

Step-1

To find all solutions to the equation $e^{ix} = -1$, and all solutions to the equation $e^{i\theta} = i$

It is known that;

$$e^{i\theta} = \cos \theta + i \sin \theta$$

Where, θ is arbitrary angle;

$$\cos \theta = -1 \text{ For } \theta = (2k+1)\pi$$

$$\sin \theta = 1 \text{ For } \theta = (2k+1)\frac{\pi}{2}, \text{ where } k \text{ is even that is } k = 0, 2, 4, \dots$$

Step-2

Now, consider the equation $e^{ix} = -1$

This implies;

$$e^{ix} = \cos(2k+1)\pi + i \sin(2k+1)\pi \text{ for } k = 0, 1, 2, \dots$$

$$e^{ix} = e^{i(2k+1)\pi} \text{ for } k = 0, 1, 2, \dots$$

$$ix = i(2k+1)\pi \text{ or } k = 0, 1, 2, \dots$$

This implies;

$$x = (2k+1)\pi \text{ for } k = 0, 1, 2, \dots$$

Step-3

Now, consider another equation $e^{i\theta} = i$

This implies;

$$e^{i\theta} = 0 + 1i$$

$$e^{i\theta} = \cos(2k+1)\frac{\pi}{2} + i \sin(2k+1)\frac{\pi}{2} \text{ for } k = 0, 2, 4, \dots$$

$$e^{i\theta} = e^{i(2k+1)\frac{\pi}{2}} \text{ for } k = 0, 2, 4, \dots$$

$$i\theta = i(2k+1)\frac{\pi}{2} \text{ or } k = 0, 2, 4, \dots$$

This implies;

$$\theta = (2k+1)\frac{\pi}{2} \text{ for } k = 0, 2, 4, \dots$$