Step-1

Projection onto a line:

If a is a vector, then every point on a is a multiple of *a*.

So, the projection of a vector b onto a is $p = \hat{x}a$ such that the line from b to the closest point $p = \hat{x}a$ is the perpendicular to a.

It is given by;

$$
\begin{aligned}
p &= \hat{x}a \\
&= \frac{a^T b}{a^T a} a
\end{aligned}
$$

Step-2

Given that q_1, q_2 and q_3 are orthonormal.

So, it follows that

$$
\begin{aligned}
q_1^T q_2 &= q_1^T q_3 \\
&= q_2^T q_3 \\
&= 0 \qquad \text{â€¦â€¦ (1)}
\end{aligned}
$$

Also,

$$
\begin{aligned}
q_1^T q_1 &= q_2^T q_2 \\
&= q_3^T q_3 \\
&= 1
\end{aligned}
$$

Suppose $xq_1 + yq_2$ is the combination closest to q_3.

Then by the above definition, it follows that $xq_1 + yq_2$ is perpendicular to q_3

By (1), obtain $xq_1 + yq_2 = 0$

Thus, the only vector which is a linear combination of q_1, q_2 and perpendicular to q_3 is $\boxed{\text{zero.}}$

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