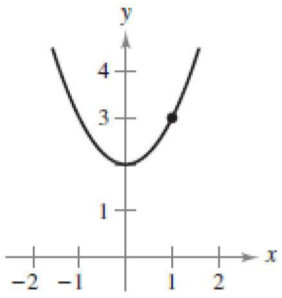
Exercise

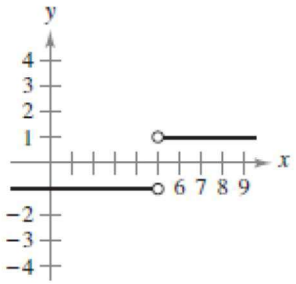
A. Limit of a function

1. Use the graph to find the limit (if it exists). If the limit does not exist, explain why.

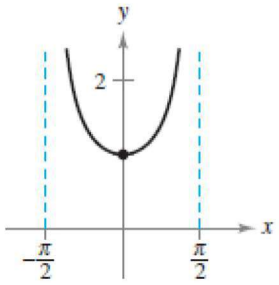
(a) $\lim_{x \to 1} (x^2 + 2)$



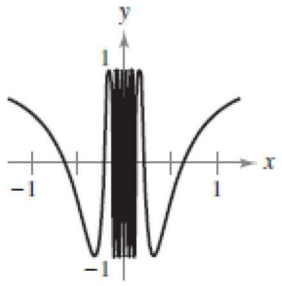
(b) $\lim_{x \to 5} \frac{|x - 5|}{x - 5}$



(c) $\lim_{x \to 0} \sec x$

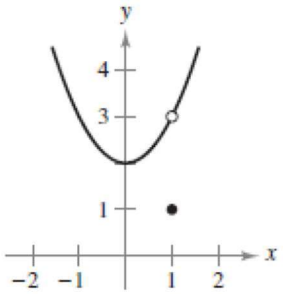


(d) $\lim_{x \to 0} \cos \frac{1}{x}$

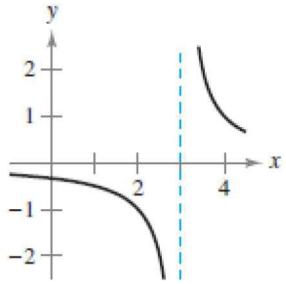


(e) $\lim_{x \to 1} f(x)$

$$f(x) = \begin{cases} x^2 + 2, & x \neq 1 \\ 1, & x = 1 \end{cases}$$

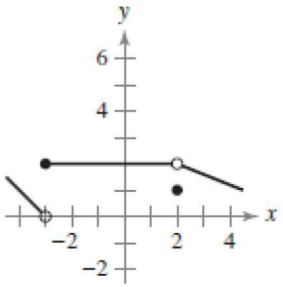


(f) $\lim_{x \to 3} \frac{1}{x - 3}$

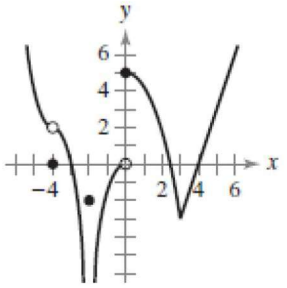


2. For each graph of function $f(x)$, identify the values of c for which $\lim_{x \to c} f(x)$ does not exist. Explain your answer.

(a)



(b)



B. Limit Laws

3. Find the limit.

(a) $\lim_{x\to -3}(2x^2+4x+1)$

(b) $\lim_{x\to 1}(3x^3-2x^2+4)$

(c) $\lim_{x\to 2}\dfrac{1}{x}$

(d) $\lim_{x\to -3}\dfrac{2}{x+2}$

(e) $\lim_{x\to 1}\dfrac{x-3}{x^2+4}$

(f) $\lim_{x\to 3}\dfrac{2x-3}{x+5}$

(g) $\lim_{x\to 7}\dfrac{5x}{\sqrt{x+2}}$

(h) $\lim_{x\to 3}\dfrac{\sqrt{x+1}}{x-4}$

(i) $\lim_{x\to 3}\sqrt{x+1}$

(j) $\lim_{x\to 4}\sqrt[3]{x+4}$

(k) $\lim_{x\to -4}(x+3)^2$

(l) $\lim_{x\to 0}(2x-1)^3$

4. Use the information to evaluate the limit.

$\lim_{x\to c} f(x) = 2$

$\lim_{x\to c} g(x) = 3$

(a) $\lim_{x\to c}[5g(x)]$

(b) $\lim_{x\to c}[f(x)+g(x)]$

(c) $\lim_{x\to c}[f(x)g(x)]$

(d) $\lim_{x\to c}\dfrac{f(x)}{g(x)}$

$\lim_{x\to c} f(x) = 4$

(a) $\lim_{x\to c}[f(x)]^3$

(b) $\lim_{x\to c}\sqrt{f(x)}$

(c) $\lim_{x\to c}[3f(x)]$

(d) $\lim_{x\to c}[f(x)]^{3/2}$

5. Evaluate the limit (if it exists).

(a) $\lim_{h\to 0}\dfrac{(2+h)^2-4}{h}$

(b) $\lim_{h\to 0}\dfrac{(2+h)^3-8}{h}$

(c) $\lim_{x\to 5}\dfrac{x-5}{x^2-25}$

(d) $\lim_{x\to 2}\dfrac{2-x}{x^2-4}$

(e) $\lim_{x\to -3}\dfrac{x^2+x-6}{x^2-9}$

(f) $\lim_{x\to 4}\dfrac{x^2-5x+4}{x^2-2x-8}$

(g) $\lim_{x\to 0}\dfrac{\sqrt{x+5}-\sqrt{5}}{x}$

(h) $\lim_{x\to 0}\dfrac{\sqrt{2+x}-\sqrt{2}}{x}$

(i) $\lim_{x\to 4}\dfrac{\sqrt{x+5}-3}{x-4}$

(j) $\lim_{x\to 3}\dfrac{\sqrt{x+1}-2}{x-3}$

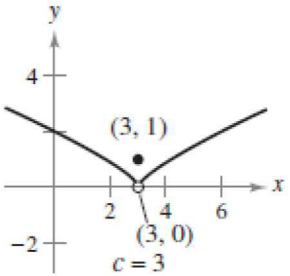
(k) $\lim_{x\to 16}\dfrac{4-\sqrt{x}}{x-16}$

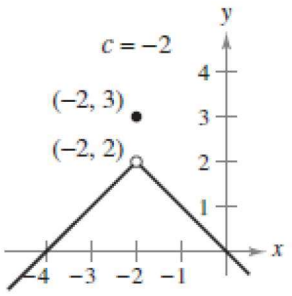
(l) $\lim_{x\to 1}\dfrac{x^2-1}{\sqrt{x}-1}$

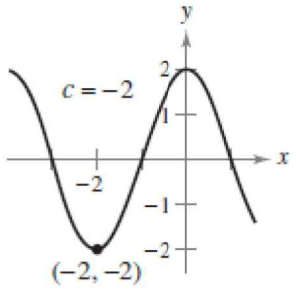
C. One-Sided Limits

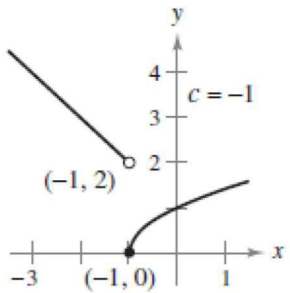
6. For each graph determine the limit.

(a) $\lim_{x\to c^+} f(x)$ (b) $\lim_{x\to c^-} f(x)$ (c) $\lim_{x\to c} f(x)$









7. Find the limit (if it exists). If it does not exist, explain why.

(a) $\lim_{x\to 2} f(x)$, where $f(x) = \begin{cases} x^2 - 4x + 6, & x < 2 \\ -x^2 + 4x - 2, & x \geq 2 \end{cases}$

(b) $\lim_{x\to 1} f(x)$, where $f(x) = \begin{cases} x^3 + 1, & x < 1 \\ x + 1, & x \geq 1 \end{cases}$

(c) $\lim_{x\to 1} f(x)$ where $f(x) = \begin{cases} x, & x < 1 \\ 2, & x = 1 \\ 2x - 1, & x > 1 \end{cases}$

D. Infinite Limits and vertical asymptotes

8. Find each limit. State the vertical asymptote, if any.

(a) $\lim_{x\to 2^+} \frac{x-3}{x-2}$

(b) $\lim_{x\to 1^+} \frac{2+x}{1-x}$

(c) $\lim_{x\to 3^+} \frac{x^2}{x^2-9}$

(d) $\lim_{x\to 4^-} \frac{x^2}{x^2+16}$

9. Find the vertical asymptote (if any) of the function.

(a) $f(x) = \frac{1}{x^2}$

(b) $f(x) = \frac{4}{(x-2)^3}$

(c) $h(x) = \frac{x^2-2}{x^2-x-2}$

(d) $g(x) = \frac{2+x}{x^2(1-x)}$

(e) $f(x) = \frac{x^2}{x^2-4}$

(f) $f(x) = \frac{-4x}{x^2+4}$

(g) $g(t) = \frac{t-1}{t^2+1}$

(h) $h(s) = \frac{2s-3}{s^2-25}$

(i) $f(x) = \frac{8}{(x-10)^2}$

(j) $g(x) = 1 + \frac{2}{x}$

E. Limit at infinity and horizontal asymptotes

10. Find each limit. State the horizontal asymptote, if any.

(a) $\lim_{x\to\infty} \frac{x^2+2}{x^3-1}$

(b) $\lim_{x\to\infty} \frac{3-2x}{3x^3-1}$

(c) $\lim_{x\to\infty} \frac{x^2+2}{x^2-1}$

(d) $\lim_{x\to\infty} \frac{3-2x}{3x-1}$

(e) $\lim_{x\to\infty} \frac{x^2+2}{x-1}$

(f) $\lim_{x\to\infty} \frac{3-2x^2}{3x-1}$

(g) $\lim_{x\to\infty} \frac{3x^3+2}{9x^3-2x^2+7}$

(h) $\lim_{x\to\infty} \frac{2x-1}{3x+2}$

(i) $\lim_{x\to\infty} \left(4 + \frac{3}{x}\right)$

(j) $\lim_{x\to\infty} \frac{x}{x^2-1}$

(k) $\lim_{x\to-\infty} \left(\frac{1}{2}x - \frac{4}{x^2}\right)$

(l) $\lim_{x\to-\infty} \frac{5x^2}{x+3}$

F. Limit involving trigonometric functions

11. Determine the limit of the trigonometric function (if it exists).

(a) $\lim_{x\to 0} \dfrac{\sin x}{5x}$

(b) $\lim_{x\to 0} \dfrac{3(1-\cos x)}{x}$

(c) $\lim_{x\to 0} \dfrac{\sin x(1-\cos x)}{2x^2}$

(d) $\lim_{\theta\to 0} \dfrac{\cos\theta \tan\theta}{\theta}$

(e) $\lim_{x\to 0} \dfrac{\sin^2 x}{x}$

(f) $\lim_{x\to 0} \dfrac{\tan^2 x}{x}$

(g) $\lim_{h\to 0} \dfrac{(1-\cos h)^2}{h}$

(h) $\lim_{\phi\to \pi} \phi \sec\phi$

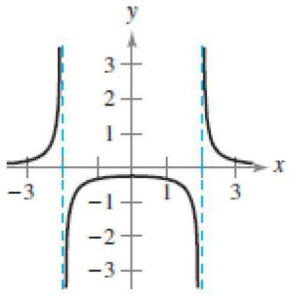
(i) $\lim_{x\to \pi/2} \dfrac{\cos x}{\cot x}$

(j) $\lim_{x\to \pi/4} \dfrac{1-\tan x}{\sin x - \cos x}$

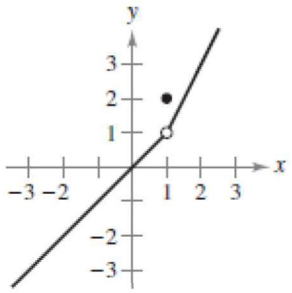
G. Continuity

12. Discuss the continuity of each function.

(a) $f(x) = \dfrac{1}{x^2-4}$



(b) $f(x) = \begin{cases} x, & x < 1 \\ 2, & x = 1 \\ 2x-1, & x > 1 \end{cases}$



13. Is the function continuous at the given number, a?

(a) $f(x) = \begin{cases} \dfrac{1}{x-1} & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$ $\quad a = 1$

(b) $f(x) = \begin{cases} e^x & \text{if } x < 0 \\ x^2 & \text{if } x \geqslant 0 \end{cases}$ $\quad a = 0$

(c) $f(x) = \begin{cases} \dfrac{x^2-x}{x^2-1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$ $\quad a = 1$

14. Find the x-values (if any) at which f is not continuous.

(a) $f(x) = x^2 - 2x + 1$

(b) $f(x) = \dfrac{x-3}{x^2-9}$

(c) $f(x) = \dfrac{1}{x^2+1}$

(d) $f(x) = \dfrac{x+2}{x^2-3x-10}$

(e) $f(x) = 3x - \cos x$

(f) $f(x) = \dfrac{x-1}{x^2+x-2}$

(g) $f(x) = \dfrac{x}{x^2-x}$

(h) $f(x) = \cos\dfrac{\pi x}{2}$

(i) $f(x) = \begin{cases} \dfrac{\cos x - 1}{x}, & x < 0 \\ 5x, & x \geq 0 \end{cases}$

(j) $f(x) = \begin{cases} \frac{1}{2}x + 1, & x \leq 2 \\ 3 - x, & x > 2 \end{cases}$

(k) $f(x) = \begin{cases} -2x, & x \leq 2 \\ x^2 - 4x + 1, & x > 2 \end{cases}$

l) $f(x) = \begin{cases} -2x + 3, & x < 1 \\ x^2, & x \geq 1 \end{cases}$

Exercise: Topic 3

A. Limit of a function

1. (a) 3
   (b) Does not exist
   (c) 1
   (d) Does not exist
   (e) 3
   (f) Does not exist

2. (a) $x=-3$
   (b) $x=-2, x=0,$

B. Limit Laws

3. (a) 7 (b) 5
   (c) 1/2 (d) -2
   (e) -2/5 (f) 3/8
   (g) 35/3 (h) -2
   (i) 2 (j) 2
   (k) 1 (l) -1

4. (a) 15 (a) 64
   (b) 5 (b) 2
   (c) 6 (c) 12
   (d) 2/3 (d) 8

5. (a) 4 (b) 12
   (c) 1/10 (d) -1/4
   (e) 5/6 (f) 1/2
   (g) $\frac{1}{2\sqrt{5}}$ (h) $\frac{1}{2\sqrt{2}}$
   (i) 1/6 (j) 1/4
   (k) -1/8 (l) 4

C. One-Sided Limits

6.
Graph 1: (a) 0 (b) 0 (c) 0
Graph 2: (a) 2 (b) 2 (c) 2
Graph 3: (a) -2 (b) -2 (c) -2
Graph 4: (a) 0 (b) 2
(c) Does not exist

7. (a) 2 (b) 2 (c) 1

D. Infinite Limits and vertical asymptotes

8. (a) $-\infty$ (does not exist), $x=2$
   (b) ∞ (does not exist), $x=1$
   (c) ∞ (does not exist), $x=3, x=-3$
   (d) 1/2

9. (a) $x=0$ (b) $x=2$
   (c) $x=2, x=-1$ (d) $x=0, x=1$
   (e) $x=-2, x=2$ (f) -
   (g) - (h) $s=-5, s=5$
   (i) $x=10$ (j) $x=0$

E. Limit at infinity and horizontal asymptotes

10.
   (a) $0, y=0$ (b) $0, y=0$
   (c) $1, y=1$ (d) $-2/3, y=-2/3$
   (e) ∞ (f) $-\infty$
   (g) $1/3, y=1/3$ (h) $2/3, y=2/3$
   (i) $4, y=4$ (j) $0, y=0$
   (k) $-\infty$ (l) $-\infty$

F. Limit involving trigonometric functions

11. (a) 1/5 (b) 0
    (c) 0 (d) 1
    (e) 0 (f) 0
    (g) 0 (h) $-\pi$
    (i) 1 (j) -2

G. Continuity

12. (a) Discontinuous at $x=-2, 2$
    (b) Discontinuous at $x= 1$

13. (a) Discontinuous
    (b) Discontinuous
    (c) Discontinuous

14. (a) (b) $x= -3, 3$
    (c) (d) $x= -2, 5$
    (e) (f) $x= -2, 1$
    (g) $x= 0, 1$ (h)
    (i) (j) $x= 2$
    (k) $x= 2$ (l)