Step-1

A matrix N is normal if it commutes with N^H.

i.e., if $NN^H = N^H N$, then N is normal.

Suppose N is a matrix with ortho normal eigen vectors.

So, we have $U^{-1}NU = \Lambda$

Or, $N = U\Lambda U^{-1}$

To show N commutes with itself, let us consider $NN^H = \left(U\Lambda U^{-1}\right)\left(U\Lambda U^{-1}\right)^H$

$=\left(U\Lambda U^{-1}\right)\left(\left(U^{-1}\right)^H \Lambda^H U^H\right)$

$=U\Lambda\left(\left(U^{-1}\right)\left(U^{-1}\right)^H\right)\Lambda^H U^H$ â€¦â€¦ (1)

Since U is the unitary matrix, we have $U^H = U^{-1}$ and so,

$\left(U^{-1}\right)\left(U^{-1}\right)^H = U^{-1}U = I$ â€¦â€¦ (2)

$= UU^{-1}$

$= \left(U^{-1}\right)^H\left(U^{-1}\right)$ â€¦â€¦ (3)

Step-2

Using (2) in (1), we get $NN^H = U\left(\Lambda\Lambda^H\right)U^H$

Since Λ is a diagonal matrix, we follow that $\Lambda\Lambda^H = \Lambda^H\Lambda$

So, $NN^H = U\left(\Lambda^H I\Lambda\right)U^H$

$$=U\Lambda^H\left(U^{-1}U\right)\Lambda U^H$$
$$=\left(\left(U^H\right)^H\Lambda^H U^{-1}\right)\left(U\Lambda U^H\right)$$
$$=\left(\left(U^H\right)^H\Lambda^H U^H\right)\left(U\Lambda U^H\right)$$
$$=\left(U\Lambda U^H\right)^H\left(U\Lambda U^H\right)$$
$$=N^H N$$

Therefore, N is a normal matrix.