TMA1101 Calculus, T1, 2017/18

Tutorial 4 (Differentiation)

(You should practice writing proper steps.)

1. (a) Use the formal definition of the derivative to compute $f'(2)$ when $f(x)=\dfrac{1-x}{2x}$.

   (b) Do the same to find $f'(x)$.

2. Find the slope of the tangent to the graph of the given function at the given point. Then obtain an equation for that tangent.

-3, f(x)=4, y=-3x+16   1/3, f(x)=3, y=1/3 x +5/3

   (a) $f(x)=\dfrac{x}{x-3}$, at $x=4$ (b) $f(x)=\sqrt{2x+1}$, at $x=4$

3. Find the following limits.

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   (a) $\lim\limits_{\theta\to\frac{\pi}{6}}\dfrac{\sin\theta-\frac{1}{2}}{\theta-\frac{\pi}{6}}$ (b) $\lim\limits_{x\to 0}\dfrac{e^x-1}{x}$ (c) $\lim\limits_{t\to 1}\dfrac{\ln t}{t-1}$ (d) $\lim\limits_{h\to 0}\dfrac{(1+h)^{2014}-1}{h}$

4. Suppose *u* and *v* are functions of *x* that are differentiable at *x* = 0 and that $u(0)=1, u'(0)=2, v(0)=4, v'(0)=6$.

   Find the values of the following derivatives at $x=0$.

   (a) $\dfrac{d}{dx}(uv)$ (b) $\dfrac{d}{dx}\left(\dfrac{u}{v}\right)$ (c) $\dfrac{d}{dx}\left(\dfrac{v}{u}\right)$ (d) $\dfrac{d}{dx}(3u^2-4v)$ 6u-4=2

14   1/8   -2   8

5. Find the derivatives.

   [*Write proper steps showing the basic differentiation rules used.*]

17/(2x+5)^2   (tan x)(cos x - sin x) + (sin x + cos x)(sec2x)

   (a) $f(x)=\dfrac{3x-1}{2x+5}$ (b) $y=\tan x(\sin x+\cos x)$ (c) $h(x)=\dfrac{1+\sqrt{x}}{x+3}$

   (d) $v=(1+t)(1+t^2)$ (e) $g(x)=\dfrac{2x+1}{x^2-3}$ (f) $r=3\left(\sqrt{\theta}+\dfrac{1}{\sqrt{\theta}}\right)$

   (g) $p(x)=\dfrac{e^x}{e^x+1}$ (h) $y=\dfrac{\sin x}{1+\cos x}$ (i) $y=x\ln x-xe^x$

   (j) $y=x^2\sin x+3x\cos x-e^x\cos x$

6. Use the chain rule to differentiate: [*Show the steps in using the chain rule.*]

   (a) $\sin 5x$ (b) $\cos 5x$ (c) $\tan 5x$ (d) $\sec 5x$ (e) e^{5x}

   (f) $\ln(5x)$ [*Can you obtain the derivative of* $\ln(5x)$ *without using the chain rule*?]

7. (a) Let $y=(g\circ f)(x)$ where $f(x)=\sqrt{1-x^2}$ and $g(x)=x^3$. Find the derivative $\dfrac{dy}{dx}$.

   (b) Let $y=(f\circ g)(x)$ where $f(x)=\sqrt{1-x^2}$ and $g(x)=x^3$. Find the derivative $\dfrac{dy}{dx}$.

   (c) Find the derivative of $y=x(2x+3)^{1101}$.

   (d) Find the derivative of $y=\sqrt{(\sin x)^4+1}$.

8. Differentiate the following functions. (Show proper steps in using product, quotient and/or chain rules of differentiation. *Practice in writing proper steps will help improve learning*.)

(a) $g(x) = 8x^{\frac{3}{4}} + 4x^{\frac{1}{4}} - x^{-\frac{1}{3}}$

(b) $y = x^3(1-3x)^{\frac{1}{4}}$

(c) $y = \dfrac{x}{\sqrt{x^2+1}}$

(d) $f(x) = 5\sin 3x \cos x$

(e) $f(x) = \dfrac{1-\sin x}{x^4}$

(f) $y = \sin^3(\sin^2 x)$

(g) $y = \sqrt{x^3 - 2x^2 + x - 7}$

(h) $y = \tan x - \sec x$

(i) $y = \dfrac{\tan x}{1+\tan x}$

(j) $y = e^x(\cos x + 2x)$

(k) $y = \sqrt{1 + xe^{-4x}}$

(l) $y = \dfrac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}}$

(m) $f(x) = (4x\sin x - 1)^3$

(n) $y = \sin(\cos(2x-5))$

(o) $y = e^{x\tan\sqrt{x}}$

(p) $y = e^{x^2+x}$

9. Find the first and second derivatives of $y = (1+u)(1+u^2)(1+u^3)$.

10. Determine the specified derivative for each of the given functions.

(a) Find $f^{(3)}(x)$ for $f(x) = x^4 - 10x^2$

(b) Find $f^{(2)}(x)$ for $f(x) = e^{2x}\sin 2x$

(c) Find $y^{(4)}$ for $y = x^4 - 7x^2 - 4x + 2$

11. (a) Find $\dfrac{dy}{dx}$ if $y = y(x)$ is defined implicitly as follows.

(i) $x^2y + 3y - x + 1 = 0$

(ii) $y = (x^2+1)^x$ [*Hint*: Take logarithms of both sides.]

(iii) $\cos^2(y+1) = 2\sin x$

(iv) $ye^x + xe^y = xy$

(b) Show that the point (1, 3) lies on the curve $x^2y + xy^2 - xy = 9$.
Find the gradient of the tangent to the curve at the point (1, 3).
Then find an equation for the tangent.

(c) Find an equation of the tangent to the curve $x^3 + 7xy + y^4 = 1$ at the point (−1, 2).

12. (a) Find the absolute maximum and minimum values of the function
$f(x) = 3x^4 - 4x^3 + 2$ on the closed interval [−1, 2].

(b) Find the absolute maximum and minimum values of the function
$f(x) = 3x^4 - 4x^3 + 2$ on the closed interval [−1, 8].

13. Find the local maximum and minimum values of the following functions and where they occur.

(a) $f(x) = 4x^3 + 12x^2 + 12x + 1$ (b) $f(x) = \dfrac{2x+5}{x^2-4}$

(c) $f(x) = \dfrac{x}{x^2+4}$ (d) $f(x) = \sin x + \cos x$ for $0 \le x \le 2\pi$.

14. Find all the local extreme values of each function on the given interval.
On what intervals is the function increasing or decreasing?
Sketch a graph of the function.
Identify the points on the graph where the extrema occur and include their coordinates.
Identify also the absolute extrema.
[You need to find the critical points, decide on types of local extrema, check also at the endpoints, and finally decide on the absolute extrema. If you read up on concavity and inflection points, you may be able to sketch a better graph.]
(a) $f(x) = x^2 - 6x + 9$, $0 \le x \le 5$
(b) $f(x) = x^3 - 3x^2 - 24x + 1$, $-5 \le x \le 5$
(c) $f(x) = \sin(x + \dfrac{\pi}{4})$, $0 \le x \le \dfrac{5\pi}{2}$

15. Find the following limits. For those that do not exist, see if you can write as $+\infty$ or $-\infty$. [Before applying l'Hopital's rule, make sure the conditions are right.]

(a) $\displaystyle\lim_{x\to\infty} \frac{5x^2 - 3x + 1}{3x^2 - 5}$ (b) $\displaystyle\lim_{x\to 0} \frac{3e^{2x} - 3}{\sin x}$ (c) $\displaystyle\lim_{x\to 0} \frac{3^x - 1}{x}$

(d) $\displaystyle\lim_{x\to\frac{\pi}{2}} \frac{1 - \sin x}{x - \frac{\pi}{2}}$ (e) $\displaystyle\lim_{x\to k} \frac{3\sqrt{x} - 3\sqrt{k}}{x - k}$ [k is a positive number]

(f) $\displaystyle\lim_{x\to+\infty} \frac{\ln(1+e^x)}{1+x}$ (g) $\displaystyle\lim_{x\to 1^+} \left(\frac{3}{\ln x} - \frac{3}{x-1} \right)$ (h) $\displaystyle\lim_{x->0} \frac{\sin x - \cos x}{x^4}$

(i) $\displaystyle\lim_{x\to\infty} \frac{e^x - x}{e^{2x} - 1}$ (j) $\displaystyle\lim_{x\to\infty} \frac{x(x-1)(x+1)}{x^3 - \sqrt{x}}$ (k) $\displaystyle\lim_{x\to\infty} \frac{e^{2x} - x}{e^x - 1}$

[For (g), you may not see the usual indeterminate form $\dfrac{0}{0}$ or $\dfrac{\infty}{\infty}$.]

[Go back to earlier questions on limits to see where l'Hopital's rule can be applied.]

(nby, Jun 2017)