

## Step-1

Consider  $V$  be a vector space. A subspace is itself a vector space under the same operation as defined on the vector space  $V$ .

## Step-2

(a)

Consider the subspace of  $\mathbf{R}^3$ . Objective is to describe the subspace of  $\mathbf{R}^3$  spanned by the two vectors  $(1,1,-1)$  and  $(-1,-1,1)$ .

For this consider  $\alpha \in \mathbf{R}^3$  then,

$$\begin{aligned}\alpha &= a(1,1,-1) + b(-1,-1,1) \\ &= a-b, a-b, -a+b\end{aligned}$$

Thus, the spanning space is equal to  $\{(a-b), a-b, -a+b \mid a, b \in \mathbf{R}\}$ .

**This is a line  $\mathbf{R}^3$ .**

## Step-3

(b)

Consider the subspace of  $\mathbf{R}^3$ . Objective is to describe the subspace of  $\mathbf{R}^3$  spanned by the three vectors  $(0,1,1)$  and  $(1,1,0)$  and  $(0,0,0)$ .

**The subspace of  $\mathbf{R}^3$  spanned by the three vectors  $(0,1,1)$  and  $(1,1,0)$  and  $(0,0,0)$  is a plane in  $\mathbf{R}^3$ .**

## Step-4

(c)

Consider the subspace of  $\mathbf{R}^3$ . Objective is to describe the subspace of  $\mathbf{R}^3$  spanned by the columns of a 3 by 5 echelon matrix with 2 pivots.

This is a plane in  $\mathbf{R}^3$ .

## Step-5

(d)

Consider the subspace of  $\mathbf{R}^3$ . Objective is to describe the subspace of  $\mathbf{R}^3$  spanned by all vectors with positive component.

**The span of all vectors with positive component will generate  $\mathbf{R}^3$ .**