar{c} + abc + a\bar{b}c$

$= abc + ab\bar{c} + a\bar{b}c$

Convert the SOP expressions into **standard SOP form**

domain abcd

$a + a = a$

$2^2 = 4$ (cd) $2^2 = 4$ (ab)

d. $ab + cd$

$= ab(c + \bar{c}) + cd(a + \bar{a})$

$= abc + ab\bar{c} + acd + \bar{a}cd$

$= abc(d + \bar{d}) + ab\bar{c}(d + \bar{d}) + acd(b + \bar{b}) + \bar{a}cd(b + \bar{b})$

$= abcd + abc\bar{d} + ab\bar{c}d + ab\bar{c}\bar{d} + abcd + a\bar{b}cd + \bar{a}bcd + \bar{a}\bar{b}cd$

$= abcd + abc\bar{d} + ab\bar{c}d + ab\bar{c}\bar{d} + a\bar{b}cd + \bar{a}bcd + \bar{a}\bar{b}cd$

Convert the SOP expressions into **standard SOP form**

d. $ab + cd$

$= abcd + abc\bar{d} + ab\bar{c}d + ab\bar{c}\bar{d}$
$+a\bar{b}cd + \bar{a}bcd + \bar{a}\bar{b}cd$

ab × × ← don't care

× × cd

	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$			1	
$\bar{a}b$			1	
ab	$ab\bar{c}\bar{d}$ 1	$ab\bar{c}d$ 1	$abcd$ 1	$abc\bar{d}$ 1
$a\bar{b}$			1	

THEORY BASED QUESTIONS

QUESTION 4

Convert the SOP expressions into **standard SOP form**

d. $ab + cd$

$= abcd + abc\bar{d} + ab\bar{c}d + ab\bar{c}\bar{d}$

$+a\bar{b}cd + \bar{a}bcd + \bar{a}\bar{b}cd$

INPUT				OUTPUT
A	B	C	D	Y
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	1
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	1
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	

Convert the SOP expressions into **standard SOP form**

Domain ABCD

c $2^1 = 2$

e. abd

$= abd(c + \bar{c})$

$= abcd + ab\bar{c}d$

Convert the SOP expressions into **standard SOP form**

f. $a + bd$ (3, 2)

Domain : ABCD

$= a(b + \bar{b}) + bd(a + \bar{a})$

$= ab + a\bar{b} + abd + \bar{a}bd$

$= ab(c + \bar{c}) + a\bar{b}(c + \bar{c}) + abd(c + \bar{c}) + \bar{a}bd\ (c + \bar{c})$

$= abc + ab\bar{c} + a\bar{b}c + a\bar{b}\bar{c} + abcd + ab\bar{c}d + \bar{a}bcd + \bar{a}b\bar{c}d$

$= abc(d + \bar{d}) + ab\bar{c}(d + \bar{d}) + a\bar{b}c(d + \bar{d}) + a\bar{b}\bar{c}(d + \bar{d}) + abcd + ab\bar{c}d + \bar{a}bcd + \bar{a}b\bar{c}d$

← 8 term →

$= abcd + abc\bar{d} + ab\bar{c}d + ab\bar{c}\bar{d} + a\bar{b}cd + a\bar{b}c\bar{d} + a\bar{b}\bar{c}d + a\bar{b}\bar{c}\bar{d} + abcd + ab\bar{c}d + \bar{a}bcd + \bar{a}b\bar{c}d$

← 4 →

$= abcd + abc\bar{d} + ab\bar{c}d + ab\bar{c}\bar{d} + a\bar{b}cd + a\bar{b}c\bar{d} + a\bar{b}\bar{c}d + a\bar{b}\bar{c}\bar{d} + \bar{a}bcd + \bar{a}b\bar{c}d$

Convert the SOP expressions into **standard SOP form**

f. $a + bd$

$= abcd + abc\bar{d} + ab\bar{c}d + ab\bar{c}\bar{d} + a\bar{b}cd + a\bar{b}c\bar{d} + a\bar{b}\bar{c}d + a\bar{b}\bar{c}\bar{d} + \bar{a}bcd$

$+ \bar{a}b\bar{c}d$

a x x x

$2^3 = 8$

x b x d

$2^2 = 4$

	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$				
$\bar{a}b$		1	1	
ab	1	1	1	1
$a\bar{b}$	1	1	1	1

THEORY BASED QUESTIONS

QUESTION 4

Convert the SOP expressions into **standard SOP form**

f. $a + bd$

$= abcd + abc\bar{d} + ab\bar{c}d + ab\bar{c}\bar{d} + a\bar{b}cd + a\bar{b}c\bar{d} + a\bar{b}\bar{c}d + a\bar{b}\bar{c}\bar{d} + \bar{a}bcd + \bar{a}b\bar{c}d$

INPUT				OUTPUT
A	B	C	D	Y
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

Convert the standard **SOP expressions** into standard **POS form**

	INPUT			OUTPUT
	A	B	C	Y
	0	0	0	**0**
	0	0	1	**0**
	0	1	0	**0**
$\bar{a}bc$	0	1	1	**1**
$a\bar{b}\bar{c}$	1	0	0	**1**
$a\bar{b}c$	1	0	1	**1**
	1	1	0	**0**
abc	1	1	1	**1**

a. $\overset{111}{abc} + \overset{101}{a\bar{b}c} + \overset{100}{a\bar{b}\bar{c}} + \overset{011}{\bar{a}bc}$

sum term

$0+0+0 = A+B+C$

$0+0+1 = A+B+\bar{C}$

$0+1+0 = A+\bar{B}+C$

$1+1+0 = \bar{A}+\bar{B}+C$

The equivalent **POS expression** is

$$(a+b+c)(a+b+\bar{c})(a+\bar{b}+c)(\bar{a}+\bar{b}+c)$$

Convert the standard **SOP expressions** into standard **POS form**

	INPUT			OUTPUT
	A	B	C	Y
	0	0	0	**0**
$\bar{a}\bar{b}c$	0	0	1	**1**
	0	1	0	**0**
	0	1	1	**0**
	1	0	0	**0**
$a\bar{b}c$	1	0	1	**1**
	1	1	0	**0**
abc	1	1	1	**1**

b. $abc + a\bar{b}c + \bar{a}\bar{b}c$

The equivalent **POS expression** is

$$(a+b+c)(a+\bar{b}+c)(a+\bar{b}+\bar{c})(\bar{a}+b+c)(\bar{a}+\bar{b}+c)$$

Convert the standard **SOP expressions** into standard **POS form**

	INPUT			OUTPUT
	A	B	C	Y
	0	0	0	**0**
	0	0	1	**0**
	0	1	0	**0**
	0	1	1	**0**
	1	0	0	**0**
$a\bar{b}c$	1	0	1	**1**
$ab\bar{c}$	1	1	0	**1**
abc	1	1	1	**1**

c. $abc + a\bar{b}c + ab\bar{c}$

The equivalent **POS expression** is

$$(a+b+c)(a+b+\bar{c})(a+\bar{b}+c)(a+\bar{b}+\bar{c})(\bar{a}+b+c)$$

THEORY BASED QUESTIONS

QUESTION 5

Convert the standard **SOP expressions** into standard **POS form**

d. $abcd + abc\overline{d} + ab\overline{c}d + ab\overline{c}\overline{d} + a\overline{b}cd + \overline{a}bcd + \overline{a}\overline{b}cd$

The equivalent **POS expression** is

$(a + b + c + d)(a + b + c + d')(a + b + c' + d)$
$(a + b' + c + d)(a + b' + c + d')(a + b' + c' + d)$
$(a' + b + c + d)(a' + b + c + d')(a' + b + c' + d)$

INPUT				OUTPUT
A	B	C	D	Y
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

THEORY BASED QUESTIONS

QUESTION 5

Convert the standard **SOP expressions** into standard **POS form**

e. $abcd + ab\bar{c}d$

$2^4 = 16$

2 SOP ; 14 POS

The equivalent **POS expression** is

$(a + b + c + d)(a + b + c + d')(a + b + c' + d)$
$(a + b + c' + d')(a + b' + c + d)(a + b' + c + d')$
$(a + b' + c' + d)(a + b' + c' + d')(a' + b + c + d)$
$(a' + b + c + d')(a' + b + c' + d)(a' + b' + c + d)$
$(a' + b' + c' + d) + (a' + b + c' + d')$

INPUT				OUTPUT
A	B	C	D	Y
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

THEORY BASED QUESTIONS

QUESTION 5

Convert the standard **SOP expressions** into standard **POS form**

f. $abcd + abc\bar{d} + ab\bar{c}d + ab\bar{c}\bar{d} + a\bar{b}cd + a\bar{b}c\bar{d} + a\bar{b}\bar{c}d + a\bar{b}\bar{c}\bar{d} + ab\bar{c}d + \bar{a}bcd + \bar{a}b\bar{c}d$

The equivalent **POS expression** is

$(a+b+c+d)(a+b+c+d')(a+b+c'+d)$
$(a+b+c'+d')(a+b'+c+d)(a+b'+c'+d)$

INPUT				OUTPUT
A	B	C	D	Y
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

QUESTION 6

Derive the truth table for the following **standard SOP expressions**

a. $A\bar{B}C + \bar{A}B\bar{C} + ABC$

$A\bar{B}C$ 101

$\bar{A}B\bar{C}$ 010

ABC 111

INPUT			OUTPUT
A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Derive the truth table for the following **standard SOP expressions**

b. $\bar{X}\bar{Y}\bar{Z} + \overline{XYZ} + XY\bar{Z} + X\bar{Y}Z + \bar{X}YZ$

INPUT			OUTPUT
A	B	C	Y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

QUESTION 6

Derive the truth table for the following **standard SOP expressions**

c. $\overline{A}B + AB\overline{C} + \overline{A}\overline{C} + A\overline{B}C$

INPUT			OUTPUT
A	B	C	Y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

QUESTION 6

Derive the truth table for the following **standard SOP expressions**

c. $\overline{A}B + AB\overline{C} + \overline{A}\overline{C} + A\overline{B}C$

INPUT			OUTPUT
A	B	C	Y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Derive the truth table for the following **standard SOP expressions**

$2^3 \quad 2^2 \quad 2^2 \quad 2^1 \qquad 8+4+4+2=18$

d. $\bar{X} + Y\bar{Z} + WZ + X\bar{Y}Z$

$= \bar{X}(W + \bar{W}) + Y\bar{Z}(W + \bar{W}) + WZ(X + \bar{X}) + X\bar{Y}Z\ (W + \bar{W})$

$= \bar{X}W + \bar{X}\bar{W} + WY\bar{Z} + \bar{W}Y\bar{Z} + WXZ + W\bar{X}Z + WX\bar{Y}Z + \bar{W}X\bar{Y}Z$

$= \bar{X}W(Y + \bar{Y}) + \bar{X}\bar{W}(Y + \bar{Y}) + WY\bar{Z}(X + \bar{X}) + \bar{W}Y\bar{Z}(X + \bar{X}) + WXZ(Y + \bar{Y}) + W\bar{X}Z(Y + \bar{Y}) + WX\bar{Y}Z + \bar{W}X\bar{Y}Z$

$= W\bar{X}Y + W\bar{X}\bar{Y} + \bar{W}\bar{X}Y + \bar{W}\bar{X}\bar{Y} + WXY\bar{Z} + W\bar{X}Y\bar{Z} + \bar{W}XY\bar{Z} + \bar{W}\bar{X}Y\bar{Z} + WXYZ + WX\bar{Y}Z + W\bar{X}YZ + W\bar{X}\bar{Y}Z + WX\bar{Y}Z + \bar{W}X\bar{Y}Z$

$= W\bar{X}Y + W\bar{X}\bar{Y} + \bar{W}\bar{X}Y + \bar{W}\bar{X}\bar{Y} + WXY\bar{Z} + W\bar{X}Y\bar{Z} + \bar{W}XY\bar{Z} + \bar{W}\bar{X}Y\bar{Z} + WXYZ + WX\bar{Y}Z + W\bar{X}YZ + W\bar{X}\bar{Y}Z + WX\bar{Y}Z + \bar{W}X\bar{Y}Z$

Continue .....

THEORY BASED QUESTIONS

QUESTION 6

Derive the truth table for the following **standard SOP expressions**

$2^3 = 8$

d. $\overline{X} + Y\overline{Z} + WZ + X\overline{Y}Z$

? $\overline{X}$? ?

? 0 ? ?

1 0

? ? $y\overline{z}$

1 1

W ? ? Z

LSB

$2^2 = 4$

$2^3 = 8$

1 0 1

? x $\overline{y}$ z

2

INPUT				OUTPUT
W	X	Y	Z	Y
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

MMU
MULTIMEDIA UNIVERSITY

TUTORIAL 5
COMBINATIONAL LOGIC ANALYSIS 1
(PART II)

PDS0101: INTRODUCTION TO DIGITAL SYSTEMS
TRI 2, 2022-2023

QUESTION 7

Derive the truth table for the following **standard POS expressions**

Output = 0

a. $(\overline{A} + \overline{B} + \overline{C})(A + B + C)(A + \overline{B} + C)$

INPUT			OUTPUT
A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

$\bar{A} + \bar{B} + \bar{C} = 1 + 1 + 1$

$A + B + C = 0 + 0 + 0$

$A + \bar{B} + C = 0 + 1 + 0$

Derive the truth table for the following standard POS expressions

Output = 0

non-standard POS

b. $(A + B)(A + C)(A + B + C)$

$A + B + C = 0 + 0 + 0$

$A + (X) + C = 0 + X + 0$

$2^1 = 2$ sum term

$A + B + C = 0 + 0 + X$

$2^1 = 2$

	INPUT			OUTPUT
	A	B	C	Y
=1D	0	0	0	0
=1D	0	0	1	0
	0	1	0	0
	0	1	1	1
	1	0	0	1
	1	0	1	1
	1	1	0	1
	1	1	1	1

Derive the truth table for the following **standard POS expressions**

b. $(A + B)(A + C)(A + B + C)$

INPUT			OUTPUT
A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Standard SOP (output = 1)

$X = \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC$

AOI for POS $\bar{A}\bar{B} + \bar{A}\bar{C} + \bar{A}\bar{B}\bar{C}$

NOT AND / NAND = negative OR

$(A+B)(A+C)(A+B+C)$

$= \overline{(\overline{A+B})} + \overline{(\overline{A+C})} + \overline{(\overline{A+B+C})}$

NOT OR / NOR = negative AND

$= \bar{A}\bar{B} + \bar{A}\bar{C} + \bar{A}\bar{B}\bar{C}$ ※

non standard POS ⟶ standard POS

Rule 12

domain ABC

$A + BC = (A + B)(A + C)$

$A + B + C\bar{C} = (A + B + C)(A + B + \bar{C})$

$A + C + B\bar{B} = (A + C + B)(A + C + \bar{B})$

$= (A + B + C)(A + \bar{B} + C)$

$A(0) = 0$

$A(1) = A$

$A(A) = A$

$A(\bar{A}) = 0$

domain ABC

non standard → standard

$$(A + \bar{B}) = A + \bar{B} + C\bar{C}$$ rule 12

$$= (A + \bar{B} + C)(A + \bar{B} + \bar{C})$$

Derive the truth table for the following **standard POS expressions**

b. $(A + B)(A + C)(A + B + C)$

INPUT			OUTPUT
A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

THEORY BASED QUESTIONS

QUESTION 7

Derive the truth table for the following **standard POS expressions**

c. $(A+\overline{B}+C+\overline{D})(\overline{A}+B+\overline{C}+D)(\overline{A}+B+C+\overline{D})$

$(A+\overline{B}+\overline{C}+D)$

$A+\overline{B}+C+\overline{D} = 0+1+0+1$

$\overline{A}+B+\overline{C}+D = 1+0+1+0$

$\overline{A}+B+C+\overline{D} = 1+0+0+1$

$A+\overline{B}+\overline{C}+D = 0+1+1+0$

INPUT				OUTPUT
A	B	C	D	Y
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

THEORY BASED QUESTIONS

QUESTION 7

Derive the truth table for the following **standard POS expressions**

domain ABCD

non standard POS

d. $(A+\bar{B})(A+\bar{B}+\bar{C})(B+C+\bar{D})(\bar{A}+B+\bar{C}+D)$

1 + 0 + 1 + 0

convert to standard POS

$(A+\bar{B}+C\bar{C})$

$(A+\bar{B}+C)(A+\bar{B}+\bar{C})$

$(A+\bar{B}+C+D\bar{D})(A+\bar{B}+\bar{C}+D\bar{D})$

$(A+\bar{B}+C+D)(A+\bar{B}+C+\bar{D})(A+\bar{B}+\bar{C}+D)(A+\bar{B}+\bar{C}+\bar{D})$

non standard POS

$A+\bar{B}+x+x$

$= 0+1+x+x$

$2^2 = 4$

INPUT				OUTPUT
A	B	C	D	Y
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	
1	0	0	1	
1	0	1	0	0
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

THEORY BASED QUESTIONS

QUESTION 7

Derive the truth table for the following **standard POS expressions** domain ABCD

d. $(A + \overline{B})(A + \overline{B} + \overline{C})(B + C + \overline{D})(\overline{A} + B + \overline{C} + D)$

$A + \overline{B} + \overline{C} + x = 0 + 1 + 1 + x$

$x + B + C + \overline{D} = x + 0 + 0 + 1$

INPUT				OUTPUT
A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

QUESTION 8

Using the truth tables below, derive the **standard SOP** and **standard POS** expressions.

(a)

A B C	X
0 0 0	0
0 0 1	1
0 1 0	0
0 1 1	0
1 0 0	1
1 0 1	1
1 1 0	0
1 1 1	1

standard SOP

$$X = \overline{A}\,\overline{B}C + A\overline{B}\,\overline{C} + A\overline{B}C + ABC$$

standard POS

$$X = (A + B + C)(A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + \overline{B} + C)$$

Using the truth tables below, derive the **standard SOP** and **standard POS** expressions.

(b)

A B C	X
0 0 0	0
0 0 1	0
0 1 0	0
0 1 1	0
1 0 0	0
1 0 1	1
1 1 0	1
1 1 1	1

standard SOP

$$X = A\overline{B}C + AB\overline{C} + ABC$$

standard POS

$$X = (A + B + C)(A + B + \overline{C})(A + \overline{B} + C)(A + \overline{B} + \overline{C})$$
$$(\overline{A} + B + C)$$

Using the truth tables below, derive the **standard SOP** and **standard POS** expressions.

(c)

ABCD	X
0000	1
0001	1
0010	0
0011	1
0100	0
0101	1
0110	1
0111	0
1000	0
1001	1
1010	0
1011	0
1100	1
1101	0
1110	0
1111	0

standard SOP

$$X = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}CD + \bar{A}B\bar{C}D + \bar{A}BC\bar{D} + A\bar{B}\bar{C}D + AB\bar{C}\bar{D}$$

standard POS

$$X = (A + B + \bar{C} + D)(A + \bar{B} + C + D)\ (A + \bar{B} + \bar{C} + \bar{D})$$
$$(\bar{A} + B + C + D)\ (\bar{A} + B + \bar{C} + D)\ (\bar{A} + B + \bar{C} + \bar{D})$$
$$(\bar{A} + \bar{B} + C + \bar{D})\ (\bar{A} + \bar{B} + \bar{C} + D)\ (\bar{A} + \bar{B} + \bar{C} + \bar{D})$$

Using the truth tables below, derive the **standard SOP** and **standard POS** expressions.

(d)

ABCD	X
0000	0
0001	0
0010	1
0011	0
0100	1
0101	1
0110	0
0111	1
1000	0
1001	0
1010	0
1011	1
1100	1
1101	0
1110	0
1111	1

standard SOP

$$X = \bar{A}\bar{B}C\bar{D} + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}BCD + A\bar{B}CD + AB\bar{C}\bar{D} + ABCD$$

standard POS

$$X = (A + B + C + D)(A + B + C + \bar{D})\ (A + B + \bar{C} + \bar{D})\ (A + \bar{B} + \bar{C} + D)\ (\bar{A} + B + C + D)\ (\bar{A} + B + C + \bar{D})\ (\bar{A} + B + \bar{C} + D)\ (\bar{A} + \bar{B} + C + D)\ (\bar{A} + \bar{B} + \bar{C} + D)$$

QUESTION 9 (a)

Complete the K-maps below using gray code and label each cell with its binary value

CD \ AB	11	10	00	01
11	$ABCD$	$A\bar{B}CD$	$\bar{A}\bar{B}CD$	$\bar{A}BCD$
10	$ABC\bar{D}$	$A\bar{B}C\bar{D}$	$\bar{A}\bar{B}C\bar{D}$	$\bar{A}BC\bar{D}$
00	$AB\bar{C}\bar{D}$	$A\bar{B}\bar{C}\bar{D}$	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}B\bar{C}\bar{D}$
01	$AB\bar{C}D$	$A\bar{B}\bar{C}D$	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}B\bar{C}D$

QUESTION 9 (b)

Complete the K-maps below using gray code and label each cell with its binary value

WX \ YZ	01	11	10	00
10	$W\bar{X}\ \bar{Y}Z$	$W\bar{X}YZ$	$W\bar{X}Y\bar{Z}$	$W\bar{X}\bar{Y}\bar{Z}$
00	$\bar{W}\bar{X}YZ$	$\bar{W}\bar{X}YZ$	$\bar{W}\bar{X}Y\bar{Z}$	$\bar{W}\bar{X}\bar{Y}\bar{Z}$
01	$\bar{W}X\bar{Y}Z$	$\bar{W}XYZ$	$\bar{W}XY\bar{Z}$	$\bar{W}X\bar{Y}\bar{Z}$
11	$WX\bar{Y}Z$	$WXYZ$	$WXY\bar{Z}$	$WX\bar{Y}\bar{Z}$

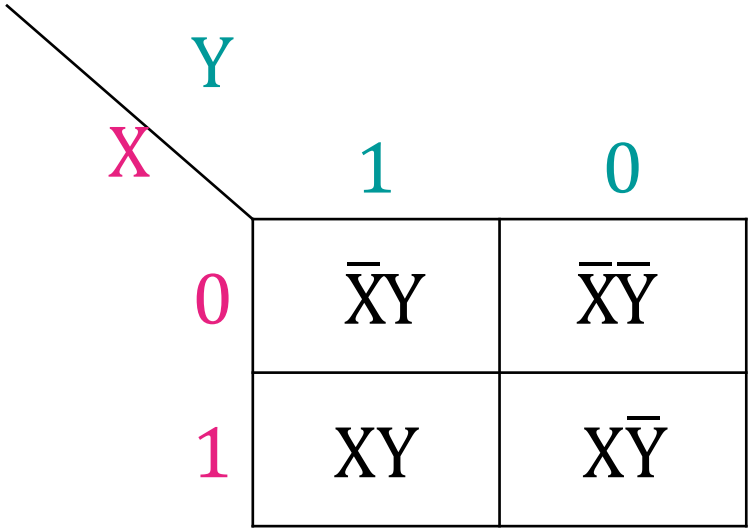
QUESTION 9 (c)

Complete the K-maps below using gray code and label each cell with its binary value

GH \ F	1	0
00	$F\bar{G}\bar{H}$	$\bar{F}\bar{G}\bar{H}$
01	$F\bar{G}H$	$\bar{F}\bar{G}H$
11	FGH	FGH
10	$FG\bar{H}$	$\bar{F}G\bar{H}$

QUESTION 9 (d)

Complete the K-maps below using gray code and label each cell with its binary value



QUESTION 10

Use K-maps to simplify each expression below to its minimum **SOP form** if possible

output = 1

a) $\overline{A}\overline{B}\overline{C} + A\overline{B}C + \overline{A}BC + AB\overline{C}$

000 101 011 110

4 x 2

AB \ C	$\overline{C}$	C
$\overline{A}\overline{B}$	1	
$\overline{A}B$		1
AB	1	
$A\overline{B}$		1

2 x 4

A \ BC	00	01	11	10
0	1		1	
1		1		1

NO SIMPLIFICATION

Use K-maps to simplify each expression below to its minimum **SOP form** if possible

non standard BE

b) $AC[B(B+\bar{C})+\bar{B}]$

non standard SOP

$= AC(B\bar{B} + B\bar{C} + \bar{B})$

$= AC(B + \bar{B} + B\bar{C})$

$= AC$ ✳

$= AC(B\bar{B} + B\bar{C} + \bar{B})$

Rule 10

$B + B\bar{C} = B$

$= AC(B + B\bar{C} + \bar{B})$

$= AC(B + \bar{B})$

standard SOP

$= ABC + A\bar{B}C$

	$\bar{C}$	C
$\bar{A}\bar{B}$		
$\bar{A}B$		
AB		1
$A\bar{B}$		1

ABC

$A\bar{B}C$

AC

$X = AC$ ✳

QUESTION 10

Use K-maps to simplify each expression below to its minimum **SOP form** if possible

Standard SOP

c) $\bar{D}\bar{E}\bar{F} + \bar{D}E\bar{F} + DE\bar{F}$

Boolean rules

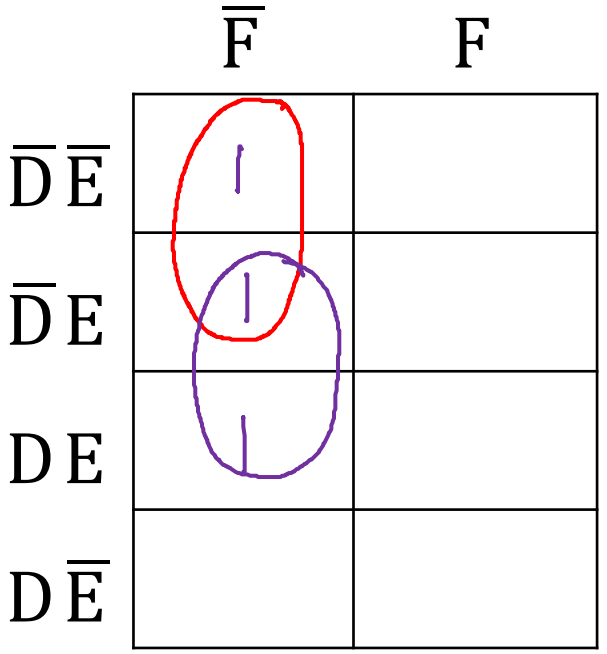
$= \bar{F}(\bar{D}\bar{E} + \bar{D}E + DE)$

$= \bar{F}(\bar{D}(\bar{E} + E) + DE)$ ($\bar{E} + E = 1$)

$= \bar{F}(\bar{D} + DE)$ rule 11 $\quad \bar{D} + DE = \bar{D} + E$

$= \bar{F}(\bar{D} + E)$

$= \bar{D}\bar{F} + E\bar{F}$ ※



$\bar{D}\bar{E}\bar{F}$

$\bar{D}E\bar{F}$

$\bar{D}\bar{F}$

$\bar{E}\bar{F}$

$E\bar{F}$

$D\bar{F}$

$X = \bar{D}\bar{F} + E\bar{F}$ ※

Use K-maps to simplify each expression below to its minimum **SOP form** if possible

non standard SOP

d) $ab + abc + a\bar{b}c$

ab ✗

standard SOP

$ab(c + \bar{c}) = abc + ab\bar{c}$

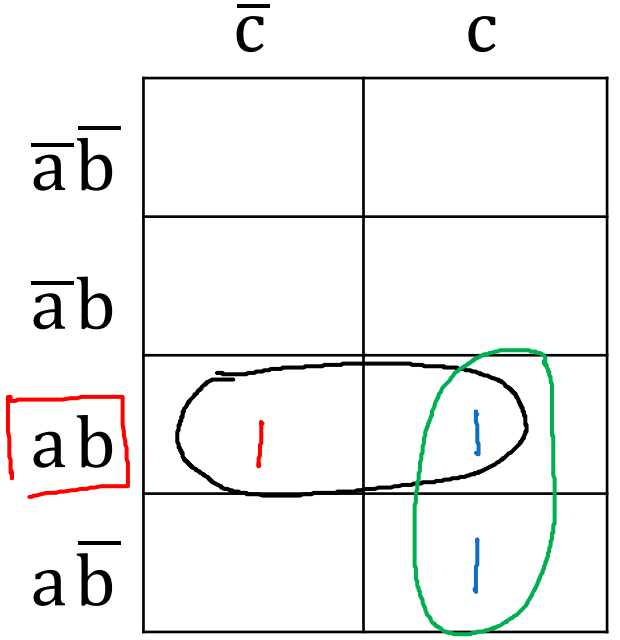
Boolean rules

$ab + abc + a\bar{b}c = ab + a\bar{b}c$ (rule 10)

$= a(b + \bar{b}c)$ (rule 11)

$= a(b + c)$

$= ab + ac$ ✓✓



ab ✗
ab ✗
ab

$X = ab + ac$ ✓✓

abc ✗
$a\bar{b}c$ ✗
ac

Use K-maps to simplify each expression below to its minimum **SOP form** if possible

non - standard SOP

domain : abc

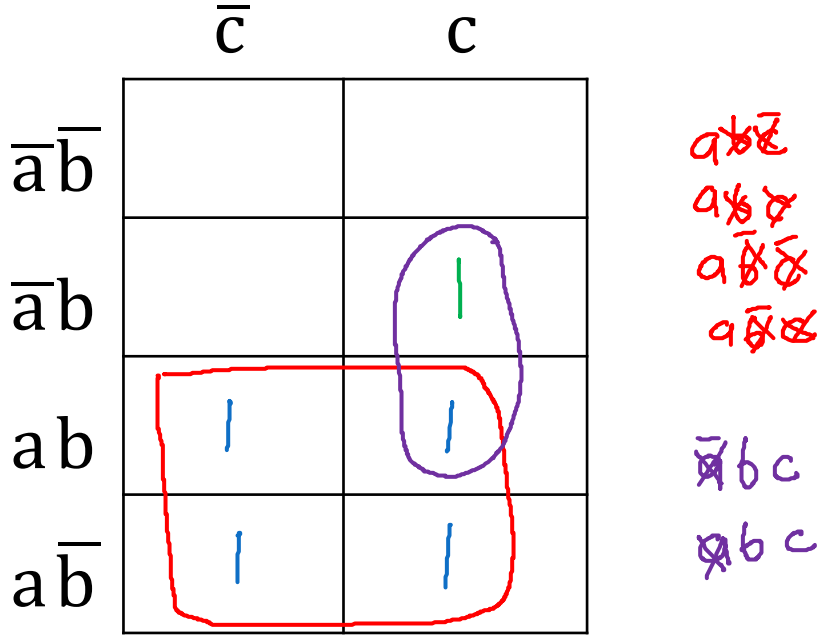
e) $a + bc$

$a\,x\,x$ → $2^2 = 4$

$x\,bc$ → $2^1 = 2$

rule 12

$a + bc$



$\bar{a}\bar{b}\bar{c}$

$a\bar{b}\bar{c}$

$a\bar{b}c$

$\bar{a}bc$

abc

$x = a + bc$ ✳

QUESTION 10

Use K-maps to simplify each expression below to its minimum **SOP form** if possible

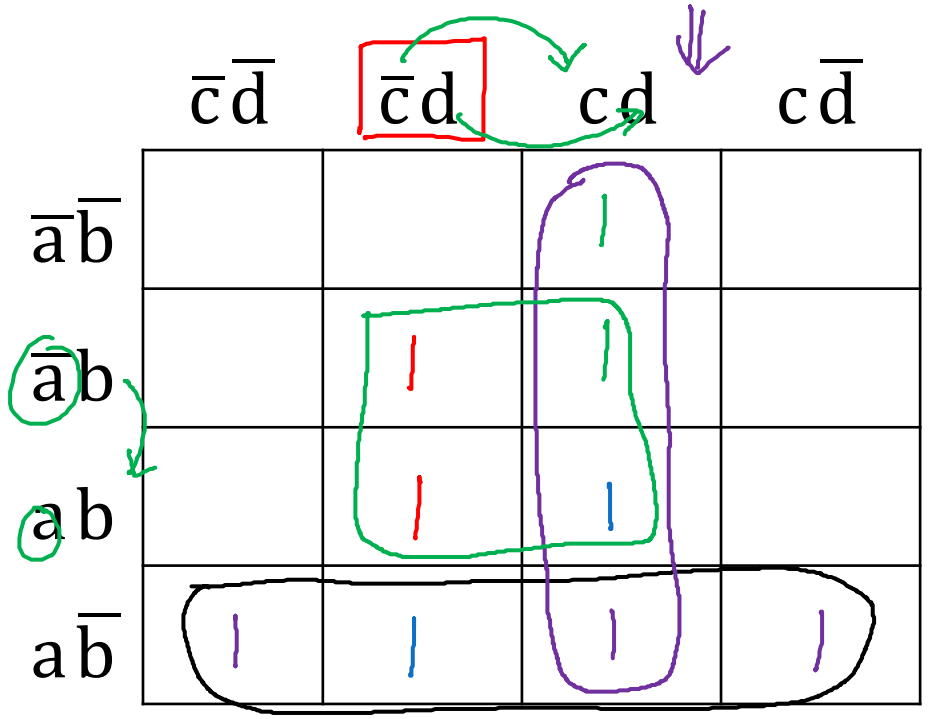
f) $a\bar{b} + a\bar{b}\bar{c}d + cd + b\bar{c}d + abcd$

Domain abcd

$a\bar{b}$ → $a\bar{b}xx$ → $2^2 = 4$

cd → $xxcd$ → $2^2 = 4$

$b\bar{c}d$ → $xb\bar{c}d$ → $2^1 = 2$



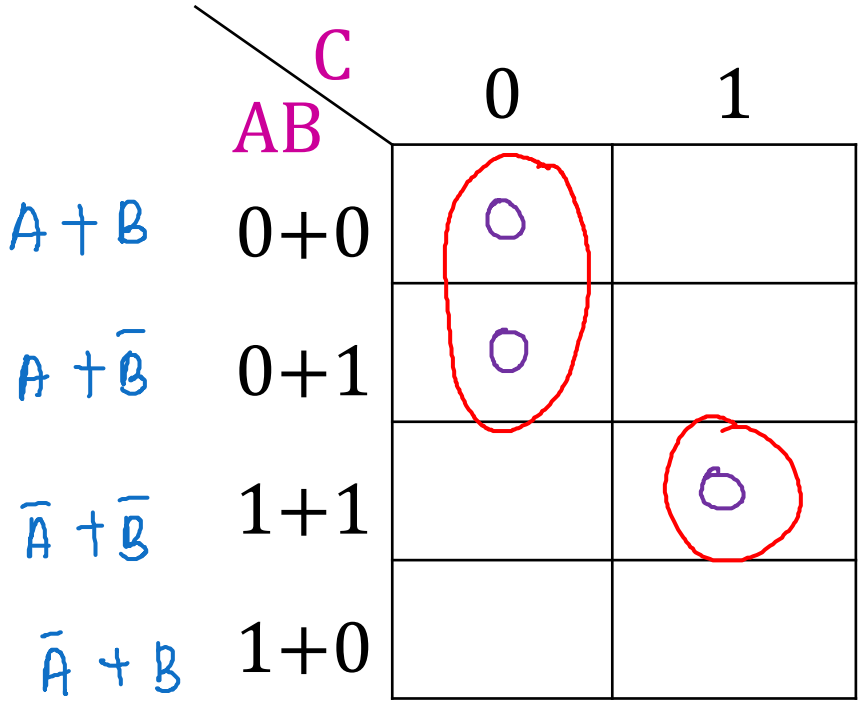
$x = a\bar{b} + cd + bd$ #

QUESTION 11

Use K-maps to find the minimum POS form for each expression below if possible.

0 + 0 + 0 0 + 1 + 0 1 + 1 + 1

a) $(A + B + C)(A + \overline{B} + C)(\overline{A} + \overline{B} + \overline{C})$



$A + \cancel{B} + C$

$A + \cancel{\overline{B}} + C$

$X = (A + C)(\overline{A} + \overline{B} + \overline{C})$

QUESTION 11

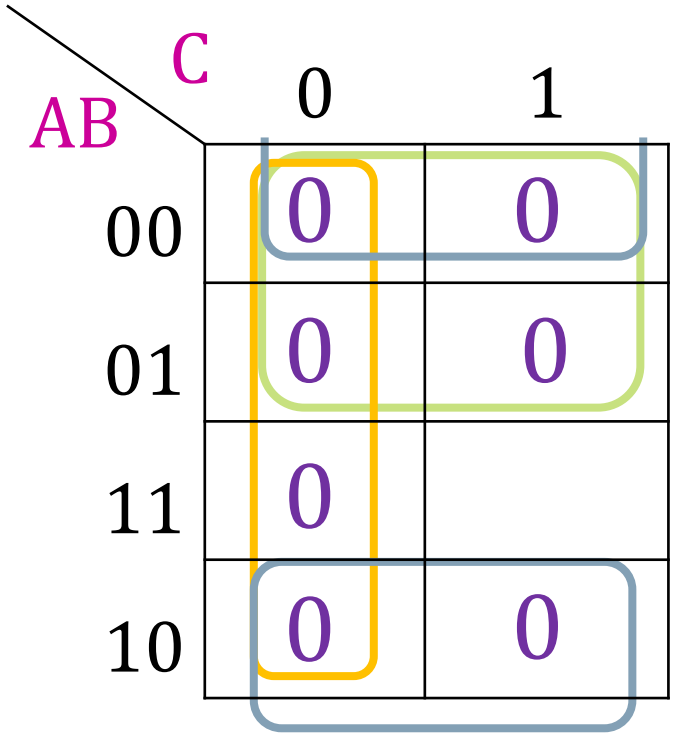
Use K-maps to find the minimum **POS form** for each expression below if possible.

b) $A(B+\bar{C})(\bar{A}+C)(A+\bar{B}+C)(A+B+\bar{C})$

$= (A+B+C)(A+\bar{B}+C)(A+B+\bar{C})(A+\bar{B}+\bar{C})$

~~$(A+B+\bar{C})$~~$(\bar{A}+B+\bar{C})(\bar{A}+B+C)(\bar{A}+\bar{B}+C)$

~~$(A+\bar{B}+C)(A+B+\bar{C})$~~



$Y = ABC$

QUESTION 11

Use K-maps to find the minimum **POS form** for each expression below if possible.

c) $(X+\bar{Y})(\bar{X}+Z)(X+\bar{Y}+\bar{Z})(\bar{X}+\bar{Y}+Z)$

$= (X+\bar{Y}+Z)\,(X+\bar{Y}+\bar{Z})(\bar{X}+Y+Z)(\bar{X}+\bar{Y}+Z)(\cancel{X+\bar{Y}+\bar{Z}})(\cancel{\bar{X}+\bar{Y}+Z})$

$0+1+0 \quad 0+1+1 \quad 1+0+0 \quad 1+1+0$

XY \ Z	0	1
00		
01	0	0
11	0	
10	0	

$Y = (X+\bar{Y})(\bar{X}+Z)$

QUESTION 11

Use K-maps to find the minimum **POS form** for each expression below if possible.

d) $(A + \overline{B} + C + \overline{D})(\overline{A} + B + \overline{C} + D)(\overline{A} + \overline{B} + \overline{C} + \overline{D})$

$0 + 1 + 0 + 1 \quad 1 + 0 + 1 + 0 \quad 1 + 1 + 1 + 1$

AB \ CD	00	01	11	10
00				
01		0		
11			0	
10				0

$$Y = (A + \overline{B} + C + \overline{D})(\overline{A} + B + \overline{C} + D)(\overline{A} + \overline{B} + \overline{C} + \overline{D})$$

QUESTION 11

Use K-maps to find the minimum **POS form** for each expression below if possible.

e) $(X + \bar{Y})(W + \bar{Z})(\bar{X} + \bar{Y} + \bar{Z})(W + X + Y + Z)$

0 0 0 0

Domain w x y z

? + $\bar{x}$ + $\bar{y}$ + $\bar{z}$

? + 1 + 1 + 1

w + ? + ? + $\bar{z}$

0 + ? + ? + 1

? + x + $\bar{y}$ + ?

0 + 1

WX \ YZ	00	01	11	10
00	0	0	0	0
01		0	0	
11			0	
10			0	0

A = $(x + \bar{y})(w + x)$ $(\bar{y} + \bar{z})(w + \bar{z})$

END DISCUSSIONS
ANY QUESTIONS ??