1 Uncertainty Analysis

1.1 Finding k_0

$$
\begin{aligned}
\mathbf{M} &= k_0\hat{\mathbf{M}} \\
\begin{pmatrix} \hat{\mathbf{M}} & \mathbf{F_1} & \mathbf{F_2} \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{D} \end{pmatrix} &= \begin{pmatrix} k_0 \\ k_1 \\ k_2 \end{pmatrix} \\
\begin{pmatrix} \hat{\mathbf{M}} & \mathbf{F_1} & \mathbf{F_2} \end{pmatrix}^{-1} &= \begin{pmatrix} \frac{-\cos(\theta_C)\sec(\theta_C+\psi)\sin(\psi)}{\hat{\mathbf{M}}_Z} & 0 & \frac{\cos(\theta_C)\sec(\theta_C+\psi)\cos(\psi)}{\hat{\mathbf{M}}_Z} \\ \sin(\theta_C)\sec(\theta_C+\psi)\sin(\psi) & 1 & -\sin(\theta_C)\sec(\theta_C+\psi)\cos(\psi) \\ \cos(\theta_C)\sec(\theta_C+\psi) & 0 & -\sin(\theta_C)\sec(\theta_C+\psi) \end{pmatrix} \\
k_0 &= \frac{-\cos(\theta_C)\sec(\theta_C+\psi)\sin(\psi)}{\hat{\mathbf{M}}_Z}D_x + \frac{\cos(\theta_C)\sec(\theta_C+\psi)\cos(\psi)}{\hat{\mathbf{M}}_Z}D_z
\end{aligned}
$$

1.2 Uncertainty in $\mathbf{M}_x$

$$
\begin{aligned}
\mathbf{M}_x &= \sin(\theta_C)\sec(\theta_C+\psi)(\cos(\psi)D_z - \sin(\psi)D_x) \\
\delta\mathbf{M}_\mathrm{x} &= \sqrt{A^2+B^2+C^2+E^2} \\
A &= [\delta\theta_C(\cos(\psi)\sec(\theta_C+\psi)(D_z\cos(\psi) - D_x\sin(\psi)))] \\
B &= \left[\delta\psi(\sin(\theta_c)\sec^2(\theta_C+\psi)(D_z\sin(\theta_C) - D_x\cos(\theta_c)))\right] \\
C &= [\delta D_z(\sin(\theta_C)\cos(\psi)\sec(\theta_C+\psi)] \\
E &= \left[\delta\psi(\sin(\theta_c)\sec^2(\theta_C+\psi)(D_z\sin(\theta_C) - D_x\cos(\theta_c)))\right]
\end{aligned}
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1.3 Uncertainty in $\mathbf{M}_y$

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\begin{aligned}
\mathbf{M}_y &= \tan(\phi_C)\cos(\theta_C)\sec(\theta_C+\psi)(\cos(\psi)D_z - \sin(\psi)D_x) \\
\delta\mathbf{M}_\mathrm{y} &= \sqrt{A^2+B^2+C^2+E^2+F^2} \\
A &= [\delta\theta_C(\tan(\phi_C)\sin(\psi)\sec(\theta_C+\psi)(D_z\cos(\psi) - D_x\sin(\psi)))] \\
B &= \left[\delta\psi(-\tan(\phi_C)\cos(\theta_c)\sec^2(\theta_C+\psi)(D_x\cos(\theta_C) - D_x\sin(\theta_c)))\right] \\
C &= [\delta D_z(\tan(\phi_C)\cos(\theta_C)\cos(\psi)\sec(\theta_C+\psi)] \\
E &= [\delta D_x(-\tan(\phi_C)\cos(\theta_C)\sin(\psi)\sec(\theta_C+\psi)] \\
F &= \left[\delta\phi_C(\sec^2(\phi_C)\cos(\theta_C)\sec(\theta_C+\psi)(D_z\cos(\psi) - D_x\sin(\psi)))\right]
\end{aligned}
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1.4 Uncetainty in $\mathbf{M}_z$

$$
\begin{aligned}
\mathbf{M}_\mathrm{z} &= \cos(\theta_C)\sec(\theta_C+\psi)(\cos(\psi)D_z-\sin(\psi)D_x) \\
\delta\mathbf{M}_\mathrm{z} &= \sqrt{A^2+B^2+C^2+E^2} \\
A &= [\delta\theta_C(\sin(\psi)\sec(\theta_C+\psi)(D_z\cos(\psi)-D_x\sin(\psi)))] \\
B &= \left[\delta\psi(-\cos(\theta_c)\sec^2(\theta_C+\psi)(D_x\cos(\theta_C)-D_x\sin(\theta_c)))\right] \\
C &= [\delta D_z(\cos(\theta_C)\cos(\psi)\sec(\theta_C+\psi)] \\
E &= [\delta D_x(-\cos(\theta_C)\sin(\psi)\sec(\theta_C+\psi)]
\end{aligned}
$$

1.5 Uncertainty in ψ

$$
\begin{aligned}
\psi &= \pi+\theta_p+\beta \\
\delta\psi &= \sqrt{[\delta\theta_p^2+\delta\beta^2}
\end{aligned}
$$

1.6 Uncertainty in θ_C

$$
\begin{aligned}
\theta_C &= \arctan\frac{u'_C}{S_C} \\
\delta\theta_C &= \sqrt{[\delta u'_C(\frac{S_C}{u'^2_C+S^2_C})]^2+[\delta S_C(\frac{u'_C}{u'^2_C+S^2_C})]^2}
\end{aligned}
$$

1.7 Uncertainty in ϕ_C

$$
\begin{aligned}
\phi_C &= \arctan\frac{v'_C}{S_C} \\
\delta\phi_C &= \sqrt{[\delta v'_C(\frac{S_C}{v'^2_C+S^2_C})]^2+[\delta S_C(\frac{v'_C}{v'^2_C+S^2_C})]^2}
\end{aligned}
$$