TMA1201 Tutorial 06 -
T4 Recursion and Induction

1. Find $f(1)$, $f(2)$, $f(3)$, and $f(4)$ if $f(n)$ is defined recursively by:
   a) $f(0) = 1$, $f(n+1) = 2^{f(n)}$, $n \geq 0$.
   b) $f(0) = 1$, $f(1) = 1$, $f(n+1) = \mathrm{f}(n) - \mathrm{f}(n-1)$, $n \geq 1$.

2. Let f_n be the number of ways (order being important) of writing n as the sums of 1s and 2s and $n \geq 1$. For example $f_4 = 5$ because 4 can be written in five ways:

   1+1+1+1   1+2+1   1+1+2   2+1+1   2+2

   a) Find the value of f_1, f_2, f_3, f_5, f_6.
   b) Construct a recursive definition of f_n.

3. State if induction or strong induction is appropriate for proving of the following statements and prove each of the statements:

   a) $1^2 + 2^2 + \ldots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$, $\forall n \geq 1$.

   b) $1 + 2^n \leq 3^n \quad \forall n \geq 1$.

   c) Given that

   $f_0 = 12$,
   $f_1 = 29$,
   $f_n = 5f_{n-1} - 6f_{n-2} \;\forall n \geq 2$

   Prove that $f_n = 5 \times 3^n + 7 \times 2^n$, $\forall n \geq 0$