

## Step-1

a)  $A$  is a square matrix, let  $A$  be an  $n$  by  $n$  matrix.

Suppose  $x \in N(A)$

Then we follow that  $Ax = 0$

Applying  $A$  on both sides, we get  $A^2x = 0$

So, it follows that  $x$  is in the null space of  $A^2$

Therefore, every vector in the null space of  $A$  is also in the null space of  $A^2$

In other words, null space of  $A^2$  contains the null space of  $A$ .

## Step-2

b) Suppose  $x$  is in the column space of  $A^2$

Then we follow that there exists a non zero vector  $b$  such that  $A^2x = b$

Applying  $A^{-1}$  on both sides, we get  $Ax = A^{-1}b$

Writing  $A^{-1}b = c$ , we are left with  $Ax = c$  where  $c$  is a non zero vector.

From this, we can say that  $x$  is in the column space of  $A$ .

Therefore, every vector in the column space of  $A^2$  is also in the column space of  $A$ .

In other words, the column space of  $A^2$  is contained in the column space of  $A$ .