FINAL EXAM
MARKING SCHEME
TRIMESTER II, 2016/2017
PMT0101 – MATHEMATICS I

Question 1 [10 marks]
Solution:

a) $\dfrac{\left(2a^{\frac{2}{3}}b^{-1}\right)^3}{2ab^{-1}}$

$$= \frac{2^3\left(a^{\frac{2}{3}}\right)^3\left(b^{-1}\right)^3}{2ab^{-1}}$$
$$= \frac{8a^2b^{-3}}{2ab^{-1}}$$
$$= 4a^{2-1}b^{-2}$$
$$= \frac{4a}{b^2} \qquad [0.5 + 0.5 + 0.5 + 0.5]$$

b) $\dfrac{\sqrt{2}-3}{3+\sqrt{2}}$

$$= \frac{\sqrt{2}-3}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}}$$
$$= \frac{3\sqrt{2}-2-9+3\sqrt{2}}{9-2}$$
$$= \frac{6\sqrt{2}-11}{7} \qquad [0.5+1+0.5]$$

c)

$$\frac{x^3-64}{x-4}$$
$$=\frac{x^3-4^3}{x-4}$$
$$=\frac{(x-4)(x^2+4x+16)}{x-4}$$
$$=x^2+4x+16 \qquad [0.5+1+0.5]$$

d)

i)
$$z=\frac{4i}{4+3i}$$
$$=\frac{4i}{(4+3i)}\frac{(4-3i)}{(4-3i)}$$
$$=\frac{16i-12i^2}{16-9i^2}$$
$$=\frac{-12(-1)+16i}{16-9(-1)}$$
$$=\frac{12+16i}{25}$$
$$=\frac{12}{25}+\frac{16}{25}i \qquad [0.5+0.5+0.5+0.5+0.5]$$

ii)
$$\bar{z}=\frac{12}{25}-\frac{16}{25}i \qquad [0.5]$$

Question 2 [10 marks]
Solution:

a) $2x^2 + (2+h)x + 2 = 0$

Exactly one solution ➔ $b^2 - 4ac = 0$

$(2+h)^2 - 4(2)(2) = 0$ [0.5]

$h^2 + 4h + 4 - 16 = 0$

$h^2 + 4h - 12 = 0$ [0.5]

$(h+6)(h-2) = 0$ [0.5]

$h + 6 = 0$ or $h - 2 = 0$

$h = -6$ or $h = 2$ [0.5]

b) $\dfrac{x+5}{x-1} \le 0$

		-5		1	
Sign of $(x+5)$	---	+++	+++	[0.5]
Sign of $(x-1)$	---	---	+++	[0.5]
Sign of $\dfrac{x+5}{x-1}$	+++	---	+++	[0.5]

The solution set is $[-5,\ 1)$ [0.5]

iii) Domain of $h(x) = (-\infty,\ -5] \cup (1,\ \infty)$ [0.5 + 0.5]

c)

i) $|3-\pi| = -(3-\pi) = \pi - 3$ [0.5 + 0.5]

ii) $|x-\pi| = 3$

$x - \pi = \pm 3$

$x = \pi \pm 3$ [0.5 + 0.5]

d) $\sqrt{15-2x}=x$

$15-2x=x^2$

$x^2+2x-15=0$ [0.5]

$(x+5)(x-3)=0$ [0.5]

$x=-5$ *or* $x=3$ [0.5]

Checking:

When $x=-5$	When $x=3$
LHS:	LHS:
$\sqrt{15-2(-5)}$ $=\sqrt{25}$ $=5$	$\sqrt{15-2(3)}$ $=\sqrt{9}$ $=3$
RHS:	RHS:
-5	3
LHS $\neq$ RHS [0.5]	LHS=RHS [0.5]

Therefore, the only solution is $x=3$ [0.5]

Question 3 [10 marks]

a)

i)

$$
\begin{array}{r}
3x^2 - x - 2 \\
x-2\overline{)\,3x^3 - 7x^2 + 0x + 4} \\
\underline{3x^3 - 6x^2} \\
-x^2 + 0x \\
\underline{-x^2 + 2x} \\
-2x + 4 \\
\underline{-2x + 4} \\
0
\end{array}
$$

[0.5+0.5+0.5+0.5]

Quotient = $3x^2 - x - 2$ [0.5]
Remainder = 0 [0.5]

ii)

$$P(x) = x^3 - 7x^2 + 4$$
$$= (x-2)(3x^2 - x - 2) \quad [0.5]$$
$$= (x-2)(3x+2)(x-1) \quad [0.5]$$

iii)

$$(x-2)(3x+2)(x-1) = 0$$
$$x = 2 \quad or \quad x = -\frac{2}{3} \quad or \quad x = 1 \quad [1]$$

b) $f(x) = (x+2)^2(x-1)^2(x-4)$

i) Degree = 5 [0.5]

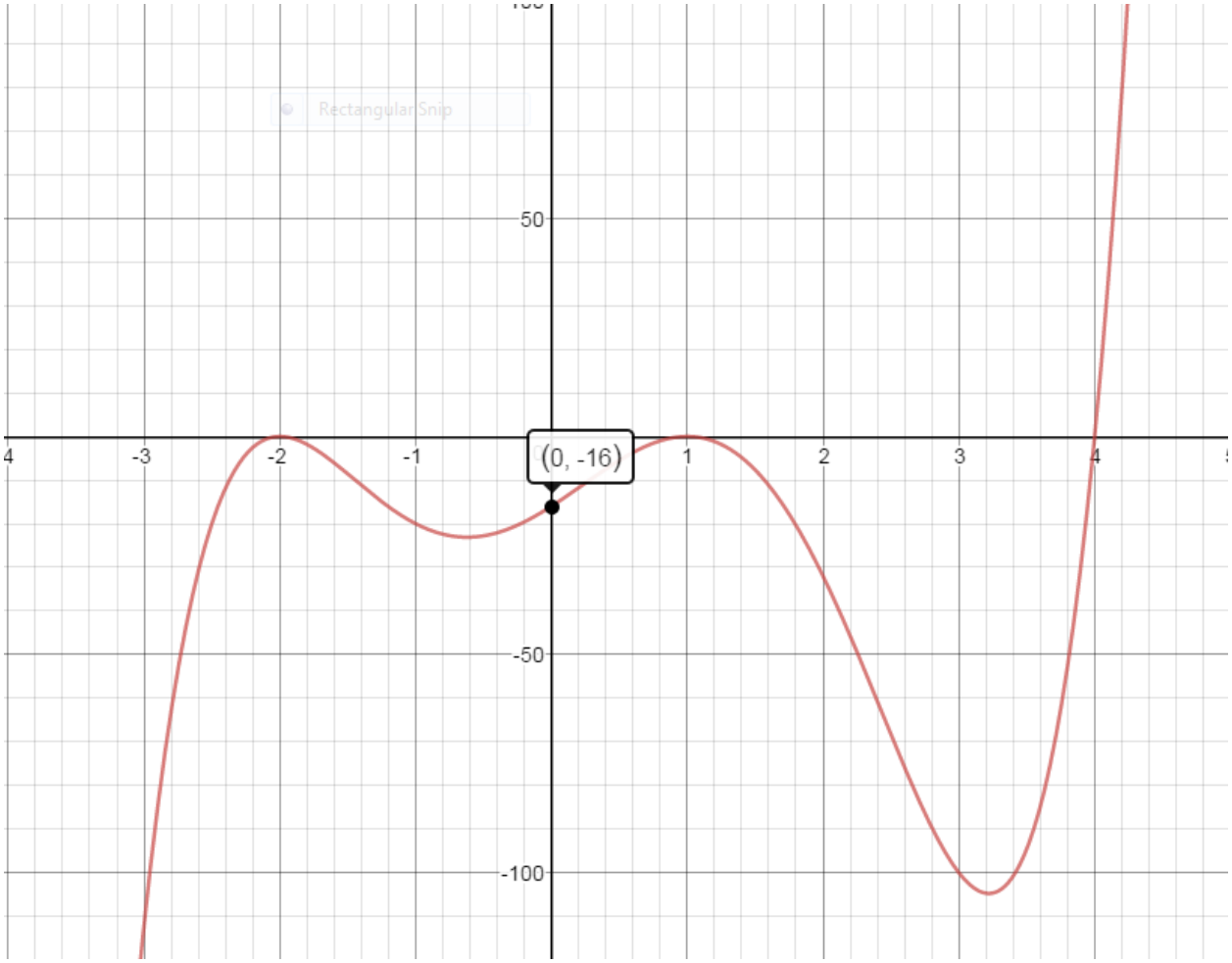
ii) T**he zeros** of f are $-2,\ 1$ and 4

Zeros	Multiplicities	Cross/Touch
-2	2	Touches
1	2	Touches
4	1	Crosses
[0.5]	[0.5]	[0.5]

iii) y-intercept, $f(0) = -16$. [0.5]

iv) As $x \to -\infty,\ y \to -\infty$ [0.5]
As $x \to \infty,\ y \to \infty$ [0.5]

v) Sketch the graph:



Shows all intercepts	[0.5]
Proper end behaviour	[0.5]
Shows correct crossing or touching at *x*-intercepts	[0.5]

Question 4 [10 marks]

a)

i) 1 [0.5]

ii) $\{3,\ 4,\ 5,\ 6\}$ [0.5]

b)

i)

$$(f \circ g)(2)$$
$$= f(g(2))$$
$$= f(2^2 + 5)$$
$$= f(9)$$
$$= \sqrt{9-3}$$
$$= \sqrt{6} \qquad [0.5 + 0.5 + 0.5]$$

ii)

Let $y = \sqrt{x-3}$

$$y^2 = x - 3$$
$$x = y^2 + 3$$
$$f^{-1}(x) = x^2 + 3 \qquad [0.5 + 0.5 + 0.5]$$

iii) Domain of $f^{-1} = [0,\ \infty)$ [0.5]

Range of $f^{-1} = [3,\ \infty)$ [0.5]

c)

$$\ln(3-k) - 1 = -1$$
$$\ln(3-k) = 0$$
$$3 - k = e^0$$
$$3 - 1 = k$$
$$k = 2 \qquad [0.5 + 0.5 + 0.5 + 0.5]$$

d)

$$2^{2x+3} = 16$$
$$2^{2x+3} = 2^4$$
$$2x + 3 = 4$$
$$2x = 1$$
$$x = \frac{1}{2} \qquad [0.5 + 0.5]$$

e) $\log_3 x = 2 - 3\log_3 y,$

$\log_3 x + \log_3 y^3 = 2$

$\log_3 xy^3 = 2$

$xy^3 = 3^2$

$x = \dfrac{9}{y^3}$ [0.5 + 0.5 + 0.5 + 0.5]

Question 5 [10 marks]

a)i)

$m = -\dfrac{1}{4}$

$\dfrac{t-0}{0-8} = -\dfrac{1}{4}$

$t = -\dfrac{1}{4} \times -8$

$t = 2$ [0.5 + 0.5]

ii)

$m_{PQ} = -\dfrac{1}{4}$

$-\dfrac{1}{4} \times m_{perpendicular} = -1$

$m_{perpendicular} = 4$ [0.5]

Equation of perpendicular line:

$y - 0 = 4(x - 8)$

$y = 4x - 32$ [0.5 + 0.5]

iii)

$T = \left(\dfrac{2(8) + 1(0)}{1+2}, \dfrac{2(0) + 1(2)}{1+2} \right)$

$= \left(\dfrac{16}{3}, \dfrac{2}{3} \right)$ [1 + 0.5]

iv)

$SP = 5$

$\sqrt{(x-8)^2 + (y-0)^2} = 5$

$x^2 - 16x + 64 + y^2 = 25$

$x^2 + y^2 - 16x + 39 = 0$ $[0.5 + 1 + 0.5]$

b)

i)

$Slope = m = \frac{20-8}{5-1} = \frac{12}{4} = 3$ $[0.5 + 0.5]$

$Y = mX + c$

$8 = 3(1) + c$

$c = 5$

Y – intercept is 5 [0.5+ 0.5]

ii)

$y = pq + px$

$\frac{y}{x} = pq\left(\frac{1}{x}\right) + p$

$Y = (pq)X + p$ [0.5]

$p = 5$ [0.5]

$pq = 3$

$q = \frac{3}{5}$ $[0.5 + 0.5]$