23/9/20

~~Vectors~~ @ Notes

1. 2 and 3 Dimensions
2. Unit Vectors
3. Dot Product
4. Cross Product
5. Equations of Lines $\}$ in 3D
6. Equations of Planes.
7. Angle Between Planes – 3D

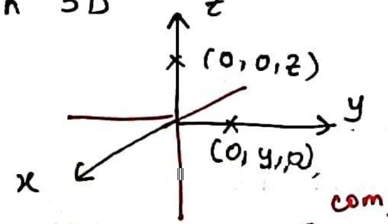
$V(V_1, V_2) \Rightarrow$ a point

$V\langle V_1, V_2\rangle \Rightarrow$ a vector.

angle bracket

component

In 3D



component form

$\overrightarrow{OA} : \langle x_1, y_1, z_1\rangle = \underbrace{x_1\underline{i} + y_1\underline{j} + z_1\underline{k}}_{\text{standard basis vectors}}$.

$\overrightarrow{OB} = \langle x_2, y_2, z_2\rangle$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= \overrightarrow{OB} - \overrightarrow{OA}$

$$\boxed{\overrightarrow{AB} = \langle x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1\rangle}$$

$\overrightarrow{AB} = \langle x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1\rangle$

② Norm of $\underline{v}$ = Length of $\underline{v}$ = $|\underline{v}|$

$|\underline{v}| = \sqrt{x^2 + y^2 + z^2}$ (mcm P.T.).

$|AB| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

③ Unit vector = $\hat{\underline{v}}$

$\hat{\underline{v}} = \dfrac{\underline{v}}{|\underline{v}|}$ $\Rightarrow$ a vector has magnitude $\underline{1}$.

④ DOT PRODUCT / SCALAR PRODUCT

$\Rightarrow$ a number / scalar. $\rightarrow$ 2 ways / method

$\underline{u} = \langle u_1, u_2, u_3\rangle$

$\underline{v} = \langle v_1, v_2, v_3\rangle$

method 1

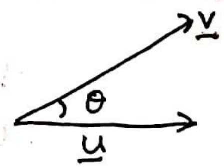
$$\boxed{\underline{u} \cdot \underline{v} = u_1v_1 + u_2v_2 + u_3v_3}$$

Also can be defined as

method 2.

$$\boxed{\underline{u} \cdot \underline{v} = |\underline{u}||\underline{v}| \cos\theta}$$

$\Rightarrow$ usually to get θ.



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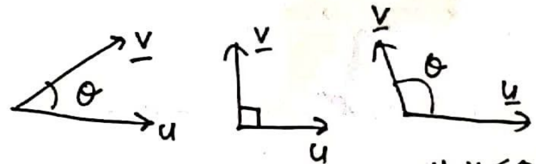
$\cos 0° = 1$

$\cos 90° = 0$

cos<0	cos>0

Notes: ve



$\underline{u} \cdot \underline{v} > 0$ positive (acute)

$\underline{u} \cdot \underline{v} = 0$ zero

$\underline{u} \cdot \underline{v} < 0$ negatif (obtuse)

To get θ.

i) Find $\underline{u} \cdot \underline{v}$

ii) Find $|\underline{u}|$ and $|\underline{v}|$

iii) Can get $\cos\theta$ & θ.

i) To show $\underline{u}$ and $\underline{v}$ are $\perp$

$\Rightarrow$ Show that $\underline{u} \cdot \underline{v} = 0$

ii) To show $\underline{u}$ and $\underline{v}$ are $\nearrow\nearrow$

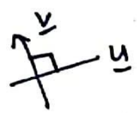
$\Rightarrow \underline{u} \cdot \underline{v} = |\underline{u}||\underline{v}|$

$\underline{u} \cdot \underline{v} = |\underline{u}||\underline{v}| \cos 0 = |\underline{u}||\underline{v}|$

$\underline{u} \cdot \underline{v} = |\underline{u}||\underline{v}|$ $\underline{u} \nearrow\nearrow \underline{v}$.

$\underline{u} \cdot \underline{v} = |\underline{u}||\underline{v}| \cos 90° = 0$

$\underline{u} \cdot \underline{v} = 0$



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CROSS / VECTOR PRODUCT

$\Rightarrow$ a vector

$\underline{u} = u_1\underline{i} + u_2\underline{j} + u_3\underline{k}$.

$\underline{v} = v_1\underline{i} + v_2\underline{j} + v_3\underline{k}$.

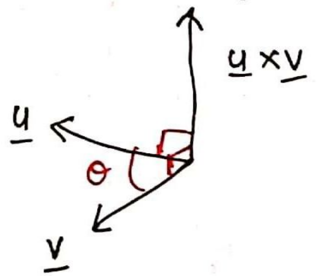
$$\underline{u}\times\underline{v} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

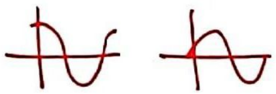
☆☆

$$\boxed{\begin{aligned} \underline{u}\times\underline{v} &= \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix}\underline{i} - \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix}\underline{j} + \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}\underline{k} \\ &\text{or} \\ \underline{u}\times\underline{v} &= \left\langle \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix}, -\begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix}, \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} \right\rangle \end{aligned}}$$

$\underline{u}\times\underline{v} \Rightarrow$ a vertor normal/ orthogonal to both $\underline{u}$ and $\underline{v}$

$\Rightarrow$ Right Hand Rule.

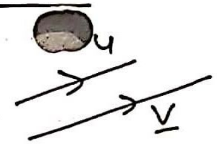




θ	0°	90°
cos θ	1	0
sin θ	0	1

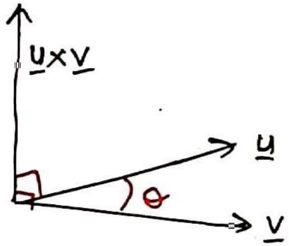
Remark

i)



$\left.\begin{matrix}\text{Parallel}\end{matrix}\right\} \Rightarrow \underline{u}\times\underline{v} = \underline{0}$

ii)



$\underline{u}$ is $\perp$ to $\underline{u}\times\underline{v}$

$\underline{v}$ is $\perp$ to $\underline{u}\times\underline{v}$

$\left.\begin{aligned} \underline{u}\cdot(\underline{u}\times\underline{v}) &= 0 \\ \underline{v}\cdot(\underline{u}\times\underline{v}) &= 0 \end{aligned}\right\}$ Dot Product.

2 ways to find angle between 2 vectors.

i) Scalar Product: (Dot) $\quad \underline{u}\cdot\underline{v} = |\underline{u}||\underline{v}|\cos\theta$

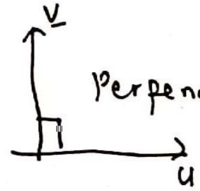
ii) Vector (Cross) Product: $\quad |\underline{u}\times\underline{v}| = |\underline{u}||\underline{v}|\sin\theta$.

$0^\circ \le \theta \le 180^\circ$

Note

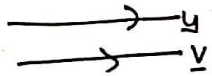
$\boxed{\cos 90^\circ = 0}$

$\sin 90^\circ = 1$



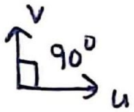
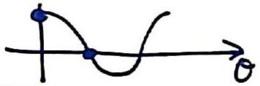
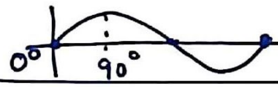
$\cos 0^\circ = 1$

$\boxed{\sin 0^\circ = 0}$



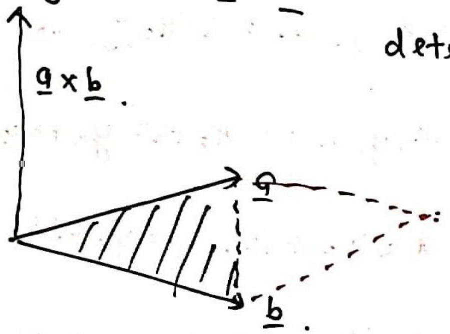
$\underline{u}\cdot\underline{v} = |\underline{u}||\underline{v}|\cos 90^\circ = 0 \Rightarrow \underline{u}\cdot\underline{v} = 0$

$|\underline{u}\times\underline{v}| = |\underline{u}||\underline{v}|\sin 0^\circ = \underline{0} \Rightarrow |\underline{u}\times\underline{v}| = \underline{0}$

	Dot or Cross Product	Parallel $\Rightarrow 0^\circ$ 	Perpendicular 
$y = \cos\theta$ $\cos 0^\circ = 1$ $\cos 90^\circ = 0$ 	DOT PRODUCT $\underline{u}\cdot\underline{v} = \lvert\underline{u}\rvert\lvert\underline{v}\rvert\cos\theta^\circ$	$\underline{u}\cdot\underline{v} = \lvert\underline{u}\rvert\lvert\underline{v}\rvert\underbrace{\cos 0^\circ}_{1}$ $\underline{u}\cdot\underline{v} = \lvert\underline{u}\rvert\lvert\underline{v}\rvert$	$\underline{u}\cdot\underline{v} = \lvert\underline{u}\rvert\lvert\underline{v}\rvert\cos 90^\circ$ $\underline{u}\cdot\underline{v} = \lvert\underline{u}\rvert\lvert\underline{v}\rvert(0) = 0$
$y = \sin\theta$ $\sin 0^\circ = 0$ $\sin 90^\circ = 1$ 	CROSS PRODUCT $\lvert\underline{u}\times\underline{v}\rvert = \lvert\underline{u}\rvert\lvert\underline{v}\rvert\sin\theta^\circ$	$\lvert\underline{u}\times\underline{v}\rvert = \lvert\underline{u}\rvert\lvert\underline{v}\rvert\sin 0^\circ$ $\lvert\underline{u}\times\underline{v}\rvert = 0$	$\lvert\underline{u}\times\underline{v}\rvert = \lvert\underline{u}\rvert\lvert\underline{v}\rvert\sin 90^\circ$ $\lvert\underline{u}\times\underline{v}\rvert = \lvert\underline{u}\rvert\lvert\underline{v}\rvert$

Note.

The length of $\underline{a} \times \underline{b}$ = area of parallelogram determine by $\underline{a}$ and $\underline{b}$



$|\underline{a} \times \underline{b}|$ = area of parallelogram.

Area of $\triangle = \frac{1}{2}|\underline{a} \times \underline{b}|$

Steps to find area of $\triangle$.

i) Find $\underline{a} \times \underline{b}$

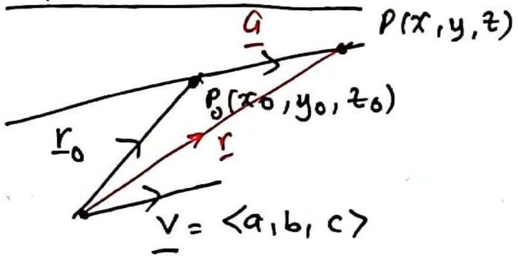
ii) Find $|\underline{a} \times \underline{b}|$

iii) Area of $\triangle = \frac{1}{2}|\underline{a} \times \underline{b}|$

$y = mx + c$

Equation of Lines

→ Vector Equation
→ Parametric Equation
→ Symmetric Equation.



Note
$P_0 = (x_0, y_0, z_0) \Rightarrow$ point
$\underline{r}_0 = \langle x_0, y_0, z_0 \rangle$
= vector.

$P(x, y, z) \Rightarrow$ any points lie on the line

$P_0(x_0, y_0, z_0) \Rightarrow$ a specific point

$\underline{V} = \langle a, b, c \rangle \Rightarrow$ parallel vector
$\Rightarrow$ direction of line L

} like gradient ⇒ find this first.

direction numbers.

$y = mx + c$

$\underline{r} = \underline{r}_0 + \underline{a}$

$\underline{r} = \underline{r}_0 + t\underline{v}$

$y = mx + c$

$\underline{r} = t\underline{V} + \underline{r}_0$

direction vector

$\langle x, y, z \rangle = \langle x_0, y_0, z_0 \rangle + t\langle a, b, c \rangle$

$\langle x, y, z \rangle = \langle x_0 + at, y_0 + t.b, z_0 + tc \rangle \Rightarrow$ vector eq.

$x = x_0 + at, \quad y = y_0 + bt, \quad z = z_0 + ct \Rightarrow$ parametric equation.

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c} \Rightarrow \text{symmetric equation.}$$

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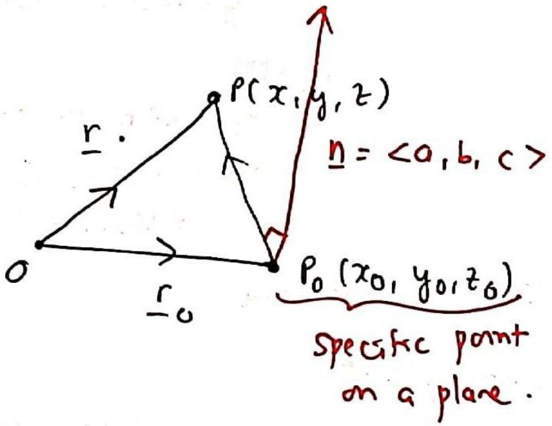
Equation of PLANES



O, P, P_0 lie on a plane.

$\underline{n}$ perpendicular to the plane

$$\overrightarrow{P_0P} = \overrightarrow{P_0O} + \overrightarrow{OP}$$

$$= \overrightarrow{OP} - \overrightarrow{OP_0}$$

$$= \langle x, y, z\rangle - \langle x_0, y_0, z_0\rangle$$

$$= \langle x - x_0, y - y_0, z - z_0\rangle$$

$$\underline{n} = \langle a, b, c\rangle$$

By dot product,

$$\underline{n} \cdot \overrightarrow{P_0P} = 0$$

$$\langle a, b, c\rangle \langle x - x_0, y - y_0, z - z_0\rangle = 0$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

expand and finally we get

$$\boxed{ax + by + cz + d = 0}$$

equation of a plane.

example

$$P_0 = (\overset{x_0}{3}, \overset{y_0}{-1}, \overset{z_0}{7})$$

$$\underline{n} = \langle \underset{a}{4}, \underset{b}{2}, \underset{c}{-5}\rangle$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$4(x-3) + 2(y+1) + (-5)(z-7) = 0$$

$$4x - 12 + 2y + 2 - 5z + 35 = 0$$

$$\boxed{4x + 2y - 5z + 25 = 0}$$

Equation of a straight line

V, P, S

$$\underline{r} = \underline{r_0} + t\underline{v}$$

$$\langle x, y, z\rangle = \langle x_0, y_0, z_0\rangle + t\langle a, b, c\rangle$$

$$\langle x, y, z\rangle = \langle x_0 + at, y_0 + bt, z_0 + ct\rangle$$

$$x = x_0 + at, \quad y = y_0 + bt, \quad z = z_0 + ct$$

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c} = t$$

Eq. of a Plane: $n \overrightarrow{P_0P} = 0$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$