

## Step-1

Suppose  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\det(A - \lambda I) = \begin{vmatrix} a - \lambda & b \\ c & d - \lambda \end{vmatrix} = 0$$

$$= (a - \lambda)(d - \lambda) - bc$$

$$= \lambda^2 - (a + d)\lambda + (ad - bc)$$

## Step-2

Given that  $|A - \lambda I| = \lambda^2 - 9\lambda + 20$  and  $a + d = 9, ad - bc = 20$

We get,  $a + d = 9, ad - bc = 20$

There can be infinitely many integers which satisfy these equations.

Let us consider the ordered pairs of values of  $a$  and  $d$

$(1, 8), (2, 7), (3, 6), \dots$  which are positive and integer choices.

When  $a = 1, d = 8$ , we have the choices for  $b$  and  $c$  obtained by  $bc = 12$

This gives  $b = -6, c = 2$  or  $b = -3, c = 4$  or  $b = -4, c = 3$  or  $b = 6, c = -2$

So, some possible choices of matrices are  $\begin{bmatrix} 1 & -4 \\ 3 & 8 \end{bmatrix}, \begin{bmatrix} 1 & 4 \\ -3 & 8 \end{bmatrix}, \begin{bmatrix} 1 & -6 \\ 2 & 8 \end{bmatrix}, \begin{bmatrix} 1 & 6 \\ -2 & 8 \end{bmatrix}$