

PDS0101 Introduction to Digital Systems

Tutorial 4 SAMPLE SOLUTIONS

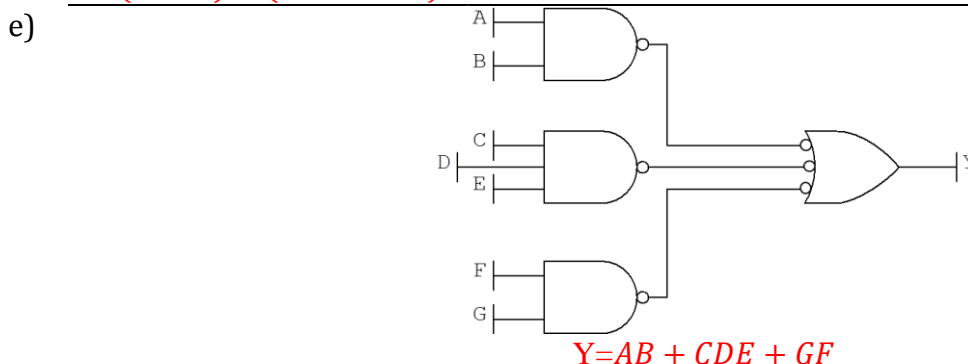
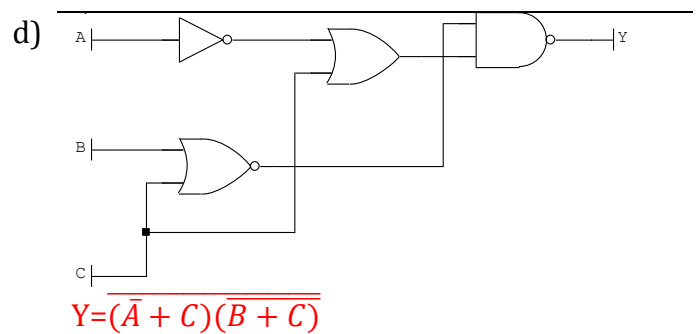
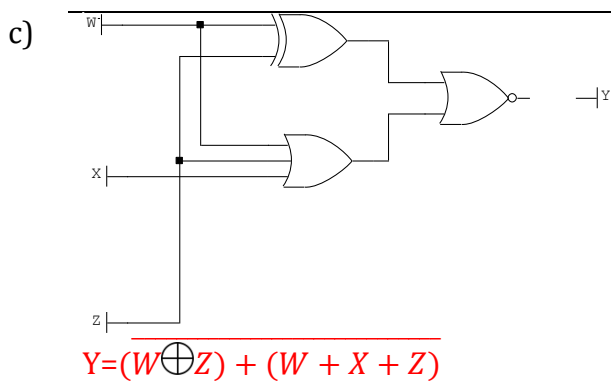
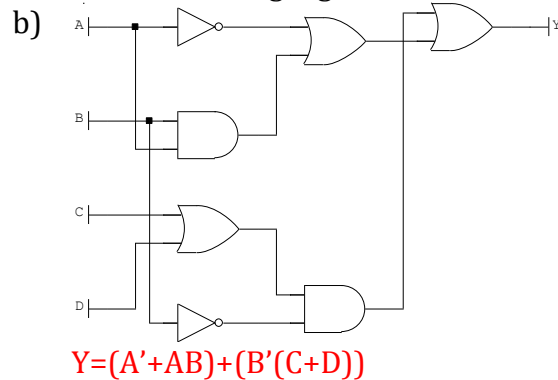
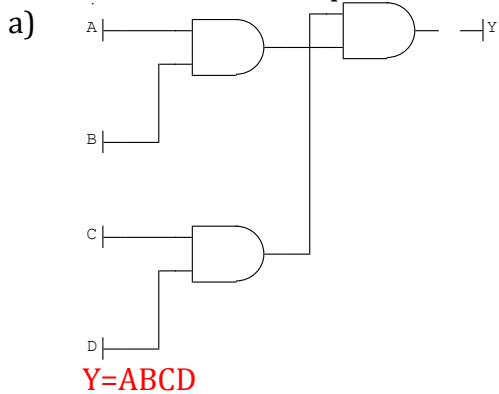
Tutorial outcomes

By the end of today's tutorial, you should be able to

- apply the basic laws and rules of boolean algebra
- apply DeMorgan's theorems to boolean expressions
- simplify boolean expressions using boolean algebra

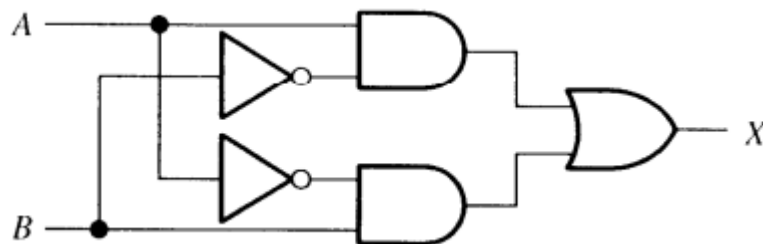
Theory based questions

1. Write the boolean expressions for the outputs from the following logic circuits

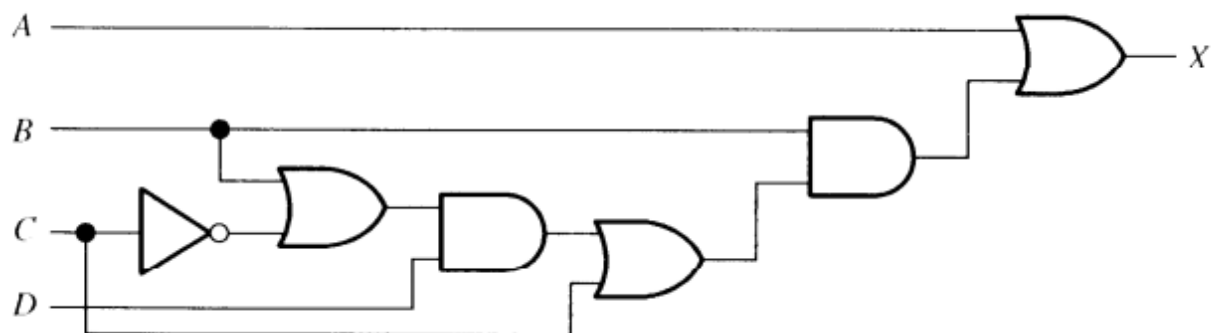


2. Draw the logic circuit represented by the following boolean expressions

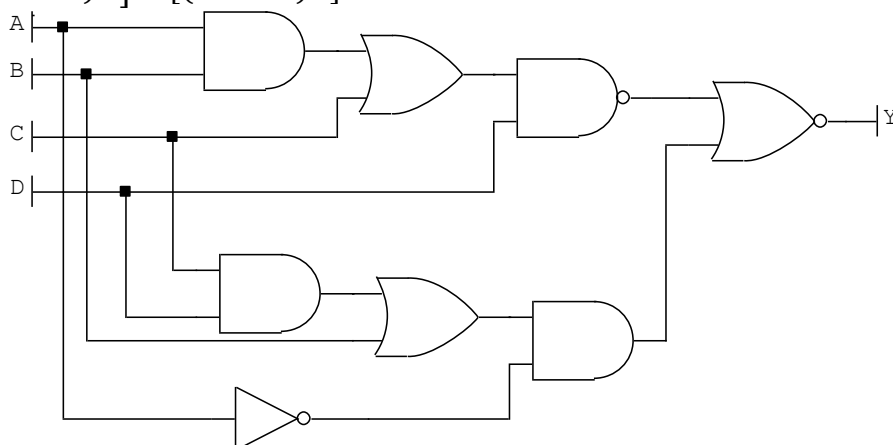
a) $Y = A\bar{B} + \bar{A}B$



b) $Y = A + B[C + D(B + \bar{C})]$



c) $Y = \overline{(AB + C)D} + [(CD + B)\bar{A}]$



3. Define and describe De Morgan's first and second theorem

See notes

4. Apply DeMorgan's theorems to the following expressions

a) $\overline{(A + \bar{B})} = \bar{A}\bar{\bar{B}} = \bar{A}B$

b) $\overline{\bar{A}B} = \bar{\bar{A}} + \bar{B} = A + \bar{B}$

c) $\overline{(A + \bar{B} + C + \bar{D})} + \overline{(ABC\bar{D})} = \bar{A}\bar{\bar{B}}\bar{C}\bar{\bar{D}} + \bar{A}\bar{B}\bar{C}\bar{D} = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A} + \bar{B} + \bar{C} + \bar{D}$

d) $\overline{(A\bar{B}(C + \bar{D}))} = \overline{A\bar{B}} + \overline{(C + \bar{D})} = \bar{A} + B + \bar{C}\bar{D}$

$$\begin{aligned}
& \overline{\overline{(ABC)(EFG)} + \overline{(HIJ)(KLM)}} = \overline{\overline{ABC} + \overline{EFG} + \overline{HIJ} + \overline{KLM}} \\
& = \overline{\overline{ABC} + \overline{EFG} + \overline{HIJ} + \overline{KLM}} = \overline{(\overline{ABC})(\overline{EFG})(\overline{HIJ})(\overline{KLM})} \\
\text{e)} & = \overline{(\overline{A} + \overline{B} + \overline{C})(\overline{E} + \overline{F} + \overline{G})(\overline{H} + \overline{I} + \overline{J})(\overline{K} + \overline{L} + \overline{M})} \\
& \overline{(A+B)(C+D)(E+F)(G+H)} \\
\text{f)} & = \overline{(A+B)(C+D)(E+F)(G+H)} = \overline{ABCDEFGH}
\end{aligned}$$

Applied knowledge questions

6. Using boolean algebra rules, simplify the following expressions as much as possible

a) $A(\bar{A} + AB)$

$$\begin{aligned}
A' + AB &= A'(B + B') + AB \\
&= A'B + A'B' + AB \\
&= (A' + A)B + A'B' \\
&= B + A'B' \\
&= \mathbf{A' + B}
\end{aligned}$$

b) $(A + \bar{B})(A + C)$

$$\begin{aligned}
(A+B')(A+C) &= AA+AC+AB'+B'C \\
&= A+AC+AB'+B'C \\
&= A(1+C+B')+B'C \\
&= \mathbf{A+B'C}
\end{aligned}$$

c) $AB + (\bar{A} + \bar{B})C + AB$

$$\begin{aligned}
&= AB + (A' + B')C \\
&= AB + (AB)'C \\
&= \mathbf{AB+C}
\end{aligned}$$

d) $\bar{A}B + \bar{A}B\bar{C} + \bar{A}BCD + \bar{A}B\bar{C}\bar{D}E$

$$\begin{aligned}
&= A'B(1+C+CD+C'D'E) \\
&= \mathbf{A'B}
\end{aligned}$$

e) $(a' + b')(a + b)$

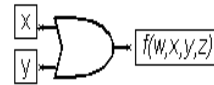
$$\begin{aligned}
&= a'a + b'b + a'b + ab' \\
&= \mathbf{a'b + ab'}
\end{aligned}$$

f) $(a' + b' + c')(a + b + c)$ ← optional may require lecture 5

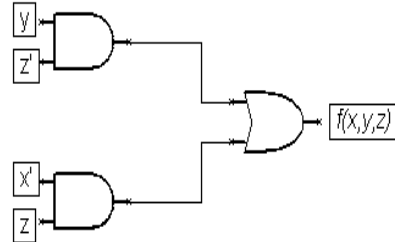
$$\begin{aligned}
&= \overline{a+b+c} + a'b + ac' + ab' + b'c + ac' + bc' \\
&= a'b + a'c + ab' + b'c + ac' + bc' \\
&= a'b(c+c') + a'c(b+b') + ab'(c+c') + b'c(a+a') + ac'(b+b') + bc'(a+a') \\
&= a'bc + a'bc' + a'bc + a'b'c + ab'c + ab'c' + ab'c + a'b'c + abc' + ab'c' + abc' + a'bc' \\
&= a'bc + a'bc' + a'b'c + ab'c + ab'c' + abc' \\
&= a'bc + ab'c' + (a'+a)bc' + (a'+a)b'c = a'bc + ab'c' + bc' + b'c \\
&= \mathbf{a'b + ab' + bc' + b'c}
\end{aligned}$$

7. Using boolean algebra rules, simplify the following functions and draw the resulting logic circuit of f

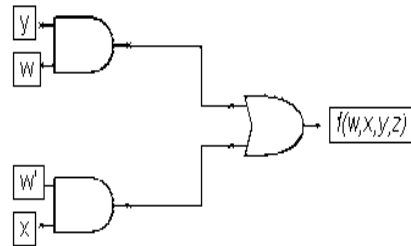
$$\begin{aligned}
 (a) \quad f(w, x, y, z) &= x + xyz + \bar{x}yz + wx + \bar{w}x + \bar{x}y + \bar{x}yw \\
 &= x(1 + yz + w + \bar{w}) + \bar{x}y(z + 1 + w) \\
 &= x + \bar{x}y \\
 &= x + y
 \end{aligned}$$



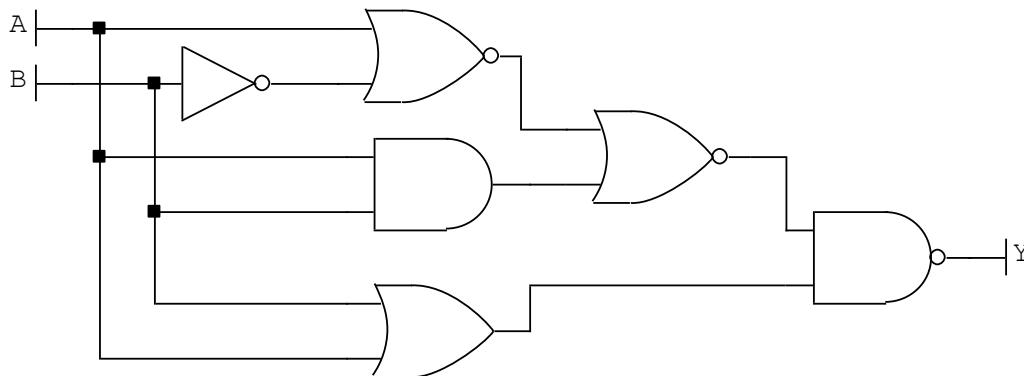
$$\begin{aligned}
 (b) \quad f(x, y, z) &= y\bar{z}(\bar{z} + \bar{z}x) + (\bar{x} + \bar{z})(\bar{x}y + \bar{x}z) \\
 &= y\bar{z} + \bar{x}y + \bar{x}z + \bar{x}y\bar{z} + \bar{x}z\bar{z} \\
 &= y\bar{z} + \bar{x}y + \bar{x}z \\
 &= y\bar{z} + \bar{x}y\bar{z} + \bar{x}yz + \bar{x}z \\
 &= y\bar{z} + \bar{x}z
 \end{aligned}$$



$$\begin{aligned}
 (c) \quad f(w, x, y, z) &= (w + x)(\bar{w} + y)(x + y + z) \\
 &= (wy + \bar{w}x + xy)(x + y + z) \\
 &= wxy + wy + wyz + \bar{w}x + \bar{w}xy + \bar{w}xz + xy + xyz \\
 &= wy(x + 1 + z) + \bar{w}x(1 + y + z) + xy(1 + z) \\
 &= wy + \bar{w}x + xy \\
 &= wy + \bar{w}x + \bar{w}xy + wxy \\
 &= wy + \bar{w}x
 \end{aligned}$$

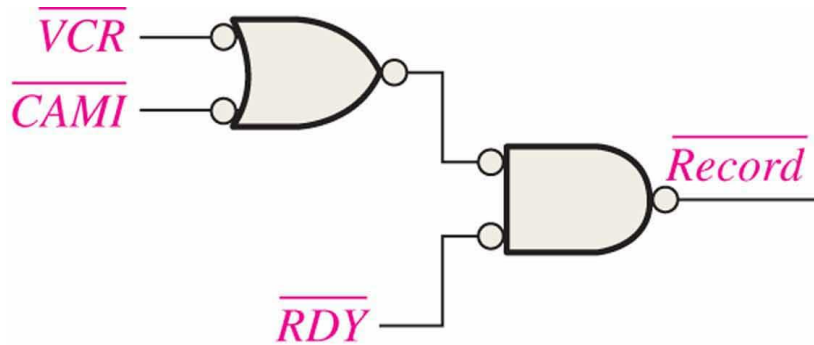


8. Derive the truth table for the circuits shown below



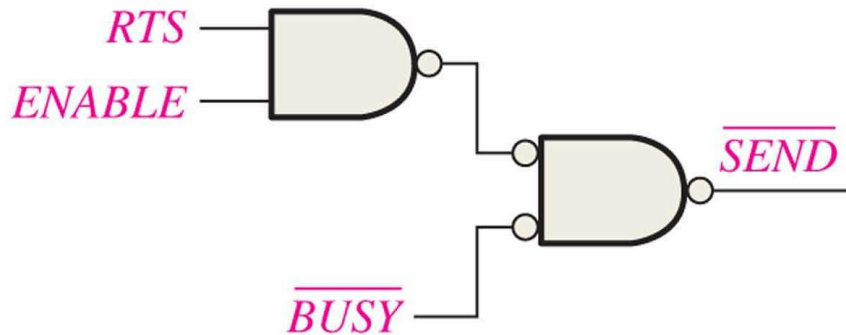
a)

INPUTS		OUTPUT
A	B	Y
0	0	1
0	1	1
1	0	0
1	1	1



b)

INPUTS			OUTPUT
\overline{VCR}	\overline{CAMI}	\overline{RDY}	\overline{RECORD}
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1



c)

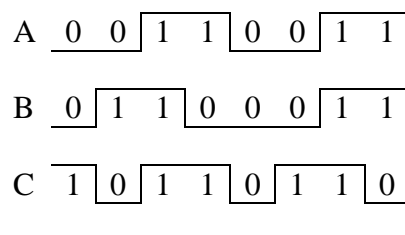
INPUTS			OUTPUT
RTS	$ENABLE$	\overline{BUSY}	\overline{SEND}
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

9. The Boolean expression of a logic circuit is given as:

$$Y = A\overline{B}C + \overline{A}B(C + \overline{D}) + (A \oplus C) + \overline{A + \overline{B} + D}$$

(a) Draw the implementation of the logic circuit given above

(b) If the inputs A, B, C and D are varying according to the timing diagram below, what should be waveform at Y?



D 0 1 0 0 1 1 1 0

