CMA6134 - Tutorial 5C

1. Compute the first two steps of the Jacobi and the Gauss-Seidel Methods with starting vector $[0,\dots,0]$.

   (a) $\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

   (b) $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$

   (c) $\begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 5 \end{bmatrix}$

2. Consider the system of equations,

$$\begin{aligned} -w + 11x - y + 3z &= 25 \\ 2w - x + 10y &= -11 \\ 3x - y + 8z &= -11 \\ 10w - x + 2y &= 6 \end{aligned}$$

   Solve the linear system by the Gauss-Seidel Method using four iterations, beginning with the approximate solution $[w, x, y, z] = [0, 0, 0, 0]$. Give your answer correct to four decimal places.

3. Given the following matrix,

$$\begin{aligned} 2x - y - 6z &= -2 \\ 4x + y - z &= 13 \\ x - 5y - z &= -8 \end{aligned}$$

   (a) Write down the coefficient matrix for the following system of equations. Is the coefficient matrix diagonally dominant?

   (b) Solve the following system of equations by using the Jacobi Method starting with $[x, y, z] = [0, 0, 0]$. Stop the iteration when the absolute relative approximate errors for *x*, *y*, and *z* are less than the tolerance 0.8.

   (c) Solve the following system of equation by using the Gauss-Seidel Method starting with $[x, y, z] = [0, 0, 0]$. Stop the iteration when the absolute relative approximate errors for *x*, *y*, and *z* are less than the tolerance 0.8.

4. Determine whether the Jacobi and Gauss-Seidel methods converge for all the initial guesses for the following system of linear equations.

$$2u - v = 4$$
$$-u + 2v = 5$$

5. Is $\lambda = 2$ an eigenvalue of $\begin{bmatrix} 3 & 2 \\ 3 & 8 \end{bmatrix}$?

6. Find the characteristic equation and the eigenvalues of the matrices.

(a) $\begin{bmatrix} 2 & 7 \\ 7 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$

(d) $\begin{bmatrix} 5 & 3 \\ -4 & 4 \end{bmatrix}$

7. Given matrix, $M = \begin{bmatrix} 3 & 2 \\ 3 & -2 \end{bmatrix}$, find its characteristic equation, eigenvalues, and eigenvectors (as basis).

8. Is $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$ an eigenvector of $\begin{bmatrix} -3 & 1 \\ -3 & 8 \end{bmatrix}$? If so, find the eigenvalue.

9. Find the basis for the eigenspace corresponding to each listed eigenvalue.

(a) $A = \begin{bmatrix} 5 & 0 \\ 2 & 1 \end{bmatrix}, \lambda_1 = 1, \lambda_2 = 5$

(b) $A = \begin{bmatrix} 4 & -2 \\ -3 & 9 \end{bmatrix}, \lambda = 10$