

Step-1

(a)

Consider the following vectors:

$$(1,3,2), (2,1,3), (3,2,1)$$

The objective is to determine whether the above set of vectors is dependent (or) independent.

Step-2

To prove the dependency of vectors, write the above vectors as the columns of a matrix as follows:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -7 \\ 0 & -1 & -5 \end{bmatrix} \text{ (By row operations } R_2 - 3R_1 \rightarrow R_2, R_3 - 2R_1 \rightarrow R_3 \text{)}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -5 \\ 0 & -5 & -7 \end{bmatrix} \text{ (By row operation } R_2 \leftrightarrow R_3 \text{)}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -5 \\ 0 & 0 & 18 \end{bmatrix} \text{ (By row operation } R_3 - 5R_2 \rightarrow R_3 \text{)}$$

In the above row echelon form of the matrix, all rows are non-zero.

Therefore, the vectors $(1,3,2), (2,1,3), (3,2,1)$ form linearly independent set.

Step-3

(b)

Consider the following vectors:

$$(1,-3,2), (2,1,-3), (-3,2,1)$$

The objective is to determine whether the above set of vectors is dependent (or) independent.

Step-4

To prove the dependency of vectors, write the above vectors as the columns of a matrix as follows:

$$A = \begin{bmatrix} 1 & 2 & -3 \\ -3 & 1 & 2 \\ 2 & -3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & -3 \\ 0 & 7 & -7 \\ 0 & -7 & 7 \end{bmatrix} \quad (\text{By row operations } R_2 + 3R_1 \rightarrow R_2, R_3 - 2R_1 \rightarrow R_3)$$

$$= \begin{bmatrix} 1 & 2 & -3 \\ 0 & 7 & -7 \\ 0 & 0 & 0 \end{bmatrix} \quad (\text{By row operation } R_3 + R_2 \rightarrow R_3)$$

In the above row echelon form of the matrix, only two rows are non-zero (the last row is zero).

Therefore, the vectors $(1, -3, 2), (2, 1, -3), (-3, 2, 1)$ form linearly dependent set.