## Step-1

Given the matrix $A=\begin{bmatrix}2 & -1\\ -1 & 2\end{bmatrix}$, with Eigen values $\lambda_1=1$ and $\lambda_2=3$ and the inverse of this matrix $A^{-1}=\frac{1}{3}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}$. The initial guess is $u_0=\begin{bmatrix}3\\ 4\end{bmatrix}$

We need to compare three inverse power steps and need to find the limiting vector.

The inverse power method is $u_{k+1}=A^{-1}u_k$,

When $k=0$ then,

$$\begin{aligned}u_1 &= A^{-1}u_0\\ &=\frac{1}{3}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}3\\ 4\end{bmatrix}\\ &=\frac{1}{3}\begin{bmatrix}10\\ 11\end{bmatrix}\\ &=\begin{bmatrix}3.333\\ 3.667\end{bmatrix}\end{aligned}$$

Therefore, $\boxed{u_1=\begin{bmatrix}3.333\\ 3.667\end{bmatrix}}$.

## Step-2

When $k=1$ then,

$$\begin{aligned}u_2 &= A^{-1}u_1\\ &=\frac{1}{3}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}3.333\\ 3.667\end{bmatrix}\\ &=\begin{bmatrix}3.444\\ 3.556\end{bmatrix}\end{aligned}$$

Therefore, $\boxed{u_2=\begin{bmatrix}3.444\\ 3.556\end{bmatrix}}$.

## Step-3

When $k=2$ then,

$$u_3 = A^{-1}u_2$$
$$= \frac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 3.444 \\ 3.556 \end{bmatrix}$$
$$= \begin{bmatrix} 3.481 \\ 3.518 \end{bmatrix}$$

Therefore, $\boxed{u_3 = \begin{bmatrix} 3.481 \\ 3.518 \end{bmatrix}}$.

## Step-4

We follow that (1, 1) and (1, 2) entries of $u_k$ approaches equally to 3.5 as $k$ becoming alarmingly high and tends to infinity.

So,

$$\lim_{k\to\infty} u_k = \begin{bmatrix} 3.5 \\ 3.5 \end{bmatrix}$$
$$= 3.5\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Therefore, the multiple of the Eigen vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is $\boxed{3.5}$.