

## Step-1

4764-1.5-22E AID: 124

RID: 175 | 3/15/12

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ 11 \end{pmatrix}$$

We have the system  $Ax = b$  is

Applying the forward elimination method, we get  $Ux = c$  is

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ 2 \end{pmatrix}$$

To find the matrix  $L$ , we apply the reverse process to identity matrix  $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Adding row 1 to row 2 and adding row 1 to row 3

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

Adding 2 times row 2 to row 3 gives

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$$

## Step-2

$$Lc = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} 5 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ 21 \end{pmatrix}$$

Now

Since by matrix multiplication

$Lc = b$  solves for  $(5, 7, 21)$

## Step-3

Further,  $Ux = c = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ 2 \end{pmatrix}$  can be written as the system of equations from below as

$$z = 2$$

$$y + 2z = 2$$

$$x + y + z = 5$$

Consequently,  $z = 2, y = -2$  and  $x = 5$

So,  $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \\ 2 \end{pmatrix}$  satisfies the equation  $Ux = c$ .