Exercise

1. Find $f(x)$ for the following functions.

a) $f'(x) = 1 - 6x, \quad f(0) = 8$

b) $f'(x) = 8x^3 + 12x + 3, \quad f(1) = 6$

c) $f'(t) = \sqrt{t}(6 + 5t), \quad f(1) = 10$

d) $f'(x) = 3\cos x, \quad f(0) = 1$

2. Evaluate the integrals.

a) $\int_{-1}^{3} x^5 \, dx$

b) $\int_{-2}^{5} 6 \, dx$

c) $\int_{2}^{8} (4x + 3) \, dx$

d) $\int_{0}^{4} (1 + 3y - y^2) \, dy$

e) $\int_{1}^{5} \frac{1}{x} \, dx$

f) $\int_{0}^{-1} 5e^x dx$

g) $\int_{0}^{\frac{\pi}{2}} \cos\theta \, d\theta$

h) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin x \, dx$

i) $\int_{-2}^{2} (3u + 1)^2 \, du$

j) $\int_{0}^{2} (2v + 5)(3v - 1) \, dv$

3. Find the integral using technique of substitution.

a) $\int 2x(x^2 + 4)^4 \, dx$

b) $\int (3x - 2)^{20} dx$

c) $\int_{0}^{2} x^2 (x^3 + 5)^9 \, dx$

d) $\int_{0}^{2} (2 - x)^6 dx$

e) $\int \frac{1 + 4x}{\sqrt{1 + x + 2x^2}} \, dx$

f) $\int_{-3}^{0} \frac{x}{(x^2 + 1)} dx$

g) $\int \cot x \, dx$

h) $\int 2 \sin x \cos^4 x \, dx$

i) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{-\cos x}{\sin^2 x} \, dx$

j) $\int_{0}^{\frac{\pi}{3}} \sec^3 x \tan x \, dx$

k) $\int (2x - 2) e^{-x^2 + x} dx$

l) $\int_{0}^{4} \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx$

4. Find the integrals using integration by parts. `

(a) $\int x\cos 5x\ dx$ (b) $\int_0^{\pi/2} x\sin x\, dx$

(c) $\int xe^{-x} dx$ (d) $\int_0^2 xe^{2x} dx$

(e) $\int \ln x\ dx$ (f) $\int_1^4 x\ln x dx$

(g) $\int x^2 \sin 5x\ dx$ (h) $\int (2x-1)\cos x\ dx$

5. Find the integrals using integration by partial fractions.

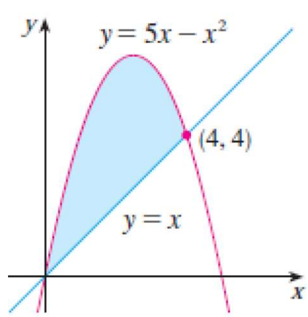
(a) $\int \frac{5x-5}{x^2+2x-3}\ dx$ (b) $\int \frac{3x+11}{(x^2-x-6)}\ dx$

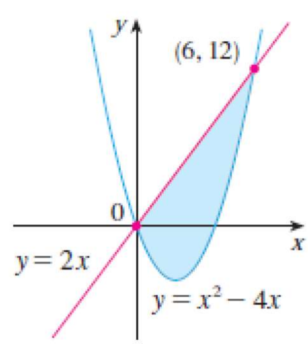
(c) $\int \frac{x}{x^2+2x-3}\ dx$ (d) $\int \frac{x+4}{x^2-3x+2}\ dx$

6. Sketch the region corresponding to each definite integral. Then evaluate each integral.

(a) $\int_{-1}^{1} x^2\ dx$ (b) $\int_0^4 (x+1)\, dx$ (c) $\int_1^3 2x\, dx$

7. Find the area of the shaded region.





8. Sketch the region bounded by the curves. Find the area of the region by integrating respect to *x*.

(a) $y = x + 1,\quad y = 9 - x^2,\quad x = -1,\quad x = 2$

(b) $y = \sin x,\ y = x,\quad x = \pi/2,\quad x = \pi$

(c) $y = (x-2)^2,\quad y = x$

(d) $y = x^2 - 2x,\quad y = x + 4$

9. Find the **volume** of the solid obtained by rotating the region enclosed by the curves $y = \sqrt{x}$, $x = 9$ and $y = 0$ is rotated about the *x*-axis.

10. Sketch the region bounded by $y = x^3$, $y = 27$ and $x = 0$. Hence, find the **volume** of the solid obtained by rotating the region about $y-$axis.

Exercise: Topic 5

1. (a) $f(x) = x - 3x^2 + 8$ (b) $f(x) = 2x^4 + 6x^2 + 3x - 5$
(c) $f(t) = 4t^{3/2} + 2t^{5/2} + 4$ (d) $f(x) = 3\sin(x) + 1$

2. (a) 728/6 (b) 42
(c) 138 (d) 20/3
(e) ln 5 (f) $= 5/e - 5$
(g) 1 (h) 0
(i) $\boxed{4}$ Correct:52 (j) 32

3. (a) $\frac{(x^2+4)^5}{5} + c$ (b) $\frac{(3x-2)^{21}}{63} + c$
(c) $\frac{13^{10}-5^{10}}{30}$ (d) 128/7
(e) $2\sqrt{1 + x + 2x^2} + c$ (f) $-\frac{1}{2}\ln 10$
(g) $\ln|\sin x| + c$ (h) $-\frac{2}{5}\cos^5 x + c$
(i) 1/2 (j) 7/3
(k) **Correction**: $\int (2x - 2)e^{(-x^2+2x)}\, dx$ $ans: -e^{(-x^2+2x)} + c$
(l) $2e^2 - 2$

4. (a) $\frac{1}{5}x\sin 5x + \frac{1}{25}\cos 5x + c$ (b) $[-x\cos x + \sin x]_0^{\pi/2} = 1$
(c) $-xe^{-x} - e^{-x} + c$ (d) $\left[\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}\right]_0^2 = \frac{3}{4}e^4 + \frac{1}{4}$
(e) $x\ln x - x + c$ (f) $\left[\frac{1}{2}x^2\ln x - \frac{1}{4}x^2\right]_1^4 = 8\ln x - \frac{15}{4}$
(g) $-\frac{1}{5}x^2\cos 5x + \frac{2}{25}x\sin 5x + \frac{2}{125}\cos 5x + c$
(h) $(2x - 1)\sin x + 2\cos x + c$

5. (a) $5\ln|x + 3| + c$ (b) $4\ln|x - 3| - \ln|x + 2| + c$
(c) $\frac{1}{4}\ln|x - 1| + \frac{3}{4}\ln|x + 3| + c$ (d) $-5\ln|x - 1| + 6\ln|x - 2| + c$

6. *Note: Sketch region as in the example page 16*
(a) 2/3 (b) 12 (c) 8

7. Area = *32/3 unit*^{2} , Area = *36 unit*^{2}

8. (a) $39/2$ *unit*^{2} (b) $\frac{3\pi^2}{8} - 1$ *unit*^{2} (c) $\frac{9}{2}$ *unit*^{2} (d) $\frac{125}{6}$ *unit*^{2}

9. $\frac{81}{2}\pi$ *unit*^{3}

10. $\frac{729}{5}\pi$ *unit*^{3}