PDS0101 Introduction to Digital Systems

Tutorial 5 SAMPLE SOLUTIONS

NOTE

Answers shown may be only one of many possibilities available. Please approach your tutor if you have an alternative answer of which you are not sure of. Any errors and omissions in the answers are deeply regretted.

Theory based questions

1. Identify which of the following expressions are in proper SOP and POS forms.
   a) $AB + CD\bar{E}$ ✓
   b) $AB + CD + \overline{BF}$ ✗
   c) $\bar{A}B + \bar{C}D\bar{E} + CA$ ✓
   d) $A(B + CD)$ ✗
   e) $((A + B)(C + D + E))'$ ✗
   f) $(\bar{A} + B)(A + \bar{B} + C)$ ✓
   g) $(W + \bar{X})(Y + \bar{Z})$ ✓
   h) $A(B + C)(E + \bar{D} + F)$ ✓
   i) $(H + I + J)(K + \bar{L})$ ✓

   Expressions a and c are already in SOP form
   Expressions f, g, h and i are in POS form
   Expressions b and e are not in any form because of the overbar extending multiple variables
   Expression d is neither in POS or SOP form as shown but can be made into SOP

2. For all the expressions in (3) determine the *domain* of each expression
   Domain of a Boolean expression is the set of variables contained in either complemented or uncomplemented form
   a) ABCDE   f) ABC
   b) ABCDF   g) WXYZ
   c) ABCDE   h) ABCDEF
   d) ABCD   i) HIJKL
   e) ABCDE

3. Convert the following expressions to SOP form

a) $(a + b)(c + \bar{b})$	b) $(a + \bar{b}c)c$	c) $(a + c)(ab + ac)$
ac+bc+ab'	ac+b'c	ab+ac
d) $ab + cd(a\bar{b} + cd)$	e) $ab(\bar{b}\bar{c} + bd)$	f) $a + b[ac + (b + \bar{c})d]$
ab+cd	abd	a+bd

   * only final answers shown – use boolean algebra to work out how to obtain answer

4. Convert the SOP expressions from (3) into standard SOP form

ac+bc+ab'	ac+b'c	ab+ac
=abc+ab'c' +a'bc+ab'c	=ab'c+abc +a'b'c	=abc+abc' +ab'c
ab+cd	abd	a+bd
=abcd+abcd'+abc'd+abc'd'+ a'b'cd+a'bcd+ab'cd	=abcd+abc'd	=ab'c'd'+ab'c'd+ab'cd'+ ab'cd+abc'd'+abc'd+abcd' +abcd+a'bc'd+a'bcd

5. Convert the standard SOP expressions from (4) into standard POS form

(a+b+c)(a+b+c') (a'+b'+c)(a+b'+c)	(a+b+c)(a+b'+c)(a+b'+c') (a'+b+c)(a'+b'+c)	(a+b+c)(a+b+c')(a+b'+c) (a+b'+c')(a'+b+c)
(a+b+c+d) (a+b+c+d') (a+b+c'+d) (a+b'+c+d) (a+b'+c+d') (a+b'+c'+d) (a'+b+c+d) (a'+b+c+d') (a'+b+c'+d)	(a+b+c+d) (a+b+c+d') (a+b+c'+d) (a+b+c'+d') (a+b'+c+d) (a+b'+c+d') (a+b'+c'+d) (a+b'+c'+d') (a'+b+c+d) (a'+b+c+d') (a'+b+c'+d) (a'+b'+c+d) (a'+b'+c'+d) (a'+b+c'+d')	(a+b+c+d) (a+b+c+d') (a+b+c'+d) (a+b+c'+d') (a+b'+c+d) (a+b'+c'+d)

6. Derive the truth table for the following standard SOP expressions

a) $A\bar{B}C + \bar{A}B\bar{C} + ABC$

A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

b) $\bar{X}\bar{Y}\bar{Z} + \bar{X}\bar{Y}Z + XY\bar{Z} + X\bar{Y}Z + \bar{X}YZ$

X	Y	Z	Q
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

c) $\bar{A}B + AB\bar{C} + \bar{A}C + ABC$

A	B	C	X
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

d) $\bar{X} + Y\bar{Z} + WZ + X\bar{Y}Z$

W	X	Y	Z	Q
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

7. Derive the truth table for the following (standard) POS expressions

a) $(\bar{A} + \bar{B} + \bar{C})(A + B + C)(A + \bar{B} + C)$

A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

b) $(A + B)(A + C)(A + B + C)$

A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

c) $(A + \bar{B} + C + \bar{D})(\bar{A} + B + \bar{C} + D)(\bar{A} + B +$
    $C + \bar{D})(A + \bar{B} + \bar{C} + D)$

A	B	C	D	X
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

d) $(A + \bar{B})(A + \bar{B} + \bar{C})(C + B + \bar{D})(\bar{A} +$
    $B + \bar{C} + D)$

A	B	C	D	X
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

8. Using the truth tables below, derive the standard SOP and standard POS expressions

(a) $X = \overline{AB}C + A\overline{BC} + \overline{A}B\overline{C} + ABC$
     $X = (A + B + C)(A + \bar{B} + C)(A + \bar{B} + \bar{C})(\bar{A} + \bar{B} + C)$

(b) $X = AB\overline{C} + \overline{AB}C + ABC$
     $X = (A + B + C)(A + B + \overline{C})(A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + B + C)$

(c) $X = \overline{ABC}D + \overline{AB}CD + \overline{A}BC\overline{D} + \overline{A}BCD + AB\overline{CD} + A\overline{B}CD + ABC\overline{D}$
     $X = (A + B + \overline{C} + D)(A + \overline{B} + C + D)(A + \overline{B} + \overline{C} + \overline{D})(\overline{A} + B + C + D)(\overline{A} + B + \overline{C} + D)$
     $(\overline{A} + B + \overline{C} + \overline{D})(\overline{A} + \overline{B} + C + D)(\overline{A} + \overline{B} + \overline{C} + D)(\overline{A} + \overline{B} + \overline{C} + \overline{D})$

(d) $X = \overline{ABC}\overline{D} + \overline{AB}C\overline{D} + \overline{A}B\overline{C}D + \overline{A}BCD + A\overline{B}CD + AB\overline{C}D + ABCD$
     $X = (A + B + C + D)(A + B + C + \overline{D})(A + B + \overline{C} + \overline{D})(A + \overline{B} + \overline{C} + D)(\overline{A} + B + C + D)$
     $(\overline{A} + B + C + \overline{D})(\overline{A} + B + \overline{C} + D)(\overline{A} + \overline{B} + C + \overline{D})(\overline{A} + \overline{B} + \overline{C} + D)$

9. Complete the Karnaugh maps below using *gray code* and label each cell with its binary value

a)

CD \ AB	11	01	00	10
11	ABCD	A'BCD	A'B'CD	AB'CD
10	ABCD'	A'BCD'	A'B'CD'	AB'CD'
00	ABC'D'	A'BC'D'	A'B'C'D'	AB'C'D'
01	ABC'D	A'BC'D	A'B'C'D	AB'C'D

b)

WX \ YZ	01	11	10	00
10	WX'Y'Z	WX'YZ	WX'YZ'	WX'Y'Z'
00	W'X'Y'Z	W'X'YZ	W'X'YZ'	W'X'Y'Z'
01	W'XY'Z	W'XYZ	W'XYZ'	W'XY'Z'
11	WXY'Z	WXYZ	WXYZ'	WXY'Z'

c)

F GH	1	0
00	FG'H'	F' G'H'
01	FG'H	F' G'H
11	FGH	F'GH
10	FGH'	F'GH'

d)

x y	1	0
0	x'y	x'y'
1	xy	xy'

10. Use K-maps to simplify each expression below to its minimum SOP form if possible

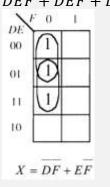a) $\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + AB\bar{C}$

AB \ C	0	1
00	(1)	
01		(1)
11	(1)	
10		(1)

No simplification

b) $AC[B(B + \bar{C}) + \bar{B}]$

AB \ C	0	1
00		
01		
11		(1)
10		1

$X = AC$

c) $\bar{D}\bar{E}\bar{F} + \bar{D}E\bar{F} + DE\bar{F}$

DE \ F	0	1
00	1	
01	1	
11	1	
10		

$X = \bar{D}\bar{F} + E\bar{F}$

d) $ab + abc + a\bar{b}c$

$AB + \bar{A}BC + ABC = AB(C + \bar{C}) + \bar{A}BC + ABC$
$\qquad = ABC + AB\bar{C} + \bar{A}BC + ABC$
$\qquad = ABC + \bar{A}BC + AB\bar{C}$

AB \ C	0	1
00		
01		
11	1	1
10		1

(a) $X = AB + AC$

e) $a + bc$

$A + BC = A(B + \bar{B})(C + \bar{C}) + (\bar{A} + A)BC = (AB + A\bar{B})(C + \bar{C}) + (\bar{A} + A)BC$
$\qquad = ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}BC + ABC$
$\qquad = ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}BC$

AB \ C	0	1
00		
01		1
11	1	1
10	1	1

$X = A + BC$

f) $a\bar{b} + a\bar{b}\bar{c}d + cd + b\bar{c}d + abcd$

$A\bar{B} + A\bar{B}\bar{C}D + CD + B\bar{C}D + ABCD$
$= A\bar{B}(C + \bar{C})(D + \bar{D}) + A\bar{B}\bar{C}D + (A + \bar{A})(B + \bar{B})CD + (A + \bar{A})B\bar{C}D + ABCD$
$= A\bar{B}CD + A\bar{B}\bar{C}D + A\bar{B}C\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}D + ABCD + \bar{A}BCD + A\bar{B}CD + ABCD$
$\quad + \bar{A}\bar{B}CD + AB\bar{C}D + \bar{A}B\bar{C}D + ABCD$
$= A\bar{B}CD + A\bar{B}\bar{C}D + A\bar{B}C\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}D + \bar{A}B\bar{C}D + \bar{A}\bar{B}CD + AB\bar{C}D + \bar{A}BCD + A\bar{B}\bar{C}D$
$= ABCD + \bar{A}BCD + \bar{A}\bar{B}CD + A\bar{B}CD + A\bar{B}\bar{C}D + AB\bar{C}D + \bar{A}B\bar{C}D + A\bar{B}C\bar{D} + A\bar{B}\bar{C}\bar{D}$

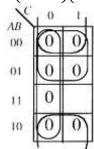| AB \ CD	00	01	11	10
00			1	
01		1	1	
11		1	1	
10	1	1	1	1

$X = A\bar{B} + CD + BD$

11. Use K-maps to find the minimum POS form for each expression below if possible

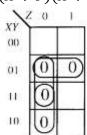a) $(A + B + C)(A + \bar{B} + C)(\bar{A} + \bar{C} + \bar{B})$

AB\C	0	1
00	0	
01	0	
11		0
10		

$X = (A + C)(\bar{A} + \bar{B} + \bar{C})$

b) $A(B + \bar{C})(\bar{A} + C)(A + \bar{B} + C)(A + B + \bar{C})$

AB\C	0	1
00	0	0
01	0	0
11	0	
10	0	0

Y = ABC

c) $(X + \bar{Y})(\bar{X} + Z)(X + \bar{Y} + \bar{Z})(\bar{X} + \bar{Y} + Z)$

XY\Z	0	1
00		
01	0	0
11	0	
10	0	

Y = (X+Y') (X'+Z)

$W = (X + \bar{Y})(\bar{X} + Z)(\bar{Y} - Z)$

d) $(A + \bar{B} + C + \bar{D})(\bar{A} + B + \bar{C} + D)(\bar{A} + \bar{B} + \bar{C} + \bar{D})$

AB\CD	00	01	11	10
00				
01		0		
11			0	
10				0

$X = (A + \bar{B} + C + \bar{D})(\bar{A} + B + \bar{C} + D)(\bar{A} + \bar{B} + \bar{C} + \bar{D})$

e) $(X + \bar{Y})(W + \bar{Z})(\bar{X} + \bar{Y} + \bar{Z})(W + X + Y + Z)$

WX\YZ	00	01	11	10
00	0	0	0	0
01		0	0	
11			0	
10			0	0

$Q = (W + \bar{Z})(W + X)(\bar{X} + \bar{Y})(\bar{Z} + X + \bar{Y})$