Mathematics: Applications and interpretation HL / SL **Mathematics: Analysis and approaches HL / SL** **Exploration—initial planning**	**Form A**

This form should be completed

and returned to your teacher by**: Monday, August 28th 8:00am**

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1. Topic

If one of n stickers comes out randomly when buying a Pokemon bread, and I want to collect all n stickers, at least how many breads should I buy?

- Markov's inequality
- linearity of expectation
- briefly derivations
- Coupon collector's problem

2. Area of interest

Since the start of 2023, Pokemon Bread was one of the hottest issue due to the ssil/sticker(띠부띠부씰) inside the bread packet. Although it was popular, the bread packet wasn't able to be seen easily in convenient stores since the supply was way lower compared to the demand.

The main reason it was popular was not the bread themselves but the stickers inside. Everyone, including myself, desired to collect all the stickers. In total, there are 160 types of stickers that have different Pokemons drawn on each one.

3. Reason for choice

Because I was one of the people who wanted to collect all stickers, I chose to mathematically model how many Pokemon bread I need to purchase in minimum in order to collect all 160 stickers. (Although this will be impossible in real life)

It was interesting that it is easy to collect new stickers when I first buy Pokemon breads, but as I collect more stickers, the possibility that I will gain a new type of sticker will decrease. However, at last, the number of Pokemon breads I must buy will converge to a specific number.

4. Aim of the exploration: The exploration should meet its aim.

a. Calculate the minimum number of Pokemon bread that needs to be purchased in order to collect all 160 stickers and calculate the price (money needed)
b. Derive the most economic way to collect all stickers by comparing 3 methods:
    i. Buy Pokemon bread
    ii. Buy Pokemon bread for some but purchase stickers independently online
    iii. Purchase all stickers online
c. Obtain a more realistic scope of collecting stickers by assuming that the probability of each sticker showing up may not be uniform

5. Overall plan

Methodology / Mathematical Approaches

Try to be creative with approaches and put the details that can drive your exploration forward.

<table>
<tr><td>Approach Idea #1</td><td>Approach Idea #2</td></tr>
<tr><td>Probability and Expected values
https://www.youtube.com/watch?v=3mu47FWEuqA - The Coupon Collector Problem
This is the equation that will be used in order to find the expected value of breads I must buy to collect all 160 types of stickers
$P_i = \frac{n-(i-1)}{n} = \frac{n-i+1}{n}$
$E[X_i] = \frac{1}{P_i} = \frac{n}{n-i+1}$
$E[X] = E(x_1 + x_2 + \ldots + x_n)$
$= E(x_1) + E(x_2) + \ldots + E(x_n)$
$= \frac{1}{p_1} + \frac{1}{p_2} + \ldots \frac{1}{p_n} = \sum_{k=1}^{n} \frac{n}{n-k+1}$
$= \frac{n}{n} + \frac{n}{n-1} + \ldots + \frac{n}{1} = n \sum_{k=1}^{n} \frac{1}{k}$
$P(x \geq a) \leq \frac{E[X]}{a}$
Here, P_i is the probability of an event ‘i’ occurring out of ‘n’ possible events. ‘i’ is assumed as an integer of 1<i<n.
It shows the probability distribution for a uniformly distributed discrete random variable. Since I assume that each sticker has the same probability of obtaining it through buying any Pokemon bread, I can use this equation based on the assumption.
This equation is derived from the idea that the sum of outcomes is always equal to 1. This sum is equally distributed among all outcomes.
Expected value of a random variable shows the average value I would obtain if I conduct the same experiment several times. The second equation shows the relationship between expected value and random variables.</td><td>Instead of buying bread, what if I purchase the stickers separately online → what would be the maximum number of stickers to buy in order to be economical?
https://taeyeobv.tistory.com/entry/%EC%83%88%EB%A1%9C%EB%82%98%EC%98%A8-%ED%8F%AC%EC%BC%93%EB%AA%AC-%EB%9D%A0%EB%B6%80%EB%9D%A0%EB%B6%80%EC%94%B0-%EC%8B%9C%EC%84%B8%EB%8A%94-%EC%96%BC%EB%A7%88%EC%9D%BC%EA%B9%8C%EC%98%A4%EB%A5%98%EC%94%B0-%ED%8F%AC%ED%95%A8
This website shows the current price of each sticker. Although the price of stickers are different, I will calculate the average price.
$Average\ price = \frac{sum\ of\ all\ price}{total\ number\ of\ stickers\ (160)}$
By combining with approach 1, I will find the most economical method.
→ how many pieces of bread should I buy and how many stickers should I buy separately to spend minimal money in collecting all stickers?</td></tr>
<tr><td>Approach Idea #3</td><td>Approach Idea #4</td></tr>
</table>

When I assume that the probability of stickers coming out aren't uniform?

According to 'experts' who 'professionally' collect stickers and post articles on the internet (just like one in approach 2), some stickers are more rare and hard to obtain. Based on the assumption that stickers with higher price have greater rarity, I will calculate the probability independently.

More rare stickers will be harder to obtain, which means that I will have to buy more Pokemon bread in order to get that sticker. As seen in approach 1 and 2, there are 2 methods to gain the sticker.

a. obtain rare sticker by purchasing more Pokemon bread
b. obtain the rare sticker by purchasing the online selling stickers.

However, we should also note that there is a great possibility that expensive, rare stickers have a higher probability of fraud. For example, if I buy a sticker online and that might have a deficiency (being not complete etc.)

Therefore, my method will be seen as below:

1. obtain rare sticker by purchasing more pokemon bread
2. obtain rare stickers by purchasing online stickers, which has the probability of "X" getting fraud stickers. → may have to buy another one

By comparing the price of these two methods, I will derive the minimal money needed.

This will allow me to gain a more realistic scope about collecting stickers in an economical way.

What if there aren't any breads in the convenient store?
(Approach 1 + the probability of Pokemon bread being in the convenient store)

The convenience stores sometimes have Pokemon bread, but often, all of them are sold out.

The probability of Pokemon bread being absent in the convenience store will be obtained by surveying 5 convenience stores closest to my house.

Probability will be calculated by:

1. visit each store 1 time a day in a set time
2. check the presence of pokemon bread
3. repeat 10 times and calculate the probability by:

$$P_c = \frac{number\ of\ occasions\ when\ the\ Pokemon\ bread\ was\ present}{10}$$

This method will be effective when the probability of obtaining bread in the store is only high enough. consider the effort I have to put in (if I fail, I am wasting my time)

Then, calculate the actual probability of getting all the stickers.

6. Details of stimulus such as ideas, research, experiment, material and/or data collection.

If you require us to use data, please attach the raw data to your proposal.

Included in the approach part

7. School library resources to be used.

Not needed