Step-1

Consider the function,

$$F_1(x,y) = \frac{1}{4}x^4 + x^2y + y^2$$

And,

$$F_2(x,y) = x^3 + xy - x$$

Objective is to determine the second derivative matrices A_1 and A_2.

So, first consider,

$$F_1(x,y) = \frac{1}{4}x^4 + x^2y + y^2$$

Take the partial derivatives,

$$\frac{\partial F_1(x,y)}{\partial x} = x^3 + 2xy$$

$$\frac{\partial^2 F_1(x,y)}{\partial x^2} = 3x^2 + 2y$$

And,

$$\frac{\partial^2 F_1(x,y)}{\partial x \partial y} = 2x$$

Next consider,

$$\frac{\partial F_1(x,y)}{\partial y} = x^2 + 2y$$

$$\frac{\partial^2 F_1(x,y)}{\partial y^2} = 2$$

And,

$$\frac{\partial^2 F_1(x,y)}{\partial y \partial x} = 2x$$

Step-2

The second derivative matrix A_1 is calculated as follows:

$$A_1 = \begin{bmatrix} \frac{\partial^2 F_1(x,y)}{\partial x^2} & \frac{\partial^2 F_1(x,y)}{\partial x \partial y} \\ \frac{\partial^2 F_1(x,y)}{\partial y \partial x} & \frac{\partial^2 F_1(x,y)}{\partial y^2} \end{bmatrix}$$

On substitution $\frac{\partial^2 F_1(x,y)}{\partial x^2}, \frac{\partial^2 F_1(x,y)}{\partial x \partial y}, \frac{\partial^2 F_1(x,y)}{\partial y \partial x}$ and $\frac{\partial^2 F_1(x,y)}{\partial y^2}$ in the above values the matrix A_1 becomes,

$$A_1 = \begin{bmatrix} 3x^2 + 2y & 2x \\ 2x & 2 \end{bmatrix}$$

Step-3

Next, consider the function,

$$F_2(x,y) = x^3 + xy - x$$

Take the partial derivatives of F_2,

$$\frac{\partial F_2(x,y)}{\partial x} = 3x^2 + y - 1$$
$$\frac{\partial^2 F_2(x,y)}{\partial x^2} = 6x$$

And,

$$\frac{\partial^2 F_1(x,y)}{\partial x \partial y} = 1$$

Also,

$$\frac{\partial F_2(x,y)}{\partial y} = x$$
$$\frac{\partial^2 F_2(x,y)}{\partial y^2} = 0$$

And,

$$\frac{\partial^2 F_2(x,y)}{\partial y \partial x} = 1$$

So, the second derivative matrix A_2 is calculated as,

$$A_2 = \begin{bmatrix} \dfrac{\partial^2 F_2(x,y)}{\partial x^2} & \dfrac{\partial^2 F_2(x,y)}{\partial x \partial y} \\ \dfrac{\partial^2 F_2(x,y)}{\partial y \partial x} & \dfrac{\partial^2 F_2(x,y)}{\partial y^2} \end{bmatrix}$$

On substitution,

$$A_2 = \begin{bmatrix} 6x & 1 \\ 1 & 0 \end{bmatrix}$$

Step-4

Next objective is to determine the minimum point of F_1 and the saddle point of F_2.

For this equate $\dfrac{\partial F_1(x,y)}{\partial x}$ to zero and $\dfrac{\partial F_1(x,y)}{\partial y}$ equal to zero. That is,

$\dfrac{\partial F_1(x,y)}{\partial x} = 0$ and $\dfrac{\partial F_1(x,y)}{\partial y} = 0$

This implies that,

$$x^3 + 2xy = 0$$

Step-5

And,

$$x^2 + 2y = 0$$

Solve these equations simultaneously to get $x = 0$ and $y = 0$ as the minimum point for F_1.

Step-6

Next, find the saddle point for F_2. Note that, a point is called a saddle point of a function of two variable if, $\dfrac{\partial f}{\partial x} = 0, \dfrac{\partial f}{\partial y} = 0$ and $\dfrac{\partial^2 f}{\partial x^2} \times \dfrac{\partial^2 f}{\partial y^2} - \left[\dfrac{\partial^2 f}{\partial x \partial y}\right]^2 < 0$ at that point. So, in order to determine the saddle point for F_2 consider,

$$\frac{\partial F_2(x,y)}{\partial x} = 3x^2 + y - 1 = 0$$
$$\frac{\partial F_2(x,y)}{\partial y} = x = 0$$

And,

$$\frac{\partial^2 F_2(x,y)}{\partial x^2} \times \frac{\partial^2 F_2(x,y)}{\partial y^2} - \left[\frac{\partial^2 F_1(x,y)}{\partial x \partial y}\right]^2 < 0$$

Solve this simultaneously to get $x = 0 \text{ and } y = 1$ as the required saddle point.

Hence, the minimum point for F_1 is $\boxed{x = 0 \text{ and } y = 0}$ and the saddle point for F_2 is $\boxed{x = 0 \text{ and } y = 1}$.