Chapter 2 Exersises

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1 2.1

Prove -1 is odd. Using the original 2k + 1 to create any integer.

$$\begin{aligned}-1 &= 2k + 1\\ -2 &= 2k\\ -1 &= k\end{aligned}$$

Since k is equal to -1, it is a odd integer.

2 2.3

Assume a,b are odd integers.

$$\begin{aligned}a &= 2n + 1, n \in Z\\ b &= 2m + 1, m \in Z\\ ab &= (2n + 1)(2m + 1)\\ ab &= 4mn + 2n + 2m + 1\\ ab &= 2(2mn + n + m)\\ j &= 2mn + n + m, j \in Z\\ ab &= 2j + 1\end{aligned}$$

3 2.5

Prove by condition $\sqrt[3]{2}$ is irrational, by proving its irrational. Assume a, b, (integers) where either a or b are even.

$$\begin{aligned}\sqrt[3]{2} &= \frac{a}{b}\\ \sqrt[3]{2} * b &= a\\ (\sqrt[3]{2} * b)^3 &= a^3\\ 2b^3 &= a^3\\ a &= 2k\\ 2b^3 &= (2k)^3\\ 2b^3 &= 8k^3\\ 2b^3 &= 4(2k^3)\\ \sqrt[3]{2} &= \frac{2b^3}{2(4k)^3}\end{aligned}$$

Since, a and b have a common factor of two meaning they are even. This is the condiriction stating that they cant have a common factor other than one.

4 2.7

Prove that on a seven-sided die, its likely to fall on any one of its faces.
We start with a regular 6 sided dice, but then we enlogate one of the sides, ensuring all sides are equal to ensure fairness. This would create a no bias situation and would only allow for one 'side' for the die to fall.

5 2.9

5.1 a

x, y, k, c, d are integers.

$$
\begin{aligned}
c &= x^2 \\
d &= y^2 \\
cd &= x^2y^2 \\
cd &= (xy)^2 \\
k &= xy \\
cd &= k^2
\end{aligned}
$$

5.2 b

x, y, k, c, d are integers

$$
\begin{aligned}
cd &= k^2 \\
c &= x^2 \\
d &= y^2 \\
x &\neq y \\
k &= xy
\end{aligned}
$$

5.3 c

Counterexample
x, y, k, c, d are integers

$$\begin{aligned} c &= x^2 \\ d &= y^2 \\ c &> d \\ x^2 &> y^2 \\ \sqrt{x^2} &> \sqrt{y^2} \\ \pm x &> \pm y \end{aligned}$$

The last statemnt is false, unless they are in the same state with $\pm$.

6 2.11

The first part looks good, exepct the last two label them as a, b.

6.1 a

You cant conclude x + y ¿ 0, because x - y ¿ 0, which makes x ¿ 0 or x ¡ 0. Meaning y could be negitive.

6.2 b

Since the previous section a, could be wrong. With negitive integers of y.

7 2.13

7.1 a

$$\forall x \in \mathbb{R} | x > 0 \implies \exists a, b \in \mathbb{R} | a \neq b \wedge x = a^2 \wedge x = b^2$$

7.2 b

$$\forall n \in \mathbb{Z} | n > 0 \iff \exists k \in \mathbb{Z} | n = 2k \implies \exists p, q \in \mathbb{Z} | p \text{ is prime } \wedge q \text{ is prime } \wedge n = p + q$$

8 2.15

There is only two conditions, you either have 3 friends, or dont have three friends. Now piddgenhole principle states if we map the group of people to the amount of conditions (2), either group will have atleast 3 (k) members.

$$k = \left\lceil \frac{5}{2} \right\rceil$$
$$k = 3$$