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二、

D A B

三、

(1) $x-y+2z=\pm 3$ (2) $dx-dy$ (3) $y+z=3$ (4) 0

解:

设 $x-y+2z=a$.

$\nabla f = (2x,-4y,2z)$

$\vec{n}=(1,-1,2)$

$$\begin{cases}2cx=1\\ -4cy=-1\\ 2cz=2\end{cases} \Rightarrow \begin{cases}x=\frac{1}{2c}\\ y=\frac{1}{4c}\\ z=\frac{1}{c}\end{cases}$$

$x^2-2y^2+z^2=2$

$\therefore \frac{9}{8c^2}=2 \quad c=\pm\frac{3}{4}$

代入 $x-y+2z$ 解得 $a=\pm 3$

四、 $3\sin\theta = 1+\sin\theta \quad \therefore \sin\theta=\frac{1}{2} \quad \theta=\frac{\pi}{6} \quad \theta=\frac{5\pi}{6}$

$$A=\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{1}{2}(r_1^2-r_2^2)\,d\theta = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{1}{2}\left((3\sin\theta)^2-(1+\sin\theta)^2\right)d\theta=\pi$$

五、 $\int |\vec{v}|\,dt$

$\vec{r}(t)=(12\sin t)\vec{i}-(12\cos t)\vec{j}+5t\vec{k}$

$\vec{v}(t)=12\cos t\,\vec{i}+12\sin t\,\vec{j}+5\vec{k}$

$|\vec{v}(t)|=13$

$A(0,-12,0) \Rightarrow t=0$

$\left|\int_0^x 13\,dt\right|=26\pi \Rightarrow x=\pm 2\pi$

$\therefore (0,-12,\pm 10\pi)$

六、 $\lim\limits_{n\to\infty}\sqrt[n]{|a_n|}=|2x|<1 \quad x\in\left(-\frac{1}{2},\frac{1}{2}\right)$

① $x=-\frac{1}{2} \quad a_n=\frac{(-1)^n}{\ln(n+2)} \quad \because \lim\limits_{n\to\infty}\frac{1}{\ln(n+2)}=0$ 且 $\frac{1}{\ln(n+2)}\geq\frac{1}{\ln(n+2+1)}$

$\therefore \sum\limits_{n=0}^{\infty}\frac{(-1)^n}{\ln(n+2)}$ 收敛 $\quad |a_n|=\frac{1}{\ln(n+2)}>\frac{1}{n} \quad \therefore \sum\limits_{n=0}^{\infty}\frac{(-1)^n}{\ln(n+2)}$ 条件收敛.

② $x=\frac{1}{2} \quad a_n=\frac{1}{\ln(n+2)} \quad \sum\limits_{n=0}^{\infty}\frac{1}{\ln(n+2)}$ 发散

$\therefore \left[-\frac{1}{2},\frac{1}{2}\right)$

七、 用泰勒展开和二项级数定理分别展开 $\cos(\sin x)$ 与 $\sqrt{1-x^2}$.

$\cos(\sin x)-\sqrt{1-x^2}=\frac{1}{3}x^4+o(x^5)$ （注 若令 $\cos(\sin x)=1$ 则会丢掉一些东西）是错的.

$\therefore\ a=4\quad b=\frac{1}{3}$

八、内部: $f(x,y)=e^{-x^2-y^2}(x^2+2y^2)$

$$\begin{cases}\frac{\partial f}{\partial x}=2xe^{-x^2-y^2}(1-x^2-2y^2)=0\\ \frac{\partial f}{\partial y}=2ye^{-x^2-y^2}(2-x^2-2y^2)=0\end{cases}$$

$$\begin{cases}x=0\\y=0\end{cases}\quad\begin{cases}x=0\\y=\pm1\end{cases}\quad\begin{cases}x=\pm1\\y=0\end{cases}$$

外圈: $x^2+y^2=4\quad f(x,y)=e^{-4}(4+y^2)$

$\therefore (\pm2,0)\quad(0,\pm2)$

$f(0,0)=0\quad f(0,\pm1)=2e^{-1}$

$f(\pm1,0)=e^{-1}\quad f(\pm2,0)=4e^{-4}$

$f(0,\pm2)=8e^{-4}\quad f_{max}=2e^{-1}$

$f_{min}=0.$

九.

$z=1\quad z=\sqrt{x^2+y^2}.$

$x=\rho\sin\varphi\cos\theta$

$y=\rho\sin\varphi\sin\theta$

$z=\rho\cos\varphi\quad\because z=1\quad\therefore\rho=\frac{1}{\cos\varphi}$

$$\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{\frac{1}{\cos\varphi}}\rho\cos\varphi\,\rho\cdot\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta$$

$$=\frac{2\pi}{15}(2\sqrt{2}-1).$$

十:

$$\int_L\sin2x\,dx+2(x^2-1)y\,dy=\int_0^{\pi}\sin2x\,dx+2(x^2-1)\sin x\cos x\,dx\quad\begin{pmatrix}y=\sin x\\ dy=\cos x\,dx\end{pmatrix}$$

$$=\int_0^{\pi}x^2\sin2x\,dx\quad（分步积分）$$

$$=-\frac{\pi^2}{2}$$

十一:

（自己写的 大家可以验证一下）

与教材 P1023 例9 相似

$$\nabla\times\vec{F}=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\ \frac{\partial}{\partial x}&\frac{\partial}{\partial y}&\frac{\partial}{\partial z}\\ y&xz&x^2\end{vmatrix}=(-x)\vec{i}+(-2x)\vec{j}+(z-1)\vec{k}.$$

$\vec{n}=\frac{1}{\sqrt{3}}(1,1,1).$ ~~令十=~~

$d\sigma=\sqrt{1+1+1}\,dx\,dy=\sqrt{3}\,dx\,dy.$

更正：-5/6

$$\therefore\iint_S\nabla\times\vec{F}\cdot\vec{n}\,d\sigma=\int_0^1\int_0^{1-x}(-4x-y)\,dy\,dx=-\frac{13}{6}$$

十二.

$\nabla\cdot\vec{F}=2(x+y+z).$

$$\int_0^{2\pi}\int_0^{2}\int_0^{1}2(r\cos\theta+r\sin\theta+z)\,dz\,r\,dr\,d\theta$$

$$=\int_0^{2\pi}\int_0^{2}(2r(\cos\theta+\sin\theta)+1)\,r\,dr\,d\theta=\int_0^{2\pi}\int_0^{2}(2r^2\cos\theta+2r^2\sin\theta+r)\,dr\,d\theta=4\pi$$