CMA6134 - Tutorial 5A

1. Write a system of equations that is equivalent to the given vector equation.

(a) $x_1 \begin{bmatrix} 4 \\ -1 \\ 7 \\ -4 \end{bmatrix} + x_2 \begin{bmatrix} -5 \\ 3 \\ -5 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 7 \\ -8 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 0 \\ -7 \end{bmatrix}$

(b) $x_1 \begin{bmatrix} 6 \\ -1 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} -3 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -7 \\ 5 \end{bmatrix}$

2. Write a vector equation that is equivalent to the given system of equations.

$$\begin{aligned} x_2 + 5x_3 &= 0 \\ 4x_1 + 6x_2 - x_3 &= 0 \\ -x_1 + 3x_2 - 8x_3 &= 0 \end{aligned}$$

3. Use the definition of $A\mathbf{x}$ to write the matrix equation as a vector equation.

$$\begin{bmatrix} 5 & 1 & -8 & 4 \\ -2 & -7 & 3 & -5 \end{bmatrix} \begin{bmatrix} 5 \\ -1 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} -8 \\ 16 \end{bmatrix}$$

4. Write the system first as a vector equation then as a matrix equation.

$$\begin{aligned} 3x_1 + x_2 - 5x_3 &= 9 \\ x_2 + 4x_3 &= 0 \end{aligned}$$

5. Determine which matrices are in reduced row echelon form and which others are only in row echelon form.

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

6. Row reduce the matrix to reduced row echelon form. Circle the pivot positions in the final matrix and in the original matrix, and list the pivot column.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix}$$

7. The augmented matrix of a linear system has been reduced by row operations to the form shown. **Continue with the appropriate row operations** for solution, if any.

(a) $\begin{bmatrix} 1 & 7 & 3 & | & -4 \\ 0 & 1 & -1 & | & 3 \\ 0 & 0 & 0 & | & 1 \\ 0 & 0 & 1 & | & -2 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -1 & 0 & 0 & | & -4 \\ 0 & 1 & -3 & 0 & | & -7 \\ 0 & 0 & 1 & -3 & | & -1 \\ 0 & 0 & 0 & 2 & | & 4 \end{bmatrix}$

8. Write the augmented matrix for the linear system that corresponds to the matrix equation $A\mathbf{x} = \mathbf{b}$. Then solve the system and write the solution as a vector.

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 5 \\ -2 & -4 & -3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -2 \\ 2 \\ 9 \end{bmatrix}$$

9. Determine if the system has a nontrivial solution.

(a) $\begin{aligned} 2x_1 - 5x_2 + 8x_3 &= 0 \\ -2x_1 - 7x_2 + x_3 &= 0 \\ 4x_1 + 2x_2 + 7x_3 &= 0 \end{aligned}$

(b) $\begin{aligned} -3x_1 + 5x_2 - 7x_3 &= 0 \\ -6x_1 + 7x_2 + x_3 &= 0 \end{aligned}$

10. Solve by back substitution:

(a)
$$\begin{aligned} 3x - 4y + 5z &= 2 \\ 3y - 4z &= -1 \\ 5z &= 5 \end{aligned}$$

(b)
$$\begin{aligned} x - 2y + z &= 2 \\ 4y - 3z &= 1 \\ -3z &= 3 \end{aligned}$$

11. Use Gaussian Elimination to solve the systems:

(a)
$$\begin{aligned} 2x - 3y &= 2 \\ 5x - 6y &= 8 \end{aligned}$$

(b)
$$\begin{aligned} x + 2y &= -1 \\ 2x + 3y &= 1 \end{aligned}$$

(c)
$$\begin{aligned} -x + y &= 2 \\ 3x + 4y &= 15 \end{aligned}$$