FINAL EXAM
MARKING SCHEME
TRIMESTER I, 2016/2017
PMT0101 – MATHEMATICS I

Question 1 [10 marks]
Solution:

a) $\dfrac{2y-6}{y^2-9} \div \dfrac{y-3}{y+3}$

$= \dfrac{2(y-3)}{y^2-3^2} \times \dfrac{y+3}{y-3}$

$= \dfrac{2(y-3)}{(y-3)(y+3)} \times \dfrac{y+3}{y-3}$

$= \dfrac{2}{y-3}$ [1 + 0.5 + 0.5]

b)

$\dfrac{1+\sqrt{5}}{3+2\sqrt{5}}$

$= \dfrac{(1+\sqrt{5})}{(3+2\sqrt{5})} \cdot \dfrac{(3-2\sqrt{5})}{(3-2\sqrt{5})}$ [0.5]

$= \dfrac{3-2\sqrt{5}+3\sqrt{5}-2(5)}{9-4(5)}$ [0.5 + 0.5]

$= \dfrac{-7+\sqrt{5}}{-11}$ [0.5]

c) $(a+2b)^3$

$= (a)^3 + 3a(2b)^2 + 3a^2(2b) + (2b)^3$

$= a^3 + 12ab^2 + 6a^2b + 8b^3$ [1 + 1]

d)

$x\sqrt{50y^2} + \sqrt{200x^2y^2}$

$= xy\sqrt{25\times 2} + xy\sqrt{100\times 2}$

$= 5xy\sqrt{2} + 10xy\sqrt{2}$

$= (15\sqrt{2})xy$ [1 + 0.5 + 0.5]

e) $\dfrac{3}{2-i}+\dfrac{1}{2+i}$

$$=\frac{3(2+i)+2-i}{(2-i)(2+i)}$$
$$=\frac{6+3i+2-i}{4-i^2}$$
$$=\frac{8+2i}{4-(-1)}$$
$$=\frac{8}{5}+\frac{2}{5}i \qquad [0.5+0.5+0.5+0.5]$$

Question 2 [10 marks]
Solution:

a) $2x^2 - 3x + 4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(4)}}{2(2)}$$

$$x = \frac{3 \pm \sqrt{9 - 32}}{4}$$

$$x = \frac{3 \pm \sqrt{-23}}{4}$$

$$x = \frac{3}{4} \pm \frac{i\sqrt{23}}{4}$$ [0.5 + 0.5 + 0.5 + 0.5]

b) $$\frac{x-1}{(x-2)(x+3)} > 0$$

Let

$x - 1 = 0$, $x - 2 = 0$, $x + 3 = 0$

$x = 1$, $x = 2$, $x = -3$

	-3		1		2	
Sign of $(x-1)$	---		---		+++	+++
Sign of $(x-2)$	---		---		---	+++
Sign of $(x+3)$	---		+++		+++	+++
Sign of $\frac{x-1}{(x-2)(x+3)}$	---		+++		---	+++
	[0.5]		[0.5]		[0.5]	[0.5]

The solution set is $(-3, 1) \cup (2, \infty)$ [0.5+0.5]

c) $\left|\frac{1}{2}x-6\right|<4$

$-4<\frac{1}{2}x-6<4$

$2<\frac{1}{2}x<10$

$4<x<20$ $[0.5+0.5+0.5]$

The solution set is $(4,\ 20)$ [0.5]

d) $\sqrt{x-2}=8-x$

$\left(\sqrt{x-2}\right)^2=(8-x)^2$

$x-2=64-16x+x^2$

$x^2-16x-x+64+2=0$

$x^2-17x+66=0$

$(x-6)(x-11)=0$

$x=6$ *or* $x=11$ [0.5 + 0.5 + 0.5]

Check the solution:

When $x=6$ LHS: $\sqrt{6-2}=\sqrt{4}=2$ RHS: $8-6=2$ LHS=RHS [0.5]	When $x=11$ LHS: $\sqrt{11-2}=\sqrt{9}=3$ RHS: $8-11=-3$ LHS $\neq$ RHS [0.5]

Therefore, the solution is $x=6$ [0.5]

Question 3 [10 marks]
Solution:

a)
i)

$$2x-3=0$$
$$x=\frac{3}{2}$$

$$Domain:\left(-\infty,\ \frac{3}{2}\right)\cup\left(\frac{3}{2},\ \infty\right) \qquad [0.5+0.5]$$

ii)

$$f(5)=5$$
$$\frac{4(5)+k}{2(5)-3}=5$$
$$20+k=5(7)$$
$$k=15 \qquad [0.5+0.5+0.5]$$

iii) Let

$$y=\frac{4x+15}{2x-3}$$
$$y(2x-3)=4x+15$$
$$2xy-3y=4x+15$$
$$2xy-4x=3y+15$$
$$x(2y-4)=3y+15$$
$$x=\frac{3y+15}{2y-4}$$
$$\therefore \quad f^{-1}(x)=\frac{3x+15}{2x-4} \qquad [0.5+0.5+0.5+0.5+0.5]$$

b) $f(x) = 2(x+1)^2(x-1)^2(x-3)$

i) Degree = 5 [0.5]

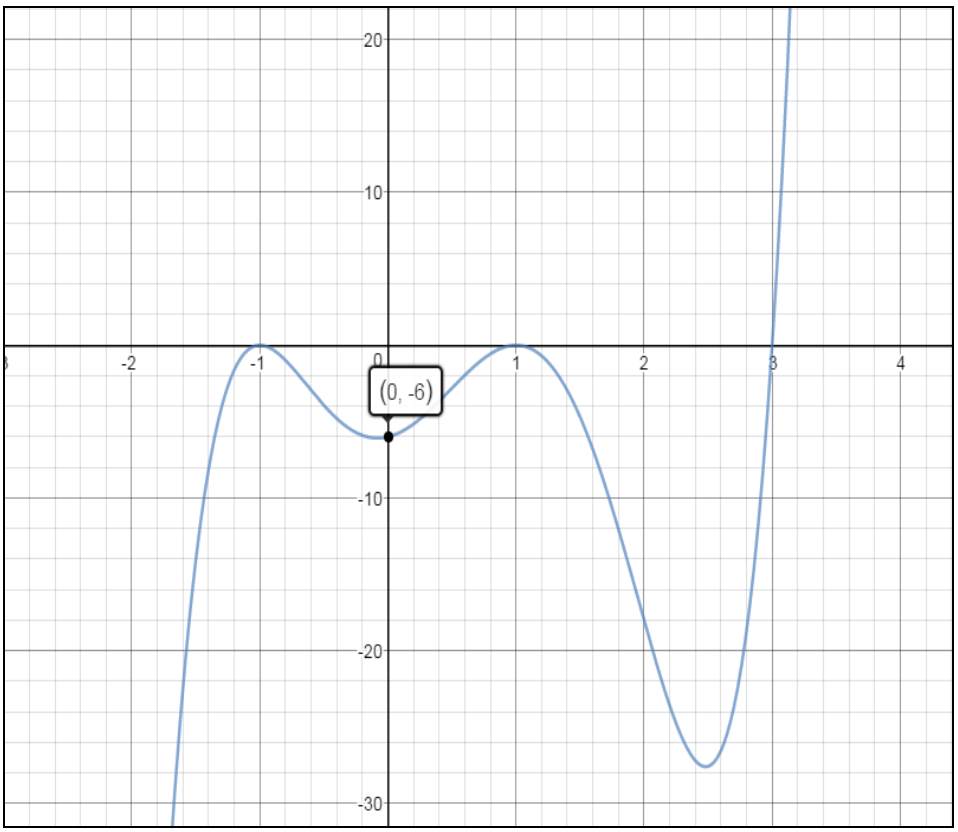
ii) The zeros of f are $-1,\ 1$ and 3

Zeros	Multiplicities	Cross/Touch
-1	2	Touch
1	2	Touch
3	1	Cross
[0.5]	[0.5]	[0.5]

iii) y-intercept, $f(0) = -6$ [0.5]

iv) As $x \to -\infty,\ y \to -\infty$ [0.5]
As $x \to \infty,\ y \to \infty$ [0.5]

v) Sketch the graph:



Shows all intercepts [0.5]
Proper end behaviour [0.5]
Shows correct crossing or touching at x-intercepts [0.5]

Question 4 [10 marks]
Solution:

a)

$$
\begin{array}{r}
\quad -2x^2 - 5x + 3 \\
x+3\overline{)\,-2x^3 - 11x^2 - 12x + 9} \\
-\ \underline{-2x^3 - 6x^2} \\
\quad -5x^2 - 12x \\
-\ \underline{-5x^2 - 15x} \\
\quad 3x + 9 \\
-\ \underline{3x + 9} \\
\quad ..0
\end{array}
\qquad \Big\} \ [0.5+0.5+0.5+0.5]
$$

Quotient = $-2x^2 - 5x + 3$ [0.5]
Remainder = 0 [0.5]

b)

$f(x) = x^3 - 2x + 4$
Divisor : Compare $x - 1$ with $x - c$ $\Longrightarrow c = 1$
Remainder = $f(1) = 1^3 - 2(1) + 4 = 3$ $[0.5 + 0.5]$

c)

$$\frac{1}{2}\log_{10} 25 - 2\log_{10} 3 + 2\log_{10} 6$$

$$= \log_{10} 25^{\frac{1}{2}} - \log_{10} 3^2 + \log_{10} 6^2$$

$$= \log_{10}\left(\frac{5 \times 36}{9}\right)$$

$$= \log_{10} 20 \qquad [1 + 0.5 + 0.5]$$

d) $2^{2x} = 5$

$$\log 2^{2x} = \log 5$$
$$2x \log 2 = \log 5$$
$$x = \frac{\log 5}{2 \log 2} \qquad [0.5 + 0.5 + 1]$$

e)

$$3 = 3^{-2\left(\frac{1}{2}\right)+k}$$
$$3^1 = 3^{-1+k}$$
$$1 = -1 + k$$
$$k = 2 \qquad [0.5 + 0.5 + 0.5 + 0.5]$$

Question 5 [10 marks]
Solution:

a)i)

$$M = \left(\frac{0+2}{2}, \frac{6+0}{2} \right) = (1,\ 3) \quad [0.5 + 0.5]$$

ii)

$$m_{AB} = \frac{6-0}{0-2} = -3 \qquad [0.5 + 0.5]$$

iii)

$$m_{AB} \times m_{perpendicular} = -1$$
$$-3 \times m_{perpendicular} = -1$$
$$m_{perpendicular} = \frac{1}{3} \qquad [0.5 + 0.5]$$

iv)

$$y - 3 = \frac{1}{3}(x - 1)$$
$$y = \frac{1}{3}x - \frac{1}{3} + 3$$
$$y = \frac{1}{3}x + \frac{8}{3} \qquad [0.5 + 0.5]$$

b)

$$PQ = 4$$
$$\sqrt{(x+2)^2 + (y-5)^2} = 4$$
$$x^2 + 4x + 4 + y^2 - 10y + 25 = 16$$
$$x^2 + y^2 + 4x - 10y + 13 = 0 \qquad [1 + 1 + 1]$$

c)
i)

$$\sqrt{x}\left(y\sqrt{x} = \frac{5}{\sqrt{x}} - 2 \right)$$
$$xy = 5 - 2\sqrt{x}$$
$$xy = -2\sqrt{x} + 5 \qquad [0.5 + 0.5]$$

ii)

$$p = 5 \qquad [0.5]$$

When $\sqrt{x} = 4$, $xy = q$ [0.5]

$$q = -2(4) + 5$$
$$q = -3 \qquad [0.5 + 0.5]$$