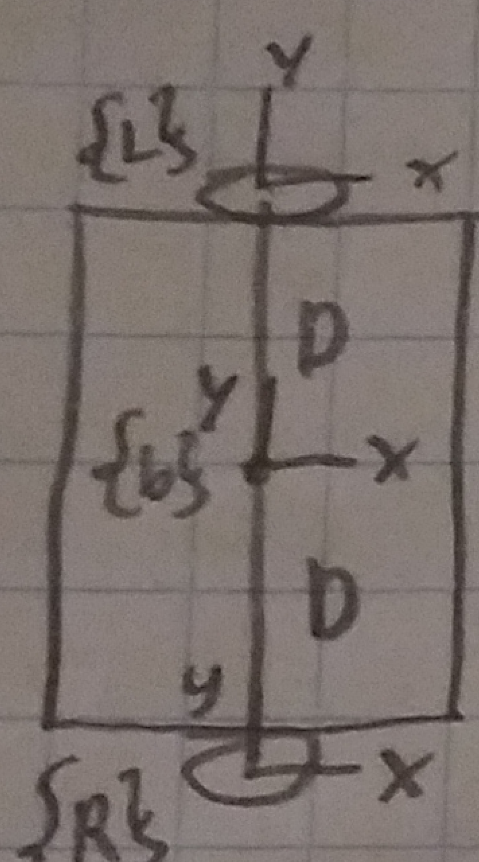
This document is a summary of forward and inverse kinematics for the differtial drive robot.

Inverse Kinematics: $V_b \rightarrow \dot{\phi}_L, \dot{\phi}_R$

Let the robot be



where D = wheel_track / 2
r = wheel_radius

We can use the adjoints A_{LB} and A_{RB} to convert V_b to V_L, V_R

$$(1)\ V_L = A_{Lb} V_b \Rightarrow \begin{bmatrix} \dot{\theta}_L \\ V_{xL} \\ V_{yL} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_b \\ V_{xb} \\ V_{yb} \end{bmatrix} \Rightarrow \begin{matrix} \dot{\theta}_L = \dot{\theta}_b \\ V_{xL} = -D\dot{\theta}_b + V_{xb} \\ V_{yL} = V_{yb} \end{matrix}$$

$$(2)\ V_R = A_{Rb} V_b \Rightarrow \begin{bmatrix} \dot{\theta}_R \\ V_{xR} \\ V_{yR} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_b \\ V_{xb} \\ V_{yb} \end{bmatrix} \Rightarrow \begin{matrix} \dot{\theta}_R = \dot{\theta}_b \\ V_{xR} = D\dot{\theta}_b + V_{xb} \\ V_{yR} = V_{yb} \end{matrix}$$

We can relate V_L, V_R to $\dot{\phi}_L, \dot{\phi}_R$

(3) $V_{xL} = r\dot{\phi}_L$; $V_{yL} = 0$ ← we have conventional wheels, thus a twist with V_{yi} component is Invalid

(4) $V_{xR} = r\dot{\phi}_R$; $V_{yR} = 0$ ←

Combine eq (1) + (3) and (2) + (4) to get:

$$\boxed{(5)\ \dot{\phi}_L = (-D\dot{\theta}_b + V_{xb})/r \qquad (6)\ \dot{\phi}_R = (D\dot{\theta}_b + V_{xb})/r}$$

Forward Kinematics $\dot{\phi}_L, \dot{\phi}_R \rightarrow V_b$

for inverse kinematics, an H matrix can be used to calculate $\dot{\phi}_L, \dot{\phi}_R$ from V_b

$$\begin{bmatrix} \dot{\phi}_L \\ \dot{\phi}_R \end{bmatrix} = HV_b \text{ , where } H = \frac{1}{r}\begin{bmatrix} -D & 1 & 0 \\ D & 1 & 0 \end{bmatrix}$$

We can find V_b but calculating H^{+}, the Moore-Penrose Pseudoinverse

$$V_b = H^{+}u \text{ where } H^{+} = \frac{r}{2}\begin{bmatrix} -1/D & 1/D \\ 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{\phi}_L \\ \dot{\phi}_R \end{bmatrix}$$

This results in

(7) $\dot{\theta}_b = r/2D\,(\dot{\phi}_R - \dot{\phi}_L)$

(8) $v_{xb} = r/2\,(\dot{\phi}_R + \dot{\phi}_L)$

We don't care for v_{yb} since none of velocities of conventional wheels will contribute to v_{yb}.

To apply forward kinematics, we can apply the adjoint between the body and world frame

(9) $V_w = Adj_{wb}(\theta_b, 0, 0)V_b$

(10) $x_w \mathrel{+}= v_{xw}$, $y_w \mathrel{+}= v_{yw}$, $\theta_w \mathrel{+}= \dot{\theta}_w$