

Digital Logic Design

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Preface

This is my notes on digital logic design.

Resources

Some relevant resources:

- [EECS 270 - Logic Design \(University of Michigan\)](#)
- [Digital Design and Computer Architecture \(ETH Zurich\)](#)

Textbooks:

- J. F. Wakerly, Digital Design: Principles and Practices, 4th ed., Prentice-Hall.
- J. P. Hayes, Introduction to Digital Logic Design, Addison-Wesley.
- C. H. Roth, Jr., Fundamentals of Logic Design.
- R. H. Katz, Contemporary Logic Design, Prentice-Hall.
- D. Thomas, P. Moorby, The Verilog Hardware Description Language.

1 Introduction

1.1 Perspective

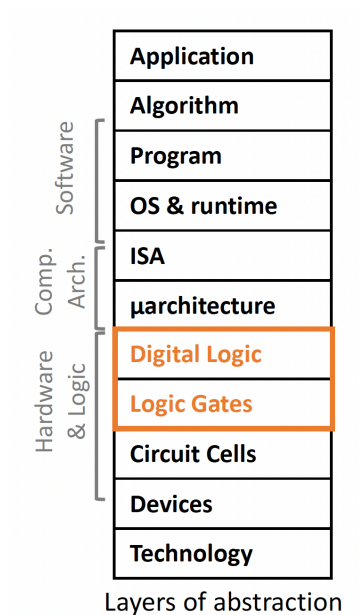


Figure 1.1: Digital Logic Design in the Computing Stack (figure from EECS270-W24)

i Note 1: Definition - Digital

Digital signals represent information as *discrete* values, typically binary values where two valid states exist: 0 (low, false) or 1 (high, true).

This notebook focuses on the design of **digital circuits**. We study both the logic/math used to build digital systems (Boolean algebra), as well as the circuit design implications (transistors, timing, etc).

1.2 (Very) High-Level Digital Circuit Design Flow

Insert high-level design flow figure here

The design flow of a digital circuit starts off with a problem statement or design specification. Digital circuits are then described by the designer in a **Hardware Description Language (HDL)**, most commonly **Verilog/SystemVerilog** or **VHDL**. The design is then simulated with a **testbench**, which feeds the design with test inputs. During **simulation**, we can use tools to inspect the state of the signals in the circuit to analyze, debug, and evaluate the design. At this point, such a **behavioral** description of the design merely describes the functionality and not yet its physical implementation. **Synthesis** maps the behavioral description of the design into **netlist** of standard cells, which indicates the physical mapping to circuit components. The **place and route** process then physically places the netlist of cells and routes the wires to connect the components, generating a hardware implementation.

This notebook will cover basic Verilog. For more in-depth notes on Verilog/SystemVerilog, please refer to my other notebook.

2 Binary Basics

2.1 Analog vs Digital

In contrast to the *discrete* **digital** signals, **analog** signals are *continuous*. Signals from the physical world are inherently analog (e.g. sound, light, temperature, voltage). However, modern computing systems are primarily digital because of several key advantages:

- Reliability: Provides more noise resistance since it operates at low or high levels
- Digitized signals can represent analog values with good precision given enough digits
- Ease of data storage, transmission, and compression
- Digital circuit components are more cost-effective and scalable compared to analog components

2.2 Why Binary?

- Storing/transmitting binary values is much easier than three or more values
- Binary switches are easier, more robust, and more noise tolerant in circuit implementation

Note that digital = binary!

2.3 Data Encoding

Numbers are encoded in a system using digits and powers of a base number. In simpler terms, each position of a number represents a quantity. And the digit in each position indicates how many of that quantity there are in the number.

A **bit** is a binary digit. The total number of integers that can be represented with n bits is 2^n .

The maximum (unsigned) decimal number that can be represented with n bits is Max Value = $2^n - 1$. This range can be generalized to other bases:

$$\begin{array}{l}
 \begin{array}{c} 2 \ 1 \ 0 \\ 249_{10} \end{array} = 2 * 10^2 + 4 * 10^1 + 9 * 10^0 \\
 \begin{array}{c} 2 \ 1 \ 0 \\ 371_8 \end{array} = 3 * 8^2 + 7 * 8^1 + 1 * 8^0 = 249_{10} \\
 \begin{array}{c} 7 \ 6 \ 5 \ 4 \ 3 \ 2 \ 1 \ 0 \\ 11111001_2 \end{array} = 1 * 2^7 + 1 * 2^6 + 1 * 2^5 + 1 * 2^4 + 1 * 2^3 + 0 * 2^2 \\
 \quad \quad \quad + 0 * 2^1 + 1 * 2^0 = 249_{10}
 \end{array}$$

Figure 2.1: Data encoding in base 10, 8, and 2. Figure from EECS270-W24

i Note 2: Equation - Max Unsigned Decimal Value with n Digits

$$\text{Max Value} = \text{base}^n - 1$$

Common number systems:

- Base-16: **Hexadecimal**
- Base-10: **Decimal**
- Base-8: **Octal**
- Base-2: **Binary**

The number of bits n needed to represent an unsigned decimal number x is given below:

i Note 3: Equation - Number of Bits to Represent Unsigned Decimal Number

$$n = \text{ceil}(\log_2(x + 1))$$

where the $\text{ceil}()$ function is a ceiling function that rounds up to the nearest integer.

2.3.1 Conversions

2.3.1.1 Decimal - Binary

To convert from decimal to binary:

- Step 1: Divide the given number repeatedly by 2 until you get 0 as the quotient.
- Step 2: Write the remainders in reverse order.

Step 1	Quotient	Remainder
212/2	106	0
106/2	53	0
53/2	26	1
26/2	13	0
13/2	6	1
6/2	3	0
3/2	1	1
1/2	0	1

Step 2: 11010100

Figure 2.2: Example decimal to binary conversion

2.3.2 Hex - Octal - Binary

Hexadecimal and octal have bases that are powers of 2, which makes conversion much simpler. Since hex is base 16, which is 2^4 , we can split each hex digit into 4 bits when converting to binary, conversely group every 4 bits into 1 hex digit. Similarly, an octal digit corresponds to 3 bits.

*TODO: Add hex conversion chart.

2.3.3 ASCII

Text can also be encoded by numbers. ASCII is a common character encoding standard that represents a character in 8 bits.

3 Boolean Algebra

3.1

4 Combinational Logic

i Note 4: Definition - Combinational Logic

Combinational Logic: output is a pure function of the present input only.

4.1

4.2 Transistors

4.3 Transistors to Gates

4.4 Transistor Scaling

5 Timing

5.1

6 Sequential Logic

6.1

7 Finite State Machines

7.1

8 Digital Arithmetic

8.1

9 Memories

10 Summary

In summary...

References