Mapping the Shape of the U.S. Economy: A Topological Data Analysis Approach with BallMapper

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ABSTRACT

Topological Data Analysis is (TDA) is a developing data analysis method which gained popularity starting in the early 21st century. Currently, a large body of TDA research utilizes the traditional Mapper algorithm. Some goals of this paper is to show and entry level path into TDA as well as expand the body of literature using Ball Mapper–a traditional Mapper adjacent algorithm. Using U.S. macroeconomic data from the Bureau of Economic Analysis (BEA), the Federal Reserve Bank (FRB), and the Bureau of Labor Statistics (BLS), we achieve our goal by first showcasing Ball Mapper's use in Exploratory Data Analysis. We then show how we can effectively use Ball Mapper in combination with other derived economic measures: the Fisher Equation, PPI-CPI Gap, and Wage Growth. Our findings suggest that Ball Mapper is a useful data analysis tool to look at large multidimensional datasets and extract meaningful relationships in insights about our data quickly [...]

INTRODUCTION

Topological Data Analysis (TDA) is an emerging field of data analysis that is increasing in popularity. Broadly, traditional TDA applications use two tools: an algorithm called Mapper whose output is represented by a mathematical graph and an analysis technique called Persistence Homology.[1][A] These two tools are not necessarily dependent on one another and do provide information about data in their own right. Moreover, the combination of Mapper and Persistence Homology is what forms the central argument for TDA: data contains an underlying shape, and this shape can provide us with qualitative, and sometimes quantitative, insights about large multidimensional datasets.[2]

As a helpful general framework, Mapper can be thought of the visual side of TDA, and Persistent Homology as a the underlying engine. For this analysis we will be focusing on the "visual side" of TDA, the graph creation. Specifically, we are examining various macroeconomic indicators of the United States of America (U.S.) using a Mapper-adjacent algorithm called Ball Mapper(BM). Ball Mapper is of particular interest to us because it reduces the parameters compared to traditional Mapper, allowing for easier implementation. Further, this reduction in parameters removes some of the barriers to start learning TDA; i.e. needed background and coding knowledge.

In traditional Mapper, one must pass the data through three stages, all with their own parameters, in order to generate a graph. Ball Mapper however only needs one parameter before producing a graph.[3] An important trade off between Ball Mapper and traditional Mapper is the following: BM reduces the number of steps needed to produce similar outcomes of traditional Mapper, but one loses control of fine tuning these output. Additionally, traditional Mapper presents itself as needing a minimum mathematical background, introductory course in Topology, or conceptual knowledge of data science methods and algorithms to understand it. Although it is not impossible to learn TDA without the aforementioned backgrounds, without them, TDA might seem unnecessarily complicated for data analysis, or impossible to decipher.

One major motivation for this paper came from a paper by Dłotko, et al.[4] This paper was a two pronged macroeconomic analysis using Ball Mapper. The first was a dataset of five macroeconomic variables from 16 countries from the 1870s-2017. The authors were

[A] A mathematical graph, from Graph Theory, is a diagram that consits of nodes and edges.

interested in comparing how countries have evolved over time, their transformation from the Great Depression Era, and various views on wealth and inequality. The second prong was to look at relationship between private credit growth and GDP of various countries. This paper was the only on in the literature that we could find which combines both macroeconomics and Ball Mapper.

If successful this analysis will expand the small body of literature whose main focus is applications with Ball Mapper.[1] We also hope that it will serve as an on-ramp for anyone interested in TDA but feels inundated with jargon upon early stage researching.

Using U.S. macroeconomic data from the Bureau of Economic Analysis (BEA), the Federal Reserve Bank (FRB), and the Bureau of Labor Statistics (BLS), we plan to achieve these objectives in two stages. The first is to show Ball Mapper's use in exploratory data analysis (EDA) by looking at structural observations, coloration, as well as size variation of our graph. Then using what we showed in our first stage, we extend that framework to other derived macroeconomic measures: The Fisher Equation,[5,6] PPI-CPI Gap,[B7] and Real-Wage Growth.[8C]

METHODS AND PROCEDURES

Data Selection & Preparation

We pulled data from three publicly available data from U.S. government sources: The Bureau of Economic Analysis (BEA), The Federal Reserve Board (FRB), and The Bureau of Labor Statistics (BLS).[9,10] The data were gathered using R using two application programming interfaces (APIs): one for data from the BEA, and the other from the Federal Reserve Economic Data (FRED) API.[11,12D] We additionally used recession dates based on business cycle contractions and expansions provided by the National Bureau of Economic Research (NBER). These agencies were selected because of they are authoritative sources for U.S. economic data. Their widespread use in both the private and public sectors gives us high confidence in the accuracy and integrity of the data.[13]

Data Series	Series Abbreviation	Year Start	Year End	Frequency	Series	API
Gross Domestic Product	GDP	1930	2024	Annual	Table 1.1.1	BEA
Personal Income & Its Disposition	PID	1948	2024	Annual	Table 2.1	BEA
Foreign Direct Investment	FDI	2014	2024	Annual	Direct Investment and MNE	BEA
Federal Funds Rate	FFR	1955	2025	Annual	RIFSPFFNA	FRED
Employment Cost Index	ECI	2001	2025	Quarterly	ECIALLCIV	FRED
Consumer Price Index	CPI	1947	2025	Monthly	CPIAUCSL	FRED
New Privately-Owned Housing Units Started	Housing Starts	1959	2025	Monthly	HOUST	FRED
Producer Price Index - Finished Goods	PPI	1947	2025	Monthly	WPSFD49207	FRED
Unemployment Rate	Unrate	1948	2025	Monthly	UNRATE	FRED

Table 1: *Housing Starts is considered a flow because it is provided as a Seasonally Adjusted Annual Rate (SAAR).

Table 1 shows all the data series we considered for this analysis. We excluded Employment Cost Index (ECI) and Foreign Direct Investment (FDI) due to their limited availability of years. Of the remaining data series, New Privately-Owned Housing Units Started (Housing Starts) had the smallest range and so set our lower bound for the range years used (1960-2024). Our upper bound was the latest full calendar year that was available. We wanted to represent the economy from various vantage points. i.e. the broad economy, policy makers, consumers, business owners, etc. Table 2 shows the data series that we felt were representative of the economy as a whole and had data for our time period of interest. Within each of these series we used a subset of columns to construct our final dataset for analysis. To see what parts of the economy we determined each series plays, the column Functional Description, can be found Table 2.

For this analysis we are focusing on an annual time frame and some data transformation was needed. Looking at Table 1, you can see that not all our data was provided on an annual basis. Table 2 shows a summary of the transformation taken and the details for each follow.

[B] Producer Price Index: Finished Goods less Consumer Price Index. Details on data transformation in Methods and Procedures section.
[C] Compesation less Inflation (CPI). Details on data transformation in Methods and Procedures section.
[D] FRED aggregates data from national and international sources, as well as public and private sources.

The first transformation we did was on Personal Income and Its Disposition (subsequently, Personal Income) is reported in nominal dollars. Since nominal dollars do not account for inflation, we used the Consumer Price Index (CPI) we transformed our values to Real Dollars (Equation 1).[14] We then took the log difference (Equation 2) from the output from Equation 1 to get an approximate percentage change from the preceding year.

$$\text{Real Dollars} = \frac{\text{Nominal Dollars}}{\text{CPI}} \times 100 \tag{1}$$

Calculating the year-over-year percent change using log differences provides us with a few advantages. First, taking the log stabilizes the variance and makes exponential trends across the series more linear. Second, the difference between log values approximates the true percentage change well for small to moderate values. Further, the log difference allows for symmetric percent changes: increases or decreases from year to year are of equal magnitude.[15] An interesting question that has not been explored to our knowledge is the impact on linearized data and Ball Mapper's distance metric, the Euclidean distance. Further exploration of this question will be touched on in the discussion section.

$$\Delta \ln(\text{Level}) = [\ln(\text{Level}_t) - \ln(\text{Level}_{t-1})] \times 100 \tag{2}$$

Our next transformations were applied to Housing Starts, Producer Price Index - Finished Goods (PPI), and CPI. All three of these sources were only provided on a monthly time frame, so to get an annual value for them we used the simple arithmetic mean. Once these data were in annual form, we then found the percent change from the previous year using the log difference (Equation 2).

Housing Starts data is given in a seasonally adjusted annual rate (SAAR). To get this number the U.S. Census first takes the raw monthly number of housing starts reported to their survey. Then they adjust that number to account for seasonal patterns (e.g. summertime booms or winter lulls), and finally they multiply it by 12. So how we interpret this final number: for the reporting month, it is the number of houses that would be built at the end of a 12-month period.[16]

Series Abbreviation	Economic Role	Functional Descriptioin	Transformation Applied
GDP	Growth Composition	Consumption, Investment, Gov., and Trade	None (Source in % Change)
PID	Income Structure	Wages, Entrepreneurship, Gov. Transfers	Real Adjustment & Log Difference
UnRate	Labor Market Dynamics	Labor Distress Level and Trend	None (Annual Rate) & Simple Difference
FFR	Monetary Policy	Cost of Capital and Borrowing Conditions	None (Annual Rate) & Simple Difference
PPI	Supply-Side Signal	Producer Costs & Measures Cost-Push Inflation	12-month Avg. & Log Difference
CPI	Demand-Side Signal	Cost of Living & Measures Demand-Pull Inflation	12-month Avg. & Log Difference
Housing Starts	Leading Indicator	Physical Residential Production	12-month Avg. & Log Difference

Table 2: All years for analysis are 1960-2024.

Both PPI and CPI are economic indexes and represent a weight average of the certain good they respectively track. Although PPI has many different series, the one use for this analysis tracks the price the producers receives from the physical goods that it sells to business or consumers. An important distinction to note about the PPI used here, is that the index captures goods which are finished or final in the economic sense. Colloquially, it is similar to the wholesale price a retailer might pay to a producer before marking up for retail sales. However, it also includes goods which are sold to business, think fully assembled computers or fleet vehicles.[17] CPI on the other hand captures the consumer side and the retail price. That is, the cost of the good after profit margin, transportation cost, and other intermediary costs are calculated into a goods price sold at the register.[18] Broadly, both have been associated with measuring inflation for producers and consumers. The more nuanced understanding is that index changes can reflect many things: push-cost inflation, demand-pull inflation, supply shocks, monetary policy inflation, and wage-price spirals are just a few examples. It is our hope that Ball Mapper might show the distinctive separation in the various aforementioned reasons for the PPI and CPI indexes changing from year to year. [SAM: I am worried about this ending sentence and that if I say it here I will need to do it for all the other data.]

BallMapper

In depth descriptions of Ball Mapper's theory have been covered in various papers.[3,19,20] As the aim of our paper is to provided a introductory foray into Ball Mapper and TDA in general, we will omit some of the more technical details below but the previously mentioned references provide a more in-depth explanation.

Theoretically, Ball Mapper supposes we are given a dataset X with K dimensions, N observations and some metric d. Then to create a Ball Mapper graph, our goal is to construct a collection of balls, let's call it B, with a constant radius, ϵ, such that every observation of our dataset is covered by at least one ball in B. Mathematically, this is written as follows: Let (X, d) be a metric space where X is our dataset and d is a distance metric on X. For a fixed radius $\epsilon > 0$, let $C \subseteq X$ be a set of center points. Then for each $c \in C$, we define a ball $b(c, \epsilon) = \{x \in X : d(c, x) < \epsilon\}$. Our collection then is $B = \{b(c, \epsilon) : c \in C\}$ and $\bigcup_{b \in B} b = X$.

In practice, our dataset X is called a Point Cloud. Our N observations or rows are normally referenced as points, and our K dimensions, the columns of X, are denoted variables. Since Ball Mapper is know to handle large multidimensional datasets well, it is common to have point clouds with dimensions $K > 2$. [Think I need a ref. here about BM and large datasets?] Ball Mapper was originally written in R, but later on was created for use in Python. For this analysis we used R and the Ball Mapper package aptly named "BallMapper".[21]

To construct the Ball Mapper (BM) graph, the balls of our graph are created in an iterative manner and not a predetermined manner like the theory defines. If we pre-defined a subset of ball centers, it would take away from the driving idea behind TDA–that data has shape–as well as start moving into the territory of other clustering methods such as K-means where you selected a number of clusters based on statistical properties of the data. This iterative process allows our point cloud to cluster in a "natural" way based on on our choice on epsilon and distance metric.

$$d(x_i, x_j) = \sqrt{(x_{i_1} - x_{j_1})^2 + (x_{i_2} - x_{j_2})^2 + ... + (x_{i_k} - x_{j_k})^2} < \epsilon \tag{3}$$

The iterative ball creation is carried out in a linear fashion from the first row to the last of the point cloud. The result of this linear selection process is a pseudo-random creation of ball centers. How this works is that it takes the first point, let's call it c_1, of the point cloud and assigns it as the center of the first node in our graph. Then using the default distance metric, the Euclidean Distance (Equation 3), it compares every point in the point cloud to c_1 to create $b_1 = b(c_1, \epsilon) = \{x \in X : d(c_1, x) < \epsilon\}$. After this first ball is created, Ball Mapper looks for the next row in the point cloud that is not a member of the first ball, and assigns it as the next center. This process continues until all the points are in a ball and the union of balls consists of our whole point cloud.

Hopefully, it is clear now why this process can be described as pseudo-random because it relies a great deal on the point cloud as well as the epsilon. In fact, the Ball Mapper graph changes for epsilons sufficiently large enough to change at least one point's inclusion or exclusion to a ball; the graph can also change for unique orders of the point cloud rows. This is touch on in greater detail in the Discussion.

The last step to the Ball Mapper graph is to draw the edges if they exists. The way the BM algorithm is structured, it allows for the possibility of points being in multiple balls–also called nodes. This intersection of points between nodes is what creates an edge, and the number of share points between nodes indicates an edge strength. So, once we have all the nodes, edges, and edge strengths known, we then can construct a Ball Mapper graph. The Ball Mapper algorithm described above can be broken down as follows:

1. Select a point $x_i \in X$ and assign it as c_i.
2. Given $\epsilon > 0$, construct a ball $b_i(c_i, \epsilon)$ with a center c_i and radius ϵ by checking if x_j satisfies $d(c_i, x_j) < \epsilon : x_j \in X$ and d is the distance metric (Equation 3).
3. Place b_i in a set B.
4. Select another x_n and repeat step 1-3 such that $x_n \neq x_j : x_j \in b_i$.
5. Draw an edge between $b_i, b_j \in B : b_i \neq b_j$ if $b_i \cap b_j \neq \emptyset$, weighting the edge based on the size of the intersection.

Implementation

To use Ball Mapper one only needs three parts: a point cloud, a variable to color by, and an epsilon. For this analysis, our point cloud was made up of 16 variables and 65 points, each representing a year from 1965-2024. Two of the variables, *Unemployment Change* and *Fed Rate Change*, were created before any transformation took place. We elected to create these because the Unemployment Rate (Unemployment) and the Federal Funds Rate (Fed Rate) were the only two dimensions which did not have a transformation and thus did not have a year-over-year change like our other variables. Having the rate of change for both Unemployment and the Fed Rate is also important because it gives us additional information beyond the intrinsic value of each of the measures. Both measures on their

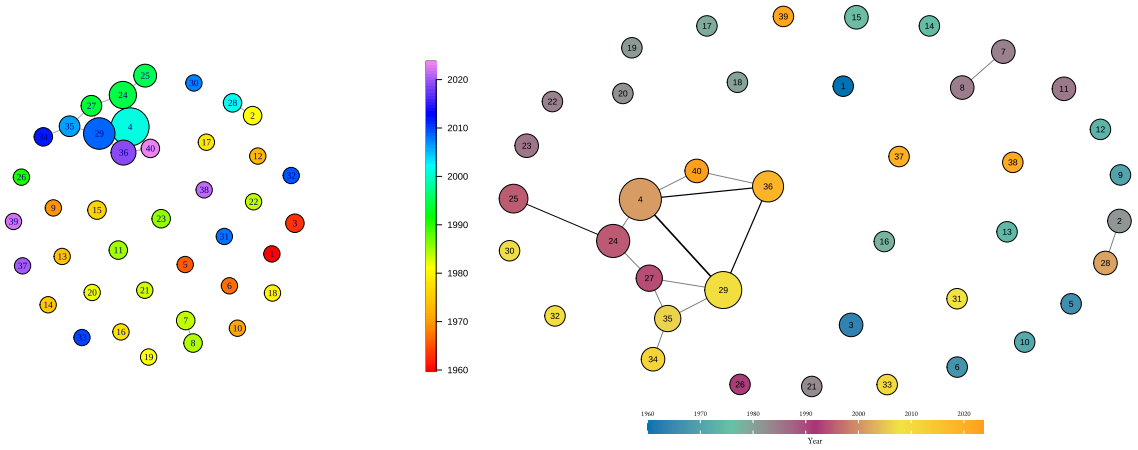
own give us a current state at large: the Unemployment Rate telling us how many who are not working but want to,[22] and the Fed Rate signalling the structural cost of capital.[23]

To our knowledge there is no generally accepted way of choosing epsilon in the literature. Since Ball Mapper constructs maps by selecting a point and creating a ball with a radius of epsilon, this is should in theory, given an epsilon small enough, produce a graph such that every point is a node of only itself. Conversely, this should lead to and epsilon big enough such that all of our points are included in one node. For our analysis here, this was indeed the case. So to find an appropriate epsilon we first started by finding lower and upper bounds.

What we look for in our lower bound is a graph with an epsilon small enough make every point in our point cloud a node of one point, a singleton node, not connected to any point. For our upper bound we do the converse, we look for an epsilon so large where it creates a singular node that houses all of our points. Additionally, to reduce some of the computational time one can look for a lower bound where a connected component started to form, and an upper bound where we had a connected component of only two nodes. This cuts off tail end graphs that generally don't provide much substantial insight into the data.

For our analysis we found an initial lower and upper bound represented by the interval $[0.38 - 0.90]$. We then wrote a function to generate around 100 graphs, reviewed them, and narrowed the upper and lower bounds to $[0.40 - 0.70]$. During this narrowing process we were looking for interesting features: connected components forming or dissolving, flares coming off of any components, and notable sizing or coloration patterns. The resulting value of this process was 0.474. We thought it contained all the aforementioned features we look for. This final epsilon is used for all the graphs generated in the analysis.

Before describing some visual changes we made to the Ball Mapper output graph,it is important to note that Ball Mapper will output the same graph unless at least one of two things changes: the size of the point cloud changes or the order of the point cloud. The former intuitively makes sense, the algorithm is creating epsilon radius balls so adding or subtracting any data will yield a different amount of points in at least one ball. Depending on the data being changed or number of points, the structure of the output might not change much. As an example, when we were deciding on which PPI series to use, we had a multitude of choices. Initially we used PPI - All Commodities but found when reading it is susceptible to double counting and used PPI - Finished Goods instead.[17]



(a) Ball Mapper output using standard function (ColorIgraphPlot)

(b) Ball Maper output using new graph output (ggraph)

Figure 1: Ball Mapper Graph colored by Year with Epsilon set to 0.511.

The standard Ball Mapper package in R produces a sufficient graph for analysis. The output object of the Ball Mapper algorithm is a series of nodes, edges, edge weights, and other graph properties created by the package *igraph*.[24] Separate from the igraph object, a graphing function that uses R's base plot function and default coloring scheme produces the visual graph. This is an advantage because when packages are written in Base R they are usable to anyone who can install R. We however felt that this graph output could be improved upon and recognized that we could use alternative, and more comprehensive packages to create a graph, *ggraph*.[25]

Using the same underlying output igraph object from Ball Mapper, we created the same graph in ggraph but changed the visual representation and added another helpful piece of information, edge weight, that was given but not used in the default BM graph. Looking

at Figure 1, we have the standard Ball Mapper graph output on the left (Figure 1a), and our new ggraph output on the right (Figure 1b). The most prominent changes is the spread of the network and coloration.

Looking at Figure 1b, having the nodes spread out helps us see the structure of our graph more clearly as well as the edge weight between nodes. We see that highlighting the edge strength presents a potential insight between certain nodes. Although our data is relatively small, consisting of 65 rows, one could imagine the advantage Figure 1b might have with a data set much larger that this one (e.g. looking at our time frame on a quarterly or monthly basis). Additionally, we changed the color palette for accessibility and readability.

For Figure 1b and Figure 2, we use Year as our initial coloring variable. We started with this variable because Dłotko, et al.[4] provided a framework for this type of analysis. In this paper the authors started by coloring their graphs by Year to initially understand the shape of their data. Later, in the second half of their analysis, they produced more graphs each colored by the point cloud's variables. Since we have 16 dimensions to our point cloud, an analysis of that size is outside the scope of this paper and is the the subject for another. An additional note about Dłotko, et al., when they colored by the variable 'Year', it was not a variable in their point cloud. Mentioned in the intro, we recognized the usefulness of this technique and extended this idea to our point cloud. Specifically, we calculated variables from out data that we thought represented different economic lenses: the Fisher Equation (Figure 3), PPI-CPI Gap (Figure 4), and Real Wage Growth (Figure 5).

$$\text{Real Interest Rate} \approx \text{Fed Rate} - \text{Inflation} \tag{4}$$

The Fisher Equation (Equation 4) is a mathematical relationship to approximate Real Interest Rates. That is, it represents the true "feeling" of interest rates by taking into account inflation. Business will "feel" the Real Rate when the Fisher Equation is positive and act accordingly because the cost of capital is rising. Similarly, consumers will "feel" the rate when the Fisher equation presents a negative number, signaling that inflation is outpacing the Fed Rate. An important note about the Fisher equation, the approximation does not hold well when nominal interest rates (Fed Rate) are relatively high and when time spans are short.[6]

We calculated the PPI-CPI Gap by taking the the difference between PPI - Finished Goods and CPI (inflation). As a quick reminder, PPI represents an "at-cost" revenue (before adding margin, transportation, etc) that producers receive, and CPI is a common measure of consumer inflation. It is important to note that although CPI is used for general consumer inflation, the Federal Reserve prefers Personal Consumption Expenditures (PCE) because it tracks more normal consumer behavior such s substitution of products. The predominate question for this measure was: if producers or consumers are experiencing inflation, can our graph show the primary driving force behind the complex nature that is inflation?

Similar in nature to the the PPI-CPI gap, we took the difference between Compensation and Inflation for Real Wage Growth. With this measure, we were interested on the relationship between people's annual compensation and inflation, i.e. are wages were keeping up with inflation. Generally, for this relationship, positive numbers indicate that wages are growing faster than inflation, and vice versa if it is negative–effectively people are taking a pay cut.

RESULTS

Exploratory Data Analysis

There are two general categories of nodes when interpreting Ball Mapper outputs: connected or not connected. Figure 2 shows three different connected nodes (connected components), and many nodes not connected to any others (singletons). These singletons suggests that the data is unlike all the other data points and indicates possible outliers in the dataset.[4] The three connected components, there is one large and two small components. Moving clockwise, starting with the large component and will label them C1,[E] C2,[F] and C3[G] for conversational ease.

Looking at C1, we note that one large difference between it and C2 and C3 is that it has more than two nodes. Since it does, any nodes which are not directly connected by an edge (e.g. Nodes 24 and 4), can be interpreted as alike in some fashion, but also signal a significant difference within the component.[4] What is unclear, though is if path length between two nodes is commensurate with the similarity in data and something that could be explored for future research. [THINK ABOUT MOVING THIS PARAGRPAH TO DISCUSSION, AT LEAST THIS SENTENC(BEFORE)] This also applies the nodes of large size that are connected. We have not

[E] Nodes: 4, 24, 25, 27, 29, 34, 35, 40
[F] Nodes: 7 and 8
[G] Nodes: 2 and 28

found any literature that supports this idea that proximity is commensurate with relatedness. What we can conclude about size is that the size of the node indicates the number of points that nodes has. Similarly, our edge strength is shown this ways with a thicker line representing more shared points between two nodes.

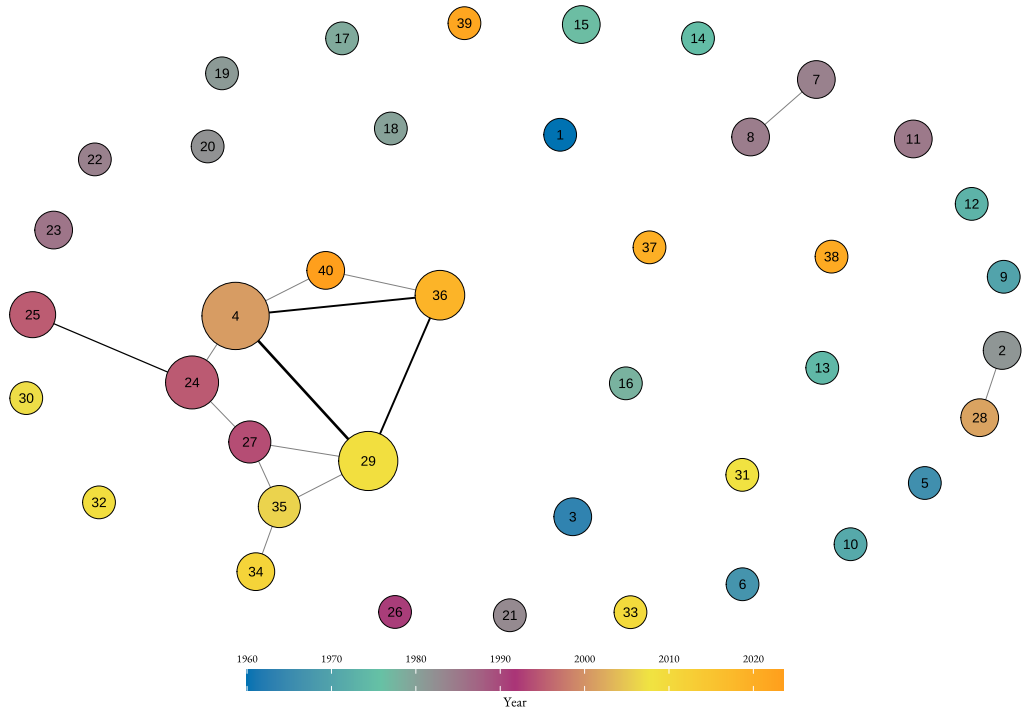


Figure 2: Ball Mapper Graph colored by Year with Epsilon set to 0.474.

Most often coloration of Ball Mapper graph is used to highlight a variable of interest, or certain know structures in the data. However, we also will look for any patterning (e.g. a smooth gradient from one end of the graph to the other), or nodes which do not conform to patterning. For our map some coloration of interest is the three sub-groups of colors in C1 as well as the mismatching colors for C3. It is important to remember when looking at coloration that the value of the color is an average value and is susceptible to large variances.

Since out graph consists of 40 nodes and three components we will provide all the singletons in a table (Table 3), and talk more in depth about our components below.

Node	Year(s)	Node	Year(s)
1	1960	19	1981
3	1962, 1965	20	1982
5	1966	21	1983
6	1967	22	1984
9	1970	23	1985, 1986
10	1971	26	1991
11	1972, 1998	30	2007
12	1973	31	2008
13	1974	32	2009
14	1975	33	2010
15	1976, 1977	37	2020
16	1978	38	2021
17	1979	39	2022
18	1980		

Table 3: Singlton nodes and the years they contain.

Still using Figure 2, we first will examine C2 closer. Adding to some of our initial observations we see that C2's nodes are colored very similarly, and are of the same size. As a relative measure, using Table 3, we look at our singletons and notice that many only hold one point. Thus comparing C2 nodes we should expect around one to two points in each of the nodes, as well as C3. When looking at what points C2 contains we find the years 1968 and 1969 in nodes 7 and 8, respectively, and 2000 being in both nodes, creating the edge. C2 is a good example where coloring by average value can be misleading. Conversely, looking at C3, we first notice that both nodes are colored differently. This inconsistent coloration could be meaningful if we had a strong global coloration pattern, however since C3 is such a small component, much like a small sample size, does not provide us with a lot of further detail than observation. Looking at the nodes we find that node 2 contains 1961 and node 28 contains 2001, with 2002 being our bridge year between the two.

As mentioned above, C1 contains three subgroups of different colors. For the left flare in C1 (nodes 25, 24, 27), all the years are in the range of 1987-2006. These three nodes have 13 points total and a interquartile range (IQR) being between 1990 and 1996, and is consistent with this flare's coloration. Moving towards the bottom flare and connection with the main body of C1 (nodes 34, 35, 29), visual inspection tells us we should expect to find years from the mid 2000's to the 2010's. Upon inspection we also find that these three nodes contain 13 points, however the IQR and range are larger with the IQR being 2004-2014 and range being 1993-2018. Next looking at nodes 36 and 40 we find that they consist of 7 points with a range 2016-2024, a much tighter range than the two previous sub groups. Our last node, node 4, has a bit of all of the sub groups. It points range from 1963 to 2024, with and IQR of 1996-1026, much larger. Indeed, we find the coloration consistent with its wide range, but we also see how wide ranges can be misleading.

Focusing on the structure of C1 there are two distinct features that we notice: the three largest nodes are all connected to each other (nodes 4, 29, and 36), and the sweeping arm on the left side of the component that contains two flares at each end (nodes 25, 24, 27, 35, and 34). The three large nodes that form a triangle, they also have the strongest strength between one another relative any other two nodes. The next strongest connection is between nodes 25 and 24. Between the three nodes we find that 2017 and 2018 are the only years which reside in all three nodes. The middle portion of our sweeping arm (nodes 24, 27, 35) contain most points during the late 80s and the early-mid 90s. The flares (25 and 24) are rather different than the middle nodes. Node 25 has a wide range of years, skipping from 1990 to 1995 and then to 2006. Node 34 on the other hand has years 2011 and 2012, a very tight grouping of years.

Fisher Effect

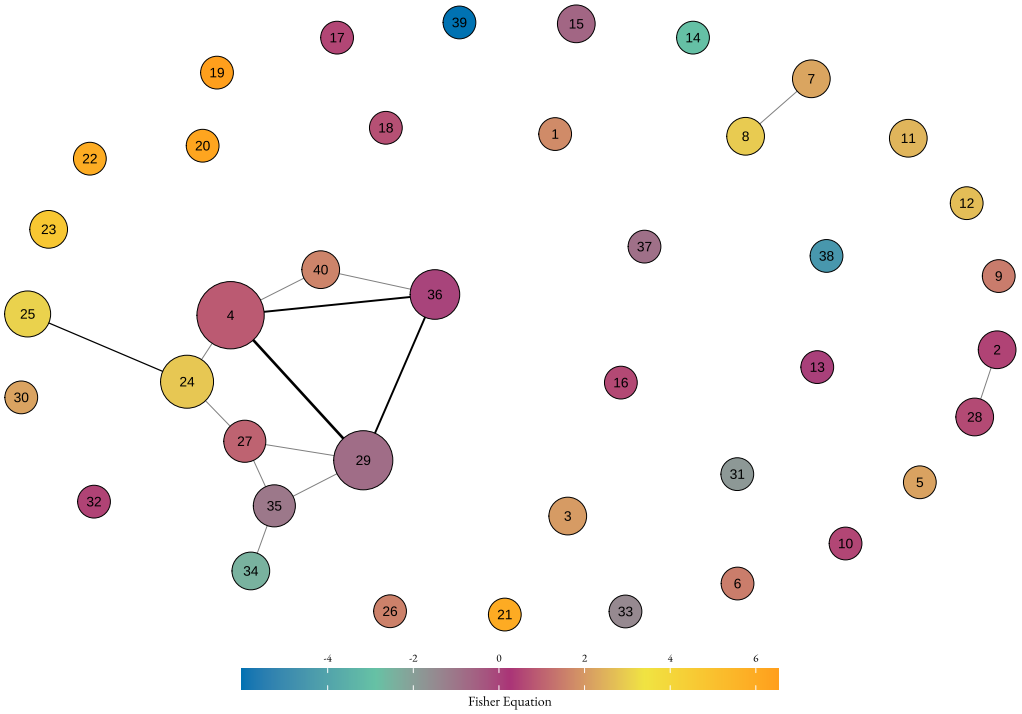


Figure 3: Ball Mapper Graph colored by Fisher Equation with Epsilon set to 0.474.

The most prominent nodes that stand out will be those on the far ends of our scale on the bottom of Figure 3. Looking at Equation 4, we see that when the Fed Rate is lower than Inflation, we get negative values, and vice versa when the Fed Rate is later than Inflation.

Singleton nodes 19-22 have relatively high values (average of 6.178%) consist of years 1981-1984. This directly lines up with the experience "Volcker Shock".[26] On the opposite end, moving from the twelve o'clock position clockwise, we can visually see nodes 39, 14, 38, 31, 33, 34 (2022, 1975, 2021, 2008, 2010, 2011-2012 respectively) show a negative value (Equation 4).

We also observe that in C1, most nodes along with C3 seem to have Real Interest Rates showing between -1% and 2%. To contrast this, we see that nodes 25, 24, and C2 look to be above 2% but not above 4-5%.

PPI-CPI Gap

Similar to Figure 3, and subsequently Figure 5, we see that singleton nodes tend to be colored at the extreme ends and our components having mostly a consistent coloring. Node 39, 39, and 13 (2022, 2021, 1974) show high positive values indicating higher PPI values relative to Inflation. On the opposite end, the nodes which represent highest inflation relative to PPI values are nodes 23, 22, 20, 37, 9, 2, 28, 26, and 32 (1985-86, 1984, 1982, 2020, 1970, 1961 and 2002, 2001-02, 1991, and 2009 respectively).

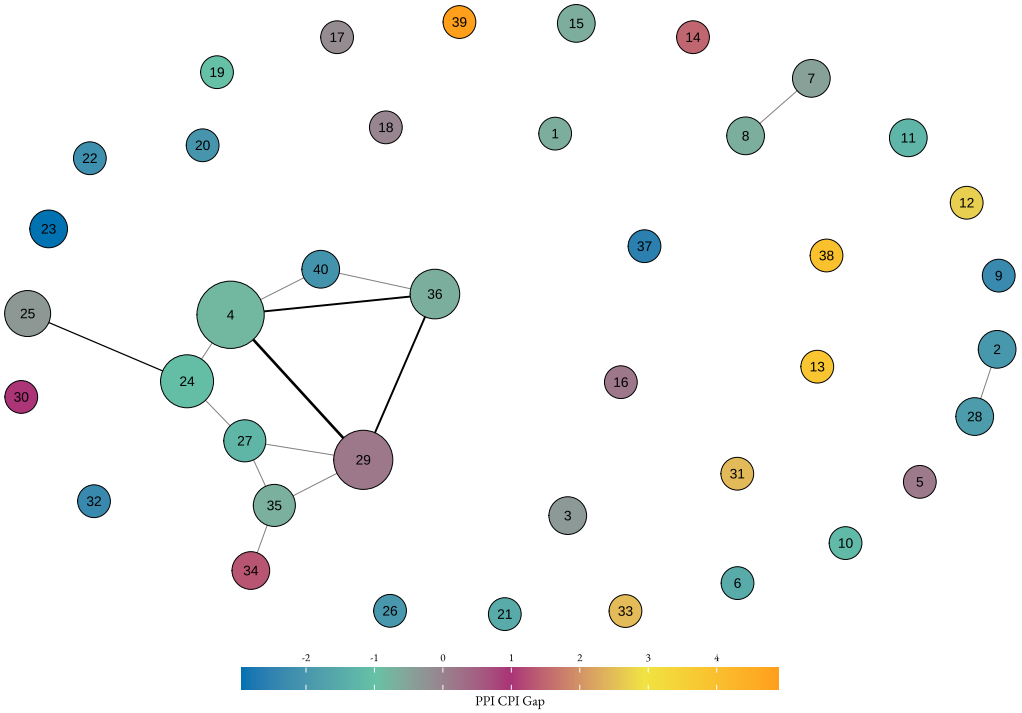


Figure 4: Ball Mapper Graph colored by PPI less CPI with Epsilon set to 0.474.

Wage Growth

Looking at nodes 19, 17, 18, 39, 14, and 13,[H] we see that these are the worst in terms of inflation outpacing people's compensation. Indeed, when we look at the years contained in the nodes, they are years which had record breaking inflation - The 1970s Oil Embargo, The Volcker Shock, and 2022 once all the stimulus cash made its way through the economy. On our positive end we have nodes 11, 5 and 3 which consist of years 1972 and 1998, 1966, as well as 1962 and 1965. What is interesting that we don't see in Figure 3 or Figure 4 is that the legend shows a long tail with our zero point being on the far right end. Notice, most of our points are shades of yellow (around zero or little below) but we have very few values that represent negative double digits.

[H] 1981, 1979, 1980, 2022, 1975, 1974 respectively

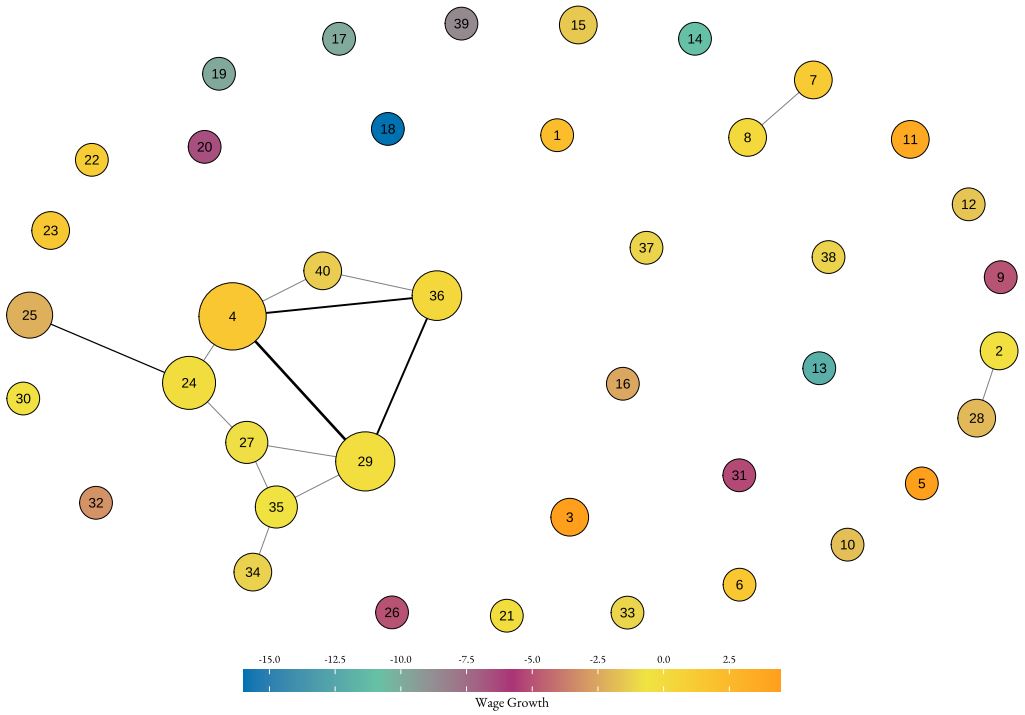


Figure 5: Ball Mapper Graph colored by Wage Growth with Epsilon set to 0.474.

DISSCUSSION

When first looking our graph, it is hard not to notice C1 and then see that there are two other components. What was so striking about C1 was that there seem to be two shapes, a triangle and a arc that seems to be attached to the triangle's left side. Although we have not found any literature that explores the relationship between nodes and edge strength (how many points they share), it was something immediately of interest. Since our point cloud was made up of economic data, we considered that these nodes (4, 29, and 36) could represent a certain extended time period or "phase" of the economy. [EXTEND PARAGRAPH TO EXTEND PHASE OF ECONOMY IDEA] Further, we extended this idea to the arc on the left side of C1 with the two flares, as well as C2 and C3.

Starting with C1 we found that nodes 4, 29, 36 largely represented different expansionary states of the economy. Looking at our complete data set [UPDATE: need to put in blurb about "full" data set in the data section.] we find that for all points in our triangle, all years

To hark back to our discussion of C1, when we investigate nodes 4, 24, and 26 we indeed notice a pattern with the three generally: for the whole year the US was in an expansionary state. Node 4 shows us an "ideal" version of the economy. We have personal income and compensation slightly higher than inflation, sitting around 2%. Unemployment is a little elevated but historically not unusually high. For Nodes 24 and 26 we see two different sides of large economic shocks. Node 24 shows us years where the economy is starting to get "over it's skis". That is, it is starting growing faster than it can keep up with but it's not necessarily too late. Node 26 on the other hand consists of years following right after recessions or points where the economy was over heating. Most of the years in Node 26, follow what economist call a "Jobless Recovery".[27] Usually marked by growth returning to the economy but unemployment being "sticky" and not falling as business starts to take off. For the flares and smaller points that are connect each of our large nodes in C1, we find that some years are in both the smaller node and larger one, which creates our edge. However they also contain additional years which are related nodes 4, 24, or 26 but instead describe transition years to these large nodes.

C2 represents a textbook definition of demand-pull inflation. The formation of this component seems to be driven by the distinct and parallel macroeconomic structure of each point. 1968, 1969, and 2000 were a culmination of factors which led the economy into a state of "too much money chasing too few goods."[28] In the late 1960s, the demand was primarily driven by an injection of cash from Johnson's "Great Society" policies and spending on the Vietnam War during the mid-1960s.[29] Conversely, in 2000 demand accelerated due to the private sector increasing their capital expenditures in anticipation of the digital age.[30]

Additionally, demographic and psychological factors played a role during these years' demand shocks. The 1960s saw the Baby Boomer generation hitting their prime working-age years. This rise in the workforce and subsequent creation of households led to an increase in consumption and lower unemployment - signs of a healthy economy. 2000 presented a similar outcome of high consumption via a different mechanism. As mentioned previously, corporate spending was accelerating. This led to high stock valuations and, in turn, high stock market returns –

Talk about the semi-randomization of BM. If any data in the point cloud is modified in anyway, this ball creation process will change..

and a general rise in asset values. Known today as the "Wealth Effect", this led consumers to increase their spending, placing more upward pressure on demand.[31]

The small component (C2), nodes 2 and 27, are of interest because it tells us us that these related nodes are somehow distinctly different from C1 and all other nodes. Seeing a small components like this sometimes can indicate outliers in data. In our case, this could be years where large economic events happened such as a recessions or extraordinary growth. However, we also can see this behavior in singleton nodes much like we have Figure 2. When we look at the coloration of the singleton nodes, we see indication of outlier events such as The Great Recession and the COVID-19 Pandemic.

Further this short time period not only is represented by interesting about why these nodes are colored is that these nodes see to show the impact of Volker's decision to raise interest and the lag that took place when raising rates.[32]

In these year rates were already extremely low, or the economy just came off a huge economic shock. In 1975 the fed aggressively lowered rates due to climbing inflation and high unemployment rates from the 1973-'74 oil embargo.[33] In 2021-'22 the COVID-19 Pandemic stimulus checks were going to cause inflation but the fed did not want to raise rates too aggressively.

Positive values indicated that businesses are having their revenue rise higher than consumer inflation. This could mean various things: corporate margins are expanding, they are passing through cost to customers, or.... # CONCLUSION

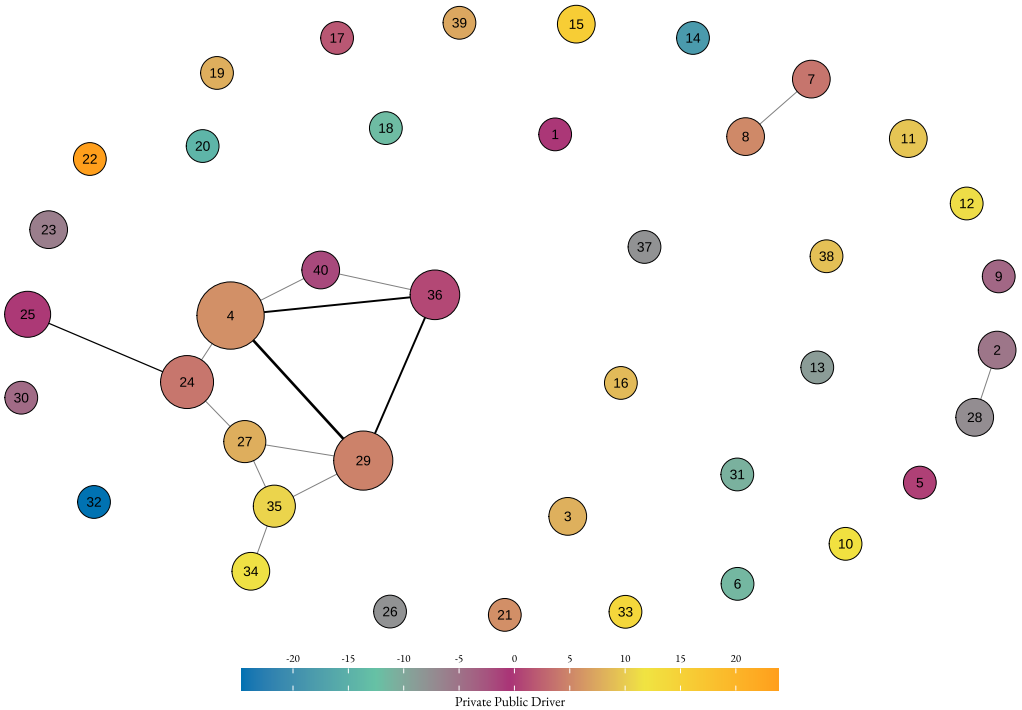


Figure 6: Ball Mapper Graph colored by Private Public Engine with Epsilon set to 0.474.

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Discussion

In our section we briefly went over two ways to interpret TDABM graphs. As seen in many other, longer papers written by Dlotko and colleagues, there is a lot of room to expound our TDABM graph here,though that is beyond the scope of this paper. In general, though, we can see that TDBM can give insight into our data that we might not see otherwise. Seeing that there is data that is not like the others gives an indication that there is something to investigate. Consider our comparison of the 08' - 09' financial crisis and the 2020 COVID Pandemic: We see learned that the years following the financial crisis were similar economically. Another way to view this is, clustering revels that the financial crisis was a singular problem that spurred the collapse of the financial system. Conversely, the COVID pandemic was a singular issue but exposed many different weaknesses in our current world.

For the future development of TDABM, and TDA in general, there are many possibilities of future research and one glaring downside. The downside to TDA is that it is a high barrier to entry to understand the methodologies inner workings. Those with a technical background will have an easier time, but nonetheless a higher barrier than methods such as linear regressions. Subject wise, TDA has an advantage in some of the social sciences because it can be somewhat of a bridge between qualitative and quantitative analyses. TDABM has been used to analyze the Brexit Vote data, seeking to understand quantitative and qualitative motivations behind its outcome. TDA also has the advantages of dimension reduction which could be useful to the life-sciences. Fields such as genetics, chemistry, and biology have many fields of data, and being able to consolidate it into an understandable form could benefit the body of knowledge greatly.

Dimension/Variable	Data Series
Year	All
PPI Finished Change	Producer Price Index - Finished Goods
Personal Income	Personal Income & Its Disposition
Compensation	Personal Income & Its Disposition
Entrepreneurship	Personal Income & Its Disposition
Transfers	Personal Income & Its Disposition
Consumption	Gross Domestic Product
Domestic Investment	Gross Domestic Product
Government Spending	Gross Domestic Product
Exports	Gross Domestic Product
Imports	Gross Domestic Product
Inflation	Consumer Price Index
Unemployment	Unemployment Rate
Unemployment Change	Unemployment Rate
Fed Rate	Federal Funds Rate
Fed Rate Change	Federal Funds Rate
Housing Change	New Privately-Owned Housing Units Started

Table 4: Variables inculde in BallMapper Point Cloud