Duke University – Spring 2025

ME 321 Mechanical Engineering Analysis for Design

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Due: April 23, 2025

Contents

1 Introduction

Welcome to PumpCo! You've got the money and we've got pumps! And now, thanks to us, the structure to support them! The team was tasked with designing and prototyping a support system for a centrifugal pump mounted on an aluminum frame. This platform needs to hold the pump in place, handle the loading from a belt-driven pulley system, and transfer those loads safely into the frame through a pair of support arms, all while being cost effective. The setup is meant to be modular, easy to install and remove, and strong enough to handle real world conditions in post-flood environments.

The system had to meet several constraints. The pump needed to deliver 150 GPM to a 60-foot head with 65% efficiency at 1800 RPM. It had to be mounted to the platform using a fixed 3.75" by 3" hole pattern, with the 12.5 lb load centered at that location. Belt forces from a 10" pitch diameter pulley located 18.56" from the frame with a minimum return side tension of 200 N. The platform and support arms had to attach to the frame using removable clevis pins, and the entire structure couldn't exceed 24" in height. The maximum tip deflection had to stay under 1 mm, and the design needed a minimum factor of safety of 1.5 against both yield and buckling. All components were to be corrosion resistant and fabricated under $150 in raw material costs.

The approach started with modeling the key loads acting on the platform including pump weight, pulley tension, and support/wall connections. From there, the team used static force analysis, stress calculations, and deflection estimates to define the initial geometry. Once that was in place, the team ran simulations and iterated the design based on performance, manufacturability, and cost. The final prototype uses 6061 and 6063 Aluminum I-beams, hollow rectangular tubes, 90° angled aluminum, bolts, and clevis pin connections.

The remainder of this report is structured as follows: Section 2 covers the force and moment calculations that guided the overall design. Section 3 walks through the stress analysis and identifies critical failure locations. Section 4 focuses on failure determination, including factors of safety, deflection, and buckling. Section 5 explains the design iteration process and final material choices. Section 6 covers the FEA setup and results. Section 7 covers design changes the team made after validation. Section 8 presents the CAD model and drawings. Section 9 displays physical prototype results. Section 10 presents the discussion. The report wraps up with Section 11 with conclusions and Section 12 appendix.

2 Equilibrium Calculations

2.1 Equilibrium Equations

To determine unknown support forces and wall reactions, we apply static equilibrium equations for forces and moments in all directions and axes. The unknowns include support forces S_1, S_2, and wall reactions in the x- and z-directions ($\mathrm{Wall}_{X1}, \mathrm{Wall}_{X2}, \mathrm{Wall}_{Z}$).

The following equilibrium equations are used. It should be noted that the sums of forces, moments, and torques in each direction sum up to zero, symbolizing a static case:

$$\begin{aligned}
\textbf{Vertical Force Balance:} \quad & T_1 + T_2 - W_{\text{pump}} - S_1 \sin\theta - S_2 \sin\theta \\
& - W_{\text{beam}} + \text{Wall}_Z = 0 \qquad (1) \\
\textbf{Horizontal Force Balance:} \quad & S_1 \cos\theta + S_2 \cos\theta + \text{Wall}_{X1} + \text{Wall}_{X2} = 0 \qquad (2) \\
\textbf{Moment About the Pump:} \quad & M_G + (T_1 + T_2 - W_{\text{pump}}) L_{\text{pump-wall}} \\
& - (S_1 \sin\theta + S_2 \sin\theta) L_{\text{support}} \\
& - W_{\text{beam}} \cdot \frac{L_{\text{beam}}}{2} = 0 \qquad (3) \\
\textbf{Torque About Platform Axis:} \quad & T_G - \left(S_1 \sin\theta + \frac{\text{Wall}_Z}{2}\right) \frac{W_{\text{beam}}}{2} \\
& + \left(S_2 \sin\theta + \frac{\text{Wall}_Z}{2}\right) \frac{W_{\text{beam}}}{2} = 0 \qquad (4) \\
\textbf{Moment in Horizontal Plane:} \quad & (S_1 \cos\theta - S_2 \cos\theta + \text{Wall}_{X1} - \text{Wall}_{X2}) \\
& \cdot \frac{W_{\text{beam}}}{2} = 0 \qquad (5)
\end{aligned}$$

Using a Python code, S_1, S_2, $Wall_{X2}$, $Wall_{X2}$, and $Wall_z$ were solved. These values can be found in Table 1. Using the forces, a VM diagram was made, assisting with further stress analysis as explained in Section 3.

Finally, the reaction forces in the x and z directions at the connections can be computed as:

$$S_{1x} = -S_1 \cos\theta \qquad S_{1z} = -S_1 \sin\theta \qquad (6)$$

$$S_{2x} = -S_2 \cos\theta \qquad S_{2z} = -S_2 \sin\theta \qquad (7)$$

,where θ is the angle that the support arm makes with the platform. The forces provided in the equations above can be shown in the free body diagram below.

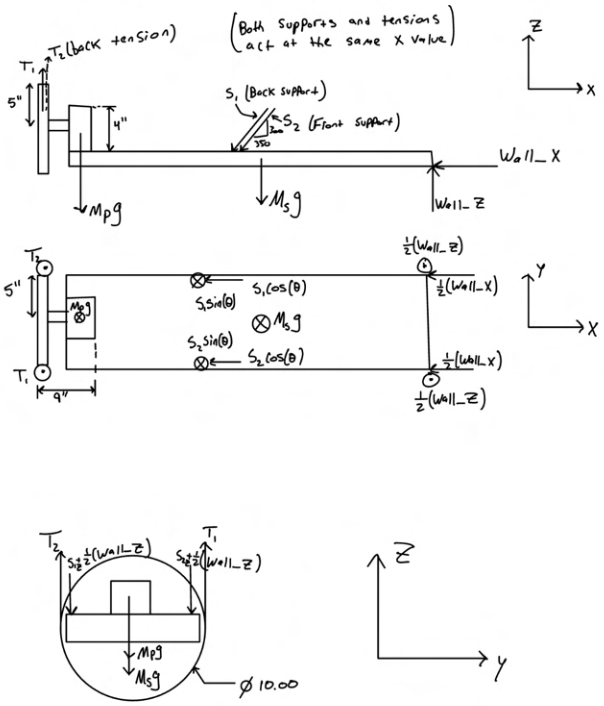

Figure 1: Free Body Diagrams for Initial Design

These provide the net reaction forces at the bolted connections due to the applied pulley forces, pump weight, and beam weight under the given angle θ.

Quantity	**Value**
Moment at Point G	743.873 lbf-in
Torque at Point G	122.592 lbf-in
Force S_1	279.323 lbf
Force S_2	226.130 lbf
Force Wall_{X1}	-178.818 lbf
Force Wall_{X2}	-144.765 lbf
Force Wall_{Z}	290.725 lbf
S_1 Wall Connection X	-178.818 lbf
S_1 Wall Connection Z	-214.582 lbf
S_2 Wall Connection X	-144.765 lbf
S_2 Wall Connection Z	-173.718 lbf

Table 1: Calculated Reaction Forces and Moments

The resulting VM Diagram from the calculated reaction forces in Table 1 is shown in Figure 2.

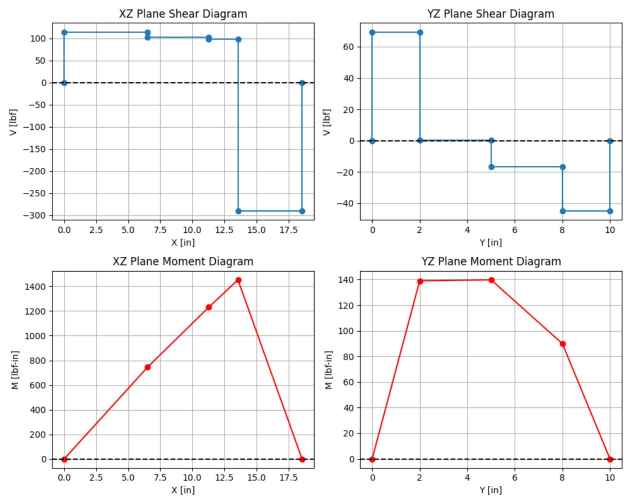

Figure 2: Python Generated VM Diagrams

2.2 Power and Pulley Force Equations

Next, the fluid and mechanical power is calculated which will be integral in solving the tensions about the pulley. These calculations guide the final results for the reaction forces and thus is valuable for this analysis. The power needed by the pump is driven by the height that the water must be driven up to. The fluid power P_{Fluid} delivered by a pump is given by:

$$P_{\text{Fluid}} = \frac{\rho \cdot g \cdot Q_{\text{ft}} \cdot h}{550} \tag{8}$$

where:

- ρ is the fluid density [lb/ft^3]
- g is the acceleration due to gravity [ft/s^2]
- Q_{ft} is the volumetric flow rate [ft^3/s]
- h is the head [ft]
- 550 is the conversion factor from ft·lb/s to horsepower

The mechanical power required from the motor, considering pump efficiency η_p, is:

$$P_{\text{Mech}} = \frac{P_{\text{Fluid}}}{\eta_p} \tag{9}$$

The tension in the return side of the pulley system T_2 is given by:

$$T_2 = \frac{P_{\text{Mech}} \cdot 550}{\left(\frac{2\pi \cdot \text{RPM}}{60}\right) \cdot \left(\frac{r_{\text{pulley}}}{12}\right)} + T_1 \tag{10}$$

where:

- P_{Mech} is the mechanical power [HP]
- RPM is the rotational speed [rev/min]
- r_{pulley} is the pulley radius [in]
- T_1, T_2 are tensions in the belt [lbf]

The moment on the platform created by the tensions exerted on the pulley can be calculated by multiplying the tensions by the distance to the pump's attachment location on the platform. This is shown below:

$$M_G = (T_1 + T_2) \cdot (L_{\text{pulley-wall}} - L_{\text{pump-wall}}) \tag{11}$$

The torque exerted on the platform is solved by multiplying the difference in the tension by the radius of the pulley, shown below:

$$T_G = (T_2 - T_1) \cdot r_{\text{pulley}} \tag{12}$$

The final calculated powers, pulley tensions, moment, and torque is summarized in Table 2 below.

Quantity	Value
Fluid Power	2.276 hp
Mechanical Power	3.501 hp
Pulley Tension 1	44.962 lbf
Pulley Tension 2	69.480 lbf
Moment at Point G	743.873 lbf-in
Torque at Point G	122.592 lbf-in

Table 2: Calculated Mechanical Parameters

3 Stress Analysis

3.1 Identification of Stress Types from Combined Loading

The platform undergoes the following loading modes:

- **Torsion** from shaft torque transmitted through pulley tension. This creates a uniform torsion throughout the platform, however, the greatest torsion is the middle of the longer side of the rectangular cross section.
- **Bending** due to distributed pump weight and tension loads from pulley. This bending can be analyzed through a VM diagram which is shown in Figure 2.
- **Axial Loading** from support arm reactions in the longitudinal direction. This reaction is compressive, meaning the failure more will either be shear/tear out through the pin holes or buckling. Thus, both failure modes were analyzed.
- **Transverse Shear** from support and pulley loads acting on the cross section. The magnitude across the platform changes but across the length of the platform as well as across the cross section.

Support arms experience:

- **Axial Compression** as they are two-force members. This reaction is compressive, meaning the failure more will either be shear/tear out through the pin holes or buckling. Thus, both failure modes were analyzed.

Pins at the wall and support connections undergo:

- **Direct Shear** only. This is due to the 'scissors-like' loading produced between the interface of the platform and the suppport arms.

3.2 Stress Concentrations and Factors

Stress concentrations arise primarily at:

- **Through-holes** on the platform for bolt attachments.
- **Pin holes** on the support arms.

Stress concentration factors K_t used:

- **Table A-15-2**: Platform bending: $K_t = 2.1$ (hole between support arm connections)
- **Table A-15-1**: Platform axial: $K_t = 2.6$ (hole in platform)
- **Table A-15-12**: Support arm pin: $K_t = 3.5$ (tear-out/bearing stress)

3.3 Critical Stress Locations

The critical locations identified include:

- **Top-middle of platform** between support connections: Max bending and torsion.
- **Pin holes** on platform: stress concentration due to hole connection with support arm.

Figure 3 depicts these two stress concentration locations.

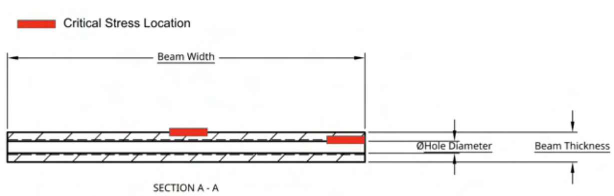

Figure 3: Symbolic diagram of platform indicating critical stress locations

3.4 Nominal Stress Expressions

The nominal stresses calculated through a python program can be summarized by the following equations. These stresses are shear and normal, caused by torsion, transverse, bending, and axial loadings. Platform:

$$\sigma_{\text{bending}} = \frac{Mc}{I}$$
$$\sigma_{\text{axial}} = \frac{F_x}{A}$$
$$\tau_{\text{torsion}} = \frac{Tr}{J}$$
$$\tau_{\text{transverse}} = \frac{VQ}{It}$$

Support Arm:

$$\sigma_{\text{nominal}} = \frac{F}{A}$$

Pin:

$$\tau = \frac{V}{A}$$

3.5 Max Shear and Principal Stress (Mohr's Circle)

Incorporating K_t, as provided in Section 3.2 into nominal stresses to produce the final stresses located around the hole in the platform:

$$\sigma_x = K_t \cdot \sigma_{\text{axial}}$$
$$\sigma_y = 0$$
$$\tau_{xy} = K_t \cdot \tau_{\text{shear}}$$

Then:

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

For pure shear pins:

$$\tau_{\max} = \tau_{xy}$$
$$\sigma_{1,2} = \pm\tau_{xy}$$

Note: These stresses are then used for Von Mises and Factor of Safety calculations.

4 Failure Determination

4.1 Factor of Safety

This section outlines the symbolic derivation of bending, axial, torsional, and combined stresses at critical locations on the pump platform structure, as well as corresponding deflection and factor of safety calculations.

Von Mises Equivalent Stress

To evaluate yielding under combined stress, the von Mises criterion is used. For a general case combining axial, bending, and torsional stresses:

$$\sigma' = \frac{1}{\sqrt{2}} \cdot \sqrt{\left(\sigma_{\text{bending,max}} - \sigma_{\text{axial,max}}\right)^2 + 6 \cdot \tau_{\text{max}}^2} \tag{13}$$

This value represents an equivalent axial stress experienced at the critical stress location within the platform.

Principal and Shear Stress (Location 1)

Using Mohr's circle relationships, the principal and maximum shear stresses are computed as:

$$C = \frac{\sigma_{\text{bending,max}} + \sigma_{\text{axial,max}}}{2} \tag{14}$$

$$R = \sqrt{C^2 + \tau_{\text{max}}^2} \tag{15}$$

$$\tau_{\text{max}} = R \tag{16}$$

$$\sigma_{\text{principal,max 1,2}} = C + / - R \tag{17}$$

Factor of Safety

Assuming a ductile material, the factor of safety (FoS) is computed using the von Mises equivalent stress:

$$\text{FoS} = \frac{S_y}{\sigma'} \tag{18}$$

where S_y is the yield strength of the material.

4.2 Max Deflection

The total deflection at the tip of the platform is a combination of:

- **Deflection due to pulley tensions, pump weight, and resultant moment about point G (location of pump attachment on the platform):**

$$\delta_G = \frac{P_G \cdot L_{\text{pump-wall}}^2 (3L_{\text{beam}} - L_{\text{pump-wall}})}{6EI} + \frac{M_G \cdot L_{\text{beam}}^2}{2EI} \tag{19}$$

  where P_G is the load acted on point G, $L_{\text{pump-wall}}$ is the distance from pump to wall, and M_G is the moment about point G.

- **Deflection due to support forces:**

$$\delta_{\text{Support Forces}} = \frac{P_{\text{EF}} \cdot L_{\text{support}}^2 (3L_{\text{beam}} - L_{\text{support}})}{6EI} \tag{20}$$

- **Total deflection:**

$$\delta_{\text{tip}} = \delta_G + \delta_{\text{Support Forces}} \tag{21}$$

4.3 Critical Buckling Load

This section outlines the buckling calculations for the support arm (Segment B–F), accounting for the geometry of the arm.

Geometric Properties of the Support Arm

The cross-sectional area A and moment of inertia I of the rectangular cross-section are given by:

$$A = (w - d) \cdot t \quad \text{(accounts for pin hole diameter } d\text{)} \tag{22}$$

$$I_{xx} = \frac{w^3 \cdot t}{12} \tag{23}$$

$$I_{yy} = \frac{w \cdot t^3}{12} \tag{24}$$

where: w is the width of the support arm, t is the thickness, d is the hole diameter.

To prevent failure by buckling, the critical buckling load P_{cr} for a short column is determined using Euler's formula, adjusted for geometry and boundary conditions:

$$P_{\text{cr}} = \frac{\pi^2 E I}{\left(\frac{L_{\text{eff}}}{r}\right)^2} \tag{25}$$

Where: E is Young's modulus, I is the moment of inertia about the weak axis, $r = \sqrt{\frac{I}{A}}$ is the radius of gyration, L_{eff} is the effective column length, adjusted by an end condition coefficient C (e.g., $C = 1$ for pinned-pinned, $C = 0.5$ for fixed-fixed ends).

In symbolic terms:

$$r = \sqrt{\frac{I}{A}} \tag{26}$$

$$\left(\frac{L_{\text{eff}}}{r}\right)^2 = \left(\frac{L_{\text{support}}}{r}\right)^2 \tag{27}$$

$$P_{\text{cr}} = \frac{\pi^2 E A r^2}{L_{\text{support}}^2} \tag{28}$$

Alternatively, if you directly substitute and simplify:

$$P_{\text{cr}} = b \cdot h \cdot C \cdot \frac{\pi^2 E}{\left(\frac{L_{\text{support}}}{h/\sqrt{12}}\right)^2} \tag{29}$$

This assumes a rectangular cross section with width b, height h, and uses the approximation $r = \frac{h}{\sqrt{12}}$ for the axis of buckling.

5 Design Iteration

5.1 Initial Dimensions

Component	Dimension	Value (inches)
Pump Platform	Length	18.00
Pump Platform	Width	6.00
Pump Platform	Thickness	0.50
Pump Platform	Length from End to CoM	5.94
Pump Platform	Support Arm Screw Diameter	0.201
Pump Platform	Length from Edge to CoM	3.00
Support Arm	Length	8.246
Support Arm	Width	0.750
Support Arm	Thickness	0.25
Support Arm	Platform Screw Diameter	0.201
Support Arm	Center of Pin Hole from Edge	0.375
Pins	Diameter	0.25

Table 3: Initial Dimensions of Components

5.2 Pump Platform

Dimensions and properties:

Compute nominal stresses:

$$\sigma_{\text{bend}} = \frac{M_{\max} c}{I_{\text{eff}}} = 3182.23 \text{ psi}, \quad \sigma_{\text{axial}} = \frac{S_1 + S_2}{A} = 174.324 \text{ psi}, \quad \tau = \frac{T_G}{k\, b\, t^2} = 254.605 \text{ psi}.$$

With concentration factors:

$$\sigma_{\text{bend,max}} = K_b \sigma_{\text{bend}} = 6682.68 \text{ psi}, \quad \sigma_{\text{axial,max}} = K_a \sigma_{\text{axial}} = 453.242 \text{ psi}.$$

Von Mises and principal/shear:

$$\sigma_{\text{vm},1} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_{\text{bend,max}} - \sigma_{\text{axial,max}})^2 + 6\tau^2} = 4426.89 \text{ psi},$$

$$C = \tfrac{\sigma_{\text{bend,max}} + \sigma_{\text{axial,max}}}{2} = 3567.96 \text{ psi}, \quad R = \sqrt{C^2 + \tau^2} = 3577.03 \text{ psi},$$
$$\tau_{\max} = R = 3577.03 \text{ psi}, \quad \sigma_{\text{principal,max}} = C + R = 7144.99 \text{ psi},$$

Parameter	Value
L	18.00 in
b	6.00 in
t	0.50 in
d	0.201 in
E	1.0×10^7 psi
S_y	4.00×10^4 psi
K_b	2.1
K_a	2.6
k	0.321
$A = (b - d)t$	2.8995 in^2
$I = \frac{b\,t^3}{12}$	0.06250 in^4
$I_{\text{eff}} = \frac{b}{12}(t^3 - d^3)$	0.05844 in^4
$c = \frac{t}{2}$	0.25 in
$M_{\max}$	743.8734 lbf·in
T_G	122.5923 lbf·in
S_1	279.3227 lbf
S_2	226.1295 lbf

Table 4: Platform: geometric, material, section properties, and applied loads

$$\text{FoS} = \frac{S_y}{\sigma_{\text{vm},1}} = 9.036.$$

At bolt holes (Location 2), transverse shear dominates ($\tau_{\text{trans}} = 575.7$ psi):

$$\sigma_{\text{vm},2} = \frac{1}{\sqrt{2}}\sqrt{\sigma^2_{\text{axial,max}} + 6\tau^2_{\text{trans}}} = 1047.38 \text{ psi}.$$

Using

$$P_G = W_{\text{pump}} - T_1 - T_2 = 12.50 - 44.9618 - 69.4803 = -101.9421 \text{ lbf},$$

$$P_{EF} = S_{1z} + S_{2z} = -212.9015 + (-172.0374) = -384.9389 \text{ lbf},$$

$$E = 1.0 \times 10^7 \text{ psi}, \quad I = \frac{b\,t^3}{12} = 0.06250 \text{ in}^4, \quad M_G = 743.8734 \text{ lbf}-\text{in}.$$

The tip deflection due to the pump load is

$$\delta_G = \frac{P_G\,L^2_{\text{pt}}\,(3L_{\text{beam}} - L_{\text{pt}})}{6EI} + \frac{M_G\,L^2_{\text{beam}}}{2EI} = 0.02699 \text{ in},$$

and due to the supports

$$\delta_{\text{support}} = \frac{P_{EF}\, L_{\text{support}}^2\, (3L_{\text{beam}} - L_{\text{support}})}{6EI} = -0.12575 \text{ in.}$$

Hence the net deflection is

$$\delta_{\max} = \delta_G + \delta_{\text{support}} = -0.09876 \text{ in.}$$

5.3 Support Arm

Parameter	**Value**
L	8.246 in
b	0.750 in
t	0.25 in
d	0.201 in
$A = (b - d)t$	0.13725 in^2
$I = \frac{b^3 t}{12}$	0.008789 in^4
E	1.0×10^7 psi
S_y	4.00×10^4 psi
K_t	3.5

Table 5: Support arm: geometric, section, and material properties

Axial stress:

$$\sigma_0 = \frac{S_1}{A} = 2035.14 \text{ psi}, \quad \sigma_x = K_t\, \sigma_0 = 7122.98 \text{ psi}.$$

Von Mises (no shear):

$$\sigma_{\text{vm}} = \sigma_x = 7122.98 \text{ psi}, \quad n = \frac{S_y}{\sigma_{\text{vm}}} = 5.613.$$

Euler Buckling (pinned–pinned ends, $K = 1$):

$$P_{\text{cr}} = \frac{\pi^2\, E\, I}{(K\, L)^2} = \frac{\pi^2 \times 1.0 \times 10^7 \,\text{psi} \times 0.008789 \,\text{in}^4}{(1 \times 8.246 \,\text{in})^2} = 1.300 \times 10^4 \text{ lbf},$$

$$n_{\text{buckling}} = \frac{P_{\text{cr}}}{S_1} = \frac{1.300 \times 10^4}{279.3227} = 46.55.$$

Deflection:

$$\delta_{\text{arm}} = \frac{S_1\,L}{A\,E} = \frac{279.3227\,\text{lbf} \times 8.246\,\text{in}}{0.13725\,\text{in}^2 \times 1.0 \times 10^7\,\text{psi}} = 0.001678 \text{ in} = 0.0426212 \text{ mm}.$$

5.4 Support & Wall Pins

Parameter	**Value**
D	0.25 in
r	0.125 in
$A = \pi r^2$	0.04909 in^2
S_y	5.10×10^4 psi

Table 6: Pin: geometry and material properties

Support-pin shear:

$$\tau_{\text{support}} = \frac{S_1}{A} = 5690.31 \text{ psi}, \quad n_{\text{support}} = \frac{0.577\,Sy}{\tau_{\text{support}}} = 4.056.$$

Wall-pin shear (resultant of $F_x = -178.82$, $F_z = 290.73$ lbf):

$$V_{\text{wall}} = \sqrt{(-178.82)^2 + (290.73)^2} = 345.45 \text{ lbf}, \quad \tau_{\text{wall}} = \frac{V_{\text{wall}}}{A} = 7036.29 \text{ psi},$$

$$n_{\text{wall}} = \frac{0.577\,Sy}{\tau_{\text{wall}}} = 4.184.$$

5.5 Material Selection

Support Arm Considerations (Fig. 4)

Per the requirements, the support arms needed to be resistant to water. Our optimization of this component was based on the material's machinability and buckling strength since these would likely be the weak points of the design. Additional considerations were yield strength and price.

Pump Platform Considerations (Fig. 5)

The pump platform considerations were largely the same as the support arm considerations, except that buckling was not as critical of a failure mode. Because of this, our optimization was performed on stiffness and machining speed. Another optimization was performed on Machining speed and Density, but this was decided to be less critical.

Pin Considerations (Fig. 30)

The pins required the same water resistance as the other components, but also needed to have superior shear strength since the primary load applied to this component is a pure shearing load.

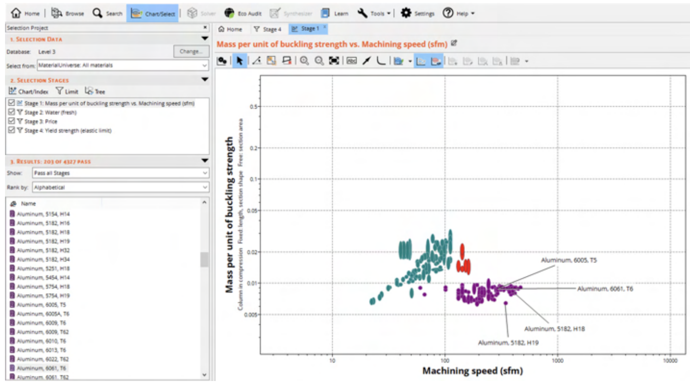

Figure 4: Material Section for Support Arm

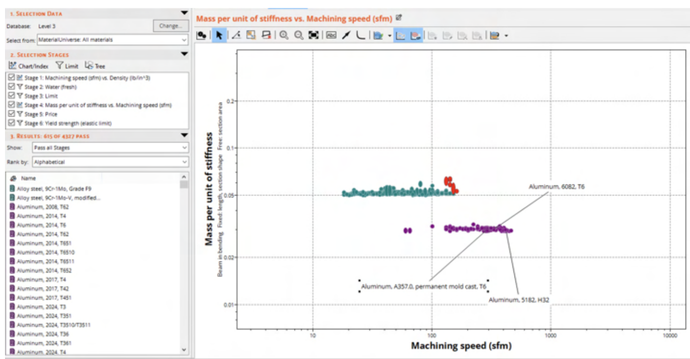

Figure 5: Material Section for Pump Platform

5.6 Iteration

Many significant changes were made to the design following the initial iteration. Through a combination of finite element analysis (FEA) and critical mechanical reasoning, the team identified inefficiencies in the original concept and opted to pivot toward a frame-based construction using more lightweight structural elements. Instead of relying on bulky, solid components, the new design strategically utilized efficient geometries such as I-beams, rectangular tubes, and angle bars—selected based on principles of structural efficiency learned in class. This approach allowed the design to maintain strength and rigidity while significantly reducing overall weight by placing material only where it was structurally necessary.

Moreover, this shift in design philosophy streamlined the fabrication process. By standardizing components and avoiding complex machining, the team minimized the required manufacturing operations. Drilling and reaming were the only processes needed for assembly, which not only saved time in the machine shop but also reduced the potential for fabrication errors. Overall, this redesign enhanced both the performance and manufacturability of the system, demonstrating the

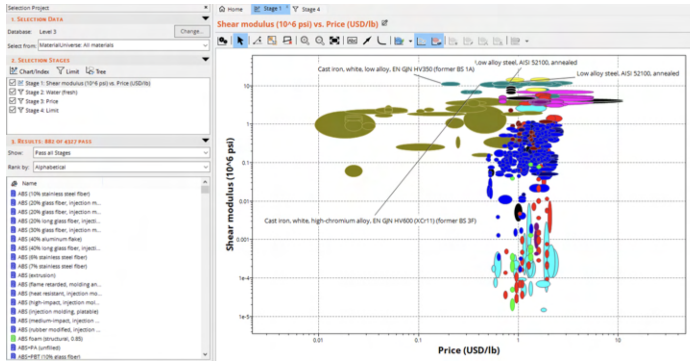

Figure 6: Material Section for Pin

value of iterative design informed by engineering analysis and practical constraints. Drawings for the final design can be found in Section 8. This process took a single major iteration, with smaller iterations once the second design had been mocked up.

5.7 Component Cost

The material costs for the final designs are outlined in Table 7. Extra material was purchased in case there were any errors made during manufacturing

Description	Unit Cost	Design Cost	Total Cost
Architectural 6063 Aluminum H-Bar (6 ft)	\$20.09	\$8.65	\$20.09
Multipurpose 6061 Aluminum Rect. Tube (2 ft)	\$14.84	\$12.06	\$29.68
Multipurpose 6061 Aluminum 90° Angle (4 ft)	\$8.55	\$3.89	\$8.55
Black-Oxide Alloy Steel Socket Head Screw	\$0.25	\$0.49	\$12.36
Shoulder-Style Ring-Grip Quick-Release Pin (0.5 in)	\$3.48	\$6.96	\$6.96
Shoulder-Style Ring-Grip Quick-Release Pin (1 in)	\$3.82	\$7.64	\$7.64
	Cost	**\$39.69**	**\$85.28**

Table 7: Bill of Materials: Unit, Design, and Total Costs

6 Finite Element Validation

6.1 FEA Setup

Due to the complexity of the loading conditions introduced by the pump, a simplification was implemented in the finite element analysis (FEA) to ensure the model remained tractable and interpretable. Rather than attempting to simulate the distributed and offset forces directly, the net effect of the pump loading was approximated by applying an equivalent resultant force at the bottom-center of the pump. This location was selected because it represents the structural interface where loads are transferred into the support frame.

Recognizing the importance of the pump base in resisting bending moments and contributing to overall system stability, we modeled a rigid plate beneath the pump to simulate its mechanical behavior. This approach allowed us to capture the load transfer without modeling the internal stiffness of the pump, which was assumed rigid as per design documentation.

At each pin joint, a revolute (pin) mate was applied in the FEA model, which constrained translational degrees of freedom while allowing rotation—accurately reflecting physical behavior. For fastened connections involving screws, a fixed joint was used between contacting surfaces. This was justified based on the high clamping force from the fasteners, and the fact that each structural element had at least two connection points, effectively preventing rotation or displacement in the assembly. For the screw connections between the arms and the beams, a washer was used between the components with a screw, so a cylindrical and planar mate were applied to this location.

The structural frame and the pump body were excluded from detailed analysis since they were both assumed to behave as rigid bodies. Pins were also omitted from the FEA model; instead, their performance was assessed through hand calculations using simple shear theory to determine the factor of safety under expected loads.

The pump's mounting configuration and loading conditions resulted in four primary loads being applied in the simulation and are shown in Fig. 7. These included a tensile force and an accompanying moment from the belt tension acting on the pulley. Additionally, asymmetric loading on either side of the pump introduced a secondary moment. A third moment arose due to the offset location of the applied load relative to the pump's center of mass. Finally, the weight of the pump itself was included as a vertical force acting downward, and gravitational acceleration was applied globally to account for the self-weight of all components.

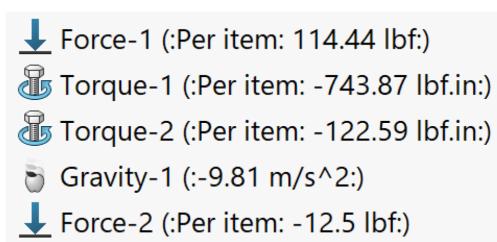

Figure 7: Applied Loads

6.2 FEA Results

6.3 Convergence Analysis

The convergence plot in Fig. 13 illustrates how increasing mesh density improves the accuracy of maximum stress calculations. To generate this plot, a mesh refinement study was performed

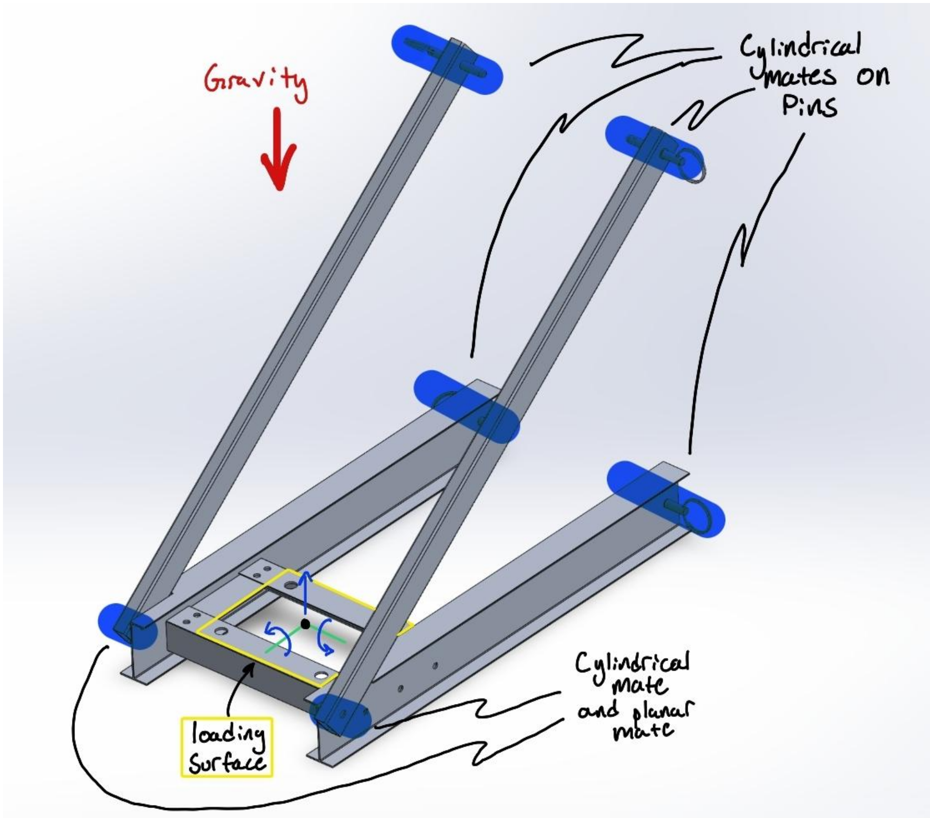

Figure 8: Isometric view of platform assembly

where simulations were run on progressively finer meshes. For each trial, the maximum stress and platform deflection were recorded, as shown in Table 8.

The study was terminated once stress values changed by less than 5% between trials, indicating convergence. This ensures the results are not significantly affected by mesh resolution, balancing accuracy with computational efficiency.

A convergence study is necessary in FEA to verify that the mesh is sufficiently refined to capture true structural behavior. Without it, stress results may appear reasonable but be inaccurate. Some initial fluctuations in stress were observed before stabilization, likely due to finer meshes resolving geometric features and stress concentrations more accurately.

Table 8: Mesh Convergence Results: Stress (psi) and Deflection (mm)

Trial	# Elements	Platform (psi)	Arm 1 (psi)	Arm 2 (psi)	Pin (psi)	Deflection (mm)
1	3343	19077.7	5205.2	1884.8	25381.6	3.986
2	4898	13893.9	3799.8	1077.0	21828.2	2.941
		-27.2%	*-27.0%*	*-42.8%*	*-14.0%*	*-26.2%*
3	9503	16326.8	3852.0	1279.2	20080.0	3.061
		+17.5%	*+1.4%*	*+18.8%*	*-8.0%*	*+4.1%*
4	11394	14866.1	3099.3	1012.5	19801.1	2.709
		-8.9%	*-19.6%*	*-20.8%*	*-1.4%*	*-11.5%*
5	16354	13999.0	3275.9	1065.1	23687.7	2.739
		-5.9%	*+5.7%*	*+5.2%*	*+19.6%*	*+1.1%*
6	18118	14604.0	3591.6	1123.6	22348.5	2.937
		+4.3%	*+9.7%*	*+12.8%*	*-5.7%*	*+7.2%*
7	18890	—	3372.5	1057.2	23718.8	2.808
			-6.1%	*-5.9%*	*+6.1%*	*-4.4%*
8	19728	—	3289.3	1086.8	23056.7	—
			-2.5%	*+2.8%*	*-2.8%*	

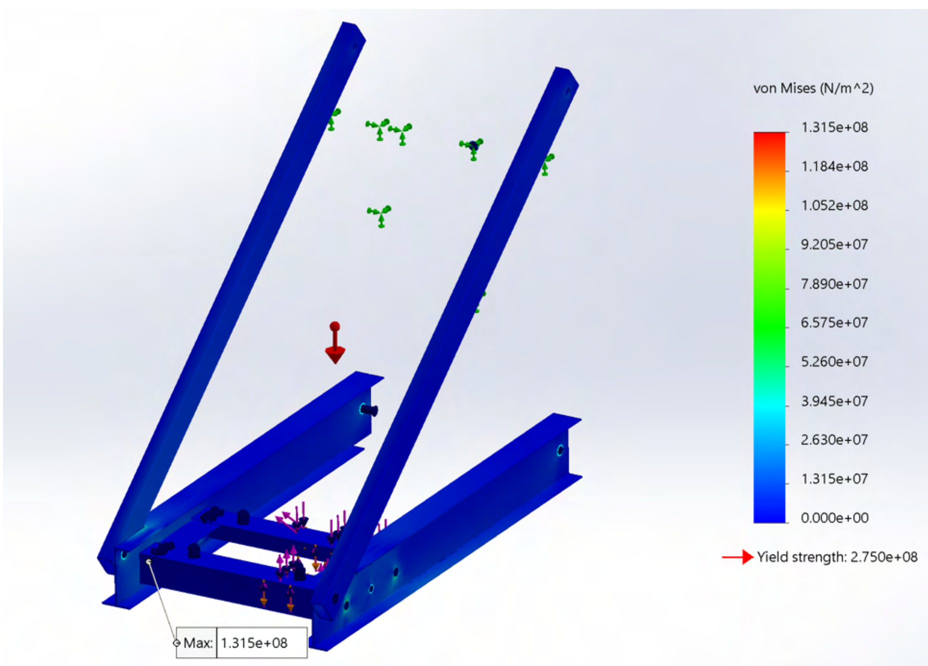

Figure 9: Solidworks Von Mises Contour for Full Structure

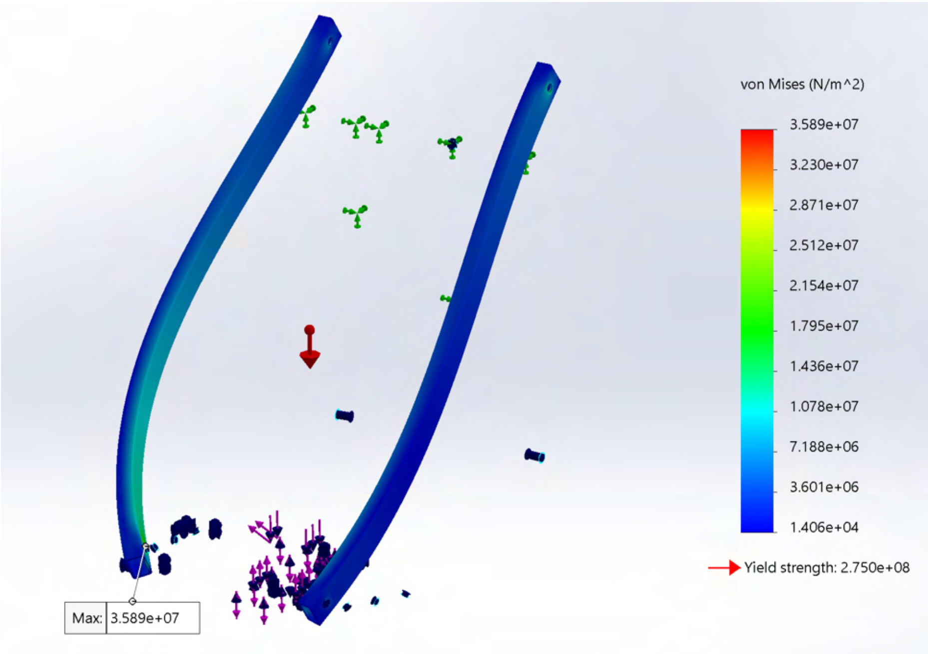

Figure 10: Solidworks Von Mises Contour for Support Arm

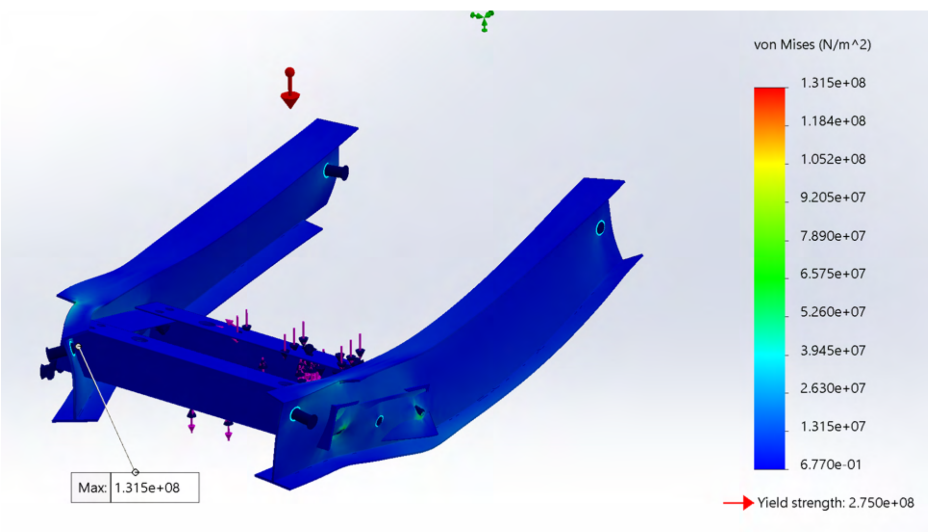

Figure 11: Solidworks Von Mises Contour for Platform

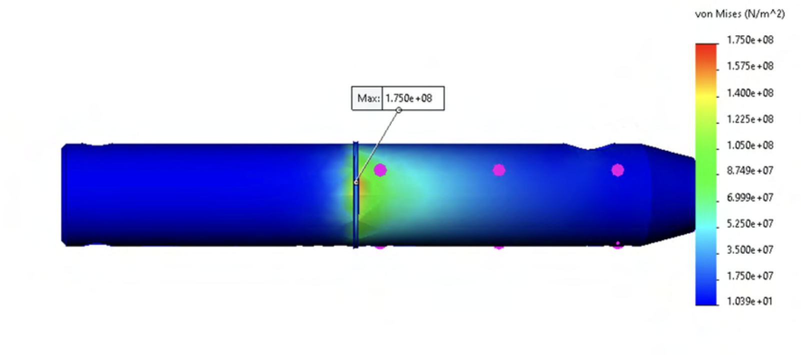

Figure 12: Solidworks Von Mises Contour for Pin

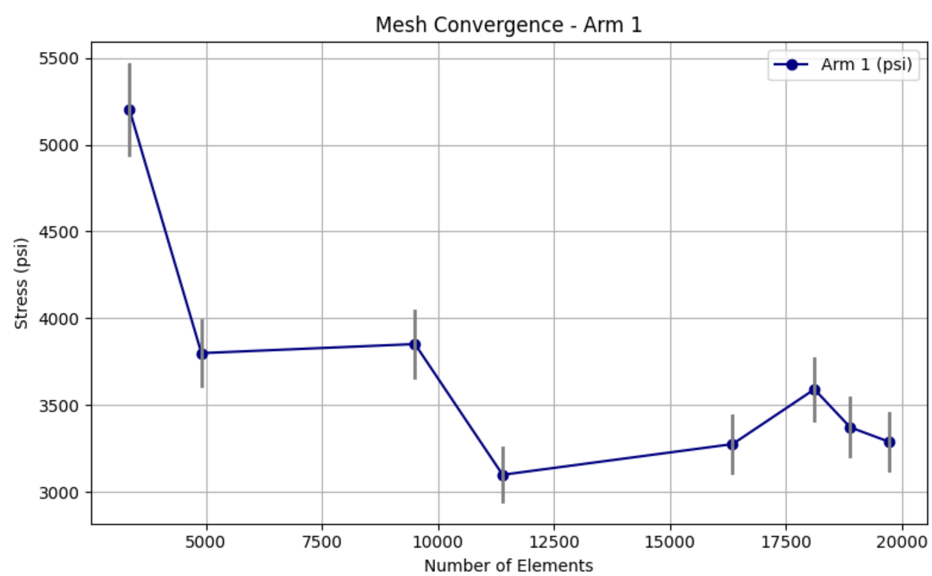

Figure 13: Support Arm 1 Convergence

6.4 Questions

1.**Using the material properties and stress values from the FEA simulations, which components are likely to be the first to fail? How does this result compare to what you expected from the selected "unique" components in GA2?**

Looking at the results of the convergence analysis in Table 1, the pins have the highest stress with 19760 psi, while the arms and platform have stresses of 1086 and 14604 psi, respectively. This matches what was expected from GA2 as the pins also had the highest calculated stress. However, since the pins are made out of stee, while the other components are made from aluminum, it likely won't fail first and instead the platform will fail first. Steel has a significantly greater yield strength than aluminum.

Component	Material	Max Stress (psi)
Support Arm	Aluminum 6061	3289.3
Platform	Aluminum 6061	14604
Pin	AISI Steel 316	23057

2.**Where is the highest stress location in each "unique" component of your design? How does this compare to what you expected from GA2?**

In GA2, the highest stress locations were found to be at the top of the cross section between the support arm connections. In the support arms, the highest stress was located to be at the pin holes due to the induced stress concentrations. In the pins attached to the frame, the highest stress is located at the top pins, at the cross section directly in contact with the frame and support arm. These finding match up with the results from the validation FEA.

3.**What is the calculated deflection in the platform and where does it occur? How does this compare to GA2?**

In the hand calculations completed in GA2, the deflection in the platform calculated using superposition was 2.441 mm at the tip of the platform. The FEA ran on SolidWorks calculated the deflection at the tip of the platform to be 2.807 mm. This is about a percent difference of 13%.

4.**If you had more time for design iteration this semester, would you make any changes? Why or why not?**

If we had more time for design iterations, we would do deeper research into how the support arm placement on the frame and platform can be optimized to reduce weight while minimizing deflection and stresses. For the most part, the support arms for the final iteration were placed at the tip of the platform because we expected that point to have the most deflection. However, this introduced greater weight in the support arms and possibly increased the stress towards the center of the platform.

7 Design Changes

Many of the design changes made between iterations 1 and 2 have previously been covered in Section 5.6. Iteration 1 relied on bulky solid components, but after submitting GA2, several factors prompted a shift to a lighter, frame-based design.

Material limitations and FEA results revealed that much of the original material was unnecessary for structural integrity. This led to a redesign using efficient structural shapes like I-beams and tubing, reducing weight without compromising strength. FEA also highlighted stress concentrations, which were addressed through geometry adjustments.

Machinability played a major role as well—complex milling was replaced with simpler operations like drilling and reaming. Additionally, some calculation errors in GA2, such as bolt sizing and load assumptions, were corrected during this stage.

8 CAD and Drawings

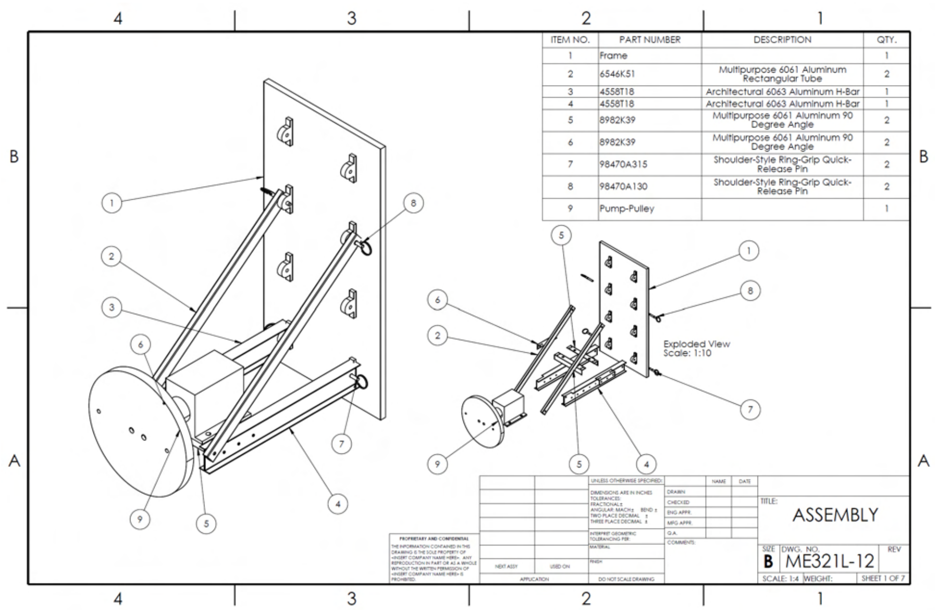

Figure 14: Assembly Drawing

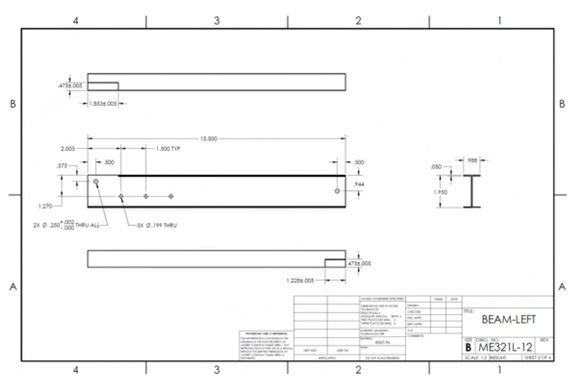

Figure 15: Left Beam Drawing

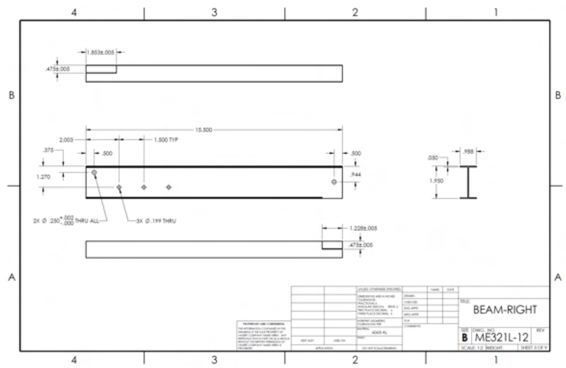

Figure 16: Right Beam Drawing

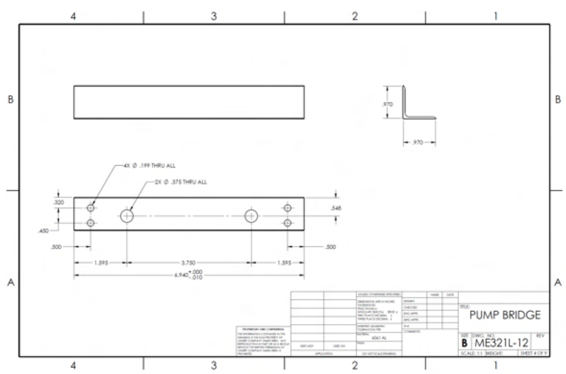

Figure 17: Pump Bridge Drawing

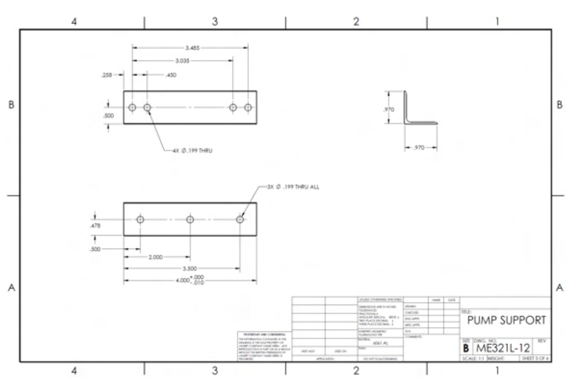

Figure 18: Pump Support Drawing

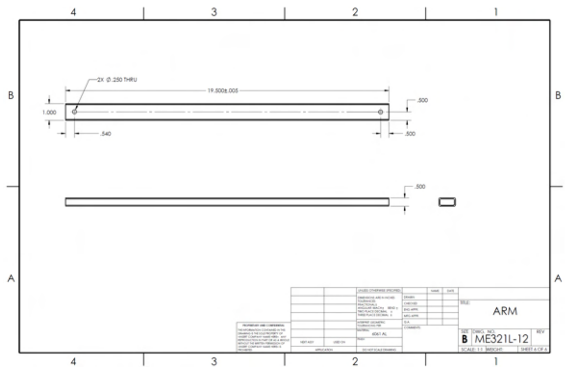

Figure 19: Arm Drawing

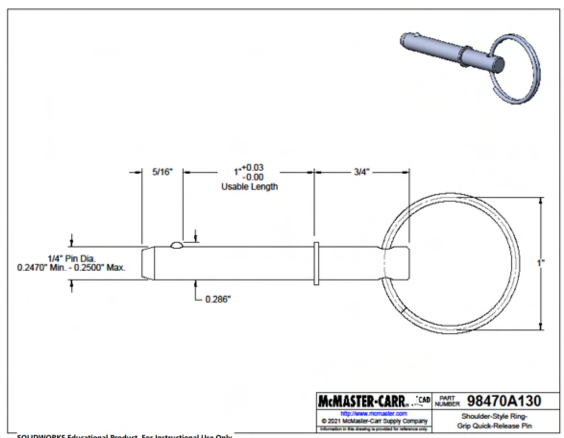

Figure 20: Pin 1 Drawing

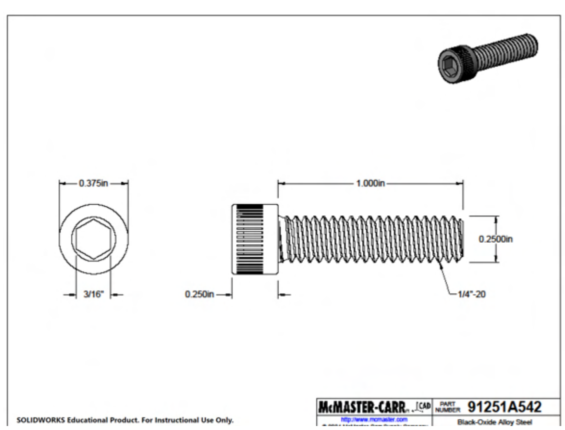

Figure 21: Pin 2 Drawing

Figure 22: Left I-Beam

Figure 23: Right I-Beam

Figure 24: Left Pump Bridge

Figure 25: Right Pump Bridge

Figure 26: Left Pump Support

Figure 27: Right Pump Bridge

Figure 28: Left Support Arm

Figure 29: Right Support Arm

9 Physical Prototype Results

Recorded Weight (lbs)	1.8625
Prototype attached (yes/no)	yes
Recorded Height (in)	14
Recorded Load (lbf)	115
Recorded Deflection (in)	0.0984

Table 9: Physical Prototype Data Collection Table

Figure 30: Physical Prototype in Frame Set Up

10 Discussion

At the start of testing, our platform did not properly interface with the test setup due to an overlap between the support arms and the surrounding frame. This interference prevented a secure fit. To address the issue, we carefully filed down the top corner of the support arms. This modification was done cautiously, removing only a minimal amount of material to avoid creating stress concentrations that could compromise the structural integrity and lead to fatigue failure. After the adjustment, the platform fit securely within the testing setup and functioned as intended.

The platform successfully withstood the maximum applied load based on our design specifications for the pulley system. Throughout the test, there were no signs of structural failure or deformation beyond what was expected. This confirmed that our material selection and structural geometry were sufficient to support the applied forces without exceeding the design limits.

In terms of deflection, the platform performed well, with a measured deflection of 2.5 mm under full load. This value was below the allowable limit of 3 mm, indicating that the structure met the deflection criteria. This performance validated our stiffness assumptions and showed that the platform was not only strong but also rigid enough to maintain functional alignment under load.

Our finite element analysis (FEA) had predicted a factor of safety greater than one and no failure under the expected loads. The physical performance of the platform matched these predictions, demonstrating that the simulation results were accurate and reflective of real-world behavior. The consistency between calculated safety margins, FEA output, and experimental results provided strong validation for our design process.

An unexpected challenge we encountered was the initial fitment issue caused by the interference between the support arms and the frame. While this was not anticipated during the design phase, we were able to overcome it through a minor geometric adjustment. Additionally, the overall fabrication and assembly process went smoothly, largely because we prioritized manufacturability and ease of construction during the design stage. Only minimal machining was required, specifically, drilling a few holes to connect support members to the I-beams and to secure the platform to the frame. This approach minimized complications during assembly and contributed to the overall success of the testing process.

11 Conclusion

Over the course of the semester, our team designed, analyzed, and prototyped a structural support platform for a centrifugal pump system under practical loading conditions and machinability constraints. The design needed to satisfy set requirements including remaining lightweight, modular, strong, and manufacturable within a $150 material budget. Through this process, we demonstrated use of many engineering principles, including static equilibrium, beam theory, stress analysis, material selection, and deflection. Through an iterative design approach, we were able to build a strong, lightweight, and cost effective final product.

Our analysis first began with force, moment, and torque calculations to determine support reactions and different loads existing on the platform and frame. We developed a symbolic model using Python to generate shear, bending, and torsional stress values along with their respective diagrams. To identify critical stress locations we used stress concentration factors from Shigley's tables. Von Mises stress and principal stress equations were used to assess yielding, and all results were validated with hand calculated factors of safety. We also evaluated the support arms for compressive buckling using the Euler buckling equation. Deflection at the platform tip was estimated with superposition and double checked through FEA, where our results showed a strong agreement (within 22% of each other) with analytical predictions.

We also learned to balance theoretical concepts taught in lecture with practicality. Initially, our design relied on heavy aluminum stock, but after stress analysis and FEA showed large amounts of material usage and, therefore, longer machining times, we moved to a more efficient frame design using I-beams and tubes. This experience showed how structure and the geometry of the beam can be just as effective as solid stock for resisting loads. Additionally, we gained insight into how small details like, hole placements and the support arm layout can have large impacts on performance and manufacturability.

This project provided us a strong example of the full design cycle, starting from open ended requirements, to building a symbolic model, then validating it with practical dimensions and loads, to ultimately delivering a working prototype.

12 Appendix

12.1 Initial Dimensions

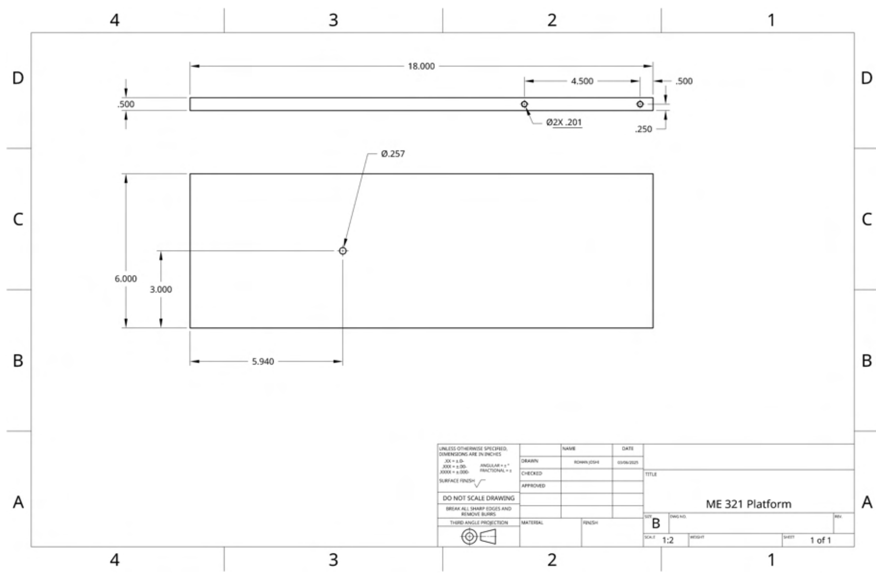

Figure 31: Initial Platform Drawing

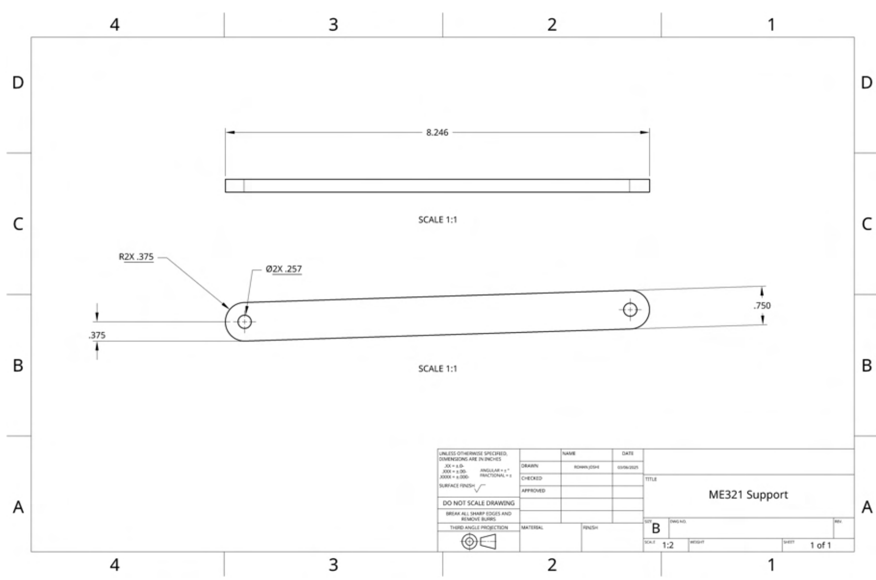

Figure 32: Initial Support Drawing

12.2 Code

12.2.1 Load Calculation

```
import math
import numpy as np
from sympy import symbols, Eq, solve
import matplotlib.pyplot as plt
import itertools

def calculate_fluid_mechanical_power(Q_ft, rho, g, h, n_p):
    P_Fluid = (rho * g * Q_ft * h) / 550
    P_Mech = P_Fluid / n_p
    return P_Fluid, P_Mech

def calculate_pulley_tension(P_Mech, RPM, Pulley_rad, T1):
    T2 = ((P_Mech * 550) / ((RPM * 2 * np.pi / 60) * (Pulley_rad / 12))) + T1
    return T2

def calculate_moment_and_torque(T1, T2, L_Pulley_to_Wall, L_Pump_to_Wall):
    M_G = (T1 + T2) * (L_Pulley_to_Wall - L_Pump_to_Wall)
    T_G = (T2 - T1) * (Pulley_rad)
    return M_G, T_G

def solve_forces(T1, T2, W_beam, W_pump, theta, L_Pump_to_Wall, L_support, L_beam, Width_beam, M_G, T_G):
    S1, S2, Wall_X1, Wall_X2, Wall_Z = symbols('S1 S2 Wall_X1 Wall_X2 Wall_Z')
    eq1 = Eq(T1 + T2 - W_pump - S1 * math.sin(theta) - S2 * math.sin(theta) - W_beam + Wall_Z, 0)
    eq2 = Eq(S1 * math.cos(theta) + S2 * math.cos(theta) + (Wall_X1 + Wall_X2), 0)
    eq3 = Eq(M_G + ((T1+T2-W_pump) * L_Pump_to_Wall)
        - ((S1 * math.sin(theta) + S2 * math.sin(theta)) * L_support) - (W_beam * (L_beam/2)), 0)
    eq4 = Eq(T_G - ((S1*np.sin(theta) + (Wall_Z/2)) * (Width_beam/2))
        + ((S2*np.sin(theta) + (Wall_Z/2)) * (Width_beam/2)), 0)
    eq5 = Eq((S1*np.cos(theta) - S2*np.cos(theta) + Wall_X1 - Wall_X2)*0.5*Width_beam, 0)
    soln = solve((eq1, eq2, eq3, eq4, eq5), (S1, S2, Wall_X1, Wall_X2, Wall_Z))

    # print(soln)
    return soln

def calculate_connection_reactions(soln, theta):
    S1_ConnectionX = -soln[symbols('S1')] * math.cos(theta)
    S1_ConnectionZ = -soln[symbols('S1')] * math.sin(theta)
    S2_ConnectionX = -soln[symbols('S2')] * math.cos(theta)
    S2_ConnectionZ = -soln[symbols('S2')] * math.sin(theta)
    return S1_ConnectionX, S1_ConnectionZ, S2_ConnectionX, S2_ConnectionZ


# CONSTRAINTS
Q = 150  # Volume flow rate [GPM]
Q_ft = Q / 7.48 / 60  # Volume flow rate [ft^3/sec]
n_p = 0.65  # Pump Efficiency
h = 60  # Final Height [ft]
rho = 1.94  # slugs/ft3
g = 32.174  # Acceleration Due to Gravity
W_pump = 12.5  # Weight of Pump [lbf]
RPM = 1800  # RPM of Motor
L_Pump_to_Wall = 9.56 + 2.5  # Distance from Pump to Wall [in]
L_Pulley_to_Wall = 18.56  # Distance from Pulley to Wall [in]

# FREE VARIABLES
L_mounting = 6  # Distance between mounting studs [in]
L_support = 5  # Placement [in]
L_beam = 18 # Placement [in]
Width_beam = 6  # [in]
Thick_beam = 0.5  # [in]
density_beam = 0.1  # [lb/in^3]
```

```
# # FREE VARIABLES
# L_mounting = 6  # Distance between mounting studs [in]
# L_support = 2.00  # Placement [in]
# L_beam = 14.56  # Placement [in]
# Width_beam = 6.00  # [in]
# Thick_beam = 0.38  # [in]

"""
# UNCOMMENT FOR USER INPUT
L_mounting = float(input(print("Enter Distance between mounting studs [in]: ")))
L_support = float(input(print("Enter Distance between support attachment and wall [in]: ")))
L_beam = float(input(print("Enter Length of Beam [in]: ")))
Width_beam = float(input(print("Enter Width of Beam [in]: ")))
Thick_beam = float(input(print("Enter Thickness of Beam [in]: ")))
density_beam = float(input(print("Enter Density of Beam [lb/in^3]: ")))
"""

# Calculated Values
W_beam = density_beam * L_beam * Width_beam * Thick_beam  # Weight of Beam [lbf]
theta = np.arctan(L_mounting / L_support)

P_Fluid, P_Mech = calculate_fluid_mechanical_power(Q_ft, rho, g, h, n_p)
print(f"Fluid Power: {P_Fluid:.6f} hp \nMechanical Power: {P_Mech:.6f} hp")

T1 = 44.9618  # Pulley Tension 1 [lbf]
Pulley_rad = 10 / 2  # [in]
T2 = calculate_pulley_tension(P_Mech, RPM, Pulley_rad, T1)
print(f"Pulley Tension 1: {T1:.6f} lbf")
print(f"Pulley Tension 2: {T2:.6f} lbf")

M_G, T_G = calculate_moment_and_torque(T1, T2, L_Pulley_to_Wall, L_Pump_to_Wall)
print(f"Moment at Point G: {M_G:.6f} lbf-in")
print(f"Torque at Point G: {T_G:.6f} lbf-in")

soln = solve_forces(T1, T2, W_beam, W_pump, theta, L_Pump_to_Wall, L_support, L_beam, Width_beam, M_G, T_G)
print(f"Force S1: {soln[symbols('S1')]:.6f} lbf")
print(f"Force S2: {soln[symbols('S2')]:.6f} lbf")
print(f"Force Wall_X1: {soln[symbols('Wall_X1')]:.6f} lbf")
print(f"Force Wall_X2: {soln[symbols('Wall_X2')]:.6f} lbf")
print(f"Force Wall_Z: {soln[symbols('Wall_Z')]:.6f} lbf")

S1_ConnectionX, S1_ConnectionZ, S2_ConnectionX, S2_ConnectionZ = calculate_connection_reactions(soln, theta)
print(f"Force S1 Wall Connection X: {S1_ConnectionX:.6f} lbf")
print(f"Force S1 Wall Connection Z: {S1_ConnectionZ:.6f} lbf")
print(f"Force S2 Wall Connection X: {S2_ConnectionX:.6f} lbf")
print(f"Force S2 Wall Connection Z: {S2_ConnectionZ:.6f} lbf")
```

12.2.2 Shear and Bending Plots

```
s1 = soln[symbols('S1')]
s2 = soln[symbols('S2')]
wall_X1 = soln[symbols('Wall_X1')]
wall_X2 = soln[symbols('Wall_X2')]
wall_Z = soln[symbols('Wall_Z')]

x_vals_xz = [
    0, 0,
    L_Pulley_to_Wall-L_Pump_to_Wall, L_Pulley_to_Wall-L_Pump_to_Wall,
    L_Pulley_to_Wall-L_beam/2, L_Pulley_to_Wall-L_beam/2,
    L_Pulley_to_Wall - L_support, L_Pulley_to_Wall - L_support,
    L_Pulley_to_Wall, L_Pulley_to_Wall
]
```

```
shear_xz = [
    0,
    T1 + T2, T1 + T2,
    (T1 + T2 - W_pump), (T1 + T2 - W_pump),
    (T1 + T2 - W_pump) - W_beam, (T1 + T2 - W_pump) - W_beam,
    (T1 + T2 - W_pump) - W_beam - s1*np.sin(theta)-s2*np.sin(theta),
    (T1 + T2 - W_pump) - W_beam - s1*np.sin(theta)-s2*np.sin(theta),
    (T1 + T2 - W_pump) - W_beam - s1*np.sin(theta)-s2*np.sin(theta) + wall_Z
]

x_vals_yz = [
    0, 0,
    Pulley_rad-Width_beam/2, Pulley_rad-Width_beam/2,
    Pulley_rad, Pulley_rad,
    Pulley_rad+Width_beam/2, Pulley_rad+Width_beam/2,
    Pulley_rad*2, Pulley_rad*2
]

shear_yz = [
    0,
    T2, T2,
    T2 - s1*np.sin(theta) + wall_Z/2, T2 - s1*np.sin(theta) + wall_Z/2,
    T2 - s1*np.sin(theta) + wall_Z/2 - W_beam - W_pump, T2 - s1*np.sin(theta) + wall_Z/2 - W_beam - W_pump,
    T2 - s1*np.sin(theta) + wall_Z/2 - W_beam - W_pump - s2*np.sin(theta) + wall_Z/2,
    T2 - s1*np.sin(theta) + wall_Z/2 - W_beam - W_pump - s2*np.sin(theta) + wall_Z/2,
    T2 - s1*np.sin(theta) + wall_Z/2 - W_beam - W_pump - s2*np.sin(theta) + wall_Z/2 + T1
]

unique_x_xz, indices_xz = np.unique(x_vals_xz, return_index=True)
unique_x_yz, indices_yz = np.unique(x_vals_yz, return_index=True)

shear_xz_unique = np.array(shear_xz)[indices_xz]
shear_yz_unique = np.array(shear_yz)[indices_yz]

moment_xz = np.zeros_like(unique_x_xz, dtype=float)
moment_yz = np.zeros_like(unique_x_yz, dtype=float)

moment_xz[0] = shear_xz_unique[0] * unique_x_xz[0]
moment_yz[0] = shear_yz_unique[0] * unique_x_yz[0]

for i in range(1, len(unique_x_xz)):
    dx = unique_x_xz[i] - unique_x_xz[i - 1]
    moment_xz[i] = moment_xz[i - 1] + shear_xz_unique[i] * dx

for i in range(1, len(unique_x_yz)):
    dy = unique_x_yz[i] - unique_x_yz[i - 1]
    moment_yz[i] = moment_yz[i - 1] + shear_yz_unique[i] * dy


fig, axes = plt.subplots(2, 2, figsize=(10, 8))

# XZ Plane Shear Diagram
axes[0, 0].plot(x_vals_xz, shear_xz, marker="o", linestyle="-")
axes[0, 0].axhline(0, color="k", linestyle="--")
axes[0, 0].set_xlabel("X [in]")
axes[0, 0].set_ylabel("V [lbf]")
axes[0, 0].set_title("XZ Plane Shear Diagram")
axes[0, 0].grid(True)

# XZ Plane Moment Diagram
axes[1, 0].plot(unique_x_xz, moment_xz, marker="o", linestyle="-", color="r", label="Moment (M_xz)")
axes[1, 0].axhline(0, color="k", linestyle="--")
axes[1, 0].set_xlabel("X [in]")
axes[1, 0].set_ylabel("M [lbf-in]")
axes[1, 0].set_title("XZ Plane Moment Diagram")
axes[1, 0].grid(True)
```

```
# YZ Plane Shear Diagram
axes[0, 1].plot(x_vals_yz, shear_yz, marker="o", linestyle="-")
axes[0, 1].axhline(0, color="k", linestyle="--")
axes[0, 1].set_xlabel("Y [in]")
axes[0, 1].set_ylabel("V [lbf]")
axes[0, 1].set_title("YZ Plane Shear Diagram")
axes[0, 1].grid(True)

# YZ Plane Moment Diagram
axes[1, 1].plot(unique_x_yz, moment_yz, marker="o", linestyle="-", color="r", label="Moment (M_yz)")
axes[1, 1].axhline(0, color="k", linestyle="--")
axes[1, 1].set_xlabel("Y [in]")
axes[1, 1].set_ylabel("M [lbf-in]")
axes[1, 1].set_title("YZ Plane Moment Diagram")
axes[1, 1].grid(True)


# axes[0, 0].set_yticklabels([])
# axes[1, 0].set_yticklabels([])
# axes[0, 1].set_yticklabels([])
# axes[1, 1].set_yticklabels([])

plt.tight_layout()
plt.show()


M_max= np.max(np.abs(moment_xz))
print(f"Peak bending moment in XZ likely at the support connection: (x,y)
    = ({unique_x_xz[np.argmax(np.abs(moment_xz))]:.3}, {np.max(np.abs(moment_xz)):.6})")
print(f"Peak bending moment in YZ likely in the middle of the beam: (x,y)
    = ({unique_x_yz[np.argmax(np.abs(moment_yz))]:.3}, {np.max(np.abs(moment_yz)):.6})")
```

12.2.3 Platform Strength Calculations

```
import numpy as np

#Platform Stresses:Focused on the Cross Section between Support Connections
Thick_beam= 0.5
Width_beam = 6
L_support = 4.9
#Geometric Constraints
c = Thick_beam/2 #distance to outer fiber

d=0.25 #size of hole
I_EF = (Width_beam/12)*(Thick_beam**3-d**3)
I = (Width_beam*Thick_beam**3)/12
A = (Width_beam-d)*Thick_beam #cross section with pin hole accounted for
k = 0.321

#Material Properties
E = 10000000 #psi for alum 6061
Sy = 40000 #psi for alum 6061

#Stress Concentration Factors
k_bending = 2.1 #Table A-15-2
k_axial = 2.6 #Table A-15-12

#Nominal Stresses for location 1
sigma_bending = (M_max)*(c)/I_EF
sigma_axial = ((S2_ConnectionX+S1_ConnectionX)/A)
torsion = T_G/(k*Width_beam*Thick_beam**2)

#max stresses _ accounting for stress concentrations
sigma_bending_max = k_bending * sigma_bending
```

```
sigma_axial_max = k_axial * sigma_axial
torsion_max = torsion
sigma_bending_max = float(sigma_bending_max)
sigma_axial_max = float(sigma_axial_max)
torsion = float(torsion)
sigma1 = 1/np.sqrt(2) * np.sqrt((sigma_bending_max - sigma_axial_max)**2 + 6*(torsion)**2)

print(f"Max Bending Stress for Location 1: {sigma_bending_max:.6f} lbf/in^2")
print(f"Max Axial Stress for Location 1: {sigma_axial_max:.6f} lbf/in^2")
print(f"Max Torsion Stress for Location 1: {torsion_max:.6f} lbf/in^2")
print(f"Max von mises stress at Location 1: {sigma1:.6f} lbf/in2")

V_max = 850
transverse = 575.7  # lbf/in^2
print(f"Max Transverse Shear Stress for Location 2: {transverse:.6f} lbf/in^2")
print(f"Max Axial Stress for Location 2: {sigma_axial_max:.6f} lbf/in^2")

sigma2 = 1/np.sqrt(2) * np.sqrt( sigma_axial_max**2 + 6*(transverse)**2 )
print(f"Max von mises stress at Location 2: {sigma2:.6f} lbf/in^2")

#Max deflection in pump platform
P_G = W_pump-T1-T2
P_EF = S1_ConnectionZ+S2_ConnectionZ #CHECK SIGN

delta_G = ((P_G*L_Pump_to_Wall**2)*(3*L_beam-L_Pump_to_Wall))/(6*E*I) + (M_G*L_beam**2)/(2*E*I)
delta_supports = ((P_EF*L_support**2)*(3*L_beam-L_support))/(6*E*I)
delta_max = delta_G + delta_supports
print(f"Max Deflection in Pump Platform: {delta_max:.6f} in")

#Max shear and principal stress calcs
C = (sigma_bending_max+sigma_axial_max)/2
R= np.sqrt(((sigma_bending_max+sigma_axial_max)/2)**2 + torsion**2)
shear_max = R
principal_max = C + R
print(f"Max Shear for Location 1: {shear_max:.6f} lbf/in^2")
print(f"Max Principal Stress for Location 1: {principal_max:.6f} lbf/in^2")

#Factor of Safety for Platform
FoS = Sy/(sigma1) #ductile material so ductile criterion
print(f"Factor of Safety for Platform: {FoS:.6f}")
```

12.2.4 Support Arm Stresses

```
import numpy as np
import math

#Support Arm Stresses

#Geometric Constraints
#Mounting hole 0 - 3in, 1 - 9in, 2 - 15in, 3 - 21in
mounting_hole = 2
Support_arm_length = math.sqrt((mounting_hole*L_mounting)**2+L_support**2)
Support_arm_thick = 0.25
Support_arm_width = 0.75
Support_arm_effective_l_pinned = 1.0  #pinned-pinned
Support_arm_effective_l_fixed = 0.5 #fixed-fixed
Support_arm_area = (Support_arm_width - d) * Support_arm_thick  #accounting for pin hole
Support_arm_I_xx = (Support_arm_width**3 * Support_arm_thick) / 12
Support_arm_I_yy = (Support_arm_width * Support_arm_thick**3) / 12

#Force in Support Arm B-F
S1 = soln[symbols('S1')]

#Nominal Stress
```

```
Support_arm_sigma_0 = S1 / Support_arm_area

#Stress Concentration Factors
Support_arm_d_w_ratio = d / Support_arm_width
Support_arm_L_w_ratio = (Support_arm_length - Support_arm_width / 2) / Support_arm_width
print(f"Support arm length: {Support_arm_length:.6f}")
print(f"d/w ratio: {Support_arm_d_w_ratio:.6f}")
print(f"L/w ratio: {Support_arm_L_w_ratio:.6f}")
Support_arm_K_t = 3.5 #Table A-15-12

#Maximum Stresses accounting for Stress Concentration
Support_arm_sigma_x = Support_arm_K_t * Support_arm_sigma_0

#Stress Concentration
sig_xx_BF = Support_arm_sigma_x
sig_yy_BF = 0
tau_xy_BF = 0

#Mohr's Circle
CMC_BF = (sig_xx_BF + sig_yy_BF)/2
RMC_BF = math.sqrt(0.5*(sig_xx_BF-sig_yy_BF)**2 + 0.5*tau_xy_BF**2)
sig1_BF = CMC_BF + RMC_BF
sig2_BF = CMC_BF - RMC_BF
tauMax_BF = RMC_BF

#Ductile Failure (von Mises Distortion Energy)
sig_von_BF = math.sqrt(sig1_BF**2 + sig2_BF**2 - sig1_BF*sig2_BF)
#Factor of safety
n_BF = Sy/sig_von_BF

# Buckling
C_arm2 = 1
b_BF = min(Support_arm_thick, Support_arm_width)
h_BF = max(Support_arm_thick, Support_arm_width)
I_BF = (b_BF*h_BF**3)/12
A_BF = Support_arm_thick*Support_arm_width
k_BF = math.sqrt(I_BF/A_BF)
l_BF = Support_arm_length
lk_BF_1 = ((2*(math.pi**2)*C_arm2*E)/Sy)**0.5


Pcr_BF = b_BF*h_BF*C_arm2*(math.pi**2)*E/(l_BF/(h_BF/math.sqrt(12)))**2


#Factors of Safety
#Critical buckling is larger than sigma_1
Support_arm_n_buckling = Pcr_BF / S1

#Maximum Deflection
Support_arm_delta_max = (S1 * Support_arm_length) / (Support_arm_area * E)

print(f"Nominal Stress: {Support_arm_sigma_0:.6f} lbf/in^2")
print(f"Max Stress (with K_t): {Support_arm_sigma_x:.6f} lbf/in^2")
print(f"Max Shear Stress: {tauMax_BF:.6f} lbf/in^2")
print(f"Principal Stresses: Sigma_1 = {sig1_BF:.6f}, Sigma_2 = {sig2_BF:.6f}")
print(f"Critical Buckling Load: {Pcr_BF:.6f} lbf")
print(f"Factor of Safety Buckling: {Support_arm_n_buckling:.6f}")
print(f"Factor of Safety (Von Mises): {n_BF:.6f}")
print(f"Max Deflection: {Support_arm_delta_max:.6f} in")
```

12.2.5 Pin Strength Calculations

```
#pin in support
#dimensions
r = 0.120
```

```
A =  np.pi * r**2

#material properties
Sy = 51000 #yield strength for 6061 alum in PSI
#shear force
V_support = soln[symbols('S1')]

#shear stress calcs
tau_support = V_support/A
print(f"Max Shear Stress: {tau_support:.6f} lbf/in^2")

#pin at wall
V_wall = np.sqrt((169.237966)**2+(875.941312)**2)
print(V_wall)
tau_wall = V_wall/A
print(f"Max Shear Stress: {tau_wall:.6f} lbf/in^2")


Fos_SupportPin = 0.577*Sy/(tau_support)
print(f"Factor of Safety for Pin in Support: {Fos_SupportPin:.6f}")
Fos_WallPin = 0.577*Sy/(tau_wall)
print(f"Factor of Safety for Pin in Wall: {Fos_WallPin:.6f}")
```