Step-1

Subtract row 1 from rows 2, 3, and 4. This gives:

$$\begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & b-a & c-a & c-a \\ 0 & b-a & c-a & d-a \end{bmatrix}$$

Step-2

Subtract row 2 from rows 3 and 4. This gives:

$$\begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & c-b & d-b \end{bmatrix}$$

Step-3

Subtract row 3 from row 4. This gives:

$$\begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & 0 & d-c \end{bmatrix}$$

This is an upper triangular matrix.

Step-4

Note that

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & 0 & d-c \end{bmatrix}=\begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}$$

Thus, we have $A = LU$, where $\boxed{L=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}}$ and $\boxed{U=\begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & 0 & d-c \end{bmatrix}}$.

Step-5

Now rank of A is equal to rank of $\begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & 0 & d-c \end{bmatrix}$. If $b-a=0$, then the second column will be identical with the first column. If $c-b=0$, then the third column will be identical with the second column. If $d-c=0$, then the fourth column will be identical with the third column.

We want all the columns independent. Thus, following conditions should be satisfied:

$$b-a \neq 0$$
$$c-b \neq 0$$
$$d-c \neq 0$$

Therefore, the columns of A will be independent if $\boxed{b-a \neq 0}$, $\boxed{c-b \neq 0}$, and $\boxed{d-c \neq 0}$. This is same as $\boxed{b \neq a}$, $\boxed{c \neq b}$, and $\boxed{d \neq c}$.