Obs & Stats HW 7

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Comparison of Student's t-test and χ^2 Hypothesis Tests

1. With $z_i = x_i - y_i$ and $\sigma_z^2 = \sigma_x^2 + \sigma_y^2$, the χ^2 statistic is given by

$$\chi_N^2 = \frac{\sum z_i^2}{\sigma_z^2} \tag{1}$$

   Calculating χ^2 and the probability to exceed (PTE), we obtain

$$\chi_N^2 = 16.14 \qquad \text{DOF} = 10 \qquad \text{PTE} = 0.096 \tag{2}$$

   The PTE is greater than the chosen $\alpha = 0.05$, and so the null hypothesis cannot be rejected at the 95% level.

2. The Student's t statistic for two samples with equal size and variance is defined as

$$t = \frac{\mu_z}{\sqrt{\sigma_z^2/N}} \tag{3}$$

   Calculating t and PTE, and losing one degree of freedom for the mean,

$$t = 2.88 \qquad \text{DOF} = 9 \qquad \text{PTE} = 0.008 \tag{4}$$

   The PTE is less than the chosen $\alpha = 0.05$, and so the null hypothesis can be rejected at the 95% level.

3. The t test is used to determine if two samples have the same mean, with the null hypothesis being that they have the same mean. The χ^2 test is used to determine if there is a relationship between two variables, with the null hypothesis being that they are not related. Thus, the χ^2 test is not as useful for cases like this one, but it is very useful for testing the concurrence of a model with data. Contrariwise, a χ^2 test would be all but useless for testing a batch of beer, which is where the t test truly shines.