

## Step-1

Let us consider the group of  $n \times n$  permutation matrices. Every matrix has  $n$  rows and  $n$  columns. Therefore, for the first row, we have  $n$  choices for the number 1. Once 1 is fixed, all other places are to be filled by 0's.

Then for the second row, we have  $n-1$  choices, for the third row, we have  $n-2$  choices and so on! Finally, for the last row we have only one choice.

Thus, in all there are  $n!$  distinct permutation matrices possible.

## Step-2

(b) From the above discussion, it is clear that there are  $3! = 6$  permutation matrices in the group of  $3 \times 3$  permutation matrices. Call this group as  $G$ .

Let  $P$  be any  $3 \times 3$  permutation matrix.

Consider  $P, P^2, P^3, \dots$  and so on!

It is clear that all these elements belong to  $G$ .

## Step-3

We have shown that  $G$  contains six elements. Therefore, the elements  $P, P^2, P^3, \dots$  cannot be all distinct.

Further, it should be clear that  $P^6 = I$ , for any  $P \in G$ .

## Step-4

Therefore, we have  $k=6$ , so that  $P^k = I$ .