

Step-1

Given that $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

We need to find out whether the given matrices have determinants 0, 1, 2, or 3.

Step-2

Consider

$$\det(A) = |A|$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix}$$

$$= - \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \text{ (Since determinant changes sign when two rows are exchanged)}$$

$$= 1 \text{ (Since the determinant of the identity matrix is 1)}$$

Thus, $\boxed{\det(A) = 1}$

Step-3

Consider

$$\det(B) = |B|$$

$$= \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

We find $\det(B)$ by cofactor expansion along the first row.

Therefore,

$$\begin{aligned}\det B &= \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} - \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} \\ &= (-1) - (1) \\ &= -2\end{aligned}$$

Thus, $\boxed{\det B = -2}$

Step-4

Consider

$$\begin{aligned}\det C &= |C| \\ &= \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}\end{aligned}$$

$= 0$ (Since the rows of the matrix C are identical)

Thus, $\boxed{\det C = 0}$

Thus, out of the 3 matrices A , B and C , only A and C have the determinants 1 and 0.