

Step-1

Let λ be an eigenvalue of the matrix A and x be the respective eigenvector of A .

Then we have $Ax = \lambda x$ (1)

We have to show that $|\lambda| \leq \|A\|$

Step-2

By the definition of the norm of a matrix whether symmetric or not, we have $\|A\| = \max \frac{\|Ax\|}{\|x\|}$

$$\Rightarrow \|A\| \geq \frac{\|Ax\|}{\|x\|}$$

$$\Rightarrow \|A\| \geq \frac{\|\lambda x\|}{\|x\|} \quad (\text{Since by (1)})$$

Since λ is a scalar, we follow that $\|\lambda x\| = |\lambda| \|x\|$

Using this in the above inequality, we get $\|A\| \geq \frac{|\lambda| \|x\|}{\|x\|}$

$$\Rightarrow |\lambda| \leq \|A\|$$

Hence $\boxed{|\lambda| \leq \|A\|}$, where λ is an eigenvalue of A .