Data Structures & Algorithms

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Preface

Notes on Data Structures & Algorithms, illustrated in C++.

Resources

Some relevant resources:

- EECS 280 - Programming and Intro Data Structures (University of Michigan)
- EECS 281 - Data Structures and Algorithms (University of Michigan)
- EECS 376 - Foundations of Computer Science
- Data Structures & Algorithms - Google Tech Dev Guide
- EECS 281 References
- The Algorithms

Practice Resources:

- LeetCode
- NeetCode
- Blind 75
- Grind 75
  - About Grind 75
- Codeforces

Interview Resources:

- Leetcode Patterns
- Learning Resources - Reddit-wiki-programming
- AlgoMonster
- Tech Interview Handbook
  - Study Cheatsheet
- Zero To Mastery

C++ Guides:

- learncpp.com

Books:

- Algorithms - Jeff Erickson
- Cracking the Coding Interview

1 Introduction

1.1 Perspective

Note 1: Definition - Definition goes here

Definition is defined by: definition.

This notebook contains programming basics, data structures, and algorithms. The language of choice is C++, and concepts from C++ STL are also covered.

1.2 Data Structures

1.3 Algorithm Families

- Brute-Force
- Greedy
- Divide and Conquer
- Dynamic Programming
- Backtracking
- Branch and Bound

2 Foundations

2.1 Resources

3 Pointers

4 Arrays

5 Strings

6 Streams and IO

7 Abstract Data Type

Note 2: Definition - Abstract Data Type

An **Abstract Data Type (ADT)** combines data with valid operations and their behaviors on stored data

- e.g. insert, delete, access
- ADTs define an **interface**

Meanwhile, a **data structure** provides a concrete implementation of an ADT.

8 Object-Oriented Programming

9 Dynamic Memory

10 Linked List

Linked list is a **non-contiguous** data structure with efficient **insertion** and **deletion**. Often used to implement other data structures like stacks, queues, or deques.

Main types of linked lists include:

- Singly linked list
- Doubly linked list
- Circular linked list

10.1 Summary

	Array	Linked List
Access	**best** time complexity **O(1)** **average** time complexity **O(1)** **worst** time complexity **O(1)**	**best** time complexity **O(1)** **average** time complexity **O(n)** **worst** time complexity **O(n)**
Insertion	**best** time complexity **O(1)** **average** time complexity **O(n)** **worst** time complexity **O(n)**	**best** time complexity **O(1)** **average** time complexity **O(n)** **worst** time complexity **O(n)**
Memory Overhead	If array size is chosen well, the array uses less memory	Additional memory is required for pointers
Memory Efficiency	May contain unused memory	Can change dynamically in size resulting in no wasted memory

10.2 C++ Example

Structure of a singly linked list node:

```
class IntList {
private:

    struct Node {
        int datum; // contains the element of the node
        Node *next; // points to the next node in the list
    }

public:
    Node *first;
    Node *last;
};
```

We can set the next pointer of the last node to `nullptr` as a sentinel value.

A more complete implementation:

```
template <typename T>
class LinkedList {
private:
    struct Node {
        T datum; // contains the element of the node
        Node *next; // points to the next node in the list
    }

    Node *first;
    Node *last;

public:
    // Constructor builds an empty list
    LinkedList() : first(nullptr) {}

    bool isEmpty() const {
        return first == nullptr;
    }

    // Return by reference the first element;
    T & front() {
        assert(!empty());
        return first->datum;
    }

    // Push a new node to the front
    void push_front(T datum) {
        Node *p = new Node;
        p->datum = datum;
        p->next = first;
        first = p;
    }

    void push_back(T datum) {
        Node *p = new Node;
        p->datum = datum;
        p->next = nullptr;
        if (empty()) {
            first = last = p;
        } else {
            last->next = p;
            last = p;
        }
    }

    // Pop the front node
    void pop_front() {
        assert(!empty());
        Node *victim = first;
        first = first->next;
        delete victim;
    }

    // Printing the linked list to os
    void print(ostream &os) const {
        for (Node *np = first; np; np = np->next) {
            os << np->datum << " ";
        }
    }

    void pop_all() {
        while (!empty()) {
            pop_front();
        }
    }

    void push_all(const LinkedList &other) {
        for (Node *np = other.first; np; np = np->next) {
            push_back(np->datum);
        }
    }
    // - The Big Three for LinkedList -

    // Destructor
    ~LinkedList() {
        pop_all();
    }

    // Copy constructor
    LinkedList(const LinkedList &other) : first(nullptr), last(nullptr) {
        push_all(other);
    }

    // Assignment Operator
    LinkedList & operator=(const LinkedList &rhs) {
        if (this == &rhs) { return *this; }
        pop_all();
        push_all(rhs);
        return *this;
    }

};
```

11 Iterators

11.1 References

- Geeks for Geeks Iterators in STL

11.2 Iterators in C++ STL

Iterator Declaration

```
<type>::iterator it;
```

Can then be initialize by assigning some valid iterator. If we already have an iterator to be assigned at the time of delcaration, then we can skip the type declaration using the auto keyword.

`auto it = iter`

Example:

```
vector<int> arr = {1, 2, 3, 4, 5};

vector<int>::iterator first = arr.begin();

vector<int>::iterator last = arr.end();

while(first != last) {
    cout << *first << " ";
    first++;
}
```

Iterator Function

Return Value

begin()

Returns an iterator to the beginning of container.

end()

Returns an iterator to the theoretical element just after the last element of the container.

cbegin()

Returns a constant iterator to the beginning of container. A constant iterator cannot modify the value of the element it is pointing to.

cend()

Returns a constant iterator to the theoretical element just after the last element of the container.

rbegin()

Returns a reverse iterator to the beginning of container.

rend()

Returns a reverse iterator to the theoretical element just after the last element of the container.

crbegin()

Returns a constant reverse iterator to the beginning of container.

crend()

Returns a constant reverse iterator to the theoretical element just after the last element of the container.

11.2.1 Iterator Operations

```
*it;            // Access
*it = new_val;  // Update
it++;           // post-increment
++it;           // pre-increment
it--;
--it;
it + int_val;   // can add or subtract by int val or another iterator
// Comparing two iterators also works (e.g. !=, <=, etc)
```

12 Recursion

12.1 Recurrence Relations

Recurrence relation describes the way a problem depends on a subproblem

12.1.1 Solving Recurrences: Linear

$$T(n) = \begin{cases} c_0, & \text{if } n = 0 \\ T(n-1) + c_1, & \text{if } n > 0 \end{cases}$$

Recurrence: $T(n) = T(n-1) + c$

Complexity: $\Theta(n)$

12.1.2 Solving Recurrences: Logarithmic

$$T(n) = \begin{cases} c_0, & \text{if } n = 0 \\ T(\frac{n}{2}) + c_1, & \text{if } n > 0 \end{cases}$$

Recurrence: $T(n) = T(n/2) + c$

Complexity: $\Theta(logn)$

12.1.3 Master Theorem

A.k.a Master Method

Let $T(n)$ be a monotonically increasing function that satisfies:

$$T(n) = aT(\frac{n}{b}) + f(n)$$

$T(1) = c_0$ or $T(0) = c_0$

where $a \geq 1$, $b > 1$. If $f(n) \in \Theta(n^c)$, then:

$$T(n) \in \begin{cases} \Theta(n^{\log_b a}) & \text{if } a > b^c \\ \Theta(n^c \log n) & \text{if } a = b^c \\ \Theta(n^c) & \text{if } a < b^c \end{cases}$$

When NOT to use Master Theorem:

- $T(n)$ is not monotonic, such as $T(n) = sin(n)$
- $f(n)$ is not a polynomial, e.g. $f(n) = 2^n$
- b cannot be expressed as a constant, e.g. $T(n) = T(\sqrt{}(sin(n)))$

12.1.4 Fourth Condition

A fourth condition that allows polylogarithmic functions:

13 Function Objects, Functors

14 Error Handling & Exceptions

15 Stacks, Queues, Deque, Priority Queue

15.1 Stack

15.1.1 Stack - Interface

Stack is a data structure that supports insertion/removal in Last In, First Out (LIFO) order

Stack ADT Interface:

Method	Description
`push(object)`	Add object to top of the stack
`pop()`	Remove top element
`object &top()`	Return a reference to top element
`size()`	Number of elements in stack
`empty()`	Checks if stack has no elements

15.1.2 Stack Implementation

A stack can be implemented with an array/vector or linked list

15.1.3 Stack in STL

```
#include <stack>
std::stack<>
```

The underlying containers are `std::deque<>` (by default), and `std::list<>, std::vector<>` (optional).

15.2 Queue

15.2.1 Queue - Interface

Queue is a data structure that supports insertion/removal in First In, First Out (FIFO) order

15.2.2 Queue Implementation

15.2.3 Queue in STL

```
#include <queue>
std::queue<>
```

The underlying containers are `std::deque<>` (by default), and `std::list<>` (optional).

15.3 Deque

15.3.1 Deque - Interface

Deque is an abbreviation of Double-Ended Queue

```
#include <deque>
std::deque<>
```

Main methods:

- `push_front()`
- `pop_front()`
- `front()`
- `push_back()`
- `pop_back()`
- `back()`
- `size()`
- `empty()`
- Random Access: `[]` or `.at()`

STL incudes constant time `operator[]()`

15.3.2 Deque - Implementation

Circular Buffer

Doubly-linked list

15.4 Priority Queue

Each datum in the priority queue is paired with a priority value (usually numbers, should be comparable). Supports **insertion**, **inspection** of data, and **removal** of datum with highest priority.

For elements of the same priority, they are served according to their order in the queue (following FIFO order).

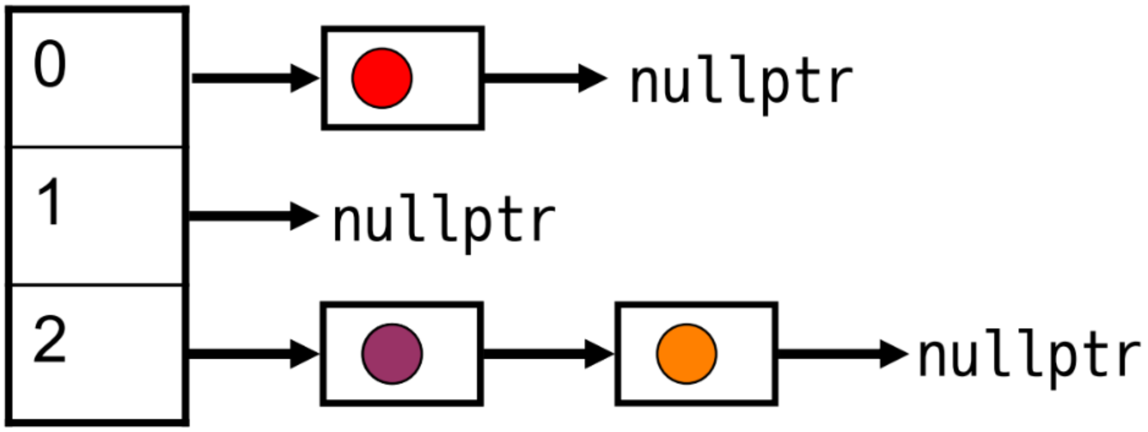


Figure 15.1: Example Priority Queue. Source: EECS 281. Lower numbers here indicate higher priority. Top element would be red node. At priority 2, orange node would be served before purple.

15.4.1 ADT - Interface

Method	Description
`push(object)`	Add object to the priority queue
`pop()`	Remove highest priority element
`const object &top()`	Return a reference to highest priority element
`size()`	Number of elements in priority queue
`empty()`	Checks if priority queue has no elements

15.4.2 Priority Queue Implementations

Priority queues can be implemented with many data structures. Heap is a common implementation.

	Insert	**Remove**
Unordered sequence container	Constant	Linear
Sorted sequence container	Linear	Constant
Heap	Logarithmic	Logarithmic
Array of linked lists (for priorities of small integers)	Constant	Constant

15.4.3 C++ Priority Queue

`std::priority_queue<>`

By default, uses `std::less<>` to determine priority. A default priority queue is a "max-PQ", where the largest element has highest priority. To implement a "min-PQ", use `std::greater<>`. Custom comparator (function object) needed if the elements cannot be compared with std less/greater.

Max PQ (`std::less<>`):

`std::priority_queue<T> myPQ;`

PQ with custom comparator type, `COMP`:

`std::priority_queue<T, vector<T>, COMP> myPQ;`

Key methods:

- `push()`
- `top()`
- `empty()`
- `pop()`

Example Usage (Source):

```
priority_queue<int> pq;
int arr[6] = { 10, 2, 4, 8, 6, 9 };
for (int i = 0; i < 6; i++) {
    pq.push(arr[i]);
}

while (!pq.empty()) {
    cout << pq.top() << ' ';
    pq.pop();
}

// Creating a min-heap (rather than a default max-heap)
//      vector<int> is the internal container used
priority_queue <int, vector<int>, greater<int>> gq;
```

Manual priority queue implementation with standard library functions:

```
#include <algorithm>
std::make_heap();
std::push_heap();
std::pop_heap();
```

16 Generating Permutations

We can generate permutations by “juggling with stacks and queues”

Essentially, given N elements, we want to generate all N element permutations.

Main ingredients:

- One recursive function
- One stack
- One queue

Technique: move elements between the two containers

```cpp
// Helper function for printing
template <typename T>
ostream &operator<<(ostream &out, const vector<T> &v) {
    // display contents of a vector on a single line
    for (auto &el : v) {
        out << el << ' ';
    }
    return out;
}

// Implementation
template <typename T>
void genPerms(vector<T> &perm, deque<T> &unused) {
    if (unused.empty()) {
        // Base case: we have reached a permutation when unused is empty
        //    i.e. a full permutation has been formed
        cout << perm << '\n';
        return;
    }

    for (size_t k = 0; k != unused.size(); ++k) {
        perm.push_back(unused.front());   // Pick the first element from unused
        unused.pop_front();               // Remove this element from unused
        genPerms(perm, unused);           // Recursively generate permutation
        unused.push_back(perm.back());     // Restore this element to unused
        perm.pop_back();                   // Remove it from the permutation
    }
}

// Example Usage
int main() {
    size_t n = 16;

    vector<size_t> perm;
    deque<size_t> unused(n);
    iota(unused.begin(), unused.end(), 1); // Fills unused with consecutive numbers starting
    genPerms(perm, unused);

    return 0;
}
```

Explanation of `genPerms()`:

- The function iterates over `unused` and chooses each element one by one as it fills up a permutation
- The chosen element is moved from `unused` to `perm` (backtracking)
- The function is recursively called to generate the remaining permutation (as each call picks another element from `unused`)
- After the recursion returns, the removed element is restored to `unused`
- Time complexity: $O(n!)$ since it generates all permutations

Another Implementation of `genPerms`

```
template <typename T>
void genPerms(vector<T> &path, size_t permLength) {
    if (permLength == path.size()) {
        // Do something with the path
        return;
    }
    if (!promising(path, permLength))
        return;
    for (size_t i = permLength; i < path.size(); ++i) {
        swap(path[permLength], path[i]);
        genPerms(path, permLength + 1);
        swap(path[permLength], path[i]);
    }
}
```

16.0.1 STL next_permutation()

The STL has function `std::next_permutation()`

Example Usage

```cpp
#include <algorithm>
#include <iostream>
#include <string>

int main()
{
    std::string s = "aba";

    do
    {
        std::cout << s << '\n';
    }
    while (std::next_permutation(s.begin(), s.end()));

    std::cout << s << '\n';
}
```

17 Complexity Analysis

17.1 References

- AlgoMonster - Runtime Summary

17.2 Runtime Overview

17.3 Common Time Complexities

Notation	Name
O(1)	Constant
O(log n)	Logarithmic
O(n)	Linear
O(n log n)	Loglinear, Linearithmic
$O(n^2)$	Quadratic
$O(n^3)$, O(n), ...	Polynomial
O(c)	Exponential
O(n!)	Factorial
O(2^(2))	Doubly Exponential

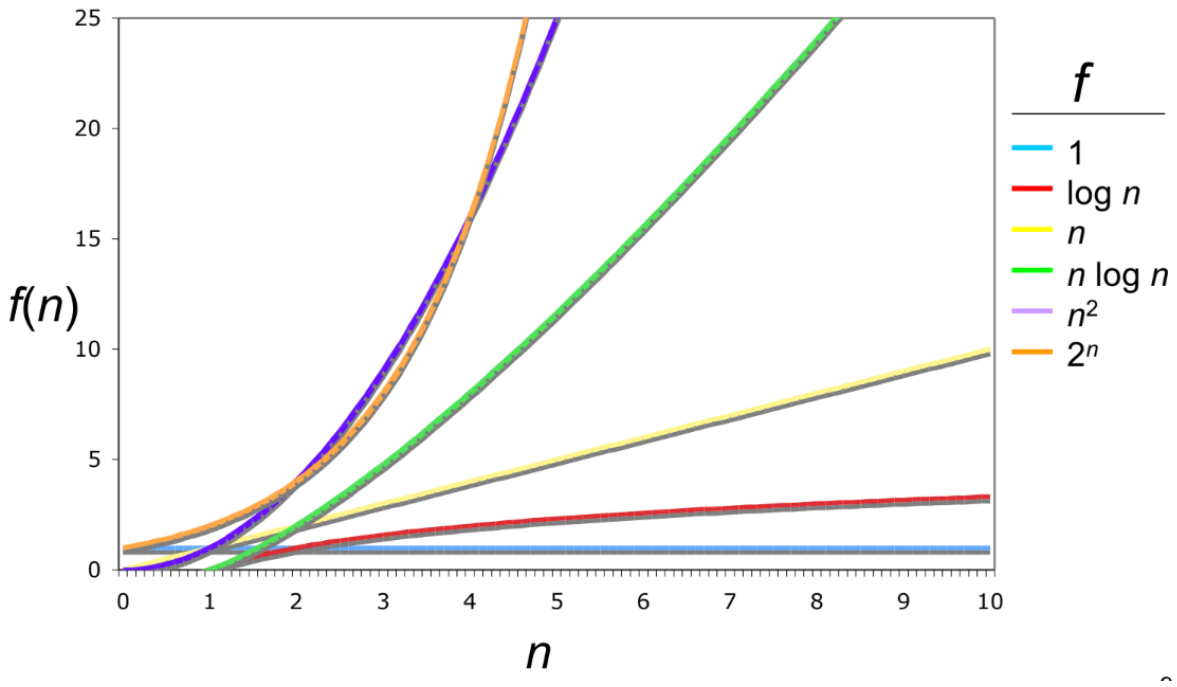


Figure 17.1: Graphing Complexities. Source: EECS 281.

17.3.1 O(1) - Constant

Constant time complexity. Could be

- Hashmap lookup
- Array access and update
- Pushing and popping elements from a stack
- Finding and applying math formula

17.3.2 O(log(N)) - Logarithmic

$log(N)$ grows very slowly

In coding interviews, log(N) typically means:

- Binary search or variant
- Balanced binary search tree lookup
- Processing the digits of a number

Unless specified, typically log(N) refers to $log_2(N)$

Example C++:

```cpp
int N = 100000000;
while (N > 0) {
  // some constant operation
  N /= 2;
}
```

Many mainstream relational databases use binary trees for indexing by default, thus lookup by primary key in a relational database is log(N).

17.3.3 O(N) - Linear

Linear time typically means looping through a linear data structure a constant number of times. Most commonly, this means:

- Going through array/linked list
- Two pointers
- Some types of greedy
- Tree/graph traversal
- Stack/Queue

Example C++:

```cpp
for (int i = 1; i <= N; i++) {
  // constant time code
}

for (int i = 1; i < 5 * N + 17; i++) {
  // constant time code
}

for (int i = 1; i < N + 538238; i++) {
  // constant time code
}
```

17.3.4 O(K log(N))

- Heap push/pop K times. When you encounter problems that seek the “top K elements”, you can often solve them by pushing and popping to a heap K times, resulting in an O(K log(N)) runtime. e.g., K closest points, merge K sorted lists.
- Binary search K times.

Since K is constant this kind of isn’t its own time complexity and can be grouped with O(log(N))

17.3.5 O(N log(N)) - Log-Linear

- Sorting. The default sorting algorithm’s expected runtime in all mainstream languages is N log(N). For example, java uses a variant of merge sort for object sorting and a variant of Quick Sort for primitive type sorting.
- Divide and conquer with a linear time merge operation. Divide is normally log(N), and if merge is O(N) then the overall runtime is O(N log(N)). An example problem is smaller numbers to the right.

17.3.6 O(N^2) - Quadratic

- Nested loops, e.g., visiting each matrix entry
- Many brute force solutions

```
for (int i = 1; i <= N; i++) {
  for (int j = 1; j <= N; j++) {
    // constant time code
  }
}
```

17.3.7 O(2^N) - Exponential

Grows very rapidly. Often requires memoization to avoid repeated computations and reduce complexity.

- Combinatorial problems, backtracking, e.g. subsets
- Often involves recursion and is harder to analyze time complexity at first sight

E.g.: A recursive Fibonacci algorithm is $O(2^N)$

```
int Fib(int n) {
  if (n == 0 || n == 1) {
    return 1;
  }
  return Fib(n - 1) + Fib(n - 2);
}
```

17.3.8 O(N!) - Factorial

Grows very very rapidly. Only solvable by computers for small N. Often requires memoization to avoid repeated computations and reduce complexity.

- Combinatorial problems, backtracking, e.g. permutations
- Often involves recursion and is harder to analyze time complexity at first sight

17.4 Big-O, Big-Theta, and Big-Omega

	Big-O (O)	**Big-Theta (Θ)**	**Big-Omega (Ω)**
Defines	Asymptotic upper bound	Asymptotic tight bound	Asymptotic lower bound
Definition	$f(n) = O(g(n))$ if and only if there exists an integer n_0 and a real number c such that for all $n \geq n_0$, $f(n) \leq c \cdot g(n)$	$f(n) = \Theta(g(n))$ if and only if there exists an integer n_0 and real constants c_1 and c_2 such that for all $n \geq n_0$: $c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$	$f(n) = \Omega(g(n))$ if and only if there exists an integer n_0 and a real number c such that for all $n \geq n_0$, $f(n) \geq c \cdot g(n)$
Mathematical Definition	$\exists n_0 \in \mathbb{Z}, \exists c \in \mathbb{R}: \forall n \geq n_0, f(n) \leq c \cdot g(n)$	$\Theta(f(n)) = O(f(n)) \cap \Omega(f(n))$	$\exists n_0 \in \mathbb{Z}, \exists c \in \mathbb{R}: \forall n \geq n_0, f(n) \geq c \cdot g(n)$
$f_1(n) = 2n + 1$	$O(n)$ or $O(n^2)$ or $O(n^3)$...	$\Theta(n)$	$\Omega(n)$ or $\Omega(1)$
$f_2(n) = n^2 + n + 5$	$O(n^2)$ or $O(n^3)$...	$\Theta(n^2)$	$\Omega(n^2)$ or $\Omega(n)$ or $\Omega(1)$

17.5 Amortized Time Complexity

18 STL

19 Trees

A **graph** consists of **nodes/vertices** connected together by **edges**. Each node can contain some data.

A **tree** is

1. A connected graph (nodes + edges) without cycles
2. A graph where any 2 nodes are connected by a unique shortest path

The two definitions above are equivalent.

Types of Trees:

- (Simple) tree
- Rooted tree

In a **directed tree**, we can identify **child** and **parent** relationships between nodes.

In a **binary tree**, a node has at most two children.

Terminology:

- **Root**: the topmost node in the tree
- **Parent**: Immediate predecessor of a node
- **Child**: Node where current node is parent
- **Ancestor**: parent of a parent (closer to root)
- **Descendent**: child of a child (further from root)
- **Internal node**: a node with children
- **Leaf node**: a node without children

```
template <class Item>
struct Node {       // a binary tree node
    Node *left;     // pointer to left child
    Node *right;    // pointer to right child
    Item item;      // data or KEY
}
```

Tree Properties

Height:

height(empty) = 0

height(node) = max(heght(left_child), height(right_child)) + 1

Size:

size(empty) = 0

size(node) = size(left_child) + size(right_child) + 1

Depth:

depth(empty) = 0

depth(node) = depth(parent) + 1

19.1 Complete (Binary) Trees

Note 3: Definition - Complete Binary Tree

Complete Binary Tree: every level, except possibly the last, is completely filled, and all nodes are as far left as possible

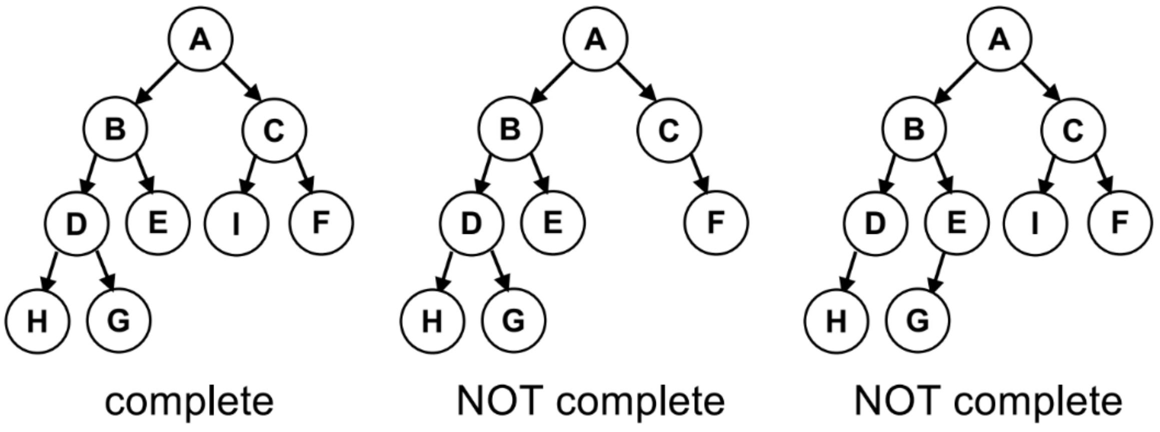


Figure 19.1: Source: EECS 281.

A complete binary tree can be stored efficiently in a growable array (vector) by indexing nodes according to level-ordering

- Completeness ensures no gaps in the array, thus easily indexable
- Index start at 1 (to make math easier)
    - To achieve 1-based indexing with 0-indexed vector, just add a dummy element at position 0 and ignore it

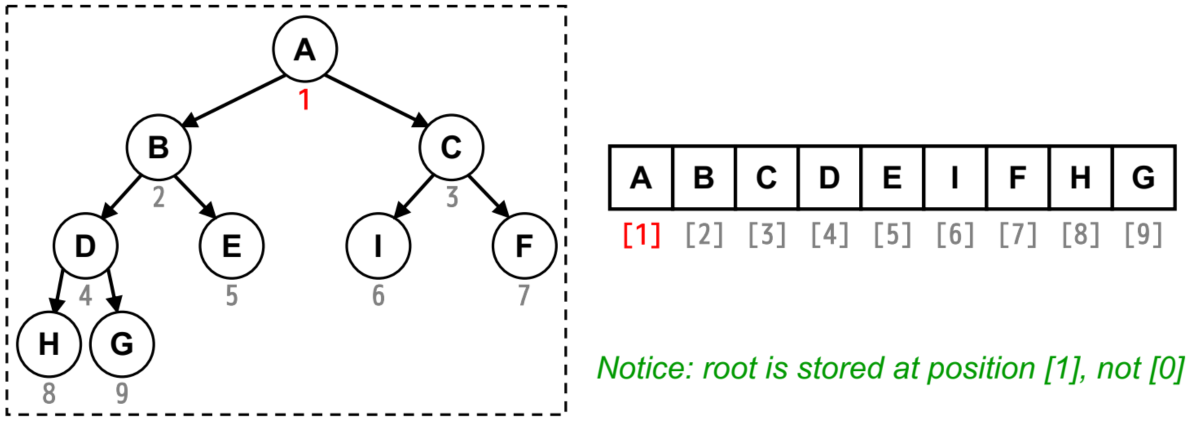


Figure 19.2: Source: EECS 281.

19.2 Binary Search Trees (BST)

Each node in a BST has at most two children, with the left child containing values less than the parent and the right child containing values greater than the parent.

Allows for efficient searching, insertion, and deletion.

E.g. finding the max/min is as simple as finding the rightmost or leftmost node. E.g., rightmost can be found by keeping on accessing the right child until a node no longer has a right child.

19.3 Tree Traversals

In-Order traversal visits:

1. left subtree
2. current node
3. right subtree

Pre-Order traversal visits:

1. current node
2. left subtree

3. right subtree

Post-Order traversal visits:

1. left subtree
2. right subtree
3. current node

Level-Order traversal visits nodes in order of increasing depth in tree. Same as bradth-first traversal.

C++ example:

```
template <class KEY>
struct Node {
    KEY key;
    Node *left = nullptr;
    Node *right = nullptr;
};  // Node{}

void preorder(Node *p) {
    if (!p) return;
    visit(p->key);
    preorder(p->left);
    preorder(p->right);
}  // preorder()

void postorder(Node *p) {
    if (!p) return;
    postorder(p->left);
    postorder(p->right);
    visit(p->key);
}  // postorder()

void inorder(Node *p) {
    if (!p) return;
    inorder(p->left);
    visit(p->key);
    inorder(p->right);
}  // inorder()
```

19.4 AVL Tree

20 Heaps

20.1 References

- AlgoMonster - Heap Fundamentals
- Geeks for Geeks - STL Heap

20.2 Heap? Priority Queue?

Priority Queue is an **Abstract Data Type**, and Heap is the concrete data structure we use to implement a priority queue. Source

20.3 Heap - Definitions & Properties

Definition: A tree is **(max) heap-ordered** if each node's priority is not greater than the priority of the node's parent

Definition: A **heap** is a set of nodes with keys (priorities) arranged in a complete heap-ordered tree, represented as an array

Property: No node in a heap has a key (priority) larger than the root's key

A heap has two crucial properties (representation invariants):

1. Completeness
2. Heap-ordering

These two properties can be leveraged to create an efficient priority queue and an efficient sorting algorithm using a heap!

Loose definition: data structure that gives easy access to the most extreme element, e.g., maximum or minimum

"Max Heap": heap with largest element being the "most extreme"

"Min Heap": heap with smallest element being the "most extreme"

Heaps use complete (binary) trees* as the underlying structure, but are often implemented with arrays

Note: Not to be confused with binary search* trees.

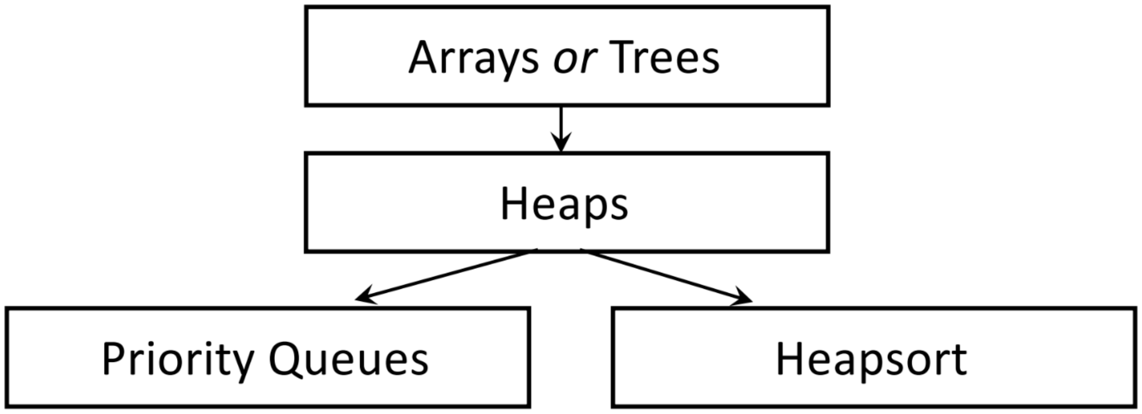


Figure 20.1: Source: EECS 281

20.4 Modifying Heaps

20.4.1 Increasing Node Priority

If the priority of a node in the heap is increased, we need to fix **bottom-up**:

1. Pass index k of array element with increased priority
2. Swap the node's key with the parent's key until:
    - a. the node has no parent (it is the root), or
    - b. the node's parent has a higher (or equal) priority key

```
Code goes here
```

20.4.2 Decreasing Node Priority

fix **top-down**

20.5 Heap in C++

20.5.1 make_heap()

`std::make_heap()` is used to convert the given range in a container to a heap. Max heap by default. Use a custom comparator to change it to a min heap.

```cpp
// Initializing a vector
std::vector<int> v1 = { 20, 30, 40, 25, 15 };

// Converting vector into a heap
std::make_heap(v1.begin(), v1.end());

std::cout << v1.front() << '\n'; // Displays max element of heap
```

20.5.2 push_heap()

`std::push_heap(begin_iterator, end_iterator)` sorts the heap after insertion. Can you `push_back()` of vector class to insert.

```cpp
vector<int> vc{ 20, 30, 40, 10 };
make_heap(vc.begin(), vc.end());

vc.push_back(50);
push_heap(vc.begin(), vc.end()); // now the heap is sorted
```

Time Complexity: $O(logN)$

Note: The `push_heap()` function should only be used after the insertion of a single element at the back. Undefined for random insertion or for building a heap.

20.5.3 pop_head()

`pop_heap()` is used to move largest element of the heap to the end of the heap, so then a `pop_back()` can be used to remove the element

```cpp
pop_heap(vc.begin(), vc.end()); // moves largest element to the end
vc.pop_back(); // removes element at the end
```

Time Complexity: $O(logN)$

20.5.4 sort_heap()

`sort_heap()` is used to sort the heap in ascending order using heapsort.

```
sort_heap(v1.begin(), v1.end());
```

Time Complexity: $O(NlogN)$

20.5.5 is_heap()

`is_heap()` can be used to check whether a given range of the container is a heap. Default checks for max heap.

20.5.6 is_heap_until()

`is_heap_until()`

20.6 Priority Queue - Heap Implementation

Definition: A **priority queue** is a data structure that supports the following three basic operations:

- push(): **insertion** of a new node
- top(): **inspection** of highest priority node
- pop(): **removal** of highest priority node

Priority queue is useful for many algorithms: e.g., Dijkstra's, heapsort, sorting in reverse order

Priority queues are often implemented using heaps because the same time complexity of insertion/removal.

20.6.1 Insertion & Deletion

TODO

20.6.2 Priority Queue - Summary

Another summary can be found in the Stacks, Queues, Priority Queues page.

Priority Queue Type	Insertion Complexity	Inspection Complexity	Removal Complexity	Notes
Unordered Array PQ	O(1)	O(n)	O(n), or O(1)	-
Sorted Array PQ	O(n)	O(1)	O(1)	-
Heap	O(log n) using `fixUp()`	O(1)	O(log n) using `fixDown()`	Must maintain heap property

20.7 Heapify

20.8 Heapsort

21 Searching

21.1 Linear Search

21.2 Binary Search

Searching in a sorted container.

Asymptotic Complexity = O(log n)

21.2.1 C++ Implementation

Basic Implementation:

```
int bsearch(double a[], double val, int left, int right) {
    // n = right - left = size of a[]
    while (right > left) {                          // loops at most k times
        int mid = left + (right - left) / 2;    // find midpoint of current range
        if (val == a[mid]) return mid;          // midpoint is the search val, return it

        if (val < a[mid]) right = mid;          // target val is less than midpoint, update r
        else left = mid + 1;                    // target val is greater than midpoint, updat
    }

    return -1;      // val not found
}
```

Optimization - 2 comparisons half the time:

```
int bsearch(double a[], double val, int left, int right) {
    // n = right - left = size of a[]
    while (right > left) {                          // loops at most k times
        int mid = left + (right - left) / 2;    // find midpoint of current range

        if (val > a[mid]) left = mid + 1;
        else if (val < a[mid]) right = mid;
        else return mid;
    }

    return -1;      // val not found
}
```

Optimization - always 2 comparisons per loop (requires `val` to be present in `a[]`):

```
int bsearch(double a[], double val, int left, int right) {
    // n = right - left = size of a[]
    while (right > left) {
        int mid = left + (right - left) / 2;    // find midpoint of current range

        if (val > a[mid]) left = mid + 1;       // target val greater than midpoint, update l
        else right = mid;                       // target val <= midpoint, update right bound
    }

    return left;
}
```

21.2.2 STL Binary Search

`binary_search()` returns a `bool`

To find locations (iterators):

- `lower_bound()` First item not less than target
- `upper_bound()` First item greater than target
- `equal_range()` Pair(lb, ub), all items equal to target

21.3 Tree Search

21.3.1 Depth-First Search (DFS)

21.3.2 Breadth-First Search (BFS)

21.3.3 Dijkstra's Algorithm

21.3.4 Floyd's Algorithm

21.3.5 Bellman-Ford

Can handle negative weight edges that Dijkstra's cannot.

22 Sets

22.1 Set Operations

- Union ($\cup$): in one set or the other (OR)
- Intersection ($\cap$): in both sets (AND)
- Set Difference ($\setminus$): in one and not the other (AND-NOT)
- Symmetric Difference ($\div$): in only one (XOR)
- addElement (+)
- isElement ($\in$)
- isEmpty

22.1.1 set_union()

Method	Asymptotic Complexity
`initialize()`	O(1) or O(n log n)
`clear()`	O(1) or O(n)
`isMember()`	O(log n)
`copy()`	O(n)
`set_union()`	O(n)
`set_intersection()`	O(n)

22.2 Set in C++

22.2.1 References

Geeks for Geeks - C++ STL Set

```cpp
std::set<data_type> set_name;

set<int> my_set;
set<int> my_set = {1, 4, 5, 12};
```

```
my_set.insert(10);

for (auto& num : my_set) {
    std::cout << num << ' ';
}

set<int>::iterator it;

my_set.erase(12);

// returns an iterator to the element, otherwise returns my_set.end()
my_set.find(5);
```

22.3 Union-Find

23 Sorting

Elementary Sorts	**High-Performance Sorts**
Bubble Sort	Quicksort
Selection Sort	Merge Sort
Insertion Sort	Heapsort
Counting Sort *(int, few different keys)*	Radix Sort *(int, char, ...)**

```
bool operator<(const T&, const T&) const
```

STL sorting is done with iterators:

```
#include <algorithm>
vector<int> a{4, 2, 5, 1, 3};
std::sort(begin(a), end(a)); // sorts in ascending order
```

23.1 Types

Non-Adaptive Sort

Adaptive Sort

23.2 Bubble Sort

23.3 Selection Sort

23.4 Insertion Sort

23.5 Counting Sort

23.6 Quick Sort

High-level idea:

1. Choose a pivot element from the array
2. Partition array into two subarrays
    1. One containing elements smaller than the pivot
    2. One containing elements larger than the pivot
3. Recursively apply quick sort to each subarray until the entire array is sorted

Average time complexity: O(n log n) Worst-case time complexity: O(n^2)

Worst case comes when the pivot element is selected as the max/min element of the entire array, thus the pivot doesn't do much. Can avoid by picking a good pivot, such as by using the median or a random element.

Quick sort is an ***in-place*** algorithm, requiring no additional memory beyond the original array.

23.7 Merge Sort

23.8 Heapsort

Refer to heaps notes.

23.9 Radix Sort

24 Dictionary and Hash Maps

aka Hash Tables

24.1 Dictionary Abstract Data Type

A container of items (key/value pairs) that supports two basic operations:

- Insert a new item
- Search (retrieve) an item with a given key

Operations:

24.2 Hash Maps / Hash Tables

24.2.1 Hashing - Definition

24.2.2 Collision Resolution

24.2.3 Hash Maps in C++

C++ hash table containers:

- `unordered_set<>`, `unordered_multiset<>`
- `unordered_map<>`, `unordered_multimap<>`

Example:

```
#include <unordered_map> // among other includes...
unordered_map<string, int> months;
string month;

months["January"] = 31;
months["February"] = 28;
...
```

```
// Looking up an item
month = "March";
auto it = months.find(month);
if (it == months.end()) {
    // month not found
} else {
    cout << it->first << " has " << it->second << " days\n";
}

// .count() counts how many elements with KEY there are, and since unordered_map has no dupli
//      it essentially is the same as .find() but it returns a size_type
months.count("February");
```

When NOT to use hash tables:

- Significant space overhead of nested containers
- Every access computes hash function
- O(n) worst-case time (STL implementations)
- When keys are small ints (use bucket arrays)
- For static data, set membership, or lookup (consider sorting + binary search)
- Key-value storage but traversal is needed and not lookup

25 Graphs

25.1 Definitions & Terminologies

25.2 Representations

Adjacency Matrix, Distance Matrix, Adjacency List

25.2.1 Adjacency List

Can be represented by a 2D vector. The first dimension indexes the vertex index, then the inner vector contains all the adjacent vertices.

25.3 Shortest Path

Single-source shortest path, can solve with:

- DFS
    - Only works on trees
- BFS
    - Works for unweighted edges (or when all edges have same weight)
- Dijkstra's
    - Works for weighted edges

25.4 Traveling Salesman Problem

Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?

I.e. given an undirected weighted graph, find a path that visits every node and returns to the first node with minimum total cost.

25.5 Minimum Spanning Tree

Given: edge-weighted, undirected graph $G = (V, E)$

Find: subgraph $T = (V, E'), E' \subseteq E$ such that:

- All vertices are pair-wise connected
- The sum of all edge weights in T is minimal

If there's a cycle in T, remove edge with highest weight

Therefore T must be a tree (since it must have no cycles)

Two algorithms to find MST:

- Prim's
- Kruskal's

25.5.1 Prim's Algorithm

25.5.2 Kruskal's Algorithm

25.6 Topological Sort

Topological sorting (of a DAG) is a linear ordering of vertices such that for every directed edge u-v, u comes before v

There can be more than one topological sorting for a graph.

Topological sort can be done using DFS or BFS

Commonly used for:

- scheduling tasks or events based on dependencies
- detect cycles in a directed graph
- solve problems with precedence constraints

25.6.1 Topological Sort with DFS

Essentially uses post-order DFS: In DFS we print a vertex then recursively call DFS for adjacent vertices, whereas to perform topological sort, we need to print a vertex before its adjacent vertices (since we need to maintain topological ordering)

Approach:

- Initialize stack and a `visited` array of size `n`
- For each unvisited vertex in the graph:
    - Call DFS on the vertex
        - Inside DFS function
            - Mark vertex as visited
            - Recursively call DFS for all unvisited neighbors of this vertex
            - Push vertex onto stack
- Once all vertices have been visited, pop all elements from the stack to the output vector
- Output vector is a topological sorting of the graph

25.6.1.1 C++ Implementation

Geeks for Geeks Reference

```
// Function to perform DFS and topological sorting
void topologicalSortUtil(int v, vector<vector<int> >& adj,
    vector<bool>& visited, stack<int>& st) {

    // Mark the current node as visited
    visited[v] = true;

    // Recur for all adjacent vertices
    for (int i : adj[v]) {
        if (!visited[i])
            topologicalSortUtil(i, adj, visited, st);
    }

    // Push current vertex to stack which stores the result
    st.push(v);
}

// Function to perform Topological Sort
vector<int> topologicalSort(vector<vector<int>>& adj) {
    int V = adj.size();

    // Stack to store the result
    stack<int> st;
    vector<bool> visited(V, false);

    // Call the recursive helper function to store
    // Topological Sort starting from all vertices one by
    // one
    for (int i = 0; i < V; i++) {
        if (!visited[i])
            topologicalSortUtil(i, adj, visited, st);
    }

    vector<int> ans;

    // append contents of stack
    while (!st.empty()) {
        ans.push_back(st.top());
        st.pop();
    }

    return ans;
}

int main() {

    // Graph represented as an adjacency list
    vector<vector<int>> adj = {{1}, {2}, {}, {1, 2}};

    vector<int> ans = topologicalSort(adj);

    for (auto node: ans) {
        cout << node << " ";
    }
    cout << endl;

    return 0;
}
```

25.6.2 Topological Sort with BFS

Aka Kahn's Algorithm

26 Brute-Force & Greedy Algorithms

26.1 Brute-Force

Definition:

26.2 Greedy Algorithms

Definition: At every decision step, makes the locally optimal decision.

To guarantee correctness/optimality, must show that locally optimal solutions lead to globally optimal solution

26.3 Summary

Brute-force

- Solve problem in simplest way
- Generate entire solution set, pick best
- Will give optimal solution with (typically) poor efficiency

Greedy

- Make local, best decision, and don't look back
- May give optimal solution with (typically) 'better' efficiency
- Depends upon 'greedy-choice property'
- Global optimum found by series of local optimum choices

27 Divide and Conquer, Dynamic Programming

27.1 Divide and Conquer

Repeatedly divide a problem into smaller *non-overlapping* problems (preferably of equal size)

- Often recursive
- Often involve log n complexities
- Top down approach
- E.g. binary search, quicksort

Alternatively, there is also "Combine and Conquer" Algorithms, that are bottom-up: start with smallest subproblem possible, then combine increasingly large subdomains until the given problem size.

- E.g. Merge Sort

27.2 Dynamic Programming

Remembers partial solutions (memoization) of *overlapping* subproblems

Solve small subproblems first and store the results, then use these results when needed.

Can use bottom-up or top-down approach

27.2.1 Simple Example - Fibonacci

Top-Down DP:

```
uint64_t fib(uint32_t n) {
    // Array of known Fibonacci numbers. Start out with 0, 1,
    // and the rest get automatically initialized to 0.
    // MAX_FIB + 1 used to account for 0-indexing
    static uint64_t fibs[MAX_FIB + 1] = { 0, 1 };

    // Doesn't fit in 64 bits, so don't even bother computing
    if (n > MAX_FIB)
        return 0;

    // Is already in array, so look it up
    if (fibs[n] > 0 || n == 0)
        return fibs[n];

    // Currently unknown, so calculate and store it for later
    fibs[n] = fib(n - 1) + fib(n - 2);
    return fibs[n];
}
```

Bottom-Up DP:

```
uint64_t fibBU(uint32_t i) {
    uint64_t f[MAX_N];
    i = min(i, MAX_N - 1);
    f[0] = 0;
    f[1] = 1;
    for (size_t k = 2; k <= i; k++)
        f[k] = f[k - 1] + f[k - 2];
    return f[i];
}
```

27.2.2 Simple Example - Binomial Coefficient

27.2.3 General Approach

Top-Down:

- Save known values as they are calculated
- Generally preferred because it can be more naturally transformed from recursive solution, the order of computing the subproblems takes care of itself, and it may not need to compute all subproblems

Bottom-Up:

- Precompute values from base case up towards the solution. Note it will compute all subproblems, regardless if it is needed.

27.2.4 Example - Knight Moves

27.2.5 Example - Knapsack Problem

27.2.5.1 Problem Definition

A knapsack has capacity M. An item has various weights and values.

Problem: Find the maximum value of items that can be packed into the knapsack with N items that does not exceed capacity M.

Many variations of the problem exist:

- Each item is unique (the standard formulation), aka 0-1 Knapsack Problem. Must take (1) or leave (0) an item
- Finite amount of each item
- Infinite number of copies of each item
- Fractional amount of item can be taken

27.2.5.2 Bottom-Up Example

Approach:

- For each item:
    - Can fit AND improves overall value? Take the item
    - Too large to fit in current capacity (or doesn't improve value)? Leave behind

Subproblem: What is the maximum value of items at this index with this capacity?

Thus, will use two nested loops:

1. Loop through all items
2. Look through knapsack sizes from 0 to M

	1	2	3	4	5

Safe:

	1	2	3	4	5
Size	1	2	5	6	7
Value	1	6	18	22	28

Knapsack:

Safe Item # \ Knapsack Size	0	1	2	3	4	5	6	7	8	9	10	11
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	1	1	1	1	1	1	1	1
2	0	1	6	7	7	7	7	7	7	7	7	7
3	0	1	6	7	7	18	19	24	25	25	25	25
4	0	1	6	7	7	18	22	24	28	29	29	40
5	0	1	6	7	7	18	22	28	29	34	35	40

C++ implementation:

```
uint32_t knapsackDP(const vector<Item> &items, const size_t m) {
    const size_t n = items.size();  // Get the number of items
    vector<vector<uint32_t>> memo(n + 1, vector<uint32_t>(m + 1, 0)); // DP table, initialize

    for (size_t i = 0; i < n; ++i) {          // Iterate over items
        for (size_t j = 0; j < m + 1; ++j) { // Iterate over capacities (0 to m)
            if (j < items[i].size)            // If item doesn't fit in knapsack
                memo[i + 1][j] = memo[i][j]; // Inherit value from previous row
            else                              // If item fits in the knapsack
                memo[i + 1][j] = max(memo[i][j],
                                     memo[i][j - items[i].size] + items[i].value);
                                              // Either take it or leave it (maximize value)
        }
    }

    return memo[n][m];  // Maximum value obtainable with `n` items and capacity `m`
}
```

Time complexity: $O(MN)$

Explanation:

- Table construction:
  - i represents the number of items that is being considered, i.e. considering all items 0 to index i
  - j represents the current knapsack capacity
  - Each cell of the DP table stores the max value that can be obtained using the first i items within capacity j
- Algorithm:
  - Start with no items and an empty knapsack
  - Go through each item and each capacity (double for loop)
    - If the item fits and increases overall value: take it
      - `memo[i][j]` is the value of the previous row, i.e. not taking the item
      - `memo[i][j - items[i].size()] + items[i].value` is the value of taking this item (value of knapsack with capacity that just reaches current capacity if you take this item + this item's value)
    - Otherwise, don't take the item: keep value from previous row
- Subproblem:
  - `memo[i][j]` stores the best value possible using only the first i items and knapsack of size j

Reconstructing the solution:

- included items improve a smaller solution, while excluded items don't
- if a smaller solution + an item is $\geq$ a full solution without the item, it is included. Otherwise, exclude
- the items taken can be reconstructed from the completed memo by working backwords from (n, m) to $(0, 0)$

C++ implementation of the reconstruction:

```cpp
vector<bool> knapDPReconstruct(const vector<Item> &items,
                               const vector<vector<uint32_t>> &memo,
                               const size_t m) {
    const size_t n = items.size();  // Number of items
    size_t c = m;                   // Current capacity (tracking remaining space)
    vector<bool> taken(n, false);   // Boolean vector to track selected items

    for (int i = n - 1; i >= 0; --i) {  // Iterate in reverse to reconstruct solution
```

```
        if (items[i].size <= c) {       // If the item fits in the remaining capacity
            if (memo[i][c - items[i].size] + items[i].value >= memo[i][c]) {
                taken[i] = true;        // Mark item as taken
                c -= items[i].size;     // Reduce the remaining capacity
            }
        }
    }

    return taken; // Return boolean array indicating chosen items
}
```

28 Backtracking & Branch and Bound Algorithms

Can be used to solve the following types of algorithm problems:

- **Constraint satisfaction problems**
  - Stop when a satisfying solution is found
    - one solution is sufficient
  - Can rely on **backtracking algorithms**
  - E.g. sorting, mazes, spanning tree, puzzles, GRE/analytical
- **Optimization problems**
  - Usually cannot stop early (since we need to find a global min/max)
  - Must develop a set of possible solutions
    - "feasibility set"
    - Then pick best solution
  - Can rely on **branch and bound algorithms**
  - E.g. giving change, MST
-

28.1 Backtracking Algorithms

Consider all possible outcomes of each decision, but prune searches that do not satisfy constraints

This prevents us from having to perform exhaustive search, however the search space is still large.

- Branch on every possibility
- Maintain one or more "partial solutions"
- Check every partial solution for validity
  - If a partial solution violates some constraint, prune it since it makes no sense to go further, i.e. backtrack

28.1.1 General Form

Pseudocode:

```
Algorithm checknode(node v)
    if (promising(v))
        if (solution(v))
            write solution*
        else
            for each node u adjacent to v
                checknode(u)
```

Alternate Form:

```
Algorithm checknode(node v)
    if (solution(v))
        write solution*
    else
        for each node u adjacent to v
            if (promising(u))
                checknode(u)
```

`solution(v)` - checks 'depth' of the solution (constraint satisfaction)

`promising(v)` - checks whether should be pruned. Different for each application

`checknode(v)` - called only if partial solution is both promising and not a solution (i.e. branch to check further)

Often the most difficult part is determining `promising()`

28.1.2 Example: M-Coloring

Given:

- n (number of nodes)
- m (number of colors)
- W[0...n)[0...n) (adjacency matrix), where W[i][j] is true iff i is connected to node j

Find all possible colorings of graph represented by `int vcolor[0...n)`, where `vcolor[i]` is the color associated with node `i`

Pseudocode:

```
Algorithm m_coloring(index i = 0)
    if (i == n)
        print vcolor(0) thru vcolor(n - 1)
        return
    for (color = 0; color < m; color++)
        vcolor[i] = color
        if (promising(i))
            m_coloring(i + 1)

bool promising(index i)
    for (index j = 0; j < i; ++j)
        if (W[i][j] and vcolor[i] == vcolor[j])
            return false
```

promising is essentially where we check partial solution for validity. If it is not promising, we don't go further, i.e. prune this branch.

28.1.3 Example: n Queens

Can n queens be placed on a 1×1 board so that it doesn't threaten another?

28.2 Branch and Bound Algorithms

Essentially extends backtracking to optimization problems

Minimizing a function with this property:

- a partial solution is pruned if its cost $\geq$ cost of best known complete solution
- e.g. length of a path

The property is esentially saying that if the cost of a partial solution is too big, drop this partial solution (because cannot be optimal)

28.2.1 General Form

Pseudocode:

```
Algorithm checknode(Node v, Best currBest)
    Node u
    if (promising(v, currBest))
        if (solution(v)) then
            update(currBest)
        else
            for each child u of v
                checknode(u, currBest)
    return currBest
```

`solution()` - checks ‘depth’ of solution (constraint satisfaction)

`update()` - if the new solution is better than the current solution, update the known current best solution

`checknode()` - called only if promising and not solution (extend and go further to this branch)

`lowerbound()` - an estimate of the solution based on the cost so far and the underestimate of the remaining cost (bound)

`promising()` - return true when `lowerbound() < currBest`, i.e. it is promising to look into further since it is potentially (by the lowerbound) the optimal solution. Return of false results in pruning

The key idea to B&B is the **bound**, by bounding away (pruning) unpromising partial solutions

Minimizing With B&B:

- Start with an “infinity” bound
- Find first complete solution – use its cost as an upper bound to prune the rest of the search
- Measure each partial solution and calculate a lower bound estimate needed to complete the solution
- Prune partial solutions whose lower bounds exceed the current upper bound
- If another complete solution yields a lower cost – that will be the new upper bound
- When search is done, the current upper bound will be a minimal solution

Maximizing With B&B

- Start with a “zero” bound
- Find first complete solution – use its cost as a lower bound to prune the rest of the search
- Measure each partial solution and calculate an upper bound estimate needed to complete the solution
- Prune partial solutions whose upper bounds are less than the current lower bound

- If another complete solution yields a larger value – that will be the new lower bound
- When search is done, the current lower bound will be a maximal solution

28.2.2 Summary

28.3 Traveling Salesperson Problem (TSP)

28.3.1 Definition

Hamiltonian Cycle -

TSP is a optimization problem that asks to find the Hamiltonian cycle with least weight

We can apply both backtracking and B&B to TSP.

29 Trie

A.k.a. Prefix Tree, Radix Tree, or Digital Tree

30 Summary

In summary...

31 Tips on Solving DS&A Questions

31.1 Problem Solving Flowchart

31.2 DS&A Roadmap

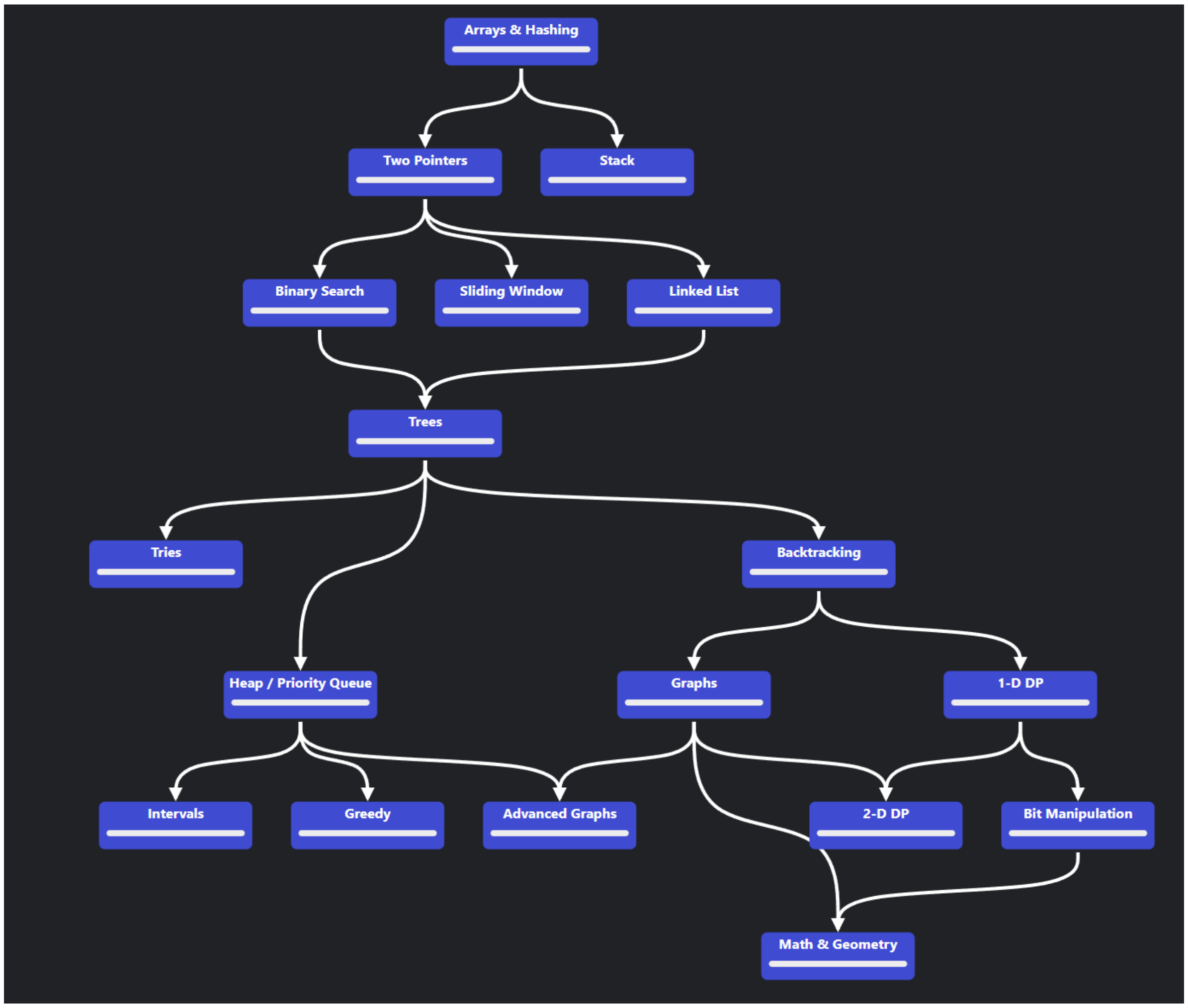
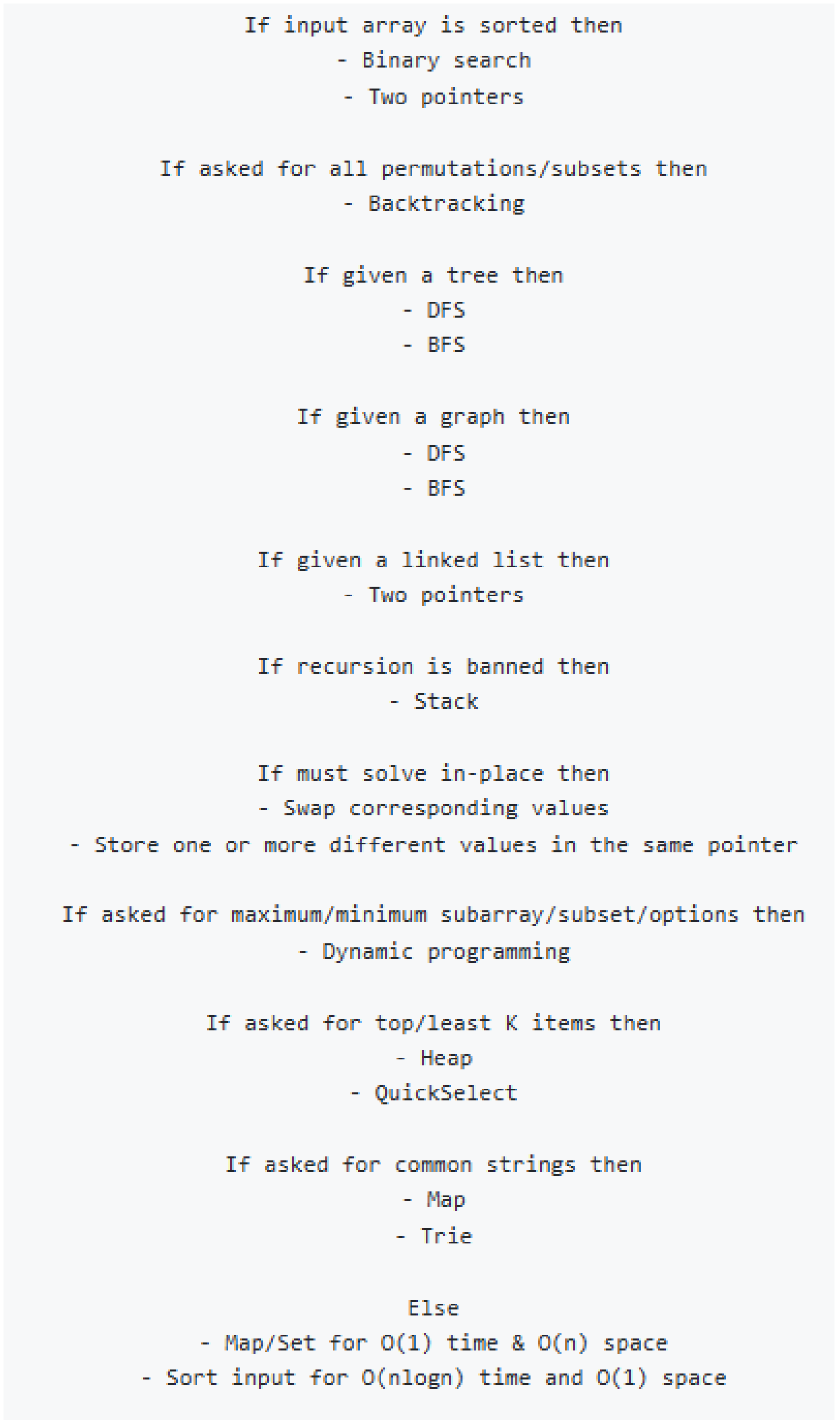


Figure 31.2: A Roadmap for studying. Source



```
If input array is sorted then
- Binary search
- Two pointers

If asked for all permutations/subsets then
- Backtracking

If given a tree then
- DFS
- BFS

If given a graph then
- DFS
- BFS

If given a linked list then
- Two pointers

If recursion is banned then
- Stack

If must solve in-place then
- Swap corresponding values
- Store one or more different values in the same pointer

If asked for maximum/minimum subarray/subset/options then
- Dynamic programming

If asked for top/least K items then
- Heap
- QuickSelect

If asked for common strings then
- Map
- Trie

Else
- Map/Set for O(1) time & O(n) space
- Sort input for O(nlogn) time and O(1) space
```

Figure 31.1: Tips on problem approach.Image Source

31.3 Problem Flowchart

31.4 ROI

31.5 “Academic” Algorithms

According to AlgoMonster, some **algorithms** that are very rarely/almost never asked in interviews:

- Minimal spanning tree: Kruskal’s algorithm and Prim’s algorithm
- Minimum cut: Ford-Fulkerson algorithm
- Shortest path in weight graphs: Bellman-Ford-Moore algorithm
- String search: Boyer-Moore algorithm

31.6 Keyword to Algo

AlgoMonster provides a convenient “Keyword to Algorithm” summary:

“Top k”

- Heap
    - E.g. K closest points

“How many ways..”

- DFS
    - E.g. Decode ways
- DP
    - E.g. Robot paths

“Substring”

- Sliding window
    - E.g. Longest substring without repeating characters

“Palindrome”

- two pointers: Valid Palindrome
- DFS: Palindrome Partitioning
- DP: Palindrome Partitioning II

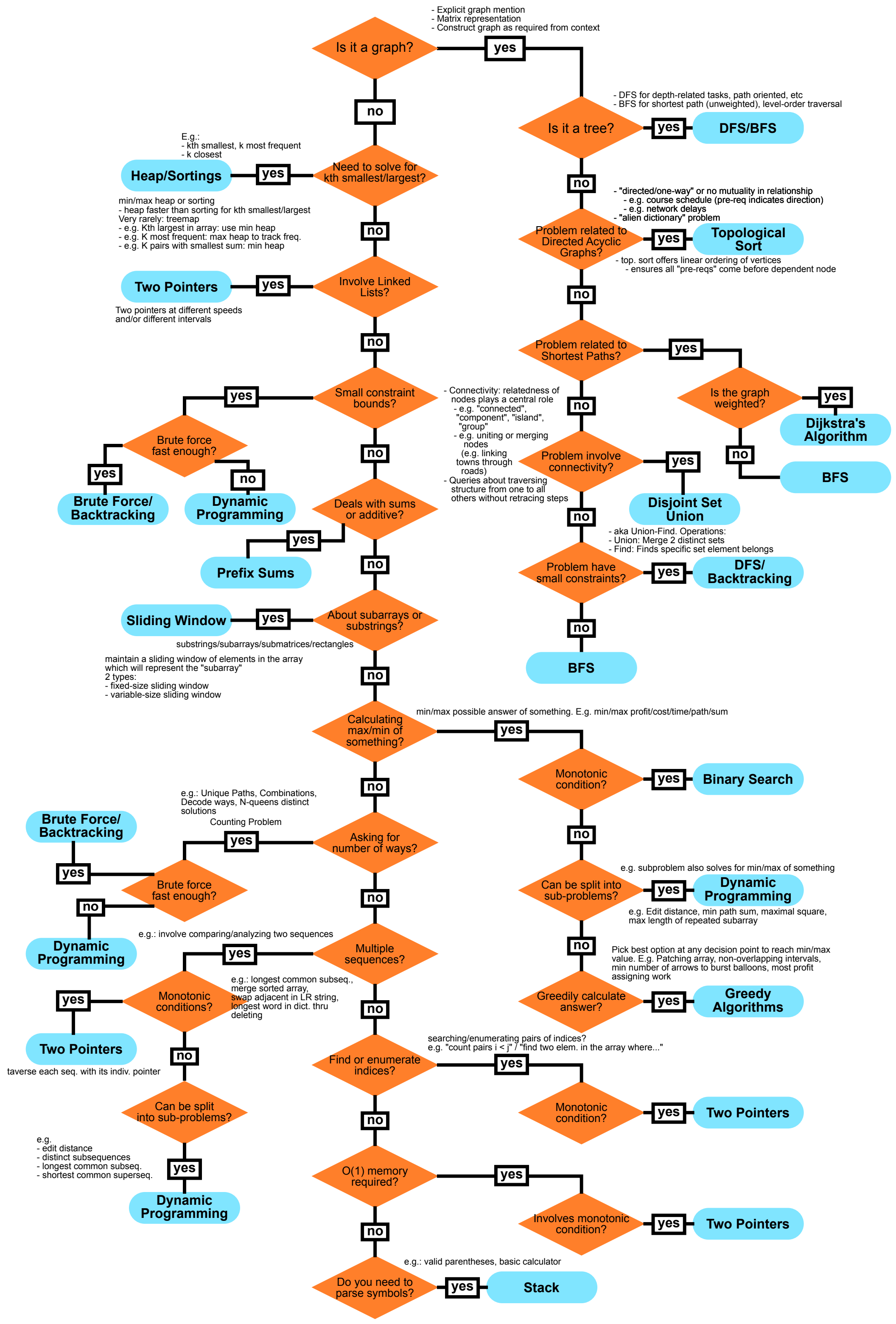
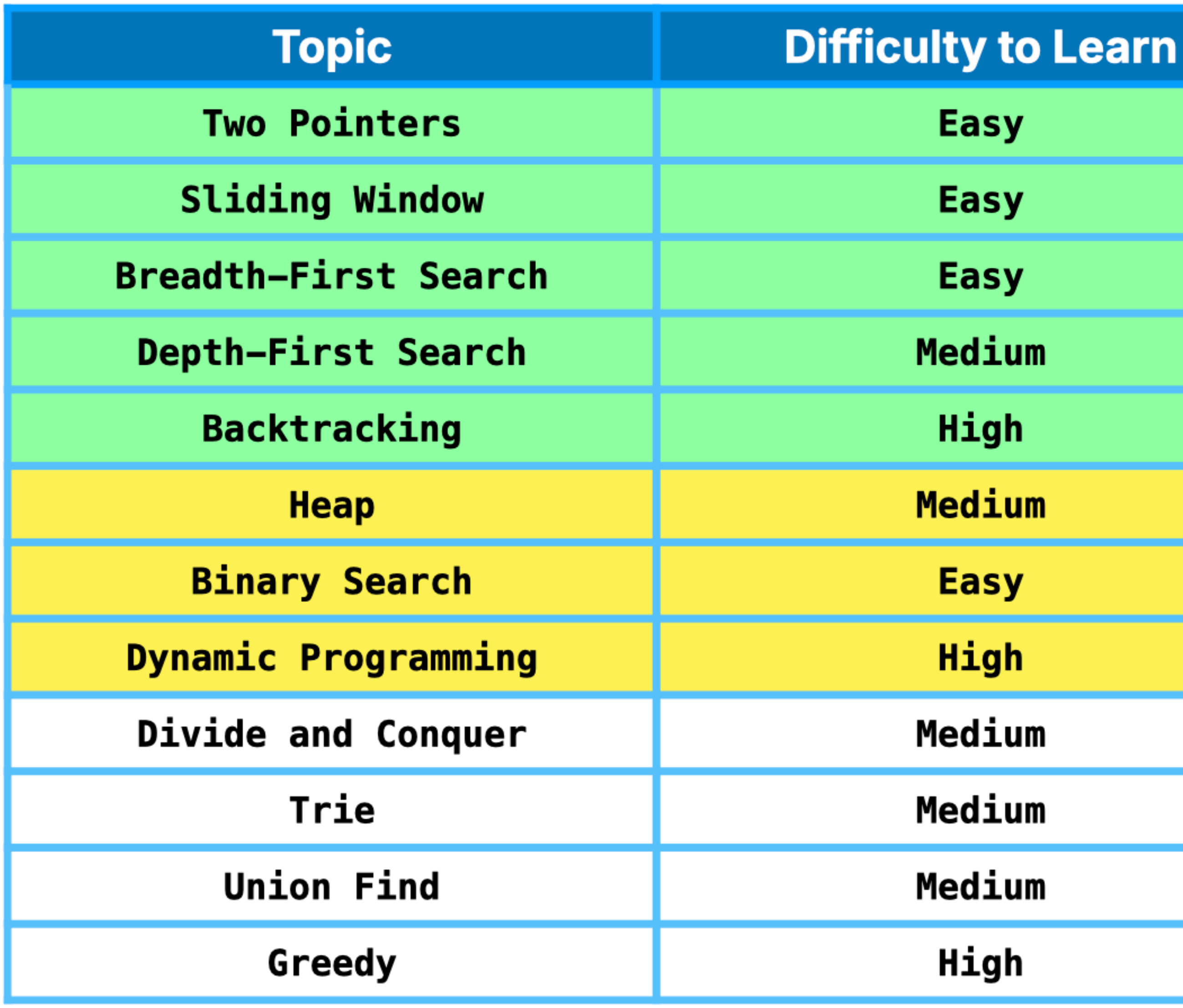

Is it a graph?
- Explicit graph mention
- Matrix representation
- Construct graph as required from context
yes
no
Is it a tree?
- DFS for depth-related tasks, path oriented, etc
- BFS for shortest path (unweighted), level-order traversal
yes
DFS/BFS
no
Problem related to Directed Acyclic Graphs?
- "directed/one-way" or no mutuality in relationship
- e.g. course schedule (pre-req indicates direction)
- e.g. network delays
- "alien dictionary" problem
yes
Topological Sort
- top. sort offers linear ordering of vertices
- ensures all "pre-reqs" come before dependent node
no
Problem related to Shortest Paths?
yes
Is the graph weighted?
yes
Dijkstra's Algorithm
no
BFS
no
Problem involve connectivity?
- Connectivity: relatedness of nodes plays a central role
- e.g. "connected", "component", "island", "group"
- e.g. uniting or merging nodes (e.g. linking towns through roads)
- Queries about traversing structure from one to all others without retracing steps
yes
Disjoint Set Union
- aka Union-Find. Operations:
- Union: Merge 2 distinct sets
- Find: Finds specific set element belongs
no
Problem have small constraints?
yes
DFS/ Backtracking
no
BFS
Need to solve for kth smallest/largest?
E.g.:
- kth smallest, k most frequent
- k closest
yes
Heap/Sortings
min/max heap or sorting
- heap faster than sorting for kth smallest/largest
Very rarely: treemap
- e.g. Kth largest in array: use min heap
- e.g. K most frequent: max heap to track freq.
- e.g. K pairs with smallest sum: min heap
no
Involve Linked Lists?
yes
Two Pointers
Two pointers at different speeds and/or different intervals
no
Small constraint bounds?
yes
Brute force fast enough?
yes
Brute Force/ Backtracking
no
Dynamic Programming
no
Deals with sums or additive?
yes
Prefix Sums
no
About subarrays or substrings?
substrings/subarrays/submatrices/rectangles
yes
Sliding Window
maintain a sliding window of elements in the array which will represent the "subarray"
2 types:
- fixed-size sliding window
- variable-size sliding window
no
Calculating max/min of something?
min/max possible answer of something. E.g. min/max profit/cost/time/path/sum
yes
Monotonic condition?
yes
Binary Search
no
Can be split into sub-problems?
e.g. subproblem also solves for min/max of something
yes
Dynamic Programming
e.g. Edit distance, min path sum, maximal square, max length of repeated subarray
no
Greedily calculate answer?
Pick best option at any decision point to reach min/max value. E.g. Patching array, non-overlapping intervals, min number of arrows to burst balloons, most profit assigning work
yes
Greedy Algorithms
no
Asking for number of ways?
e.g.: Unique Paths, Combinations, Decode ways, N-queens distinct solutions
Counting Problem
yes
Brute force fast enough?
yes
Brute Force/ Backtracking
no
Dynamic Programming
no
Multiple sequences?
e.g.: involve comparing/analyzing two sequences
yes
Monotonic conditions?
e.g.: longest common subseq., merge sorted array, swap adjacent in LR string, longest word in dict. thru deleting
yes
Two Pointers
taverse each seq. with its indiv. pointer
no
Can be split into sub-problems?
e.g.
- edit distance
- distinct subsequences
- longest common subseq.
- shortest common superseq.
yes
Dynamic Programming
no
Find or enumerate indices?
searching/enumerating pairs of indices?
e.g. "count pairs i < j" / "find two elem. in the array where..."
yes
Monotonic condition?
yes
Two Pointers
no
O(1) memory required?
yes
Involves monotonic condition?
yes
Two Pointers
no
Do you need to parse symbols?
e.g.: valid parentheses, basic calculator
yes
Stack

Figure 31.3: A problem solving flowchart based on AlgoMonster's flowchart. Open the SVG in a new tab and zoom in/out for better viewing.



Topic	Difficulty to Learn
Two Pointers	Easy
Sliding Window	Easy
Breadth-First Search	Easy
Depth-First Search	Medium
Backtracking	High
Heap	Medium
Binary Search	Easy
Dynamic Programming	High
Divide and Conquer	Medium
Trie	Medium
Union Find	Medium
Greedy	High

Figure 31.4: Studying to Maximizing ROI according to AlgoMonster.

"Tree"

- shortest, level-order
  - BFS: Binary Tree Level-Order Traversal
- else: DFS: Max Depth

"Parentheses"

- Stack: Valid Parentheses

"Subarray"

- Sliding window: Maximum subarray sum
- Prefix sum: Subarray sum
- Hashmap: Continuous subarray sum

Max subarray

- Greedy: Kadane's Algorithm

"X Sum"

- Two pointer: Two sum

"Max/longest sequence"

- Dynamic programming, DFS: Longest increasing subsequence
- mono deque: Sliding window maximum

"Minimum/Shortest"

- Dynamic programming, DFS: Minimal path sum
- BFS: Shortest path

"Partition/split … array/string"

- DFS: Decode ways

"Subsequence"

- Dynamic programming, DFS: Longest increasing subsequence
- Sliding window: Longest increasing subsequence

"Matrix"

- BFS, DFS: Flood fill, Islands
- Dynamic programming: Maximal square

"Jump"

- Greedy/DP: Jump game

"Game"

- Dynamic programming: Divisor game, Stone game

"Connected component", "Cut/remove" "Regions/groups/connections"

- Union Find: Number of connected components, Redundant connections

Transitive relationship

- If the items are related to one another and the relationship is transitive, then chances are we can build a graph and use BFS or Union Find.
  - string converting to another, BFS: Word Ladder
  - string converting to another, BFS, Union Find: Sentence Similarity
  - numbers having divisional relationship, BFS, Union Find: Evaluate Division

"Interval"

- Greedy: sort by start/end time and then go through sorted intervals Interval Pattern

32 Problems & Explanations

32.1 Array

32.1.1 Two Sum

Difficulty: Leetcode Easy

(Blind 75)

Classic problem

- Naive Solution: Brute force approach - O(n^2)
- Optimization: Use hash table

Approach 1: Brute Force

Approach 2: Hash Table

High-level idea: use the hash table to find whether the number needed to meet the target with the current number is already in the array (that we have iterated through so far).

Example 1 - Two-pass hash table

Example 2 - One-pass hash table

32.1.2 Contains Duplicate

Difficulty: Leetcode Easy

(Blind 75)

Approaches:

- Brute force: compare every pair of numbers in the array
- Sort the array, then check for consecutive duplicates
- Hashing
    - Build hash set
    - Build hash table, find duplicates of the same key
    - The above two methods have same time complexity

32.1.3 Maximum Subarray

Difficulty: Leetcode Medium

(Blind 75)

At a glance:

- Notice that this is an optimization problem: Finding the sum of the **maximum** subarray
- Notice how this could be split into overlapping subproblems

Clarification: Note that a subarray must be a contiguous part of the array, unlike the longest common subsequence problem (where characters can be deleted).

Approach:

- Dynamic programming
    - Use a 1D DP table, at each index store the sum of max subarray up until that index in the array
        - At each index, add current number to the sum
            - If current sum > max sum so far, update max sum
            - If the current sum becomes negative, then reset to 0 to start a new subarray
            - This is because you always want to start a subarray at a positive number to maximize the sum

32.1.4 3 Sum

Difficulty: Leetcode Medium

(Blind 75)

Concepts: Array, Two Pointers, Sorting

Approaches:

- Naive approach: Brute force by testing all triplets in the array
- Set + Two Pointer Approach
    - Sort the array
    - Create a set from the array to *store* unique triplets found that sum to 0
    - Iterate through the array with index i from 0
        - Initialize two pointers, one pointer j at $i + 1$ and the other k at the end of the array
        - While

        - check sum of i, j, k is 0. If it is add the triplet to the set, increment j and decrement k
        - if the sum is <0, increment j. if the sum is >0, decrement k
        - *these work because the array is sorted
    - Turn the set to a vector for final output
- Hash map approach
    - Hash indices of all elements into a hash map

32.2 Binary / Bit Manipulation

32.2.1 Sum of Two Integers

32.2.2 Number of 1 Bits

32.2.3 Counting Bits

32.2.4 Missing Number

32.2.5 Reverse Bits

32.3 Dynamic Programming

32.3.1 Coin Change

Difficulty: Leetcode Medium

(Blind 75)

Leetcode

Solution Reference

Problem: Given integer array `coins` (which contain coins of different denominations) and integer `amount` of money, return the **fewest number of coins needed to make up the amount.** If the amount cannot be made up, return -1. Assume infinite amount of each kind of coin.

At first glance:

- Notice that this is an optimization problem: finding the **fewest** number of coins
- Then, notice how this could be split into subproblems: finding the fewest number of coins needed to make up amount m, for an amount $\leq$ `amount` (target amount)

- Notice how this is very similar to 0-1 knapsack problem.
- Strategy: Dynamic programming, memoize subproblems. A first solution could use a 2D DP table, then an optimization could make this 1D to save memory

Approach:

- Since we want to find the *fewest*, we know we can initialize the memo with `INT_MAX` (doesn't necessarily need to be implemented this way, but easy to think about)
- Build a 1D memo, where `memo[i]` indicates the fewest number of coin needed to make up amount i
- Key insight: to build up amount i with minimum number of coin, we can look at $memo[i - coin[j]]$ for each coin j
  - This tells us the fewest number of coins needed to build up an amount before we take this coin, and that taking this coin can build up to this amount
  - We take the minimum of $memo[i - coin[j]] + 1$, where $+1$ is to include taking this current coin denom. Note that we need to prevent out of bounds access for `coin[j]` incase it is too large

Example Walkthrough:

Assume coins `[1, 2, 5]` and `amount = 6`

1. Initialize memo: `{0, INT_MAX, INT_MAX, INT_MAX, INT_MAX, INT_MAX, INT_MAX}`
2. At index `i=1`, we try to build up amount 1 with all the coins we have
   1. Try using coin of val 1 (coin[0]). To build up `i-coin[0]` (amount 0), we need 0 coins.
   2. No other coin values are in bounds, so use coin[0], adding a 1 to this amount
   3. `{0, 1, INT_MAX, INT_MAX, INT_MAX, INT_MAX, INT_MAX}`
3. At index `i=2`, build up amount 2 with all the coins we have
   1. Try using coin[0]. To build up amount `2-coin[0]` (amount 1), we needed 1 coin. Thus to make up 2 with a value 1 coin, we use this coin and add 1
   2. Try using coin[1]. To build up amount `2-coin[1]` (amount 0), we needed 0 coins. Thus to make up 2 with a value 2 coin, we use this coin and add 1
   3. We cannot coin[2].
   4. Minimum of the coins is to use 1 coin. Thus 1 is the value for this amount
   5. `{0, 1, 1, INT_MAX, INT_MAX, INT_MAX, INT_MAX}`
4. At index `i=3`
   1. `{0, 1, 1, 2, INT_MAX, INT_MAX, INT_MAX}`
5. At index `i=4`
   1. `{0, 1, 1, 2, 2, INT_MAX, INT_MAX}`

6. {0, 1, 1, 2, 2, 1, INT_MAX}
7. {0, 1, 1, 2, 2, 1, 2}

If the final cell has INT_MAX, it means it cannot be made up with these coins, and we should instead return -1 per the problem spec.

C++ Implementation:

```
class Solution {
public:
    int coinChange(vector<int>& coins, int amount) {
        // Create DP table
        int memo[amount + 1];
        memo[0] = 0; // initialize first val to 0 since 0 coins make up amount 0

        // Sort the coins so we can access them from small to large values
        sort(begin(coins), end(coins));

        // Walk through the DP table, for each amount i
        for (int i = 1; i < amount + 1; i++) {
            memo[i] = INT_MAX;  // default value, since we want to find min val, we start wi
            for (int c : coins) {
                if (i - c < 0) break;       // if this coin is too large to make up current a

                if (memo[i - c] != INT_MAX) // Only use previously used cell
                    memo[i] = min(memo[i], 1 + memo[i - c]);
                    // Choose the minimum between using the current coin c, otherwise keep IN
            }
        }

        // Return -1 if amount cannot be made up
        return memo[amount] == INT_MAX ? -1 : memo[amount];
    }
};
```

32.3.2 Longest Common Subsequence

Difficulty: Leetcode Medium

(Blind 75)

Leetcode

Leetcode Solution Reference

Subsequence: a string that can be generated from an original string with some (or no) characters deleted without changing the relative ordering of the remaining characters

Problem: Given two strings, return the length of the **longest common subsequence**

At first glace:

- Notice that this is an optimization problem: finding the **longest**
- The problem can be divided into overlapping subproblems -> use dynamic programming

Approach:

- Define the DP table, use a 2D table where each dimension is the length of each string (text1.size() by text2.size()):
  - At each cell, store the longest common subsequence up to that position/index of each string
    - e.g. `memo[i][j]` represents the longest common subsequence length of subtrings `text1[0:i-1]` and `text2[0:j-1]`
  - Fill the table in a bottom-up approach
    - Recurrence Relation:
      - if character at `[i-1]` and `[j-1]` match, add 1 to the length `memo[i][j] = memo[i-1][j-1] + 1`
      - If character does not match, use longest length from either substring's previous char: `memo[i][j] = max(memo[i-1][j], memo[i][j-1])`
- Bottom right cell of the table `memo[length1][length2]` contains the answer

Complexity: O(`length1` × `length2`) in both time and space

C++ Implementation:

```
class Solution {
public:
    int longestCommonSubsequence(string text1, string text2) {
        int text1Length = text1.size();
        int text2Length = text2.size();

        // Create 2D DP table to store lengths of common subsequence at each index
        int memo[text1Length + 1][text2Length + 1];

        // Initialize the 2D array with zeros
        memset(memo, 0, sizeof memo);

        // Loop through both strings and fill the dp array.
        for (int i = 1; i <= text1Length; ++i) {
            for (int j = 1; j <= text2Length; ++j) {
                // If current characters match, add 1 to the length of the sequence
                if (text1[i - 1] == text2[j - 1]) {
                    memo[i][j] = memo[i - 1][j - 1] + 1;
                } else {
                    // If current characters do not match, take the max len
                    // of either skipping the current char of either strings
                    memo[i][j] = max(memo[i - 1][j], memo[i][j - 1]);
                }
            }
        }

        // bottom-right cell which contains answer
        return memo[text1Length][text2Length];
    }
};
```

32.3.3 Combination Sum

Difficulty: Leetcode Medium

Concepts: Array, Dynamic Programming

Note:

- Ordering of the combination matters!

32.4 Graph

32.4.1 Course Schedule

Difficulty: Leetcode Medium

Concepts: DFS, BFS, Graph, Topological Sort

Problem: There are `numCourses` courses that must be taken, labeled from 0 to `numCourses - 1`. Given an array `prerequisites` where `prerequisites[i] = [ai, bi]`, where bi must be taken before taking ai. Return true if you can finish all courses. Otherwise, return false. Pair [0, 1], indicates that to take course 0, you have to first take course 1.

At a glance:

- Notice that this is essentially a directed graph
- Equivalent to finding whether a cycle exists, since if a cycle exists it is impossible to take all classes

Approach:

- Topological sort via DFS (or BFS)
- `pair<int, int>` is not convenient, can transform to adjacency list

Example C++ Implementation with DFS (Source):

```
class Solution {
public:
    bool canFinish(int numCourses, vector<pair<int, int>>& prerequisites) {
        graph g = buildGraph(numCourses, prerequisites);
        vector<bool> todo(numCourses, false), done(numCourses, false);
        for (int i = 0; i < numCourses; i++) {
            if (!done[i] && !acyclic(g, todo, done, i)) {
                return false;
            }
        }
        return true;
    }
private:
    typedef vector<vector<int>> graph;

    graph buildGraph(int numCourses, vector<pair<int, int>>& prerequisites) {
        graph g(numCourses);
        for (auto p : prerequisites) {
            g[p.second].push_back(p.first);
        }
        return g;
    }

    bool acyclic(graph& g, vector<bool>& todo, vector<bool>& done, int node) {
        if (todo[node]) {
            return false;
        }
        if (done[node]) {
            return true;
        }
        todo[node] = done[node] = true;
        for (int v : g[node]) {
            if (!acyclic(g, todo, done, v)) {
                return false;
            }
        }
        todo[node] = false;
        return true;
    }
};
```

32.4.2 Number of Islands

Approach:

- Starting from land, all cells of the island can be found by doing DFS / BFS

32.4.3 Longest Consecutive Sequence

Difficulty: Leetcode Medium

Concepts: Array, Hash Table, Union Find

Problem: Given an unsorted array of integers `nums`, return the length of the longest consecutive elements sequence. You must write an algorithm that runs in O(n) time.

At a glance:

Approaches:

- Brute Force: array + triple nested loops
- Brute Force Optimized: array + sorting
- hash table: use array + hash table (unordered map)
- hash optimized: use array + hash table (unordered set)

Brute Force Approach

Brute Force Optimized

Hash Table (Unordered Map)

- First pass builds a hash table of the given array (int key to bool value)
- Second pass tries to mark the start of potential sequences
  - goes through each number and checks if the predecessor (number-1) exists
  - If predecessor exists, mark the current number as false in the array
- Third pass tries to find the longest sequence

  - Go through the hash table and on a true (start of a sequence)
    * Keep checking consecutive numbers and whether they exist in the hash table
  - Count how long each sequence extends and update curr_max if logner than what it has seen

32.5 Interval

32.5.1 Merge Intervals

32.6 Linked List

32.6.1 Reverse a linked list

Classic problem

32.6.2 Merge K Sorted Lists

32.7 Matrix

32.8 String

32.8.1 Longest Substring without Repeating Characters

32.8.2 Valid Palindromes

Classic

32.8.3 Valid Parentheses

Classic

32.9 Tree

32.10 Heap

32.10.1 Merge K Sorted Lists

Difficulty: Leetcode Hard

32.10.2 Top K Frequent Elements

Difficulty: Leetcode Medium

Concepts: Array, Hash Table, Divide and Conquer, Sorting, Heap (Priority Queue), Bucket Sort, Counting, Quickselect

Approach

- First use hash table to find the frequency of each element
- Then use a priority queue to essentially build a sorted list of the element's frequency

Example C++ Solution (Source):

```
class Solution {
public:
    vector<int> topKFrequent(vector<int>& nums, int k) {
        // Build hash table that stores the frequency of each number
        unordered_map<int,int> map;
        for(int num : nums){
            map[num]++;
        }

        vector<int> res;
        // pair<first, second>: first is frequency,  second is number
        //    Recall that C++ pair compares the first element first, then compares the secon
        priority_queue<pair<int,int>> pq;

        // Go over every number that occurred in the given vector
        for(auto it = map.begin(); it != map.end(); it++){

          // Insert a pair into the
            pq.push(make_pair(it->second, it->first));
            if(pq.size() > (int)map.size() - k){
              //  If the priority queue size is larger than the target top K
                res.push_back(pq.top().second);   // insert the highest freq number from the
                pq.pop();                         // pop from the top of the PQ
            }
        }

        return res;
    }
};
```

32.10.3 Find Median from Data Stream

Difficulty: Leetcode Hard

Concepts: Two Pointers, Sorting, Heap (Priority Queue), Data Stream