The Pidgenhole Principle

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1 Equations

$$f : X \to Y,\ |X| > |Y| \Rightarrow \exists\ x_1, x_2 \in X : x_1 \neq x_2 \wedge f(x_1) = f(x_2) \tag{1}$$

$$f : X \to Y, |X| > k|Y| \Rightarrow x_1 \ldots, x_{k+1} \in X \wedge f(x_1) = \ldots = f(x_{k+1}) \tag{2}$$

$$f : X \to Y, y \in Y : f(x) = y, |X| > k|Y| \Rightarrow \left\lceil \frac{|X|}{|Y|} \right\rceil = k \tag{3}$$

2 Floor

The floor $\lfloor x \rfloor$ isnt really used in pidgenhole, but it takes real numbers converts them to the less than or equal integer to x.

3 Pidgenhole Principle

The pidgenhole principle states if you have more things on the left and try to generalize them to the right you will end up with more or one thing in the right. Thats why its called pidgenhole principle.

Equation Equation No. 1 is the pidgenhole principle equation.

3.1 Equation Breakdown

The First Part The first part $f : X \to Y$ is saying declare function f, : is a such that operator, that takes set X and maps it to set Y. A set holds distinct members of any type.

The Second Part The second part $|X| > |Y|$ is saying that if the size/cardnaility of set X is greater than set Y. The cardnality of a set is the count of how many items.

The Third Part The third part $\Rightarrow \exists$ is stating that if the previous condition was true, that implies exists, $\Rightarrow$ means implies and $\exists$ is the exists.

The Fourth Part The fourth part $x_1, x_2 \in X$: says if x_1 and x_2 are members of set X such that .... Remember : is the such that symbol.

The Fifth Part The fifth part $x_1 \neq x_2 \wedge$ says if x_1 and x_2 do not equal eachother. $\wedge$ is the and symbol like & in english.

The Sixth Part The sixth part $f(x_1) = f(x_2)$ is saying that the two resultants are equal to eachother. If we remeber to part one that f maps x to y, that means $y = y$

3.2 Example

If you have 10 people and 9 booths, how do you prove there would more than 1 in a booth?

Answer You would use pidgenhole principle that says there would be atleast two people in one booth. Remember that $f(x_1) = f(x_2)$ would be in the same booth.

4 Extended Pidgenhole Principle

The point of this extended pidgenhole principle is to find by how many pidgens would actually be in a hole. It uses k to store this value, k is a nonnegtive number.

Equation Equation No. 2 is the pidgenhole principle extended equation.

Equation Alternitive Equation No. 3 is the altentive equation that directly finds k.

4.1 Equation Breakdown

The First Part The first part $f : X \to Y$ is saying declare function f, : is a such that operator, that takes set X and maps it to set Y. A set holds distinct members of any type.

The Second Part The second part $|X| > |Y|$ is saying that if the size/cardnaility of set X is greater than set Y. The cardnality of a set is the count of how many items.

The Third Part The third part $\Rightarrow$ is stating that if the previous condition was true, $\Rightarrow$ means implies.

The Fourth Part The fourth part $x_1 \ldots, x_{k+1} \in X$ is creating a finite set that members are in set X. That means there isk atleast k + 1 distinct members of set X.

The Fifth Part The Fifth part $\wedge$ means and. Which means condition and condition are true then the final result is true.

The Sixth Part he sixth part $f(x_1) = \ldots = f(x_{k+1})$ is stating that for all $f(x_{k+1})$ are all equal to eachother, which is true and the max size/carnaility of this sequence is k.

4.2 Example

If you have 10 people and 9 booths, at most how many people are in a booth?

Answer You would use extended pidgenhole principle that says there would be atleast two people in one booth. We can use the alternitive to find k by subsituting we get $\lceil \frac{10}{9} \rceil = 2$