

Step-1

Given quadratic is $z = 4x^2 + 12xy + cy^2$.

Comparing with $ax^2 + 2bxy + cy^2$,

So, $a = 4$, $2b = 12$, $c = c$.

For saddle point the condition is,

$$ac - b^2 < 0$$

$$\Rightarrow (4)(c) - (6)^2 < 0$$

$$\Rightarrow 4c - 36 < 0$$

$$\Rightarrow c < 9$$

Thus if $\boxed{c > 9}$, $4x^2 + 12xy + cy^2$ is positive definite and hence graph of z is a bowl.

Step-2

If $c < 9$,

$$z = 4x^2 + 12xy + cy^2$$

$$= (2x + 3y)^2$$

Thus if $\boxed{c = 9}$, then the graph of z is a trough staying at zero on this line $2x + 3y = 0$.