# Countable Sets

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## 7.1

If we use the natrual set of $\mathbb{N}_0$ and $\mathbb{N}_1$ Where the subscript determines if they either start at zero or one. We know that $\mathbb{N}_0$ always has one less member than $\mathbb{N}_0$ even know $\mathbb{N}_1 \in \mathbb{N}_0$

## 7.3

Jhonny assumtions about T and S make it the same as the example in 7.1. The size of the sets are not equal making it impossible where a bijection cannot exist, A -¿ B $\wedge$ B -¿ A mapping

## 7.5

### 7.5 (a)

We are going to assume that $A = \mathbb{R}$ and $B = \mathbb{R} - \{0\}$ Both of these sets are uncountable because they are based on real numbers. But when they are subtracted $A - B$ They produce a set of $\{0\}$ which is a countable set.

### 7.5 (b)

This is also true because if we use our 7.1 example. $\mathbb{N}_0 - \mathbb{N}_1$ will produce a countable set of $\{0\}$

### 7.5 (c)

This is false, we can think of this as a string where you cannot see the end. To determine the size of a power set, you have to see the end of the string. This is due to the size of power sets being expressed as $2^{|A|}$ where A is the set.

### 7.5 (d)

False, imagine the set $A = \{w, x, y\}$ and $B = \{x, y, z\}$ The uninon of A and B would be $C = w, x, y, z$ which C is a finite set and countable. Now imagine if the variables were a finite set. They would still be countable.

### 7.5 (e)

See sub section 7.5 d.

### 7.5 (f)

False, an emptyset isnt the only element they could share. An example, create an infinte uncountable set. Then create the same one but missing one value.

The intersection between the two wouldnt contain an emptyset unless both sets are empty.

## 7.7

### 7.7 (a)

To prove this we will repserent the real numbers in $[0, 1]$ as $x \in X, y \in Y | X, Y \in \mathbb{R}$ We then will construct a real number by using $z = 0.x_1y_1x_2y_2 \ldots x_{n+1}y_{n+1}$

We can use z to prove that $z \leftrightarrow r \mid r \in \mathbb{R}$ where r is also in the interval of $[0, 1]$

Creating a bidirectional releationship between z and r.

### 7.7 (b)

The above subsection should work for proving pairs as long as $r$ becomes like $z$ and without the interval requirement.

## 7.9

### 7.9 (a)

Countable infinite. The reason is that we know that there are finite sets, but we dont know how many finite sets are included in all. We also know that the size of the set has a bijection relationship with each book, making it countable.

### 7.9 (b)

Countable infinite. The reason stems from sub section a, you can still map them to a natural number.

### 7.9 (c)

Countable infinite. This question is just a rewording of sub section a.

### 7.9 (d)

Uncountable infinte. You can always use a infinte table to determine a new number by using existing ones you already know.

### 7.9 (e)

Uncountable infinite. Because for the creation of all sets divisible by 17, you have to use a powerset of integers, which is uncountable.

### 7.9 (f)

Finite. The only even prime number is two making a finite set of one.

### 7.9 (g)

Countable infinite. You can just create a mapping of $N^2$ which would still be a natural number set, and is also infinite countable by having a bijection with natrual numbers.

### 7.9 (h)

Uncountable infinite. If you want all the set of all possible functions. Their can be many sets that create 0, 1 and the power set that can create that would create a uncountable infinte set.

## 7.11

To fix Jhonny, we must just add D to the new set. This ensures that we include it so it can properly be mapped complelty and keeps a bidrectional relationship.