

Step-1

Let z be half way between x and y .

We have to show that Az is halfway between Ax and Ay .

Step-2

Given that every straight line remains straight after a linear transformation.

Let A be a linear major matrix.

Since z is the halfway between x and y .

So

$$\begin{aligned} z &= \frac{x+y}{2} \\ &= \frac{x}{2} + \frac{y}{2} \end{aligned}$$

Step-3

Now

$$\begin{aligned} Az &= A\left(\frac{x+y}{2}\right) \\ &= A\left(\frac{x}{2} + \frac{y}{2}\right) \\ &= A\left(\frac{x}{2}\right) + A\left(\frac{y}{2}\right) \quad (\text{since } A \text{ linear transformation}) \\ &= \frac{1}{2}(Ax + Ay) \quad (\text{since } A \text{ linear transformation}) \end{aligned}$$

Hence Az is halfway between Ax and Ay .