We will wait 5 minutes for the rest to join in. Class will start at 9:05 AM.

WAVES
CHAPTER 5

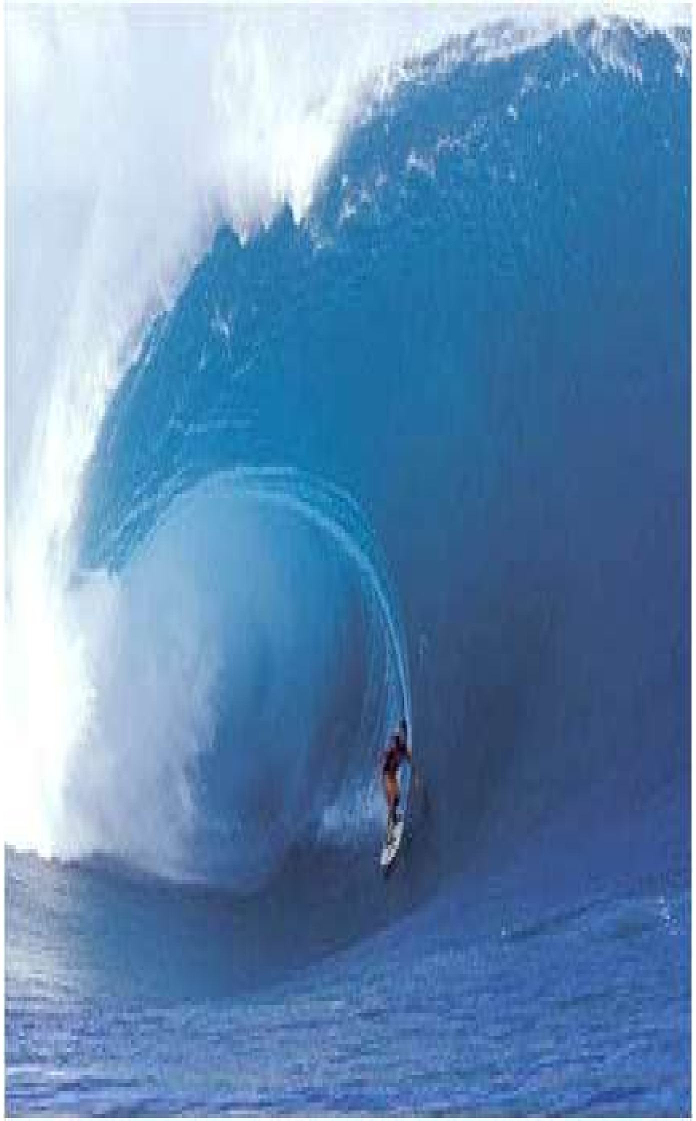
CONTENTS

AT THE END OF THIS CHAPTER YOU SHOULD BE ABLE TO:

- Describe wave motion in terms of various parameters and identify different types of waves.
- Explain various wave phenomena.
- Define transverse and longitudinal wave.
- Understand and use the terms displacements, amplitude, phase difference, period, frequency, wavelength and speed.
- Describe the formation and characteristics of superposition and interference of wave.

5.1 DEFINITION OF WAVE

- A wave motion is a mean of transferring between energy from one point to another without there being any transfer of matter between the points
- Waves are oscillations that can propagated (spread out) from a source carrying energy.
- All waves carry energy and momentum



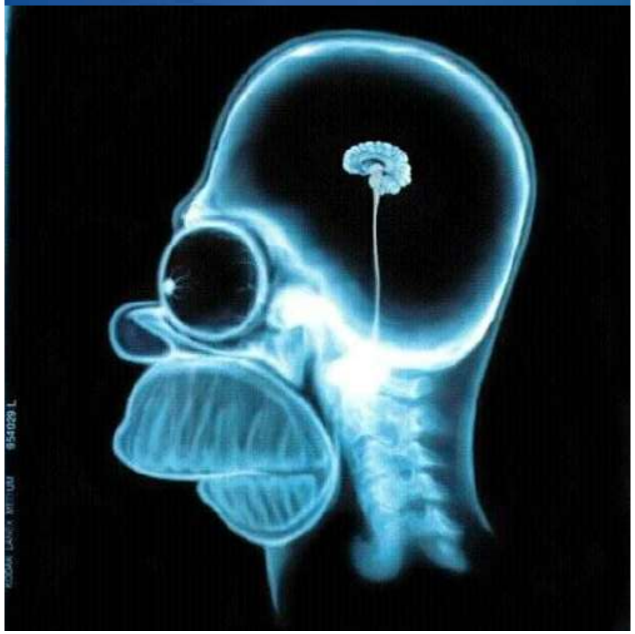
5.2 MECHANICAL AND ELECTROMAGNETIC WAVES

Mechanical waves

- It requires a material medium for their propagation.
- For instance, water waves, sound waves, waves in stretched strings

Electromagnetic waves

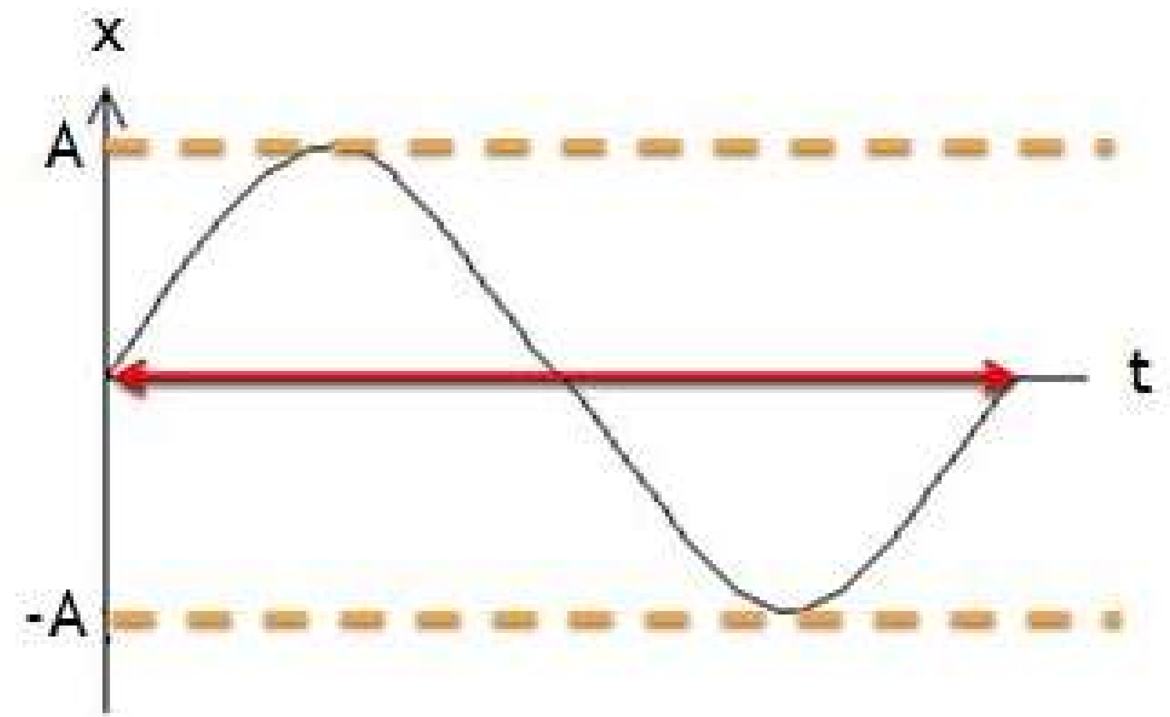
- It requires no medium to exist, can travel through a vacuum
- For Instance, light, radio and X-ray



5.3 TYPES OF WAVES

- From chapter 4,

Displacement Vs. Time



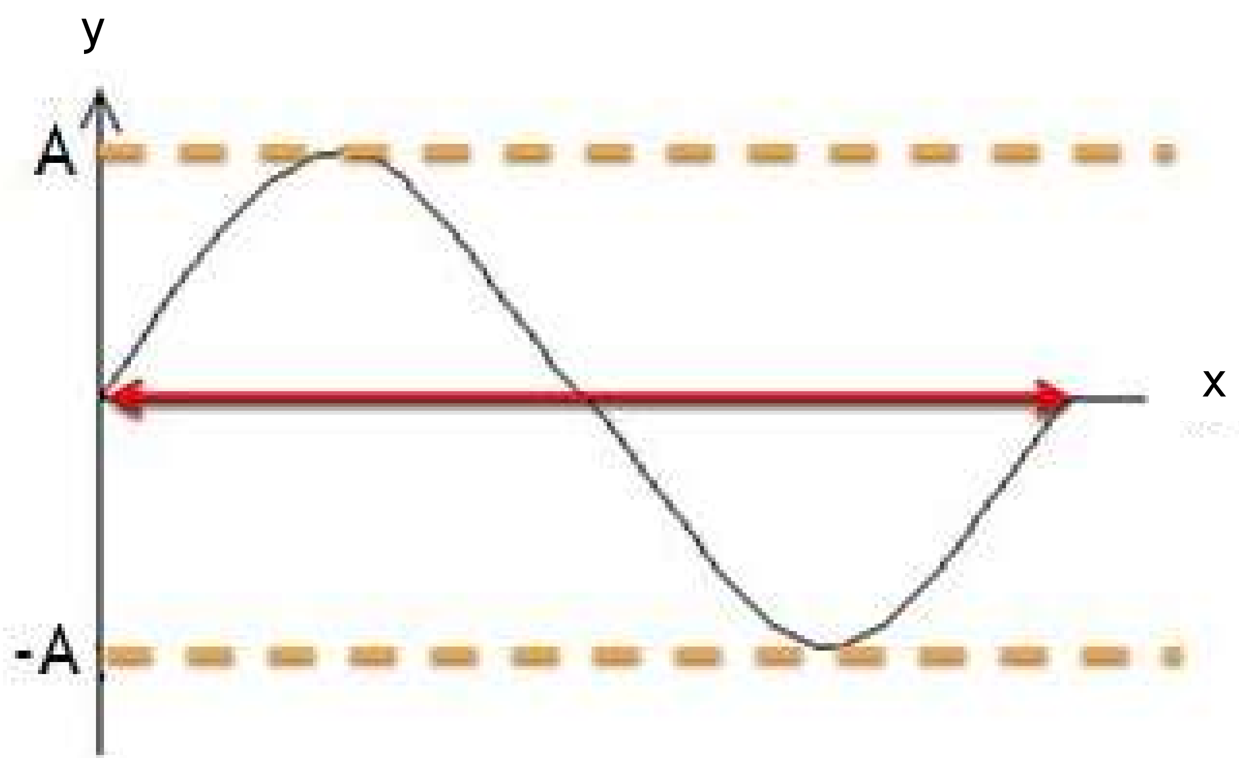
$$x = A\sin\omega t$$

5.3 TYPES OF WAVES

$$y = A\sin\frac{2\pi}{\lambda}x$$

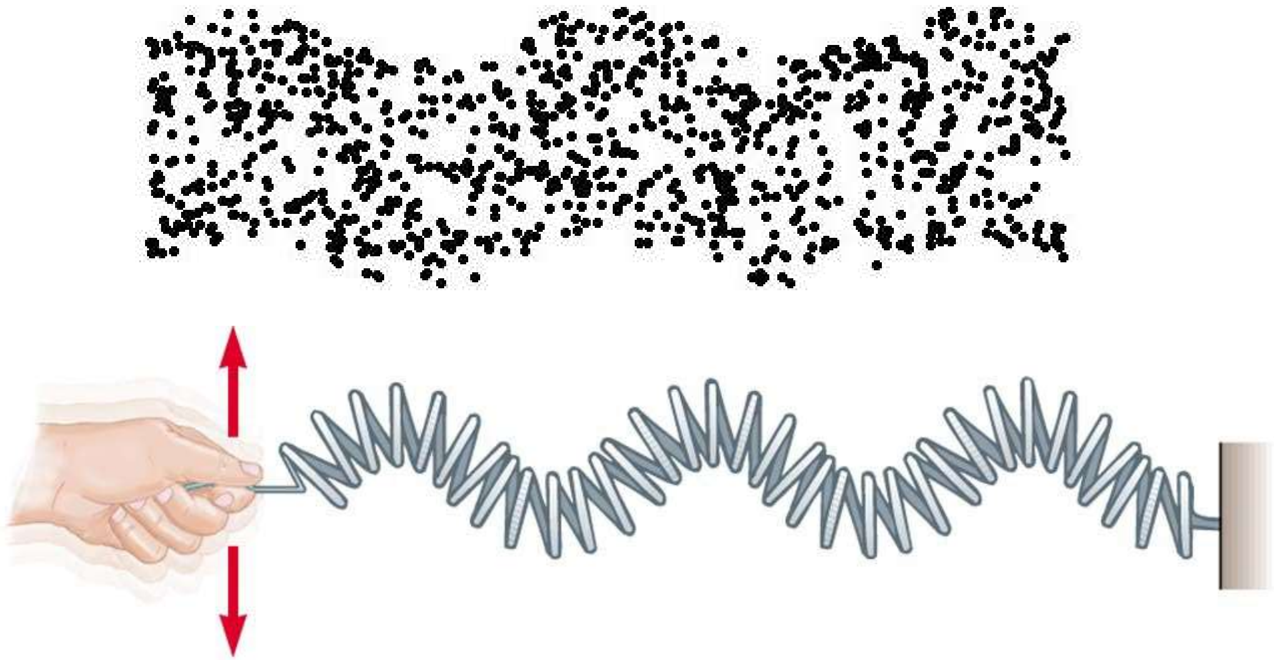
- This chapter

Displacement Vs. Distance



5.3 TYPES OF WAVES

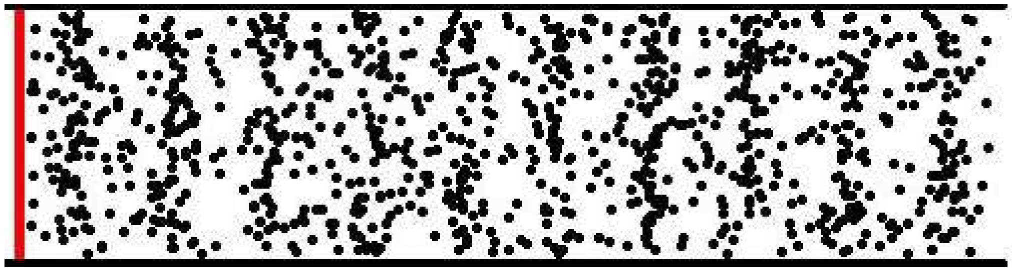
- Transverse Wave
  - the particle displacement is perpendicular to the direction of wave propagation. Eg: Light wave, microwave.

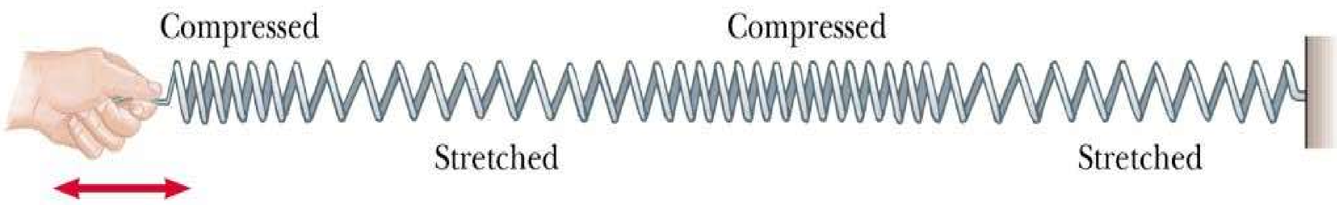


(a) Transverse wave

5.3 TYPES OF WAVES

- Longitudinal Wave
  - the particle displacement is parallel to the direction of wave propagation. Eg: Sound wave, ultrasound wave.

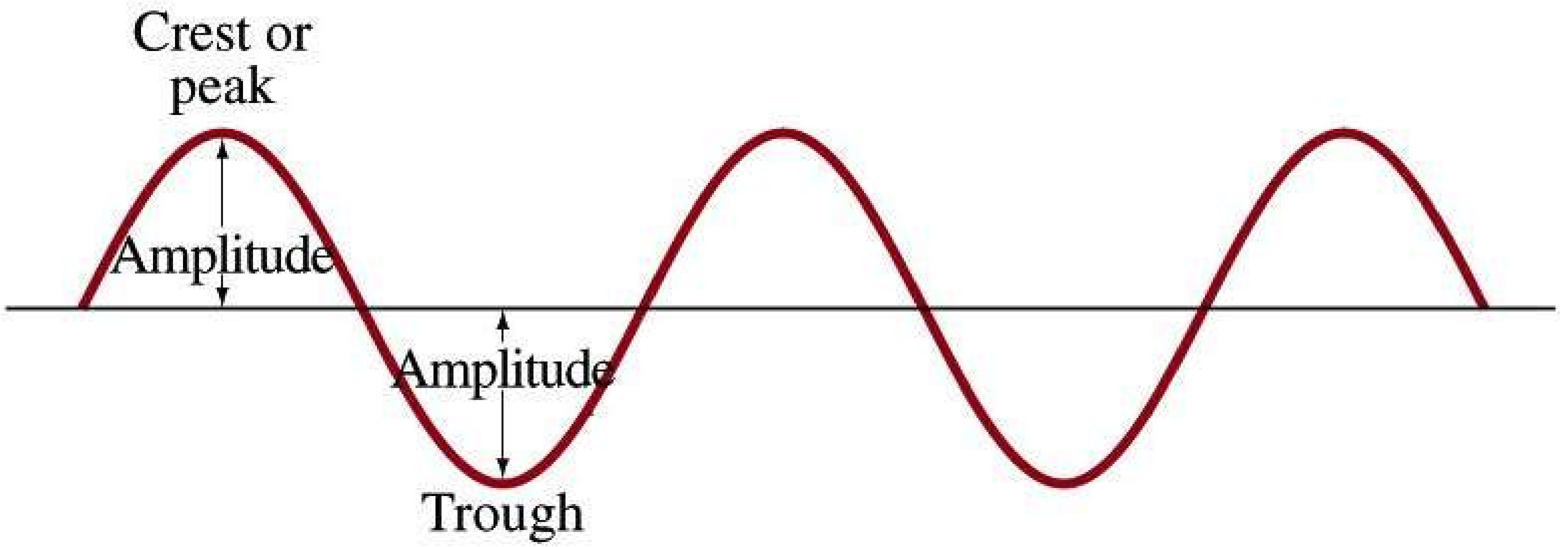




(b) Longitudinal wave

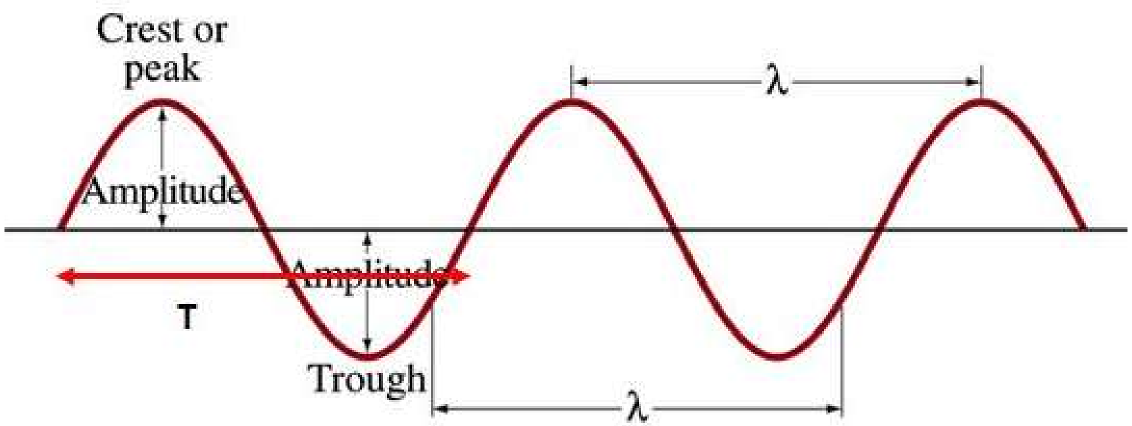
5.4 PROPERTIES OF SINUSOIDAL WAVES (REVIEW AND ADD ON)

- **Amplitude, A**
  - The magnitude of the maximum displacement of any particle from its equilibrium position.



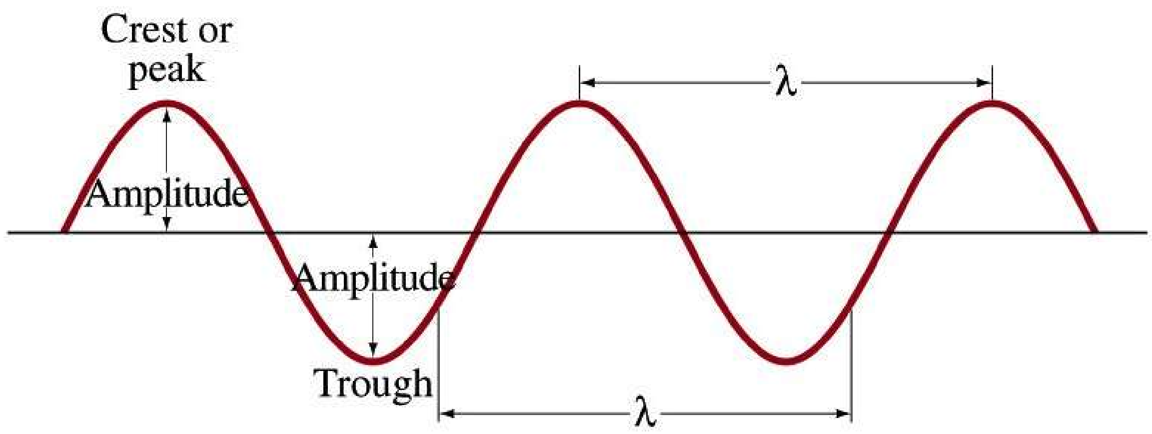
5.4 PROPERTIES OF SINUSOIDAL WAVES (REVIEW AND ADD ON)

- Period, T
  - The time taken for the wave to travel a distance of one wavelength
  - the time taken for any particle to undergo a complete oscillation
- Frequency, f
  - The number of wavelengths per second which pass a given point.



5.4 PROPERTIES OF SINUSOIDAL WAVES (REVIEW AND ADD ON)

- **Wavelength, λ**
  - **How far wave has travel after 1 period**
  - The distance between two adjacent crests or troughs.
  - the distance between any two successive particle that are in phase; that is, at identical



$$\lambda = \frac{v}{f}$$

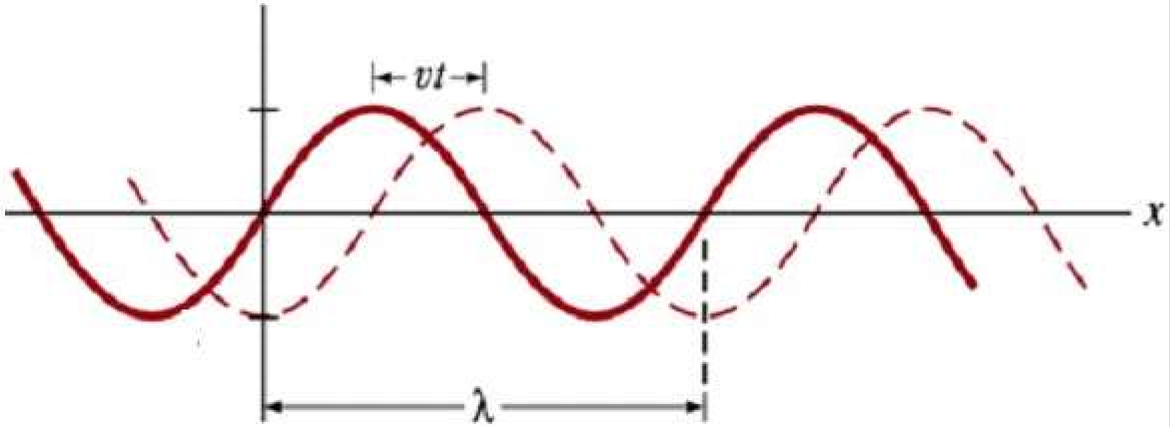
COMPARISON FUNCTION WITH FREQUENCY AND WAVELENGTH

$$y = A \sin \frac{2\pi}{\lambda} x$$

5.5 TRAVELLING WAVES AND SINUSOIDAL WAVES

- If the wave *moves to the right* with a velocity *v*, the wave function at some time later, t , is

$$y = A\sin\frac{2\pi}{\lambda}\left(x - vt\right)$$



5.5 TRAVELLING WAVES AND SINUSOIDAL WAVES

$$\boxed{y = A\sin\frac{2\pi}{\lambda}(x+vt)}$$

What does x+vt mean here ?

5.5 TRAVELLING WAVES AND SINUSOIDAL WAVES

- As we know the travelling waves is $\boxed{y = A\sin\frac{2\pi}{\lambda}(x - vt)}$

$$
\boxed{
\begin{aligned}
y &= A\sin\frac{2\pi}{\lambda}(x - vt) \\
y &= A\sin\left(\frac{2\pi}{\lambda}x - \frac{2\pi v}{\lambda}t\right) \\
y &= A\sin\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{\lambda}f\lambda.t\right) \\
y &= A\sin\left(\boxed{\frac{2\pi}{\lambda}}x - 2\pi ft\right) \\
y &= A\sin(kx - \omega t)
\end{aligned}
}
$$

$$v = f\lambda$$

$$= \frac{\lambda}{T}$$

Wave Number

$$k = \frac{2\pi}{\lambda}$$

angular frequency

$$\omega = \frac{2\pi}{T}$$

$$= 2\pi f$$

5.5 TRAVELLING WAVES AND SINUSOIDAL WAVES

- The general travelling waves equation is

$$\boxed{y = A\sin(kx - \omega t)}$$

- The vertical displacement , y is zero at x=0 and t=0

5.5 TRAVELLING WAVES AND SINUSOIDAL WAVES

- The wave is moving to the right

$$\boxed{y = A\sin(kx - \omega t)}$$

- The wave is moving to the left

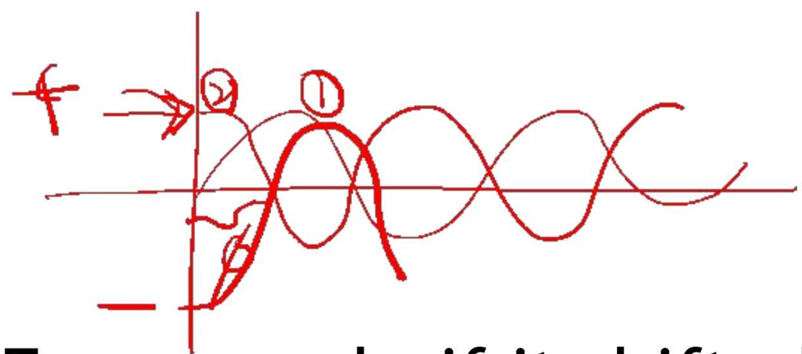
$$\boxed{y = A\sin(kx + \omega t)}$$

5.5 TRAVELLING WAVES AND SINUSOIDAL WAVES

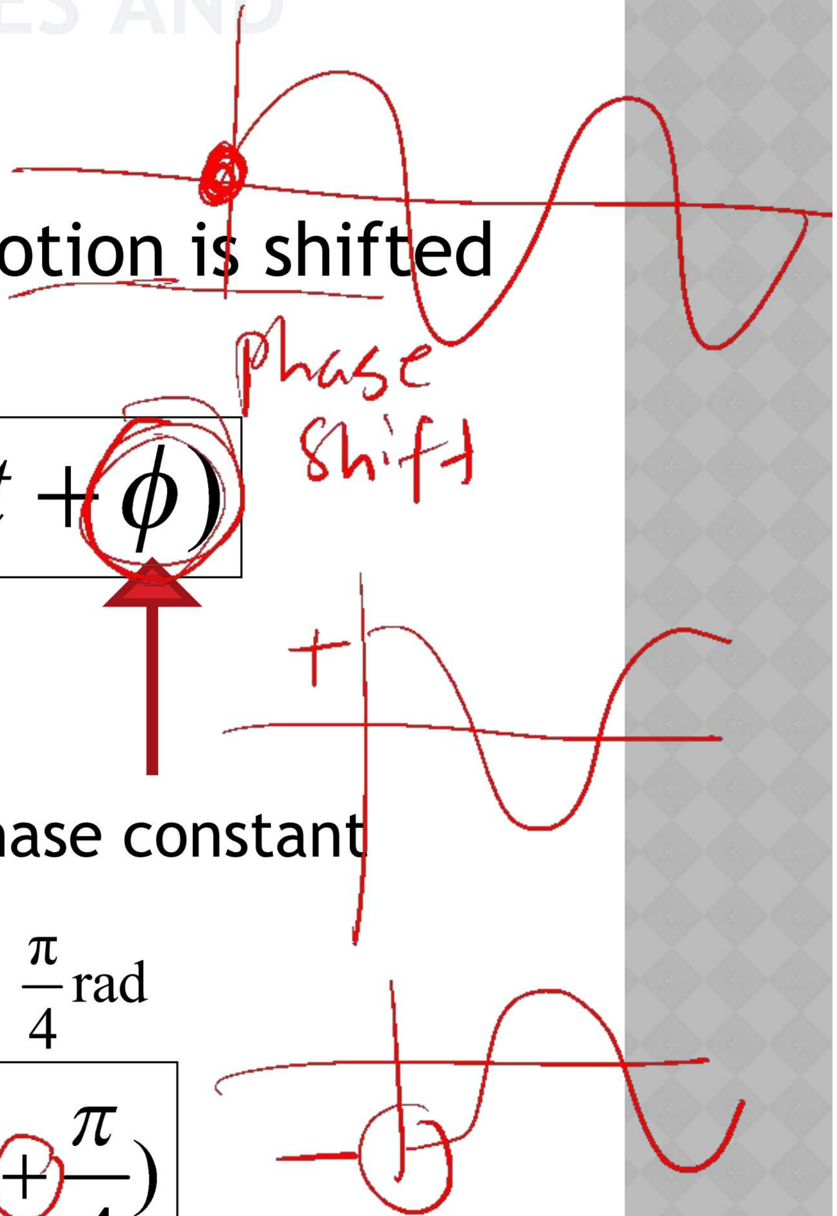
- If the Angle by which the motion is shifted from y=0, at t=0 and x=0

phase shift

$$y = A\sin(kx - \omega t + \phi)$$



Phase constant



- For example if it shifted to $\frac{\pi}{4}$ rad

$$y = A\sin(kx - \omega t + \frac{\pi}{4})$$

5.5 TRAVELLING WAVES AND SINUSOIDAL WAVES

- Example

ω = ang. vel.
= ang. frequency.

$\omega = 2\pi f$

$\omega = 2\pi(8) = 16\pi = 50.27$ rad/s

$k = \frac{2\pi}{\lambda}$ $\frac{rad}{m}$.

$k = \frac{2\pi}{0.4}$

$k = 5\pi$

$k = 15.71$ rad/m

A sinusoidal wave travelling to the right has the following characteristics:

- Amplitude, A = 15 cm,
- Wavelength, λ = 40 cm ,
- Frequency, f = 8.00 Hz.
- Vertical displacement: y (x) = 15 cm (at t = 0 and x = 0)

$T = \frac{1}{f} = \frac{1}{8} = 0.125$ s

$v = f\lambda = 8(0.4) = 3.2$ m/s

a) Find *k*, ω, T and *v*.

b) Determine the phase constant , Φ and write the wave function of the wave

5.5 TRAVELLING WAVES AND SINUSOIDAL WAVES

a) Find *k*

$$\boxed{\begin{aligned} k &= \frac{2\pi}{\lambda} \\ &= \frac{2\pi}{40cm} \\ &= 0.157 rad / cm \end{aligned}}$$

5.5 TRAVELLING WAVES AND SINUSOIDAL WAVES

a) Find ω

$$\boxed{\begin{aligned}\omega &= \frac{2\pi}{T} \\ &= 2\pi f \\ &= 50.3 rad / s\end{aligned}}$$

a) Find T

$$\boxed{\begin{aligned} T &= \frac{1}{f} \\ &= \frac{1}{8.00Hz} \\ &= 0.125s \end{aligned}}$$

5.5 TRAVELLING WAVES AND SINUSOIDAL WAVES

a) Find V

$$\boxed{\begin{aligned} v &= f\lambda \\ &= (8.00Hz)(40cm) \\ &= 320cms^{-1} \end{aligned}}$$

5.5 TRAVELLING WAVES AND SINUSOIDAL WAVES

b) Determine the phase constant , Φ

K =0.157 rad/cm

ω = 50.3 rad/s

T = 0.125 s

V = *320 cm/s*

$$
\begin{aligned}
y &= A\sin(kx-\omega t+\varphi) \\
15cm &= (15cm)\sin(0-0+\varphi) \\
15cm &= (15cm)\sin(\varphi) \\
\sin\varphi &= 1 \\
\varphi &= 90^{o} \\
&= \frac{\pi}{2}
\end{aligned}
$$

b) and write the wave function of the wave

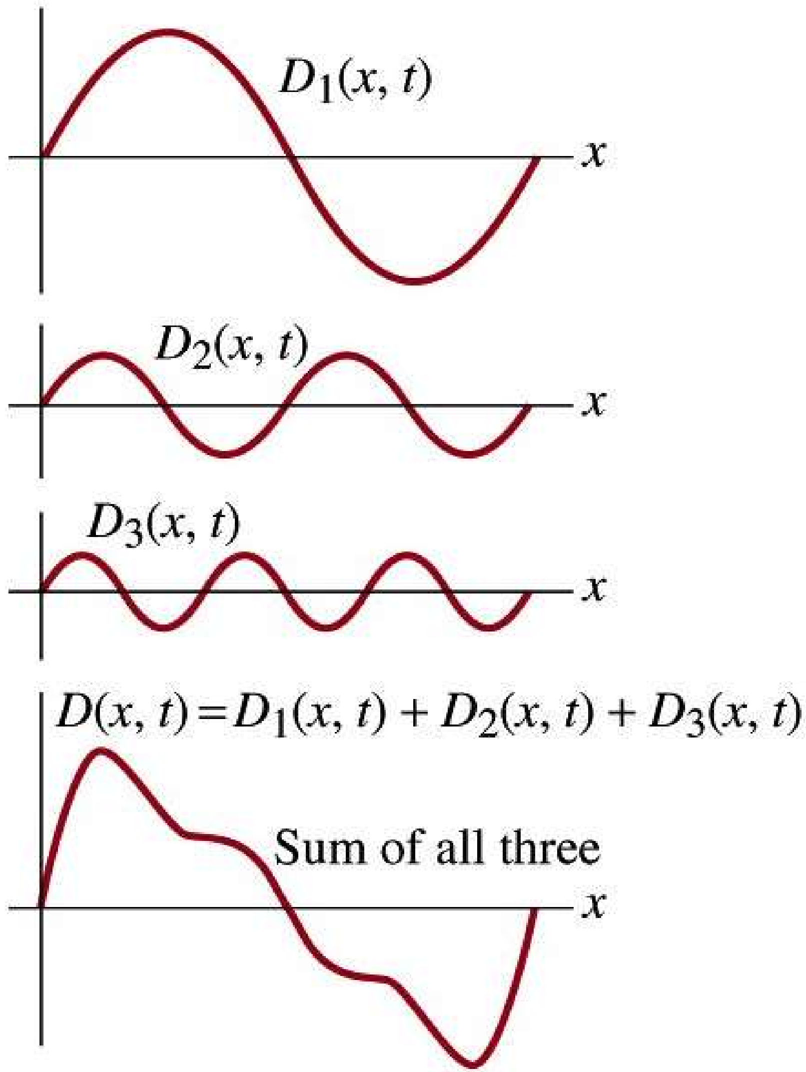
$$y = A\sin(kx - \omega t + \phi)$$

$$y = (15cm)\sin(kx - \omega t + \frac{\pi}{2})$$

$$y = (15cm)\sin[0.157\, rad / cm)x - (50.3 rad / s)t + \frac{\pi}{2}]$$

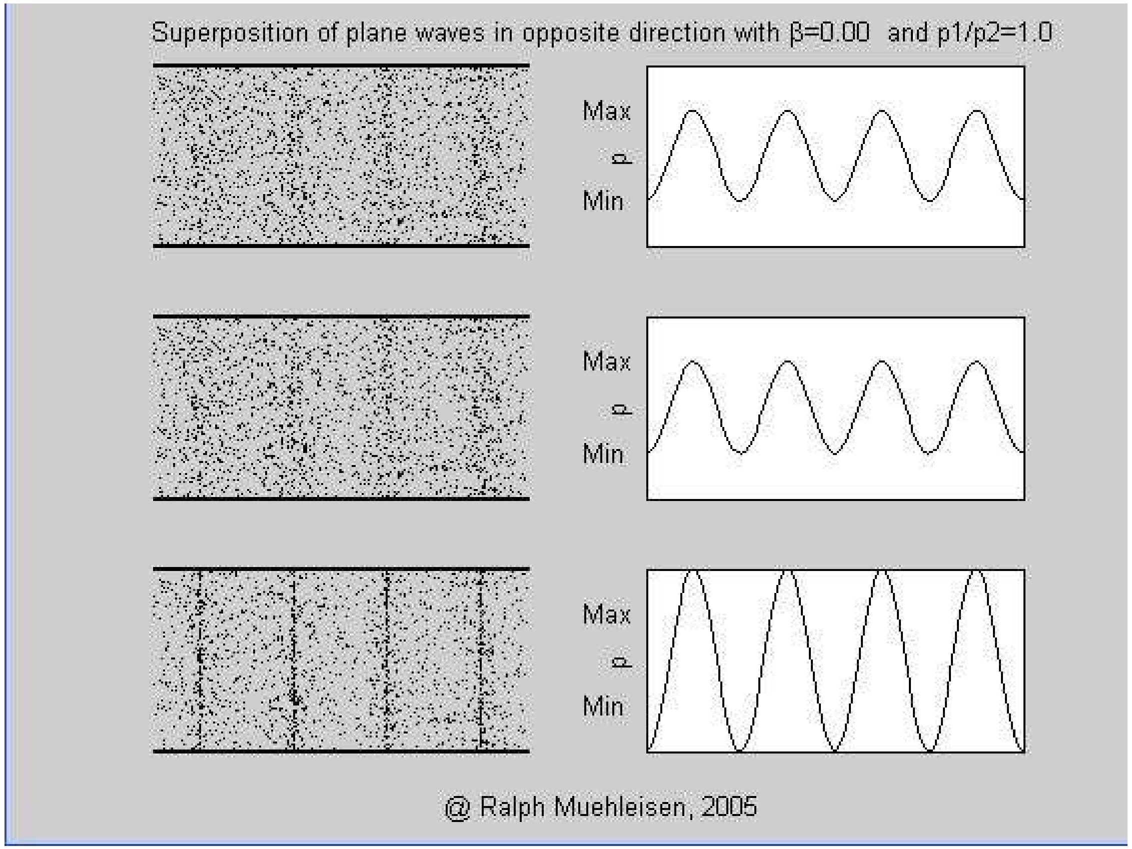
5.6 SUPERPOSITION AND INTERFERENCE OF WAVES

- The principle of superposition states that whenever two or more travelling waves are moving in the same region the **total displacement at any point is equal to the vector sum of their individual displacements at that point.**



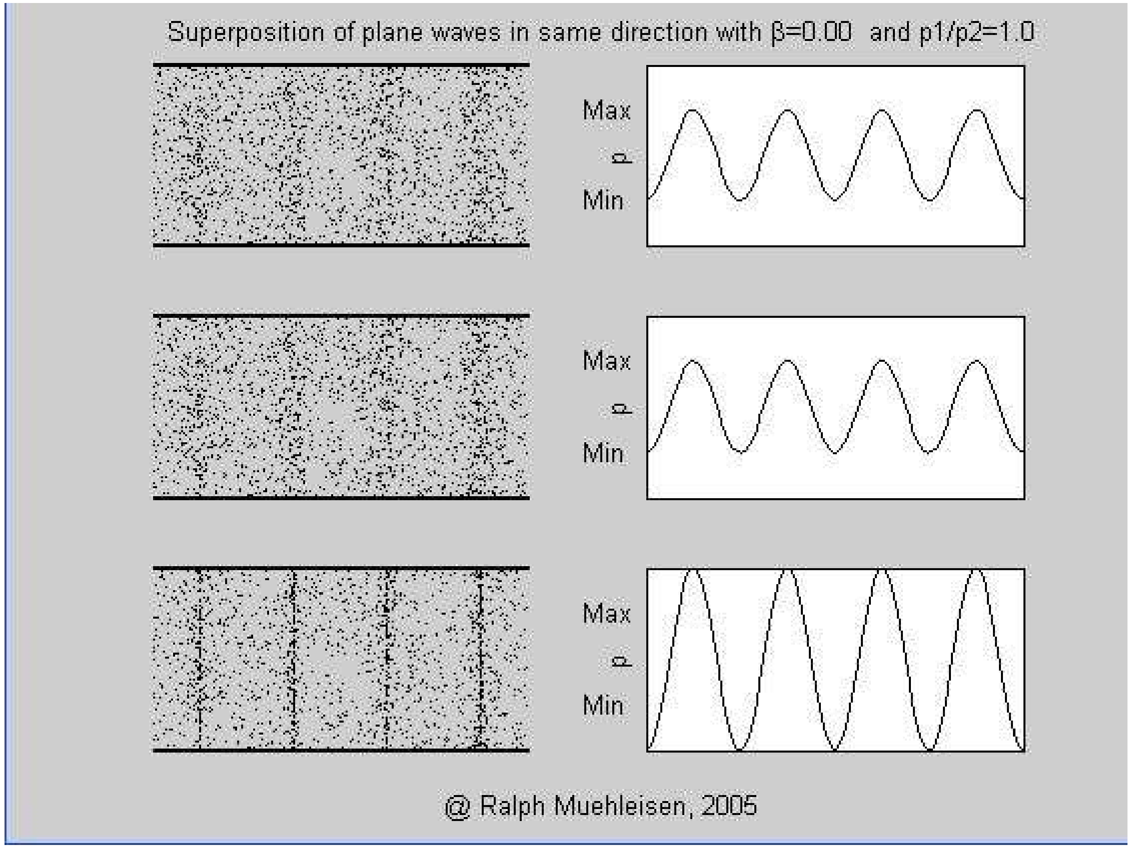
5.6 SUPERPOSITION AND INTERFERENCE OF WAVES

- **Superposition of Planes Waves in Opposite Directions**



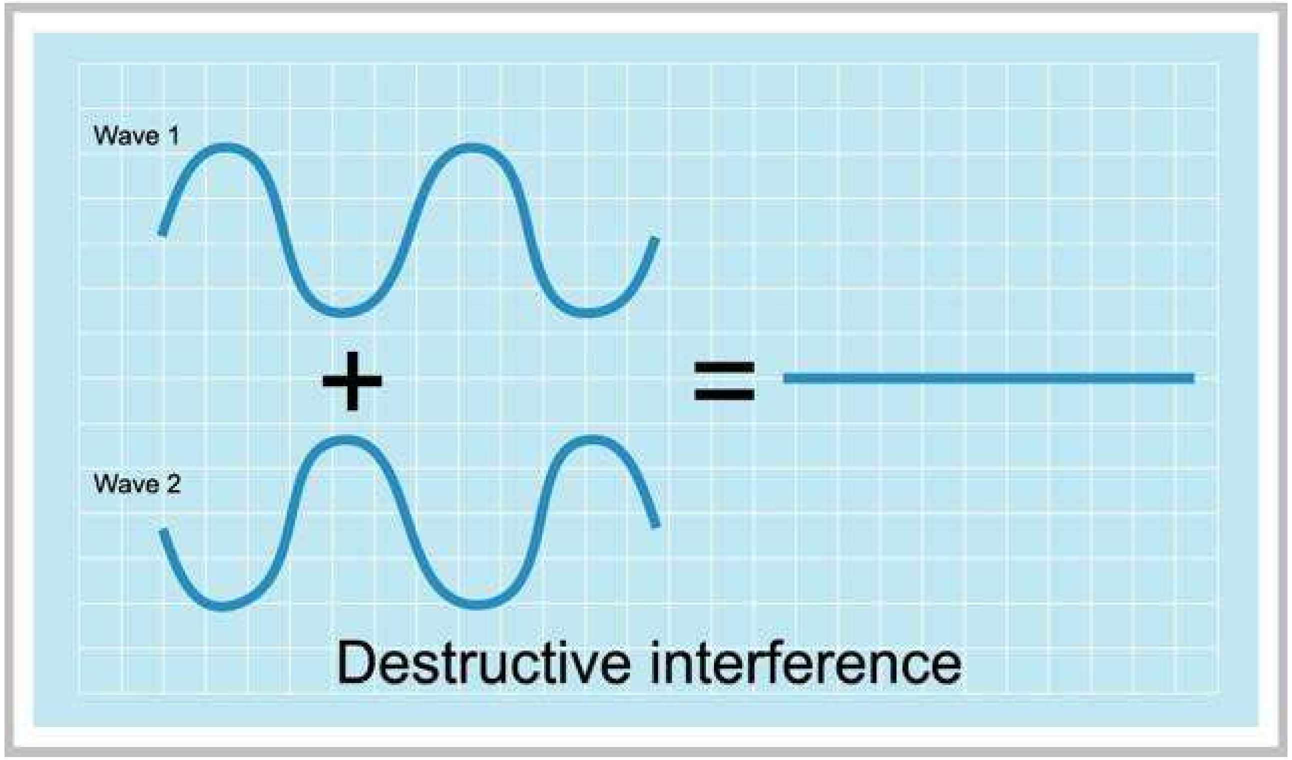
5.6 SUPERPOSITION AND INTERFERENCE OF WAVES

- **Superposition of Planes Waves in Same Direction**



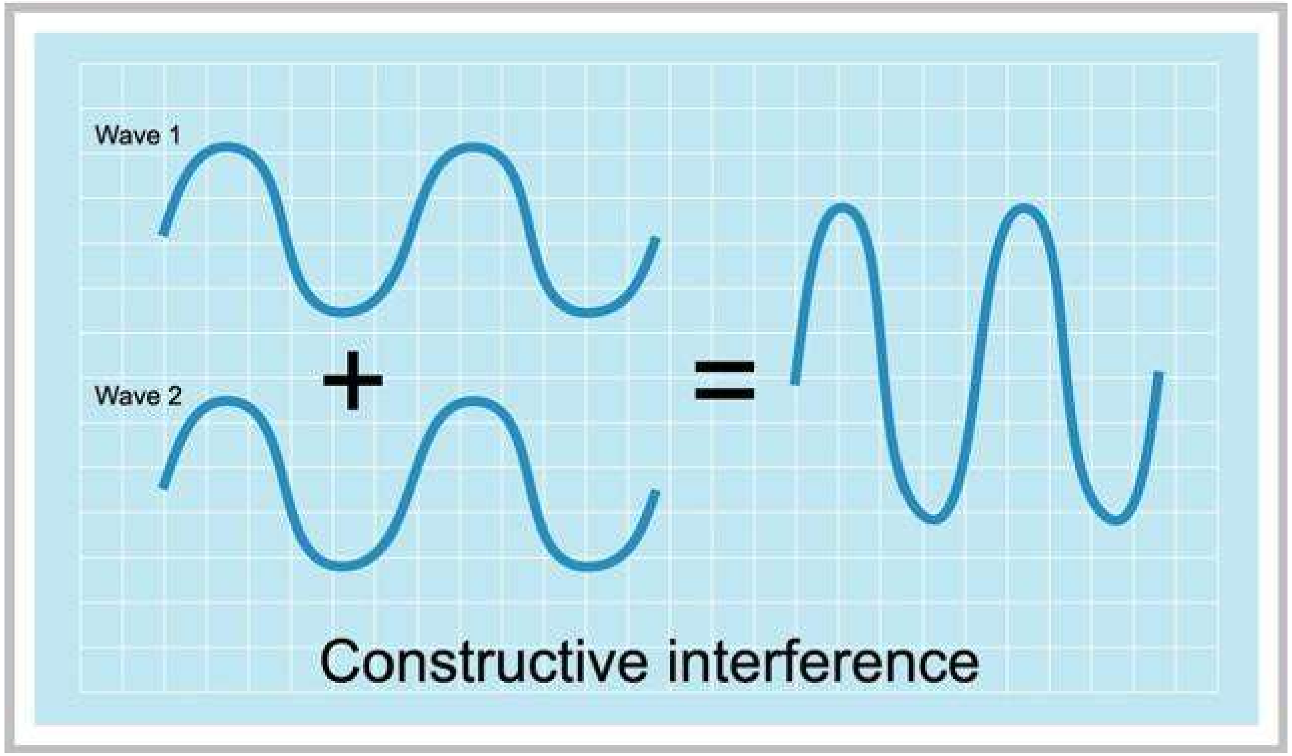
5.6 SUPERPOSITION AND INTERFERENCE OF WAVES

- If the amplitude of the combined waves smaller than that of any of the individual waves, it is called destructive interference.



5.6 SUPERPOSITION AND INTERFERENCE OF WAVES

- If the amplitude of the combined waves greater than that of any of the individual waves, it is called constructive interference.

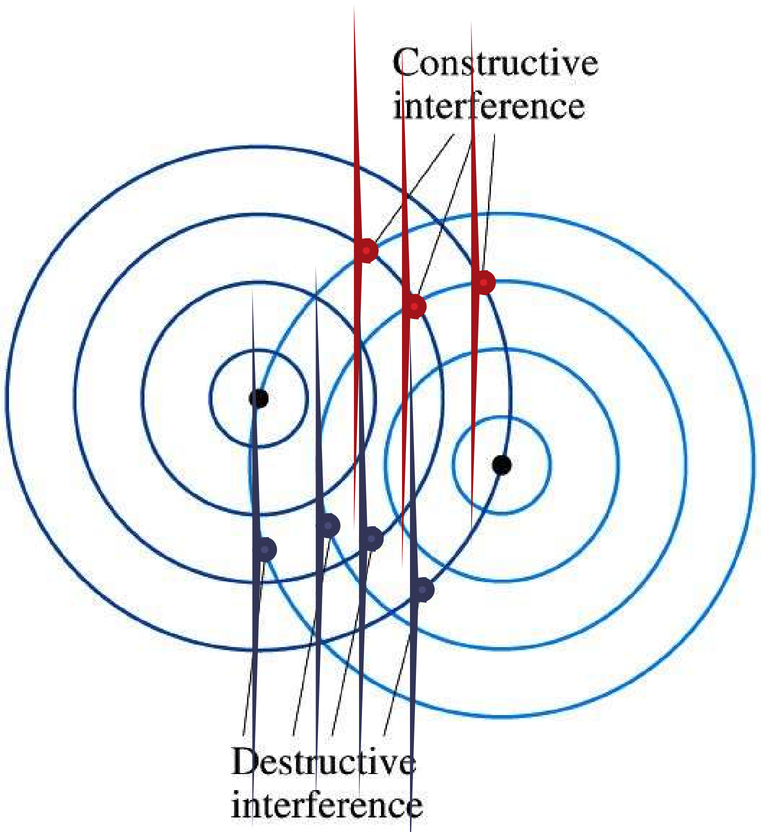


5.6 SUPERPOSITION AND INTERFERENCE OF WAVES

- The total destructive interference takes place if two waves of the same frequency and amplitude are completely 180° out of phase (they have phase difference of π radian; the crest of one wave is aligned with the trough of the other and vice versa)
- The total constructive interference takes place if two waves of the same frequency and amplitude are exactly in phase (they have phase difference of zero; the crest of one wave is aligned with the crest of the other)
- The intermediate interference takes place if the phase difference has an arbitrary value somewhere between 0 and π radian.(the resultant wave has an amplitude whose value is somewhere between 0 and 2A)

5.6 SUPERPOSITION AND INTERFERENCE OF WAVES

- Sketch showing interference between two wave sources



(b)

5.7 SUPERPOSITION AND INTERFERENCE OF SINUSOIDAL WAVES

- Type 1: Travelling Sinusoidal waves
  - Two sinusoidal waves travelling in the same direction in a medium.
  - The waves are travelling to the right and have the same frequency, wavelength, and amplitude but differ in phase.

$$\boxed{y_1 = A\sin(kx - \omega t)} \quad \boxed{y_2 = A\sin(kx - \omega t - \phi)}$$

$$\boxed{\begin{aligned} y &= y_1 + y_2 \\ &= A\sin(kx - \omega t) + A\sin(kx - \omega t - \phi) \end{aligned}}$$

5.7 SUPERPOSITION AND INTERFERENCE OF SINUSOIDAL WAVES

- The resultant wave function

$$\boxed{\begin{aligned} y &= y_1 + y_2 \\ &= A\sin(kx - \omega t) + A\sin(kx - \omega t - \phi) \\ &= A[\sin(kx - \omega t) + \sin(kx - \omega t - \phi)] \end{aligned}}$$

- According to trigonometric identity:

$$\boxed{\sin\alpha + \sin\beta = 2\cos\left(\frac{\alpha - \beta}{2}\right)\sin\left(\frac{\alpha + \beta}{2}\right)}$$

- Let

$$\boxed{\begin{aligned} \alpha &= kx - \omega t \\ \beta &= kx - \omega t - \phi \end{aligned}}$$

5.7 SUPERPOSITION AND INTERFERENCE OF SINUSOIDAL WAVES

So,

$$
\begin{aligned}
y &= A[\sin(kx-\omega t)+\sin(kx-\omega t-\phi)] \\
&= A\left[2\cos\left(\frac{kx-\omega t-(kx-\omega t-\phi)}{2}\right)\sin\left(\frac{kx-\omega t+kx-\omega t-\phi}{2}\right)\right] \\
&= A\left[2\cos\left(\frac{kx-\omega t-kx+\omega t+\phi}{2}\right)\sin\left(\frac{kx-\omega t+kx-\omega t-\phi}{2}\right)\right] \\
&= A\left[2\cos\left(\frac{\phi}{2}\right)\sin\left(\frac{2kx-2\omega t-\phi}{2}\right)\right] \\
&= A\left[2\cos\left(\frac{\phi}{2}\right)\sin\left(kx-\omega t-\frac{\phi}{2}\right)\right] \\
&= 2A\cos\left(\frac{\phi}{2}\right)\sin\left(kx-\omega t-\frac{\phi}{2}\right)
\end{aligned}
$$

5.7 SUPERPOSITION AND INTERFERENCE OF SINUSOIDAL WAVES

$$y = \boxed{2A\cos\left(\frac{\phi}{2}\right)}\sin\left(kx - \omega t - \frac{\phi}{2}\right)$$

- The resultant wave function
  - Same frequency and wavelength
  - Difference
    - Phase constant is $\frac{\phi}{2}$
    - Its amplitude is $2A\cos\left(\frac{\phi}{2}\right)$

5.7 SUPERPOSITION AND INTERFERENCE OF SINUSOIDAL WAVES

- In constructive interference
  - If $\varphi = 0$ rad, (0°) , the two combining waves are exactly **in phase**.
  - So, $\cos(0) = 1$

$$
\begin{aligned}
y &= 2A\cos\left(\frac{\phi}{2}\right)\sin\left(kx-\omega t-\frac{\phi}{2}\right)\\
&= 2A\boxed{\cos\left(\frac{0}{2}\right)}\sin\left(kx-\omega t-\frac{0}{2}\right)\\
&= 2A\sin(kx-\omega t)
\end{aligned}
$$

- The *amplitude of the resultant wave is twice* the amplitude of either individual wave.
- Interference that produces the greatest possible amplitude is called *fully constructive interference*.
- In general, fully constructive interference occurs when $\cos\varphi = \pm 1$, which means when $\varphi = 0, 2\pi, 4\pi, \ldots$ rad.

5.7 SUPERPOSITION AND INTERFERENCE OF SINUSOIDAL WAVES

- In destructive interference
  - If $\varphi = \pi$ rad, (180°) , the two combining waves are exactly **out of phase**.
  - So,

$$\begin{aligned} y &= 2A\cos(\frac{\varphi}{2})\sin(kx-\omega t-\frac{\varphi}{2}) \\ &= 2A\boxed{\cos(\frac{\pi}{2})}\sin(kx-\omega t-\frac{\pi}{2}) \\ &= 0 \end{aligned}$$

$$\cos(\frac{\pi}{2}) = 0$$

$$\cos(\frac{3\pi}{2}) = 0$$

- The resultant wave has ***zero amplitude*** everywhere.
- Interference that produces zero amplitude is called ***fully destructive interference***.
- In general , fully destructive interference occurs when $\Phi = \pi$ rad or to any *odd multiple of* π, then $\cos(\varphi/2) = 0$

5.7 SUPERPOSITION AND INTERFERENCE OF SINUSOIDAL WAVES

- Intermediate Interference
  - If has an arbitrary value between 0 and π rad, (eg = $2\pi/3$ rad), the resultant wave has an amplitude whose value is somewhere between 0 and 2A.
  - So,

$$
\begin{aligned}
y &= 2A\cos\left(\frac{\phi}{2}\right)\sin\left(kx-\omega t-\frac{\phi}{2}\right)\\
&= 2A\cos\left(\frac{2\pi}{(3)2}\right)\sin\left(kx-\omega t-\frac{2\pi}{(3)2}\right)\\
&= 2A\cos\left(\frac{\pi}{3}\right)\sin\left(kx-\omega t-\frac{\pi}{3}\right)
\end{aligned}
$$

The amplitude $= 2A\cos\left(\frac{\pi}{3}\right)$

The phase constant is = $\frac{\pi}{3}$

5.7 SUPERPOSITION AND INTERFERENCE OF SINUSOIDAL WAVES

- Example

Two identical waves, moving in the same direction along a stretched string, interference with each other. The amplitude, A of each waves is 9.8 mm and the phase difference between them is 100°.

a) What is the amplitude , A of the resultant wave, and what type of interference occurs?

b) What phase difference (in radians) will give the resultant wave an amplitude of 4.9mm?

5.7 SUPERPOSITION AND INTERFERENCE OF SINUSOIDAL WAVES

- Solution

What is the amplitude , A of the resultant wave, and what type of interference occurs?

$$y = 2A\cos(\frac{\phi}{2})$$

$$y = 2(9.8mm)Cos(\frac{100^o}{2})$$

$$y = 12.6mm$$

Intermediate interference

5.7 SUPERPOSITION AND INTERFERENCE OF SINUSOIDAL WAVES

What phase difference (in radians) will give the resultant wave an amplitude of 4.9mm?

$$\boxed{y = 2A\cos(\frac{\phi}{2})\sin(kx - \omega t - \frac{\phi}{2})}$$

$$y = 2A\cos(\frac{\phi}{2})$$

$$4.9\,mm = 2(9.8mm)\cos(\frac{\phi}{2})$$

$$\frac{4.9}{19.6} = \cos\frac{\phi}{2}$$

$$0.25 = \cos\frac{\phi}{2}$$

$$\phi = \pm 2.636\ rad$$

$$\phi = \pm 151.0^{o}$$

5.7 SUPERPOSITION AND INTERFERENCE OF SINUSOIDAL WAVES

- Type 2: Standing waves

A standing (stationary) wave result when two waves which are travelling in opposite directions, and which have the same speed and frequency and approximately equal amplitudes, are superposed.

$$y_1 = A\sin(kx - \omega t)$$

$$y_2 = A\sin(kx + \omega t)$$

$$\begin{aligned} y &= y_1 + y_2 \\ &= A\sin(kx - \omega t) + A\sin(kx + \omega t) \\ &= A\left[\sin(kx - \omega t) + \sin(kx + \omega t)\right] \end{aligned}$$

5.7 SUPERPOSITION AND INTERFERENCE OF SINUSOIDAL WAVES

- Applying trigonometric identity

$$\sin(\alpha-\beta)+\sin(\alpha+\beta)=2\sin\alpha\cos\beta$$

$$\boxed{\alpha=kx,\quad \beta=\omega t}$$

$$\begin{aligned} y &= A\left[\sin(kx-\omega t)+\sin(kx+\omega t)\right] \\ &= A\left[2\sin kx\cos\omega t\right] \\ &= 2A\sin kx\cos\omega t \end{aligned}$$

5.7 SUPERPOSITION AND INTERFERENCE OF SINUSOIDAL WAVES

$$y = 2A \sin kx \cos \omega t$$

- Amplitude
  - $2A \sin kx$
  - the amplitude varies with position, x.

5.7 SUPERPOSITION AND INTERFERENCE OF SINUSOIDAL WAVES

- Nodes
  - points of zero amplitude

  - It will happen if

$$\sin kx = 0$$

$$kx = 0, 180, 360, 540......$$

$$kx = 0, \pi, 2\pi, 3\pi......$$

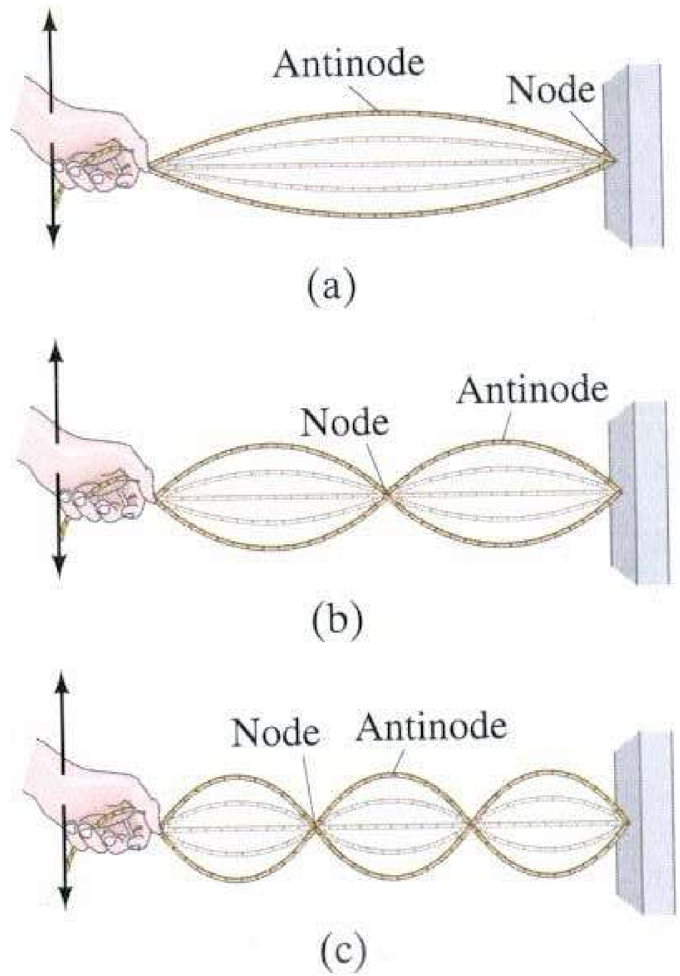
$$= n\pi \qquad \text{where n=0,1,2,...,n}$$

$$k = \frac{2\pi}{\lambda}$$

$$kx = n\pi$$

$$\frac{2\pi}{\lambda} x = n\pi$$

$$x = \frac{n\lambda}{2}$$



5.7 SUPERPOSITION AND INTERFERENCE OF SINUSOIDAL WAVES

- Antinodes
  - points of maximum amplitude

  - It will happen if

$$\sin kx = 1$$

$$kx = 90, 270, 450\ldots\ldots$$

$$kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}\ldots$$

$$= \left(n + \frac{1}{2}\right)\pi$$

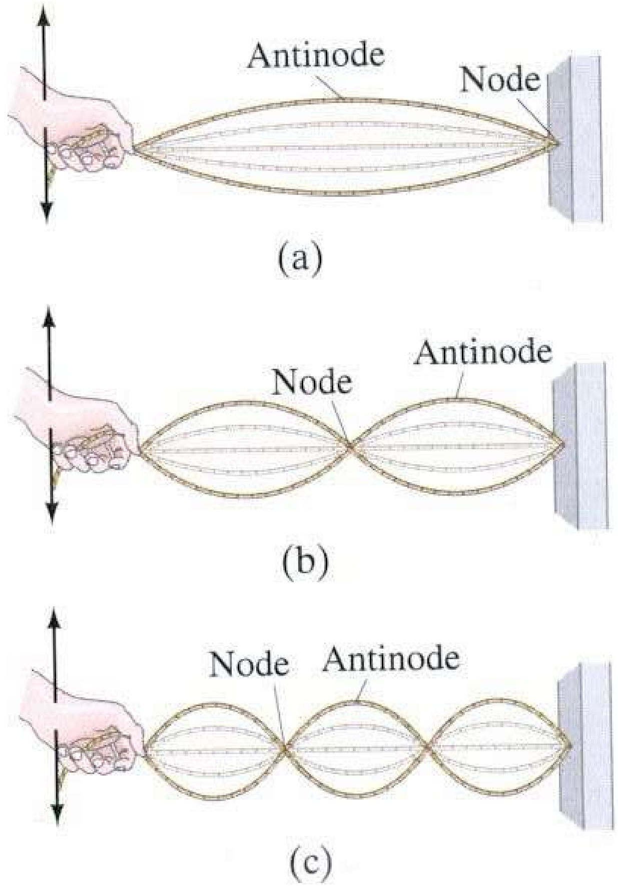
where n=0,1,2,...,n

$$k = \frac{2\pi}{\lambda}$$

$$kx = \left(n + \frac{1}{2}\right)\pi$$

$$\frac{2\pi}{\lambda}x = n\pi$$

$$x = \frac{n\lambda}{4}$$



TUTORIAL QUESTION

- Question 12
- Question 14


THANK YOU!
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