Exercise

A. Definition of derivatives

1. Use the **definition of derivative** to find the derivative.

(a) $f(x) = 3$ (b) $g(x) = -5$

(c) $f(x) = -5x$ (d) $f(x) = 3x + 2$

(e) $f(x) = x^3 - 12x$ (f) $f(x) = \dfrac{1}{x^2}$

(g) $f(x) = \dfrac{1}{x-1}$ (h) $f(x) = \sqrt{x+1}$

2. By using the formal definition of derivatives, find the *f '(a)* of the following functions.

(a) $f(x) = 5x - 1$, $a = 2$ (b) $f(x) = \sqrt{x+2}$, $a = 0$

B. Differentiation Rules

3. Find the derivative of the following functions (by derivatives rules). Show the intermediate steps.

(a) $f(x) = 50.1$ (b) $y = \sqrt[5]{x}$

(c) $g(x) = 5x^8 + 2x^2 + 7x$ (d) $g(x) = x^{-2} + 2x^{-1} + 7$

(e) $h(t) = \dfrac{1}{2t^6} - \dfrac{1}{3t^4} + \dfrac{1}{t}$ (f) $y = \dfrac{(x^2 - 2x)}{x^4}$

(g) $y = \sqrt{x}(x-1)$ (h) $f(x) = (x^3 - 1)(x+1)$

(i) $y = \dfrac{5x^3 + 2}{-4x^2 + 5}$ (j) $y = \dfrac{x^2 + 4x + 3}{\sqrt{x}}$

4. Find the derivative of the trigonometric functions.

(a) $f(x) = x + 3\sin x$ (b) $f(x) = 2\sin x - 5\tan x$

(c) $f(x) = x\sin x$ (d) $y = 2\csc x + 3\cos x$

(e) $y = \dfrac{2 - 3\sin x}{\cos x}$ (f) $y = \dfrac{1 - \cos x}{\sin x}$

5. Find the derivative of the following functions (Chain Rule).

Function	*Derivative*
$y = \tan 4x$	$y' = 4\sec^2(4x)$
$y = \sin\left(\frac{1}{4}x\right)$	$y' =$
$y = e^{4x}$	$y' =$
$y = e^{\frac{1}{4}x}$	$y' =$
$y = \ln(4x)$	$y' =$

6. Find the derivative of the following functions (by using Chain Rule). Show all the detailed steps.

(a) $F(x) = (x^3 - 4x)^7$ (b) $f(x) = \sqrt[4]{1 + 2x - x^4}$

(c) $g(x) = (2 + x^4)^{2/3}$ (d) $F(x) = \tan(\sin x)$

(e) $F(x) = \cos(x^3)$ (f) $F(x) = xe^{x^2}$

(g) $y = e^{5x}\cos(3x+1)$ (h) $f(x) = \ln(x^2 - 10)$

(i) $f(x) = \ln(\cos x)$ (j) $f(x) = \sqrt[5]{\ln x}$

7. Find the second derivative $f''(x)$ of the functions
   (a) $y = x\sin x$.
   (b) $y = x^2 + 1$

C. The derivative as a rate of change

8. **Particle motion** At time t, the position of a body moving along the s-axis is $s = t^3 - 6t^2 + 9t$ m.
   a. Find the body's acceleration each time the velocity is zero.
   b. Find the body's speed each time the acceleration is zero.
   c. Find the total distance traveled by the body from $t = 0$ to $t = 2$.

9. A particle moves with position function

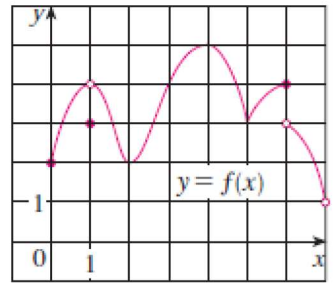
$$s = t^4 - 4t^3 - 20t^2 + 20t \qquad t \geqslant 0$$

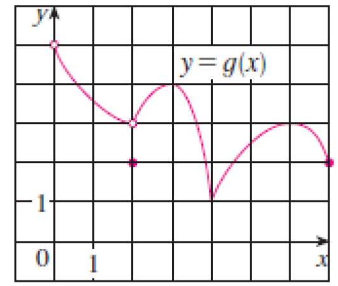
   (a) At what time does the particle have a velocity of 20 m/s?
   (b) At what time is the acceleration 0? What is the significance of this value of t?

10. If a ball is thrown vertically upward with a velocity of 80 ft/s, then its height after t seconds is $s = 80t - 16t^2$.
   (a) What is the maximum height reached by the ball?
   (b) What is the velocity of the ball when it is 96 ft above the ground on its way up? On its way down?

D. Minimum and maximum values

11. Use the graph to state the absolute and local maximum and minimum values of the function.





12. Find the critical numbers of the function.
   (a) $f(x) = 4 + \frac{1}{3}x - \frac{1}{2}x^2$
   (b) $f(x) = 2x^3 - 3x^2 - 36x$
   (c) $f(x) = x^3 + 6x^2 - 15x$
   (d) $f(x) = 2x^3 + x^2 + 2x$

13. Find the absolute maximum and absolute minimum values of f on the given interval.
   (a) $f(x) = 12 + 4x - x^2$, $[0, 5]$
   (b) $f(x) = 5 + 54x - 2x^3$, $[0, 4]$
   (c) $f(x) = 2x^3 - 3x^2 - 12x + 1$, $[-2, 3]$
   (d) $f(x) = x^3 - 6x^2 + 5$, $[-3, 5]$
   (e) $f(x) = 3x^4 - 4x^3 - 12x^2 + 1$, $[-2, 3]$
   (f) $f(x) = (x^2 - 1)^3$, $[-1, 2]$

Exercise: Topic 4

A.

1. (a) $f'(x)=0$ (b) $g'(x)=0$
   (c) $f'(x)=-5$ (d) $f'(x)=3$
   (e) $f'(x)=3x^2-12$ (f) $f'(x)=-2/x^3$
   (g) $f'(x) = -\frac{1}{(x-1)^2}$ (h) $f'(x)=\frac{1}{2\sqrt{x+1}}$

2. (a) $f'(2)=5$ (b) $f'(0) = \frac{1}{2\sqrt{2}}$

B.

3. (a) $f'(x)=0$ (b) $y' = \frac{1}{5x^{4/5}}$
   (c) $g'(x) = 40x^7 + 4x + 7$ (d) $g'(x) = \frac{-2}{x^3} - \frac{2}{x^2}$
   (e) $h'(t) = \frac{-3}{t^7} + \frac{4}{3t^5} - \frac{1}{t^2}$ (f) $y' = \frac{-2}{x^3} + \frac{6}{x^4}$
   (g) $y' = \frac{3}{2}\sqrt{x} - \frac{1}{2\sqrt{x}}$ (h) $f'(x) = 4x^3 + 3x^2 - 1$
   (i) $y' = \frac{-20x^4+75x^2+16x}{(-4x^2+5)^2}$ (j) $y' = \frac{3}{2}\sqrt{x} + \frac{2}{\sqrt{x}} - \frac{3}{2\sqrt{x}^3}$

4. (a) $f'(x) = 1 + 3\cos x$ (b) $f'(x) = 2\cos x - 5sec^2x$
   (c) $f'(x) = x\cos x + \sin x$ (d) $\frac{dy}{dx} = -2\csc x \cot x - 3\sin x$
   (e) $dy/dx = \frac{2\sin x - 3}{\cos^2 x}$ (f) $dy/dx = \frac{1-\cos x}{\sin^2 x}$

5. $y' = \frac{1}{4}\cos\frac{1}{4}x$
   $y' = 4e^{4x}$
   $y' = \frac{1}{4}e^{\frac{1}{4}x}$
   $y' = \frac{1}{x}$

6. (a) $7(x^3 - 4x)^6(3x^2 - 4)$ (b) $\frac{(2-4x^3)}{4(1+2x-x^4)^{1/2}}$
   (c) $\frac{8x^3}{3(2+x^4)^{1/3}}$ (d) $\cos x \sec^2(\sin x)$
   (e) $-3x^2\sin(x^3)$ (f) $2x^2e^{x^2} + e^{x^2}$
   (g) $-3e^{5x}\sin(3x+1) + 5e^{5x}\cos(3x+1)$ (h) $\frac{2x}{x^2-10}$
   (i) $-\tan x$ (j) $\frac{1}{5x}(\ln x)^{-4/5}$

7. (a) $2\cos x - x\sin x$ (b) 2

C.

8. (a) $a(1)=-6\ m/s^2$, $a(3)=6\ m/s^2$
   (b) $v(2)=-3\ m/s$
   (c) Total distance $=6\ m$

9. (a) $t=0,\ 5$
   (b) $t = 1 + \sqrt{\frac{13}{3}}$, velocity is constant (there is no change in velocity)

10. (a) 100 ft
    (b) Velocity on its way up: 16 ft/s
    (c) Velocity on its way down: -16 ft/s

D.

11.

	Graph 1	Graph 2
Absolute maximum	*f(4)=5*	*There is no abs max value*
Absolute minimum	*There is no abs min value*	*f(4)=1*
Local maximum	*f(4)=5 and f(6)=4*	*f(3)=4 and f(6)=3*
Local minimum	*f(1)=f(5)=3 and f(2)=2* *Note:f(0) is not a local min because it occurs at endpoint.*	*f(2)=2 and f(4)=1*

Endpoints can be either absolute max/min but not local max/min

12. (a) $x=1/3$
    (b) $x=-2,\ 3$
    (c) $x=-5,1$
    (d) *There is no critical number*

13.

Absolute maximum	Absolute minimum
$f(2)=16$	$f(5) = 7$
$f(3) =93$	$f(4) =-291$
$f(-1) =8$	$f(2) =-19$
$f(0) = 5$	$f(-3) =-76$
$f(3) =28$	$f(2) =-31$
$f(2) =27$	$f(0) =-1$