COORDINATE GEOMETRY (Adapted from "Precalculus" by Stewart et als.)

- The Coordinate Plane
- The Distance and Midpoint Formulas
- Graphs of Equations in Two Variables
- Intercepts
- Circles
- Symmetry

The ***coordinate plane*** is the link between algebra and geometry. In the coordinate plane we can draw graphs of algebraic equations. The graphs, in turn, allow us to "see" the relationship between the variables in the equation. In this section we study the coordinate plane.

The Coordinate Plane

Just as points on a line can be identified with real numbers to form the coordinate line, points in a plane can be identified with ordered pairs of numbers to form the coordinate plane or Cartesian plane.

To do this, we draw two perpendicular real lines that intersect at 0 on each line.

Usually, one line is horizontal with positive direction to the right and is called the x-axis; the other line is vertical with positive direction upward and is called the y-axis.

The point of intersection of the x-axis and the y-axis is the origin O, and the two axes divide the plane into four quadrants, labeled I, II, III, and IV in Figure 1.
*(**The points on the coordinate axes are not assigned to any quadrant**.)*

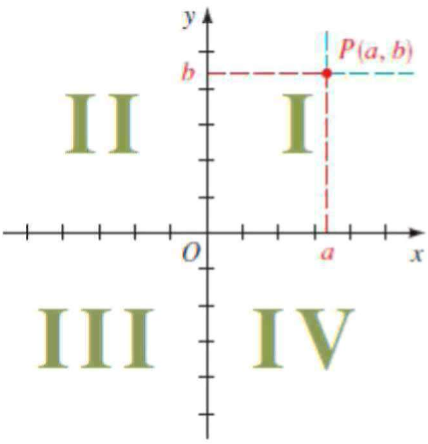


FIGURE 1

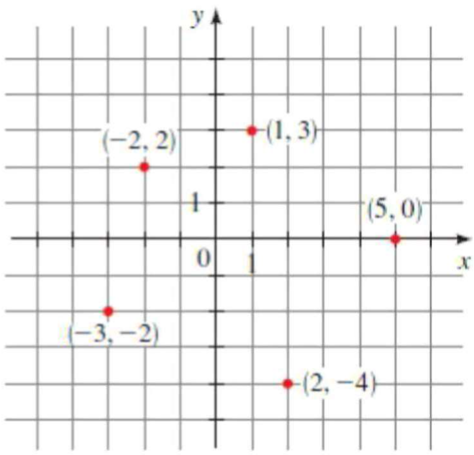


FIGURE 2

Any point P in the coordinate plane can be located by a unique ordered pair of numbers (a, b), as shown in Figure 1.

*The first number a is called the **x-coordinate** of P;*
*the second number b is called the **y-coordinate** of P.*

We can think of the coordinates of P as its "address," because they specify its location in the plane. Several points are labeled with their coordinates in Figure 2.

Example 1 – *Graphing Regions in the Coordinate Plane*
Describe and sketch the regions given by each set.
$(a)\quad \{(x,y)\,|\,x \ge 0\}\quad (b)\quad \{(x,y)\,|\,y = 1\}\quad (c)\quad \{(x,y)\,|\,|y| < 1\}$

Solution

(a) The points whose x-coordinates are 0 or positive lie on the y-axis or to the right of it, as shown in Figure 3(a).

(b) The set of all points with y-coordinate 1 is a horizontal line one unit above the x-axis, as in Figure 3(b).

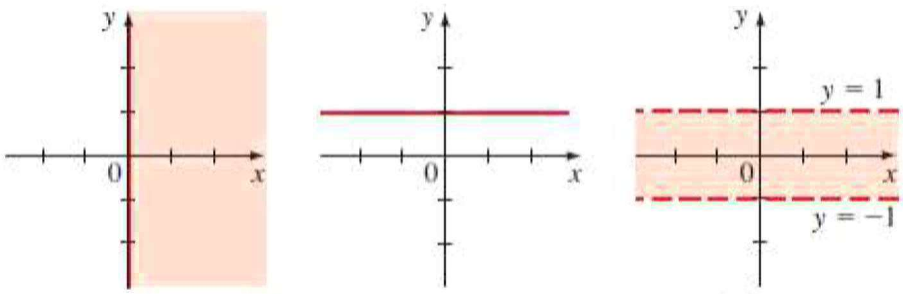
(c) We know that

$$|y| < 1 \qquad \text{if and only if} \qquad -1 < y < 1$$

So the given region consists of those points in the plane whose y-coordinates lie between –1 and 1.

Thus, the region consists of all points that lie between (but not on) the horizontal lines $y = 1$ and $y = -1$.

These lines are shown as broken lines in Figure 3(c) to indicate that the points on these lines do not lie in the set.



(a) $x \ge 0$ (b) $y = 1$ (c) $|y| < 1$

Fig.3

NOW TRY EXERCISES

Sketch the region given by the set.
$(a)\quad \{(x,y)\,|\,x \ge 3\}\quad (b)\quad \{(x,y)\,|\,y = 2\}\quad (c)\quad \{(x,y)\,|\,|x| > 4\}$

The Distance and Midpoint Formulas

We now find a formula for the distance $d(A, B)$ between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the plane.

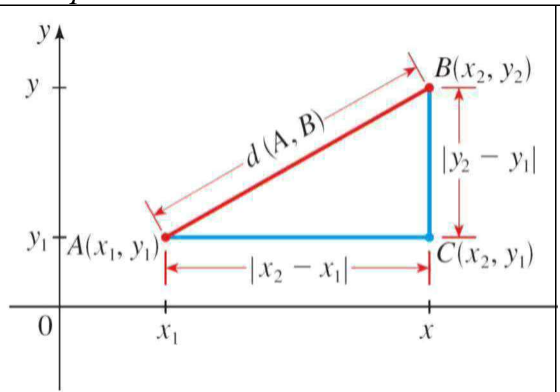


Figure 4

Since triangle ABC is a right triangle, the Pythagorean Theorem gives

$$d(A,B) = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$
$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

DISTANCE FORMULA

The distance between the points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the plane is

$$d(A, B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 2 – *Applying the Distance Formula*

Which of the points $P(1, –2)$ or $Q(8, 9)$ is closer to the point $A(5, 3)$?

Solution:

By the Distance Formula we have

$$d(P, A) = \sqrt{(5-1)^2 + [3-(-2)]^2} = \sqrt{4^2 + 5^2} = \sqrt{41}$$

$$d(Q, A) = \sqrt{(5-8)^2 + [3-9]^2} = \sqrt{(-3)^2 + (-6)^2} = \sqrt{45}$$

This shows that $d(P, A) < d(Q, A)$, so P is closer to A (see Figure 5).

NOW TRY EXERCISE

Which of the points $A(6, 7)$ or $Q(–5, 8)$ is closer to the origin?

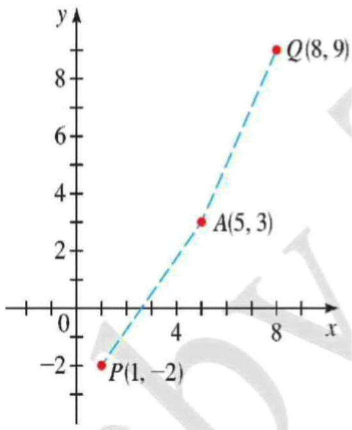
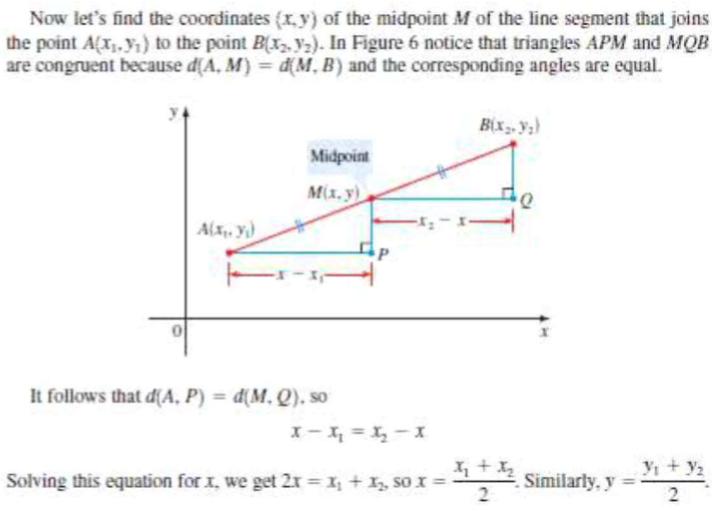


Fig. 5


Now let's find the coordinates (x, y) of the midpoint M of the line segment that joins the point $A(x_1, y_1)$ to the point $B(x_2, y_2)$. In Figure 6 notice that triangles APM and MQB are congruent because $d(A, M) = d(M, B)$ and the corresponding angles are equal.

It follows that $d(A, P) = d(M, Q)$, so

$$x - x_1 = x_2 - x$$

Solving this equation for x, we get $2x = x_1 + x_2$, so $x = \frac{x_1 + x_2}{2}$. Similarly, $y = \frac{y_1 + y_2}{2}$.

Fig. 6

MIDPOINT FORMULA

The midpoint of the line segment from $A(x_1, y_1)$ to $B(x_2, y_2)$ is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Example 3 – *Applying the Midpoint Formula*

Show that the quadrilateral with vertices P(1, 2), Q(4, 4), R(5, 9), and S(2, 7) is a parallelogram by proving that its two diagonals bisect each other. (A theorem from elementary geometry states that the quadrilateral is therefore a parallelogram.)

Solution:

If the two diagonals have the same midpoint, then they must bisect each other.(Why?)

The midpoint of the diagonal PR is

$$\left(\frac{1+5}{2},\frac{2+9}{2}\right)=\left(3,\frac{11}{2}\right)$$

and the midpoint of the diagonal QS is

$$\left(\frac{4+2}{2},\frac{4+7}{2}\right)=\left(3,\frac{11}{2}\right)$$

so each diagonal bisects the other, as shown in Figure 7.

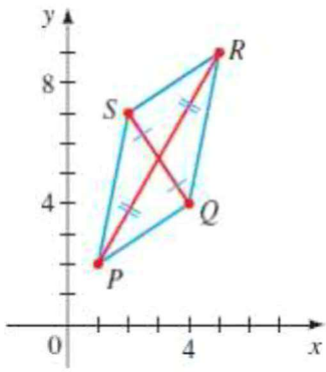


Fig. 7

NOW TRY EXERCISE

Show that the triangle with vertices $A(0,2), B(-3,-1)$, and $C(-4,3)$ is isosceles.

Graphs of Equations in Two Variables

- An equation in two variables, such as $y = x^2 + 1$, expresses a relationship between two quantities.

- A point (x, y) satisfies the equation if it makes the equation true when the values for x and y are substituted into the equation.

For example, the point (3, 10) satisfies the equation $y = x^2 + 1$ because 10 = 32 + 1, but the point (1, 3) does not, because 3 ≠ 12 + 1.

THE GRAPH OF AN EQUATION

The **graph** of an equation in x and y is the set of all points (x, y) in the coordinate plane that satisfy the equation.

Example 4 – *Sketching a Graph by Plotting Points*

Sketch the graph of the equation $2x - y = 3$.

Solution:

Solve the given equation for y to get: $y = 2x - 3$

This helps us calculate the y-coordinates in the following table.

x	$y = 2x - 3$	(x, y)
-1	-5	$(-1, -5)$
0	-3	$(0, -3)$
1	-1	$(1, -1)$
2	1	$(2, 1)$
3	3	$(3, 3)$
4	5	$(4, 5)$

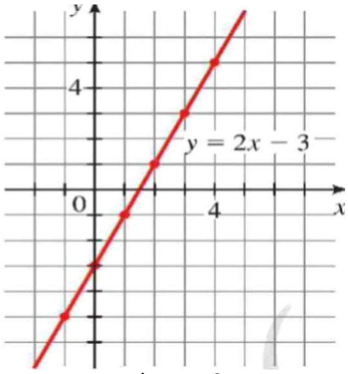


Figure 8

Of course, there are <u>*infinitely many*</u> *points on the graph, and it is impossible to plot all of them. But the more points we plot, the better we can imagine what the graph represented by the equation looks like. We plot the points we found in Figure 8; they appear to lie on a line. So we complete the graph by joining the points by a line.*

<u>Example 5</u> – *Sketching a Graph by Plotting Points*
Sketch the graph of the equation $y = x^2 - 2$.•
<u>*Solution*</u>:

x	$y = x^2 - 2$	(x, y)
-3	7	$(-3, 7)$
-2	2	$(-2, 2)$
-1	-1	$(-1, -1)$
0	-2	$(0, -2)$
1	-1	$(1, -1)$
2	2	$(2, 2)$
3	7	$(3, 7)$

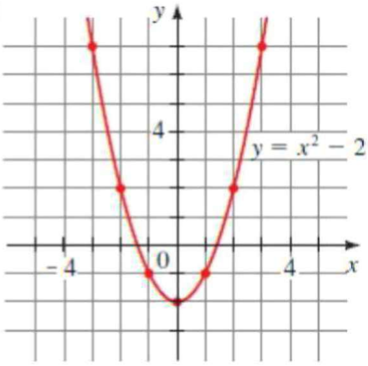


Figure 9

We find some of the points that satisfy the equation, as shown in the table above.
In Figure 9 we plot these points and then connect them by a smooth curve.

A curve with this shape is called a <u>***parabola***</u>.

<u>Example 6</u> – *Graphing an Absolute Value Equation*

Sketch the graph of the equation $y = |x|$.

x	$y = |x|$	(x, y)
-3	3	$(-3, 3)$
-2	2	$(-2, 2)$
-1	1	$(-1, 1)$
0	0	$(0, 0)$
1	1	$(1, 1)$
2	2	$(2, 2)$
3	3	$(3, 3)$

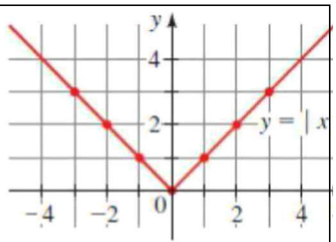


Fig.10

NOW TRY EXERCISES

Make a table of values and sketch the graph of the equation.
Find the x- and y-intercepts and test for symmetry.(<u>*See later for intercepts and symmetry*</u>.)

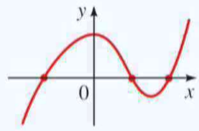
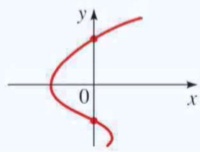
(a) $2x - y = 6$, (b) $4y = x^2$, (c) $y = 4 - |x|$

<u>Intercepts</u>

- The x-coordinates of the points where a graph intersects the x-axis are called the **<u>x-intercepts</u>** of the graph and are obtained by setting $y = 0$ in the equation of the graph.
- The y-coordinates of the points where a graph intersects the y-axis are called the **<u>y-intercepts</u>** of the graph and are obtained by setting $x = 0$ in the equation of the graph.

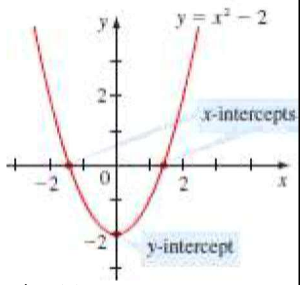
The x- and y-intercepts are numbers.

DEFINITION OF INTERCEPTS

Intercepts	How to find them	Where they are on the graph
x-intercepts: The x-coordinates of points where the graph of an equation intersects the x-axis	Set $y = 0$ and solve for x	
y-intercepts: The y-coordinates of points where the graph of an equation intersects the y-axis	Set $x = 0$ and solve for y	

<u>Example 7</u> – *Finding Intercepts*
Find the x- and y-intercepts of the graph of the equation $y = x^2 - 2$.

<u>*Solution*</u>*:*

To find the x-intercepts, we set $y = 0$ and solve for x. Thus $0 = x^2 - 2$ $x^2 = 2$ $x = \pm\sqrt{2}$ The x-intercepts are $\sqrt{2}$ and $-\sqrt{2}$.	To find the y-intercepts, we set $x = 0$ and solve for y. Thus $y = 0^2 - 2$ $y = -2$ The y-intercept is -2.	 Fig.11

NOW TRY EXERCISE

Make a table of values and sketch the graph of the equation.
Find the x- and y-intercepts and test for symmetry. (<u>*See later for symmetry*</u>.)

$$y = x^2 - 9$$

Circles

EQUATION OF A CIRCLE

An equation of the circle with center (h, k) and radius r is

$$(x - h)^2 + (y - k)^2 = r^2$$

This is called the **standard form** for the equation of the circle. If the center of the circle is the origin $(0, 0)$, then the equation is

$$x^2 + y^2 = r^2$$

EXAMPLE 8 | Graphing a Circle

Graph each equation.

(a) $x^2 + y^2 = 25$ **(b)** $(x - 2)^2 + (y + 1)^2 = 25$

SOLUTION

(a) Rewriting the equation as $x^2 + y^2 = 5^2$, we see that this is an equation of the circle of radius 5 centered at the origin. Its graph is shown in Figure 13.

(b) Rewriting the equation as $(x - 2)^2 + (y + 1)^2 = 5^2$, we see that this is an equation of the circle of radius 5 centered at $(2, -1)$. Its graph is shown in Figure 14.

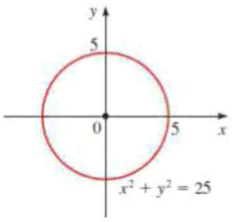


FIGURE 13

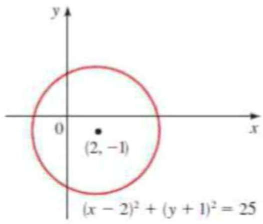


FIGURE 14

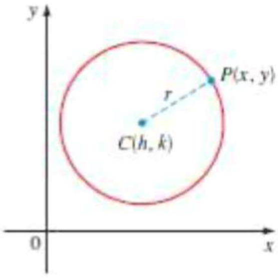


Fig.12

NOW TRY EXERCISES

Find the center and radius of the circle and sketch its graph

(a) $x^2 + y^2 = 9$, (b) $(x - 3)^2 + y^2 = 16$

Example 9 – *Finding an Equation of a Circle*

(a) Find an equation of the circle with radius 3 and center $(2, -5)$.

(b) Find an equation of the circle that has the points $P(1, 8)$ and $Q(5, -6)$ as the endpoints of a diameter.

Solution: (a) Using the equation of a circle with $r = 3$, $h = 2$, and $k = -5$, we obtain

$$(x - 2)^2 + (y + 5)^2 = 9$$

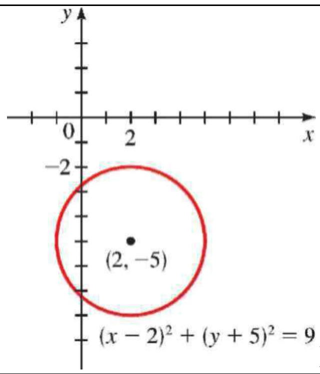


Figure 15

(b) The center is the midpoint of the diameter PQ. By the Midpoint Formula, the centre is

$$\left(\frac{1+5}{2},\frac{8-6}{2}\right)=(3,1)$$

The radius r is the distance from P to the center, so by the Distance Formula

$$r^2=(3-1)^2+(1-8)^2=2^2+(-7)^2=53$$

Therefore, the equation of the circle is

$$(x-3)^2+(y-1)^2=53$$

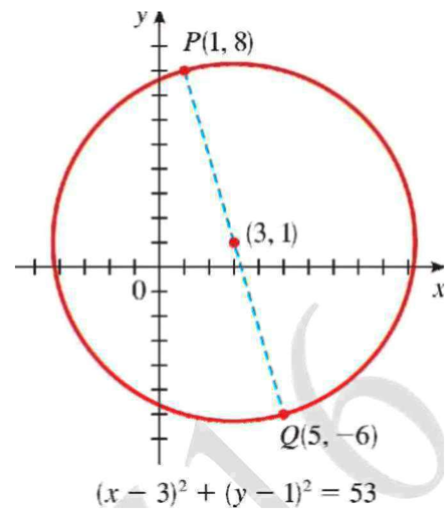


Fig.16

NOW TRY EXERCISES

Find an equation of the circle that satisfies the given conditions.
(a) Centre $(2,-1)$; radius 3   (b) Endpoints of a diameter are $P(-1,1)$ and $Q(5,9)$.

EXAMPLE 10 | Identifying an Equation of a Circle

Show that the equation $x^2+y^2+2x-6y+7=0$ represents a circle, and find the center and radius of the circle.

SOLUTION We first group the x-terms and y-terms. Then we complete the square within each grouping. That is, we complete the square for x^2+2x by adding $\left(\frac{1}{2}\cdot 2\right)^2=1$, and we complete the square for y^2-6y by adding $\left[\frac{1}{2}\cdot(-6)\right]^2=9$.

$(x^2+2x\qquad)+(y^2-6y\qquad)=-7$	Group terms
$(x^2+2x+1)+(y^2-6y+9)=-7+1+9$	Complete the square by adding 1 and 9 to each side
$(x+1)^2+(y-3)^2=3$	Factor and simplify

Comparing this equation with the standard equation of a circle, we see that $h=-1$, $k=3$, and $r=\sqrt{3}$, so the given equation represents a circle with center $(-1,3)$ and radius $\sqrt{3}$.

NOW TRY EXERCISE

Show that the equation represents a circle, and find the center and radius of the circle.

$$x^2+y^2-4x+10y+13=0$$

Symmetry

Figure 17 shows the graph of $y=x^2$. Notice that the part of the graph to the left of the y-axis is the mirror image of the part to the right of the y-axis.

The reason is that if the point (x, y) is on the graph, then so is $(-x, y)$, and these points are reflections of each other about the y-axis.

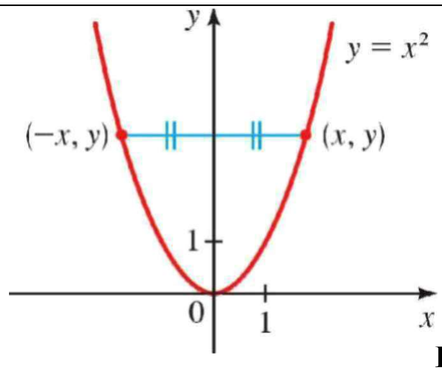
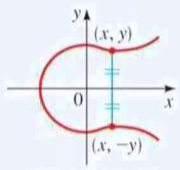
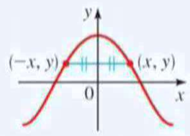
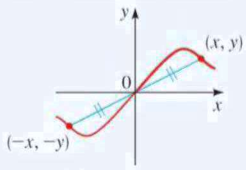


Fig. 17

- In this situation we say that the graph is **symmetric with respect to the y-axis**.
- Similarly, we say that a graph is **symmetric with respect to the x-axis** if whenever the point (x, y) is on the graph, then so is $(x, -y)$.
- A graph is **symmetric with respect to the origin** if whenever (x, y) is on the graph, so is $(-x, -y)$.

DEFINITION OF SYMMETRY

Type of symmetry	How to test for symmetry	What the graph looks like (figures in this section)	Geometric meaning
Symmetry with respect to the x-axis	The equation is unchanged when y is replaced by $-y$	 (Figures 13, 18)	Graph is unchanged when reflected in the x-axis
Symmetry with respect to the y-axis	The equation is unchanged when x is replaced by $-x$	 (Figures 9, 10, 11, 13, 17)	Graph is unchanged when reflected in the y-axis
Symmetry with respect to the origin	The equation is unchanged when x is replaced by $-x$ and y by $-y$	 (Figures 13, 19)	Graph is unchanged when rotated 180° about the origin

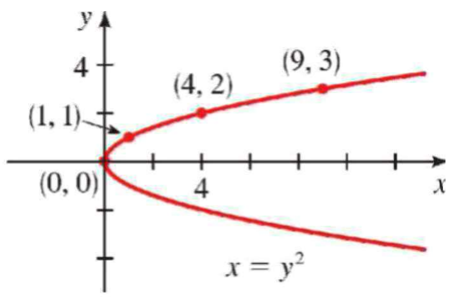
Example 11 – *Using Symmetry to Sketch a Graph*
Test the equation $x = y^2$ for symmetry and sketch the graph.
Solution:
If y is replaced by $-y$ in the equation $x = y^2$, *we get* $x = (-y)^2$

This is simplified to $x = y^2$ and so the equation is unchanged.
Therefore, the graph is symmetric about the x-axis.

But changing x to $-x$ gives the equation $-x = y^2$, which is not the same as the original equation, so the graph is not symmetric about the y-axis.

We use the symmetry about the x-axis to sketch the graph by first plotting points just for $y > 0$ and then reflecting the graph in the x-axis, as shown here.

y	$x = y^2$	(x, y)
0	0	(0, 0)
1	1	(1, 1)
2	4	(4, 2)
3	9	(9, 3)



EXAMPLE 12 | Using Symmetry to Sketch a Graph

Test the equation $y = x^3 - 9x$ for symmetry and sketch its graph.

SOLUTION If we replace x by $-x$ and y by $-y$ in the equation, we get

$$-y = (-x)^3 - 9(-x) \qquad \text{Replace } x \text{ by } -x \text{ and } y \text{ by } -y$$
$$-y = -x^3 + 9x \qquad \text{Simplify}$$
$$y = x^3 - 9x \qquad \text{Multiply by } -1$$

and so the equation is unchanged. This means that the graph is symmetric with respect to the origin. We sketch it by first plotting points for $x > 0$ and then using symmetry about the origin (see Figure 19).

x	$y = x^3 - 9x$	(x, y)
0	0	(0, 0)
1	−8	(1, −8)
1.5	−10.125	(1.5, −10.125)
2	−10	(2, −10)
2.5	−6.875	(2.5, −6.875)
3	0	(3, 0)
4	28	(4, 28)

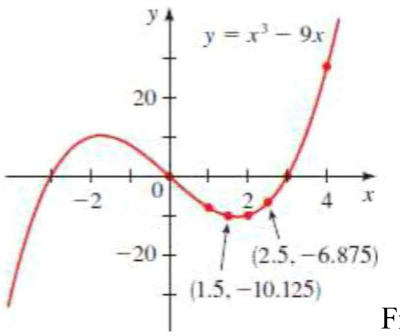


Fig.19

NOW TRY EXERCISES

Test the equation for symmetry.

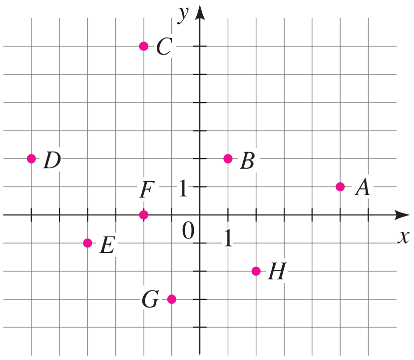
(a) $y = x^4 + x^2$ (b) $x^2 y^2 + xy = 1$

Exercise

1. Plot the given points in a coordinate plane:

$$(2, 3), (-2, 3), (4, 5), (4, -5), (-4, 5), (-4, -5)$$

2. Find the coordinates of the points shown in the figure.

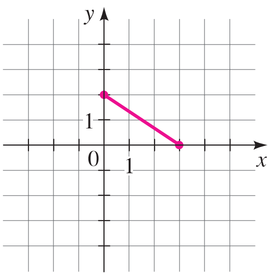


3–6 ■ A pair of points is graphed.

(a) Find the distance between them.

(b) Find the midpoint of the segment that joins them.

3.



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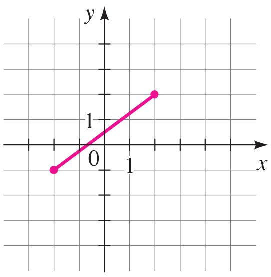

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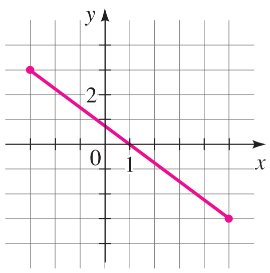
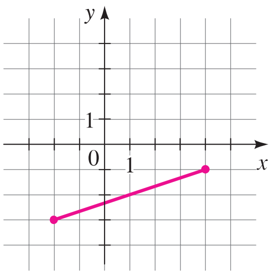


6.



7–12 ■ A pair of points is graphed.

(a) Plot the points in a coordinate plane.

(b) Find the distance between them.

(c) Find the midpoint of the segment that joins them.

7. $(0, 8), (6, 16)$

8. $(-2, 5), (10, 0)$

9. $(-3, -6), (4, 18)$

10. $(-1, -1), (9, 9)$

11. $(6, -2), (-6, 2)$

12. $(0, -6), (5, 0)$

13. Draw the rectangle with vertices $A(1, 3)$, $B(5, 3)$, $C(1, -3)$, and $D(5, -3)$ on a coordinate plane. Find the area of the rectangle.

14. Draw the parallelogram with vertices $A(1, 2)$, $B(5, 2)$, $C(3, 6)$, and $D(7, 6)$ on a coordinate plane. Find the area of the parallelogram.

15. Plot the points $A(1, 0)$, $B(5, 0)$, $C(4, 3)$, and $D(2, 3)$, on a coordinate plane. Draw the segments AB, BC, CD, and DA. What kind of quadrilateral is $ABCD$, and what is its area?

16. Plot the points $P(5, 1)$, $Q(0, 6)$, and $R(-5, 1)$, on a coordinate plane. Where must the point S be located so that the quadrilateral $PQRS$ is a square? Find the area of this square.

17–26 ■ Sketch the region given by the set.

17. $\{(x, y) \mid x \geq 3\}$

18. $\{(x, y) \mid y < 3\}$

19. $\{(x, y) \mid y = 2\}$

20. $\{(x, y) \mid x = -1\}$

21. $\{(x, y) \mid 1 < x < 2\}$

22. $\{(x, y) \mid 0 \leq y \leq 4\}$

23. $\{(x, y) \mid |x| > 4\}$

24. $\{(x, y) \mid |y| \leq 2\}$

25. $\{(x, y) \mid x \geq 1 \text{ and } y < 3\}$

26. $\{(x, y) \mid |x| \leq 2 \text{ and } |y| \leq 3\}$

27. Which of the points $A(6, 7)$ or $B(-5, 8)$ is closer to the origin?

28. Which of the points $C(-6, 3)$ or $D(3, 0)$ is closer to the point $E(-2, 1)$?

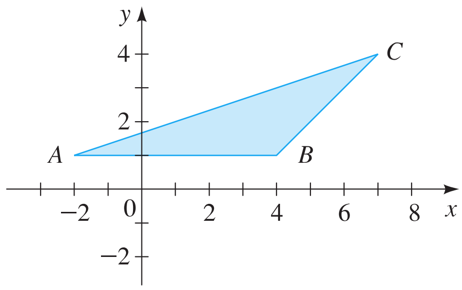
29. Which of the points $P(3, 1)$ or $Q(-1, 3)$ is closer to the point $R(-1, -1)$?

30. **(a)** Show that the points $(7, 3)$ and $(3, 7)$ are the same distance from the origin.

(b) Show that the points (a, b) and (b, a) are the same distance from the origin.

31. Show that the triangle with vertices $A(0, 2)$, $B(-3, -1)$, and $C(-4, 3)$ is isosceles.

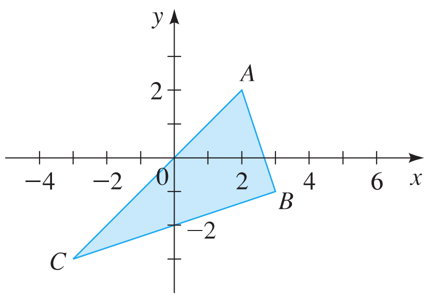
32. Find the area of the triangle shown in the figure.



33. Refer to triangle ABC in the figure.

(a) Show that triangle ABC is a right triangle by using the converse of the Pythagorean Theorem (see page 54).

(b) Find the area of triangle ABC.



34. Show that the triangle with vertices $A(6, -7)$, $B(11, -3)$, and $C(2, -2)$ is a right triangle by using the converse of the Pythagorean Theorem. Find the area of the triangle.

35. Show that the points $A(-2, 9)$, $B(4, 6)$, $C(1, 0)$, and $D(-5, 3)$ are the vertices of a square.

36. Show that the points $A(-1, 3)$, $B(3, 11)$, and $C(5, 15)$ are collinear by showing that $d(A, B) + d(B, C) = d(A, C)$.

37. Find a point on the y-axis that is equidistant from the points $(5, -5)$ and $(1, 1)$.

38. Find the lengths of the medians of the triangle with vertices $A(1, 0)$, $B(3, 6)$, and $C(8, 2)$. (A *median* is a line segment from a vertex to the midpoint of the opposite side.)

39. Plot the points $P(-1, -4)$, $Q(1, 1)$, and $R(4, 2)$, on a coordinate plane. Where should the point S be located so that the figure $PQRS$ is a parallelogram?

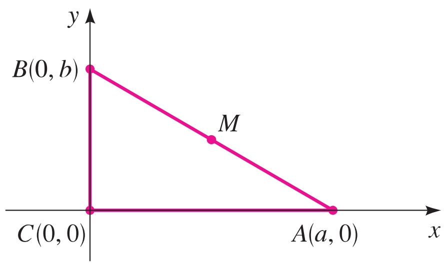
40. If $M(6, 8)$ is the midpoint of the line segment AB, and if A has coordinates $(2, 3)$, find the coordinates of B.

41. **(a)** Sketch the parallelogram with vertices $A(-2, -1)$, $B(4, 2)$, $C(7, 7)$, and $D(1, 4)$.

(b) Find the midpoints of the diagonals of this parallelogram.

(c) From part (b) show that the diagonals bisect each other.

42. The point M in the figure is the midpoint of the line segment AB. Show that M is equidistant from the vertices of triangle ABC.



43–46 ■ Determine whether the given points are on the graph of the equation.

43. $x - 2y - 1 = 0$; $(0, 0), (1, 0), (-1, -1)$

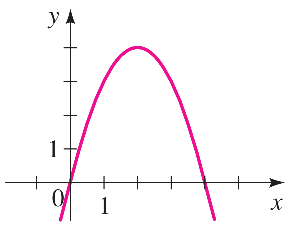
44. $y(x^2 + 1) = 1$; $(1, 1), \left(1, \frac{1}{2}\right), \left(-1, \frac{1}{2}\right)$

45. $x^2 + xy + y^2 = 4$; $(0, -2), (1, -2), (2, -2)$

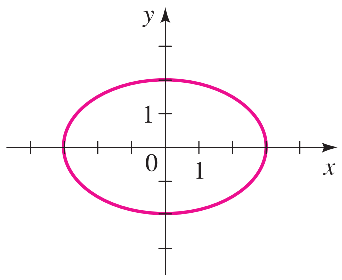
46. $x^2 + y^2 = 1$; $(0, 1), \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

47–50 ■ An equation and its graph are given. Find the x- and y-intercepts.

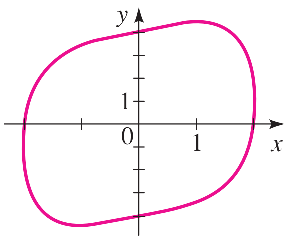
47. $y = 4x - x^2$



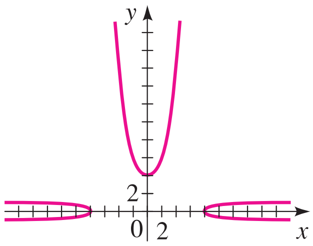
48. $\frac{x^2}{9} + \frac{y^2}{4} = 1$



49. $x^4 + y^2 - xy = 16$



50. $x^2 + y^3 - x^2y^2 = 64$



51–70 ■ Make a table of values and sketch the graph of the equation. Find the x- and y-intercepts and test for symmetry.

51. $y = -x + 4$

52. $y = 3x + 3$

53. $2x - y = 6$

54. $x + y = 3$

55. $y = 1 - x^2$

56. $y = x^2 + 2$

57. $4y = x^2$

58. $8y = x^3$

59. $y = x^2 - 9$

60. $y = 9 - x^2$

61. $xy = 2$

62. $y = \sqrt{x + 4}$

63. $y = \sqrt{4 - x^2}$

64. $y = -\sqrt{4 - x^2}$

65. $x + y^2 = 4$

66. $x = y^3$

67. $y = 16 - x^4$

68. $x = |y|$

69. $y = 4 - |x|$

70. $y = |4 - x|$

71–76 ■ Test the equation for symmetry.

71. $y = x^4 + x^2$

72. $x = y^4 - y^2$

73. $x^2y^2 + xy = 1$

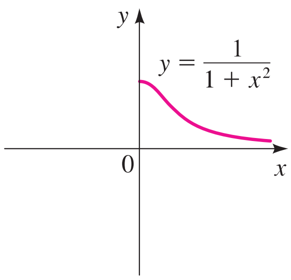
74. $x^4y^4 + x^2y^2 = 1$

75. $y = x^3 + 10x$

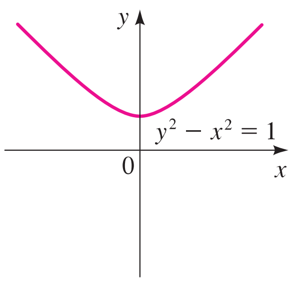
76. $y = x^2 + |x|$

77–80 ■ Complete the graph using the given symmetry property.

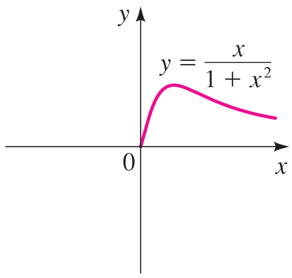
77. Symmetric with respect to the y-axis



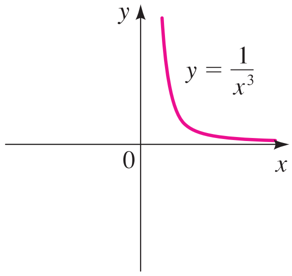
78. Symmetric with respect to the x-axis



79. Symmetric with respect to the origin



80. Symmetric with respect to the origin



81–86 ■ Find an equation of the circle that satisfies the given conditions.

81. Center $(2, -1)$; radius 3

82. Center $(-1, -4)$; radius 8

83. Center at the origin; passes through $(4, 7)$

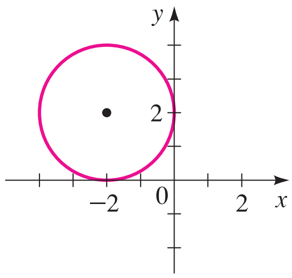
84. Endpoints of a diameter are $P(-1, 1)$ and $Q(5, 9)$

85. Center $(7, -3)$; tangent to the x-axis

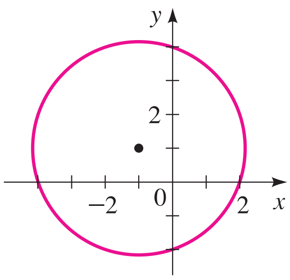
86. Circle lies in the first quadrant, tangent to both x-and y-axes; radius 5

87–88 ■ Find the equation of the circle shown in the figure.

87.



88.



89–94 ■ Show that the equation represents a circle, and find the center and radius of the circle.

89. $x^2 + y^2 - 4x + 10y + 13 = 0$

90. $x^2 + y^2 + 6y + 2 = 0$

91. $x^2 + y^2 - \frac{1}{2}x + \frac{1}{2}y = \frac{1}{8}$

92. $x^2 + y^2 + \frac{1}{2}x + 2y + \frac{1}{16} = 0$

93. $2x^2 + 2y^2 - 3x = 0$

94. $3x^2 + 3y^2 + 6x - y = 0$

95–96 ■ Sketch the region given by the set.

95. $\{(x, y) \mid x^2 + y^2 \le 1\}$

96. $\{(x, y) \mid x^2 + y^2 > 4\}$

97. Find the area of the region that lies outside the circle $x^2 + y^2 = 4$ but inside the circle

$$x^2 + y^2 - 4y - 12 = 0$$

98. Sketch the region in the coordinate plane that satisfies both the inequalities $x^2 + y^2 \le 9$ and $y \ge |x|$. What is the area of this region?

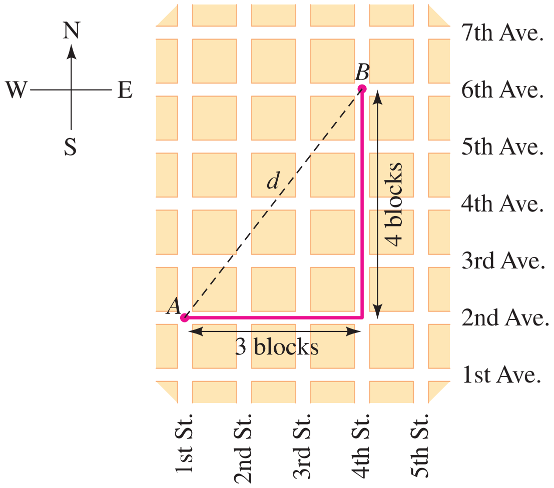
Applications

99. Distances in a City A city has streets that run north and south, and avenues that run east and west, all equally spaced. Streets and avenues are numbered sequentially, as shown in the figure. The *walking* distance between points A and B is 7 blocks—that is, 3 blocks east and 4 blocks north. To find the *straight-line* distances d, we must use the Distance Formula.

(a) Find the straight-line distance (in blocks) between A and B.

(b) Find the walking distance and the straight-line distance between the corner of 4th St. and 2nd Ave. and the corner of 11th St. and 26th Ave.

(c) What must be true about the points P and Q if the walking distance between P and Q equals the straight-line distance between P and Q?



100. Halfway Point Two friends live in the city described in Exercise 99, one at the corner of 3rd St. and 7th Ave., the other at the corner of 27th St. and 17th Ave. They frequently meet at a coffee shop halfway between their homes.

(a) At what intersection is the coffee shop located?

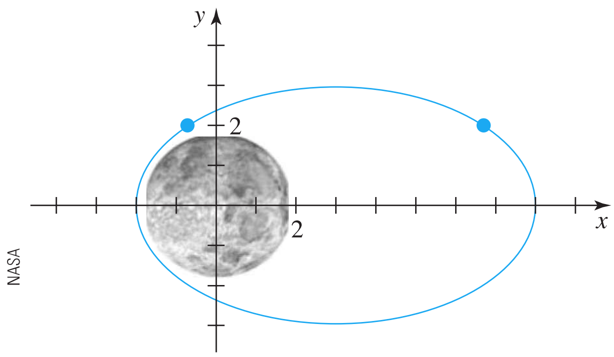
(b) How far must each of them walk to get to the coffee shop?

101. Orbit of a Satellite A satellite is in orbit around the moon. A coordinate plane containing the orbit is set up with the center of the moon at the origin, as shown in the graph, with distances measured in megameters (Mm). The equation of the satellite's orbit is

$$\frac{(x-3)^2}{25} + \frac{y^2}{16} = 1$$

(a) From the graph, determine the closest and the farthest that the satellite gets to the center of the moon.

(b) There are two points in the orbit with y-coordinates 2. Find the x-coordinates of these points, and determine their distances to the center of the moon.



Discovery • Discussion

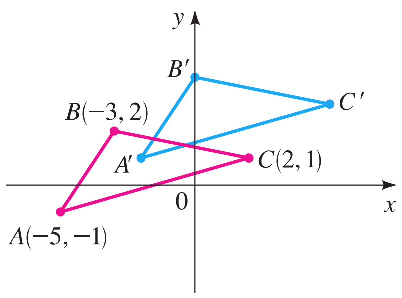
102. Shifting the Coordinate Plane Suppose that each point in the coordinate plane is shifted 3 units to the right and 2 units upward.

(a) The point $(5, 3)$ is shifted to what new point?

(b) The point (a, b) is shifted to what new point?

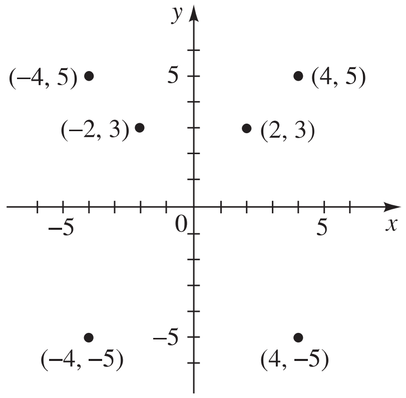
(c) What point is shifted to $(3, 4)$?

(d) Triangle ABC in the figure has been shifted to triangle $A'B'C'$. Find the coordinates of the points A', B', and C'.



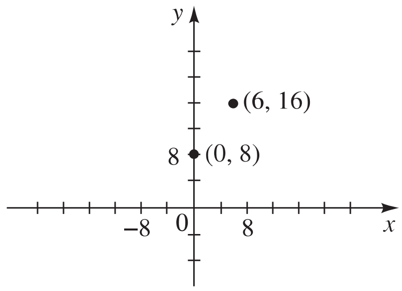
Answer

1.



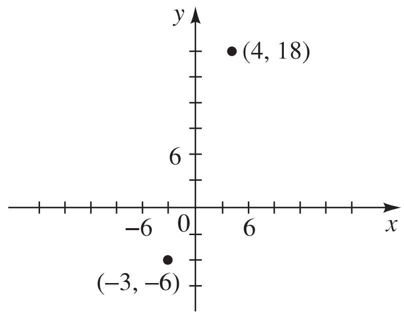
3. **(a)** $\sqrt{13}$ **(b)** $\left(\frac{3}{2}, 1\right)$ **5.** **(a)** 10 **(b)** $(1, 0)$

7. **(a)**



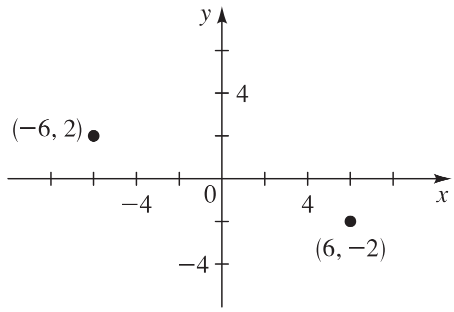
(b) 10 **(c)** $(3, 12)$

9. **(a)**



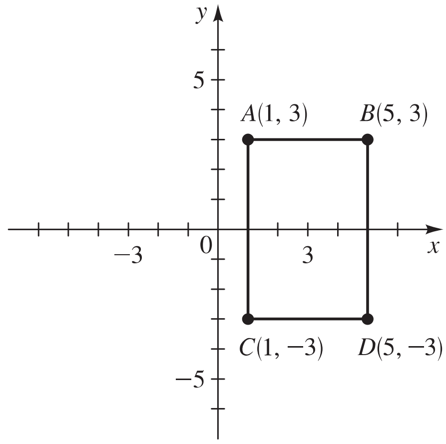
(b) 25 **(c)** $\left(\frac{1}{2}, 6\right)$

11. **(a)**

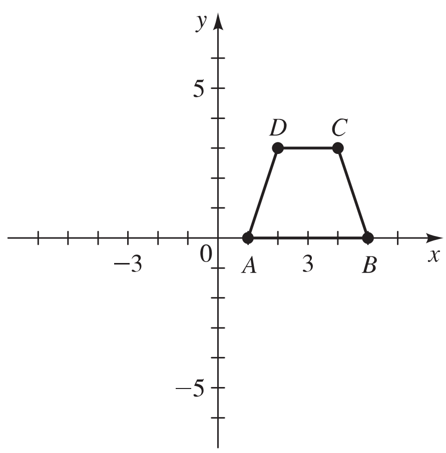


(b) $4\sqrt{10}$ **(c)** $(0, 0)$

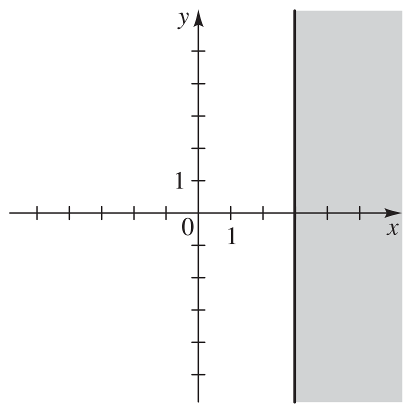
13. 24



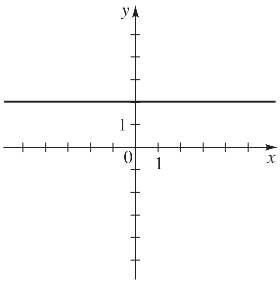
15. Trapezoid, area $= 9$



17.



19.



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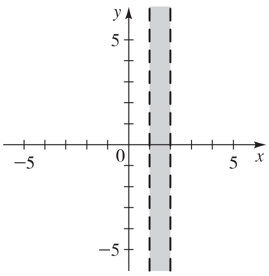
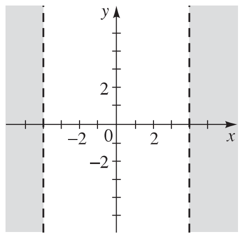
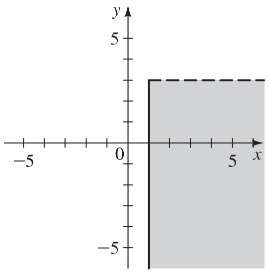


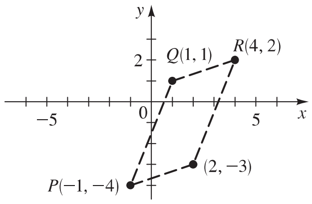
23.



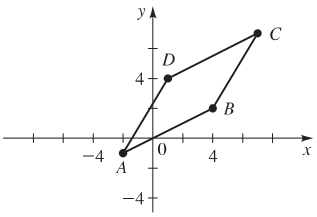
25.



27. $A(6, 7)$ **29.** $Q(-1, 3)$
33. (b) 10 **37.** $(0, -4)$
39. $(2, -3)$



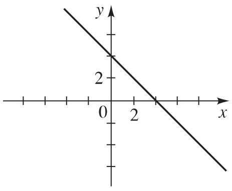
41. (a)



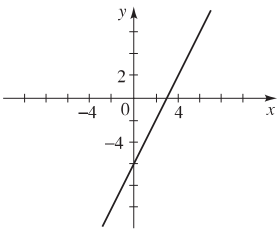
(b) $\left(\frac{5}{2}, 3\right), \left(\frac{5}{2}, 3\right)$

43. No, yes, yes **45.** Yes, no, yes
47. x-intercepts 0, 4; y-intercept 0
49. x-intercepts -2, 2; y-intercepts -4, 4

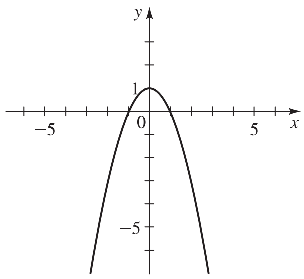
51. x-intercept 4,
y-intercept 4,
no symmetry



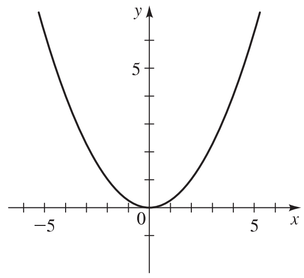
53. x-intercept 3,
y-intercept -6,
no symmetry



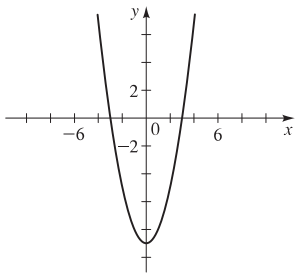
55. x-intercepts ± 1,
y-intercept 1,
symmetry about y-axis



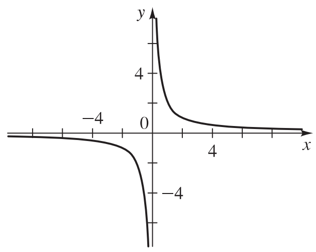
57. x-intercept 0,
y-intercept 0,
symmetry about y-axis



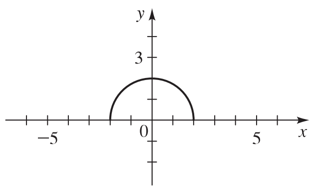
59. x-intercepts ± 3,
y-intercept -9,
symmetry about y-axis



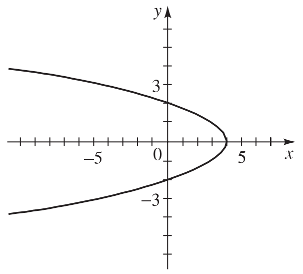
61. No intercepts,
symmetry about origin



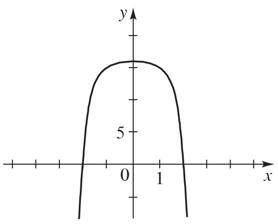
63. x-intercepts ± 2,
y-intercept 2,
symmetry about y-axis



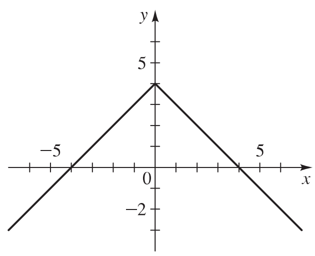
65. x-intercept 4,
y-intercepts -2, 2,
symmetry about x-axis



67. x-intercepts ± 2,
y-intercept 16,
symmetry about y-axis

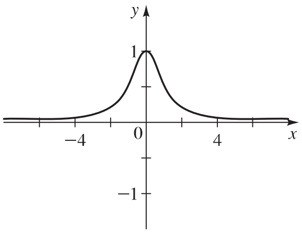


69. x-intercepts ± 4,
y-intercept 4,
symmetry about y-axis

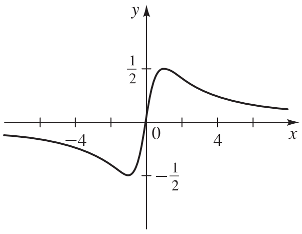


71. Symmetry about y-axis **73.** Symmetry about origin
y-axis, and origin **75.** Symmetry about origin

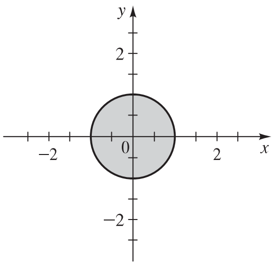
77.



79.



81. $(x - 2)^2 + (y + 1)^2 = 9$ **83.** $x^2 + y^2 = 65$
85. $(x - 7)^2 + (y + 3)^2 = 9$ **87.** $(x + 2)^2 + (y - 2)^2 = 4$
89. $(2, -5), 4$ **91.** $\left(\frac{1}{4}, -\frac{1}{4}\right), \frac{1}{2}$ **93.** $\left(\frac{3}{4}, 0\right), \frac{3}{4}$
95. **97.** 12π

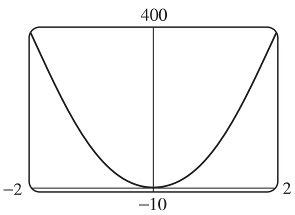


99. (a) 5 **(b)** 31; 25 **(c)** Points P and Q must either be on the same street or the same avenue. **101. (a)** 2 Mm, 8 Mm **(b)** $-1.33, 7.33$; 2.40 Mm, 7.60 Mm

Section 1.9 ■ page 109

1. (c) **3.** (c) **5.** (c)

7.



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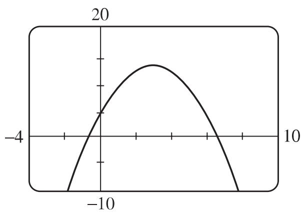

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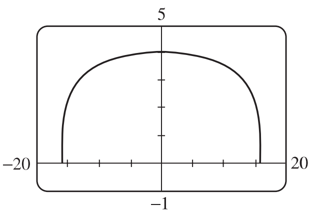

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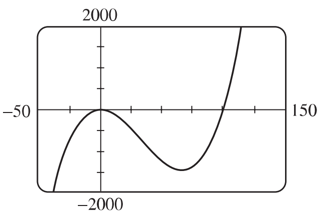

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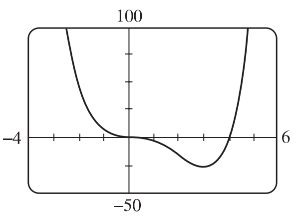

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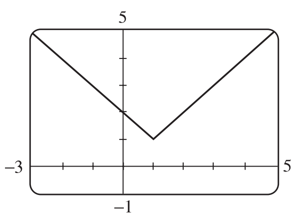

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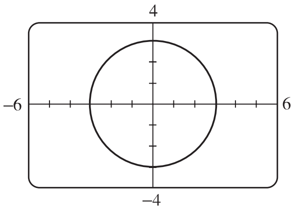

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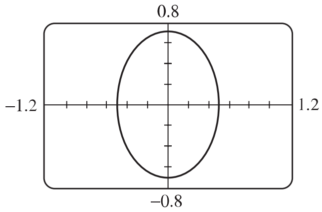
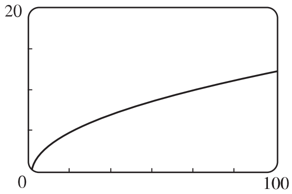
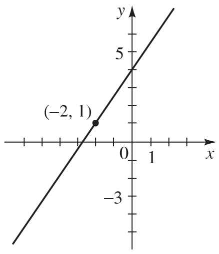


23. No **25.** Yes, 2 **27.** -4 **29.** $\frac{5}{14}$ **31.** $\pm 4\sqrt{2} \approx \pm 5.7$
33. $2.5, -2.5$ **35.** $5 + 2\sqrt[4]{5} \approx 7.99, 5 - 2\sqrt[4]{5} \approx 2.01$
37. 3.00, 4.00 **39.** 1.00, 2.00, 3.00 **41.** 1.62
43. $-1.00, 0.00, 1.00$ **45.** 2.55 **47.** $-2.05, 0, 1.05$
49. $[-2.00, 5.00]$ **51.** $(-\infty, 1.00] \cup [2.00, 3.00]$
53. $(-1.00, 0) \cup (1.00, \infty)$ **55.** $(-\infty, 0)$ **57.** 0, 0.01
59. (a) **(b)** 67 mi

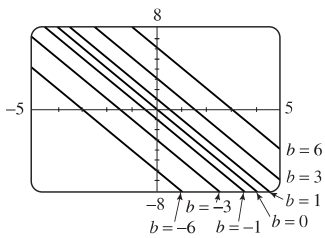


Section 1.10 ■ page 120

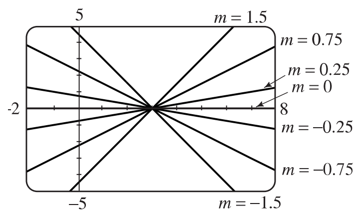
1. $\frac{1}{2}$ **3.** $\frac{1}{6}$ **5.** $-\frac{1}{2}$ **7.** $-\frac{9}{2}$ **9.** $-2, \frac{1}{2}, 3, -\frac{1}{4}$
11. $x + y - 4 = 0$ **13.** $3x - 2y - 6 = 0$
15. $x - y + 1 = 0$ **17.** $2x - 3y + 19 = 0$
19. $5x + y - 11 = 0$ **21.** $3x - y - 2 = 0$
23. $3x - y - 3 = 0$ **25.** $y = 5$ **27.** $x + 2y + 11 = 0$
29. $x = -1$ **31.** $5x - 2y + 1 = 0$
33. $x - y + 6 = 0$
35. (a) **(b)** $3x - 2y + 8 = 0$



37. They all have the same slope.



39. They all have the same x-intercept.



LINES (Adapted from "Precalculus" by Stewart et als.)

- The Slope of a Line
- Point-Slope Form of the Equation of a Line
- Slope-Intercept Form of the Equation of a Line
- Vertical and Horizontal Lines

- General Equation of a Line
- Parallel and Perpendicular Lines

<u>The Slope of a Line</u>

The slope of a line is the ratio of rise to run:

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

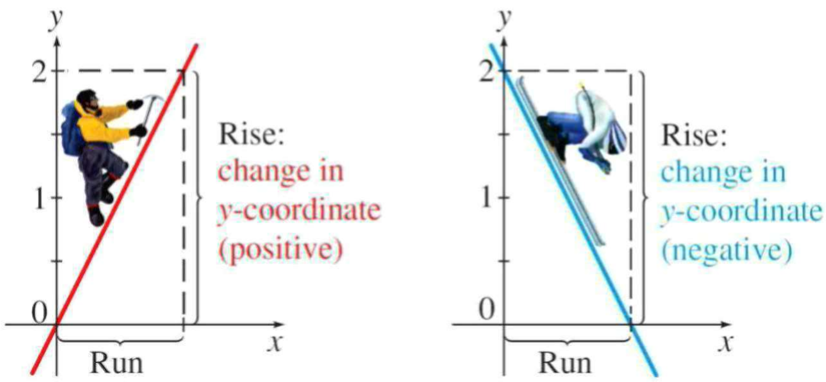


Figure 2

If a line lies in a coordinate plane, then the run is the change in the x-coordinate and the rise is the corresponding change in the y-coordinate between any two points on the line (see Figure 2).

This gives us the following definition of slope.

SLOPE OF A LINE

The **slope** m of a nonvertical line that passes through the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

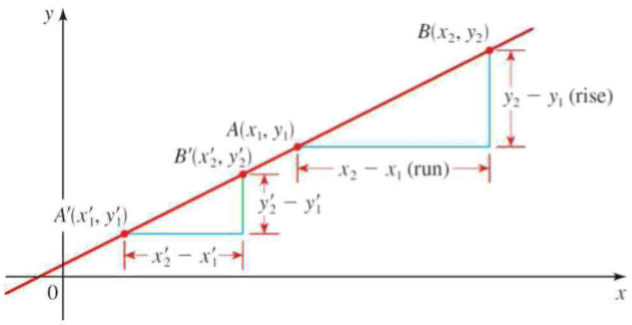
$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

The slope of a vertical line is not defined.

The slope of a vertical line is not defined.

The slope is independent of which two points are chosen on the line.

We can see that this is true from the similar triangles in Figure 3:



$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y'_2 - y'_1}{x'_2 - x'_1}$$

Figure 3

<u>Example 1</u> – *Finding the Slope of a Line through Two Points*
Find the slope of the line that passes through the points $P(2, 1)$ *and* $Q(8, 5)$.
<u>*Solution*</u>:

Since any two different points determine a line, only one line passes through these two points. From the definition
the slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 1}{8 - 2} = \frac{4}{6} = \frac{2}{3}$$

This says that for every 3 units we move to the right, the line rises 2 units.

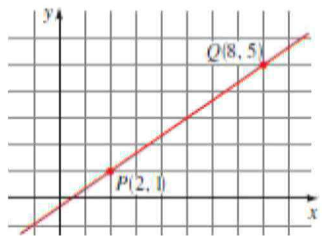


Fig. 5

NOW TRY EXERCISE

Find the slope of the line passing through $P(0, 0)$ and $Q(4, 2)$.

Point-Slope Form of the Equation of a Line

POINT-SLOPE FORM OF THE EQUATION OF A LINE

An equation of the line that passes through the point (x_1, y_1) and has slope m is

$$y - y_1 = m(x - x_1)$$

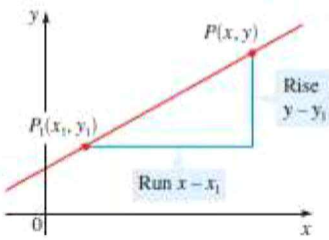


Fig.6

$$\frac{y - y_1}{x - x_1} = m$$

<u>Example 2</u> – *Finding the Equation of a Line with Given Point and Slope*
(a) Find an equation of the line through (1, –3) with slope $-\frac{1}{2}$.
(b) Sketch the line.

Solution:

(a) Using the point-slope form with $m = -\frac{1}{2}$, $x_1 = 1$, and $y_1 = -3$, *we obtain an equation of the line as*

$$y + 3 = -\tfrac{1}{2}(x - 1) \qquad \text{Slope } m = -\tfrac{1}{2}\text{, point } (1, -3)$$

$$2y + 6 = -x + 1 \qquad \text{Multiply by 2}$$

$$x + 2y + 5 = 0 \qquad \text{Rearrange}$$

(b) *The fact that the slope is* $-\frac{1}{2}$ *tells us that when we move to the right 2 units, the line drops 1 unit.*

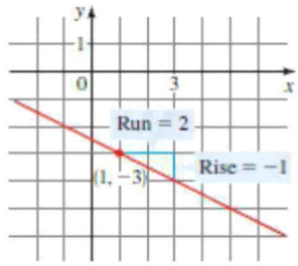


Fig.7

NOW TRY EXERCISE

Find an equation of the line that satisfies the given conditions.
Through (2, 3) ; slope 5.

EXAMPLE 3 | Finding the Equation of a Line Through Two Given Points

Find an equation of the line through the points $(-1, 2)$ and $(3, -4)$.

Solution:
The slope of the line is

$$m = \frac{-4 - 2}{3 - (-1)} = -\frac{6}{4} = -\frac{3}{2}$$

We can use *either* point, $(-1, 2)$ *or* $(3, -4)$, in the point-slope equation. We will end up with the same final answer.

Using the point-slope form with $x_1 = -1$ and $y_1 = 2$, we obtain

$$y - 2 = -\tfrac{3}{2}(x + 1) \qquad \text{Slope } m = -\tfrac{3}{2}\text{, point } (-1, 2)$$

$$2y - 4 = -3x - 3 \qquad \text{Multiply by 2}$$

$$3x + 2y - 1 = 0 \qquad \text{Rearrange}$$

NOW TRY EXERCISE

Find an equation of the line that satisfies the given conditions.
Through (2, 1) and (1, 6) ; slope 5.

Slope-Intercept Form of the Equation of a Line

Suppose a nonvertical line has slope m and y-intercept b (see Figure 8).

This means that the line intersects the y-axis at the point (0, b), so the point-slope form of the equation of the line, with x = 0 and y = b, becomes

$$y - b = m(x - 0)$$

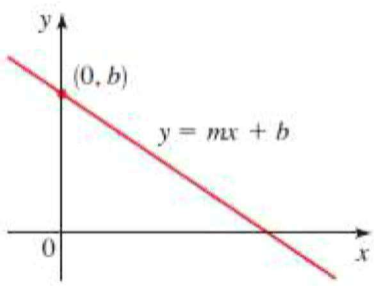


Fig.8

This simplifies to $y = mx + b$, which is called the ***slope-intercept form of the equation*** of a line.

SLOPE-INTERCEPT FORM OF THE EQUATION OF A LINE

An equation of the line that has slope m and y-intercept b is

$$y = mx + b$$

<u>Example 4</u> – *Lines in Slope-Intercept Form*
(a) Find the equation of the line with slope 3 and y-intercept –2.
(b) Find the slope and y-intercept of the line $3y – 2x = 1$.
<u>*Solution*</u>**:**
(a) Since $m = 3$ and $b = –2$, from the slope-intercept form of the equation of a line we get

$$y = 3x – 2$$

(b) We first write the equation in the form $y = mx + b$:

$$3y - 2x = 1$$
$$3y = 2x + 1 \quad \text{Add } 2x$$
$$y = \tfrac{2}{3}x + \tfrac{1}{3} \quad \text{Divide by 3}$$

From the slope-intercept form of the equation of a line,
we see that the slope is $m = \frac{2}{3}$ and the y-intercept is $b = \frac{1}{3}$.

NOW TRY EXERCISES

(a) Find an equation of the line that satisfies the given conditions.
Slope 3; y-intercept –2
(b) Find the slope and y-intercept of the line and draw its graph.
$x + 3y = 0$

<u>Vertical and Horizontal Lines</u>

VERTICAL AND HORIZONTAL LINES

An equation of the vertical line through (a, b) is $x = a$.
An equation of the horizontal line through (a, b) is $y = b$.

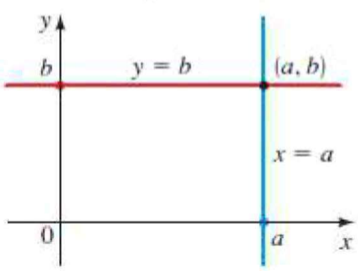


Fig.9

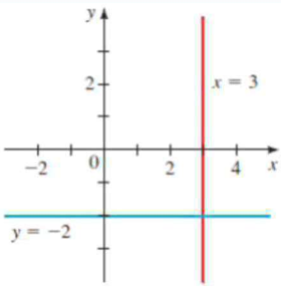


Fig.10

EXAMPLE 5 | Vertical and Horizontal Lines

(a) An equation for the vertical line through $(3, 5)$ is $x = 3$.

(b) The graph of the equation $x = 3$ is a vertical line with x-intercept 3.

(c) An equation for the horizontal line through $(8, -2)$ is $y = -2$.

(d) The graph of the equation $y = -2$ is a horizontal line with y-intercept -2.

The lines are graphed in Figure 10.

NOW TRY EXERCISES

Find an equation of the line that satisfies the given conditions.
(a) Through $(4, 5)$; parallel to the x-axis (b) Through $(-1, 2)$; parallel to the line $x = 5$

General Equation of a Line

A ***linear equation*** is an equation of the form $\mathbf{Ax + By + C = 0}$
where A, B, and C are constants and A and B are not both 0.

The equation of a line is a linear equation:

- A nonvertical line has the equation $y = mx + b$ or $-mx + y - b = 0$, which is a linear equation with $A = -m$, $B = 1$, and $C = -b$.
- A vertical line has the equation $x = a$ or $x - a = 0$, which is a linear equation with $A = 1$, $B = 0$, and $C = -a$.

Conversely, the graph of a linear equation is a line.

- If $B \neq 0$, the equation becomes $y = -\frac{A}{B}x - \frac{C}{B}$ and this is the slope-intercept form of the equation of a line (with $m = -\frac{A}{B}$ and $b = -\frac{C}{B}$).
- If $B = 0$, the equation becomes $Ax + C = 0$ or $x = -\frac{C}{A}$, which represents a vertical line.

We have proved the following.

GENERAL EQUATION OF A LINE

The graph of every **linear equation**

$$Ax + By + C = 0 \qquad (A, B \text{ not both zero})$$

is a line. Conversely, every line is the graph of a linear equation.

Example 6 – *Graphing a Linear Equation*
Sketch the graph of the equation $2x - 3y - 12 = 0$.

Solution 1:

Since the equation is linear, its graph is a line. To draw the graph, it is enough to find any two points on the line. The intercepts are the easiest points to find.

x-intercept:

Substitute $y = 0$, to get

$2x - 12 = 0$, so $x = 6$

y-intercept: Substitute $x = 0$, to get

$-3y - 12 = 0$, so $y = -4$

With these points we can sketch the graph in Figure 11.

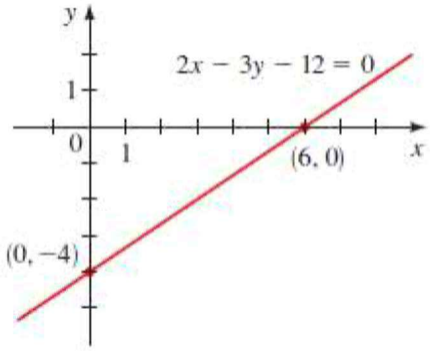


Fig.11

Solution 2:

We write the equation in slope-intercept form:

$$2x - 3y - 12 = 0$$
$$2x - 3y = 12$$
$$-3y = -2x + 12$$
$$y = \tfrac{2}{3}x - 4$$

This equation is in the form $y = mx + b$, so the slope is $m = \frac{2}{3}$

and the y-intercept is $b = -4$.

To sketch the graph, we plot the y-intercept and then move 3 units to the right and 2 units up as shown in Figure 12.

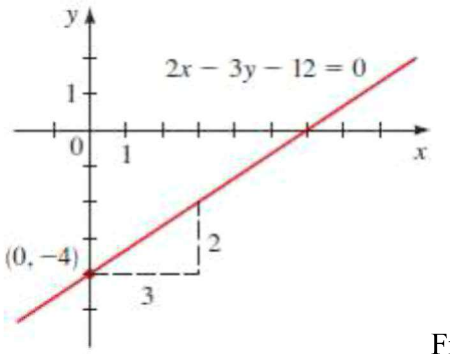


Fig.12

NOW TRY EXERCISE

Find the slope and y-intercept of the line and draw its graph.

$3x - 4y = 12$

Parallel and Perpendicular Lines

PARALLEL LINES

Two nonvertical lines are parallel if and only if they have the same slope.

PROOF Let the lines l_1 and l_2 in Figure 13 have slopes m_1 and m_2. If the lines are parallel, then the right triangles ABC and DEF are similar, so

$$m_1 = \frac{d(B, C)}{d(A, C)} = \frac{d(E, F)}{d(D, F)} = m_2$$

Conversely, if the slopes are equal, then the triangles will be similar, so $\angle BAC = \angle EDF$ and the lines are parallel.

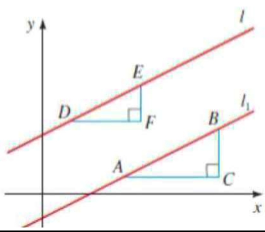


Fig.13

Example 7 – *Finding the Equation of a Line Parallel to a Given Line*
Find an equation of the line through the point (5, 2)
that is parallel to the line $4x + 6y + 5 = 0$.

Solution:
First we write the equation of the given line in slope-intercept form.

$$4x + 6y + 5 = 0$$

$$6y = -4x - 5 \qquad \text{Subtract } 4x + 5$$

$$y = -\tfrac{2}{3}x - \tfrac{5}{6} \qquad \text{Divide by 6}$$

So the line has slope $m = -\frac{2}{3}$

Since the required line is parallel to the given line, it also has slope $m = -\frac{2}{3}$

$$y - 2 = -\tfrac{2}{3}(x - 5) \qquad \text{Slope } m = -\tfrac{2}{3}, \text{ point } (5, 2)$$

$$3y - 6 = -2x + 10 \qquad \text{Multiply by 3}$$

$$2x + 3y - 16 = 0 \qquad \text{Rearrange}$$

Thus, the equation of the required line is $2x + 3y - 16 = 0$.

NOW TRY EXERCISE

Find an equation of the line that satisfies the given conditions.
Through the point (1, – 6); parallel to the line $x + 2y = 6$.

PERPENDICULAR LINES

Two lines with slopes m_1 and m_2 are perpendicular if and only if $m_1m_2 = -1$, that is, their slopes are negative reciprocals:

$$m_2 = -\frac{1}{m_1}$$

Also, a horizontal line (slope 0) is perpendicular to a vertical line (no slope).

PROOF In Figure 14 we show two lines intersecting at the origin. (If the lines intersect at some other point, we consider lines parallel to these that intersect at the origin. These lines have the same slopes as the original lines.)

If the lines l_1 and l_2 have slopes m_1 and m_2, then their equations are $y = m_1x$ and $y = m_2x$. Notice that $A(1, m_1)$ lies on l_1 and $B(1, m_2)$ lies on l_2. By the Pythagorean Theorem and its converse (see page 219), $OA \perp OB$ if and only if

$$[d(O, A)]^2 + [d(O, B)]^2 = [d(A, B)]^2$$

By the Distance Formula this becomes

$$(1^2 + m_1^2) + (1^2 + m_2^2) = (1 - 1)^2 + (m_2 - m_1)^2$$
$$2 + m_1^2 + m_2^2 = m_2^2 - 2m_1m_2 + m_1^2$$
$$2 = -2m_1m_2$$
$$m_1m_2 = -1$$

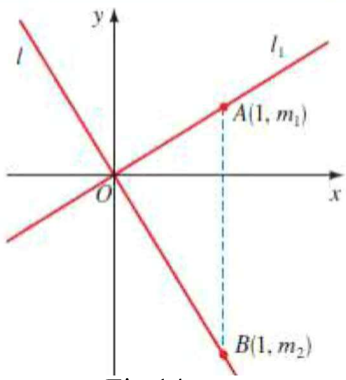


Fig.14

Example 8 – *Perpendicular Lines*

Show that the points $P(3, 3)$, $Q(8, 17)$, and $R(11, 5)$ are the vertices of a right triangle.

Solution:

The slopes of the lines containing PR and QR are, respectively,

$$m_1 = \frac{5-3}{11-3} = \frac{1}{4}$$

and $$m_2 = \frac{5-17}{11-8} = -4$$

Since $m_1 m_2 = -1$, these lines are perpendicular, so PQR is a right triangle.

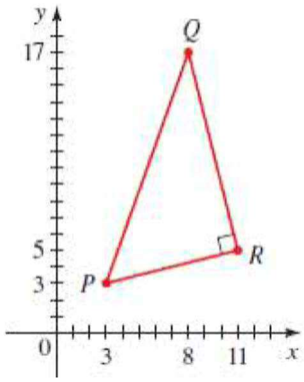


Fig.15

NOW TRY EXERCISE

Use slopes to show that points $A(1, 1)$, $B(7, 4)$, $C(5, 10)$, and $D(11, -7)$ are vertices of a parallelogram.

EXAMPLE 9 | **Finding an Equation of a Line Perpendicular to a Given Line**

Find an equation of the line that is perpendicular to the line $4x + 6y + 5 = 0$ and passes through the origin.

SOLUTION In Example 7 we found that the slope of the line $4x + 6y + 5 = 0$ is $-\frac{2}{3}$. Thus, the slope of a perpendicular line is the negative reciprocal, that is, $\frac{3}{2}$. Since the required line passes through $(0, 0)$, the point-slope form gives

$$y - 0 = \tfrac{3}{2}(x - 0) \qquad \text{Slope } m = \tfrac{3}{2}, \text{ point } (0, 0)$$

$$y = \tfrac{3}{2}x \qquad \text{Simplify}$$

NOW TRY EXERCISE

Find an equation of the line that satisfies the given conditions.

Through the point $(-1, -2)$; perpendicular to the line $2x + 5y + 8 = 0$.

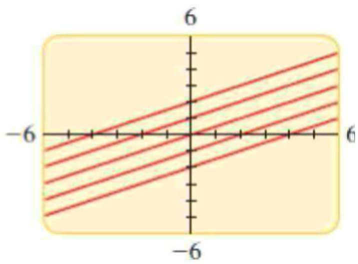
EXAMPLE 10 | **Graphing a Family of Lines**

Use a graphing calculator to graph the family of lines

$$y = 0.5x + b$$

for $b = -2, -1, 0, 1, 2$. What property do the lines share?

SOLUTION The lines are graphed in Figure 16 in the viewing rectangle $[-6, 6]$ by $[-6, 6]$. The lines all have the same slope, so they are parallel.



NOW TRY EXERCISE

Graph the given family of lines on the same axes. What do the lines have in common?

$y = -2x + b \quad \text{for } b = 0, \pm 1, \pm 3, \pm 6$

(nby, June 2015)

Exercise

1–8 ■ Find the slope of the line through P and Q.

1. $P(0,0)$, $Q(4,2)$

2. $P(0,0)$, $Q(2,-6)$

3. $P(2,2)$, $Q(-10,0)$

4. $P(1,2)$, $Q(3,3)$

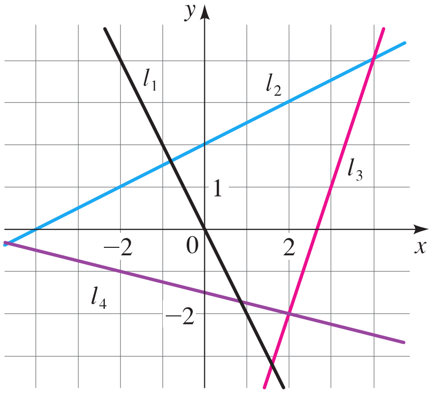
5. $P(2,4)$, $Q(4,3)$

6. $P(2,-5)$, $Q(-4,3)$

7. $P(1,-3)$, $Q(-1,6)$

8. $P(-1,-4)$, $Q(6,0)$

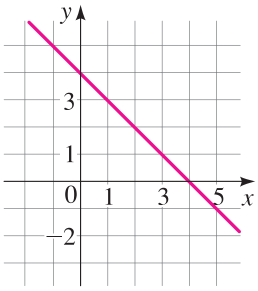
9. Find the slopes of the lines l_1, l_2, l_3, and l_4 in the figure below.



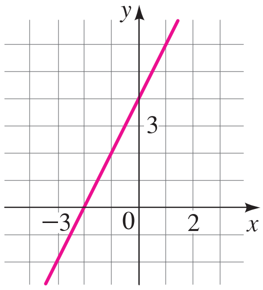
10. **(a)** Sketch lines through $(0,0)$ with slopes $1, 0, \frac{1}{2}, 2$, and -1.
(b) Sketch lines through $(0,0)$ with slopes $\frac{1}{3}, \frac{1}{2}, -\frac{1}{3}$, and 3.

11–14 ■ Find an equation for the line whose graph is sketched.

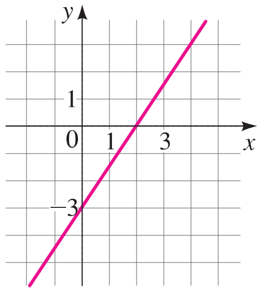
11.



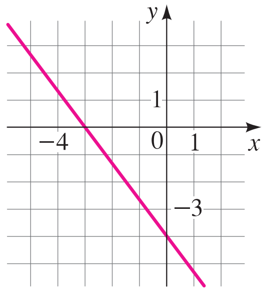
12.



13.



14.



15–34 ■ Find an equation of the line that satisfies the given conditions.

15. Through $(2, 3)$; slope 1

16. Through $(-2, 4)$; slope -1

17. Through $(1, 7)$; slope $\frac{2}{3}$

18. Through $(-3, -5)$; slope $-\frac{7}{2}$

19. Through $(2, 1)$ and $(1, 6)$

20. Through $(-1, -2)$ and $(4, 3)$

21. Slope 3; y-intercept -2

22. Slope $\frac{2}{5}$; y-intercept 4

23. x-intercept 1; y-intercept -3

24. x-intercept -8; y-intercept 6

25. Through $(4, 5)$; parallel to the x-axis

26. Through $(4, 5)$; parallel to the y-axis

27. Through $(1, -6)$; parallel to the line $x + 2y = 6$

28. y-intercept 6; parallel to the line $2x + 3y + 4 = 0$

29. Through $(-1, 2)$; parallel to the line $x = 5$

30. Through $(2, 6)$; perpendicular to the line $y = 1$

31. Through $(-1, -2)$; perpendicular to the line $2x + 5y + 8 = 0$

32. Through $(\frac{1}{2}, -\frac{2}{3})$; perpendicular to the line $4x - 8y = 1$

33. Through $(1, 7)$; parallel to the line passing through $(2, 5)$ and $(-2, 1)$

34. Through $(-2, -11)$; perpendicular to the line passing through $(1, 1)$ and $(5, -1)$

35. **(a)** Sketch the line with slope $\frac{3}{2}$ that passes through the point $(-2, 1)$.

(b) Find an equation for this line.

36. **(a)** Sketch the line with slope -2 that passes through the point $(4, -1)$.

(b) Find an equation for this line.

37–40 ■ Use a graphing device to graph the given family of lines in the same viewing rectangle. What do the lines have in common?

37. $y = -2x + b$ for $b = 0, \pm 1, \pm 3, \pm 6$

38. $y = mx - 3$ for $m = 0, \pm 0.25, \pm 0.75, \pm 1.5$

39. $y = m(x - 3)$ for $m = 0, \pm 0.25, \pm 0.75, \pm 1.5$

40. $y = 2 + m(x + 3)$ for $m = 0, \pm 0.5, \pm 1, \pm 2, \pm 6$

41–52 ■ Find the slope and y-intercept of the line and draw its graph.

41. $x + y = 3$

42. $3x - 2y = 12$

43. $x + 3y = 0$

44. $2x - 5y = 0$

45. $\frac{1}{2}x - \frac{1}{3}y + 1 = 0$

46. $-3x - 5y + 30 = 0$

47. $y = 4$

48. $4y + 8 = 0$

49. $3x - 4y = 12$

50. $x = -5$

51. $3x + 4y - 1 = 0$

52. $4x + 5y = 10$

53. Use slopes to show that $A(1, 1)$, $B(7, 4)$, $C(5, 10)$, and $D(-1, 7)$ are vertices of a parallelogram.

54. Use slopes to show that $A(-3, -1)$, $B(3, 3)$, and $C(-9, 8)$ are vertices of a right triangle.

55. Use slopes to show that $A(1, 1)$, $B(11, 3)$, $C(10, 8)$, and $D(0, 6)$ are vertices of a rectangle.

56. Use slopes to determine whether the given points are collinear (lie on a line).

(a) $(1, 1), (3, 9), (6, 21)$

(b) $(-1, 3), (1, 7), (4, 15)$

57. Find an equation of the perpendicular bisector of the line segment joining the points $A(1, 4)$ and $B(7, -2)$.

58. Find the area of the triangle formed by the coordinate axes and the line

$$2y + 3x - 6 = 0$$

59. **(a)** Show that if the x- and y-intercepts of a line are nonzero numbers a and b, then the equation of the line can be written in the form

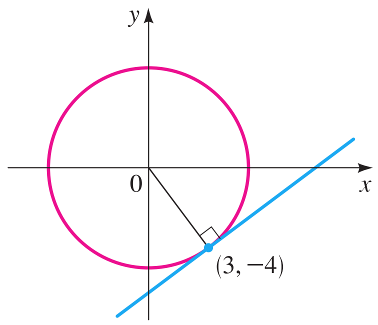
$$\frac{x}{a} + \frac{y}{b} = 1$$

This is called the **two-intercept form** of the equation of a line.

(b) Use part (a) to find an equation of the line whose x-intercept is 6 and whose y-intercept is -8.

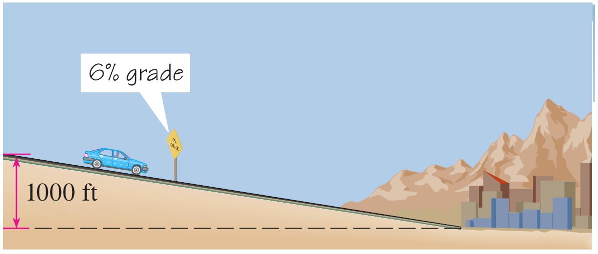
60. **(a)** Find an equation for the line tangent to the circle $x^2 + y^2 = 25$ at the point $(3, -4)$. (See the figure.)

(b) At what other point on the circle will a tangent line be parallel to the tangent line in part (a)?



Applications

61. Grade of a Road West of Albuquerque, New Mexico, Route 40 eastbound is straight and makes a steep descent toward the city. The highway has a 6% grade, which means that its slope is $-\frac{6}{100}$. Driving on this road you notice from elevation signs that you have descended a distance of 1000 ft. What is the change in your horizontal distance?



62. Global Warming Some scientists believe that the average surface temperature of the world has been rising steadily. The average surface temperature is given by

$$T = 0.02t + 8.50$$

where T is temperature in °C and t is years since 1900.

(a) What do the slope and T-intercept represent?

(b) Use the equation to predict the average global surface temperature in 2100.

63. Drug Dosages If the recommended adult dosage for a drug is D (in mg), then to determine the appropriate dosage c for a child of age a, pharmacists use the equation

$$c = 0.0417D(a + 1)$$

Suppose the dosage for an adult is 200 mg.

(a) Find the slope. What does it represent?

(b) What is the dosage for a newborn?

64. Flea Market The manager of a weekend flea market knows from past experience that if she charges x dollars for a rental space at the flea market, then the number y of spaces she can rent is given by the equation $y = 200 - 4x$.

(a) Sketch a graph of this linear equation. (Remember that the rental charge per space and the number of spaces rented must both be nonnegative quantities.)

(b) What do the slope, the y-intercept, and the x-intercept of the graph represent?

65. Production Cost A small-appliance manufacturer finds that if he produces x toaster ovens in a month his production cost is given by the equation

$$y = 6x + 3000$$

(where y is measured in dollars).

(a) Sketch a graph of this linear equation.

(b) What do the slope and y-intercept of the graph represent?

66. Temperature Scales The relationship between the Fahrenheit (F) and Celsius (C) temperature scales is given by the equation $F = \frac{9}{5}C + 32$.

(a) Complete the table to compare the two scales at the given values.

(b) Find the temperature at which the scales agree. [*Hint:* Suppose that a is the temperature at which the scales agree. Set $F = a$ and $C = a$. Then solve for a.]

C	F
−30°	
−20°	
−10°	
0°	
	50°
	68°
	86°

67. Crickets and Temperature Biologists have observed that the chirping rate of crickets of a certain species is related to temperature, and the relationship appears to be very nearly linear. A cricket produces 120 chirps per minute at 70°F and 168 chirps per minute at 80°F.

(a) Find the linear equation that relates the temperature t and the number of chirps per minute n.

(b) If the crickets are chirping at 150 chirps per minute, estimate the temperature.

68. Depreciation A small business buys a computer for \$4000. After 4 years the value of the computer is expected to be \$200. For accounting purposes, the business uses *linear depreciation* to assess the value of the computer at a given time. This means that if V is the value of the computer at time t, then a linear equation is used to relate V and t.

(a) Find a linear equation that relates V and t.

(b) Sketch a graph of this linear equation.

(c) What do the slope and V-intercept of the graph represent?

(d) Find the depreciated value of the computer 3 years from the date of purchase.

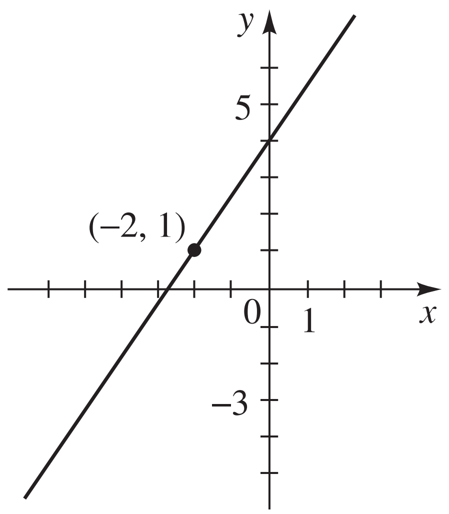
69. Pressure and Depth At the surface of the ocean, the water pressure is the same as the air pressure above the water, 15 lb/in^2. Below the surface, the water pressure increases by 4.34 lb/in^2 for every 10 ft of descent.

(a) Find an equation for the relationship between pressure and depth below the ocean surface.

(b) Sketch a graph of this linear equation.

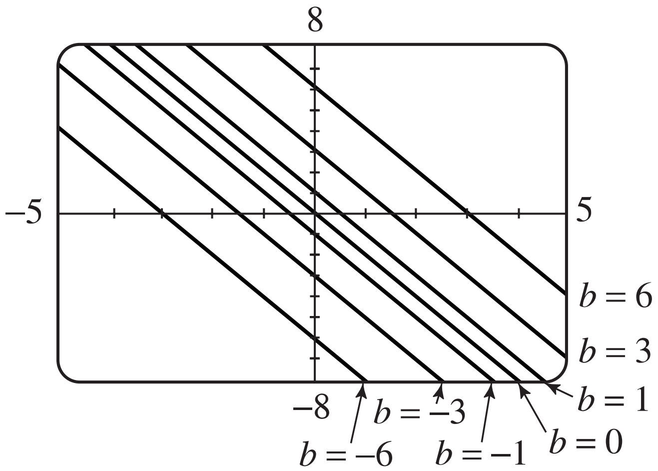
(c) What do the slope and y-intercept of the graph represent?

1. $\frac{1}{2}$ **3.** $\frac{1}{6}$ **5.** $-\frac{1}{2}$ **7.** $-\frac{9}{2}$ **9.** $-2, \frac{1}{2}, 3, -\frac{1}{4}$
11. $x + y - 4 = 0$ **13.** $3x - 2y - 6 = 0$
15. $x - y + 1 = 0$ **17.** $2x - 3y + 19 = 0$
19. $5x + y - 11 = 0$ **21.** $3x - y - 2 = 0$
23. $3x - y - 3 = 0$ **25.** $y = 5$ **27.** $x + 2y + 11 = 0$
29. $x = -1$ **31.** $5x - 2y + 1 = 0$
33. $x - y + 6 = 0$
35. (a)

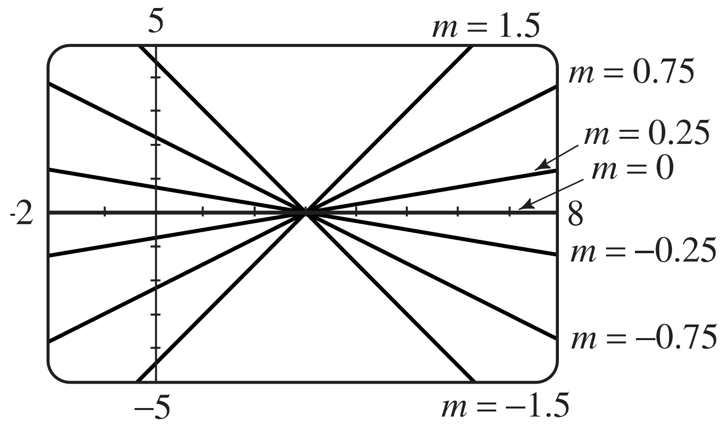


(b) $3x - 2y + 8 = 0$

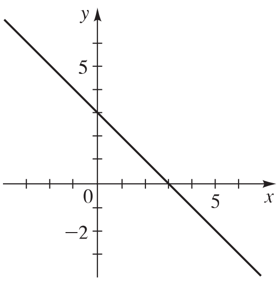
37. They all have the same slope.

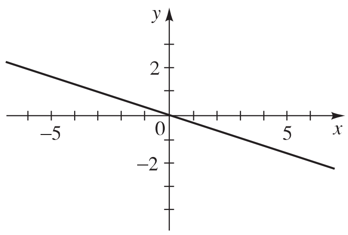


39. They all have the same x-intercept.

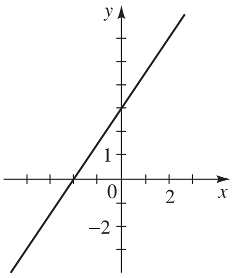


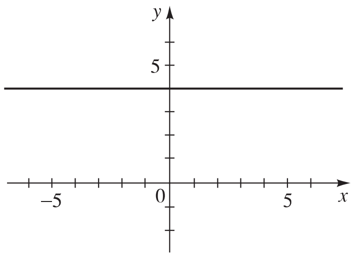
41. $-1, 3$ **43.** $-\frac{1}{3}, 0$



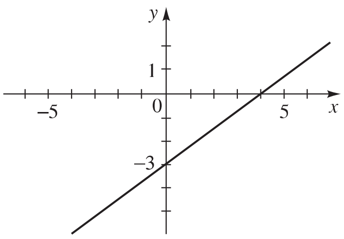


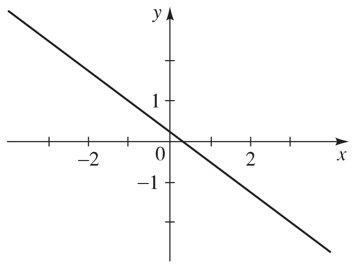
45. $\frac{3}{2}, 3$ **47.** $0, 4$





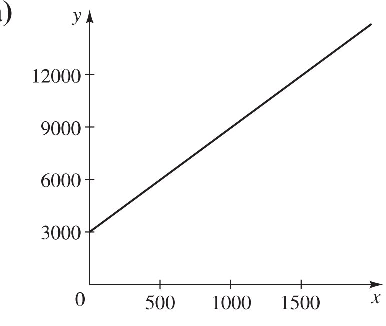
49. $\frac{3}{4}, -3$ **51.** $-\frac{3}{4}, \frac{1}{4}$





57. $x - y - 3 = 0$ **59.** **(b)** $4x - 3y - 24 = 0$
61. 16,667 ft **63.** **(a)** 8.34; the slope represents the increase in dosage for a one-year increase in age. **(b)** 8.34 mg

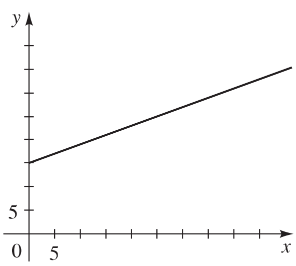
65. **(a)**



(b) The slope represents production cost per toaster; the y-intercept represents monthly fixed cost.

67. **(a)** $t = \frac{5}{24}n + 45$ **(b)** 76°F
69. **(a)** $P = 0.434d + 15$, where P is pressure in lb/in^2 and d is depth in feet

(b)



(c) The slope is the rate of increase in water pressure, and the y-intercept is the air pressure at the surface. **(d)** 196 ft

TOPIC 3.3 : COORDINATE GEOMETRY

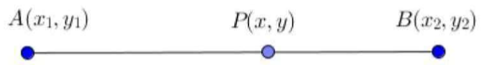
3.3.1: Internal division of Line Segment with Section Formula (in the ratio *m:n*)

This **section formula** tells us the coordinates of the points which divides a given line segment into two parts such that their lengths are in the ratio *m* : *n*.

The **midpoint of a line segment** is the point that divides a line segment in two **equal** halves. The section formula builds on it and is a more powerful tool; it locates the point dividing the line segment in any desired ratio.

Internal Divisions with Section Formula

THEOREM

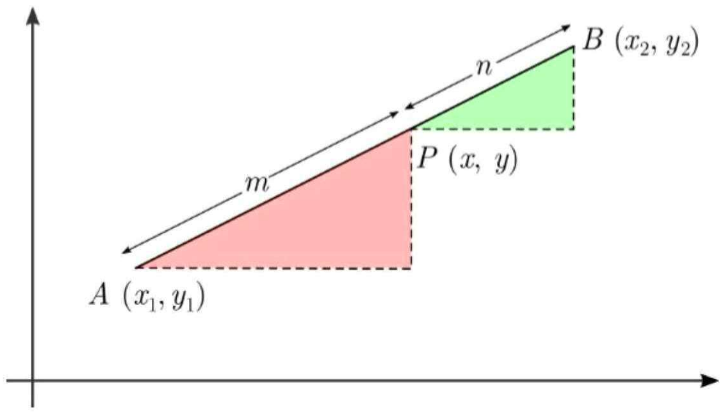


If point $P(x, y)$ lies on line segment $\overline{AB}$ (between points A and B) and satisfies $AP : PB = m : n$, then we say that P divides $\overline{AB}$ internally in the ratio $m : n$. The coordinates of the point of division has the coordinates

$$P = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right).$$

PROOF

The formula can be derived by constructing two similar right triangles as shown below. Their hypotenuses are along the line segment, and are in the ratio $m : n$.



The red and the green triangles are similar since the corresponding angles of the triangles are equal. This implies that the ratio of their corresponding sides are equal. Note that point P is $\frac{m}{m+n} \times AB$ away from A. That is,

$$\begin{aligned} x &= x_1 + \frac{m}{m+n}(x_2 - x_1) \\ &= \frac{(m+n)x_1 + mx_2 - mx_1}{m+n} \\ &= \frac{mx_2 + nx_1}{m+n} \qquad (1) \end{aligned}$$

Similarly, solving for y gives

$$y = \frac{my_2 + ny_1}{m+n} \qquad (2)$$

Therefore, from (1) and (2)

$$P(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right).$$

As a special case of internal division, if P is the midpoint of $\overline{AB}$, then it divides $\overline{AB}$ internally in the ratio $1 : 1$. Hence applying the formula for internal division and substituting $m = n = 1$, we get

$$P = \left(\frac{x_1 + y_1}{2}, \frac{x_2 + y_2}{2}\right).$$

EXAMPLE

Given $A = (-3, 1)$ and $B = (3, -6)$, what are the coordinates of the point $P = (x, y)$ which internally divides line segment $\overline{AB}$ in the ratio $1 : 2$?

Solution

EXAMPLE

Given $A = (-3, 6)$, what are the coordinates of $B = (x_2, y_2)$ if point $P = (-2, 4)$ divides line segment $\overline{AB}$ internally in the ratio $1 : 3$?

Solution

EXAMPLE

Given $A(-2, -1)$ and $B(4, 11)$, point $P(x, y)$ internally divides line segment $\overline{AB}$ in the ratio $m : n$. If P is the intersection point of $\overline{AB}$ and the y-axis, what is the value of $m : n$?

Solution

EXAMPLE

In what ratio does the point $P(-3, 7)$ divide the line segment joining $A(-5, 11)$ and $B(4, -7)$?

Solution

3.3.2: Area of Polygons and Shoelace Formula

The shoelace formula (also known as **Gauss**'s area formula and the **surveyor**'s formula) is a mathematical formula to determine the **area of a simple polygon** whose vertices are described by their Cartesian coordinates in the plane. The user cross-multiplies corresponding coordinates to find the area encompassing the polygon, and subtracts it from the surrounding polygon to find the area of the polygon within. It is called the shoelace formula because of the constant cross-multiplying for the coordinates making up the polygon, like tying shoelaces. It is also sometimes called the shoelace method. It has applications in surveying and forestry, among other areas.

The area formula is valid for any non-self-intersecting (simple) polygon, which can be convex or concave.

Definition

The formula can be represented by the expression

$$\text{Area} = \frac{1}{2}\left|\sum_{i=1}^{n} \det\begin{pmatrix} x_i & x_{i+1} \\ y_i & y_{i+1} \end{pmatrix}\right|$$

alternatively

$$\text{Area} = \frac{1}{2}\left|\sum_{i=1}^{n-1} x_i y_{i+1} + x_n y_1 - \sum_{i=1}^{n-1} x_{i+1} y_i - x_1 y_n\right|$$

$$\text{Area} = \frac{1}{2}\left|x_1 y_2 + x_2 y_3 + \ldots + x_{n-1} y_n + x_n y_1 - x_2 y_1 - x_3 y_2 - \ldots - x_n y_{n-1} - x_1 y_n\right|$$

where

n is the number of sides of the polygon,

(x_i, y_i), $i = 1, 2, \ldots, n$ are the vertices (or "corners") of the polygon, and

$x_{n+1} = x_1$ and $x_0 = x_n$, as well as $y_{n+1} = y_1$ and $y_0 = y_n$.

If the points are labeled sequentially in the counterclockwise direction, then the above determinants are positive and the absolute value signs can be omitted; if they are labeled in the clockwise direction, the determinants will be negative, and the absolute value signs can be applied.

Examples

The user must know the points of the polygon in a Cartesian plane. For example, take a triangle with coordinates {(2, 1), (4, 5), (7, 8)}. Take the first x-coordinate and multiply it by the second y-value, then take the second x-coordinate and multiply it by the third y-value, and repeat, and repeat again, until you do it for all points. This can be defined by this formula:

$$\mathbf{A}_{\text{tri.}} = \frac{1}{2}|x_1y_2 + x_2y_3 + x_3y_1 - x_2y_1 - x_3y_2 - x_1y_3|$$

for x_i and y_i representing each respective coordinate. This formula is just the expansion of those given above for the case n = 3. Using it, one can find that:

Area of the triangle

$$=\frac{1}{2}|2(5)+5(8)+7(1)-4(1)-7(5)-2(8)|$$
$$=\frac{1}{2}|10+32+7-4-35-16|$$
$$=\frac{1}{2}|-6|$$
$$=\frac{1}{2}(6)=3unit$$

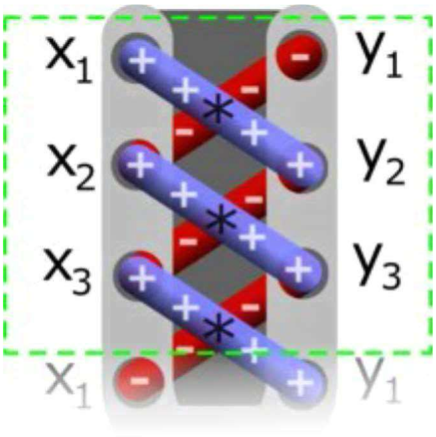
The number of variables depends on the number of sides of the polygon. For example, a pentagon will be defined up to x_5 and y_5:

$$\mathbf{A}_{\text{pent.}} = \frac{1}{2}|x_1y_2 + x_2y_3 + x_3y_4 + x_4y_5 + x_5y_1 - x_2y_1 - x_3y_2 - x_4y_3 - x_5y_4 - x_1y_5|$$

A quadrilateral will be defined up to x_4 and y_4:

$$\mathbf{A}_{\text{quad.}} = \frac{1}{2}|x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1 - x_2y_1 - x_3y_2 - x_4y_3 - x_1y_4|$$

Explanation of name



The reason this formula is called the shoelace formula is because of a common method used to evaluate it. This method uses matrices. As an example, choose the triangle with vertices (2,4), (3,−8), and (1,2). Then construct the following matrix by “walking around” the triangle and ending with the initial point.

First, draw diagonal down and to the right slashes (as shown below),

$$\begin{bmatrix} 2 & 4 \\ 3 & -8 \\ 1 & 2 \\ 2 & 4 \end{bmatrix}$$

and multiply the two numbers connected by each slash, then add all the products:
(2 × −8) + (3 × 2) + (1 × 4) = −6. Do the same thing with slashes diagonal down and to the left (shown below with downwards slashes):

$$\begin{bmatrix} 2 & 4 \\ 3 & -8 \\ 1 & 2 \\ 2 & 4 \end{bmatrix}$$

(4 × 3) + (−8 × 1) + (2 × 2) = 8. Then take the difference of these two numbers: |(−6)−(8)| = 14. Halving this gives the area of the triangle: 7. Organizing the numbers like this makes the formula easier to recall and evaluate. With all the slashes drawn, the matrix loosely resembles a shoe with the laces done up, giving rise to the formula’s name.

3.3.3: SCATTER PLOTS AND THE LINE OF BEST FIT

Scatter Plot

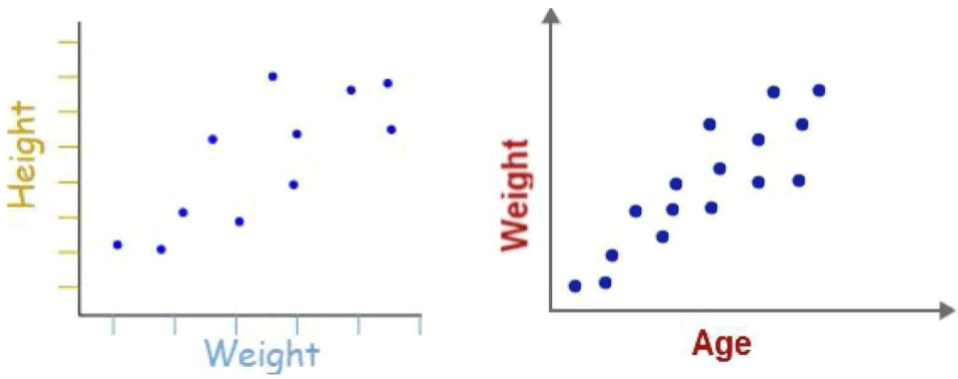
Scatter Plots are the graphs that show the relationship between two variables. It displays data points on a two-dimensional graph on Cartesian system. The variable that is independent or explanatory is plotted on *x*-axis, while another one is plotted on *y*-axis.

Scatter plots are quite useful in following condition:

- Then there is a large set of data points given
- Each set contains a pair of values
- The given data is strictly numeric.

Scatter plots immediately gives information about a large amount of data. It is also known as **scatter graph** or **scatter diagram**.

Samples of scatter plots are shown below.



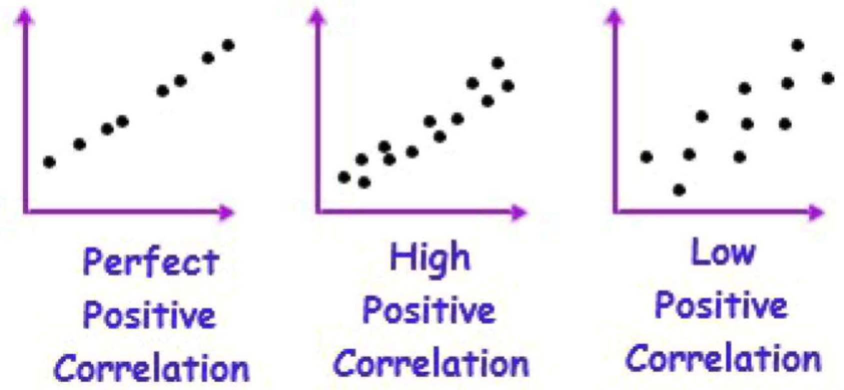
The scatter plot shows **correlation** between two variables. It provides information about how closely are two variables connected to each other. There can arise 3 situations:

- Positive Correlation
- Negative Correlation
- No correlation

A scatter plot shows **positive correlation** when the points are overall rising, moving from left to right, i.e. the values increase. There are following three types of positive correlations.

- Perfect Positive - An almost perfect straight line.
- High Positive - All points near to one another.
- Low Positive - Scattered points.

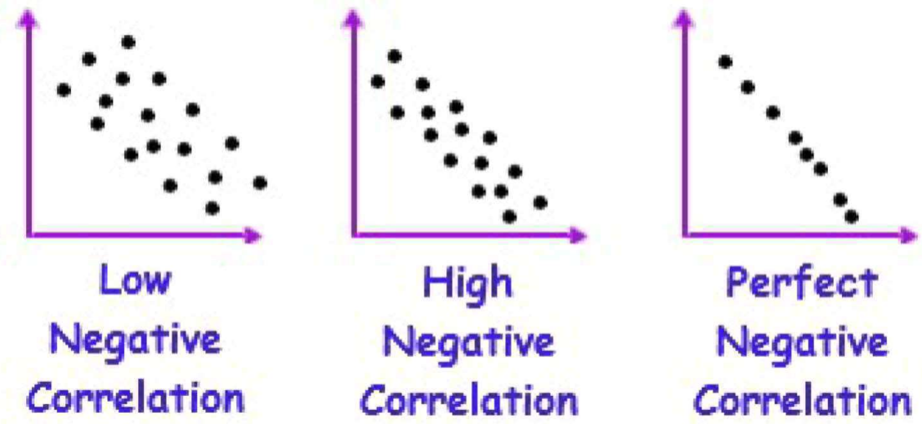
These are shown in the following diagrams below:



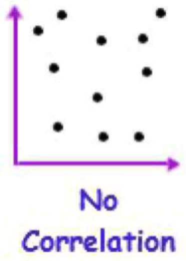
A scatter plot represents **negative correlation** when the points fall when we move from left to right, i.e. values are decreasing. There are three types of negative correlations -

- Perfect Negative - Almost a straight line.
- High Negative - Points nearer to one another.
- Low Negative - Points are scattered.

These are demonstrated below -



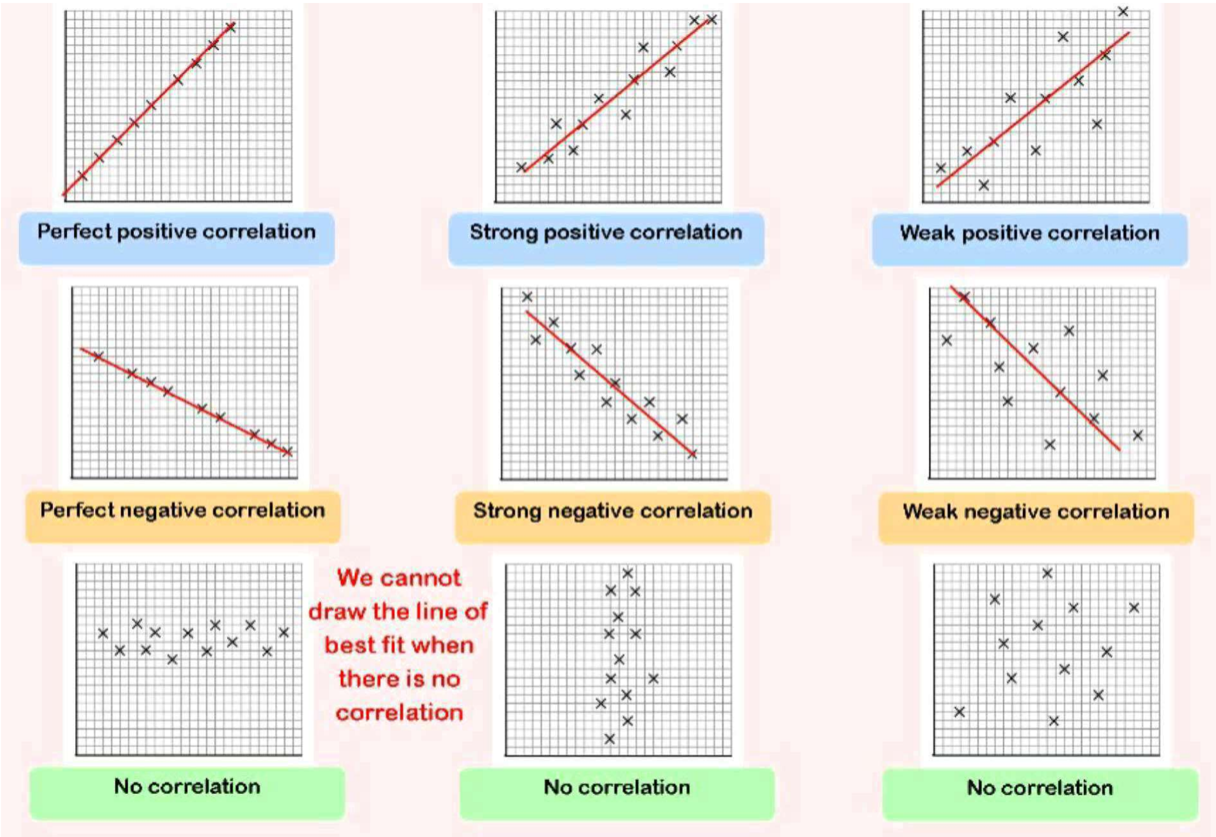
There is **no correlation** when all the points are scattered all over the graph and we are not able to conclude whether all points are overall rising or overall falling. No correlation scatter plot is shown down:



Line of Best Fit

A **line of best fit** (or "trend" line) is a straight line that best represents the data on a scatter plot. The line goes roughly through the middle of all the scatter points on a graph.
This line may pass through some of the points, none of the points, or all of the points.

The closer the points are to the line of best fit, the stronger we can say the correlation is.



Students will learn more on finding the correlation coefficient in Mathematics III later.

The line of best fit is drawn so that the points are evenly distributed on either side of the line. There are various methods for drawing this "precisely", for example by using

i) paper and pencil only,

ii) a combination of graphing calculator and paper and pencil,

iii) or solely with graphing calculator.

But we will only be expected to draw the line "by eye".

Interpolation and Extrapolation

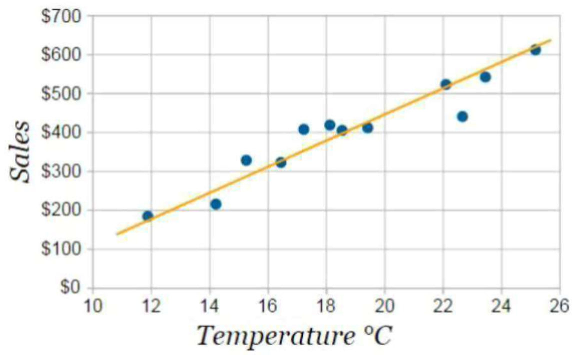
Interpolation is where we find a value inside our set of data points.

Extrapolation is where we find a value **outside** our set of data points.

Example:
The local ice cream shop keeps track of how much ice cream they sell versus the noon temperature on that day. Here are their figures for the last 12 days:

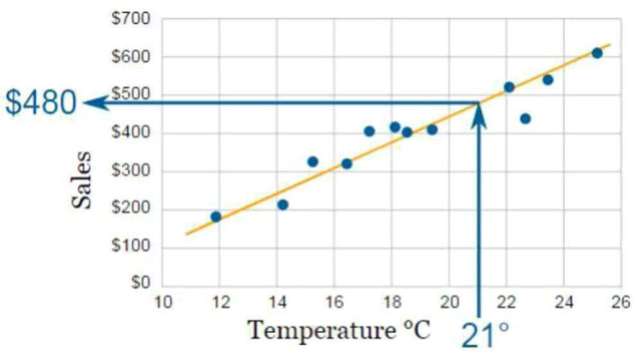
Ice Cream Sales vs Temperature	
Temperature °C	**Ice Cream Sales**
14.2°	$215
16.4°	$325
11.9°	$185
15.2°	$332
18.5°	$406
22.1°	$522
19.4°	$412
25.1°	$614
23.4°	$544
18.1°	$421
22.6°	$445
17.2°	$408

And here is the same data as a scatter plot together with the line of best fit.

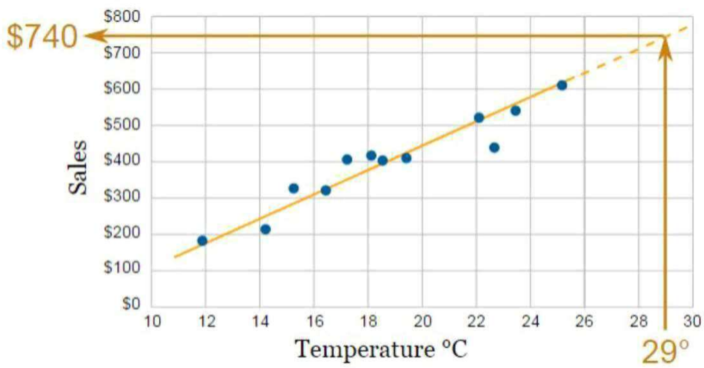


It is now easy to see that **warmer weather leads to more sales**, but the relationship is not perfect.

Here we use **linear interpolation** to estimate the sales at 21 °C.



Here we use **linear extrapolation** to estimate the sales at 29 °C (which is higher than any value we have).



Careful: **Extrapolation** can give misleading results because we are in "uncharted territory".

The line of best fit equation.

As well as using a graph (like above) we can create a formula to help us.

Example: Straight Line Equation

We can estimate a straight line equation from two points from the graph above
Let's estimate two points on the line near actual values: **(12°, $180)** and **(25°, $610).**

First, find the slope:

$$m = \frac{610-180}{25-12} = \frac{430}{13} = 33(rounded)$$

Now we put the slope and the point $(12,\ 180)$ into the point-slope formula:

$$y - 180 = 33(x - 12)$$
$$y = 33x - 396 + 180$$
$$y = 33x - 216$$

Now we can use the equation to **interpolate a sales at** $21^0 C$**:**

$$y = 33(21) - 216 = \$477$$

Now we can use the equation to **extrapolate** a sales value at $29^0 C$:

$$y = 33(29) - 216 = \$741$$

The values are too close to what we got on the graph. But that doesn't mean the are more (or less) accurate. They are all just estimates.

Don't use extrapolation too far!! What sales would you expect at $0^0 C$?

$$y = 33(0) - 216 = -\$216$$

Hmmm…Minus $216?? We extrapolated too far!!

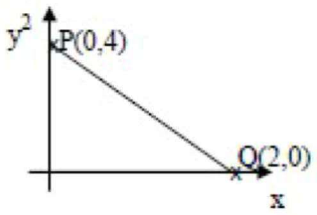
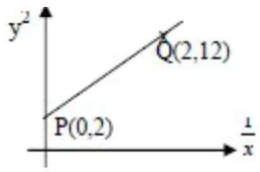
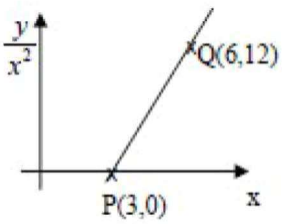
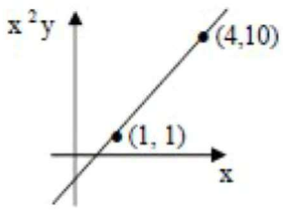
Note: we used **linear** (based on a **line**) interpolation and extrapolation, but there are many other types, for example we could use polynomials to make curvy lines, etc.

3.3.4: Converting Non-Linear Equations to Linear Form

Reduce each of the following equations to the form $Y = mX + c$, where *a* and *b* are constants.

No	Non-Linear Equation	Linear Equation	Y	X	m	c
1	$y = ax^2 + bx$					b
2	$y = ax^3 + bx^2$	$\frac{y}{x^2} = ax + b$				
3	$y = \frac{a}{x} + b$		Y			
4	$y = \frac{a}{x} + bx$					a
5	$xy = \frac{a}{x} + bx$			$\frac{1}{x^2}$		
6	$x + by = axy$		$\frac{1}{y}$			
7	$y = \frac{5}{x} - 3x$					5
8	$y = \frac{a(b-x)}{x^2}$		xy			

Given the line of best fit.

<table>
<tr>
<td>1. The diagram below shows a line of best fit for the graph of y^2 against x. Determine the non-linear equation relating y and x.

$y^2 = -2x + 4$</td>
<td>2. The diagram below shows a line of best fit for the graph of y^2 against $\frac{1}{x}$. Determine the non-linear equation relating y and x.

$y^2 = 5x + 2$</td>
</tr>
<tr>
<td>3. The diagram below shows a line of best fit for the graph of $\frac{y}{x^2}$ against x. Determine the non-linear equation relating y and x.

$\frac{y}{x^2} = 4x - 12$</td>
<td>4. The diagram below shows a line of best fit for the graph of $x^2 y$ against x. Express y in terms of x.

$y = \frac{3}{x} - \frac{2}{x^2}$</td>
</tr>
</table>

3.3.5: Locus Problem

Locus of a point $P(x,\ y)$ is a path travelled by the point which moves under a given condition.

An equation of a locus involving the distance between two points can be determined by using the distance formula.

Note : Students MUST be able to find distance between two points [using Pythagoras Theorem]

TASK : To Find the equation of the locus of the moving point P such that its distances of P from the points Q and R are equal.

Eg 1. Q(6, -5) and R(1,9)

Let P = (x,y), then PQ = PR

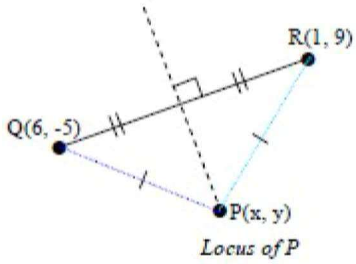
$$\sqrt{(x-6)^2+(y-(-5))^2}=\sqrt{(x-1)^2+(y-9)^2}$$

Square both sides to eliminate the square roots.

$$(x-6)^2+(y+5)^2=(x-1)^2+(y-9)^2$$

$$x^2-12x+36+y^2+10y+25=x^2-2x+1+y^2-18y+81$$

$$10x-28y+21=0$$



1. $Q(2,\ 5)$ and $R(4,\ 2)$ $4x-6y+9=0$	2. $Q(-3,\ 0)$ and $R(6,\ 4)$ $18x+8y-43=0$
3. $Q(2,\ -3)$ and $R(-4,\ 5)$ $3x-4y+7=0$	4. $Q(6,\ -2)$ and $R(0,\ 2)$ $3x-2y-9=0$

TASK : To find the equation of the locus of the moving point P such that its distances from the points A and B are in the ratio m : n

(Note : Sketch a diagram to help you using the distance formula correctly)

Eg 1. A(-2,3), B(4,8) and m : n = 1: 2

Let P = (x, y)

$$\frac{LP}{PM} = \frac{1}{2}$$

$$2LK = KM$$

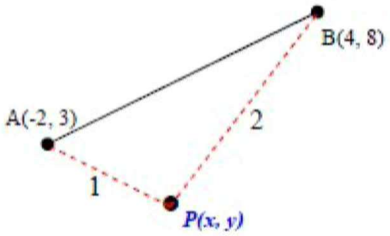
$$2\sqrt{(x-(-2))^2+(y-3)^2} = \sqrt{(x-4)^2+(y-8)^2}$$

$$(2)^2\left(\sqrt{(x+2)^2+(y-3)^2}\right)^2 = (x-4)^2+(y-8)^2$$

$$4\left((x+2)^2+(y-3)^2\right) = (x-4)^2+(y-8)^2$$

$$4x^2+16x+16+4y^2-24y+36 = x^2-8x+16+y^2-16y+64$$

$3x^2+3y^2+24x-8y-28=0$ is the equation of locus of P.



1. $A(1,\ 5)$, $B(4,\ 2)$ and $m:n=2:1$ $x^2+y^2-10x-2y+18=0$	2. $A(-3,\ 2)$, $B(3,\ 2)$ and $m:n=2:1$ $x^2+y^2-10x-4y+13=0$
3. $A(-1,\ 3)$, $B(4,\ -2)$ and $m:n=2:3$ $x^2+y^2+10x-14y+2=0$	4. $A(1,\ 5)$, $B(-4,\ -5)$ and $m:n=3:2$ $x^2+y^2+16x+26y+53=0$

EXERCISE

1. Given that EF is a straight line such that the coordinates of the points E and F are (–1, 4) and (5, 12) respectively. Find the equation of the locus of a point that moves so that it is equidistant from point E and point F.

2 Find the equation of a moving point $Q(x, y)$ which is equidistant from point $A(1, 2)$ and point $B(0, 3)$.

3 Find the equation of the straight line that passes through the point $P(-4, 1)$ and is parallel to the line $2x - 5y + 3 = 0$

4 Given that $P(h, -3)$, $Q(2, -1)$ and R (8, 1) are collinear. Find the value of h.

5 Given that the point P moves so that its distance from the point $A(-2, 3)$ is 4 units. Find the equation of the locus of the point A.

6 Given that PQR is a straight line and the coordinates of the points P and Q are (–1, 2) and (–3, –1) respectively. If $PQ : QR = 1 : 3$, find the coordinates of the point R .

7 C is a point on the straight line that joins $A(-3, 6)$ and $B(7, 1)$ such that $3AC = 2CB$, find the coordinates of the point C.

8 The coordinates of the three vertices of a triangle are (1, 5), $(-2b, a)$ and $(-2a, b)$. Given that $a + b = 0$ and the area of the triangle is 18 unit^2. Find the value of a and of b.

9 Find the equation of the straight line that passes through the point (–1, 5) and is perpendicular to the straight line that joins point (2, –6) and point (3, 8). [4 *marks*]

10 $A(2h, h)$, $B(p, t)$, and $C(2p, 3t)$ are three points on a straight line. B divides AC internally in the ratio 2 : 3. Express p in terms of t. [4 *marks*]

11 Diagram 1 shows the straight line *AB* which is perpendicular to the straight line *CB* at the point *B*.

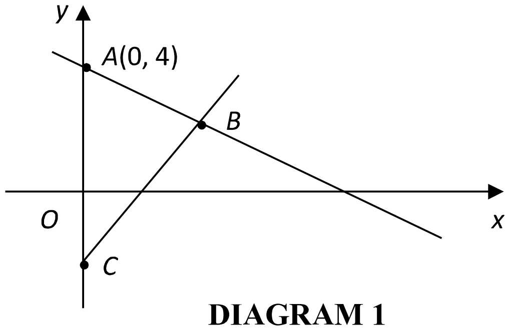


DIAGRAM 1

The equation of the straight line *CB* is $y = 2x - 1$.
Find the coordinates of *B*.

12 In Diagram 2, $\angle ABC = 90^\circ$, and the equation of the straight line *BC* is $2y + x + 6 = 0$.

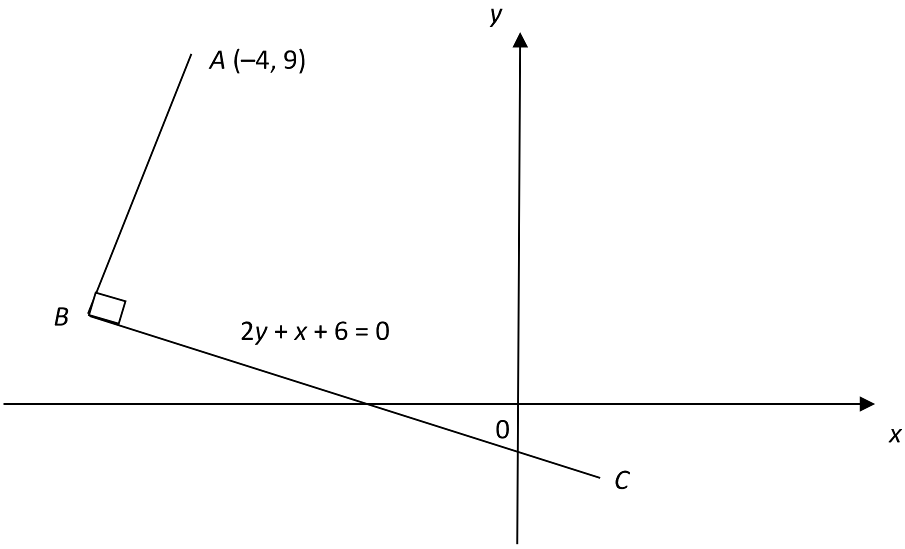


DIAGRAM 2

(a) Find
  (i) the equation of the straight line *AB*,
  (ii) the coordinates of point *B*.

(b) The straight line *AB* is extended to a point *D* such that *AB* : *BD* = 2 : 3.
  Find the coordinates of the point *D*.

(c) A point *P* moves such that its distance from point *A* is always 5 units. Find the equation of the locus of *P*.

13 *Solution to this question by scale drawing will not be accepted*.

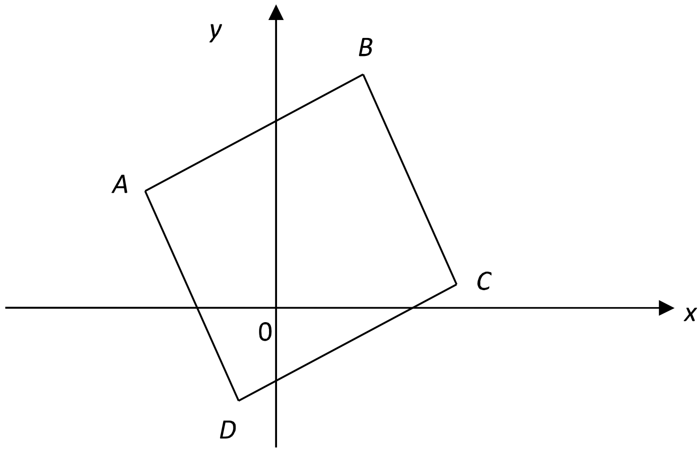


DIAGRAM 3

Diagram 3 shows a parallelogram *ABCD*. The coordinates of points *A, B* and *C* are (–4, 4), (2, 9) and (4, 1) respectively. Find

(a) the coordinates the point of intersection of the diagonals *AC* and *BD*.]

(b) the coordinates of the point *D*.

(c) the area of the parallelogram *ABCD*.

(d) the equation of the locus of a moving point *P*, such that $3AP = 2CP$.

14 *Solution to this question by scale drawing will not be accepted.*
Diagram 4 shows the triangle *AOB* where *O* is the origin. Point *C* lies on the straight line *AB*.

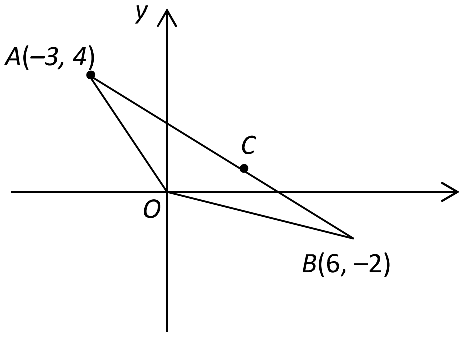


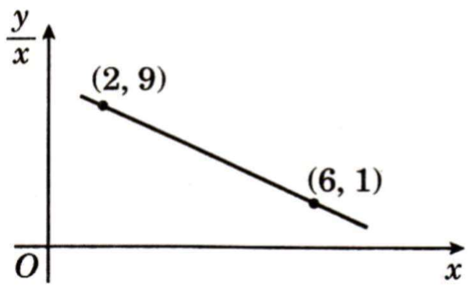
DIAGRAM 4

(a) Calculate the area, in unit^2, of triangle *AOB*.

(b) Given that $AC : CB = 3 : 2$, find the coordinates of *C*.

(c) A point *P* moves such that its distance from point *A* is always twice its distance from point *B*.
(i) Find the equation of the locus of *P*.
(ii) Hence, determine whether or not this locus intersects the *y*-axis.

15) x and y are related by the equation $y = px^2 + qx$, where p and q are constants. A straight line is obtained by plotting $\frac{y}{x}$ against x, as shown in Diagram 1. Calculate the value of p and q.



Answer	
1 $4y + 3x = 38$	13 (a) $(0, \frac{5}{2})$
2 $y = x + 2$	(b) $D(-2, -4)$
3 $5y = 2x + 13$	(c) 58 unit^2
4 $h = -4$	(d) $5x^2 + 5y^2 + 104x - 64y + 220 = 0$
5 $x^2 + y^2 + 4x - 6y - 3 = 0$	14 (a) 9 unit^2
6 $R(-9, -10)$	(b) $C(\frac{12}{5}, \frac{2}{5})$
7 $C(1, 4)$	(c) (i) $x^2 - 18x + y^2 + 8y + 45 = 0$,
8 $a = 2$, $b = -2$; $a = -2$, $b = 2$	(ii) No
9 $14y + x = 69$	15 $p = -2, q = 13$
10 $p = -2t$	
11 $B(2, 3)$	
12 (a) (i) $y = 2x + 17$	
(ii) $B(-8, 1)$	
(b) $D(-14, -11)$	
(c) $x^2 + y^2 + 8x - 18y + 72 = 0$	