OSCILLATORY MOTIONS

CONTENTS

WHAT ARE YOU GOING TO LEARN ?

- Explain what is meant by Simple Harmonic Motion
- Understand and use the terms amplitude, period, frequency, angular frequency and express the period in terms of both frequency and angular frequency for both mass-spring system and simple pendulum
- Analyze simple harmonic motion using mathematical equations
- Describe the conservation of energy and interchange between kinetic and potential energy during simple harmonic motion
- Understand the concept of simple pendulum
- Explain what is a damped and driven oscillations and give example.

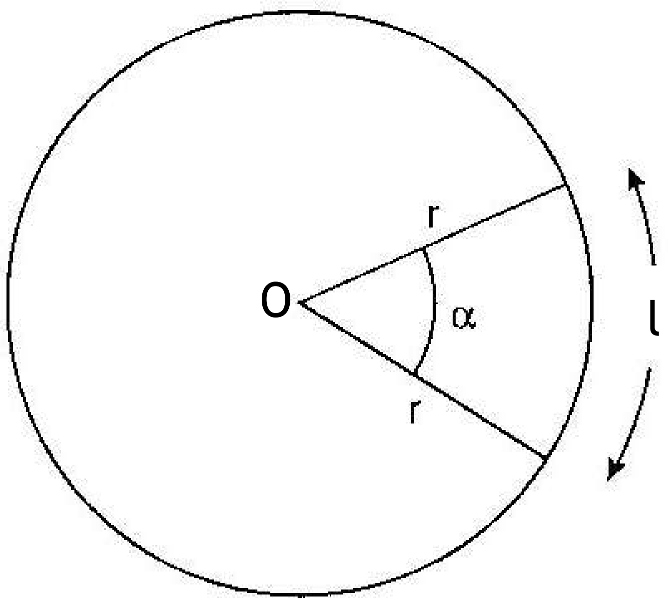
BEFORE WE START

- Review on Type of Motion
  - Translational motion
  - Rotational motion
  - Oscillatory motion (vibration motion)

We are focus on this area !!!

4.1 INTRODUCTION

- Before we go further lets review some mathematic concept which is related with this chapter



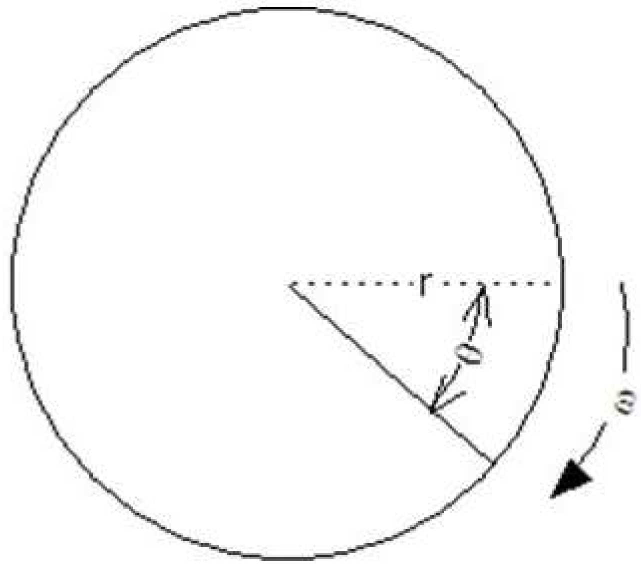
Radian:

$$\alpha = \frac{arc}{radius} = \frac{l}{r}$$

1 complete circle has 2π rad.
½ circle has π rad.

4.1 INTRODUCTION

- Angular Velocity(Angular Frequency), ω
  - Use to measure the rotation rate
  - Rate of change of radian

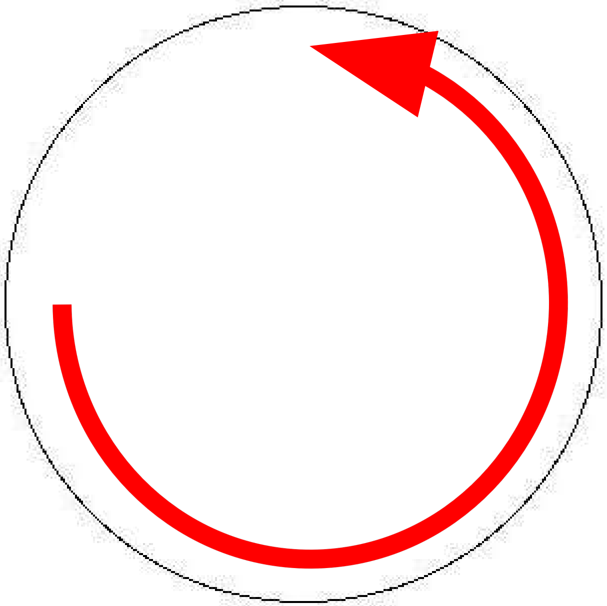


$$\omega = \frac{\Delta\theta}{\Delta t}$$

Symbol	**ω**
Units	$rad \cdot s^{-1}$

4.1 INTRODUCTION

- What is the angular frequency for one revolution ?



One revolution is equal to 2π radians, therefore,

$$\omega = \frac{2\pi}{T}$$

where

ω - angular frequency or angular speed

T - the period , taken time to complete 1 revolution.

4.1 INTRODUCTION

- **Period, T**
  - Time required for one complete revolution
  - Unit is second(s)
  - $$T = \frac{1}{f}$$

- **Frequency, f**
  - Number of repeating cycle per 1 second
  - units is Hz or s^{-1}

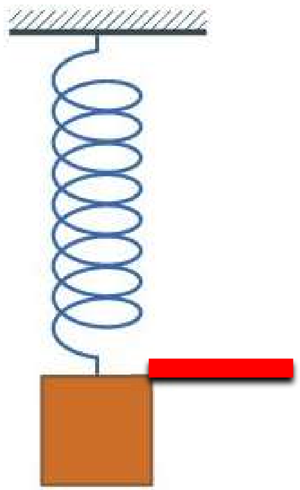
$$\therefore f = \frac{1}{T}$$

$$\omega = \frac{2\pi}{T}$$

$$= 2\pi f$$

$$f = \frac{\omega}{2\pi}$$

4.2 SIMPLE HARMONIC MOTION (SHM)



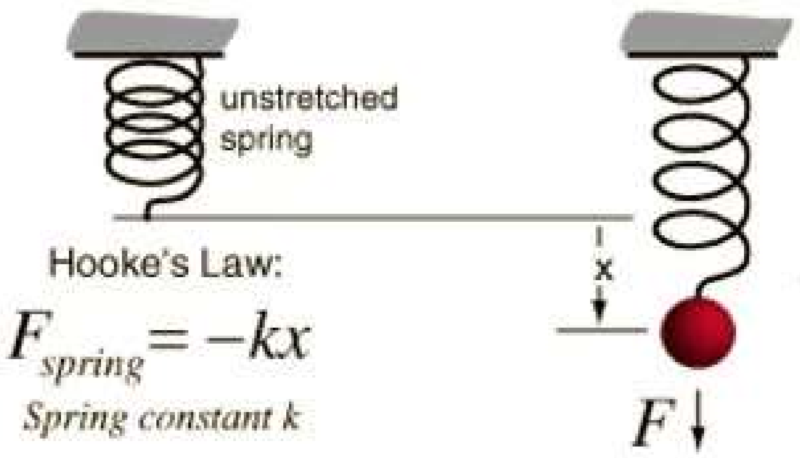
- Amplitude
  - Maximum displacement
  - Greatest distance from the equilibrium point
- One Cycle
  - Complete to-and-fro motion from initial point back to that same point
- Period
  - The time required for one complete cycle
- Frequency
  - The number of complete cycle per second

4.2 SIMPLE HARMONIC MOTION (SHM)

- Hooke's Law
  - The extension of a spring is in direct proportion with the load added to it as long as this load does not exceed the elastic limit

$$F = -kx$$

Where,
k - *spring constant* *(kgs^{-2})*
x - displacement of the end of the spring from its equilibrium position



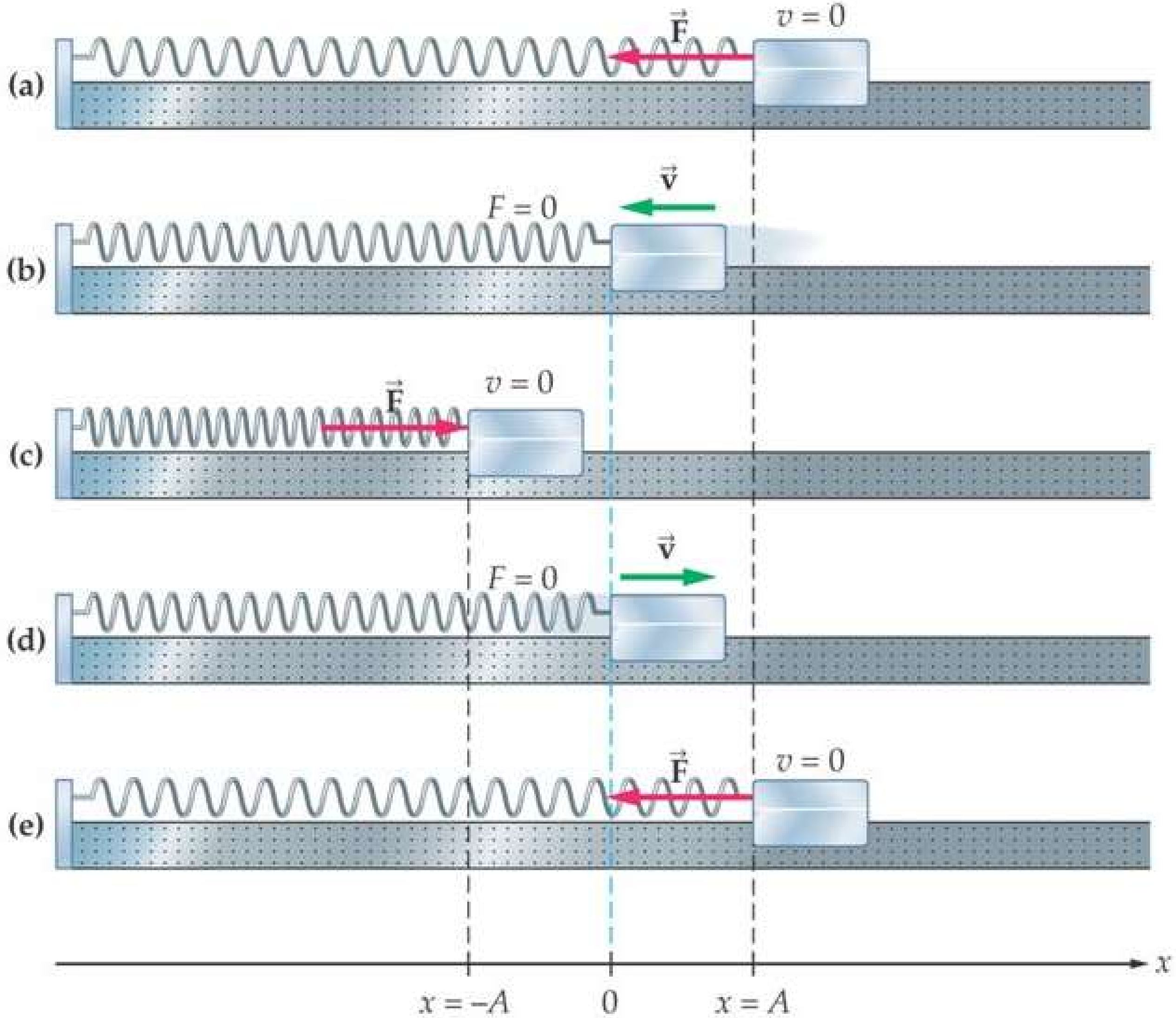
4.2 SIMPLE HARMONIC MOTION (SHM)

(b) The mass is at the equilibrium position of the spring. Here the speed has its maximum value, and the force

(e) The mass has completed one cycle of its oscillation about x = 0.

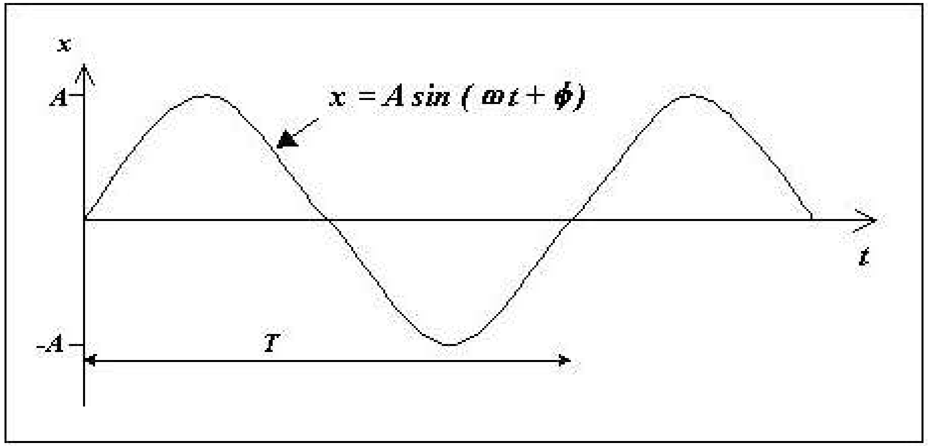
equilibrium position of the spring, with zero force acting on it and maximum speed.

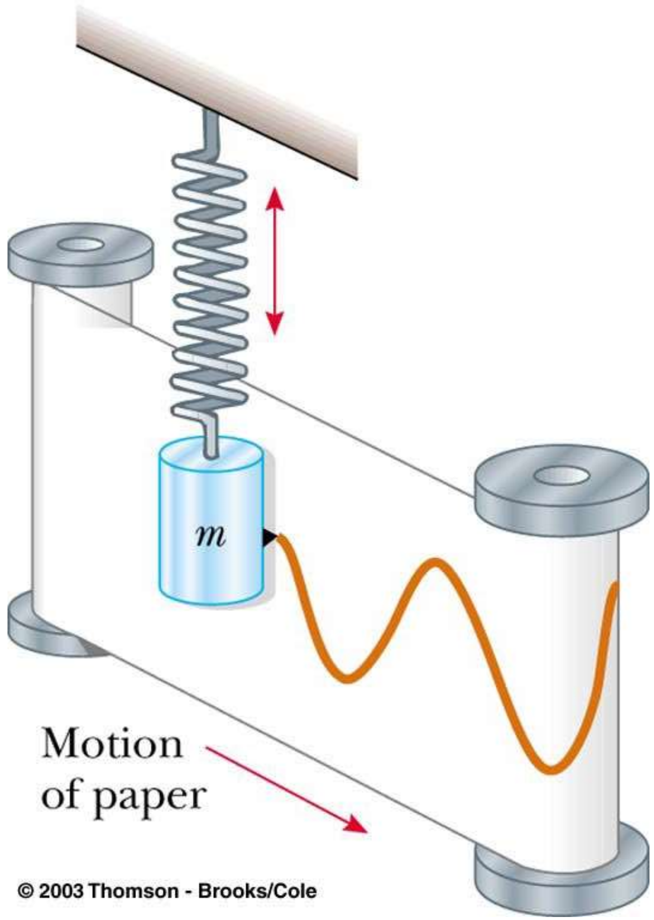
force points to the right with maximum magnitude.



4.2 SIMPLE HARMONIC MOTION (SHM)

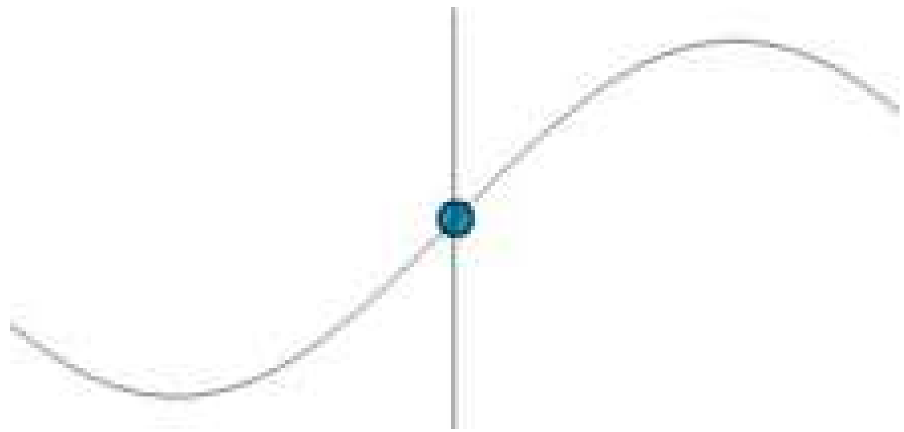
- We may represent this simple harmonic motion in graph as in



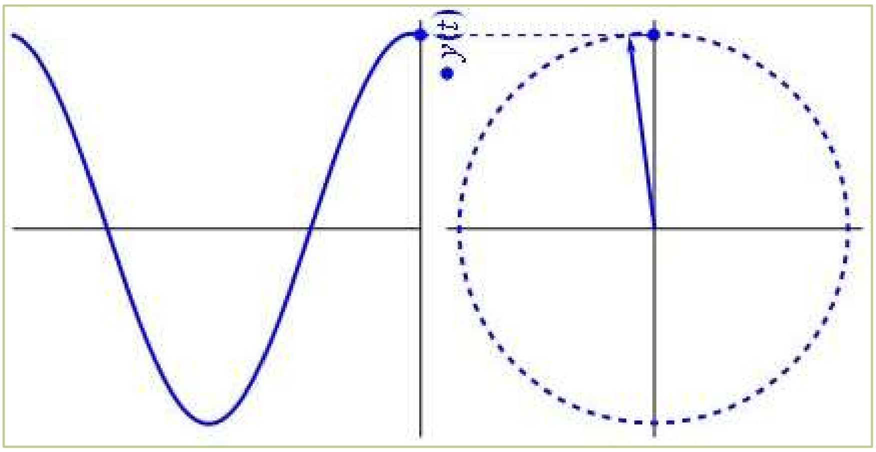


4.2 SIMPLE HARMONIC MOTION (SHM)

Reference
Point

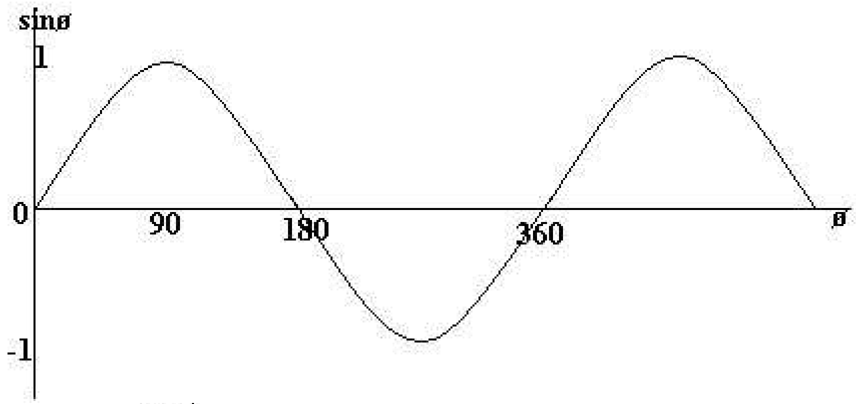


4.2 SIMPLE HARMONIC MOTION (SHM)

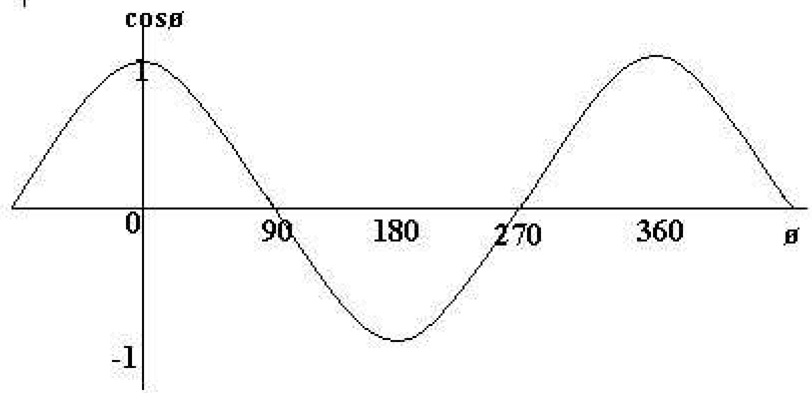


4.2 SIMPLE HARMONIC MOTION (SHM)

- The graph can be analyzed by 2 approach
  - Sin Graph

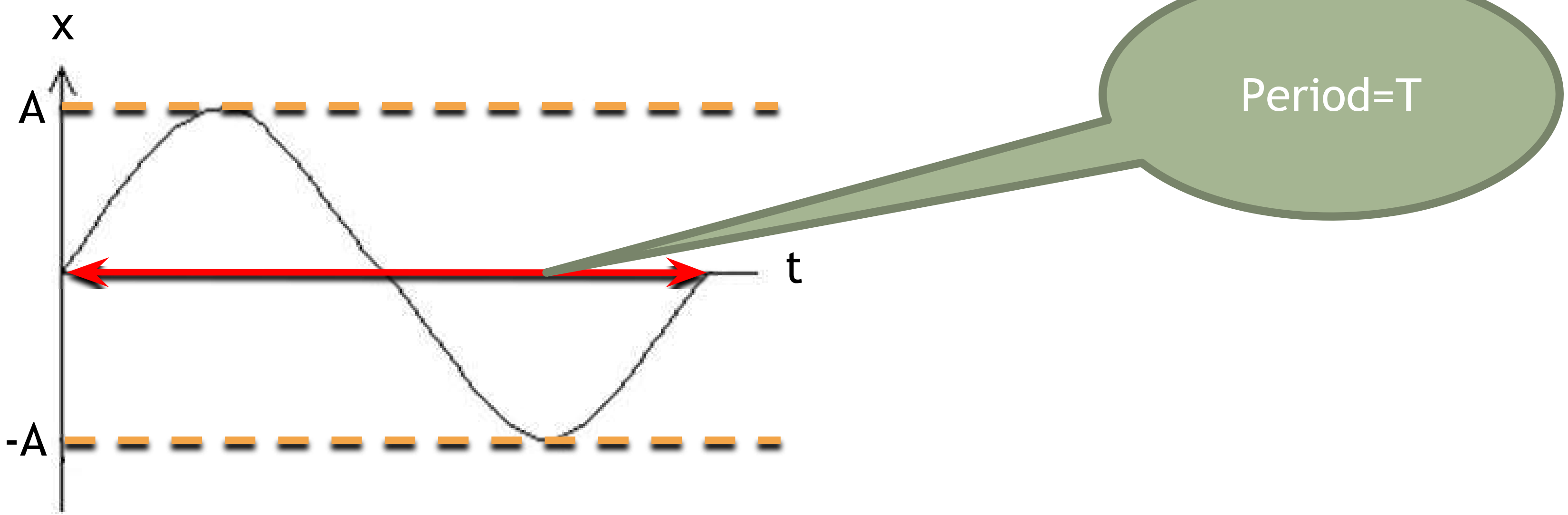


  - Cos Graph



4.2 SIMPLE HARMONIC MOTION (SHM)

SIN GRAPH



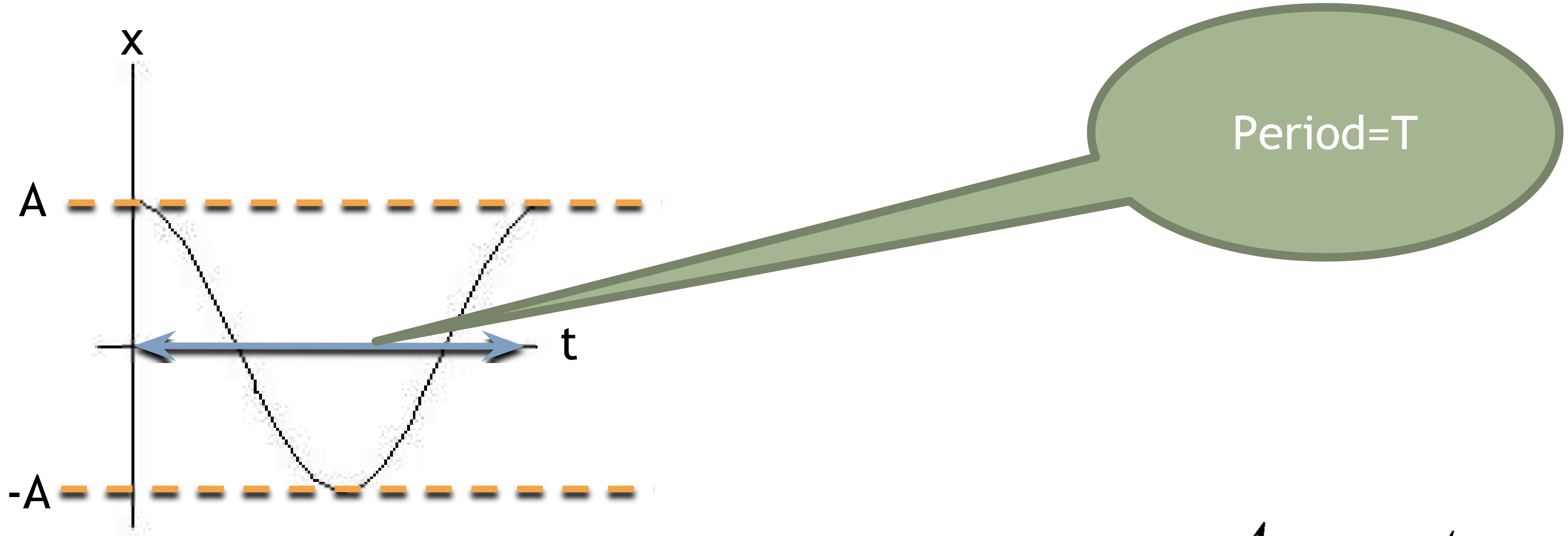
$$x = A\sin\theta$$
$$x = A\sin\omega t$$
$$x = A\sin\left(\frac{2\pi}{T}\right)t$$
$$x = A\sin(2\pi ft)$$

$$x = A\sin\omega t$$
$$v = \frac{dx}{dt}$$
$$= A\omega\cos\omega t$$
$$|V_{\max}| = A\omega$$

$$x = A\sin\omega t$$
$$a = \frac{d^2x}{dt^2}$$
$$= -A\omega^2\sin\omega t$$
$$|a_{\max}| = A\omega^2$$

4.2 SIMPLE HARMONIC MOTION (SHM)

COS GRAPH



$$x = A\cos\theta$$

$$x = A\cos\omega t$$

$$x = A\cos\left(\frac{2\pi}{T}\right)t$$

$$x = A\cos(2\pi ft)$$

$$x = A\cos\omega t$$

$$v = \frac{dx}{dt}$$

$$= -A\omega\sin\omega t$$

$$|V_{\max}| = A\omega$$

$$x = A\cos\omega t$$

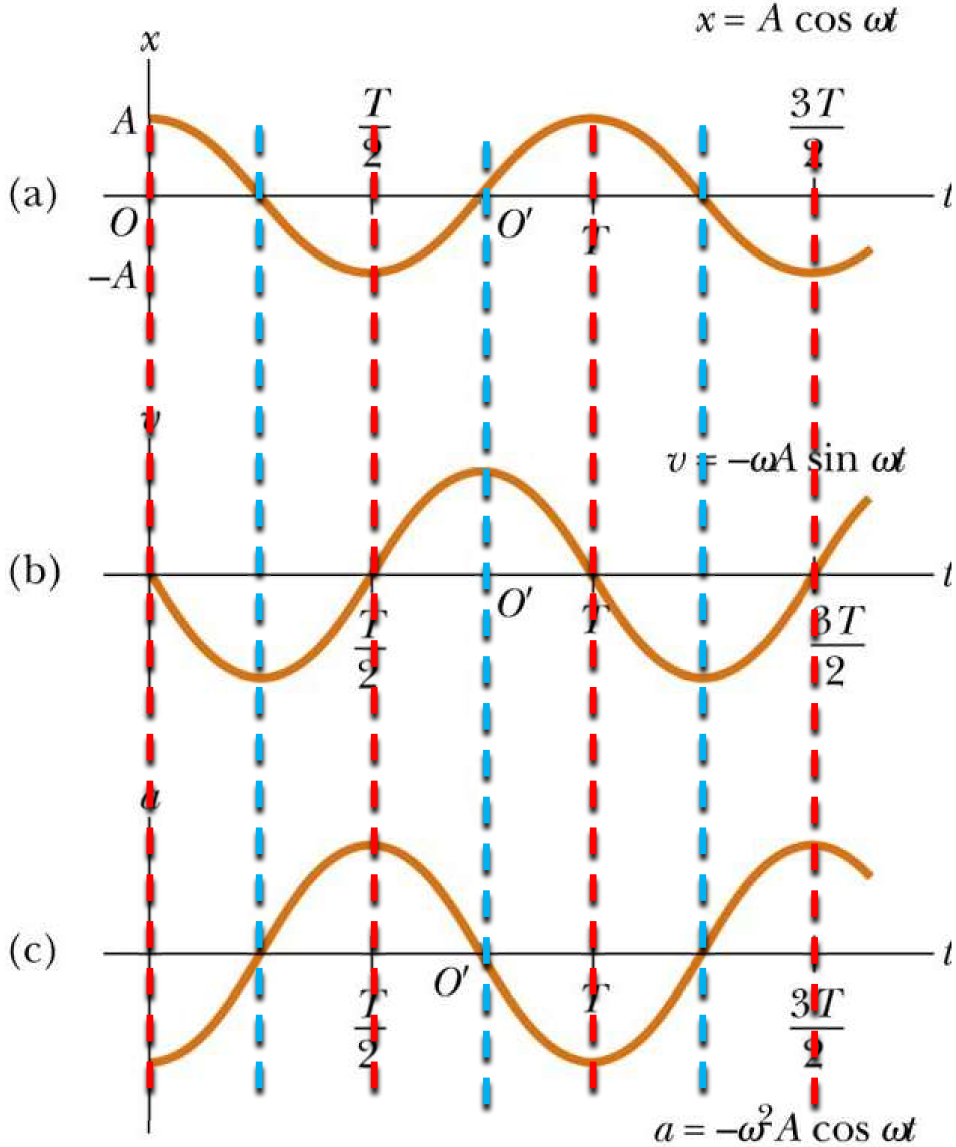
$$a = \frac{d^2x}{dt^2}$$

$$= -A\omega^2\cos\omega t$$

$$|a_{\max}| = A\omega^2$$

4.2 SIMPLE HARMONIC MOTION (SHM)

- Compare with displacement, velocity and acceleration graph, What can you tell me ?
- When x is a maximum or minimum, velocity is zero (red line)
- When x is zero, the magnitude of velocity is a maximum
- When x is a maximum in the positive direction, the acceleration is a maximum in the negative direction



4.2 SIMPLE HARMONIC MOTION (SHM)

- Period and Frequency from Circular Motion (revision)
  - Frequency, f
    - Units are cycles/second or Hertz, Hz

$$f = \frac{1}{T}$$

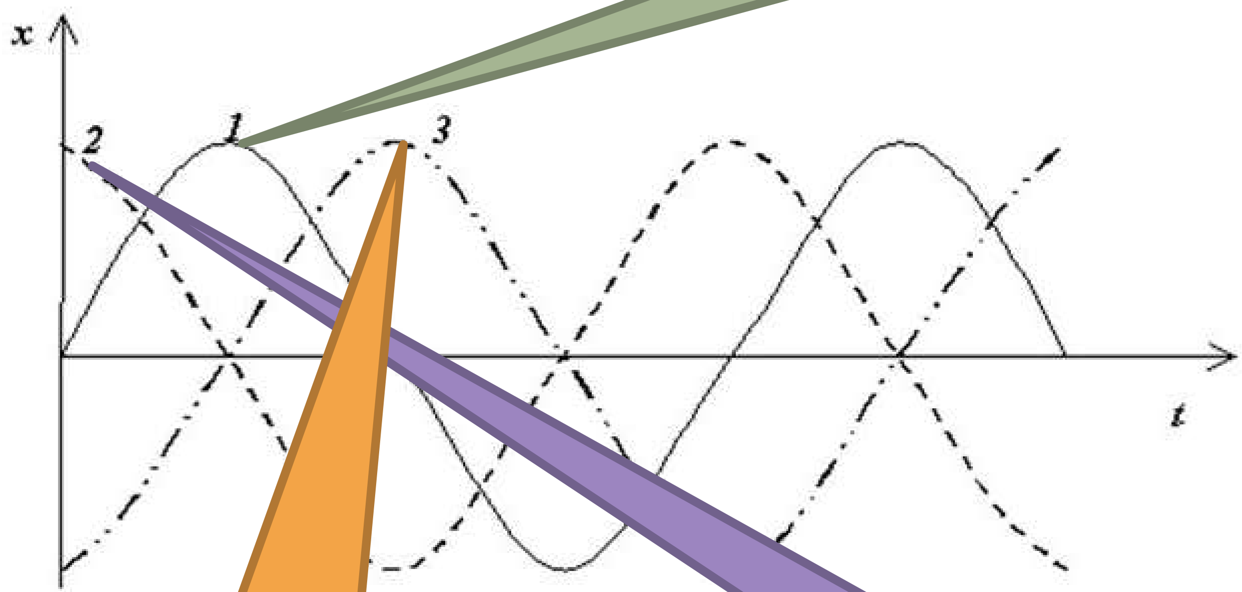
  - The angular frequency, ω

$$\omega = \frac{2\pi}{T} = 2\pi f$$

4.2 SIMPLE HARMONIC MOTION (SHM)

- Shift of the phase

$$x_1 = A\sin(\omega t)$$



$$x_3 = A\sin(\omega t - \frac{\pi}{2})$$

$$x_2 = A\sin(\omega t + \frac{\pi}{2})$$

4.2 SIMPLE HARMONIC MOTION (SHM)

- Shift of the phase
  - If the graph shift to right hand side then the formula must be

$$x = A\sin(\omega t - PhaseShift)$$

  - If the graph shift to left hand side then the formula must be

$$x = A\sin(\omega t + PhaseShift)$$

4.2 SIMPLE HARMONIC MOTION (SHM)

- Example
- An object oscillates with simple harmonic motion along the x-axis. Its displacement from the origin varies with time according to the equation:

$$x(t) = 4.00\text{m}\ \cos(\pi t)$$ t -> seconds

a) Determine the amplitude, frequency, and period of the motion
b) Calculate the velocity and acceleration of the object at any time , t .
c) Determine the position, velocity and acceleration of the object at t=1.0s
d) Determine the maximum speed and acceleration of the object.
e) Find the displacement of the body between t=0 and t=1.0s
f) What is the phase of the motion at t=2.0s?

- Solution (a)
  - Determine the amplitude, frequency, and period of the motion

$$x(t) = \underline{4.00}\,\text{m}\cos\left(\underline{\pi}\,t\right)$$

$$x(t)=\underline{A}\cos(\underline{\omega}t)$$

$$Amplitude = 4m$$

$$\omega = 2\pi f$$

$$2\pi f = \pi$$

$$f = \frac{1}{2}$$

$$= 0.5Hz$$

$$T = \frac{1}{f}$$

$$= \frac{1}{0.5}$$

$$= 2.0s$$

4.2 SIMPLE HARMONIC MOTION (SHM)

- **Solution (b)**
  - Calculate the velocity and acceleration of the object at any time ,t .

Velocity :

$$v = \frac{dx}{dt}$$

$$= -4\pi \sin(\pi t)$$

$$a = \frac{dv}{dt}$$

$$= \frac{d^2 x}{dt^2}$$

$$= -4\pi^2 \cos(\pi t)$$

- **Solution (c)**
  - Determine the position, velocity and acceleration of the object at t=1.0s

$$x(t) = 4.00\cos(\pi t)$$
$$= 4.00\cos[(\pi)(1)]$$
$$= 4.00(-1)$$
$$= -4.00m$$

$$v(t) = -4\pi\ \sin(\pi t)$$
$$v(1) = -4\pi\ \sin\left[(\pi)(1)\right]$$
$$= 0ms^{-1}$$

$$a(t) = -4\pi^2\ \cos(\pi t)$$
$$a(1) = -4\pi^2\ \cos\left[(\pi)(1)\right]$$
$$= 39.5ms^{-2}$$

- **Solution (d)**
  - Determine the maximum speed and acceleration of the object.

$$
\begin{aligned}
v_{\max} &= \omega A \\
&= (\pi)(4) \\
&= 12.6 ms^{-1}
\end{aligned}
\qquad\qquad
\begin{aligned}
a_{\max} &= \omega^2 A \\
&= \pi^2 4 \\
&= 39.5 ms^{-2}
\end{aligned}
$$

- Solution (e)
  - Find the displacement of the body between t=0 and t=1.0s

$$x_i(t) = A\cos(\pi t)$$

$$x_i(0) = A\cos(0)$$

$$= 4m$$

$$x_f(t) = A\cos(\pi t)$$

$$x_f(1) = A\cos(\pi)$$

$$= -4m$$

$$Displacement = x_f - x_i$$

$$= -4.0m - 4.0m$$

$$= -8.0m$$

- Solution (f)
  - What is the phase of the motion at t=2.0s?

$$
\begin{aligned}
Phase &= \pi t \\
&= \pi(2.0s) \\
&= 2\pi \ rad
\end{aligned}
$$

4.3 SIMPLE HARMONIC MOTION (SPRING WITH MASS)

- Extra information between Hooke's law and HMS.
  - In Hooke's law

$$F = -kx$$

  - Newton’s second law

$$F = ma$$

$$ma = -kx$$

$$a = \frac{-kx}{m}$$

$$a = \left(\frac{k}{m}\right)(-x)$$

This term is constant !!!

$$a \propto -x$$

4.3 SIMPLE HARMONIC MOTION (SPRING WITH MASS)

$$x = A\cos\omega t$$

$$a = \frac{d^2x}{dt^2}$$

$$= -A\omega^2\cos\omega t$$

$$a_{\max} = -A\omega^2\cos 0$$

$$= -A\omega^2$$

$$= -x\omega^2$$

- Compare

$$a = \left(\frac{k}{m}\right)(-x)$$

$$a_{\max} = -x\omega^2$$

$$= \omega^2(-x)$$

$$\frac{k}{m} = \omega^2$$

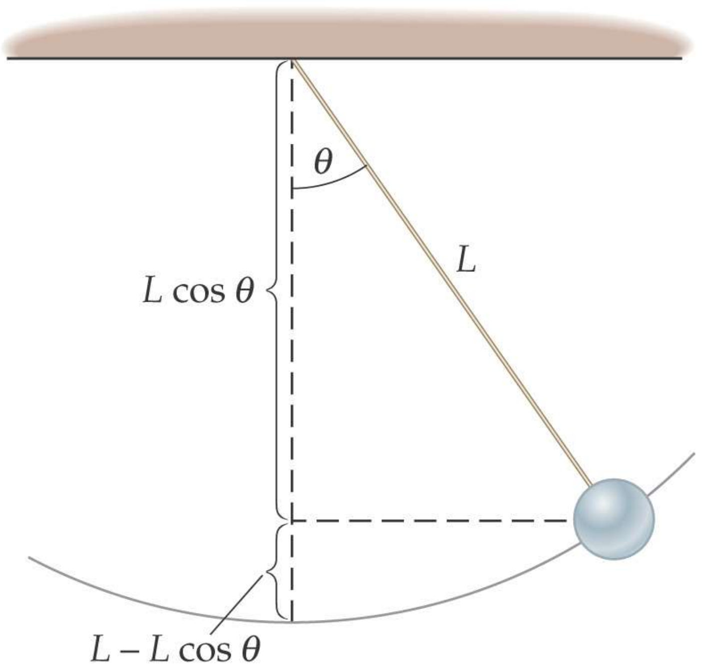
$$\omega = \sqrt{\frac{k}{m}}$$

4.3 SIMPLE HARMONIC MOTION (SPRING WITH MASS)

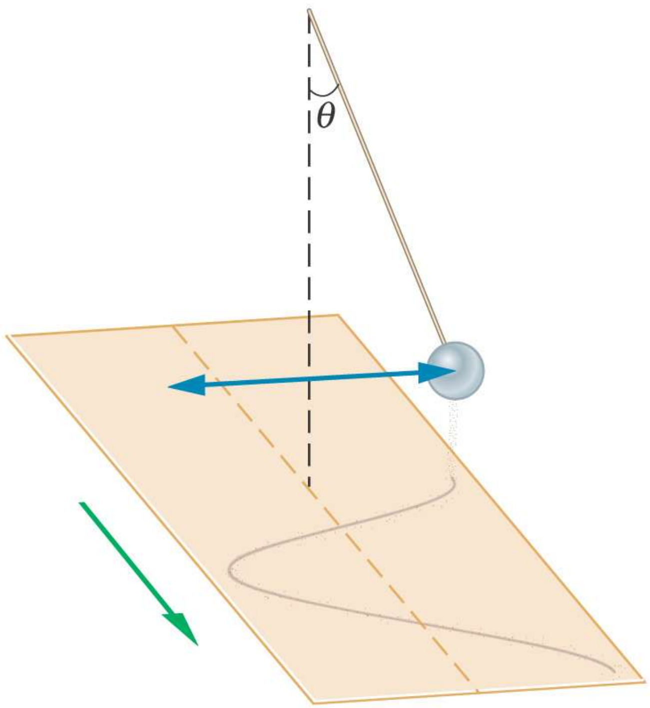
- Speed of the object at any displacement, *x*

$$\boxed{v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)}}$$

4.3 SIMPLE HARMONIC MOTION (SIMPLE PENDULUM)

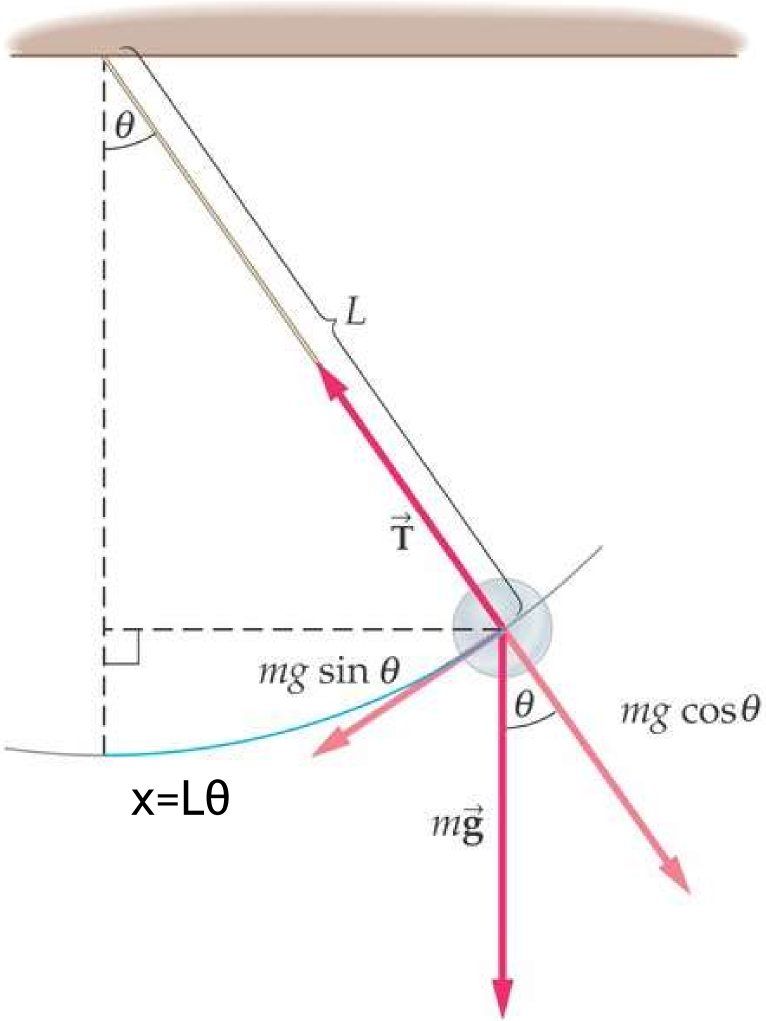


(a)



(b)

4.3 SIMPLE HARMONIC MOTION (SIMPLE PENDULUM)



- The displacement of the pendulum along the arc is

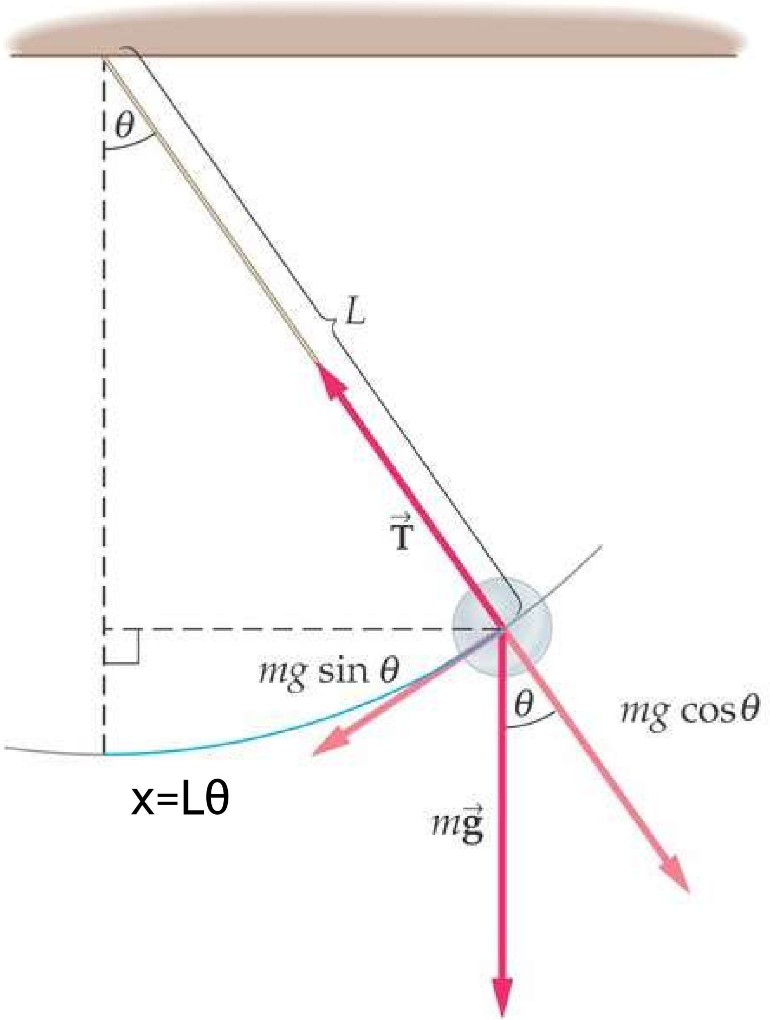
  $x = L\theta$

  Where,

  L is a constant (fixed length of the cord)

  θ is the angle that cord make with vertical (radian)

4.3 SIMPLE HARMONIC MOTION (SIMPLE PENDULUM)

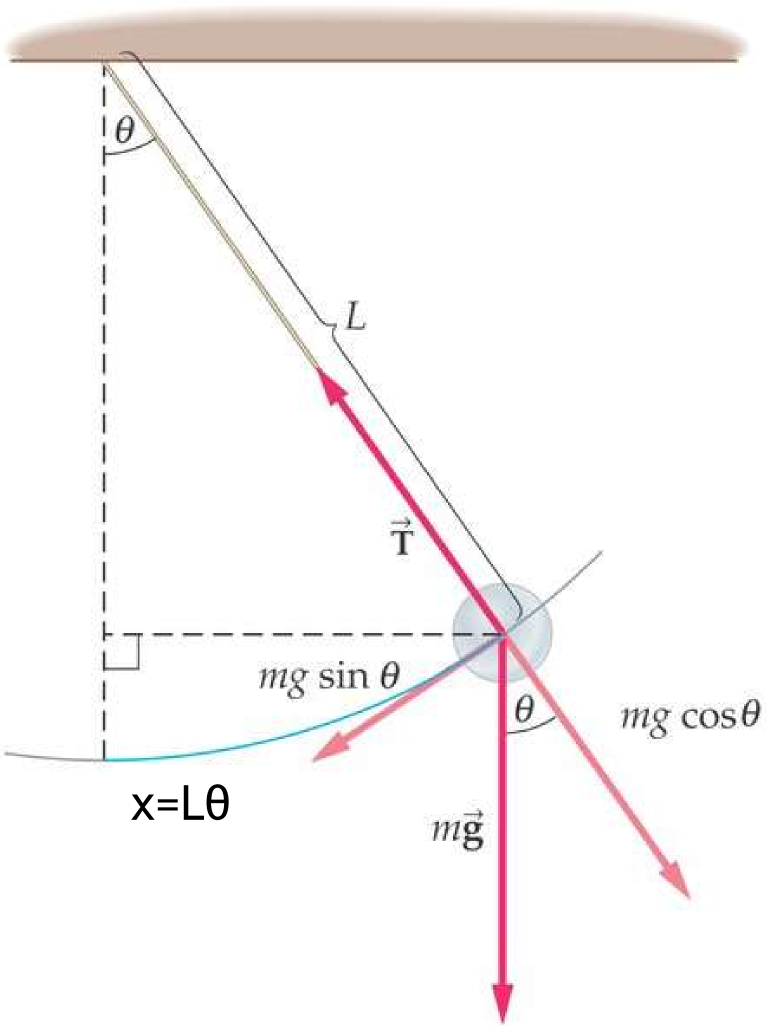


When the bob is displaced by an angle θ from the vertical, the **restoring force** is the tangential component of the weight,

$$F = -mg\sin\theta$$

The minus sign here mean that the force is in the direction opposite to the angular displacement θ.

4.3 SIMPLE HARMONIC MOTION (SIMPLE PENDULUM)



$$F = -mg\sin\theta$$

If **F is proportional to the sin θ,**

- then the motion is not a SHM.

If we assume that θ is a very small value (In general, θ < 15°),

- then sin θ is very nearly equal to θ.
- This mean that **F is proportional to the θ,** then this is a SHM

$$x = L\theta \qquad F = -mg\sin\theta$$

$$\theta = \frac{x}{L} \qquad F \approx -mg\theta$$

$$F \approx -\frac{mg}{L}x$$

4.3 SIMPLE HARMONIC MOTION (SIMPLE PENDULUM)

- Thus, we can summary that for small displacements, the motion is essentially simple Harmonic.
- Therefore the derived question fits Hooke's Law, whereby

$$F \approx \left(\frac{mg}{L}\right)(-x)$$

$$F = (k) - x$$

$$\text{So, } k = \frac{mg}{L}$$

4.3 SIMPLE HARMONIC MOTION (SIMPLE PENDULUM)

- Since $k = \dfrac{mg}{L}$
  - So,

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{mg}{Lm}} = \sqrt{\frac{g}{L}}$$

$$\omega = 2\pi f$$

$$2\pi f = \sqrt{\frac{g}{L}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

$$\omega = \frac{2\pi}{T}$$

$$\frac{2\pi}{T} = \sqrt{\frac{g}{L}}$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

4.4 ENERGY CONSERVATION

- Let say you have a mass-spring system

$$\boxed{x = A\cos(\omega t)}$$

- In frictionless condition,
  - Elastic Potential Energy (Spring),
    - $PE = \frac{1}{2}kx^2$
  - Kinetic Energy (mass attached to spring)
    - $KE = \frac{1}{2}mv^2$

4.4 ENERGY CONSERVATION

$$x = A\cos(\omega t)$$
$$= A\sin(\omega t)$$
$$v = -\omega A\sin(\omega t)$$
$$\omega^2 = \frac{k}{m}$$

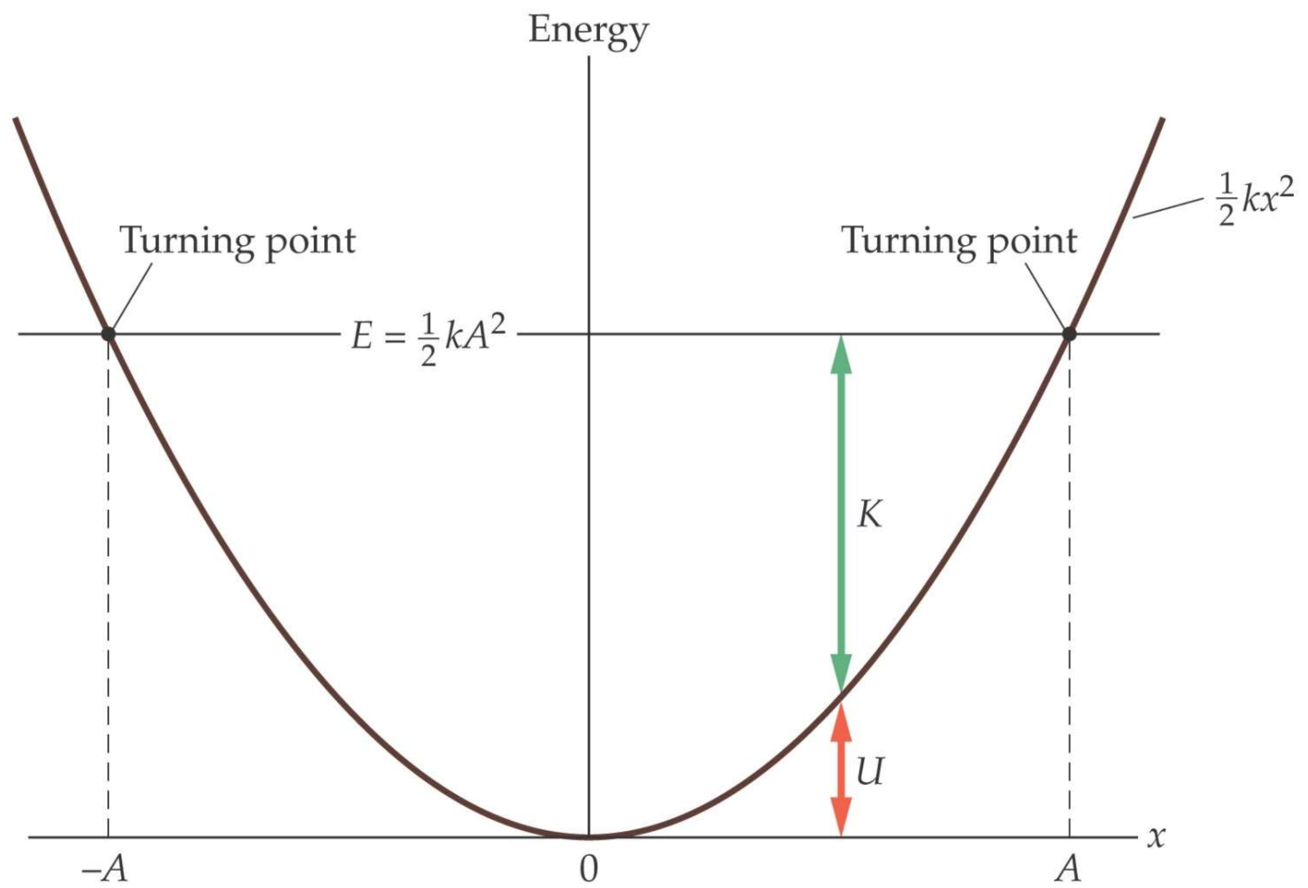
$$PE = \frac{1}{2}kx^2$$
$$= \frac{1}{2}k\left[A\cos(\omega t)\right]^2$$
$$= \frac{1}{2}kA^2\cos^2(\omega t)$$

$$KE = \frac{1}{2}mv^2$$
$$= \frac{1}{2}\left(\frac{k}{\omega^2}\right)\left(\omega^2 A^2 \sin^2(\omega t)\right)$$
$$= \frac{1}{2}KA^2\sin^2(\omega t)$$

Total Energy:

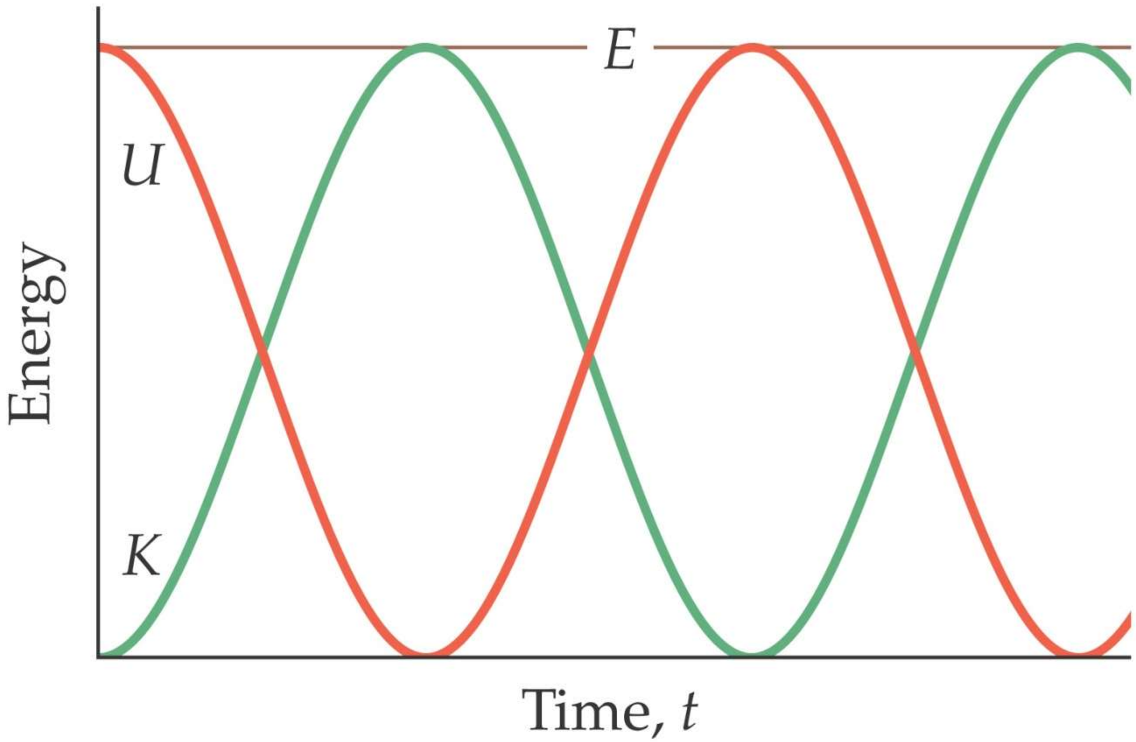
$$E = PE + KE$$
$$= \frac{1}{2}KA^2\cos^2(\omega t) + \frac{1}{2}KA^2\sin^2(\omega t)$$
$$= \frac{1}{2}KA^2\left[\cos^2(\omega t) + \sin^2(\omega t)\right]$$
$$= \frac{1}{2}KA^2[1]$$
$$= \frac{1}{2}KA^2 \quad \text{(when v=0)}$$
$$= \frac{1}{2}mv_{\max}{}^2 \quad \text{(when x=0)}$$

4.4 ENERGY CONSERVATION



The parabola represents the potential energy, U, of the spring. The horizontal line shows the total energy, E, of the system, which is constant, and the distance from the parabola to the horizontal line is the kinetic energy, K. Note that the kinetic energy vanishes at the turning points, x = A and x = -A. At these points the energy is purely potential, and thus the total energy of the system is

4.4 ENERGY CONSERVATION



The sum of the potential energy, U, and the kinetic energy, K, is equal to the (constant) total energy E at all times. Note that when one energy (U or K) has its maximum value, the other energy is zero.

4.5 DAMPED OSCILLATION

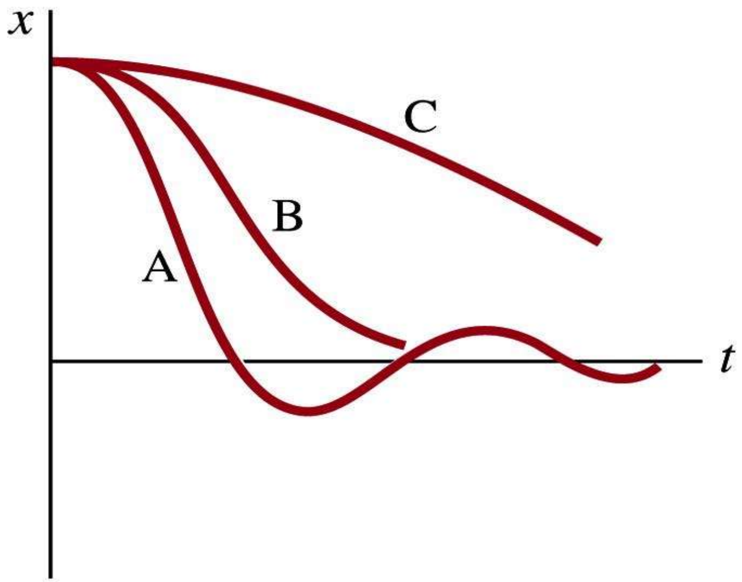
- In real time, oscillations usually damped , dies out gradually because of friction force

- The amplitude for oscillating object slowly decreases in time until the oscillation stop.

- The damping normally due to air resistance and internal resistance (within the system)

- Friction reduces the total energy of the system and the oscillation is said to be *damped*

4.5 DAMPED OSCILLATION



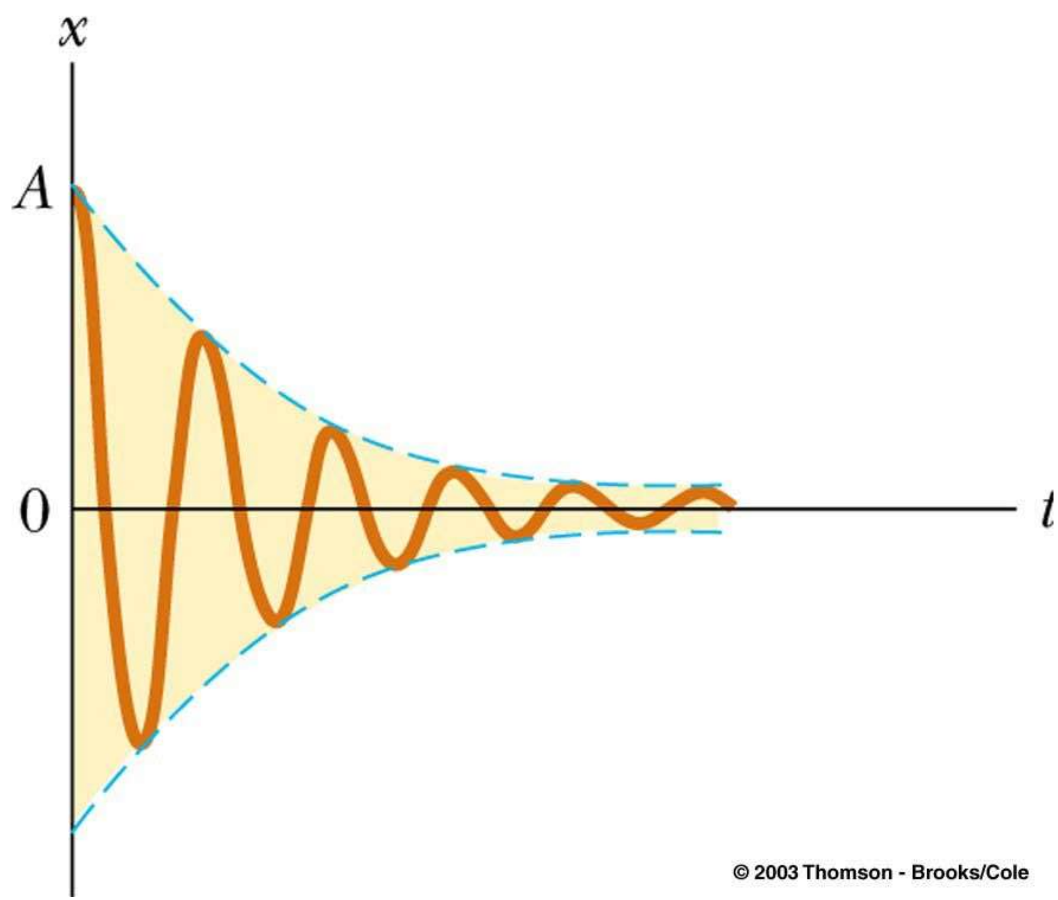
A. An under-damped oscillations, the position continues to oscillate as a function of time, but the amplitude of oscillation decreases exponentially

b) Critically damped, equilibrium is reached the quickest (no oscillations occur)

c) <u>Overdamped</u> : The damping is large that it takes a long time to reach equilibrium.

4.5 DAMPED OSCILLATION

- Damped motion varies depending on the fluid used
  - With a low viscosity fluid, the vibrating motion is preserved, but the amplitude of vibration decreases in time and the motion ultimately ceases
    - This is known as *under-damped* oscillation



4.5 DAMPED OSCILLATION

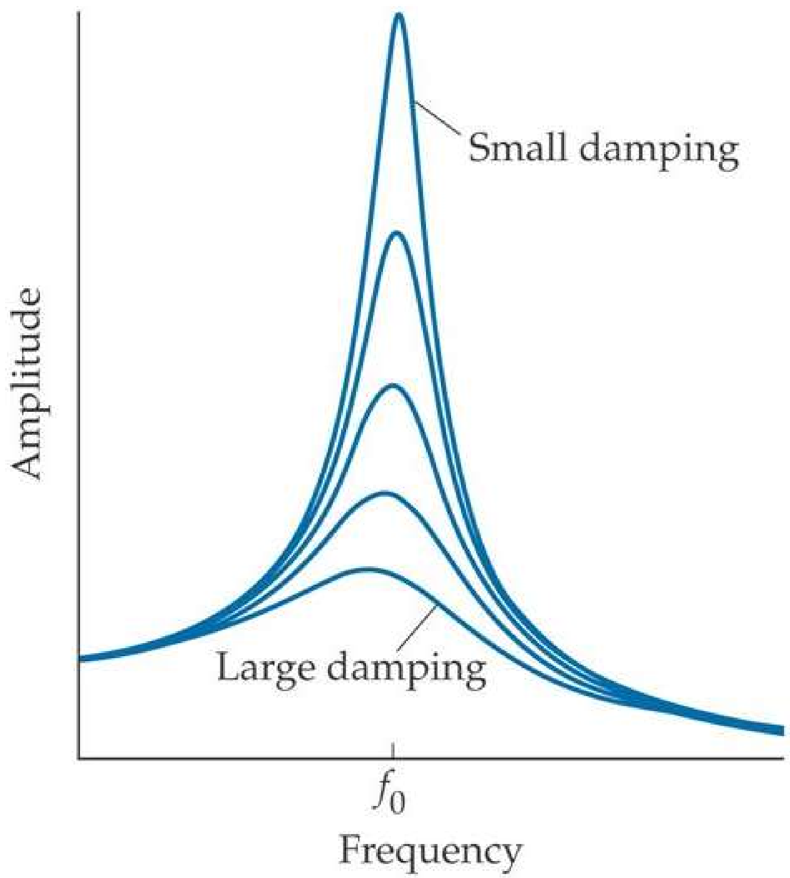
- With a higher viscosity, the object returns rapidly to equilibrium after it is released and does not oscillate
  - The system is said to be *critically damped*
- With an even higher viscosity, the piston returns to equilibrium without passing through the equilibrium position, but the time required is longer
  - This is said to be *over damped*

4.5 DAMPED OSCILLATION

- An oscillating system has its own natural frequency of oscillator and if oscillations are reinforced at this frequency, **resonance occur!!!**
- Resulting in oscillations of very large amplitude.
- Push at natural frequency displacement and velocity amplitudes will increase to large value.
- Push at other frequency (higher / lower) displacement and velocity amplitude will be smaller.
- Frequency of driving force = frequency of natural frequency , ***resonance occur !!***

4.5 DAMPED OSCILLATION

When the damping is small, the amplitude of oscillation can become very large for frequencies close to the natural frequency, f0. When the damping is large, the amplitude has only a low, broad peak near the natural frequency.




THANK
YOU
THANK