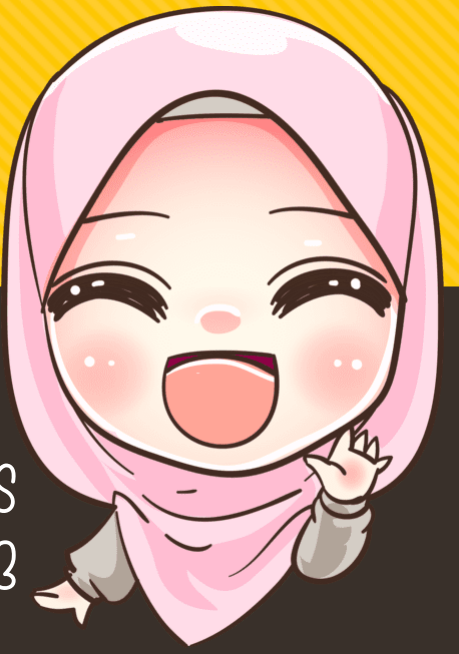
MMU
MULTIMEDIA UNIVERSITY

TUTORIAL 4
BOOLEAN ALGEBRA AND LOGIC SIMPLIFICATION



PDS0101: INTRODUCTION TO DIGITAL SYSTEMS
TRI 2, 2022-2023

BASIC RULES OF BOOLEAN ALGEBRA

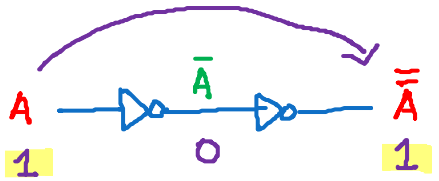
1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \bar{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \bar{A} = 0$

9. $\bar{\bar{A}} = A$
10. $A + AB = A$
11. $A + \bar{A}B = A + B$

    $\bar{A} + AB = \bar{A} + B$
12. $(A + B)(A + C) = A + BC$
13. $\overline{AB} = \bar{A} + \bar{B}$ } DMT
14. $\overline{A + B} = \bar{A}\bar{B}$

BASIC RULES OF BOOLEAN ALGEBRA

$$\bar{\bar{A}} = A$$



$2^1 = 2 < {}^{0}_{1}$

OR GATE

1. $A + 0 = A$
2. $A + A = A$
3. $A + 1 = 1$
4. $A + \bar{A} = 1$

$A + 0$

Input		Output
A	0	
0	0	0
1	0	1

$A + A$

Input		Output
A	A	
0	0	0
1	1	1

$A + 1$

Input		Output
A	1	
0	1	1
1	1	1

$A + \bar{A}$

Input		Output
A	$\bar{A}$	
0	1	1
1	0	1

BASIC RULES OF BOOLEAN ALGEBRA

AND GATE

1. $A \cdot A = A$
2. $A \cdot 1 = A$
3. $A \cdot 0 = 0$
4. $A \cdot \bar{A} = 0$

$A \cdot A$

Input		Output
A	A	
0	0	0
1	1	1

$A \cdot 1$

Input		Output
A	1	
0	1	0
1	1	1

$A \cdot 0$

Input		Output
A	0	
0	0	0
1	0	0

$A \cdot \bar{A}$

Input		Output
A	$\bar{A}$	
0	1	0
1	0	0

BASIC RULES OF BOOLEAN ALGEBRA

Rule ID

a) $A + AB = A$

b) $\bar{A} + \bar{A}B = \bar{A}$

AND - OR gate

a) $A + AB = A(1 + B^1)$

$= A(1)$

$= A$ ✱

Rule 2

$A + 1 = 1$

$B + 1 = 1$

Rule 4

$A \cdot 1 = A$

b) $\bar{A} + \bar{A}B = \bar{A}(1 + B^1)$ rule 2

rule 4

$= \bar{A}(1)$

$= \bar{A}$

✱

BASIC RULES OF BOOLEAN ALGEBRA

Rule 11

OR-AND gate

a) $A + \bar{A}B = A + B$

b) $\bar{A} + AB = \bar{A} + B$

a) $A + \bar{A}B = A + AB + \bar{A}B$

$A + AB$

$= A + B(A + \bar{A})$ (1)

$= A + B(1)$

$= A + B$ ✱

b) $\bar{A} + AB$

$\bar{A} + \bar{A}B$

$= \bar{A} + \bar{A}B + AB$

$= \bar{A} + B(\bar{A} + A)$ (1) rule 6

$= \bar{A} + B(1)$ rule 4

$= \bar{A} + B$ ✱

Rule 10

$A + AB = A$

Rule 6

$A + \bar{A} = 1$

Rule 4

$A(1) = A$

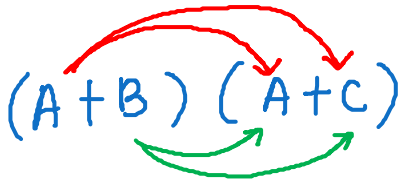
$B(1) = B$

BASIC RULES OF BOOLEAN ALGEBRA

Rule 12

$$(A+B)(A+C)=A+BC$$

OR - AND gate



rule 7

$= AA + AC + AB + BC$

$= A + AC + AB + BC$

$= A(1 + C + B) + BC$

$= A(1) + BC$

$= A + BC$ ✱

Rule 7

$A \cdot A = A$

Rule 2

$A + 1 = 1$

$C + B + 1 = 1$

Rule 4

$A \cdot 1 = A$

BASIC RULES OF BOOLEAN ALGEBRA

AND & OR GATE

1. $A + AB = A$
2. $A + \bar{A}B = A + B$

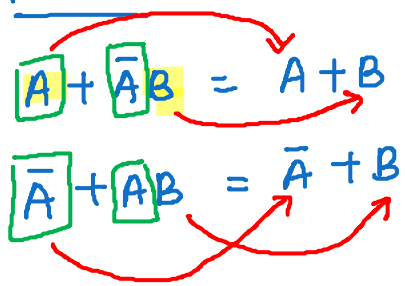
   $\bar{A} + AB = \bar{A} + B$
3. $(A+B)(A+C) = A + BC$

Rule 10

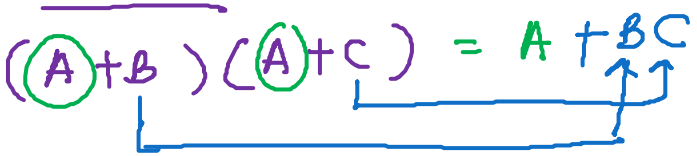
$A + AB = A$

$\bar{A} + \bar{A}B = \bar{A}$

Rule 11



Rule 12

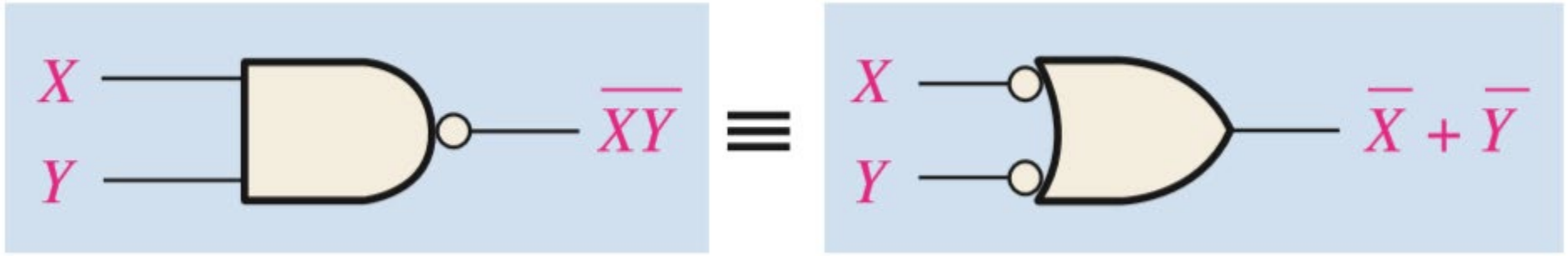


DE MORGAN'S THEOREM

(NAND)
NOT AND

negative OR

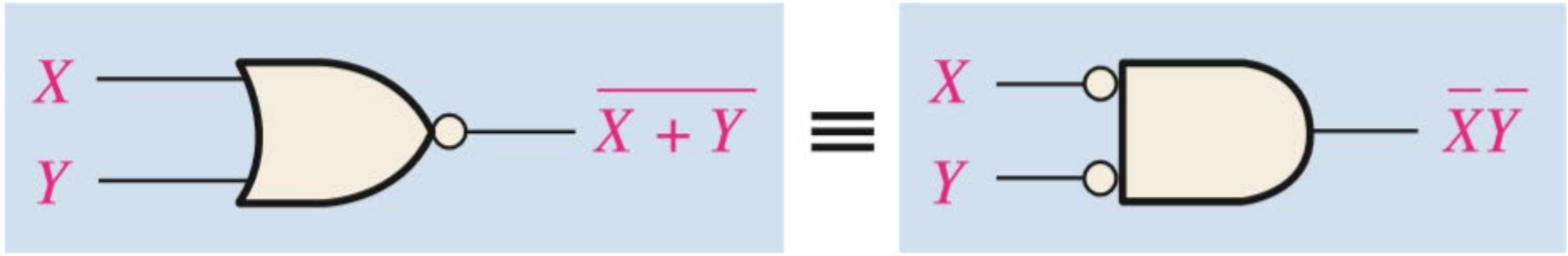
$$\overline{XY} = \overline{X} + \overline{Y}$$



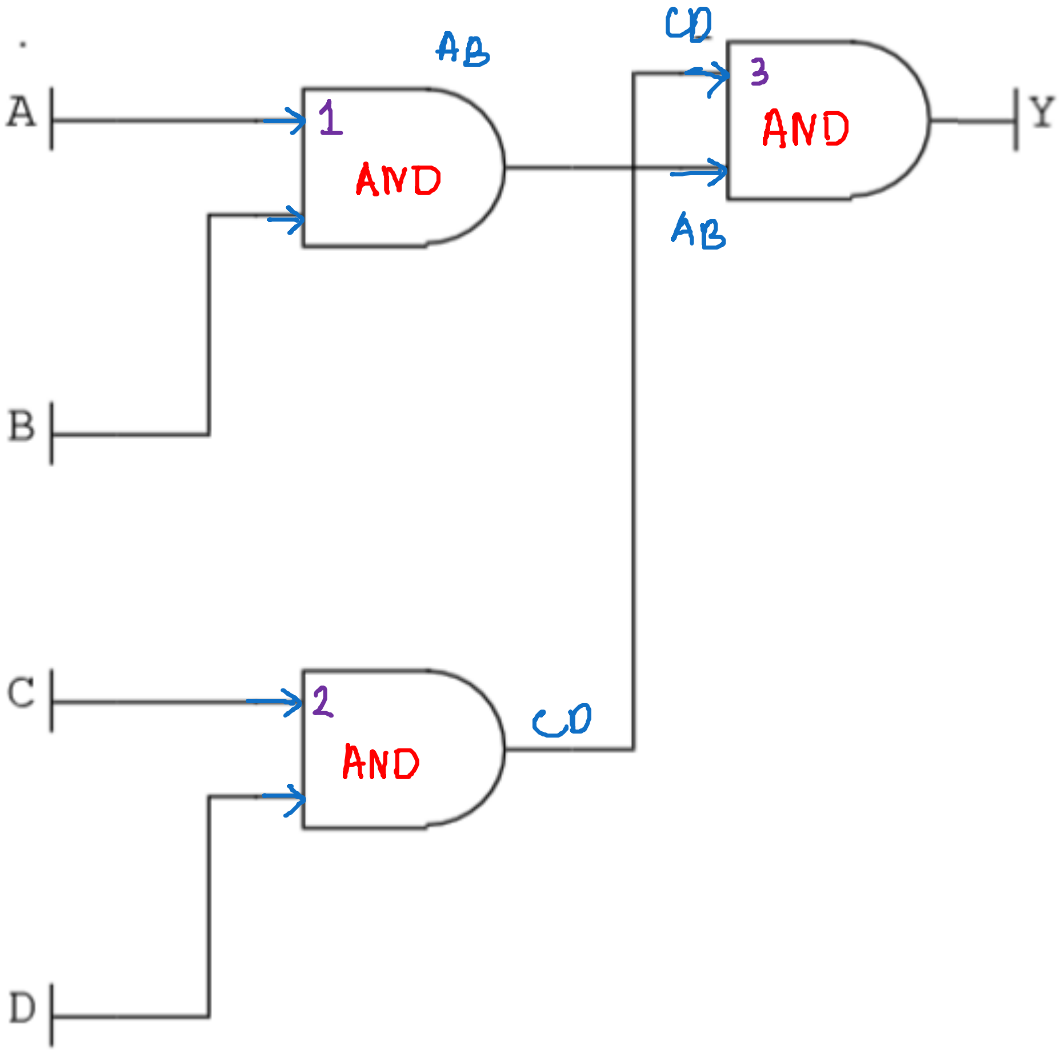
(NOR)
NOT OR

negative AND

$$\overline{X + Y} = \overline{X}\,\overline{Y}$$



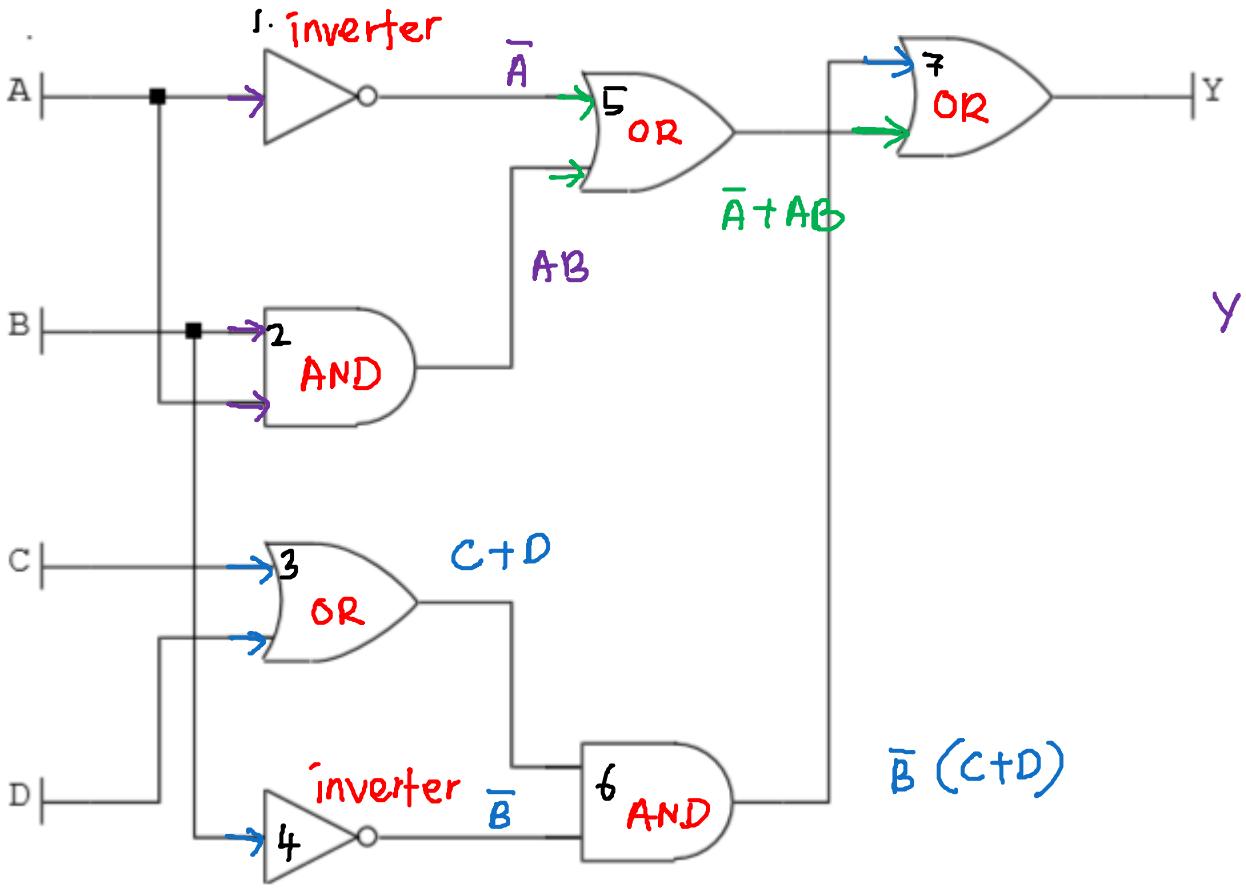
Question 1(a) : Write the **BOOLEAN EXPRESSIONS** for the outputs from the following **logic circuits**



$Y = AB \cdot CD$

$= ABCD$ ✻

Question 1(b) : Write the **BOOLEAN EXPRESSIONS** for the outputs from the following **logic circuits**



$$y = (\bar{A} + AB) + [\bar{B}(C+D)]$$

Question 1(b) : Simplify **BOOLEAN EXPRESSIONS** from the following **logic circuits**

ADDITIONAL

Method 1

$$Y = (\overline{A} + AB) + \overline{B}(C + D)$$

$$(\bar{A} + AB) + \bar{B}(C + D)$$

$$= \bar{A} + AB + \bar{B}C + \bar{B}D$$

$$= \bar{A} + B + \bar{B}C + \bar{B}D$$

$$= \bar{A} + B + C + \bar{B}D$$

$$= \bar{A} + C + B + D$$

$$= \bar{A} + B + C + D \quad \text{※}$$

Rule 11

$\bar{A} + AB = \bar{A} + B$

$A + \bar{A}B = A + B$

$B + \bar{B}C = B + C$

$B + \bar{B}D = B + D$

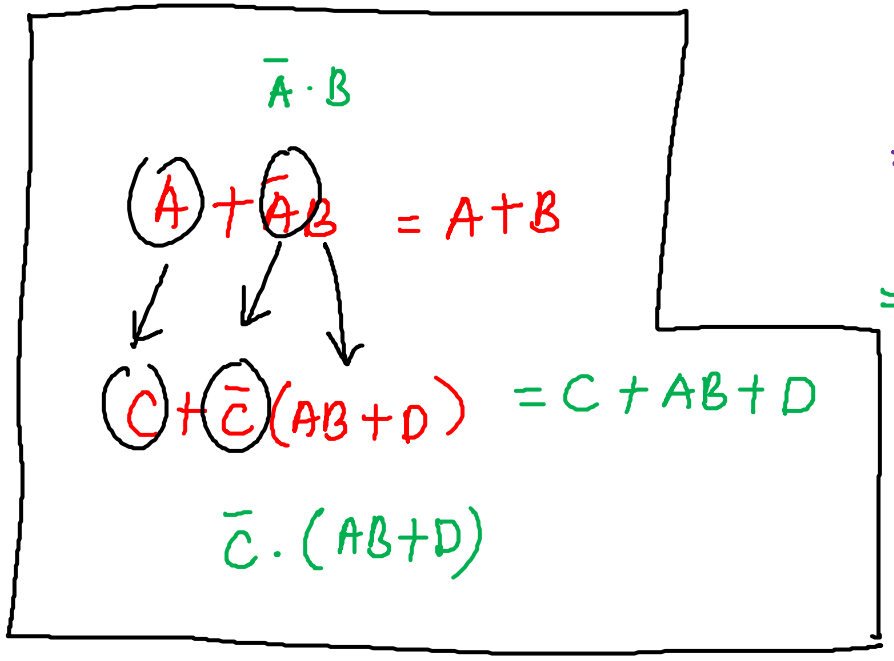
Question 1(b) : Simplify **BOOLEAN EXPRESSIONS** from the following **logic circuits**

ADDITIONAL

Method 2

$$Y = \overline{A} + AB + \overline{B}(C + D)$$

Example



$\overline{A} + AB + \overline{B}(C+D)$

$= \overline{A} + B + \overline{B}(C+D)$

$= \overline{A} + B + C + D$ #

rule 11

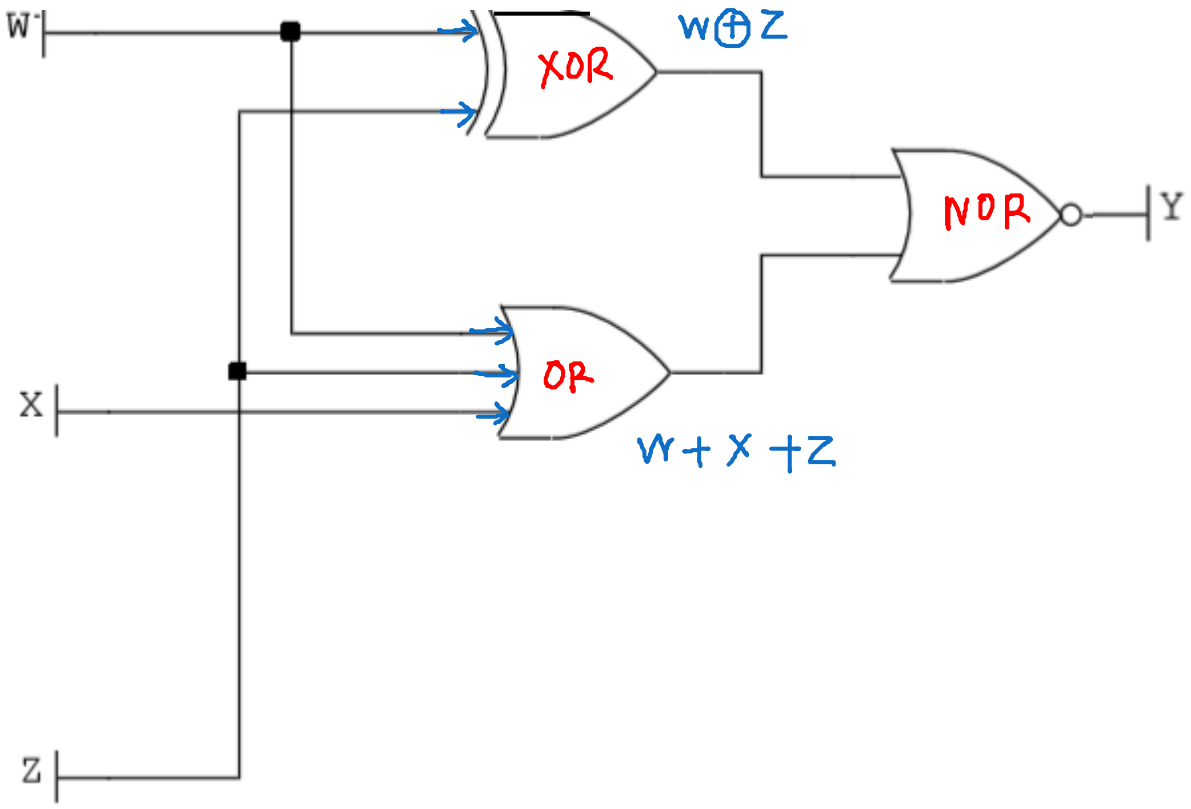
$\overline{A} + AB = \overline{A} + B$

$A + \overline{A}B = A + B$

$B + \overline{B}(C+D)$

$= B + C + D$

Question 1(c) : Write the **BOOLEAN EXPRESSIONS** for the outputs from the following **logic circuits**



not

$$Y = \overline{(W \oplus Z) + (W + X + Z)}$$

OR

Question 1(c) : Simplify **BOOLEAN EXPRESSIONS** from the following

ADDITIONAL

logic circuits

nor = negative and

$$Y = \overline{(W \oplus Z) + (W + X + Z)}$$

$= \overline{(W \oplus Z)}\,\overline{(W+X+Z)}$ — NOR = negative AND

NOR ← $= \overline{(W\bar{Z} + \bar{W}Z)}\,\overline{(W+X+Z)}$

NAND ← $= \overline{(W\bar{Z})}\,\overline{(\bar{W}Z)}\,(\bar{W}\bar{X}\bar{Z})$

$= (\bar{W}+\bar{\bar{Z}})(\bar{\bar{W}}+\bar{Z})(\bar{W}\bar{X}\bar{Z})$

$= (\bar{W}+Z)(W+\bar{Z})(\bar{W}\bar{X}\bar{Z})$

$= (\bar{W}\bar{W} + \bar{W}\bar{Z} + WZ + Z\bar{Z})(\bar{W}\bar{X}\bar{Z})$ — $Z\bar{Z} = 0$

rule 7 → $= (\bar{W}\bar{Z} + WZ)(\bar{W}\bar{X}\bar{Z})$

rule 3 → $= \bar{W}\bar{W}\bar{X}\bar{Z}\bar{Z} + W\bar{W}\bar{X}Z\bar{Z}$ — $W\bar{W} = 0$, $Z\bar{Z} = 0$

$= \bar{W}\bar{X}\bar{Z} + \bar{X}$

$= \bar{X}(\bar{W}\bar{Z} + 1)'$ $= \bar{X}$ ✱

rule 3
$A \cdot 0 = A$

rule 7
$A \cdot A = A$

rule 9
$\bar{\bar{A}} = A$
$\bar{\bar{Z}} = Z$
$\bar{\bar{W}} = W$

rule 8
$A \cdot \bar{A} = 0$
$W \cdot \bar{W} = 0$
$Z \cdot \bar{Z} = 0$

XOR

$W \oplus Z = W\bar{Z} + \bar{W}Z$

a) $\overline{W\bar{Z} + \bar{W}Z} = (\overline{W\bar{Z}})(\overline{\bar{W}Z})$

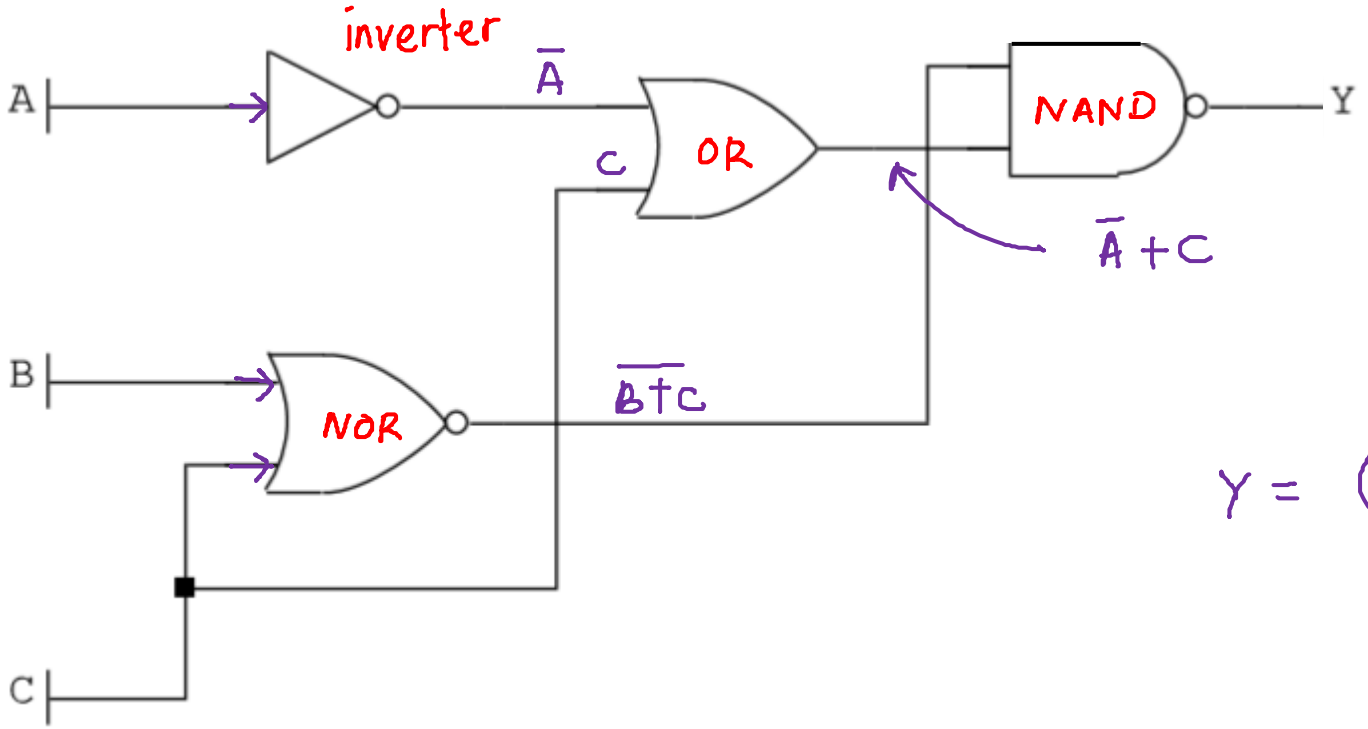
one term — OR — one term

b) $(\overline{W\bar{Z}})(\overline{\bar{W}Z}) = (\bar{W}+\bar{\bar{Z}})(\bar{\bar{W}}+\bar{Z})$

c) $\overline{W+X+Z} = \bar{W}\bar{X}\bar{Z}$ — NOR = negative AND

d) $(\bar{W}+Z)(W+\bar{Z}) = W\bar{W} + \bar{W}\bar{Z} + WZ + Z\bar{Z}$

Question 1(d) : Write the **BOOLEAN EXPRESSIONS** for the outputs from the following **logic circuits**



$$Y = \overline{(\bar{A}+C)(\overline{B+C})}$$

Question 1(d) : Simplify **BOOLEAN EXPRESSIONS** from the following **logic circuits**

ADDITIONAL

$$Y = \overline{(\overline{A} + C)(\overline{B + C})}$$

NAND = negative OR

NOR ← $= (\overline{\overline{A} + C}) + (\overline{\overline{B + C}})$

$= \overline{\overline{A}}\,\overline{C} + B + C$

$= A\overline{C} + B + C$

$= C + A + B$

$= A + B + C$ #

rule 9

$\overline{\overline{A}} = A$

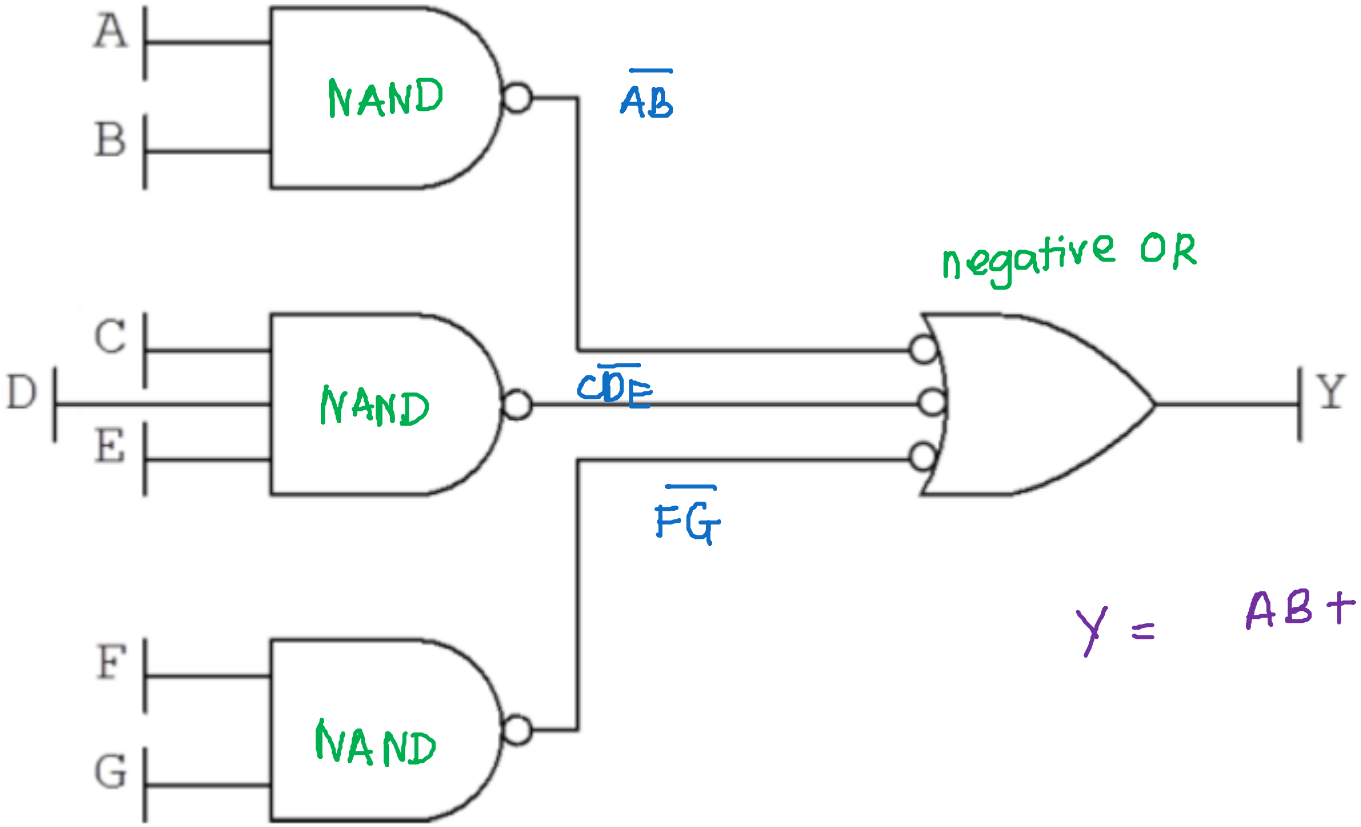
$\overline{\overline{B+C}} = B + C$

rule 11

$A + \overline{A}B = A + B$

$C + \overline{C}A = C + A$

Question 1(e) : Write the **BOOLEAN EXPRESSIONS** for the outputs from the following **logic circuits**



rule 9

$\overline{\overline{A}} = A$

$\overline{\overline{AB}} = AB$

$\overline{\overline{CDE}} = CDE$

$\overline{\overline{FG}} = FG$

$Y = AB + CDE + FG$ ✱

END DISCUSSION PART 1

ANY QUESTIONS ??