HW2 Solutions, M22A, Spring Quarter 2014, Prof. Sornborger
2.4: 2,5,6,11,14,17,21,26,31
2.5: 1,2,7,13,22,23,27,29
2.6: 1,3,4,5,7,10,12,15
2.7: 1,2,6,12,15,16

2.4: 2,5,6,11,14,17,21,26,31

2 A is a 3x5 matrix, B and 5x3, C a 5x1, and D is 3x1.
What rows or columns of matrices do you multiply to find

(a) the third column of AB
(b) the first row of AB
(c) the entry in row 3, column 4 of AB
(d) the entry in row 1, column 1 of CDE

5 Compute A^2 and A^3, make a predicition for A^5 and A^n.

$$A = \begin{bmatrix} 1, b \\ 0, 1 \end{bmatrix}$$

and

$$A = \begin{bmatrix} 2, 2 \\ 0, 0 \end{bmatrix}$$

6 Show that $(A + B)^2$ is different from $A^2 + 2AB + B^2$ when

$$A = \begin{bmatrix} 1, 2 \\ 0, 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1, 0 \\ 3, 0 \end{bmatrix}$$

11 Choose the only B so that every matrix A

(a) BA = 4A
(b) BA = 4B
(c) BA has rows 1 and 3 of A reverse and row 2 unchanged
(d) All rows of BA are the same row 1 of A.

14 True or false:

(a) if A^2 is defined then A is necessarily quire
(b) if AB and BA are defined the A and B are square
(c) If AB and BA are defined then AB and BA are square
(d) If AB = B then A =I

17 Write down a 3 by 3 matrix A whose entries are

(a) a_{ij} = minimum of i and j
(b) $a_{ij} = (-1)^{i+j}$
(c) $a_{ij} = 1/j$

21 Find all the powers $A^2, A^3, ...$ and $AB, (AB)^2 ...$ for

$$A = \begin{bmatrix} .5, .5 \\ .5, .5 \end{bmatrix}, \quad B = \begin{bmatrix} 1, 0 \\ 0, -1 \end{bmatrix}$$

26 Multiply AB using columns times rows

$$AB = \begin{bmatrix} 1,0 \\ 2,4 \\ 2,1 \end{bmatrix} \begin{bmatrix} 3,3,0 \\ 1,2,1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \begin{bmatrix} 3,3,0 \end{bmatrix} + BLANK = BLANK$$

31 With $i^2 = -1$ the product of of $A + iB$ and $x + iy$ is $Ax + iBx + iAy - By$. Use blocks to separate the real part without i from the imaginary party that multiplies i

$$\begin{bmatrix} A, -B \\ ?, ? \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} Ax - By \\ ? \end{bmatrix}$$

2.5: 1,2,7,13,22,23,27,29

1 Find the inverses of A,B,C

$$A = \begin{bmatrix} 0, 3 \\ 4, 0 \end{bmatrix}, \quad B = \begin{bmatrix} 2, 0 \\ 4, 2 \end{bmatrix}, \quad C = \begin{bmatrix} 3, 4 \\ 5, 7 \end{bmatrix}$$

2 For these permutation matrices find P^{-1} by trial and error (with 1's and 0's)

$$P = \begin{bmatrix} 0, 0, 1 \\ 0, 1, 0 \\ 1, 0, 0 \end{bmatrix}, \quad P = \begin{bmatrix} 0, 1, 0 \\ 0, 0, 1 \\ 1, 0, 0 \end{bmatrix}$$

7 If A has row 1 + row 2 = row 3 show that A is not invertible

(a) Explain why Ax = (1,0,0) cannot have a solution

(b) Which right sides (b_1, b_2, b_3) might allow a solution to Ax = b

(c) what happens to row 3 in elimination?

13 If the product $M = ABC$ of three square matrices is invertible. then B is invertible (so are A and C), find a formula for B^{-1} that involves M^{-1} and A and C.

22 Change I into A^{-1} as you reduce A to I by row operations

$$[A, I] = \begin{bmatrix} 1, 3, 1, 0 \\ 2, 7, 0, 1 \end{bmatrix}$$

and

$$[A, I] = \begin{bmatrix} 1, 4, 1, 0 \\ 3, 9, 0, 1 \end{bmatrix}$$

23 Follow the 3 by 3 text example but with plus signs in A, eliminate above and below the pivots to reduce $[A\ I]$ to $[I\ A^{-1}]$

27 Invert these matrices by the Gauss Jordan method starting with [A I]

$$A = \begin{bmatrix} 2, 1, 1 \\ 1, 2, 1 \\ 1, 1, 2 \end{bmatrix}, \quad A = \begin{bmatrix} 1, 1, 1 \\ 1, 2, 2 \\ 1, 2, 3 \end{bmatrix}$$

29 Exchange rows and continue with Gauss Jordan to find A^{-1}

$$[A\ I] = \begin{bmatrix} 0, 2, 1, 0 \\ 2, 2, 0, 1 \end{bmatrix}$$

2.6: 1,3,4,5,7,10,12,15

1.

2.7: 1,2,6,12,15,16

1 Forward elimination changes $\begin{bmatrix} 1.1 \\ 1,2 \end{bmatrix} x = b$ into triangular $\begin{bmatrix} 1,1 \\ 0,1 \end{bmatrix} x = c$

$$x + y = 5, \quad x + 2y = 7 \to x + y = 5, \quad y = 2$$

$$\begin{bmatrix} 1,1,5 \\ 1,2,7 \end{bmatrix} \to \begin{bmatrix} 1,1,5 \\ 0,1,2 \end{bmatrix}$$

That step subtracted l_{21} = BLANK times row 1 from row 2. The reverse step adds l_{21} times row 1 to row 2 . The matrix for that reverse step is $L = BLANK$. Multiply this L times the triangular system $\begin{bmatrix} 1,1 \\ 0,1 \end{bmatrix} x_1 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ to get BLANK = BLANK. In letters L multiplies Ux = c to give BLANK

2 Write down the 2 by 2 triangular systems Lc= b and Ux =c from problem 1. Check that c = (5,2) solves the first one. Find the x that solves the second one.

6 What two elimination matrices E_{21} and E_{32} put A into upper triangular form $E_{32}E_{21}A = U$? Multiply by

12

15

16