Chapter 2:
Analytic Trigonometry and Polar Coordinates (Part 1)

Lecture 07 – 25.11.2022

- Trigonometric Identities
- Trigonometric Equations

FUNDAMENTAL TRIGONOMETRIC IDENTITIES

Reciprocal Identities

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1 \qquad \tan^2 x + 1 = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

- The list of Formulae will be provided for most of the Trigonometric Identities and other important Formulae.
- Please get a copy of the list from **Google Shared Drive**, **MMLS "Files" section**, or **Google Classroom "Classwork" section**.

Even-Odd Identities

$$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x \qquad \tan(-x) = -\tan x$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \qquad \tan\left(\frac{\pi}{2} - u\right) = \cot u \qquad \sec\left(\frac{\pi}{2} - u\right) = \csc u$$

$$\cos\left(\frac{\pi}{2} - u\right) = \sin u \qquad \cot\left(\frac{\pi}{2} - u\right) = \tan u \qquad \csc\left(\frac{\pi}{2} - u\right) = \sec u$$

2.2.1.1 Simplifying Trigonometric Expressions

Identities enable us to write the same expression in different ways. It is often possible to rewrite a complicated-looking expression as a much simpler one.

Example

Simplify the expression cos t + tan t sin t.

$$\cos t + \underline{\tan t}\,\sin t$$

$$= \cos t + \left(\frac{\sin t}{\cos t}\right)\sin t$$

$$= \cos t + \frac{\sin^2 t}{\cos t}$$

$$= \frac{\cos^2 t}{\cos t} + \frac{\sin^2 t}{\cos t}$$

$$= \frac{\cos^2 t + \sin^2 t}{\cos t} = \frac{1}{\cos t} = \sec t$$

2.2.1.2 Proving Trigonometric Identities

GUIDELINES FOR PROVING TRIGONOMETRIC IDENTITIES

1. **Start with one side.** Pick one side of the equation and write it down. Your goal is to transform it into the other side. It's usually easier to start with the more complicated side.
2. **Use known identities.** Use algebra and the identities you know to change the side you started with. Bring fractional expressions to a common denominator, factor, and use the fundamental identities to simplify expressions.
3. **Convert to sines and cosines.** If you are stuck, you may find it helpful to rewrite all functions in terms of sines and cosines.

Example

Verify the identity $\cos\theta(\sec\theta - \cos\theta) = \sin^2\theta$

$$\cos\theta\,(\underline{\sec\theta} - \cos\theta) = \cos\theta\left(\frac{1}{\cos\theta} - \cos\theta\right)$$

$$= \cos\theta\left(\frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta}\right)$$

$$= \cancel{\cos\theta}\left(\frac{1-\cos^2\theta}{\cancel{\cos\theta}}\right)$$

$$= 1 - \cos^2\theta$$

$$= \sin^2\theta \quad \# \quad \text{proven.}$$

Example

Verify the identity $2\tan x \sec x = \dfrac{1}{1-\sin x} - \dfrac{1}{1+\sin x}$

$$\frac{1}{1-\sin x} - \frac{1}{1+\sin x} = \frac{\cancel{1} + \sin x - (\cancel{1} - \sin x)}{(1-\sin x)(1+\sin x)}$$

$$= \frac{2\sin x}{1-\sin^2 x}$$

$$= \frac{2\sin x}{\cos^2 x}$$

$$= 2\left(\frac{\sin x}{\cos x}\right)\left(\frac{1}{\cos x}\right)$$

$$= 2\tan x \sec x$$ = proven.

2.2.2 Addition and Subtraction Formulas

ADDITION AND SUBTRACTION FORMULAS

Formulas for sine:

$$\sin(s+t) = \sin s \cos t + \cos s \sin t$$

$$\sin(s-t) = \sin s \cos t - \cos s \sin t$$

Formulas for cosine:

$$\cos(s+t) = \cos s \cos t - \sin s \sin t$$

$$\cos(s-t) = \cos s \cos t + \sin s \sin t$$

Formulas for tangent:

$$\tan(s+t) = \frac{\tan s + \tan t}{1 - \tan s \tan t}$$

$$\tan(s-t) = \frac{\tan s - \tan t}{1 + \tan s \tan t}$$

Recall the Special-Angle Values

θ in degrees	θ in radians	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$30°$	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$45°$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$60°$	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

Example

Find the exact value of each expression

(a) $\cos 75°$ (b) $\cos \frac{\pi}{12}$

ADDITION AND SUBTRACTION FORMULAS

Formulas for sine:
$$\sin(s+t) = \sin s \cos t + \cos s \sin t$$
$$\sin(s-t) = \sin s \cos t - \cos s \sin t$$

Formulas for cosine:
$$\cos(s+t) = \cos s \cos t - \sin s \sin t$$
$$\cos(s-t) = \cos s \cos t + \sin s \sin t$$

Formulas for tangent:
$$\tan(s+t) = \frac{\tan s + \tan t}{1 - \tan s \tan t}$$
$$\tan(s-t) = \frac{\tan s - \tan t}{1 + \tan s \tan t}$$

a) $\cos 75°$

$= \cos(45° + 30°)$

$= \cos 45° \cos 30° - \sin 45° \sin 30°$

$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}\right) - \frac{1}{\sqrt{2}}\left(\frac{1}{2}\right)$

$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)$

$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}-1}{2}\right) = \frac{\sqrt{3}-1}{2\sqrt{2}}$ #

b) $\cos \frac{\pi}{12} = \cos 15°$

$= \cos(45° - 30°)$

$= \cos 45° \cos 30° + \sin 45° \sin 30°$

$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{\sqrt{2}}\left(\frac{1}{2}\right)$

$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right)$

$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}+1}{2}\right) = \frac{\sqrt{3}+1}{2\sqrt{2}}$ #

Example

Find the exact value of the expression

$$\sin 20^\circ \cos 40^\circ + \cos 20^\circ \sin 40^\circ$$

$= \sin(20^\circ + 40^\circ)$

$= \sin 60^\circ$

$= \dfrac{\sqrt{3}}{2}$ #

ADDITION AND SUBTRACTION FORMULAS

Formulas for sine:

$$\sin(s + t) = \sin s \cos t + \cos s \sin t$$

$$\sin(s - t) = \sin s \cos t - \cos s \sin t$$

Formulas for cosine:

$$\cos(s + t) = \cos s \cos t - \sin s \sin t$$

$$\cos(s - t) = \cos s \cos t + \sin s \sin t$$

Formulas for tangent:

$$\tan(s + t) = \frac{\tan s + \tan t}{1 - \tan s \tan t}$$

$$\tan(s - t) = \frac{\tan s - \tan t}{1 + \tan s \tan t}$$

2.2.2.1 Sum of Sines and Cosines

$k \sin(x+\phi)$

$= k[\sin x \cos\phi + \cos x \sin\phi]$

$= \underbrace{k\cos\phi}\sin x + \underbrace{k\sin\phi}\cos x$

$= A \sin x + B \cos x$

SUMS OF SINES AND COSINES

If A and B are real numbers, then

$$A \sin x + B \cos x = k \sin(x + \phi)$$

where $k = \sqrt{A^2 + B^2}$ and ϕ satisfies

$$\cos \phi = \frac{A}{\sqrt{A^2 + B^2}} \qquad \text{and} \qquad \sin \phi = \frac{B}{\sqrt{A^2 + B^2}}$$

$A = k\cos\phi$

$\cos\phi = \frac{A}{k} = \frac{A}{\sqrt{A^2+B^2}}$

$B = k\sin\phi$

$\sin\phi = \frac{B}{k} = \frac{B}{\sqrt{A^2+B^2}}$

Example

Write the expression $\frac{1}{2}\sin x+\frac{\sqrt{3}}{2}\cos x$ in the form of $k\sin(x+\phi)$

SUMS OF SINES AND COSINES

If A and B are real numbers, then

$$A\sin x + B\cos x = k\sin(x+\phi)$$

where $k=\sqrt{A^2+B^2}$ and ϕ satisfies

$$\cos\phi = \frac{A}{\sqrt{A^2+B^2}} \quad \text{and} \quad \sin\phi = \frac{B}{\sqrt{A^2+B^2}}$$

$A=\frac{1}{2}$

$B=\frac{\sqrt{3}}{2}$

$$\therefore k=\sqrt{\left(\tfrac{1}{2}\right)^2+\left(\tfrac{\sqrt{3}}{2}\right)^2}$$

$$=\sqrt{\tfrac{1}{4}+\tfrac{3}{4}}$$

$$=\sqrt{1}$$

$$=1$$

$$\cos\phi=\frac{A}{k}$$

$$=\frac{\left(\frac{1}{2}\right)}{1}$$

$$=\frac{1}{2}$$

$$\therefore \phi=\cos^{-1}\left(\tfrac{1}{2}\right)$$

$$=60^\circ / \tfrac{\pi}{3}$$

$$\therefore \frac{1}{2}\sin x+\frac{\sqrt{3}}{2}\cos x = \sin\left(x+\frac{\pi}{3}\right)_{\#} = \sin(x+60^\circ)_{\#}$$

Example

Express $3 \sin x + 4 \cos x$ in the form $k \sin(x + \phi)$.

SUMS OF SINES AND COSINES

If A and B are real numbers, then

$$A \sin x + B \cos x = k \sin(x + \phi)$$

where $k = \sqrt{A^2 + B^2}$ and ϕ satisfies

$$\cos \phi = \frac{A}{\sqrt{A^2 + B^2}} \quad \text{and} \quad \sin \phi = \frac{B}{\sqrt{A^2 + B^2}}$$

$A = 3$

$B = 4$

$k = \sqrt{3^2 + 4^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

$= 5$

$\cos \phi = \frac{A}{k}$

$= \frac{3}{5}$

$\therefore \phi = \cos^{-1}\left(\frac{3}{5}\right)$

$= 53.1^\circ$

$\therefore 3 \sin x + 4 \cos x = 5 \sin(x + 53.1^\circ)$

2.2.2.2 Double-Angle Formulas

DOUBLE-ANGLE FORMULAS

Formula for sine: $\sin 2x = 2 \sin x \cos x$

Formulas for cosine:

$$\begin{aligned} \cos 2x &= \cos^2 x - \sin^2 x \\ &= 1 - 2\sin^2 x \\ &= 2\cos^2 x - 1 \end{aligned}$$

Formula for tangent:

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

Note: Double angle formulas are special cases of sum of two angle formulas.

Example

If $\cos x = -\dfrac{2}{3}$ and x is in Quadrant II, find $\cos 2x$ and $\sin 2x$.

DOUBLE-ANGLE FORMULAS

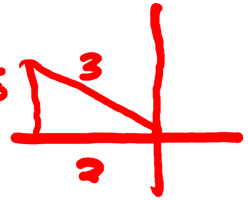
Formula for sine: $\sin 2x = 2\sin x\cos x$

Formulas for cosine:
$$\begin{aligned}\cos 2x &= \cos^2 x - \sin^2 x\\ &= 1 - 2\sin^2 x\\ &= 2\cos^2 x - 1\end{aligned}$$

Formula for tangent: $\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

$\cos x = -\dfrac{2}{3}$

$\sqrt{3^2-2^2} = \sqrt{5}$



$\sin x = \dfrac{\sqrt{5}}{3}$

OR

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x + \left(-\frac{2}{3}\right)^2 = 1$

$\sin^2 x = 1 - \frac{4}{9}$

$= \frac{5}{9}$

$\therefore \sin x = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$

i) $\cos 2x$

$= 2\cos^2 x - 1$

$= 2\left(-\frac{2}{3}\right)^2 - 1$

$= 2\left(\frac{4}{9}\right) - 1$

$= -\frac{1}{9}$ #

ii) $\sin 2x$

$= 2\sin x\cos x$

$= 2\left(\frac{\sqrt{5}}{3}\right)\left(-\frac{2}{3}\right)$

$= -\frac{4\sqrt{5}}{9}$ #

Example

Write $\cos 3x$ in terms of $\cos x$

DOUBLE-ANGLE FORMULAS

Formula for sine: $\sin 2x = 2\sin x\cos x$

Formulas for cosine: $\cos 2x = \cos^2 x - \sin^2 x$

$= 1 - 2\sin^2 x$

$= 2\cos^2 x - 1$

Formula for tangent: $\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

$$\begin{aligned}
\cos 3x &= \cos(2x + x) \\
&= \underline{\cos 2x}\,\cos x - \underline{\sin 2x}\,\sin x \\
&= (2\cos^2 x - 1)\cos x - (2\sin x\cos x)\sin x \\
&= 2\cos^3 x - \cos x - 2\underline{\sin^2 x}\,\cos x \\
&= 2\cos^3 x - \cos x - 2(1 - \cos^2 x)\cos x \\
&= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x \\
&= 4\cos^3 x - 3\cos x \;\#
\end{aligned}$$

2.2.2.3 Half Angle Formulas

HALF-ANGLE FORMULAS

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}} \qquad \cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$$

$$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$$

The choice of the $+$ or $-$ sign depends on the quadrant in which $u/2$ lies.

Note: Do not try to remember the formula but deduce it from double angle formulas by first taking $x = \frac{u}{2}$.

Example

Show that $\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}}$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$\sin x = \pm\sqrt{1-\cos^2 x}$

$= \pm\sqrt{1 - \left(\frac{\cos 2x + 1}{2}\right)}$

$= \pm\sqrt{\frac{2}{2} - \frac{\cos 2x + 1}{2}}$

$= \pm\sqrt{\frac{2 - \cos 2x - 1}{2}}$

$= \pm\sqrt{\frac{1-\cos 2x}{2}}$

$\cos 2x = 2\cos^2 x - 1$

$2\cos^2 x = \cos 2x + 1$

$\cos^2 x = \frac{\cos 2x + 1}{2}$

Let $x = \frac{u}{2}$

then $2x = u$

$\therefore \sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}}$

Example

Find the exact value of $\sin 22.5$

$$\sin 22.5 = \sin \frac{45^\circ}{2}$$

$$= \sqrt{\frac{1-\cos 45^\circ}{2}}$$

$$= \sqrt{\frac{1-\frac{1}{\sqrt{2}}}{2}}$$

$$= \sqrt{\frac{\frac{\sqrt{2}-1}{\sqrt{2}}}{2}} = \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$$ #

HALF-ANGLE FORMULAS

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}} \qquad \cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$$

$$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$$

The choice of the + or − sign depends on the quadrant in which $u/2$ lies.

2.2.2.4 Product-to-Sum Formulas

PRODUCT-TO-SUM FORMULAS

$$\sin u \cos v = \tfrac{1}{2}[\sin(u + v) + \sin(u - v)]$$

$$\cos u \sin v = \tfrac{1}{2}[\sin(u + v) - \sin(u - v)]$$

$$\cos u \cos v = \tfrac{1}{2}[\cos(u + v) + \cos(u - v)]$$

$$\sin u \sin v = \tfrac{1}{2}[\cos(u - v) - \cos(u + v)]$$

Note do not try to remember the formulas but deduce it from the sum and difference formulas

Example

Show that $\sin u \cos v = \frac{1}{2}[\sin(u+v)+\sin(u-v)]$.

$$\frac{1}{2}[\sin(u+v)+\sin(u-v)] = \frac{1}{2}[\sin u \cos v + \cancel{\cos u \sin v} + \sin u \cos v - \cancel{\cos u \sin v}]$$

$$= \frac{1}{\cancel{2}}(\cancel{2}\sin u \cos v)$$

$$= \sin u \cos v \#$$

Example

Express $\sin 3x \cos 5x$ as sum of trigonometric functions

PRODUCT-TO-SUM FORMULAS

$$\sin u \cos v = \tfrac{1}{2}[\sin(u+v) + \sin(u-v)]$$

$$\cos u \sin v = \tfrac{1}{2}[\sin(u+v) - \sin(u-v)]$$

$$\cos u \cos v = \tfrac{1}{2}[\cos(u+v) + \cos(u-v)]$$

$$\sin u \sin v = \tfrac{1}{2}[\cos(u-v) - \cos(u+v)]$$

$$\sin 3x \cos 5x$$

$$= \frac{1}{2}[\sin(3x+5x) + \sin(3x-5x)]$$

$$= \frac{1}{2}[\sin 8x + \sin(-2x)]$$

$$= \frac{1}{2}(\sin 8x - \sin 2x)$$

Remember:

$$\sin(-x) = -\sin x$$

Chapter 2:
Analytic Trigonometry and Polar Coordinates (Part 1)

Lecture 08 – 30.11.2022

- Trigonometric Identities
- Trigonometric Equations

2.2.2.5 Sum-to-Product Formulas

SUM-TO-PRODUCT FORMULAS

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$$

Note: The formula can be deduced from product-to-sum formula by letting

$$u = \frac{x+y}{2} \quad \text{and} \quad v = \frac{x-y}{2}$$

Example

Suppose $\sin u \sin v = \frac{1}{2}[\cos(u-v) - \cos(u+v)]$.

Let $u = \frac{x+y}{2}$; $v = \frac{x-y}{2}$

Show that $\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$

$$\sin u \sin v = \sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$= \frac{1}{2}\left[\cos\left(\frac{x+y}{2} - \frac{x-y}{2}\right) - \cos\left(\frac{x+y}{2} + \frac{x-y}{2}\right)\right]$$

$$= \frac{1}{2}\left[\cos\left(\frac{2y}{2}\right) - \cos\left(\frac{2x}{2}\right)\right]$$

$$= \frac{1}{2}(\cos y - \cos x)$$

$$2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right) = \cos y - \cos x$$

$\therefore \cos x - \cos y$

$= -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$ # proven.

Example

Write $\sin 7x + \sin 3x$ as a product.

$$\sin 7x + \sin 3x = 2 \sin \left(\frac{7x+3x}{2}\right) \cos \left(\frac{7x-3x}{2}\right)$$

$$= 2 \sin 5x \cos 2x$$

#

2.2.3 Trigonometric Equation

An equation that contains trigonometric functions is called a **trigonometric equation**. For example, the following are trigonometric equations:

$$\sin^2 \theta + \cos^2 \theta = 1 \qquad 2 \sin\theta - 1 = 0 \qquad \tan 2\theta - 1 = 0$$

The first equation is an *identity*—that is, it is true for every value of the variable θ. The other two equations are true only for certain values of θ.

To solve a trigonometric equation, we find all the values of the variable that make the equation true.

Example: Solve $3 \sin x = \sqrt{3} + \sin x$

$$3 \sin x - \sin x = \sqrt{3}$$

$$2 \sin x = \sqrt{3}$$

$$\sin x = \frac{\sqrt{3}}{2}$$

$$\therefore x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$= 60^\circ, 120^\circ$$

$\therefore$ the general solution:

i) $x = 60^\circ + 360^\circ k$, $k = 0, 1, 2, \ldots$

ii) $x = 120^\circ + 360^\circ k$, $k = 0, 1, 2, \ldots$

There are many possible solutions for x, i.e., $x = 60°, 120°, 420°, \ldots\ldots$ We can express the solution in terms of the ***general solution*** for x as:

$x = 60° + 360°n$, where n is any integer

or $x = 120° + 360°n$, where n is any integer

Example: Solve $\sin x + \cos x = 0$ for $0 < x < 2\pi$

$\sin x + \cos x = 0$

$\sin x = -\cos x$

$\dfrac{\sin x}{\cos x} = -\dfrac{\cos x}{\cos x}$

$\tan x = -1$

$x = \tan^{-1}(-1)$

$= 135^\circ, 315^\circ$

$\tan^{-1}(1) = 45^\circ, 215^\circ$

Example: Solve $\sin 2\theta = 1/2$ where $0 \le \theta < 2\pi$.

$$\sin 2\theta = \frac{1}{2}$$

$$2\theta = \sin^{-1}\left(\frac{1}{2}\right)$$

$$= 30^\circ, 150^\circ, 390^\circ, 410^\circ$$

$$\therefore \theta = 15^\circ, 75^\circ, 195^\circ, 205^\circ \#$$

Example: Solve $\tan^2 x + \tan x - 2 = 0$ for $0 \le x \le 2\pi$.

$\tan^{-1}(2) = 63.4°$

Let $y = \tan x$

$\tan^2 x + \tan x - 2 = 0$

$y^2 + y - 2 = 0$

$(y+2)(y-1) = 0$

$\therefore y+2 = 0$ or $y-1 = 0$

$y = -2$ $\qquad$ $y = 1$

$y = -2$

$\tan x = -2$

$\therefore x = \tan^{-1}(-2)$

$= 116.6°, 296.6°$

$y = 1$

$\tan x = 1$

$\therefore x = \tan^{-1}(1)$

$= 45°, 215°$

$\therefore$ the solution : $x = 45°, 116.6°, 215°, 296.6°$ #

Example

Solve the equation $5\sin\theta\cos\theta + 4\cos\theta = 0$

$5\sin\theta\cos\theta + 4\cos\theta = 0$

$\cos\theta(5\sin\theta + 4) = 0$

$\therefore \cos\theta = 0$ or $5\sin\theta + 4 = 0$

$\therefore \sin\theta = -\frac{4}{5}$

$\therefore \theta = 90°, 270°$

$\theta = \sin^{-1}\left(-\frac{4}{5}\right)$

$= 233.1°, 306.9°$

$\therefore$ the general solution:

i) $\theta = 90° + 360° k$

ii) $\theta = 233.1° + 360° k$

iii) $\theta = 270° + 360° k$

iv) $\theta = 306.9° + 360° k$

for $k = 0, 1, 2, \ldots$

Example

Solve the equation $1 + \sin\theta = 2\cos^2\theta$

$$1 + \sin\theta = 2\cos^2\theta$$
$$= 2(1 - \sin^2\theta)$$
$$= 2 - 2\sin^2\theta$$

$$\therefore \quad 2\sin^2\theta + \sin\theta - 1 = 0$$

Let $x = \sin\theta$

then

$$2x^2 + x - 1 = 0$$
$$(2x - 1)(x + 1) = 0$$

$\therefore$ $2x - 1 = 0$ or $x + 1 = 0$

$x = \frac{1}{2}$ $\quad$ $x = -1$

For $x = \frac{1}{2}$

$\sin\theta = \frac{1}{2}$

$\therefore \quad \theta = \sin^{-1}\left(\frac{1}{2}\right)$

$= 30°, 150°$

For $x = -1$

$\sin\theta = -1$

$\therefore \quad \theta = \sin^{-1}(-1)$

$= 270°$

$\therefore$ the general solution:

i) $\theta = 30° + 360°k$

ii) $\theta = 150° + 360°k$

iii) $\theta = 270° + 360°k$

for $k = 0, 1, 2, \ldots$

Example

Solve the equation $\sin 2\theta - \cos\theta = 0$

$$\sin 2\theta - \cos\theta = 0$$

$$2\sin\theta\cos\theta - \cos\theta = 0$$

$$\cos\theta(2\sin\theta - 1) = 0$$

$\therefore \cos\theta = 0$ or $2\sin\theta - 1 = 0$

$\sin\theta = \frac{1}{2}$

$\therefore \theta = \cos^{-1}(0)$

$= 90^\circ, 270^\circ$

$\therefore \theta = \sin^{-1}(\frac{1}{2})$

$= 30^\circ, 150^\circ$

$\therefore$ the general solution:

i) $\theta = 30^\circ + 360^\circ k$

ii) $\theta = 90^\circ + 360^\circ k$

iii) $\theta = 150^\circ + 360^\circ k$

iv) $\theta = 270^\circ + 360^\circ k$

for $k = 0, 1, 2, \ldots$

Example

Solve the equation $\sin 2\theta - \cos\theta = 0$

$$\sin 2\theta = \cos\theta$$

$$2\sin\theta\cos\theta = \cos\theta$$

$$2\sin\theta = \frac{\cancel{\cos\theta}}{\cancel{\cos\theta}} \quad 1$$

$$= 1$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = \sin^{-1}\left(\frac{1}{2}\right)$$

$$= 30^\circ, 150^\circ$$

TAKE NOTE:

X PLEASE AVOID

CANCELLING TERMS

IN SOLVING TRIGONOMETRIC

EQUATIONS !!!

THIS WILL RESULT 1 SOLUTION-SET

SHORT IN THE FINAL ANSWER

☺ ~ THE END ~ ☺