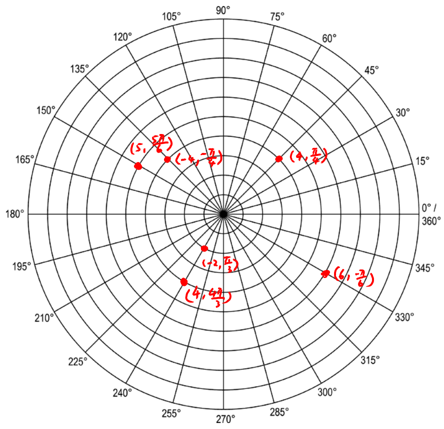
Tutorial 2 (Part 4)

Q1



Q2

a) $(r,\theta) = (4, \frac{\pi}{4})$

$(x,y) = (r\cos\theta, r\sin\theta)$

$= (4\cos(\frac{\pi}{4}), 4\sin(\frac{\pi}{4}))$

$= (4(\frac{\sqrt{2}}{2}), 4(\frac{\sqrt{2}}{2}))$

$= (2\sqrt{2}, 2\sqrt{2})$

b) $(r,\theta) = (6, \frac{-\pi}{6})$

$(x,y) = (6\cos(-\frac{\pi}{6}), 6\sin(-\frac{\pi}{6}))$

$= (6(\frac{\sqrt{3}}{2}), 6(-\frac{1}{2}))$

$= (3\sqrt{3}, -3)$

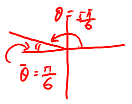
c) $(r,\theta) = (-2, \frac{\pi}{3})$

$(x,y) = (-2\cos\frac{\pi}{3}, -2\sin\frac{\pi}{3})$

$= (-2(\frac{1}{2}), -2(\frac{\sqrt{3}}{2}))$

$= (-1, -\sqrt{3})$

d) $(r,\theta) = (5, \frac{5\pi}{6})$



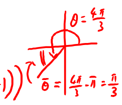
$(x,y) = (5\cos(\frac{5\pi}{6}), 5\sin(\frac{5\pi}{6}))$

$= (5(-\cos(\frac{\pi}{6})), 5\sin(\frac{\pi}{6}))$

$= (-5 \cdot \frac{\sqrt{3}}{2}, 5(\frac{1}{2}))$

$= (\frac{-5\sqrt{3}}{2}, \frac{5}{2})$

e) $(r,\theta) = (4, \frac{4\pi}{3})$



$(x,y) = (4\cos(\frac{4\pi}{3}), 4\sin(\frac{4\pi}{3}))$

$= (4(-\cos(\frac{\pi}{3})), 4(-\sin(\frac{\pi}{3})))$

$= (-4(\frac{1}{2}), -4(\frac{\sqrt{3}}{2}))$

$= (-2, -2\sqrt{3})$

f) $(r,\theta) = (-4, -\frac{\pi}{4})$

$(x,y) = (-4\cos(-\frac{\pi}{4}), -4\sin(-\frac{\pi}{4}))$

$= (-4(\frac{\sqrt{2}}{2}), -4(-\frac{\sqrt{2}}{2}))$

$= (-2\sqrt{2}, 2\sqrt{2})$

Q3

a) $(x,y) = (1,1)$

$r = \sqrt{1^2+1^2} = \sqrt{2}$

$\bar{\theta} = \tan^{-1}(|\frac{1}{1}|) = \frac{\pi}{4}$

$(x,y) = (1,1)$ is at QI

$\therefore \theta = \bar{\theta} = \frac{\pi}{4}$

$\therefore (r,\theta) = (\sqrt{2}, \frac{\pi}{4})$

b) $(x,y) = (3\sqrt{3}, -3)$

$r = \sqrt{(3\sqrt{3})^2 + (-3)^2}$

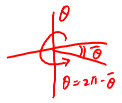
$= \sqrt{9(3)+9} = 6$

$\bar{\theta} = \tan^{-1}(|\frac{-3}{3\sqrt{3}}|)$

$= \tan^{-1}(\frac{1}{\sqrt{3}}) = \frac{\pi}{6}$

$(x,y) = (3\sqrt{3}, -\sqrt{3})$ is at QIV

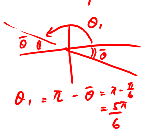
$\therefore \theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$



$\therefore (r,\theta) = (6, \frac{11\pi}{6})$

or

$(-6, \frac{5\pi}{6})$

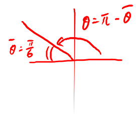


c) $(x,y) = (-\sqrt{6}, \sqrt{2})$

$r = \sqrt{(-\sqrt{6})^2 + (\sqrt{2})^2} = \sqrt{6+2}$

$= \sqrt{8}$

$\bar{\theta} = \tan^{-1}(|\frac{\sqrt{2}}{-\sqrt{6}}|) = \tan^{-1}(\frac{1}{\sqrt{3}}) = \frac{\pi}{6}$



$(x,y) = (-\sqrt{6}, \sqrt{2})$ indicate θ at QII

$\therefore \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$(r,\theta) = (\sqrt{8}, \frac{5\pi}{6})$

d) $(x,y) = (3,4)$

$r = \sqrt{3^2+4^2} = 5$

$\bar{\theta} = \tan^{-1}(|\frac{4}{3}|)$

$(x,y) = (3,4)$ indicate θ at QI

$\therefore \theta = \bar{\theta} = \tan^{-1}(\frac{4}{3})$

$\therefore (r,\theta) = (5, \tan^{-1}(\frac{4}{3}))$

e) $(x,y) = (1,-2)$

$r = \sqrt{(1)^2+(-2)^2} = \sqrt{5}$

$\bar{\theta} = \tan^{-1}(|\frac{-2}{1}|) = \tan^{-1}(2)$

$(x,y) = (1,-2)$ indicate θ at QIV

$\therefore \theta = 2\pi - \bar{\theta} = 2\pi - \tan^{-1}(2)$

$\therefore (r,\theta) = (\sqrt{5}, 2\pi - \tan^{-1}(2))$

f) $(x,y) = (0, -\sqrt{3})$

$r = \sqrt{0^2 + (-\sqrt{3})^2} = \sqrt{3}$

$(x,y) = (0, -\sqrt{3})$ indicate

$\theta = \frac{3\pi}{2}$

$\therefore (r,\theta) = (\sqrt{3}, \frac{3\pi}{2})$

Q4

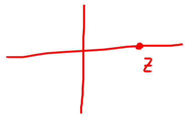
a) Let $z = -3i$

$r = \sqrt{0^2 + (-3)^2} = 3$

$\theta = \frac{3}{2}\pi$

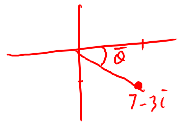
$\therefore z = 3\left(\cos\left(\frac{3}{2}\pi\right) + i\sin\left(\frac{3}{2}\pi\right)\right)$

b) Let $z = 4$



$z = 4\left[\cos(0) + i\sin(0)\right]$

c) Let $z = 7-3i$



$r = \sqrt{7^2 + (-3)^2} = \sqrt{58}$

$\bar{\theta} = \tan^{-1}\left(\frac{3}{7}\right)$

From the graph z at QIV

$\therefore \theta = 2\pi - \bar{\theta} = 2\pi - \tan^{-1}\left(\frac{3}{7}\right)$

$\therefore z = \sqrt{58}\left(\cos\theta + i\sin\theta\right)$

d) Let $z = 5+2i$

$r = \sqrt{5^2 + 2^2} = \sqrt{29}$

z in QI. Hence

$\theta = \tan^{-1}\left(\frac{2}{5}\right)$ and

$z = \sqrt{29}\left(\cos(\theta) + i\sin(\theta)\right)$

e) Let $z = -1 - \frac{\sqrt{3}}{3}i$

$r = \sqrt{(-1)^2 + \left(-\frac{\sqrt{3}}{3}\right)^2} = \sqrt{1 + \frac{1}{3}} = \frac{2}{\sqrt{3}}$

$\bar{\theta} = \tan^{-1}\left(\frac{\frac{\sqrt{3}}{3}}{1}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

z in QIII. Thus

$\theta = \pi + \bar{\theta} = \frac{7\pi}{6}$

$\therefore z = \frac{2}{\sqrt{3}}\left(\cos\left(\frac{7\pi}{6}\right) + i\sin\left(\frac{7\pi}{6}\right)\right)$

f) Let $z = \frac{-\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$

$r = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2} = 1$

$\bar{\theta} = \tan^{-1}\left(\left|\frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}\right|\right) = \tan^{-1}(1) = \frac{\pi}{4}$

z in QII

$\theta = \pi - \bar{\theta} = \frac{3\pi}{4}$

Thus

$z = \cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)$

g) Let $z = i(2-2i)$

$= 2i - 2i^2$

$= 2 + 2i$

$r = \sqrt{2^2 + 2^2} = 2\sqrt{2}$.

z in QI

$\therefore \theta = \tan^{-1}\left(\frac{2}{2}\right) = \tan^{-1}(1) = \frac{\pi}{4}$

$\therefore z = 2\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

h) Let $z = 2\sqrt{3} - 2i$

$r = \sqrt{(2\sqrt{3})^2 + (-2)^2}$

$= \sqrt{12+4} = 4$

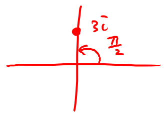
$\bar{\theta} = \tan^{-1}\left|\frac{-2}{(2\sqrt{3})}\right| = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

z is in QIV. Hence

$\theta = 2\pi - \bar{\theta} = \frac{11\pi}{6}$

$z = 4\left(\cos\left(\frac{11\pi}{6}\right) + i\sin\left(\frac{11\pi}{6}\right)\right)$

i) Let $z = 3i$



$r = \sqrt{0^2 + 3^2} = 3$

$\theta = \frac{\pi}{2}$

$\therefore z = 3\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

Q5

a) $z = \cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)$, $w = \cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)$

$zw = \cos\left(\frac{\pi}{4} + \frac{3\pi}{4}\right) + i\sin\left(\frac{\pi}{4} + \frac{3\pi}{4}\right)$

$= \cos(\pi) + i\sin(\pi)$

$= -1$

$\frac{z}{w} = \cos\left(\frac{\pi}{4} - \frac{3\pi}{4}\right) + i\sin\left(\frac{\pi}{4} - \frac{3\pi}{4}\right)$

$= \cos\left(\frac{-\pi}{2}\right) + i\sin\left(\frac{-\pi}{2}\right)$

$= -i$

b) $z = 7\left(\cos\left(\frac{9\pi}{8}\right) + i\sin\left(\frac{9\pi}{8}\right)\right)$, $w = 2\left[\cos\left(\frac{\pi}{8}\right) + i\sin\left(\frac{\pi}{8}\right)\right]$

$zw = 7 * 2\left[\cos\left(\frac{9\pi}{8} + \frac{\pi}{8}\right) + i\sin\left(\frac{9\pi}{8} + \frac{\pi}{8}\right)\right]$

$= 14\left[\cos\left(\frac{5\pi}{4}\right) + i\sin\left(\frac{5\pi}{4}\right)\right]$

$= -\frac{14}{\sqrt{2}} - \frac{14}{\sqrt{2}}i$

$\frac{z}{w} = \frac{7}{2}\left[\cos\left(\frac{9\pi}{8} - \frac{\pi}{8}\right) + i\sin\left(\frac{9\pi}{8} - \frac{\pi}{8}\right)\right]$

$= \frac{7}{2}\left[\cos\pi + i\sin\pi\right]$

$= -\frac{7}{2}$

c) $z = \frac{4}{5}(\cos 25^\circ + i\sin 25^\circ)$, $w = \frac{1}{5}(\cos 155^\circ + i\sin 155^\circ)$

$$zw = \frac{4}{5}\cdot\frac{1}{5}\left[\cos(25^\circ+155^\circ) + i\sin(25^\circ+155^\circ)\right]$$
$$= \frac{4}{25}\left[\cos(180^\circ) + i\sin(180^\circ)\right] = \frac{-4}{25}$$

$$\frac{z}{w} = \frac{\left(\frac{4}{5}\right)}{\left(\frac{1}{5}\right)}\left[\cos(25^\circ-155^\circ) + i\sin(25^\circ-155^\circ)\right]$$
$$= 4\left[\cos(-130^\circ) + i\sin(-130^\circ)\right]$$

d) $z = 4[\cos 200^\circ + i\sin 200^\circ]$, $w = 25[\cos 150^\circ + i\sin 150^\circ]$

$$zw = 4\times 25\left[\cos(200^\circ+150^\circ) + i\sin(200^\circ+150^\circ)\right]$$
$$= 100\left[\cos(350^\circ) + i\sin(350^\circ)\right]$$

$$\frac{z}{w} = \frac{4}{25}\left[\cos(200^\circ-150^\circ) + i\sin(200^\circ-150^\circ)\right]$$
$$= \frac{4}{25}\left[\cos 50^\circ + i\sin 50^\circ\right]$$

Q6

a) $z = \sqrt{2} - \sqrt{2}i$

$r_z = \sqrt{(\sqrt{2})^2 + (-\sqrt{2})^2} = 2$

$\bar{\theta}_z = \tan^{-1}\left|\frac{-\sqrt{2}}{\sqrt{2}}\right| = \tan^{-1}(1) = \frac{\pi}{4}$

z at QIV

$\therefore \theta_z = 2\pi - \bar{\theta}_z = 2\pi - \frac{\pi}{4}$

$= \frac{7\pi}{4}$

$z = 2\left[\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right]$

$w = 1 + i$

$r_w = \sqrt{1^2+1^2} = \sqrt{2}$

$\theta_w = \tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}$ (at Q1)

$w = \sqrt{2}\left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right]$

$zw = 2\sqrt{2}\left[\cos\left(\frac{7\pi}{4}+\frac{\pi}{4}\right) + i\sin\left(\frac{7\pi}{4}+\frac{\pi}{4}\right)\right]$

$= 2\sqrt{2}\left[\cos(2\pi) + i\sin(2\pi)\right]$

$= 2\sqrt{2}$

$\frac{z}{w} = \frac{2}{\sqrt{2}}\left[\cos\left(\frac{7\pi}{4}-\frac{\pi}{4}\right) + i\sin\left(\frac{7\pi}{4}-\frac{\pi}{4}\right)\right]$

$= \frac{2}{\sqrt{2}}\left[\cos\left(\frac{3\pi}{2}\right) + i\sin\left(\frac{3\pi}{2}\right)\right]$

$= \frac{-2}{\sqrt{2}}i$

$\frac{1}{z} = \frac{1}{2}\left[\cos\left(-\frac{7\pi}{4}\right) + i\sin\left(-\frac{7\pi}{4}\right)\right]$

$= \frac{-\sqrt{2}}{4} - \frac{\sqrt{2}}{4}i$

b) $z = \sqrt{3} + i$

$r_z = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = 2$

$\theta_z = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$ (Q I)

$z = 2\left[\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right]$

$w = 1 - \sqrt{3}i$

$r_w = \sqrt{1^2 + (-\sqrt{3})^2} = 2$

$\theta_w = 2\pi - \tan^{-1}\left|\frac{-\sqrt{3}}{1}\right|$

$= 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$w = 2\left[\cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{5\pi}{3}\right)\right]$

$$zw = 2\times 2\left[\cos\left(\frac{\pi}{6}+\frac{5\pi}{3}\right) + i\sin\left(\frac{\pi}{6}+\frac{5\pi}{3}\right)\right]$$
$$= 4\left[\cos\left(\frac{11\pi}{6}\right) + i\sin\left(\frac{11\pi}{6}\right)\right] = 2\sqrt{3} - 2i$$

$$\frac{z}{w} = \frac{2}{2}\left[\cos\left(\frac{\pi}{6}-\frac{5\pi}{3}\right) + i\sin\left(\frac{\pi}{6}-\frac{5\pi}{3}\right)\right]$$
$$= \cos\left(-\frac{3\pi}{2}\right) + i\sin\left(-\frac{3\pi}{2}\right) = -i$$

$$\frac{1}{z} = \frac{1}{2}\left[\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right] = \frac{\sqrt{3}}{4} - \frac{i}{4}$$

c) $z = -2i$

$r_z = 2$

$\theta_z = \frac{3\pi}{2}$

$z = 2\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

$w = -3 - 3\sqrt{3}i$

$r_w = \sqrt{(-3)^2 + (-3\sqrt{3})^2}$

$= \sqrt{9+27} = 6$

$\bar{\theta}_w = \tan^{-1}\left(\left|\frac{-3\sqrt{3}}{-3}\right|\right) = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

w in QIII

$\therefore \theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$w = 6\left[\cos\left(\frac{4\pi}{3}\right) + i\sin\left(\frac{4\pi}{3}\right)\right]$

$$zw = 2\times 6\left[\cos\left(\frac{3\pi}{2}+\frac{4\pi}{3}\right) + i\sin\left(\frac{3\pi}{2}+\frac{4\pi}{3}\right)\right]$$
$$= 12\left[\cos\left(\frac{9\pi+8\pi}{6}\right) + i\sin\left(\frac{9\pi+8\pi}{6}\right)\right]$$
$$= 12\left[\cos\left(\frac{17\pi}{6}\right) + i\sin\left(\frac{17\pi}{6}\right)\right] = -6\sqrt{3} + 6i$$

$$\frac{z}{w} = \frac{2}{6}\left[\cos\left(\frac{3\pi}{2}-\frac{4\pi}{3}\right) + i\sin\left(\frac{3\pi}{2}-\frac{4\pi}{3}\right)\right]$$
$$= \frac{1}{3}\left[\cos\left(\frac{9\pi-8\pi}{6}\right) + i\sin\left(\frac{9\pi-8\pi}{6}\right)\right]$$
$$= \frac{1}{3}\left[\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right] = \frac{\sqrt{3}}{6} + \frac{i}{6}$$

$$\frac{1}{z} = \frac{1}{2}\left[\cos\left(-\frac{3\pi}{2}\right) + i\sin\left(-\frac{3\pi}{2}\right)\right]$$
$$= \frac{i}{2}$$

d) $z = 4\sqrt{3} - 4i$

$r_z = \sqrt{(4\sqrt{3})^2 + (-4)^2}$

$= 4\sqrt{3+1} = 8$

$\bar{\theta}_z = \tan^{-1}\left|\frac{-4}{4\sqrt{3}}\right| = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

z at QIV

$\theta_z = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$\therefore z = 8\left[\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right]$

$w = 4$

$r_w = 4$

$\theta_w = 0$

$w = 4(\cos 0 + i\sin 0)$

$zw = 8 \cdot 4\left[\cos\left(\frac{11\pi}{6}+0\right) + i\sin\left(\frac{11\pi}{6}+0\right)\right]$

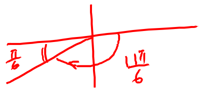
$= 32\left[\cos\left(\frac{11\pi}{6}\right) + i\sin\left(\frac{11\pi}{6}\right)\right]$

$= 16\sqrt{3} - 16i$

$\frac{z}{w} = \frac{8}{4}\left[\cos\left(\frac{11\pi}{6}-0\right) + i\sin\left(\frac{11\pi}{6}-0\right)\right]$

$= 2\left[\cos\left(\frac{11\pi}{6}\right) + i\sin\left(\frac{11\pi}{6}\right)\right] = \sqrt{3} - i$

$\frac{1}{z} = \frac{1}{8}\left[\cos\left(-\frac{11\pi}{6}\right) + i\sin\left(-\frac{11\pi}{6}\right)\right] = \frac{\sqrt{3}}{16} + \frac{i}{16}$



e) $z = -20$

$r_z = 20$

$\theta_z = \pi$

$z = 20[\cos(\pi) + i\sin(\pi)]$

$w = -2 - 2i$

$r_w = \sqrt{(-2)^2 + (-2)^2} = 2\sqrt{2}$

$\theta_w = \pi + \tan^{-1}\left|\frac{-2}{-2}\right|$ (w at QIII)

$= \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$w = 2\sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$

$zw = 20 \cdot 2\sqrt{2}\left(\cos\left(\pi + \frac{5\pi}{4}\right) + i\sin\left(\pi + \frac{5\pi}{4}\right)\right)$

$= 40\sqrt{2}\left[\cos\left(\frac{9\pi}{4}\right) + i\sin\left(\frac{9\pi}{4}\right)\right] = 40 + 40i$

$\frac{z}{w} = \frac{20}{2\sqrt{2}}\left[\cos\left(\pi - \frac{5\pi}{4}\right) + i\sin\left(\pi - \frac{5\pi}{4}\right)\right]$

$= 5\sqrt{2}\left[\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right] = 5 - 5i$

$\frac{1}{z} = \frac{1}{20}[\cos(-\pi) + i\sin(-\pi)] = -\frac{1}{20}$

Q7

a) Let $z = 1 - i$

$r = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$\theta = 2\pi - \tan^{-1}\left|\frac{-1}{1}\right|$ (z in QIV)

$= \frac{7\pi}{4}$

$\therefore z = \sqrt{2}\left[\cos\left(\frac{7\pi}{4}\right) + i\sin\left(\frac{7\pi}{4}\right)\right]$

$(1-i)^8 = (\sqrt{2})^8\left[\cos 8\left(\frac{7\pi}{4}\right) + i\sin 8\left(\frac{7\pi}{4}\right)\right]$

$= 2^4[\cos(14\pi) + i\sin(14\pi)]$

$= 16[\cos 0 + i\sin 0] = 16$

b) Let $z = 2\sqrt{3} + 2i$

$r = \sqrt{(2\sqrt{3})^2 + 2^2}$

$= 4$

$\theta = \tan^{-1}\left|\frac{2}{2\sqrt{3}}\right| = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$= \frac{\pi}{6}$

$z = 4\left[\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right]$

$(2\sqrt{3} + 2i)^5 = 4^5\left[\cos 5\left(\frac{\pi}{6}\right) + i\sin 5\left(\frac{\pi}{6}\right)\right]$

$= 1024\left[\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right]$

$= -512\sqrt{3} + 512i$

c) Let $z = \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$

$r = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$

$\theta = \pi + \tan^{-1}\left|\frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)}\right|$ (z in QIII)

$= \pi + \tan^{-1}(\sqrt{3})$

$= \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$\therefore z = 1 \cdot \left[\cos\left(\frac{4\pi}{3}\right) + i\sin\left(\frac{4\pi}{3}\right)\right]$

$\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^{15} = 1^{15}\left[\cos\left(15 \cdot \frac{4\pi}{3}\right) + i\sin\left(15 \cdot \frac{4\pi}{3}\right)\right]$

$= \cos(20\pi) + i\sin(20\pi)$

$= \cos(0) + i\sin 0 = 1$

d) Let $z = (-1+i)^{-5}$

$r = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$

$\theta = \pi - \tan^{-1}\left|\frac{1}{-1}\right|$ (z in QII)

$= \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

$z = \sqrt{2}\left[\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right]$

$(-1+i)^{-5} = (\sqrt{2})^{-5}\left[\cos\left(-5 \cdot \frac{3\pi}{4}\right) + i\sin\left(-5 \cdot \frac{3\pi}{4}\right)\right]$

$= \frac{1}{4\sqrt{2}}\left[\cos\left(-\frac{15\pi}{4}\right) + i\sin\left(-\frac{15\pi}{4}\right)\right]$

$= \frac{1}{8} + \frac{i}{8}$

e) Let $z = (-\sqrt{3} - i)$

$r = \sqrt{(-\sqrt{3})^2 + (-1)^2} = 2$

$\theta = \pi + \tan^{-1}\left|\frac{-1}{-\sqrt{3}}\right|$ (z in QIII)

$= \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$\therefore z = 2\left[\cos\left(\frac{7\pi}{6}\right) + i\sin\left(\frac{7\pi}{6}\right)\right]$

$(-\sqrt{3} - i)^{-4} = 2^{-4}\left[\cos\left(-4 \cdot \frac{7\pi}{6}\right) + i\sin\left(-4 \cdot \frac{7\pi}{6}\right)\right]$

$= \frac{1}{16}\left[\cos\left(-\frac{14\pi}{3}\right) + i\sin\left(-\frac{14\pi}{3}\right)\right]$

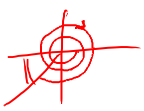
$= \frac{1}{16}\left[-\cos\left(\frac{\pi}{3}\right) + i\left(-\sin\left(\frac{\pi}{3}\right)\right)\right]$

$= \frac{1}{16}\left[-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right]$

$= \frac{-1}{32}[1 + \sqrt{3}i]$

$\pi - \frac{2\pi}{3} = \frac{\pi}{3}$

$-\frac{14\pi}{3} = -\frac{12\pi}{3} - \frac{2\pi}{3}$



f) Let $z = (-1 + \sqrt{3}i)^{-7}$

$r = \sqrt{(-1)^2 + (\sqrt{3})^2} = 2$

$\theta = \pi - \tan^{-1}\left|\frac{\sqrt{3}}{-1}\right|$ (z in QII)

$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\therefore z = 2\left[\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right]$

$(-1 + \sqrt{3}i)^{-7} = \frac{1}{2^7}\left[\cos\left(-7 \cdot \frac{2\pi}{3}\right) + i\sin\left(-7 \cdot \frac{2\pi}{3}\right)\right]$

$= \frac{1}{128}\left[\cos\left(-\frac{14\pi}{3}\right) + i\sin\left(-\frac{14\pi}{3}\right)\right]$

$= \frac{-1}{256}[1 + \sqrt{3}i]$

Q8

a) let $z = 4\sqrt{3} + 4i$

$r = \sqrt{(4\sqrt{3})^2 + 4^2}$

$= 8$

$\theta = \tan^{-1}\left|\frac{4}{4\sqrt{3}}\right|$ (z in QI)

$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

$z = 8\left[\cos\left(\frac{\pi}{6}+2n\pi\right) + i\sin\left(\frac{\pi}{6}+2n\pi\right)\right]$

Let $w = z^{\frac{1}{2}}$

$w = 8^{\frac{1}{2}}\left[\cos\left[\frac{1}{2}\left(\frac{\pi}{6}+2n\pi\right)\right] + i\sin\left[\frac{1}{2}\left(\frac{\pi}{6}+2n\pi\right)\right]\right]$

$= 2\sqrt{2}\left[\cos\left[\frac{\pi}{12}+n\pi\right] + i\sin\left[\frac{\pi}{12}+n\pi\right]\right]$

$w_0 = 2\sqrt{2}\left[\cos\left(\frac{\pi}{12}\right) + i\sin\left(\frac{\pi}{12}\right)\right]$

$w_1 = 2\sqrt{2}\left[\cos\left(\frac{\pi}{12}+\pi\right) + i\sin\left(\frac{\pi}{12}+\pi\right)\right]$

$= 2\sqrt{2}\left[\cos\left(\frac{13\pi}{12}\right) + i\sin\left(\frac{13\pi}{12}\right)\right]$

b) Let $z = 32$

$r = 32$

$\theta = 0$

$\therefore z = 32\left[\cos\left(0+2n\pi\right) + i\sin\left(0+2n\pi\right)\right]$

$= 32\left[\cos 2n\pi + i\sin(2n\pi)\right]$

let $w = z^{\frac{1}{5}}$

$= 32^{\frac{1}{5}}\left[\cos\left(\frac{2n\pi}{5}\right) + i\sin\left(\frac{2n\pi}{5}\right)\right]$

$w_0 = 2\left[\cos 0 + i\sin 0\right]$

$w_1 = 2\left[\cos\left(\frac{2\pi}{5}\right) + i\sin\left(\frac{2\pi}{5}\right)\right]$

$w_2 = 2\left[\cos\frac{4\pi}{5} + i\sin\left(\frac{4\pi}{5}\right)\right]$

$w_3 = 2\left[\cos\frac{6\pi}{5} + i\sin\left(\frac{6\pi}{5}\right)\right]$

$w_4 = 2\left[\cos\left(\frac{8\pi}{5}\right) + i\sin\left(\frac{8\pi}{5}\right)\right]$

c) Let $z = -81i$

$r = 81$

$\theta = \frac{3\pi}{2}$

$z = 81\left[\cos\left(\frac{3\pi}{2}+2n\pi\right) + i\sin\left(\frac{3\pi}{2}+2n\pi\right)\right]$

Suppose $w = z^{\frac{1}{4}}$

$w = 81^{\frac{1}{4}}\left[\cos\left(\frac{3\pi}{8}+\frac{n\pi}{2}\right) + i\sin\left(\frac{3\pi}{8}+\frac{n\pi}{2}\right)\right]$

$w_0 = 3\left[\cos\left(\frac{3\pi}{8}\right) + i\sin\left(\frac{3\pi}{8}\right)\right]$

$w_1 = 3\left[\cos\left(\frac{3\pi}{8}+\frac{\pi}{2}\right) + i\sin\left(\frac{3\pi}{8}+\frac{\pi}{2}\right)\right]$

$= 3\left[\cos\left(\frac{7\pi}{8}\right) + i\sin\left(\frac{7\pi}{8}\right)\right]$

$w_2 = 3\left[\cos\left(\frac{11\pi}{8}\right) + i\sin\left(\frac{11\pi}{8}\right)\right]$

$w_3 = 3\left[\cos\left(\frac{15\pi}{8}\right) + i\sin\left(\frac{15\pi}{8}\right)\right]$

d) Let $z = (1-i)$

$r = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$\theta = 2\pi - \tan^{-1}\left|\frac{-1}{1}\right|$ (z in QIV)

$= 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

$z = \sqrt{2}\left[\cos\left(\frac{7\pi}{4} + 2n\pi\right) + i\sin\left(\frac{7\pi}{4} + 2n\pi\right)\right]$

Let $w = z^{\frac{1}{8}}$

$w = 2^{\frac{1}{16}}\left[\cos\left(\frac{7\pi}{32} + \frac{n\pi}{4}\right) + i\sin\left(\frac{7\pi}{32} + \frac{n\pi}{4}\right)\right]$

$w_0 = 2^{\frac{1}{16}}\left[\cos\left(\frac{7\pi}{32}\right) + i\sin\left(\frac{7\pi}{32}\right)\right]$

$w_1 = 2^{\frac{1}{16}}\left[\cos\left(\frac{7\pi}{32} + \frac{\pi}{4}\right) + i\sin\left(\frac{7\pi}{32} + \frac{\pi}{4}\right)\right]$

$= 2^{\frac{1}{16}}\left[\cos\left(\frac{15\pi}{32}\right) + i\sin\left(\frac{15\pi}{32}\right)\right]$

$w_2 = 2^{\frac{1}{16}}\left[\cos\left(\frac{23\pi}{32}\right) + i\sin\left(\frac{23\pi}{32}\right)\right]$

$w_3 = 2^{\frac{1}{16}}\left[\cos\left(\frac{31\pi}{32}\right) + i\sin\left(\frac{31\pi}{32}\right)\right]$

$w_4 = 2^{\frac{1}{16}}\left[\cos\left(\frac{39\pi}{32}\right) + i\sin\left(\frac{39\pi}{32}\right)\right]$

$w_5 = 2^{\frac{1}{16}}\left[\cos\left(\frac{47\pi}{32}\right) + i\sin\left(\frac{47\pi}{32}\right)\right]$

$w_6 = 2^{\frac{1}{16}}\left[\cos\left(\frac{55\pi}{32}\right) + i\sin\left(\frac{55\pi}{32}\right)\right]$

$w_7 = 2^{\frac{1}{16}}\left[\cos\left(\frac{63\pi}{32}\right) + i\sin\left(\frac{63\pi}{32}\right)\right]$