Step-1

(a) Suppose the vectors w be the columns of A and consider $w_1 = (1,1,0)$, $w_2 = (2,2,1)$, $w_3 = (0,0,2)$, and $b = (3,4,5)$.

So we have,

$$Ax = b$$
$$\begin{bmatrix} 1 & 2 & 0 \\ 1 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

To solve for $Ax = b$, use Gaussian elimination.

$$\begin{bmatrix} 1 & 2 & 0 & | & 3 \\ 1 & 2 & 0 & | & 4 \\ 0 & 1 & 2 & | & 5 \end{bmatrix}$$

By using $R_2 \to R_2 - R_1$, we get:

$$\begin{bmatrix} 1 & 2 & 0 & | & 3 \\ 0 & 0 & 0 & | & 1 \\ 0 & 1 & 2 & | & 5 \end{bmatrix}$$

Second row represents the equation,

Step-2

$$0x_1 + 0x_2 + 0x_3 = 1$$

Step-3

By solving the equation $0x_1 + 0x_2 + 0x_3 = 1$, we get

$$0 = 1$$

As we know $0 \neq 1$, therefore, $Ax = b$ has $\boxed{\text{no solution}}$ and b $\boxed{\text{is not in it}}$.

Step-4

(b) Suppose the vectors w be the columns of A and consider $w_1 = (1,2,0)$, $w_2 = (2,5,1)0$,

Step-5

$w_3 = (0,0,2)$, and $w_4 = (0,0,0)$.

We know that the system of equation $Ax = b$ has a solution if and only if the vector *b* can be expressed as a combination of the columns of *A*. Then *b* is in the column space.

Let

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 5 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$

And

$$b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Step-6

To solve $Ax = b$, use Gaussian elimination.

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & b_1 \\ 2 & 5 & 0 & 0 & b_2 \\ 0 & 0 & 2 & 0 & b_3 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 5b_1 - 2b_2 \\ 0 & 1 & 0 & 0 & b_2 - 2b_1 \\ 0 & 0 & 1 & 0 & b_3/2 \end{array}\right]$$

Therefore, $\boxed{\text{yes there is a } b \text{ in it}}$.