# CMA6134 - Tutorial 5B

1. Find the inverse of the following matrices if they exist.

   (a) $\begin{bmatrix} 8 & 6 \\ 5 & 4 \end{bmatrix}$

   (b) $\begin{bmatrix} 8 & 5 \\ -7 & -5 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & 2 \\ 4 & 7 \end{bmatrix}$

2. Use matrix inversion to solve the system

$$\begin{aligned} 8x_1 + 6x_2 &= 2 \\ 5x_1 + 4x_2 &= -1 \end{aligned}$$

3. Solve the following systems, if solution exists, by (i) Gaussian Elimination with Back-Substitution (ii) Gauss-Jordan Elimination (iii) Matrix Inversion:

   (a)
$$\begin{aligned} x_1 + 5x_2 &= 7 \\ -2x_1 - 7x_2 &= -5 \end{aligned}$$

   (b)
$$\begin{aligned} x_2 + 4x_3 &= -5 \\ x_1 + 3x_2 + 5x_3 &= -2 \\ 3x_1 + 7x_2 + 7x_3 &= 6 \end{aligned}$$

   (c)
$$\begin{aligned} x_1 - 3x_3 &= 8 \\ 2x_1 + 2x_2 + 9x_3 &= 7 \\ x_2 + 5x_3 &= -2 \end{aligned}$$

4. Let $A = \begin{bmatrix} 1 & 2 \\ 5 & 12 \end{bmatrix}$, $\mathbf{b}_1 = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix} 1 \\ -5 \end{bmatrix}$, $\mathbf{b}_3 = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$, $\mathbf{b}_4 = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$.

   (a) Find $A^{-1}$ and use it to solve the four equations

$$A\mathbf{x} = \mathbf{b}_1, \quad A\mathbf{x} = \mathbf{b}_2, \quad A\mathbf{x} = \mathbf{b}_3, \quad A\mathbf{x} = \mathbf{b}_4$$

(b) The four equations in part(a) can be solved by the *same* set of row operations, since the coefficient matrix is the same in each case. Solve the four equations in part (a) by row reducing the augmented matrix $[A \mid \mathbf{b_1} \mid \mathbf{b_2} \mid \mathbf{b_3} \mid \mathbf{b_4}]$.

5. Find an LU factorization of the following matrices (with L unit lower triangular).

(a) $\begin{bmatrix} 2 & 5 \\ -3 & -4 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & -1 & 2 \\ -3 & -2 & 10 \\ 9 & -5 & 6 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & -6 & 3 \\ 6 & -7 & 2 \\ 1 & 7 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 3 & -5 & -3 \\ -1 & -5 & 8 & 4 \\ 4 & 2 & -5 & -7 \\ -2 & -4 & 7 & 5 \end{bmatrix}$

(e) $\begin{bmatrix} 2 & -4 & 4 & -2 \\ 6 & -9 & 7 & -3 \\ -1 & -4 & 8 & 0 \end{bmatrix}$

6. Solve the equation $A\mathbf{x} = \mathbf{b}$ using the LU factorization given for $A$.

(a)

$$A = \begin{bmatrix} 3 & -7 & -2 \\ -3 & 5 & 1 \\ 6 & -4 & 0 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -7 \\ 5 \\ 2 \end{bmatrix}$$

$$A = LU = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & -5 & 1 \end{bmatrix} \begin{bmatrix} 3 & -7 & -2 \\ 0 & -2 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

(b)

$$A=\begin{bmatrix} 2 & -1 & 2 \\ -6 & 0 & -2 \\ 8 & -1 & 5 \end{bmatrix},\ \mathbf{b}=\begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$$

$$A=LU=\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 4 & -1 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 & 2 \\ 0 & -3 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

(c)

$$A=\begin{bmatrix} 1 & -2 & -4 & -3 \\ 2 & -7 & -7 & -6 \\ -1 & 2 & 6 & 4 \\ -4 & -1 & 9 & 8 \end{bmatrix},\ \mathbf{b}=\begin{bmatrix} 1 \\ 7 \\ 0 \\ 3 \end{bmatrix}$$

(d)

$$A=LU=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -4 & 3 & -5 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 & -4 & -3 \\ 0 & -3 & 1 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

7. Solve the system by finding the LU factorization.

(a) $\begin{bmatrix} 3 & 7 \\ 6 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 1 \\ -11 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

8. Solve the equation $A\mathbf{x}=\mathbf{b}$.

$$A=LU=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 3 & 1 & 0 \\ 4 & 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}\ \text{and}\ \mathbf{b}=\begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}$$