1 Uncertainty Analysis

$$\mathbf{M} = k_0 \hat{\mathbf{M}}$$

$$\begin{pmatrix} \hat{\mathbf{M}} & \mathbf{F_1} & \mathbf{F_2} \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{D} \end{pmatrix} = \begin{pmatrix} k_0 \\ k_1 \\ k_2 \end{pmatrix}$$

$$\begin{pmatrix} \hat{\mathbf{M}} & \mathbf{F_1} & \mathbf{F_2} \end{pmatrix}^{-1} = \begin{matrix} \frac{-\cos(\theta_C)\sec(\theta_C+\psi)\sin(\psi)}{\hat{\mathbf{M}}_Z} & 0 & \frac{\cos(\theta_C)\sec(\theta_C+\psi)\cos(\psi)}{\hat{\mathbf{M}}_Z} \\ \sin(\theta_C)\sec(\theta_C+\psi)\sin(\psi) & 1 & -\sin(\theta_C)\sec(\theta_C+\psi)\cos(\psi) \\ \cos(\theta_C)\sec(\theta_C+\psi) & 0 & -\sin(\theta_C)\sec(\theta_C+\psi) \end{matrix}$$

$$k_0 = \frac{-\cos(\theta_C)\sec(\theta_C+\psi)\sin(\psi)}{\hat{\mathbf{M}}_Z} D_x + \frac{\cos(\theta_C)\sec(\theta_C+\psi)\cos(\psi)}{\hat{\mathbf{M}}_Z} D_z$$

$$\mathbf{M}_x = \sin(\theta_C)\sec(\theta_C+\psi)(\cos(\psi)D_z - \sin(\psi)D_x)$$

$$\delta\mathbf{M}_{\mathrm{x}} = \sqrt{\mathbf{A^2} + \mathbf{B^2} + \mathbf{C^2} + \mathbf{E^2}}$$

$$\mathbf{A} = [\delta\theta_C(\cos(\psi)\sec(\theta_C+\psi)(D_z\cos(\psi) - D_x\sin(\psi)))]$$

$$\mathbf{B} = [\delta\psi(\sin(\theta_c)\sec^2(\theta_C+\psi)(D_z\sin(\theta_C) - D_x\cos(\theta_c)))]$$

$$\mathbf{C} = [\delta D_z(\sin(\theta_C)\cos(\psi)\sec(\theta_C+\psi)]$$

$$\mathbf{E} = [\delta D_x(-\sin(\theta_C)\sin(\psi)\sec(\theta_C+\psi)]$$

$$\delta \mathrm{\mathbf{M}_y} = \sqrt{\mathbf{A^2 + B^2 + C^2 + E^2 + F^2}}$$

$$\mathbf{A} = [\delta\theta_C(\tan(\phi_C)\sin(\psi)\sec(\theta_C+\psi)(D_z\cos(\psi) - D_x\sin(\psi)))]$$

$$\mathbf{B} = [\delta\psi(-\tan(\phi_C)\cos(\theta_c)\sec^2(\theta_C+\psi)(D_x\cos(\theta_C) - D_x\sin(\theta_c)))]$$

$$\mathbf{C} = [\delta D_z(\tan(\phi_C)\cos(\theta_C)\cos(\psi)\sec(\theta_C+\psi)]$$

$$\mathbf{E} = [\delta D_x(-\tan(\phi_C)\cos(\theta_C)\sin(\psi)\sec(\theta_C+\psi)]$$

$$\mathbf{F} = [\delta\phi_{\mathrm{C}}(\sec^2(\phi_{\mathrm{C}})\cos(\theta_{\mathrm{C}})\sec(\theta_{\mathrm{C}}+\psi)(\mathrm{D_z}\cos(\psi) - \mathrm{D_x}\sin(\psi)))]$$

$$\mathbf{M}_{\mathrm{z}} = \cos(\theta_C)\sec(\theta_C+\psi)(\cos(\psi)D_z - \sin(\psi)D_x)$$

$$\delta \mathrm{\mathbf{M}_z} = \sqrt{\mathbf{A^2 + B^2 + C^2 + E^2}}$$

$$\mathbf{A} = [\delta\theta_C(\sin(\psi)\sec(\theta_C+\psi)(D_z\cos(\psi) - D_x\sin(\psi)))]$$

$$\mathbf{B} = [\delta\psi(-\cos(\theta_c)\sec^2(\theta_C+\psi)(D_x\cos(\theta_C) - D_x\sin(\theta_c)))]$$

$$\mathbf{C} = [\delta D_z(\cos(\theta_C)\cos(\psi)\sec(\theta_C+\psi)]$$

$$\mathbf{E} = [\delta D_x(-\cos(\theta_C)\sin(\psi)\sec(\theta_C+\psi)]$$

$$\mathbf{M}_y = \tan(\phi_C)\cos(\theta_C)\sec(\theta_C+\psi)(\cos(\psi)D_z - \sin(\psi)D_x)$$

$$\psi = \pi + \theta_p + \beta$$

$$\delta\psi = \sqrt{[\delta\theta_p^2 + \delta\beta^2}$$

$$\theta_C = \arctan \frac{u'_C}{S_C}$$

$$\delta\theta_C = \sqrt{[\delta u'_C(\frac{S_C}{u'^2_C + S^2_C})]^2 + [\delta S_C(\frac{u'_C}{u'^2_C + S^2_C})]^2}$$

$$\phi_C = \arctan \frac{v'_C}{S_C}$$

$$\delta\phi_C = \sqrt{[\delta v'_C(\frac{S_C}{v'^2_C + S^2_C})]^2 + [\delta S_C(\frac{v'_C}{v'^2_C + S^2_C})]^2}$$