

CSCE 221 Cover Page
Homework Assignment #3
Due April 23 at 23:59 pm to eCampus

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Please list all sources in the table below including web pages which you used to solve or implement the current homework. If you fail to cite sources you can get a lower number of points or even zero, read more on Aggie Honor System Office website: <http://aggiehonor.tamu.edu/>

Type of sources				
People	marrisa sam			
Web pages (provide URL)	Geeskforgeeks			
Printed material	Notes			
Other Sources				

I certify that I have listed all the sources that I used to develop the solutions/codes to the submitted work.
On my honor as an Aggie, I have neither given nor received any unauthorized help on this academic work.

Your Name Ryan Wenham

Date 4/15/2020

Homework 3 (100 points)

due April 24 at 11:59 pm to eCampus.

Write clearly and give full explanations to solutions for all the problems. Show all steps of your work.

Reading assignment:

- Balanced Binary Search Trees
- Skip Lists
- Hash Tables
- Heap and Priority Queue
- Graphs

Problems.

1. (10 points) For the following statements about red-black trees, provide a justification for each true statement and a counterexample for each false one.

(a) A subtree of a red-black tree is itself a red-black tree.

- i. False - a subtree with a red root is not a red-black tree

(b) The sibling of an external node is either external or red.

- i. True - If the sibling was black node then it would have another set of at least one red node below it and therefore violating the height property because there would be an external node that has more black ancestors than the others.

(c) There is a unique 2-4 tree associated with a given red-black tree.

- i. True, A red-black tree with two red children is a 4 node, with only one red child is a 3 node and

ii.

(d) There is a unique red-black tree associated with a given 2-4 tree

- i. False - A 3 node tree can have different representations of possibilities, so not unique 2.(10

points) Modify this skip list after performing the following series of operations: `erase(38)`, `insert(48,x)`, `insert(24,y)`, `erase(42)`. Provided the recorded coin flips for `x` and `y`.

ii. $x = 4$, $y=2$

$-\infty$	—	—	—	—	—	$+\infty$
$-\infty$	—	17	—	—	—	$+\infty$
$-\infty$	—	17	—	—	48	$+\infty$
$-\infty$	—	17	—	—	48	$+\infty$
$-\infty$	12	17	—	24	48	$+\infty$
$-\infty$	12	17	20	24	48	$+\infty$

2. (10 points) Draw the 17-entry hash table that results from using the has function: $h(k) = ((3k + 5) \bmod 11)$, to hash the keys: 12, 44, 13, 88, 23, 94, 11, 39, 20, 16, 5, assuming collisions are handled by double hashing using the secondary hash function: $h_s(k) = (7 - (k \bmod 7))$.

(a)

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
13	94			11	44	5	39	12	16	20	88		23			

(b)

3. (10 points) An airport is developing a computer simulation of air-traffic control that handles events such as landings and takeoffs. Each event has a *time-stamp* that denotes the time when the event occurs. The simulation program needs to efficiently perform the following two fundamental operations:

- Insert an event with a given time-stamp (that is, add a future event)
- Extract the event with a smallest time-stamp (that is, determine the next event to process)

Which data structure should be used for the above operations? Why? Provide big-O asymptotic complexity for each operation.

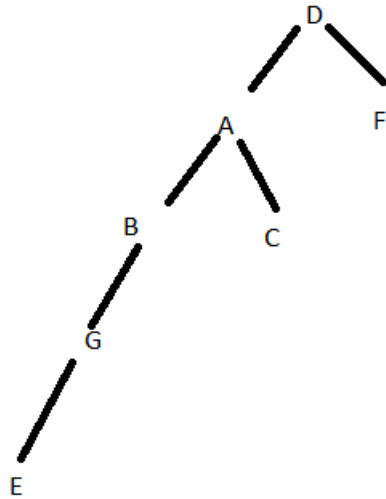
- (a) The data structure used best would be a binary search tree. Each level of the tree can be a time stamp and you insert and extract information with $O(\log n)$ time.

1. (15 points) Find the shortest path from D to all other vertices for the graph below.

(a) Illustrate the minimum priority queue at each iteration Dijkstra's algorithm.

i. D , A, C F B, G E

(b) Draw the Shortest Path Tree.



i.

(c) What is the running time of the Dijkstra's algorithm under the assumption that the graph is implemented based on an adjacency list and the minimum priority queue is implemented based on a binary heap?

i. $O(V + E)$

ii. V is vertices and E is edges

.../.../Downloads/FG_09_082.pdf

2. (15 points) Find the shortest unweighted path from D to all other vertices for the graph below. You can measure the distance from D by number of edges.

(a) Which graph algorithm can solve the problem?

i. Breadth-First algorithm

(b) Draw the Shortest Path Tree.

i. D To A : D->A

ii. D to B: D->A->B

iii. D to C: D->A->C

iv. D to G: D->A->B->G

v. D to E: D->A->C->E

vi. D to F: D->F

../../../../Downloads/FG_09_082.pdf

3. (10 points) Apply the Dijkstra's algorithm to find the shortest path from the vertex A to all the vertices in the graph below. Does the algorithm return a correct output? Justify your answer using the Dijkstra's Theorem.
- (a) First node A will be made visited out leave edges $(A,D) = 2$ and $(A,B) = 5$. Next it will $(D,B) = 2$ and it will make D visited. Now it will do $(B,C) = -2$ marking B as visited. Finally doing (C,D) as D is visited nothing will be done and C be mark visited
 - (b) A - length = 0
 - (c) B - length = 4
 - (d) C - length = 3
 - (e) D - length = 2
4. For this graph case it does return the correct output, but say (A,B) was 3 then this algorithm would fail to work and get the shortest path wrong.

../../Downloads/negative_weight.pdf

5. (20 points) There are eight small island in a lake, and the state wants to build seven bridges to connect them so that each island can be reached from any other one via one or more bridges. The cost of bridge construction is proportional to its length. The distance between pairs of islands are given in the following table.

- (a) Illustrate the Prim's algorithm using the graph below. Draw the Minimum Spanning Tree. What is the length of the bridges?

	1	2	3	4	5	6	7	8
1	-	240	210	340	280	200	345	120
2	-	-	265	175	215	180	185	155
3	-	-	-	260	115	350	435	195
4	-	-	-	-	160	330	295	230
5	-	-	-	-	-	360	400	170
6	-	-	-	-	-	-	175	205
7	-	-	-	-	-	-	-	305
8	-	-	-	-	-	-	-	-

i. 8-1: 120

ii. 8-2: 155

iii. 8-5: 170

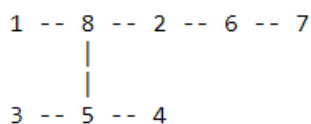
iv. 5-3: 115

v. 5-4: 160

vi. 2-6: 180

vii. 6-7: 175

Total is 1075



1. Illustrate the Kruskal's algorithm using the graph below. Draw the Minimum Spanning Tree. What is the length of the bridges?

	1	2	3	4	5	6	7	8
1	-	240	210	340	280	200	345	120
2	-	-	265	175	215	180	185	155
3	-	-	-	260	115	350	435	195
4	-	-	-	-	160	330	295	230
5	-	-	-	-	-	360	400	170
6	-	-	-	-	-	-	175	205
7	-	-	-	-	-	-	-	305
8	-	-	-	-	-	-	-	-

(a) 3-5: 115

(b) 1-8: 120

(c) 2-8: 155

(d) 4-5: 160

(e) 5-8: 160

(f) 6-7: 175

(g) 2-6: 180

Total 1075

1 -- 8 -- 2 -- 6 -- 7
|
3 -- 5 -- 4