Step-1

We need to apply row operations to produce an upper triangular U to compute the determinants of the following matrices:

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 6 & 6 & 1 \\ -1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 7 \end{bmatrix} \text{ and } \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

Step-2

(a) So, first consider

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 6 & 6 & 1 \\ -1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 2 & 0 & 1 \\ -1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 7 \end{bmatrix} \leftarrow \text{subtractign 2 times the first row from the second row}$$

Step-3

$$= \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 2 & 3 & 3 \\ 0 & 2 & 0 & 7 \end{bmatrix} \leftarrow \text{Adding first row to the third}$$

$$= \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 3 & 2 \\ 0 & 2 & 0 & 7 \end{bmatrix} \leftarrow \text{Substracting the second row from the third row}$$

Step-4

$$= \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 6 \end{bmatrix} \leftarrow \text{Substracting the second row from the fourth row}$$

$$=(1)(2)(3)(6)$$

$$=36$$

Note: if A is triangular, then det A is the product $a_{11}a_{22}...a_{nn}$ of the diagonal entries.

Step-5

(b) Now consider

$$\begin{vmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{vmatrix}$$

$$=\begin{vmatrix} 2 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 1 & 2 \end{vmatrix} \leftarrow \text{substracting the fourth row from the second and third rows}$$

Step-6

$$=\begin{vmatrix} 2 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & \frac{1}{2} & \frac{1}{2} & \frac{3}{2} \end{vmatrix} \leftarrow \text{adding } -\frac{1}{2} \text{ times the first row to the fourth.}$$

$$=\begin{vmatrix} 2 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & \frac{1}{2} & 2 \end{vmatrix} \leftarrow \text{adding } -\frac{1}{2} \text{ times the second row to the fourth.}$$

Step-7

$$=\begin{vmatrix} 2 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & \frac{5}{2} \end{vmatrix} \leftarrow \text{adding } -\frac{1}{2} \text{ times the third row to the fourth.}$$

$$= (2)(1)(1)\left(\frac{5}{2}\right)$$

$$= 5$$

Step-8

Note: if A is triangular, then det A is the product $a_{11}, a_{22} \ldots\ldots a_{nn}$ of the diagonal entries.

Step-9

Thus, $$\boxed{\det \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 6 & 6 & 1 \\ -1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 7 \end{bmatrix} = 36}$$

And

$$\boxed{\det \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} = 5}$$