

## Step-1

Consider the following matrix:

$$A = QR$$

Then,  $A^T A$  can be written as,

$$\begin{aligned} A^T A &= (QR)^T \cdot (QR) \\ &= R^T Q^T QR \end{aligned}$$

Since  $Q$  is an orthogonal matrix, then  $Q^T Q = I$ .

$$\begin{aligned} A^T A &= R^T Q^T QR \\ &= R^T R \end{aligned}$$

## Step-2

Thus, the projection  $P$  onto the column space of  $A$  is given by,

$$P = A(A^T A)^{-1} A^T \quad (1)$$

Substitute  $A = QR$  and  $A^T A = R^T R$  in (1), to get

$$\begin{aligned} P &= (QR)(R^T R)^{-1} (QR)^T \\ &= QRR^{-1} (R^T)^{-1} R^T Q^T \\ &= QRR^{-1} IQ^T \\ &= QRR^{-1} Q^T \end{aligned}$$

This implies that,

$$\begin{aligned} P &= QIQ^T \\ &= QQ^T \end{aligned}$$

Hence, the formula for the projection matrix  $P$  is  $P = QQ^T$  provided that  $R$  is invertible matrix.