Step-1

Big Formula

$$\det A = \sum_{all\ P's} \left(a_{1\alpha} a_{2\beta} \ldots a_{n\upsilon}\right) \det P$$

By using big formula, we get

$$\det\left(a_{ij}\right)_{3\times 3} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32} - a_{13}a_{22}a_{31}$$

Step-2

i) Considering $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

$$\det A = 1.0.1 + 1.1.0 + 0.1.1 - 1.1.1 - 1.1.1 - 0.0.0$$

$$= -1 - 1$$
$$= \boxed{-2}$$
$$\neq 0$$

So, columns of A are independent

Step-3

ii) Given $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

Computing det using big formula, we have $\det B = 1.5.9 + 2.6.7 + 3.4.8 - 1.6.8 - 2.4.9 - 3.7.5$

$$= 45 + 84 + 96 - 48 - 72 - 105$$

$$= \boxed{0}$$

Columns of B are dependent. Observe that second column is average of remaining columns.

So, we get $\det B = 0$

Step-4

iii) Given $C = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$

$$= \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 4 & 5 & 6 \\ 0 & 0 & 0 & 7 & 8 & 9 \end{bmatrix}$$

As columns of B are independent we get that columns of C are also dependent since 5^{th} column is average of 4^{th} and 6th columns of C.

Here we get $\det C = \boxed{0}$