

## Step-1

Let

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 3 & 1 & 0 \\ 4 & 0 & 0 \end{bmatrix}$$

$$x = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$z = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\sigma = 5$$

$$v = \begin{bmatrix} 8 \\ 4 \end{bmatrix}$$

$$H = \begin{bmatrix} -\frac{3}{5} & -\frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{bmatrix}$$

Therefore, the matrix  $U_1$  is given by,

$$U_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & H \\ 0 & \end{bmatrix}$$

$$U_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{3}{5} & -\frac{4}{5} \\ 0 & -\frac{4}{5} & \frac{3}{5} \end{bmatrix}$$

$$U_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{3}{5} & -\frac{4}{5} \\ 0 & -\frac{4}{5} & \frac{3}{5} \end{bmatrix}$$

Therefore,  $U_1^{-1}AU_1$  is given by,

$$\begin{aligned}
U_1^{-1}AU_1 &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{3}{5} & -\frac{4}{5} \\ 0 & -\frac{4}{5} & \frac{3}{5} \end{bmatrix} \begin{bmatrix} 1 & 3 & 4 \\ 3 & 1 & 0 \\ 4 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{3}{5} & -\frac{4}{5} \\ 0 & -\frac{4}{5} & \frac{3}{5} \end{bmatrix} \\
&= \begin{bmatrix} 1 & 3 & 4 \\ 0 & -\frac{9}{5} - \frac{16}{5} & -\frac{3}{5} \\ 0 & -\frac{12}{5} + \frac{12}{5} & -\frac{4}{5} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{3}{5} & -\frac{4}{5} \\ 0 & -\frac{4}{5} & \frac{3}{5} \end{bmatrix} \\
&= \begin{bmatrix} 1 & 3 & 4 \\ -5 & -\frac{3}{5} & 0 \\ 0 & -\frac{4}{5} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{3}{5} & -\frac{4}{5} \\ 0 & -\frac{4}{5} & \frac{3}{5} \end{bmatrix} \\
&= \begin{bmatrix} 1 & -\frac{9}{5} - \frac{16}{5} & -\frac{12}{5} + \frac{12}{5} \\ -5 & \frac{9}{25} & \frac{12}{25} \\ 0 & \frac{12}{25} & \frac{16}{25} \end{bmatrix} \\
U_1^{-1}AU_1 &= \begin{bmatrix} 1 & -5 & 0 \\ -5 & \frac{9}{25} & \frac{12}{25} \\ 0 & \frac{12}{25} & \frac{16}{25} \end{bmatrix}
\end{aligned}$$

## Step-2

$$U_1^{-1}AU_1 = \begin{bmatrix} 1 & -5 & 0 \\ -5 & \frac{9}{25} & \frac{12}{25} \\ 0 & \frac{12}{25} & \frac{16}{25} \end{bmatrix}$$

Thus, the tridiagonal  $U_1^{-1}AU_1$  is .