STUDENT ID NO

MULTIMEDIA UNIVERSITY

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FINAL EXAMINATION

TRIMESTER 3, 2015/2016

PMT0301 – MATHEMATICS III

(All sections/ Groups)

30 MAY 2016
2.30 p.m. – 4.30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of **THREE** printed pages excluding cover page and statistical table.
2. Answer **ALL** FOUR questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the **ANSWER BOOKLET**. All necessary working steps **MUST** be shown.

Question 1

a) Find an equation of the plane passing through the point (1, -1, 3) that is perpendicular to the planes $2x + y + 3z = -1$ and $x + 2y - 2z = 7$.

[7 marks]

b) Find a formula for the general term a_n of the following sequence, assuming that the pattern of the first few terms continues.

2, -4, 8, -16, …

[1.5 marks]

c) Write out the term containing x^5y^4 in the expansion of $(2x - y)^9$. Simplify your answer to the simplest form. [1.5 marks]

Solution :

a) Vector **u** perpendicular to plane $2x + y + 3z = -1$, **u** = <2, 1, 3>

Vector **v** perpendicular to plane $x + 2y - 2z = 7$, **v** = <1, 2, -2>

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} i & j & k \\ 2 & 1 & 3 \\ 1 & 2 & -2 \end{vmatrix}$$

$$= (-2 - 6)i - (-4 - 3)j + (4 - 1)k = \langle -8, 7, 3 \rangle$$

Equation of plane:

$$\langle -8, 7, 3 \rangle \cdot \langle x - 1, y + 1, z - 3 \rangle = 0$$
$$-8(x - 1) + 7(y + 1) + 3(z - 3) = 0$$
$$-8x + 8 + 7y + 7 + 3z - 9 = 0$$
$$-8x + 7y + 3z + 6 = 0$$

b) -4/2 = -2, 8/-4 = -2, -16/8 = -2

Geometric sequence: $a_n = 2(-2)^{n-1}$

c) the term containing x^5 : $\binom{9}{4}(2x)^5(y)^4$

$$= 4032x^5y^4$$

Question 2

a) Solve the following system of linear equations by using inverse matrix.

$$2x + y - 3z = 7$$
$$-x + 5z = -7$$
$$x - 2y + z = 1$$

[5 marks]

b) How many different ways can 2 red pens, 3 blue pens, 4 green pens and 2 yellow pens be arranged in a row? [1.5 marks]

c) Given the word "ALGORITHMS", how many 5-letter codes can be formed from the word if the code must contain the letter "A"? [1 mark]

d) How many different ways to have five couples to be seated in a row if all the male are to sit together? [1 mark]

e) An organization consists of 24 engineers from 8 different divisions with three engineers from each division. A special committee is to be formed consisting of 4 engineers. How many possible ways to form the committee if the committee members are to be from different division?

[1.5 marks]

Solution :

a) $A = \begin{bmatrix} 2 & 1 & -3 \\ -1 & 0 & 5 \\ 1 & -2 & 1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 7 \\ -7 \\ 1 \end{bmatrix}$

$$c_{11} = 10 \quad c_{12} = 6 \quad c_{13} = 2$$
$$c_{21} = 5 \quad c_{22} = 5 \quad c_{23} = 5$$
$$c_{31} = 5 \quad c_{32} = -7 \quad c_{33} = 1$$

$$|A| = (-1)(5) + (0)(5) + (5)(5) = 20$$

$$A^{-1} = \frac{1}{|A|} AdjA = \frac{1}{20} \begin{bmatrix} 10 & 5 & 5 \\ 6 & 5 & -7 \\ 2 & 5 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \\ x \end{bmatrix} = A^{-1}B$$

$$= \frac{1}{20}\begin{bmatrix} 10 & 5 & 5 \\ 6 & 5 & -7 \\ 2 & 5 & 1 \end{bmatrix}\begin{bmatrix} 7 \\ -7 \\ 1 \end{bmatrix}$$

$$= \frac{1}{20}\begin{bmatrix} 40 \\ 0 \\ -20 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$$

$$\therefore\ x = 2, \quad y = 0, \quad z = -1$$

b) $\dfrac{11!}{2!3!4!2!} = 69300$

c) ${}^{9}P_4 \times {}^{1}P_1 \times 5 = 15120$

d) $6! \times 5! = 86400$

e) ${}^{8}C_4 \times 3^4 = 5670$

Continued…

Question 3

a) The following table shows the money spent for entertainment on weekends by a sample of 40 students.

Money (in RM)	Number of students
20 – 29	5
30 – 39	9
40 – 49	14
50 – 59	8
60 – 69	4
TOTAL	**40**

i) Calculate the mean. Give your answer correct to 2 decimal places.
[1.5 marks]

ii) Calculate the median. Give your answer correct to 2 decimal places.
[2 marks]

iii) Calculate the mode. Give your answer correct to 2 decimal places.
[2 marks]

iv) Calculate the standard deviation. Give your answer correct to 2 decimal places.
[1.5 marks]

b) The light bulbs manufactured by an electrical firm have a lifespan that is normally distributed with mean 800 hours and a standard deviation of 40 hours. Find the probability that a bulb burns between 778 and 834 hours.
[3 marks]

Solution :

a)

i)
$$mean = \frac{24.5(5) + 34.5(9) + 44.5(14) + 54.5(8) + 64.5(4)}{40}$$
$$= \frac{1750}{40} = 43.75$$

ii) Location of median = $\frac{n+1}{2} = 20.5$

Class median 40 – 49

$$median = 39.5 + \frac{(20.5-14)}{14}10$$
$$= 44.14$$

iii) Class mode $40 - 49$

$$\text{mod}\, e = 39.5 + \frac{(5)}{5+6}10$$
$$= 44.05$$

iv) Standard deviation

$$= \sqrt{\frac{\sum m_i^2 f_i - \frac{\left(\sum m_i f_i\right)^2}{\sum f_i}}{\sum f_i - 1}}$$
$$= \sqrt{\frac{81840 - \frac{(1750)^2}{40}}{39}}$$
$$= 11.63$$

b) $X \sim N(800, 40^2)$

$$P(778 < X < 834)$$
$$= P\left(\frac{778-800}{40} < Z < \frac{834-800}{40}\right)$$
$$= P(-0.55 < Z < 0.85)$$
$$= P(Z < 0.85) - P(Z < -0.55)$$
$$= 0.8023 - 0.2912$$
$$= 0.5111$$

Continued…

Question 4

a) Given the following contingency table

	Mathematics (M)	Science (S)	English (E)	Total
Class A (A)	5	3	2	10
Class B (B)	3	3	4	10
Total	8	6	6	20

Find the following

i) $P(B \cup E)$ [2 marks]

ii) $P(M|A)$ [2 marks]

b) Find the probability that in tossing a fair coin three times, there will appear

i) 3 heads [1 mark]

ii) 2 tails and 1 head [1 mark]

iii) at least 1 head [1 mark]

c) Let X denotes the number of flaws on the surface of a randomly selected boiler. Assume X has a Poisson distribution with parameter $\lambda = 5$. Compute the following probabilities

i) $P(X \leq 1)$ [2 Marks]

ii) $P(X \geq 2)$ [1 Mark]

[Answer correct to 4 decimal places]

Solution:

a)

i)

$$P(B \cup E)$$
$$= P(B) + P(E) - P(B \cap E)$$
$$= \frac{10}{20} + \frac{6}{20} - \frac{4}{20} = \frac{12}{20} = \frac{3}{5}$$

ii)
$$P(M|A)$$
$$= \frac{P(M \cap A)}{P(A)}$$
$$= \frac{5/20}{10/20} = \frac{10}{20} = \frac{1}{2}$$

b)

i)
$$P(HHH)$$
$$= \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

ii)
$$P(\text{2 tails and 1 head})$$
$$= P(HTT \cup TTH \cup THT)$$
$$= P(HTT) + P(TTH) + P(THT)$$
$$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$$

iii)
$$P(\text{at least 1 head})$$
$$= 1 - P(\text{no head})$$
$$= 1 - P(TTT)$$
$$= 1 - \frac{1}{8} = \frac{7}{8}$$

c)

i)

$$
\begin{aligned}
&P(X \le 1) \\
&= P(X = 0) + P(X = 1) \\
&= e^{-5}\left(\frac{5^0}{0!} + \frac{5^1}{1!}\right) \\
&= 6e^{-5} \\
&\approx 0.0404
\end{aligned}
$$

ii)

$$
\begin{aligned}
&\mathrm{P}(\mathrm{X} \ge 2) \\
&= 1\text{-}P(X \le 1) \\
&= 1 - 0.0404 \\
&= 0.9596
\end{aligned}
$$