CMA6134 - Tutorial 6

1. Given the three following tables, find the equation of the best fit linear line using the method of least squares, $y = a + bx$

   (a) Table 1

x_i	-2	-1	0	1	2
y_i	1	2	3	3	4

   (b) Table 2

x_i	-4	-2	0	2	4
y_i	1.2	2.8	6.2	7.8	13.2

   (c) Table 3

x_i	-6	-2	0	2	6
y_i	-5.3	-3.5	-1.7	0.2	4.0

2. Two tables are given as the following.

   Table 1

x_i	1	2	3	4	5
y_i	0.6	1.9	4.3	7.6	12.6

   Table 2

x_i	4.0	4.2	4.5	4.7	5.1
y_i	102.56	113.18	130.11	142.05	167.53

   From the above tables, find

   (a) the least squares exponential of the form $y = ae^{bx}$,

   (b) the least squares exponential of the form $y = ax^b$,

   (c) the values of y when $x = 2.8$ in Table 1 and $x = 4.85$ in Table 2 for (a) and (b) respectively.

3. Given a polynomial function $f(x) = e^{\frac{-x^2}{2}}$, find:

   (a) the cubic Mclaurin polynomial, $P_3(x)$. (b) Approximate $f(x)$ when $x = 1$.

   (c) Find the absolute error and relative error in (b).

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4. A polynomial function is given as $\sqrt{x}$ at $x_0 = 1$.

    (a) Find the Taylor Polynomial of degree 3 and 5.

    (b) Calculate the absolute error at $x = 0.1$ for degree 3 and 5.

5. For a given function $f(x) = \sqrt{1+x}$, let $x_0 = 0$, $x_1 = 0.3$ and $x_2 = 0.6$.

    (a) Construct the Lagrange coefficient $L_0(x)$, $L_1(x)$, and $L_2(x)$.

    (b) Find $f(x_0), f(x_1), f(x_2)$.

    (c) Find the second interpolating polynomial $P_2(x)$.

    (d) Approximate $f(0.45)$.

6. Consider the function $f(x)$, x_0, x_1 and x_2 in Question 5.

    (a) Find the Newton polynomial, $P_2(x)$.

    (b) Use the Newton polynomial in (a) to approximate $f(0.45)$.

7. For a function f, the divided differences are given by the following table:

x_k	y_k	1st divided difference	2nd divided difference	3rd divided difference
0	0			
		1.5		
1				
2	1.5		-0.1875	
3	1.125			

    (a) Complete the table.

    (b) Find the cubic Newton Polynomial $P_3(x)$.

    (c) Approximate $f(1.5)$.