Tutorial 2 (ComplexNum+TrigIdentities)

(You should practise writing proper steps.)

1. Let z_1 and z_2 be two complex numbers given as $z_1 = 2-3i$ and $z_2 = 1+2i$.
   Compute the following.
   (a) $z_1\bar{z}_1$ 13
   (b) $z_1 z_2$ 8-i +i
   (c) $(z_1+3z_2)^2$ 16+30i
   (d) $[z_1+(1+z_2)]^2$ 15+8i -8i

2. Express the following in the form $a+bi$, where a and b are real numbers.
   (a) $\dfrac{1+4i}{5-12i}$ -43/169 + 32i/169
   (b) $(2+i)^3$ 2+11i
   (c) $3\sqrt{-50}+\sqrt{-72}$ ~~450i+72i=522i~~ 3/-(25*2) + /-(36*2)
   3(5)*/2 i + 6*/2 i
   21 */2 i
   (d) $\dfrac{1}{5-3i}-\dfrac{1}{5+3i}$ ~~i/6~~ 3/17 i
   25+9=34

3. (a) What is Euler's formula? e^o = cos o + i sin o
   e^i o

   (b) Use Euler's formula to derive the identities
   (i) $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$
   (ii) $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

4. Prove the identities
   (i) $\sin 2\theta = 2\sin\theta\cos\theta$
   (ii) $\cos 2\theta = \cos^2\theta-\sin^2\theta$
   in two ways as described below.

   (a) By setting $\alpha=\theta$ and $\beta=\theta$ in the identities
   $$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$
   $$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

   (b) By using Euler's formula and the identity $e^{i(2\theta)}=\left(e^{i\theta}\right)^2=e^{i\theta}\cdot e^{i\theta}$

   (cos o + i sin o)(cos o + i sin o)
   cos 2O + i sin 2O = cos^2 o + i (2cos o sin o) - sin^2

nby-Tri2-1516

5. Derive the identity that expressses $\sin 3\theta$ in terms of $\sin\theta$ and $\cos\theta$ by using

   Euler's formula and the identity $e^{i(3\theta)} = \left(e^{i\theta}\right)^3$. sin 3O =-(sin^3O + 3cos^2 O sin O)

   cos^3 O +3cos^2O isin O -3cosOsin^2O - isin^3O
   i(3 cos^2OsinO - sin^3O)
   sin3O = ^

6. Derive the identity that expressses $\cos 3\theta$ in terms of $\sin\theta$ and $\cos\theta$ by using

   Euler's formula and the identity $e^{i(3\theta)} = \left(e^{i\theta}\right)^3$.

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7. Express $\sin 4x \sin 5x$ in a form involving the difference of two cosines. -1/2(cos9x-cosx)

8. Express $\cos 5x \cos 2x$ in a form involving the sum of two cosines. 1/2(cos7x+cos3x)

9. Express $\sin 5x \cos 2x$ in a form involving the sum of two sines. 1/2(sin7x+sin3x)

10. Express $\cos 5x \sin 3x$ in a form involving the difference of two sines. 1/2(sin8x-sin2x)

11. Derive the subtraction formulas
    (i) $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
    (ii) $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

    in two ways as described below.

    (a) By replacing β with $-\beta$ in the following two identities you have derived earlier

    $$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
    $$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

    (b) By using Euler's formula and the identity $e^{i\alpha}\cdot e^{-i\beta} = e^{i(\alpha-\beta)}$

12. Obtain the addition formula and subtraction formula for tangent.
    Use the addition and subtraction formulas for sine and cosine to express $\tan(\alpha+\beta)$ and $\tan(\alpha-\beta)$ in terms of $\tan\alpha$ and $\tan\beta$.

    [Note that $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.]

13. Use the special angles $\alpha = \dfrac{4\pi}{3}$ and $\beta = \dfrac{\pi}{3}$ to verify that all the addition and subtraction formulas for sine, cosine and tangent hold.
    [*You may need help in understanding what this question asks.*]

(nby, Nov 2015)

nby-Tri2-1516