1) Solve for the following using these given matrices:

$$U = \begin{bmatrix} 3 & 1 & 4 \\ -2 & 0 & 1 \\ 1 & 2 & 2 \end{bmatrix} \quad W = \begin{bmatrix} 1 & 0 & 2 \\ -3 & 1 & 1 \\ 2 & -4 & 1 \end{bmatrix}$$

a) $2U$
b) $U + W$
c) $U - 3W$
d) $UW$
e) $WU$
f) $(WU)^T$

2) If we introduce matrix $V = \begin{bmatrix} 4 & -2 \\ 6 & -4 \\ 8 & -6 \end{bmatrix}$ , find

a) $(UW)V$
b) $V^T(WU)$
c) Is $UVW$ allowed?

3) For each of the following matrices, calculate $|A|$, $adj(A)$ and $A^{-1}$

a) $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$
b) $A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 1 & 1 \\ -2 & 2 & -1 \end{bmatrix}$

4) Solve the system of linear equations below by using
   (i) Gaussian Elimination
   (ii) Gauss-Jordan Elimination

a) Consistent and independent system (Note: One solution)

$$x + y - 2z = 3$$
$$2x + 5z = 11$$
$$2x + 3y = 12$$

b) Consistent and dependent system (Note: Infinite number of solutions)

$$x + 2y + z = 0$$
$$3x + 2y - z = 4$$
$$-x + 2y + 3z = -4$$

(Hint: Let $z = c$ )

c) Inconsistent system (Note: No solution)

$$2x - 2y + 6z = 10$$
$$3x - 5y + 12z = -6$$
$$4x - 4y + 12z = 18$$

5) Given the following system of linear equations:

$$\begin{aligned} x - 2y + z &= 4 \\ x - y - 3z &= -5 \\ x + 2y &= -2 \end{aligned}$$

Find the inverse matrix by using its adjoint, and hence solve the system of linear equations by using inverse method.

Answer:

1)

a) $\begin{bmatrix} 6 & 2 & 8 \\ -4 & 0 & 2 \\ 2 & 4 & 4 \end{bmatrix}$

b) $\begin{bmatrix} 4 & 1 & 6 \\ -5 & 1 & 2 \\ 3 & -2 & 3 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 1 & -2 \\ 7 & -3 & -2 \\ -5 & 10 & -1 \end{bmatrix}$

d) $\begin{bmatrix} 8 & -15 & 11 \\ 0 & -4 & -3 \\ -1 & -6 & 6 \end{bmatrix}$

e) $\begin{bmatrix} 5 & 5 & 8 \\ -10 & -1 & -9 \\ 15 & 4 & 6 \end{bmatrix}$

f) $\begin{bmatrix} 5 & -10 & 15 \\ 5 & -1 & 4 \\ 8 & -9 & 6 \end{bmatrix}$

2)

a) $\begin{bmatrix} 30 & -22 \\ -48 & 34 \\ 8 & -10 \end{bmatrix}$

b) $\begin{bmatrix} 114 & -118 & 132 \\ -78 & 78 & -82 \end{bmatrix}$

c) No

3)

a) $|A| = -7,\ adj(A) = \begin{bmatrix} -1 & -2 \\ -3 & 1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} \frac{1}{7} & \frac{2}{7} \\ \frac{3}{7} & -\frac{1}{7} \end{bmatrix}$

b) $|A| = 3,\ adj(A) = \begin{bmatrix} -3 & 5 & 2 \\ 0 & 1 & 1 \\ 6 & -8 & -5 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} -1 & \frac{5}{3} & \frac{2}{3} \\ 0 & \frac{1}{3} & \frac{1}{3} \\ 2 & -\frac{8}{3} & -\frac{5}{3} \end{bmatrix}$

4)

a) $x = 3,\ y = 2,\ z = 1$

b) $x = 2 + c,\ y = -1 - c,\ z = c$

c) No solution

5) $x = 0,\quad y = -1,\quad z = 2$