MULTIMEDIA UNIVERSITY

STUDENT ID NO

SEAT NO

VENUE:_____

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2018/2019

PMT0301 – MATHEMATICS III

(All sections/ Groups)

7 MARCH 2019
9.00 a.m. – 11.00 a.m.
(2 Hours)

Question	Marks
1	/10
2	/10
3	/10
4	/10
Total	/40

INSTRUCTIONS TO STUDENTS

1. This question paper consists of **NINE** printed pages excluding cover page and statistical table.

2. Answer **ALL** FOUR questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the **QUESTION BOOKLET**. All necessary working steps **MUST** be shown.

Question 1

(a) Find an equation of the plane that is perpendicular to the plane $x+3y+z=11$ and passes through the points $P_1(2, 3, 5)$ and $P_2(3, 2, 4)$.

[3 marks]

The normal vector to Plane $x+3y+z=5$ is $n_1=\langle 1,3,1\rangle$

$n_2=\overrightarrow{P_1P_2}=\langle 1,-1,-1\rangle$

$$n_1 \text{x}\, n_2=\begin{vmatrix} i & j & k \\ 1 & 3 & 1 \\ 1 & -1 & -1 \end{vmatrix}=\langle -2,2,-4\rangle$$

Plane of interest is perpendicular to $n_1 \text{x}\, n_2$

Equation of Plane passing through $P_1(2,3,5)$ and perpendicular to $n_1 \text{x}\, n_2$ is:

$-2(x-2)+2(y-3)-4(z-5)=0$

$-2x+2y-4z+18=0$

$2x-2y+4z=18$

(b) Find the parametric equations of the line passing through the point $(3,-1,1)$ and perpendicular to both $i+j$ and $j+k$.

[2 marks]

point $(3,-1,1)$

A parallel vector with the line, $\langle a,b.c\rangle=(i+j)\,\text{x}\,(j+k)$

$$=\begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} \quad =\langle 1,-1,1\rangle$$

The line in parametric equations

$x=x_a+at \Leftrightarrow x=3+t$

$y=y_a+bt \Leftrightarrow y=-1-t$

$z=z_a+ct \Leftrightarrow z=1+t$

Continue…

(c) Given the following system of linear equations:

$$x+2y-z=-2$$
$$x+z=0$$
$$2x-y-z=-3$$

Find the inverse matrix by using its adjoint, and hence solve the system of linear equations by using inverse method.

[5 marks]

$$A=\begin{bmatrix}1 & 2 & -1\\ 1 & 0 & 1\\ 2 & -1 & -1\end{bmatrix} \qquad B=\begin{bmatrix}-2\\ 0\\ -3\end{bmatrix}$$

$$\text{Cofactor}=\begin{bmatrix}+\begin{vmatrix}0 & 1\\ -1 & -1\end{vmatrix} & -\begin{vmatrix}1 & 1\\ 2 & -1\end{vmatrix} & +\begin{vmatrix}1 & 0\\ 2 & -1\end{vmatrix}\\ -\begin{vmatrix}2 & -1\\ -1 & -1\end{vmatrix} & +\begin{vmatrix}1 & -1\\ 2 & -1\end{vmatrix} & -\begin{vmatrix}1 & 2\\ 2 & -1\end{vmatrix}\\ +\begin{vmatrix}2 & -1\\ 0 & 1\end{vmatrix} & -\begin{vmatrix}1 & -1\\ 1 & 1\end{vmatrix} & +\begin{vmatrix}1 & 2\\ 1 & 0\end{vmatrix}\end{bmatrix}=\begin{bmatrix}1 & 3 & -1\\ 3 & 1 & 5\\ 2 & -2 & -2\end{bmatrix}$$

1st column

$$|A|=1(1)-2(-3)-1(-1)=8$$

$$AdjA=\begin{bmatrix}1 & 3 & -1\\ 3 & 1 & 5\\ 2 & -2 & -2\end{bmatrix}^T \Longrightarrow A^{-1}=\frac{1}{|A|}AdjA=\frac{1}{8}\begin{bmatrix}1 & 3 & 2\\ 3 & 1 & -2\\ -1 & 5 & -2\end{bmatrix}$$

$$x=\begin{bmatrix}x\\ y\\ x\end{bmatrix}=A^{-1}B \qquad =\frac{1}{8}\begin{bmatrix}1 & 3 & 2\\ 3 & 1 & -2\\ -1 & 5 & -2\end{bmatrix}\begin{bmatrix}-2\\ 0\\ -3\end{bmatrix}$$

$$=\frac{1}{8}\begin{bmatrix}-2+0-6\\ -6+0+6\\ 2+0+6\end{bmatrix}$$

$$=\frac{1}{8}\begin{bmatrix}-8\\ 0\\ 8\end{bmatrix}=\begin{bmatrix}-1\\ 0\\ 1\end{bmatrix}$$

x=-1, y=0, z=1

Continue…

Question 2

(a) Find the sum for $-3,-1,1,3,5...,169$. [2.5 marks]

a=-3, d=2

$$a_n = a + (n-1)d$$
$$169 = -3 + (n-1)2$$
$$169 = 2n - 5$$
$$n = \frac{174}{2} = 87$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$S_{87} = \frac{87}{2}\left[2(-3) + 86(2)\right] = 7221$$

(b) Express $0.1\overline{35}$ as a fraction. [2.5 marks]

$$0.1\overline{35} = 0.1 + 0.0\overline{35}$$
$$0.0\overline{35} = 0.035 + 0.00035 + \ldots$$

For $0.0\overline{35}$

$$a = 0.035$$
$$r = \frac{0.00035}{0.035} = 0.01$$
$$S_\infty = \frac{a}{1-r} = \frac{0.035}{1-0.01} = \frac{35}{990}$$

Therefore

$$0.1\overline{35} = 0.1 + 0.0\overline{35}$$
$$= \frac{1}{10} + \frac{35}{990} = \frac{134}{990}$$

Continue…

(c) The following data is the number of summons issued to traffic offenders at Jalan A for 12 consecutive days.

40 35 29 43 41 38 32 30 38 25 38 40

Calculate the mean, median and mode. [1.5 marks]

Mean, $\bar{x} = \frac{\sum x}{n} = \frac{429}{12} = 35.75$

Ranked data 25 29 30 32 35 38 38 38 40 40 41 43

$\therefore$ Median $= \frac{38+38}{2} = 38$

Mode = 38

(d) The following sample data have mode 4 and mean 6.5.

5 3 12 4 a (2a+b) 7 9 8 2

(i) Find the value of a and b if both must be positive integer.

(ii) Calculate the standard deviation [3.5 marks]

(i) Mean = 6.5, therefore

$$\frac{\sum x}{10} = 6.5$$

$$\frac{50+3a+b}{10} = 6.5$$

$$3a+b = 15 \text{----------} (1)$$

Mode =4, therefore

a=4 and b=3 or $2a+b=4$------(2)

solving equation (1) and (2) simultaneously yield a=11, b=-18

Since both must be positive integer, therefore a=4 and b=3

(ii)

$$S = \sqrt{\frac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1}}$$

$$= \sqrt{\frac{529 - \frac{(65)^2}{10}}{9}}$$

$$= \sqrt{11.8333}$$

$$= 3.44$$

Continue…

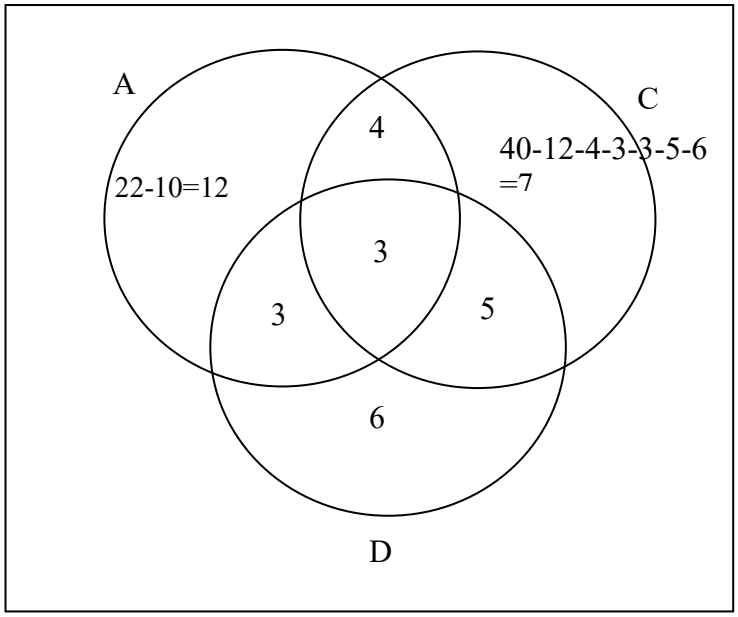
Question 3

40 FCI undergraduates have been interviewed about their choice of specialization elective subjects taken. It is found that

55% of the students took Artificial Intelligence subject
took Computer Security subject
17 took Data Mining subject
7 took Artificial Intelligence and Computer Security subjects
6 took Artificial Intelligence and Data Mining subjects
8 took Computer Security and Data Mining subjects
3 took all the three elective subjects

(a) Construct a complete Venn diagram to illustrate the above information and calculate the value of . Let A be the event of Artificial Intelligence, C be the event of Computer Security and D be the event of Data Mining.

[2 marks]



x

$= 40 - 12 - 3 - 6$ or $4 + 3 + 5 + 7$

$= 19$

Continue…

(b) Two students are selected randomly from the group who took more than one elective subjects. Find the probability that both students took Computer Security and Data Mining subjects. [1.5 marks]

$$\frac{8\times 7}{15\times 14}$$
$$=\frac{4}{15} \text{ or } 0.2667$$

(c) Find the number of ways the students can be seated in a row of five seats if three students who took only Data Mining subject and two students who took only Artificial Intelligence are chosen randomly whereby they are seated in alternate based on subjects chosen. [1.5marks]

$${}^{6}C_{3}\times {}^{12}C_{2}\times 3!\times 2!$$
$$=20\times 66\times 12$$
$$=15840$$

(d) Given a student who took Data Mining subject, find the probability that the student will take the Artificial Intelligence subject. [1 mark]

$$P\left(A|D\right)$$
$$=\frac{n\left(A\cap D\right)}{n\left(D\right)}$$
$$=\frac{6}{17} \text{ or } 0.3529$$

Continue…

(e) Are the events of A and D independent? Based on part 3(d), explain your answer by showing the proper working steps. [2 marks]

By using, $P(A|D) = P(A)$

RHS: (from part (d))

$P(A|D)$

$\frac{n(A \cap D)}{n(D)}$

$\frac{6}{17}$ or 0.3529

LHS:

$P(A) = \frac{22}{40}$ or 0.55

Therefore, $P(A|D) \neq P(A)$. As a conclusion, events A and D are **NOT** independent.

(f) Are the events of C and D mutually exclusive? Explain your answer by finding $P(C \cap D)$. [2 marks]

$P(C \cap D)$

$= \frac{n(C \cap D)}{n(S)}$

$= \frac{8}{40}$ or 0.2

Since $P(C \cap D) \neq 0$, therefore events of C and D are **NOT** mutually exclusive.

Continue…

Question 4

(a) Suppose a box contains eight red balls, seven yellow balls and five blue balls. Let X denote the number of red balls. If nine balls are selected at random with replacement, find the

(i) mean and standard deviation of the probability distribution of x.

[2.5 marks]

Given $n = 9$

$$p = \frac{8}{20} = 0.4 \qquad q = 1 - 0.4 = 0.6$$

$$\mu = np = 9 \times 0.4 = 3.6$$

$$\sigma = \sqrt{npq} = \sqrt{9 \times 0.4 \times 0.6} = \sqrt{2.16} = 1.4697$$

(ii) probability that five to seven red balls will be selected. Give your answer correct to 4 decimal places. [2 marks]

$$P(5 \le x \le 7)$$
$$= P(x = 5) + P(x = 6) + P(x = 7)$$
$$= \binom{9}{5}(0.4)^5(0.6)^4 + \binom{9}{6}(0.4)^6(0.6)^3 + \binom{9}{7}(0.4)^7(0.6)^2$$
$$= 0.1672 + 0.0743 + 0.0212$$
$$= 0.2628$$

Continue…

(b) A toll free phone number is available from 9a.m. to 9p.m. for customers to register complaints about a product purchased from a certain company. Past history indicates that an average of 0.5 calls are received per minute. What is the probability that, during a 3-minute period, between two and six phone calls will be received? Give your answer correct to 4 decimal places.

[2.5 marks]

$\mu = 1.5$ calls per 3 minutes

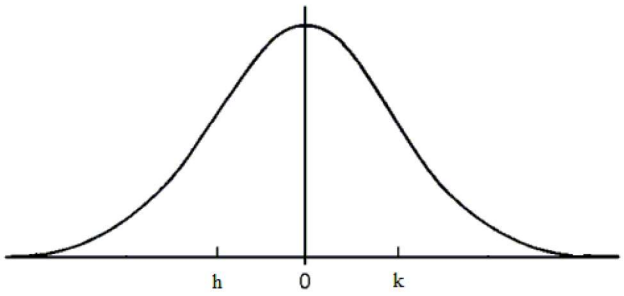
$$P(2 < x < 6)$$
$$= P(x = 3) + P(x = 4) + P(x = 5)$$
$$= \frac{1.5^3 e^{-1.5}}{3!} + \frac{1.5^4 e^{-1.5}}{4!} + \frac{1.5^5 e^{-1.5}}{5!}$$
$$= 0.1255 + 0.0471 + 0.0141$$
$$= 0.1867$$

(c) A statistical analysis of 2500 long distance telephone calls made by a company indicates that the length of these calls is normally distributed with a mean of 180 seconds and a variance of 625 seconds. If 15% of all the calls lasted less than seconds, find the value of . [3 marks]

$$P(z < h) = 0.15$$
$$P(z > k) = 0.15$$
$$P(z < k) = 1 - 0.15 = 0.85$$

From table, $k = 1.04$

$$\therefore h = -1.04$$
$$x = \mu + z\sigma$$
$$= 180 + (-1.04)(\sqrt{625})$$
$$= 154 \text{ sec}$$



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