Ryan P. March 30th, 2021

Goal: Transform the DUAL form problem:

$$\max_{\alpha} \sum_{i=1}^{n} \alpha_i - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_i \alpha_j y_i y_j \langle x_i, x_j \rangle$$

$$\text{s.t.} \quad \sum_i \alpha_i y_i = 0$$

$$C \geq a_i \geq 0$$

into the form:

$$\min_{x} \frac{1}{2} x^T P x + Q^T x$$

$$\text{s.t.} \quad Gx \leq h$$

$$Ax = b$$

to do this we will let:

$$P = \sum_{i=1}^{n}\sum_{j=1}^{n} y_i y_j \langle x_i, x_j \rangle$$

then our problem becomes:

$$\max_{\alpha} \sum_{i=1}^{n} a_i - \frac{1}{2} \alpha^T P \alpha$$

$$\text{s.t.} \quad \sum_{i=1}^{n} \alpha_i y_i = 0$$

$$\alpha_i \leq C$$

$$\alpha_i \geq 0$$

some simplification of sums gives:

$$\max_{\alpha} \ 1^T\alpha - \tfrac{1}{2}\alpha^T P \alpha$$

$$\alpha^T y = 0$$

$$\alpha_i \leq C$$

$$\alpha_i \geq 0$$

multiplying needed items by -1 to match inequalities given in required form:

$$\min_{\alpha} \ \tfrac{1}{2}\alpha^T P \alpha - 1^T \alpha$$

$$-\alpha_i \leq 0$$

$$\alpha_i \leq C$$

$$\alpha^T y = 0$$

reminder: wanted form

$$\min_{x} \ \tfrac{1}{2} x^T P x + Q^T x$$

$$\text{s.t.} \quad Gx \leq h$$

$$Ax = b$$

we will combine these 2 constraints by stacking them on top of each other.

example:

$-\alpha_i \leq 0 \quad G = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \ \wedge \ h = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\alpha_i \leq C \quad G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \ \wedge \ h = \begin{bmatrix} C \\ C \end{bmatrix}$

then combined becomes

$$G = \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \ \wedge \ h = \begin{bmatrix} 0 \\ 0 \\ C \\ C \end{bmatrix}$$

now we can see the values we need:

$$x = \alpha$$

$$P = \sum_{i=1}^{n} \sum_{j=1}^{n} y_i y_j \langle x_i, x_j \rangle$$

$$Q = -1$$

$$G = \text{stack}(\text{identity-neg} \wedge \text{identity})$$

$$h = \text{stack}(0 \wedge c)$$

$$A = y$$

$$b = 0$$