PMT0201 – MATHEMATICS II

MR. TEA BOON CHIAN

COURSE STRUCTURE

- LECTURE CLASS (ONLINE MODE) – 3 HOURS (1 + 2)
- TUTORIAL CLASS (PHYSICAL MODE) – 2 HOURS
  - MS. JULIZA BINTI MOHD JOHAR
  - MS. SURAYA BINTI MD SUYOD
  - DR. KHOR CHAI YING
  - DR. DEEPAK KUMAR

COURSE ASSESSMENTS

- QUIZZES (20%)
  - 4 QUIZZES – EACH CARRIES 5% (CONDUCTED BY TUTORS)
  - ONLINE / PHYSICAL (UPON ARRANGEMENT BY TUTORS)
- TESTS (30%)
  - 2 TESTS – EACH CARRIES 15% (CONDUCTED BY ME)
  - ONLINE / PHYSICAL (SUBJECT TO CHANGE)
- FINAL EXAMINATION (50%) - PHYSICAL MODE

COURSE CONTENT

- CHAPTER 1 – TRIGONOMETRY
- CHAPTER 2 – ANALYTIC TRIGONOMERTRY AND COMPLEX NUMBERS
- CHAPTER 3 – LIMITS AND CONTINUITY
- CHAPTER 4 – DERIVATIVES
- CHAPTER 5 - INTEGRALS

CHAPTER 1
TRIGONOMETRY (PART 1)

LECTURE 01 – 2.11.2022

2.1 Trigonometric Functions

Objectives:

- Angle Measure
- Angles in Standard Position
- Length of a Circular Arc
- Area of a Circular Sector
- Trigonometric Functions of General Angles
- Trigonometric Functions and Their Graphs
- Transformation of Sine and Cosine
- Inverse Trigonometric Functions and Their Graphs

CHAPTER OUTLINES

2.1.1 Angles Measure

A **ray** or **half-line** is a line starts at a point V extends indefinitely in one direction. The starting point V of a ray is denoted as **vertex** (Figure 1).

Figure 1

Suppose R_1 and R_2 denote the rays from O through points A and B, respectively. An **angle** AOB is the rotation of the ray R_1 onto R_2 where R_1 is called the **initial side**, and R_2 is called the **terminal side** of the angle.

If the rotation is counterclockwise, the angle is considered **positive**, and if the rotation is clockwise, the angle is considered **negative (Figure 2)**

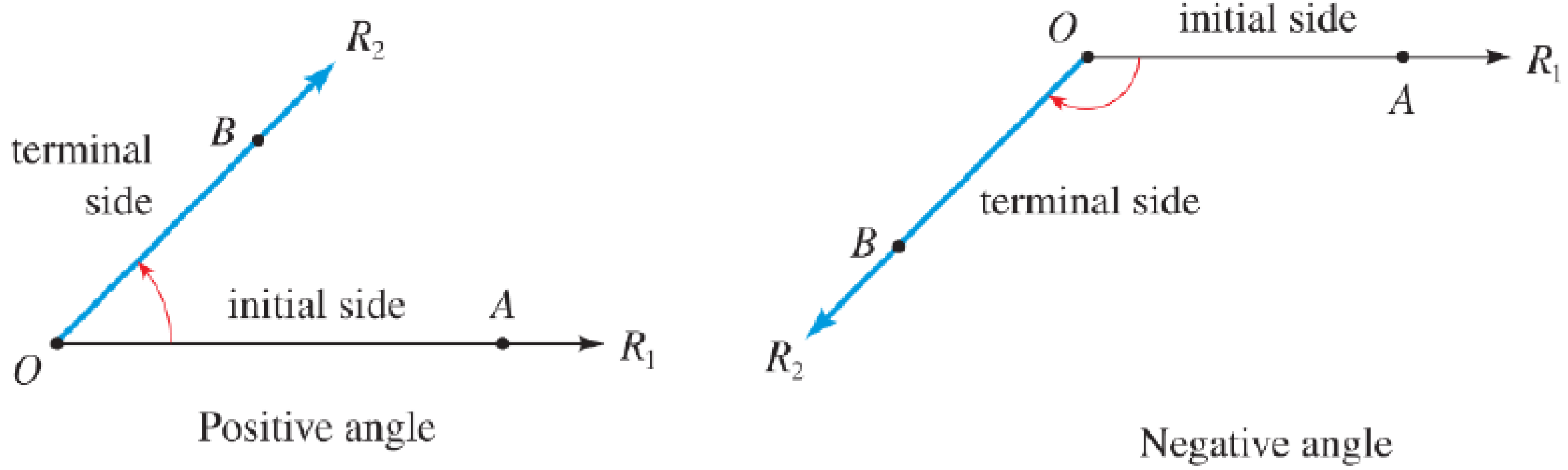


Figure 2

2.1.1.1 Angle Measure

The **measure** of an angle is the amount of rotation about the vertex required to move R_1 onto R_2. It is measured by **degree** and/or **radian**.

ONE degree measure denotes the rotation of initial side $\frac{1}{360}$ of a complete revolution.

A complete revolution is therefore 360°.

DEFINITION OF RADIAN MEASURE

If a circle of radius 1 is drawn with the vertex of an angle at its center, then the measure of this angle in **radians** (abbreviated **rad**) is the length of the arc that subtends the angle

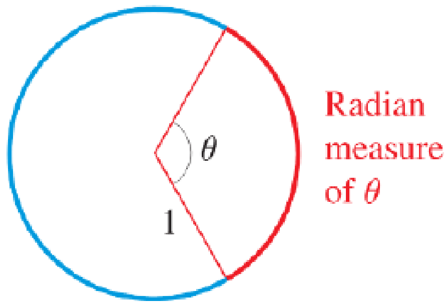


Figure 3

The circumference of the circle of radius 1 is 2π so a complete revolution has measure 2π rad.

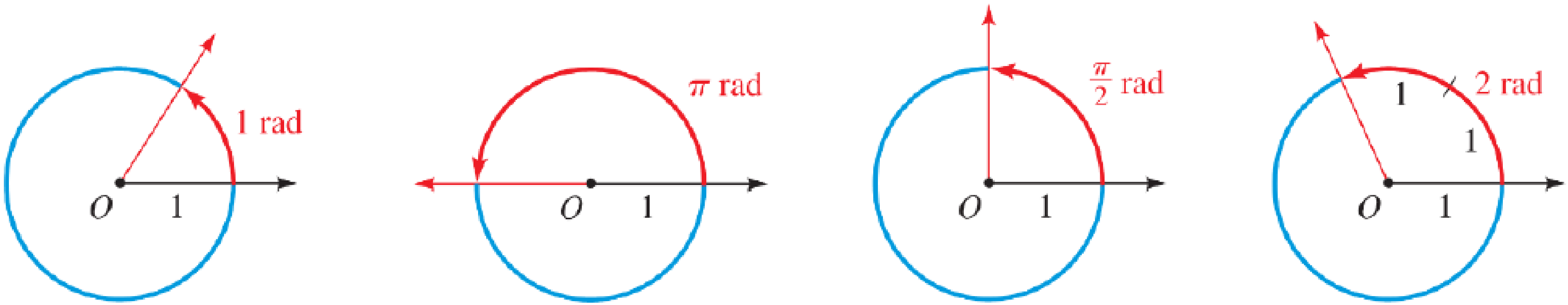


Figure 4

RELATIONSHIP BETWEEN DEGREES AND RADIANS

$$180^\circ = \pi \text{ rad} \qquad 1 \text{ rad} = \left(\frac{180}{\pi}\right)^\circ \qquad 1^\circ = \frac{\pi}{180} \text{ rad}$$

1. To convert degrees to radians, multiply by $\frac{\pi}{180}$.

2. To convert radians to degrees, multiply by $\frac{180}{\pi}$.

Degrees	0	30	45	60	90	180	360
Radians	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	2π

2.1.1.2 Angles in Standard Position

An angle is in **standard position** if it is drawn in the *xy*-plane with its vertex at the origin and its initial side on the positive *x*-axis.

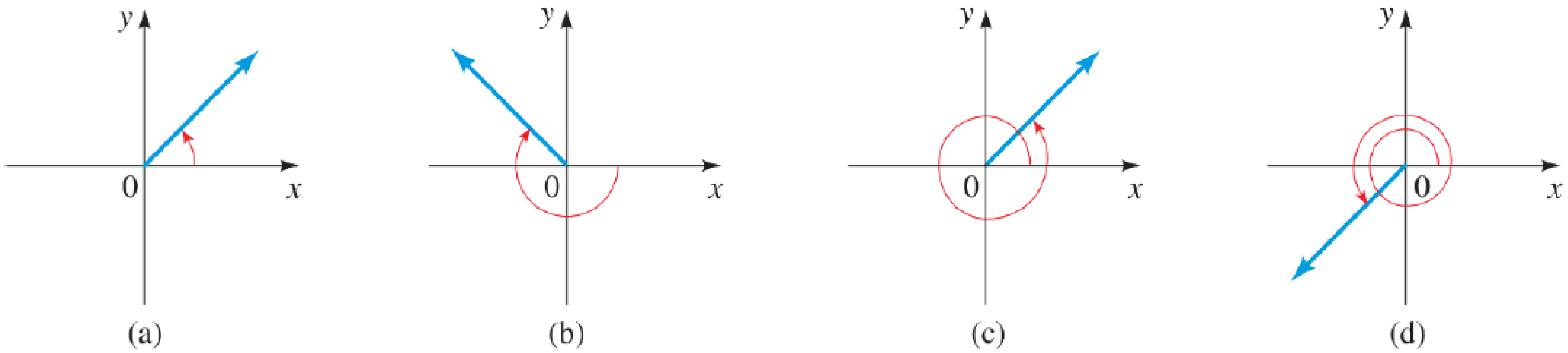


Figure 5

Two angles in standard position are **coterminal** if their sides coincide. Example Figure 5 (a) and 5(c)

Example

(a) Find angles that are coterminal with angle $\theta = 30°$ in standard position.

Solution

(a) To find positive angles that are coterminal with θ, we add any multiple of 360°. Thus

$$30° + 360° = 390° \qquad \text{and} \qquad 30° + 720° = 750°$$

are coterminal with $\theta = 30°$. To find negative angles that are coterminal with θ, we subtract any multiple of 360°. Thus

$$30° - 360° = -330° \qquad \text{and} \qquad 30° - 720° = -690°$$

are coterminal with θ. (See Figure 6.)

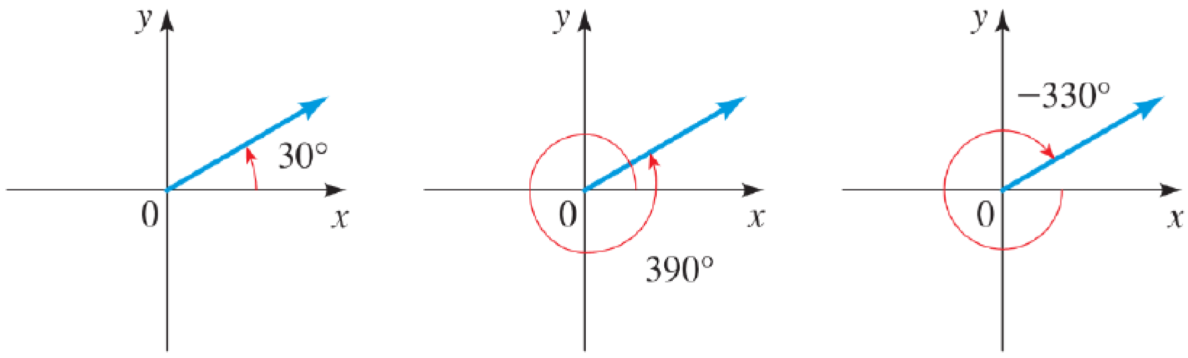


Figure 6

(b) Find angles that are coterminal with angle $\theta = \dfrac{\pi}{3}$ in standard position.

(b) To find positive angles that are coterminal with θ, we add any multiple of 2π. Thus

$$\frac{\pi}{3} + 2\pi = \frac{7\pi}{3} \qquad \text{and} \qquad \frac{\pi}{3} + 4\pi = \frac{13\pi}{3}$$

are coterminal with $\theta = \pi/3$. To find negative angles that are coterminal with θ, we subtract any multiple of 2π. Thus

$$\frac{\pi}{3} - 2\pi = -\frac{5\pi}{3} \qquad \text{and} \qquad \frac{\pi}{3} - 4\pi = -\frac{11\pi}{3}$$

are coterminal with θ. (See Figure 7.)

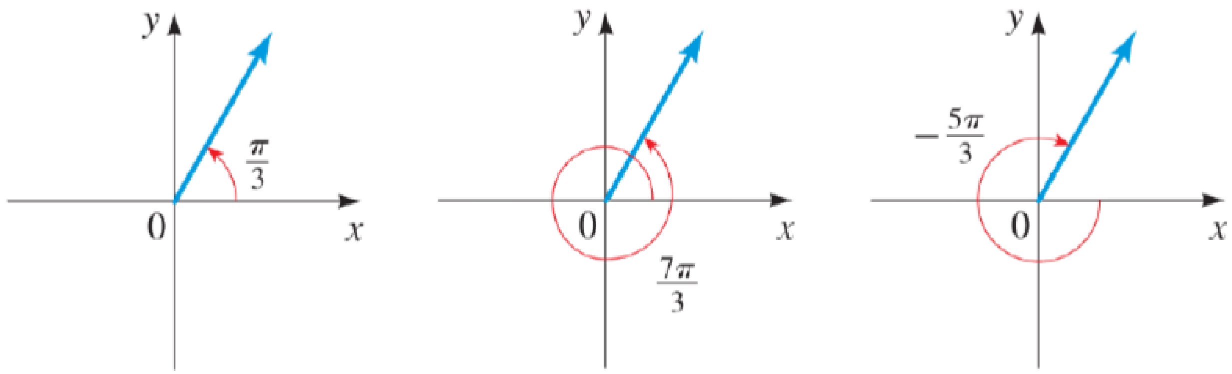


Figure 7

2.1.1.3 Length of Circular Arc

LENGTH OF A CIRCULAR ARC

In a circle of radius r, the length s of an arc that subtends a central angle of θ radians is

$$s = r\theta$$

Example

Find the length of an arc of a circle with radius 10 m that subtends a central angle of 30º.

Solution

$$30^o = \frac{\pi}{6}\text{rad} \;\; \text{thus} \;\; s = r\theta = 10\times\frac{\pi}{6} = \frac{5\pi}{3}\text{m}$$

CHAPTER 1
TRIGONOMETRY (PART 1)

LECTURE 02 – 4.11.2022

2.1.1.4 Area of a Circular Sector

AREA OF A CIRCULAR SECTOR

In a circle of radius r, the area A of a sector with a central angle of θ radians is

$$A = \frac{1}{2}r^2\theta$$

Example

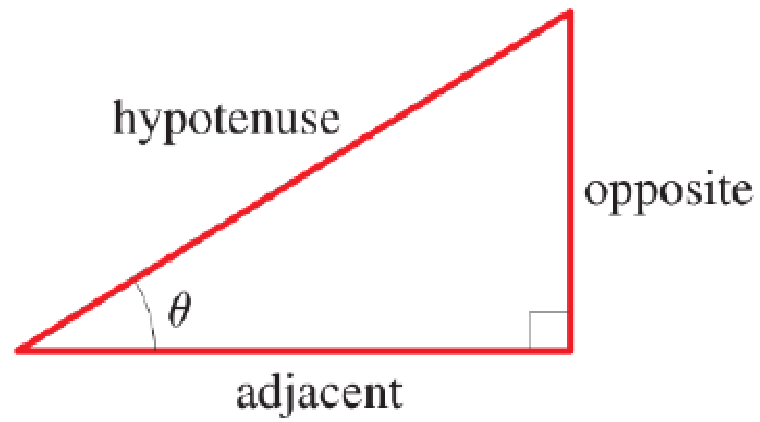
Find the area of a sector of a circle with central angle 60° if the radius of the circle is 3m.

$$A = \frac{1}{2}r^2\theta = \frac{1}{2}(3)^2\left(\frac{\pi}{3}\right) = \frac{3\pi}{2}\mathrm{m}^2$$

2.1.2 Trigonometry of Right Triangles

2.1.2.1 Trigonometric Ratios

Suppose θ denotes an acute angles of right triangle below.



The trigonometric rations are defined as follows

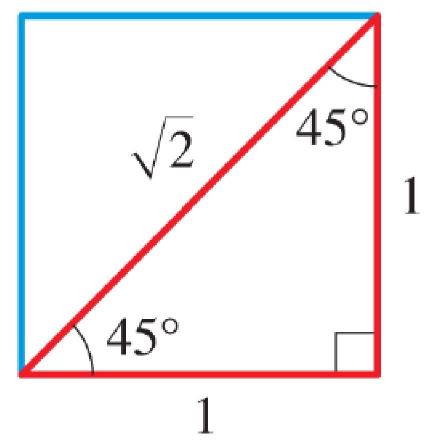
THE TRIGONOMETRIC RATIOS

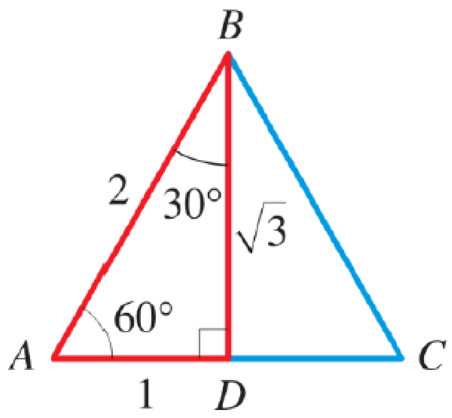
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

Special Triangles

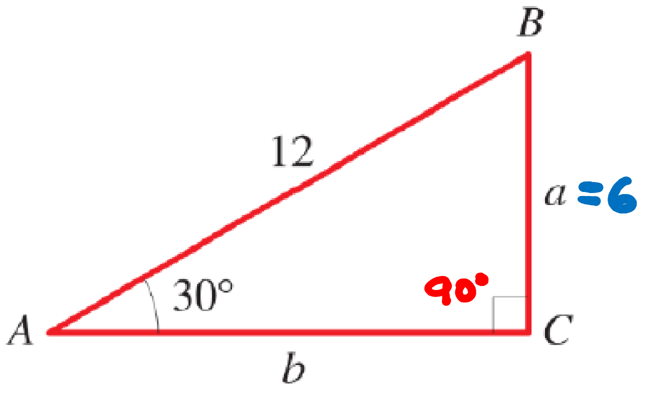
θ in degrees	θ in radians	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$30°$	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$45°$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$60°$	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$





Example

Solve triangle ABC shown below



$\measuredangle ABC = 180° - 30° - 90°$

$= 60°$

Pythagorean theorem:

$a^2 + b^2 = c^2$

$b = \sqrt{12^2 - 6^2}$

$= \sqrt{144 - 36}$

$= \sqrt{108}$

$= \sqrt{36 \times 3}$

$= 6\sqrt{3}$ #

$\sin 30° = \frac{a}{12}$

$\therefore a = 12 \sin 30°$

$= 12\left(\frac{1}{2}\right)$

$= 6$ #

$\cos 30° = \frac{b}{12}$

$\therefore b = 12 \cos 30°$

$= 12\left(\frac{\sqrt{3}}{2}\right)$

$= 6\sqrt{3}$ #

2.1.3 Trigonometric Functions of Angles

DEFINITION OF THE TRIGONOMETRIC FUNCTIONS

Let θ be an angle in standard position and let $P(x, y)$ be a point on the terminal side. If $r = \sqrt{x^2 + y^2}$ is the distance from the origin to the point $P(x, y)$, then

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x} \quad (x \neq 0)$$

$$\csc\theta = \frac{r}{y} \quad (y \neq 0) \qquad \sec\theta = \frac{r}{x} \quad (x \neq 0) \qquad \cot\theta = \frac{x}{y} \quad (y \neq 0)$$

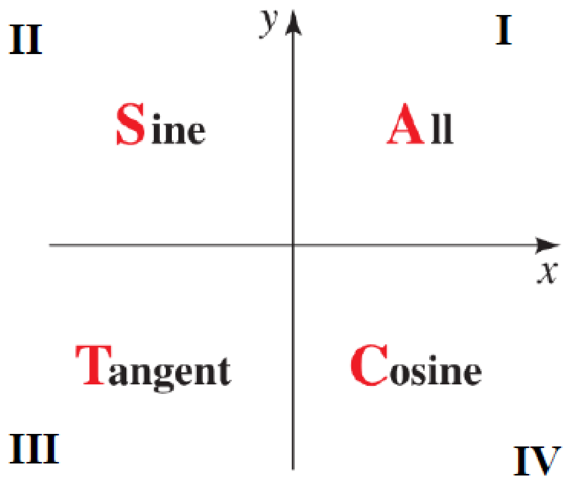
Note: At certain angle θ where x- or y-coordinate of a point on the terminal side of the angle is ZERO, some of the trigonometric functions may be undefined. This is known as **quadrantal angle**—angle that is coterminal with the coordinate axes.

2.1.3.1 Evaluating Trigonometric Functions

From the definition we see that the values of the trigonometric functions are all positive if the angle θ has its terminal side in Quadrant I. This is due to x and y are both positive. However for Quadrant II to IV not all trigonometric functions are positive.

SIGNS OF THE TRIGONOMETRIC FUNCTIONS

Quadrant	Positive Functions	Negative Functions
I	all	none
II	sin, csc	cos, sec, tan, cot
III	tan, cot	sin, csc, cos, sec
IV	cos, sec	sin, csc, tan, cot



You can remember this as "All Students Take Calculus."

Example

Find (a) $\cos 135^\circ$ and (b) $\tan 390^\circ$

a) $\cos 135^\circ = -\cos(180^\circ - 135^\circ)$

$= -\cos 45^\circ$

$= -\frac{1}{\sqrt{2}} / \frac{\sqrt{2}}{2}$ #

b) $\tan 390^\circ = \tan(390^\circ - 360^\circ)$

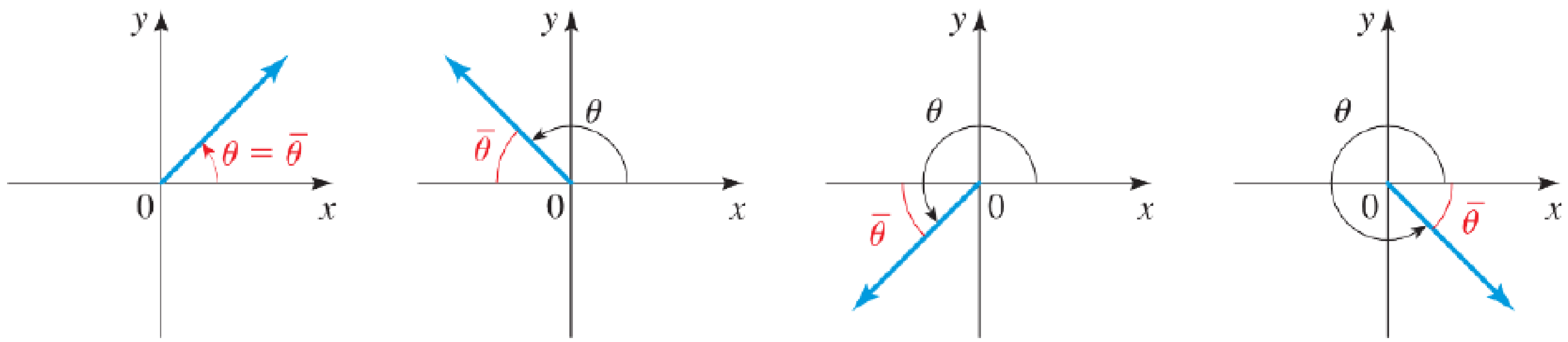
$= \tan 30^\circ$

$= \frac{1}{\sqrt{3}} / \frac{\sqrt{3}}{3}$ #

Reference angle

REFERENCE ANGLE

Let θ be an angle in standard position. The **reference angle** $\bar{\theta}$ associated with θ is the acute angle formed by the terminal side of θ and the x-axis.



Note: $\bar{\theta} = \theta$ $\quad$ $\bar{\theta} = \pi - \theta$ $\quad$ $\bar{\theta} = \theta - \pi$ $\quad$ $\bar{\theta} = 2\pi - \theta$

Example

Find the reference angle for (a) $\theta = \dfrac{5\pi}{3}$ and (b) $\theta = 870^{\circ}$.

a) $\theta = \dfrac{5\pi}{3} \times \dfrac{180^{\circ}}{\pi} = 300^{\circ} \rightarrow Q\,\text{IV}$

$\therefore \bar{\theta} = 2\pi - \dfrac{5\pi}{3}$

$= \dfrac{\pi}{3}$ #

b) $\theta = 870^{\circ}$

$= 870^{\circ} - 2(360^{\circ})$

$= 150^{\circ} \rightarrow Q\,\text{II}$

$\therefore \bar{\theta} = 180^{\circ} - 150^{\circ}$

$= 30^{\circ}$ #

EVALUATING TRIGONOMETRIC FUNCTIONS FOR ANY ANGLE

To find the values of the trigonometric functions for any angle θ, we carry out the following steps.

1. Find the reference angle $\bar{\theta}$ associated with the angle θ.
2. Determine the sign of the trigonometric function of θ by noting the quadrant in which θ lies.
3. The value of the trigonometric function of θ is the same, except possibly for sign, as the value of the trigonometric function of $\bar{\theta}$.

Example

Find (a) $\sin 240^\circ$ and (b) $\cot 495^\circ$

a) $240^\circ \to Q\,\mathrm{III}$

$$\sin 240^\circ = -\sin(240^\circ - 180^\circ)$$

$$= -\sin 60^\circ$$

$$= \frac{\sqrt{3}}{2}$$

b) $495^\circ = 495^\circ - 360^\circ$

$$= 135^\circ \to Q\,\mathrm{II}$$

$$\cot 495^\circ = \cot 135^\circ$$

$$= -\cot(180^\circ - 135^\circ)$$

$$= -\cot 45^\circ$$

$$= -\frac{1}{\tan 45^\circ}$$

$$= -1$$

Example

Find (a) $\sin\frac{16\pi}{3}$ and (b) $\sec\left(-\frac{\pi}{4}\right)$

a) $\frac{16\pi}{3} = \frac{16\pi}{3} - 4\pi = \frac{4\pi}{3} \times \frac{180}{\pi} = 240° \rightarrow Q\text{III}$

$\sin\frac{16\pi}{3} = \sin\frac{4\pi}{3}$

$= -\sin\left(\frac{4\pi}{3} - \pi\right)$

$= -\sin\frac{\pi}{3}$

$= -\frac{\sqrt{3}}{2}$ #

b) $-\frac{\pi}{4} = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4} \times \frac{180}{\pi}$

$= 315° \rightarrow Q\text{IV}$

$\sec\left(-\frac{\pi}{4}\right) = \sec\left(\frac{7\pi}{4}\right)$

$= \sec\left(2\pi - \frac{7\pi}{4}\right)$

$= \sec\frac{\pi}{4}$

$= \frac{1}{\cos\frac{\pi}{4}}$

$= \frac{1}{\left(\frac{1}{\sqrt{2}}\right)} = \sqrt{2}$ #

By even property of $\cos\theta$,

$\sec\left(-\frac{\pi}{4}\right) = \sec\frac{\pi}{4}$

2.1.3.2 Trigonometric Identities

The trigonometric functions of angles are related to each other through several important equations called **trigonometric identities**.

FUNDAMENTAL IDENTITIES

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

Example

(a) Express $\sin\theta$ in terms of $\cos\theta$

(b) Express $\tan\theta$ in terms of $\sin\theta$, where θ is in Quadrant II

a) $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\therefore \sin\theta = \pm\sqrt{1-\cos^2\theta}$ #

b) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{\sin\theta}{-\sqrt{1-\sin^2\theta}}$ #

Since $\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$\cos\theta = \pm\sqrt{1-\sin^2\theta}$

at QII, $\sin\theta$ is positive

$\therefore \cos\theta = -\sqrt{1-\sin^2\theta}$

$1 + \cot^2\theta = \csc^2\theta$

$1 + \dfrac{1}{\tan^2\theta} = \dfrac{1}{\sin^2\theta}$

$\dfrac{1}{\tan^2\theta} = \dfrac{1}{\sin^2\theta} - 1$

$= \dfrac{1-\sin^2\theta}{\sin^2\theta}$

$\tan^2\theta = \dfrac{\sin^2\theta}{1-\sin^2\theta}$

$\therefore \tan\theta = -\sqrt{\dfrac{\sin^2\theta}{1-\sin^2\theta}}$

$= -\dfrac{\sin\theta}{\sqrt{1-\sin^2\theta}}$ #

CHAPTER 1 TRIGONOMETRY (PART 1)

LECTURE 03 – 9.11.2022

Example

If $\tan\theta = \dfrac{2}{3}$ and θ is in Quadrant III, find $\cos\theta$.

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\left(\frac{2}{3}\right)^2 + 1 = \frac{1}{\cos^2\theta}$$

$$\frac{4}{9} + 1 = \frac{1}{\cos^2\theta}$$

$$\frac{13}{9} = \frac{1}{\cos^2\theta}$$

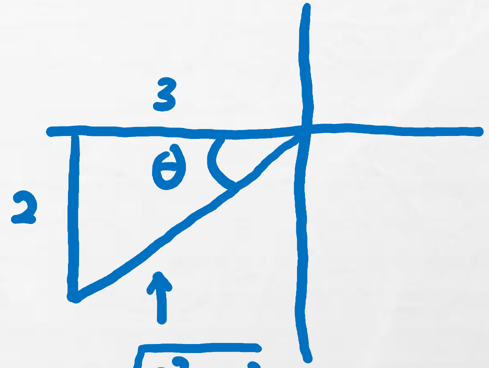
$$\cos^2\theta = \frac{9}{13}$$

$$\cos\theta = \pm\sqrt{\frac{9}{13}}$$

$$= \pm\frac{3}{\sqrt{13}}$$

at Q III, cos θ is negative,

$$\therefore \cos\theta = -\frac{3}{\sqrt{13}} \text{ \#}$$



$$\sqrt{2^2+3^2}$$
$$= \sqrt{13}$$

$$\therefore \cos\theta = -\frac{3}{\sqrt{13}} \text{ \#}$$

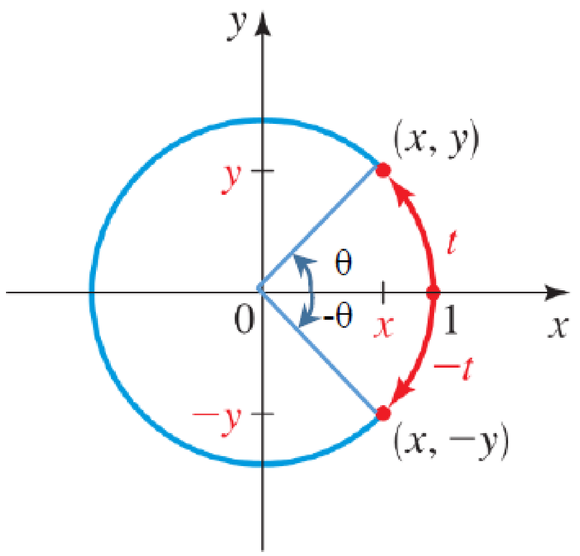
2.1.3.3 Even & Odd Properties

A function is said to be ***even*** if

$$f(-\theta) = f(\theta) \text{ for all } \theta \text{ in the domain of } f\text{, and}$$

A function is said to be ***odd*** if

$$f(-\theta) = -f(\theta) \text{ for all } \theta \text{ in the domain of } f.$$



Theorem:

$$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$$

$$\csc(-\theta) = -\csc(\theta) \qquad \sec(-\theta) = \sec(\theta) \qquad \cot(-\theta) = -\cot(\theta)$$

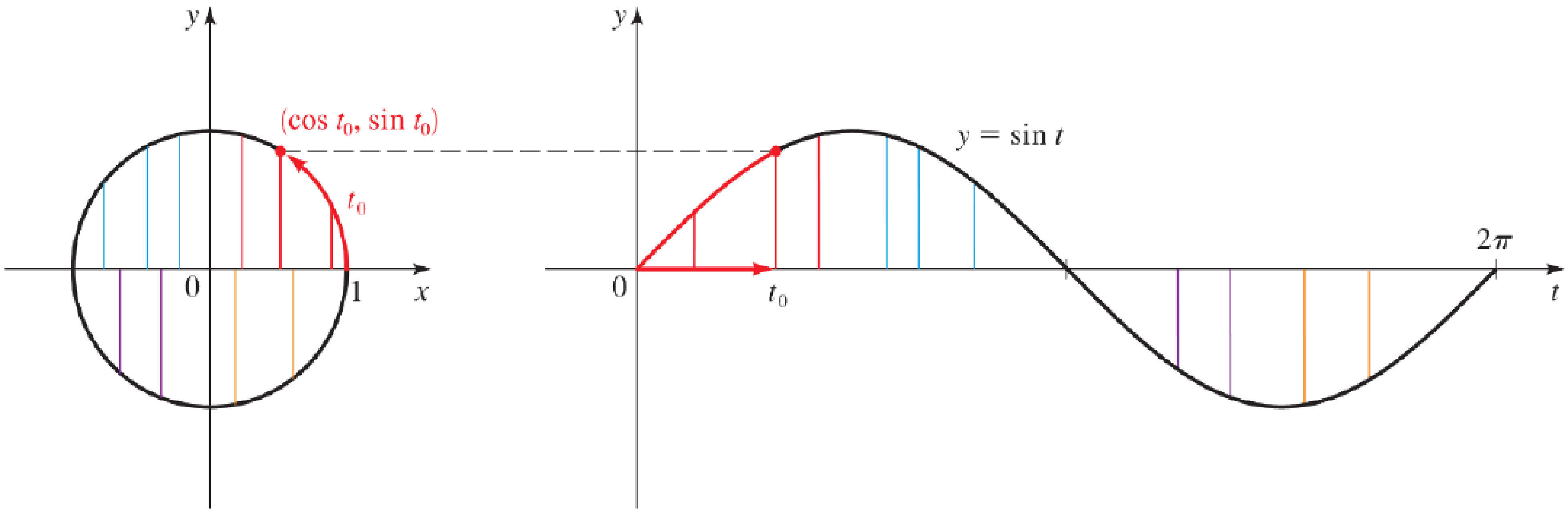
2.1.4 Trigonometric Graphs

2.1.4.1 Graph of Sine and Cosine

PERIODIC PROPERTIES OF SINE AND COSINE

The functions sine and cosine have period 2π:

$$\sin(t + 2\pi) = \sin t \qquad\qquad \cos(t + 2\pi) = \cos t$$



t	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
sin t	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	0
cos t	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1

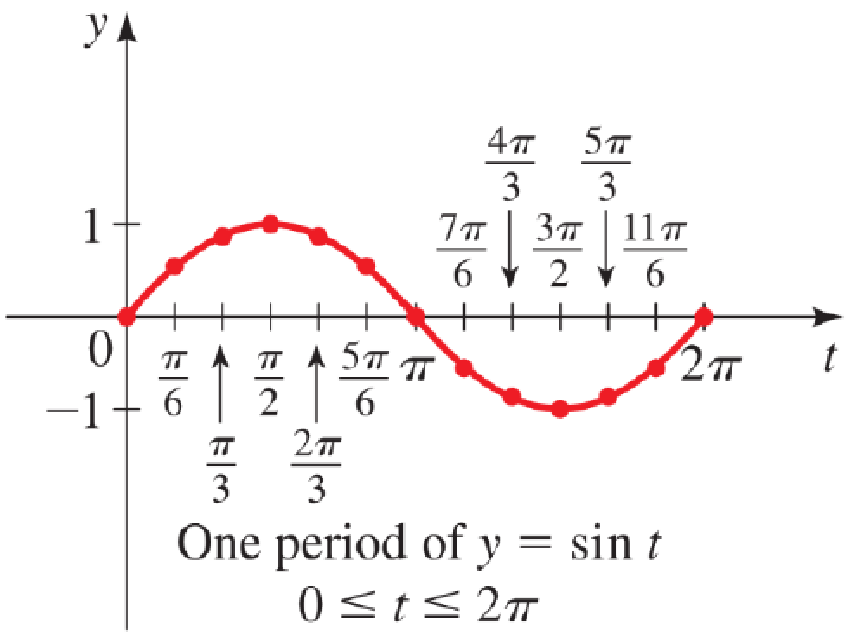


FIGURE 2 Graph of sin t

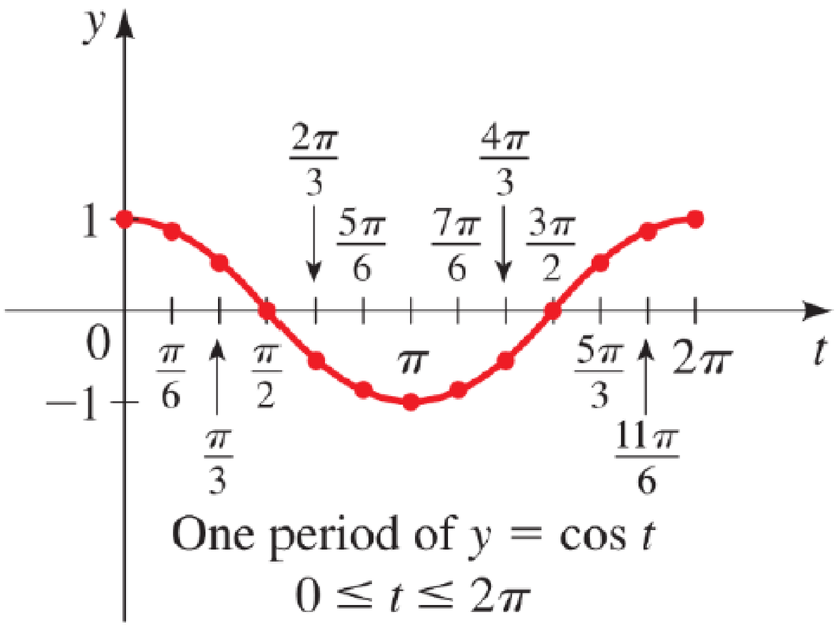


FIGURE 3 Graph of cos t

2.1.4.2 Graph transform of Sine and Cosine

SINE AND COSINE CURVES

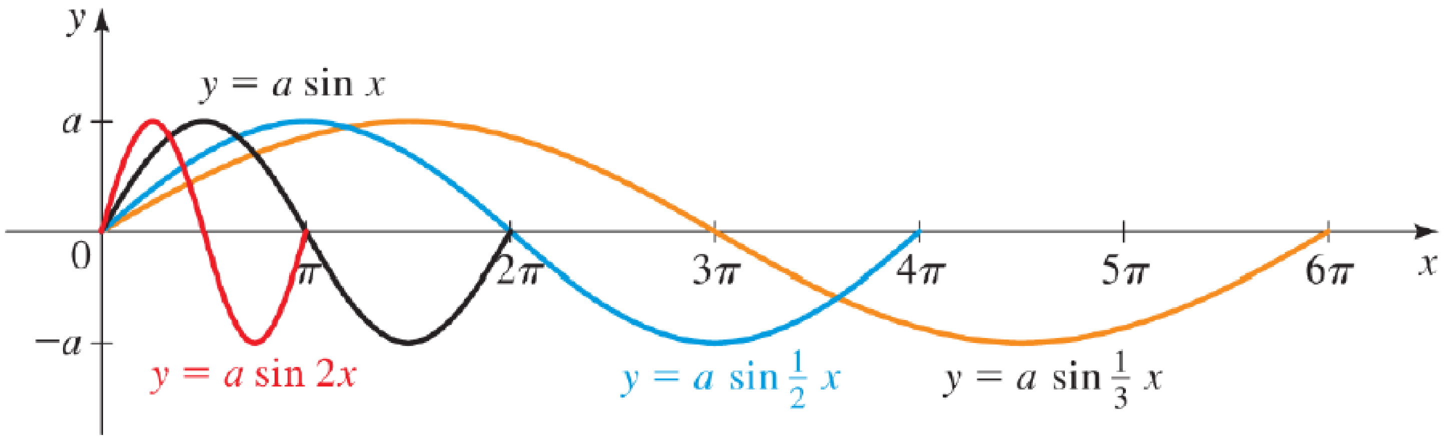
The sine and cosine curves

$$y = a \sin kx \qquad \text{and} \qquad y = a \cos kx \qquad (k > 0)$$

have **amplitude** $|a|$ and **period** $2\pi/k$.

An appropriate interval on which to graph one complete period is $[0, 2\pi/k]$.

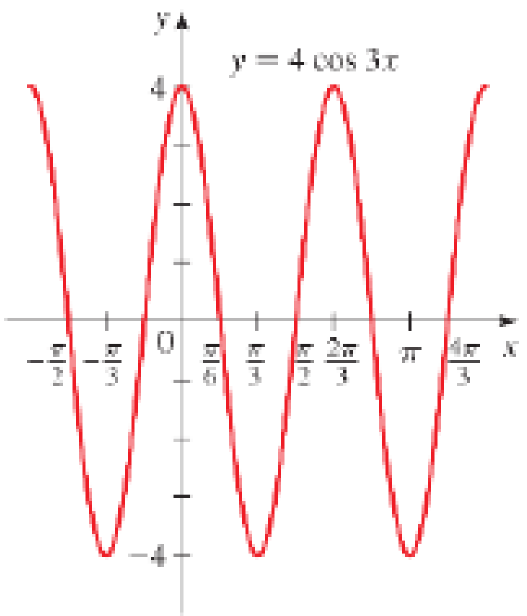
Example



Example

Find the amplitude and period of each function, and sketch its graph.

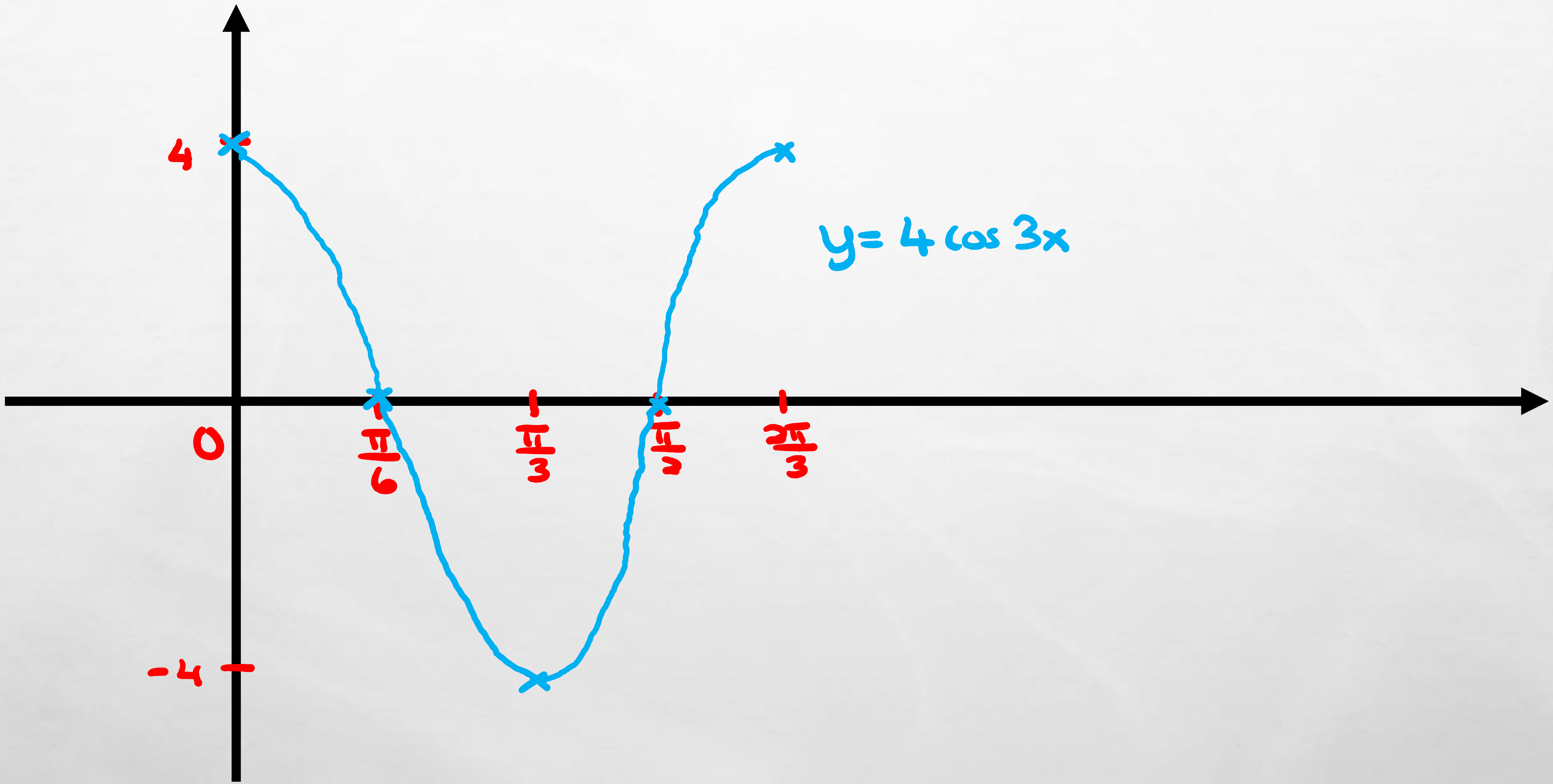
(a) $y = 4\cos 3x$



amplitude, $a = 4$ #

$\text{period} = \frac{2\pi}{k} = \frac{2\pi}{3}$ #

$\cos x$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Dividing the period by 3					
$\cos 3x$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$
$\cos 3x$	1	0	-1	0	1
$4\cos 3x$	4	0	-4	0	4

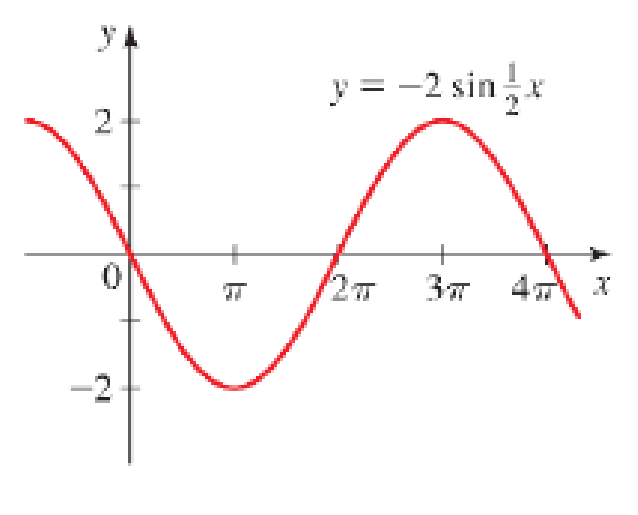


4
0
-4
π/6
π/3
π/2
2π/3
y= 4 cos 3x

Example

Find the amplitude and period of each function, and sketch its graph.

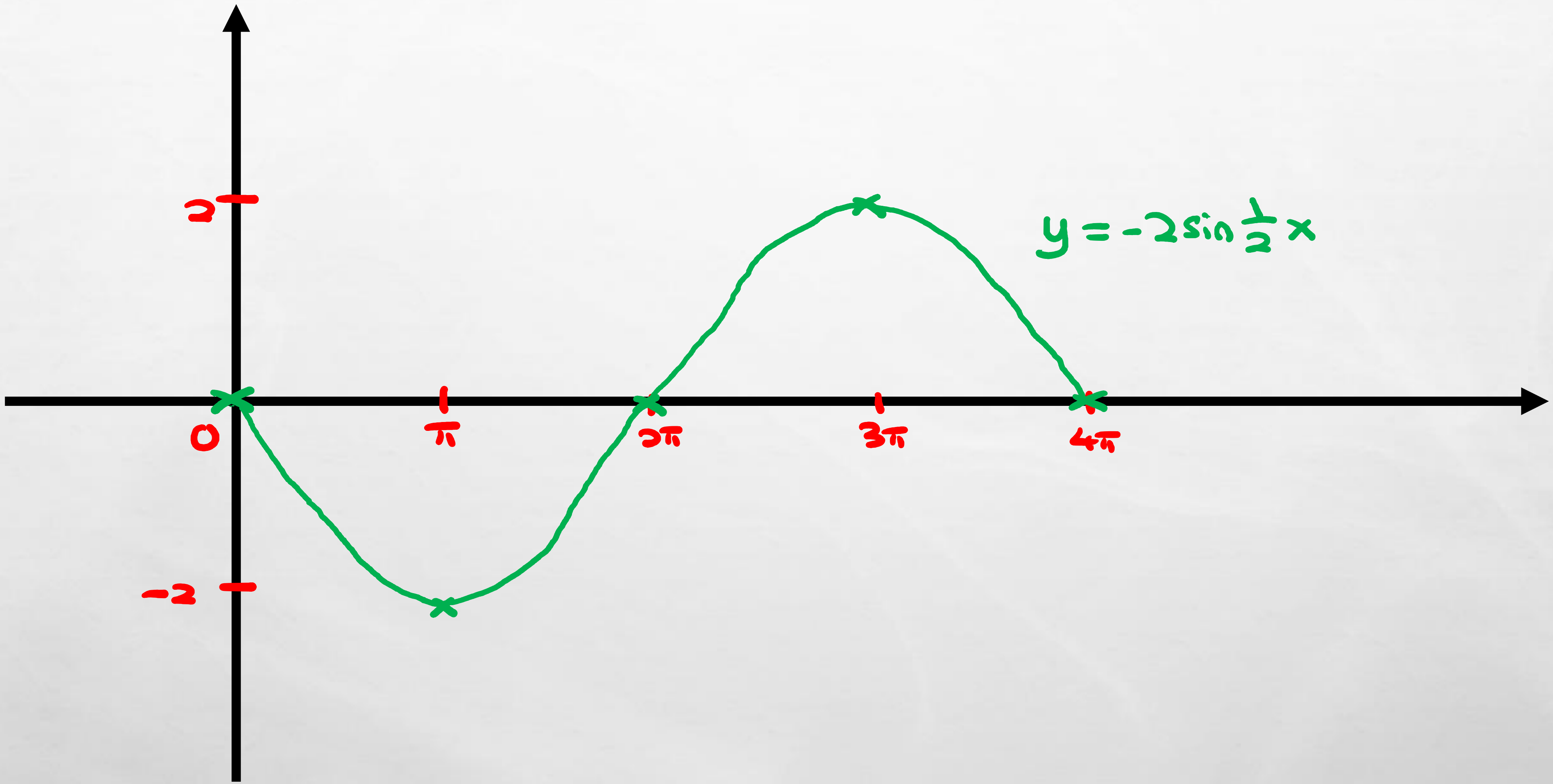
(b) $y = -2 \sin \frac{1}{2}x$



amplitude, $a = |-2| = 2$ #

period $= \frac{2\pi}{k} = \frac{2\pi}{\left(\frac{1}{2}\right)} = 4\pi$

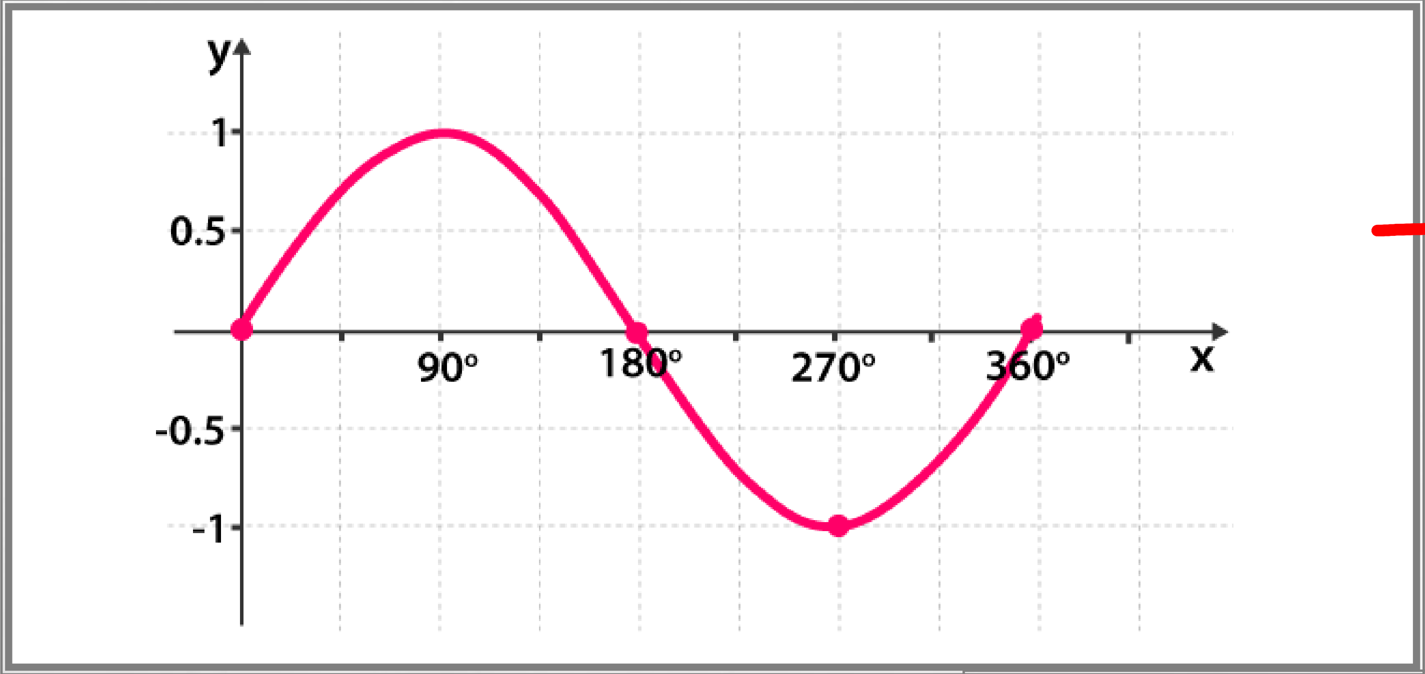
$\sin x$	$\mathbf{0}$	$\boldsymbol{\frac{\pi}{2}}$	$\boldsymbol{\pi}$	$\boldsymbol{\frac{3\pi}{2}}$	$\boldsymbol{2\pi}$
Dividing the period by 3					
$\sin \frac{1}{2}x$	0	π	2π	3π	4π
$\sin \frac{1}{2}x$	0	1	0	-1	0
$-2\sin \frac{1}{2}x$	0	-2	0	2	0



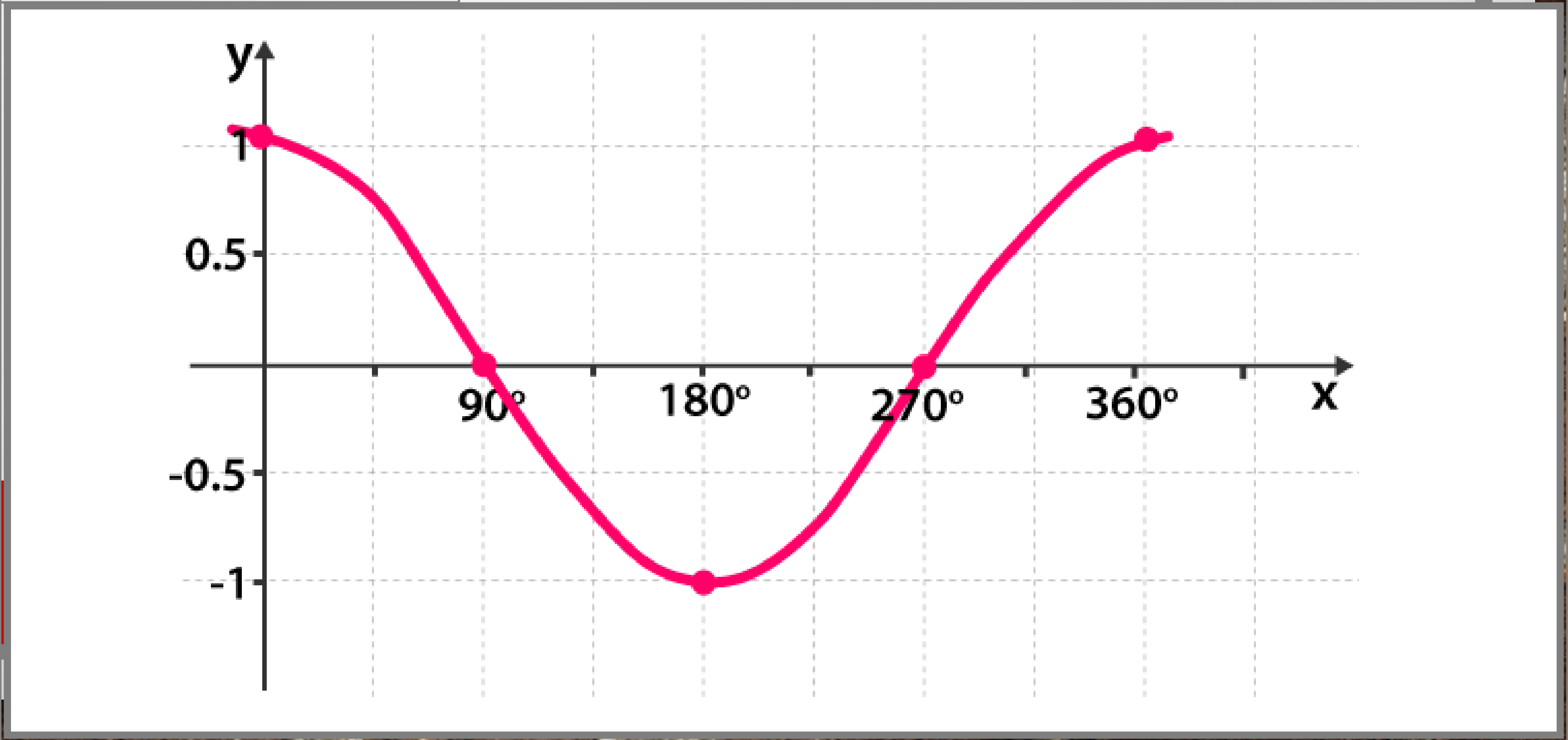
$y = -2\sin\frac{1}{2}x$
2
-2
0
π
2π
3π
4π

CHAPTER 1 TRIGONOMETRY (PART 1)

LECTURE 04 – 11.11.2022

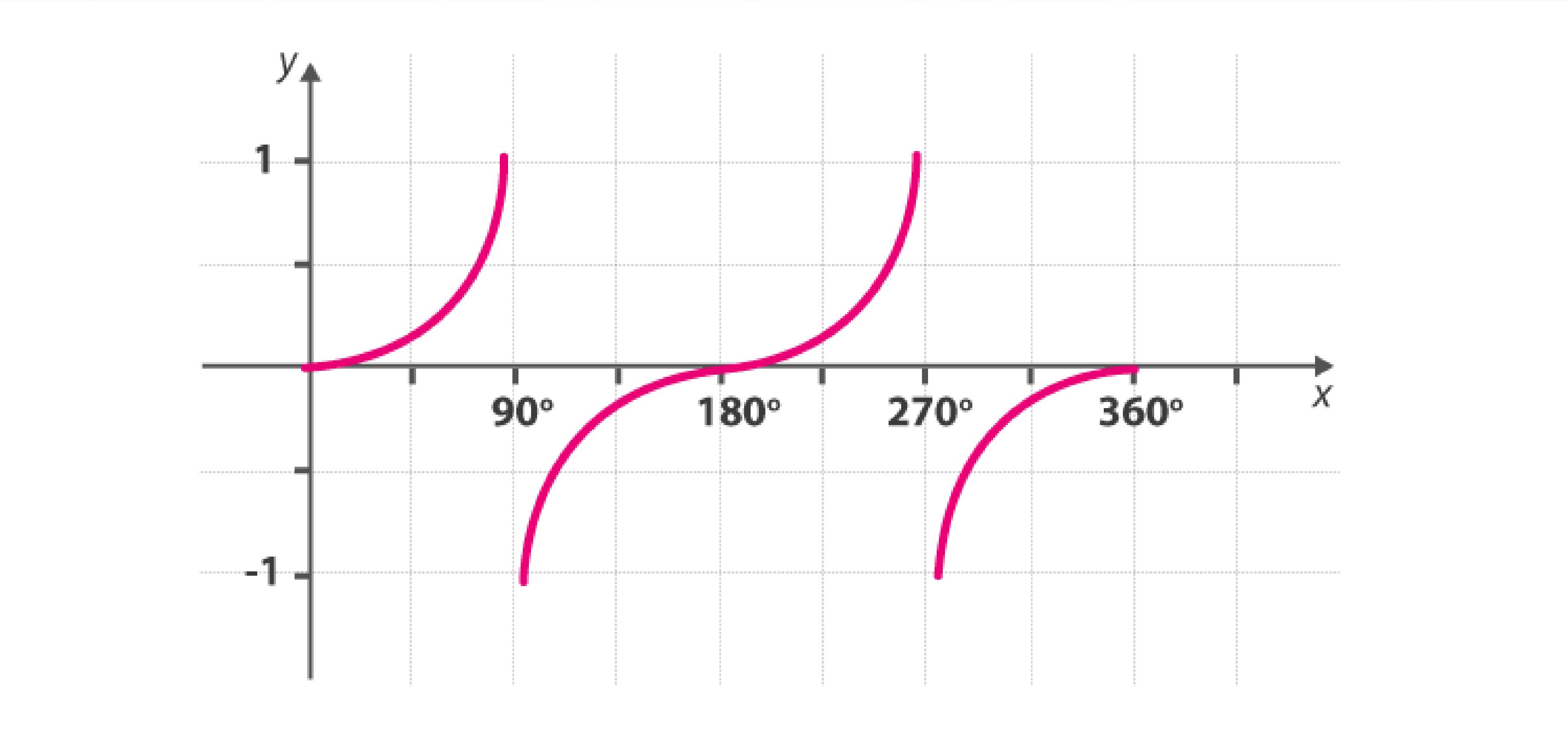


y = sin x



y = cos x

y = tan x



SHIFTED SINE AND COSINE CURVES

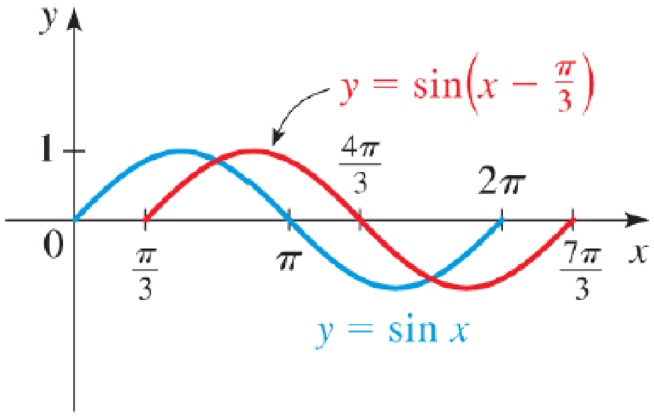
The sine and cosine curves

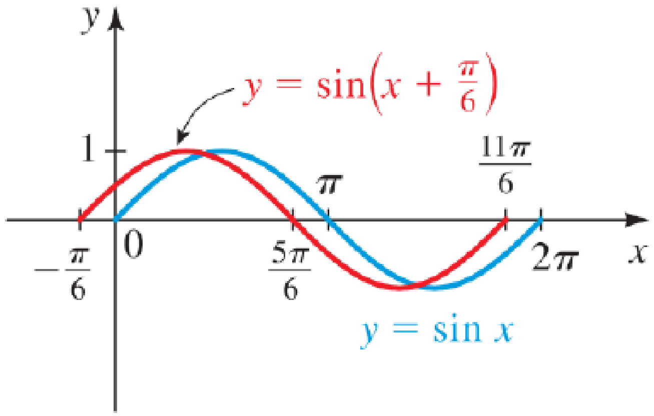
$$y = a \sin k(x - b) \qquad \text{and} \qquad y = a \cos k(x - b) \qquad (k > 0)$$

have **amplitude** $|a|$, **period** $2\pi/k$, and **phase shift** b.

An appropriate interval on which to graph one complete period is $[b, b + (2\pi/k)]$.

Example





Example

Find the amplitude, period, and phase shift of $y = 3 \sin 2\left(x - \frac{\pi}{4}\right)$, and graph one complete period.

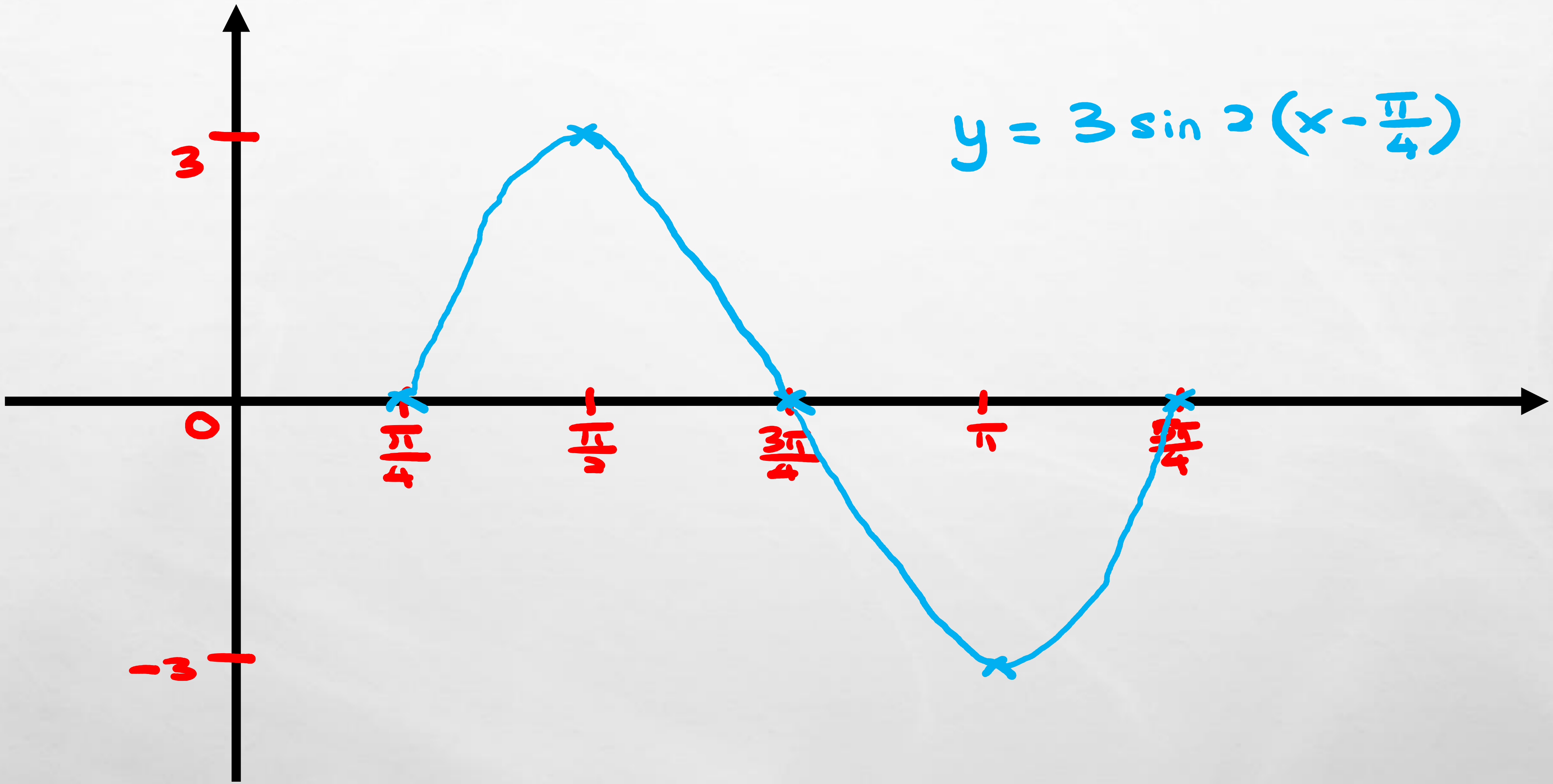
amplitude, $a = 3$ #

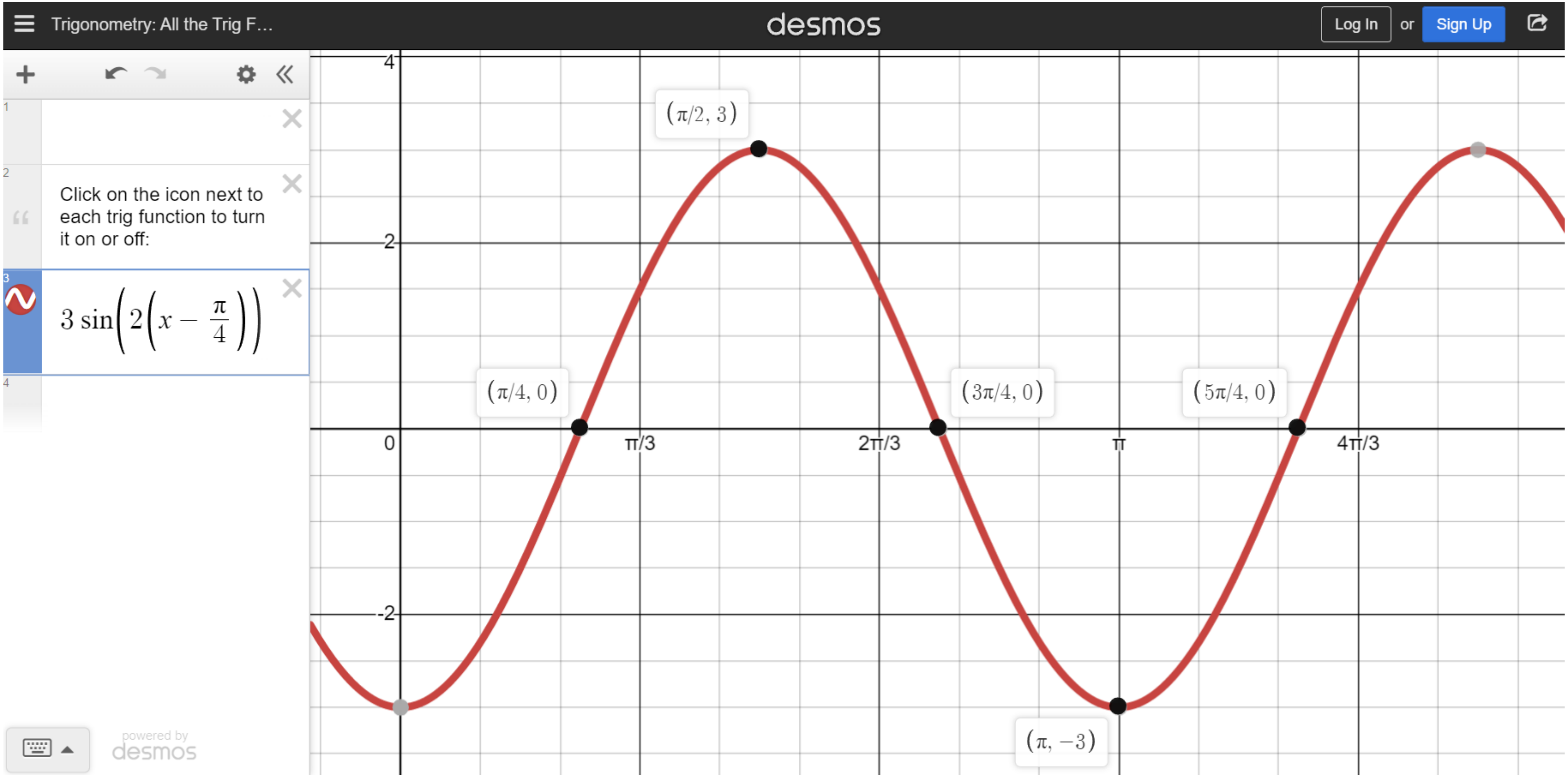
period $= \frac{2\pi}{k} = \frac{2\pi}{2} = \pi$ #

phase shift, $b = \frac{\pi}{4}$ #

$\sin \boldsymbol{x}$	$\mathbf{0}$	$\boldsymbol{\frac{\pi}{2}}$	$\boldsymbol{\pi}$	$\boldsymbol{\frac{3\pi}{2}}$	$\boldsymbol{2\pi}$
Dividing the period by 2					
$\sin 2x$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
Shifting the phase by $\frac{\pi}{4}$ **to the RIGHT** $+\frac{\pi}{4}$					
$\sin \mathbf{2}\left(\boldsymbol{x}-\boldsymbol{\frac{\pi}{4}}\right)$	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$

	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$
$\sin \mathbf{2}\left(\boldsymbol{x}-\frac{\boldsymbol{\pi}}{\mathbf{4}}\right)$	0	1	0	-1	0
$3 \sin \mathbf{2}\left(\boldsymbol{x}-\frac{\boldsymbol{\pi}}{\mathbf{4}}\right)$	0	3	0	-3	0


$y = 3\sin 2\left(x - \frac{\pi}{4}\right)$
3
0
-3
$\frac{\pi}{4}$
$\frac{\pi}{2}$
$\frac{3\pi}{4}$
π
$\frac{5\pi}{4}$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desmos
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1
2
Click on the icon next to each trig function to turn it on or off:
3
$3\sin\left(2\left(x-\frac{\pi}{4}\right)\right)$
4
powered by desmos
4
2
0
-2
π/3
2π/3
π
4π/3
(π/2, 3)
(π/4, 0)
(3π/4, 0)
(5π/4, 0)
(π, −3)

MORE EXAMPLE

- **FIND THE AMPLITUDE, PERIOD AND PHASE SHIFT OF $y = -1\cos\frac{1}{3}(x-\pi)+2$. SKETCH ONE COMPLETE PERIOD OF y.**

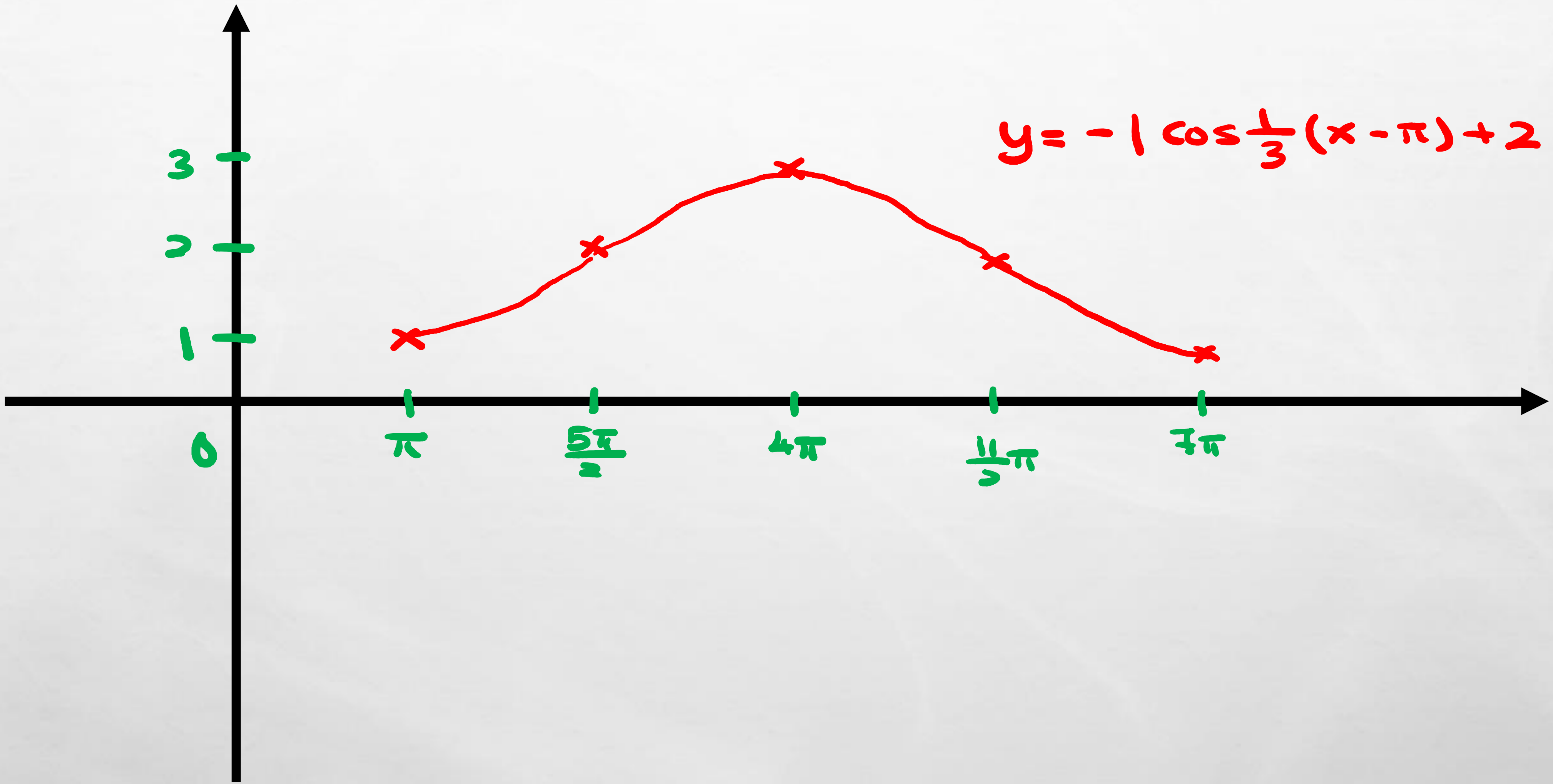
amplitude, $a = |-1| = 1$

period $= \frac{2\pi}{k} = \frac{2\pi}{\left(\frac{1}{3}\right)} = 6\pi$

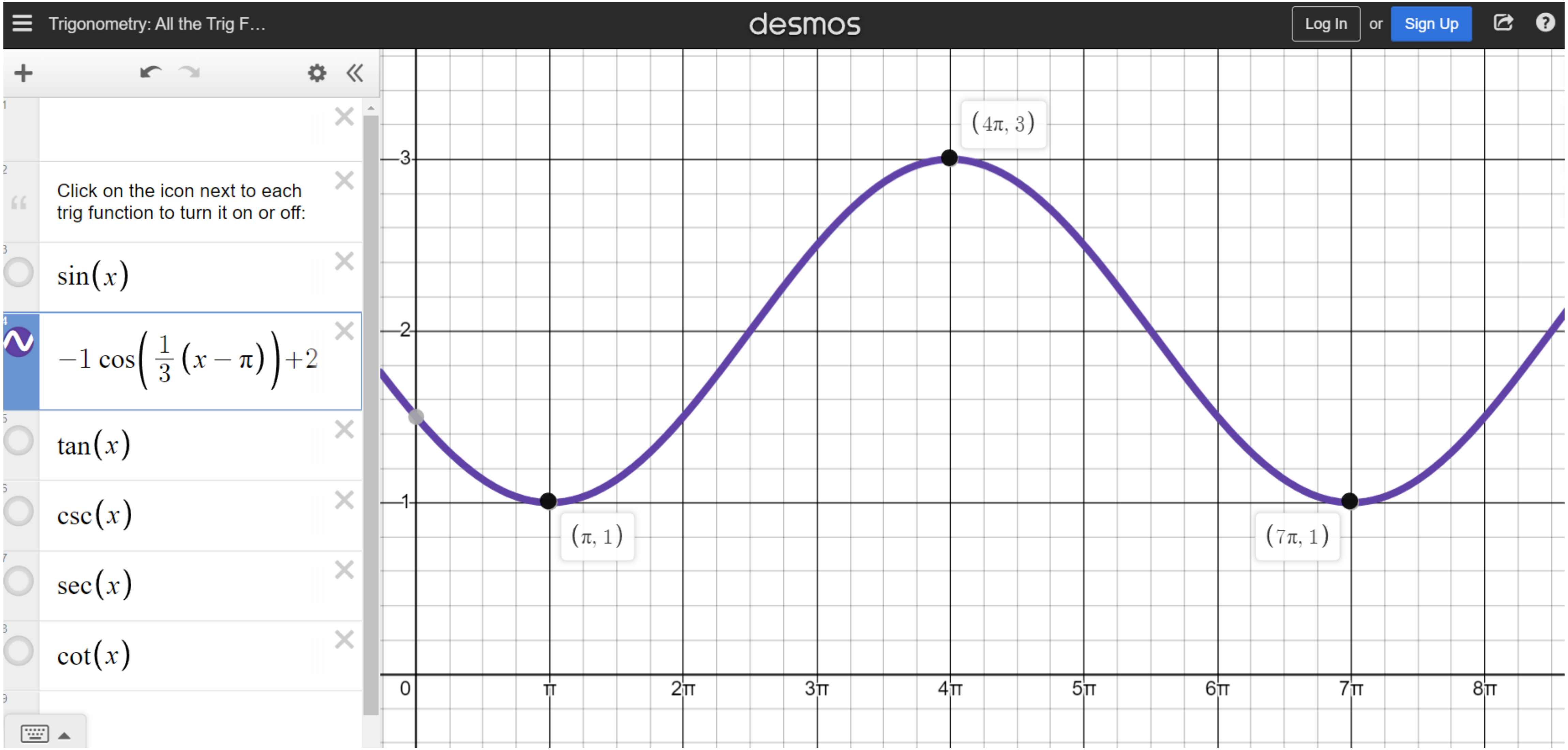
phase shift, $b = \pi$

$\cos x$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Dividing the period by $\frac{1}{3}$					
$\cos \frac{1}{3} x$	0	$\frac{3\pi}{2}$	3π	$\frac{9\pi}{2}$	6π
Shifting the phase by π **to the RIGHT** $+\pi$					
$\cos \frac{1}{3}(x - \pi)$	π	$\frac{5\pi}{2}$	4π	$\frac{11\pi}{2}$	7π

	π	$\frac{5\pi}{2}$	4π	$\frac{11\pi}{2}$	7π
$\cos\frac{1}{3}(x-\pi)$	1	0	−1	0	1
$-1\cos\frac{1}{3}(x-\pi)$	−1	0	1	0	−1
$-1\cos\frac{1}{3}(x-\pi)+2$	1	2	3	2	1



$y = -1\cos\frac{1}{3}(x-\pi)+2$
3
2
1
0
π
$\frac{5\pi}{2}$
4π
$\frac{11}{2}\pi$
7π



Trigonometry: All the Trig F…
desmos
Log In
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Click on the icon next to each trig function to turn it on or off:
sin(x)
−1 cos(1/3 (x − π)) +2
tan(x)
csc(x)
sec(x)
cot(x)
(4π, 3)
(π, 1)
(7π, 1)
3
2
1
0
π
2π
3π
4π
5π
6π
7π
8π

2.1.4.3 Graph of Tangent, Cotangent, Secant and Cosecant

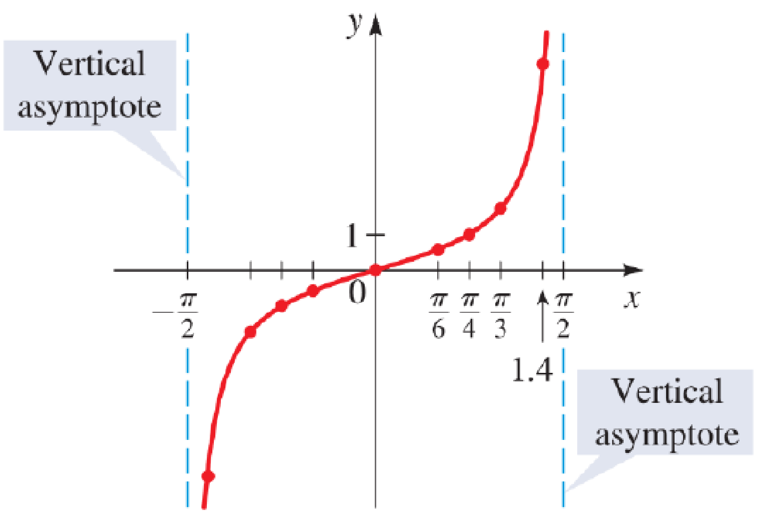
PERIODIC PROPERTIES

The functions tangent and cotangent have period π:

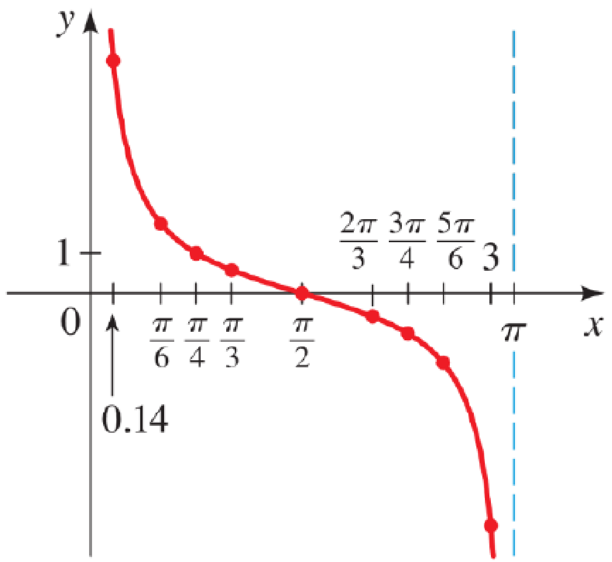
$$\tan(x+\pi) = \tan x \qquad \cot(x+\pi) = \cot x$$

The functions cosecant and secant have period 2π:

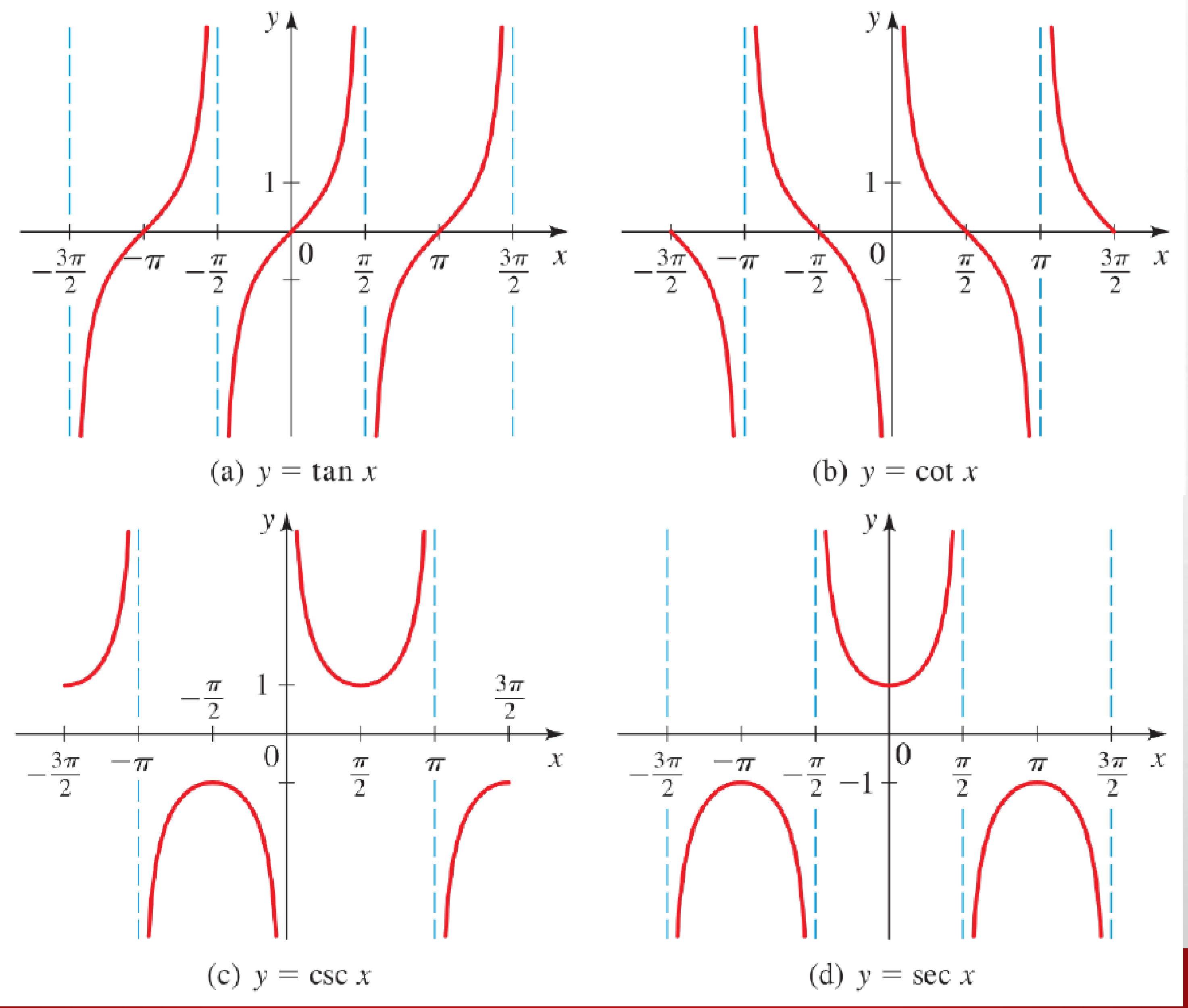
$$\csc(x+2\pi) = \csc x \qquad \sec(x+2\pi) = \sec x$$



$y = \tan x$



$y = \cot x$



(a) $y = \tan x$ (b) $y = \cot x$

(c) $y = \csc x$ (d) $y = \sec x$

2.1.5 Inverser Trigonometric Functions and Their Graphs

2.1.5.1 Definition of Inverse Function

DEFINITION OF A ONE-TO-ONE FUNCTION

A function with domain A is called a **one-to-one function** if no two elements of A have the same image, that is,

$$f(x_1) \neq f(x_2) \quad \text{whenever } x_1 \neq x_2$$

NOTE: It is equivalent to write the condition of function is one-to-one as

If $f(x_1) = f(x_2)$, then $x_1 = x_2$.

HORIZONTAL LINE TEST

A function is one-to-one if and only if no horizontal line intersects its graph more than once.

Example

Is the function $f(x) = x^3$ one-to-one?

Let $x_1 \neq x_2$, then $f(x_1) = x_1^3$ and $f(x_2) = x_2^3$

since $f(x_1) \neq f(x_2)$

$\therefore f(x)$ is one-to-one

Example

Is the function $g(x) = x^2$ one-to-one?

take $x_1 = -2$ and $x_2 = 2$

$g(-2) = (-2)^2 = 4$

$g(2) = (2)^2 = 4$

Since $g(x_1) = g(x_2)$ for $x_1 \neq x_2$,

$g(x)$ is NOT one-to-one.

DEFINITION OF THE INVERSE OF A FUNCTION

Let f be a one-to-one function with domain A and range B. Then its **inverse function** f^{-1} has domain B and range A and is defined by

$$f^{-1}(y) = x \quad \Leftrightarrow \quad f(x) = y$$

for any y in B.

INVERSE FUNCTION PROPERTY

Let f be a one-to-one function with domain A and range B. The inverse function f^{-1} satisfies the following cancellation properties:

$$f^{-1}(f(x)) = x \quad \text{for every } x \text{ in } A$$

$$f(f^{-1}(x)) = x \quad \text{for every } x \text{ in } B$$

Conversely, any function f^{-1} satisfying these equations is the inverse of f.

Example

Show that $f(x) = x^3$ and $g(x) = x^{1/3}$ are inverse of each other

$$f(x) \circ g(x) = fg(x)$$

$$= f[g(x)]$$

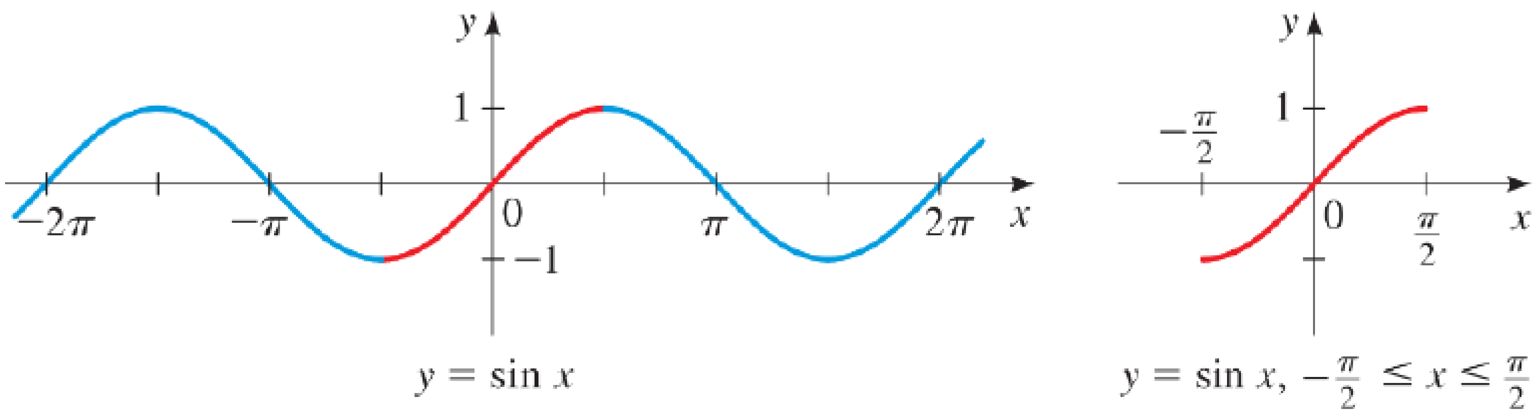
$$= f(x^{1/3})$$

$$= (x^{\frac{1}{3}})^3$$

$$= x$$

2.1.5.2 Inverse Sine Function

For a functions to have an inverse, it must be one-to-one. Hence we need to restrict the domain of trigonometric functions in order to have their inverses. Here the sine function's domain is being restricted to $-\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$.



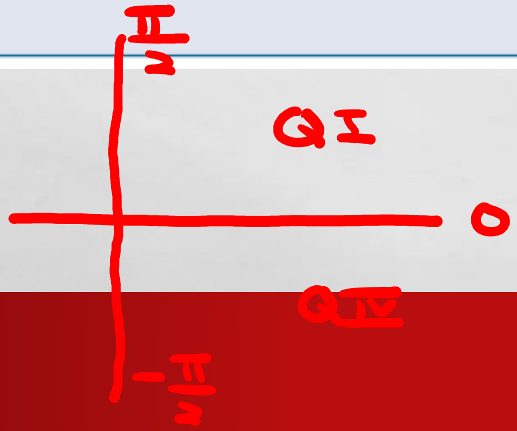
DEFINITION OF THE INVERSE SINE FUNCTION

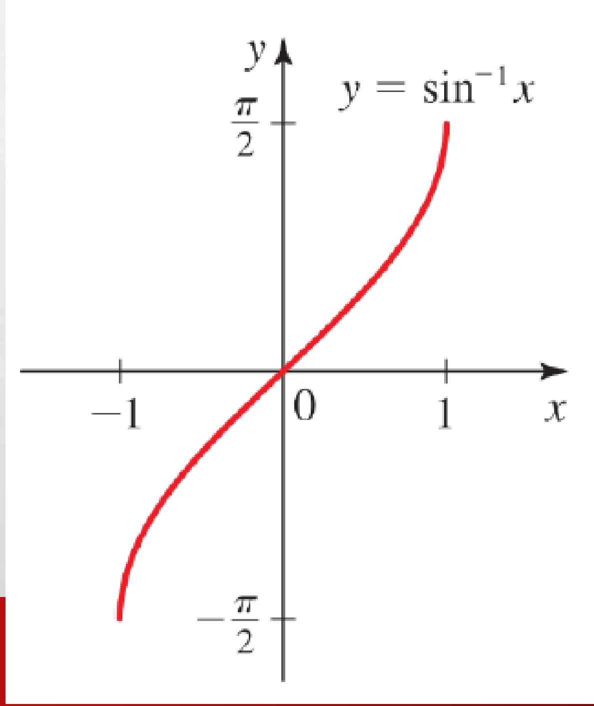
The **inverse sine function** is the function $\sin^{-1}$ with domain $[-1, 1]$ and range $[-\pi/2, \pi/2]$ defined by

$$\sin^{-1} x = y \quad \Leftrightarrow \quad \sin y = x$$

The inverse sine function is also called **arcsine**, denoted by **arcsin**.

$$\sin(\sin^{-1} x) = x \quad \text{for} \quad -1 \le x \le 1$$
$$\sin^{-1}(\sin x) = x \quad \text{for} \quad -\frac{\pi}{2} \le x \le \frac{\pi}{2}$$





Example

Find each value

(a) $\sin^{-1}\frac{1}{2}$ (b) $\sin^{-1}\left(-\frac{1}{2}\right)$ (c) $\sin^{-1}\frac{3}{2}$

arcsin $\frac{1}{2}$

$= \frac{\pi}{6} \,/\, 30^\circ$ #

arcsin $\left(-\frac{1}{2}\right)$

$= -\frac{\pi}{6} \,/\, -30^\circ$ #

arcsin $\frac{3}{2}$

= undefined

since $\frac{3}{2} > 1$ #

Example

Find each value

(a) $\sin^{-1}\left(\sin\dfrac{\pi}{3}\right)$

$= \dfrac{\pi}{3}$

(b) $\sin^{-1}\left(\sin\dfrac{2\pi}{3}\right)$

$= \dfrac{2\pi}{3}$

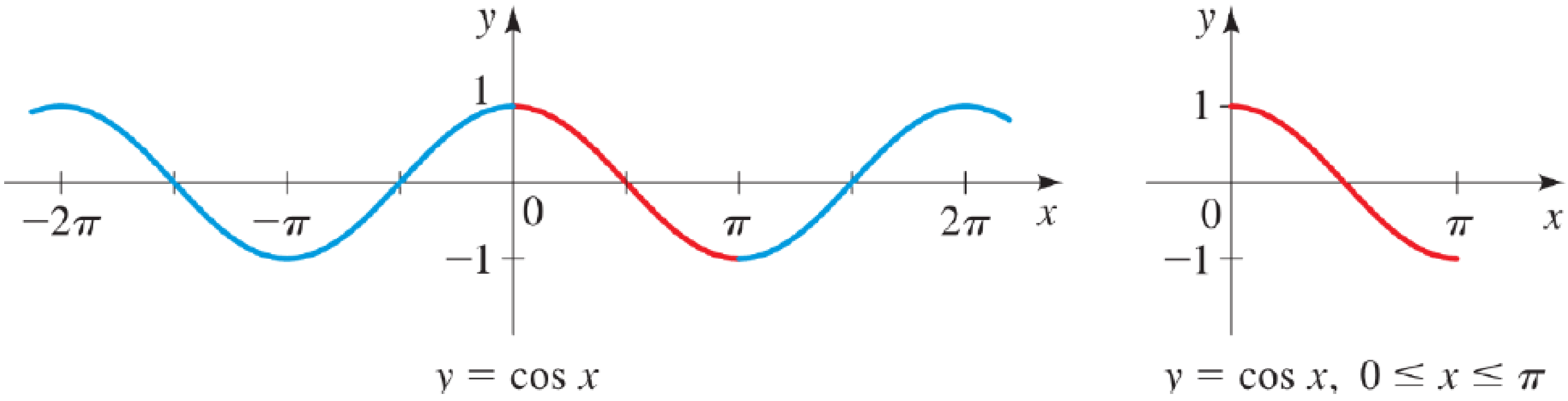
$= \left(\pi - \dfrac{2\pi}{3}\right)$

reference angle in Q I

$= \dfrac{\pi}{3}$

2.1.5.3 The Inverse Cosine Function

To get the inverse of cosine function, we restrict the function's domain to $0 \le x \le \pi$.



DEFINITION OF THE INVERSE COSINE FUNCTION

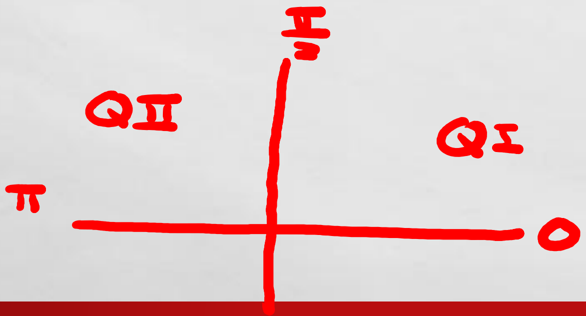
The **inverse cosine function** is the function $\cos^{-1}$ with domain $[-1, 1]$ and range $[0, \pi]$ defined by

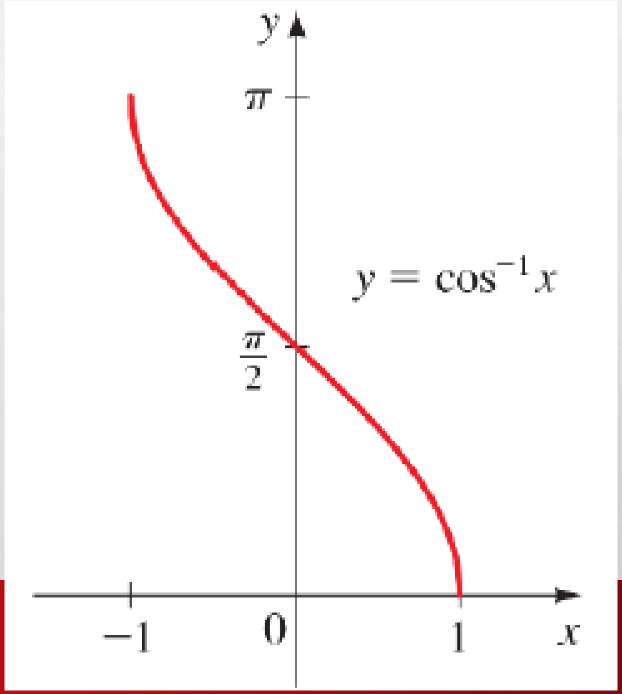
$$\cos^{-1} x = y \quad \Leftrightarrow \quad \cos y = x$$

The inverse cosine function is also called **arccosine**, denoted by **arccos**.

$$\cos(\cos^{-1} x) = x \quad \text{for} \quad -1 \le x \le 1$$

$$\cos^{-1}(\cos x) = x \quad \text{for} \quad 0 \le x \le \pi$$





Example

Find each value

(a) $\cos^{-1}\dfrac{\sqrt{3}}{2}$ (b) $\cos^{-1} 0$ (c) $\cos^{-1}\dfrac{5}{7}$

$\arccos \dfrac{\sqrt{3}}{2}$ $\arccos 0$ $\arccos \dfrac{5}{7}$

$= \dfrac{\pi}{6} / 30^\circ$ $= \dfrac{\pi}{2} / 90^\circ$ $= 0.7752 \text{ rad} /$

44.42°

Example

Find each value

(a) $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)$

$= \frac{2\pi}{3}$

(b) $\cos^{-1}\left(\cos\frac{5\pi}{3}\right)$

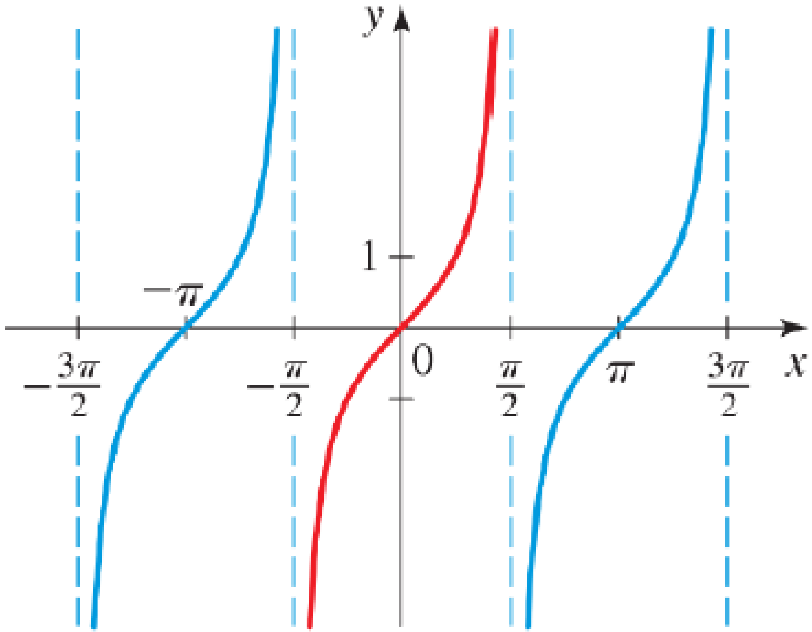
$= \frac{5\pi}{3} / 300^\circ$

reference angle

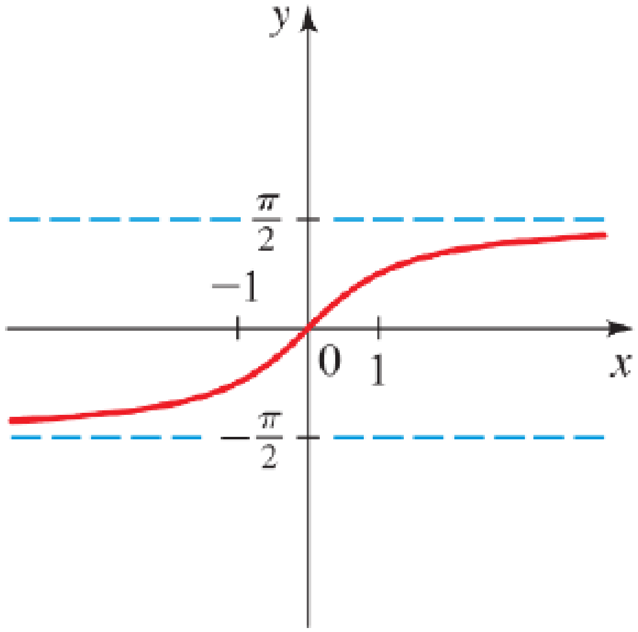
$= \left(2\pi - \frac{5\pi}{3}\right) / (360^\circ - 300^\circ)$

$= \frac{\pi}{3} / 60^\circ$

2.1.5.4 The Inverse Tangent Function



$y = \tan x,\ -\frac{\pi}{2} < x < \frac{\pi}{2}$



$y = \tan^{-1} x$

DEFINITION OF THE INVERSE TANGENT FUNCTION

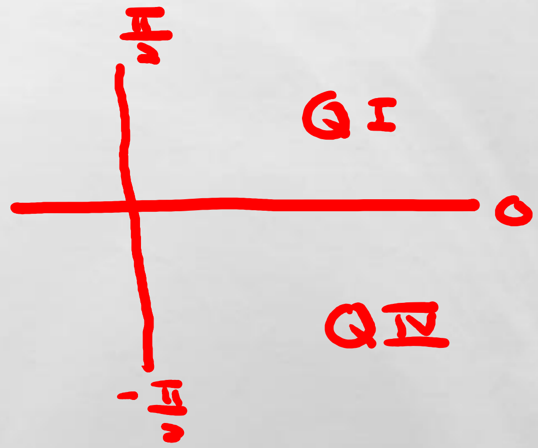
The **inverse tangent function** is the function $\tan^{-1}$ with domain $\mathbb{R}$ and range $(-\pi/2, \pi/2)$ defined by

$$\tan^{-1} x = y \quad \Leftrightarrow \quad \tan y = x$$

The inverse tangent function is also called **arctangent**, denoted by **arctan**.

$$\tan(\tan^{-1} x) = x \quad \text{for} \quad x \in \mathbb{R}$$

$$\tan^{-1}(\tan x) = x \quad \text{for} \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$$



Example

Find each value

(a) $\tan^{-1} 1$ (b) $\tan^{-1} \sqrt{3}$ (c) $\tan^{-1}(20)$

arctan 1

$= \frac{\pi}{4} / 45°$

arctan $\sqrt{3}$

$= \frac{\pi}{3} / 60°$

arctan 20

$= 87.14° / 1.5208$ rad.

☺ ~ THE END ~ ☺