## Step-1

The first column of $A$ is the vector $\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$. Its length is 1.

Therefore, the first column of the matrix $Q$ is $\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$ only. The second column of the matrix $Q$ must be of the length 1 and it should be orthogonal to the first column. Thus, the second column of $Q$ must be $\begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$.

Thus, $Q = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.

## Step-2

If we assume that the column of the matrix $A$ are $a$ and $b$, then the matrix $R$ is given by $R = \begin{bmatrix} q_1^{\mathrm{T}} a & q_1^{\mathrm{T}} b \\ 0 & q_2^{\mathrm{T}} b \end{bmatrix}$.

Thus, we get

$$\begin{aligned} R &= \begin{bmatrix} \cos^2\theta + \sin^2\theta & \cos\theta\sin\theta \\ 0 & -\sin^2\theta \end{bmatrix} \\ &= \begin{bmatrix} 1 & \cos\theta\sin\theta \\ 0 & -\sin^2\theta \end{bmatrix} \end{aligned}$$

## Step-3

Thus, we have

$$\begin{aligned} A &= Q_0 R_0 \\ &= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & \cos\theta\sin\theta \\ 0 & -\sin^2\theta \end{bmatrix} \end{aligned}$$

## Step-4

Let $A_1 = R_0 Q_0$

Thus, we get

$$
\begin{aligned}
A_1 &= R_0 Q_0 \\
&= \begin{bmatrix} 1 & \cos\theta\sin\theta \\ 0 & -\sin^2\theta \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \\
&= \begin{bmatrix} \cos\theta + \cos\theta\sin^2\theta & -\sin\theta + \sin\theta\cos^2\theta \\ -\sin^2\theta & -\sin^2\theta\cos\theta \end{bmatrix} \\
&= \begin{bmatrix} \cos\theta\left(1+\sin^2\theta\right) & -\sin\theta\left(1-\cos^2\theta\right) \\ -\sin^2\theta & -\sin^2\theta\cos\theta \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
A_1 &= \begin{bmatrix} \cos\theta\left(1+\sin^2\theta\right) & -\sin\theta\left(\sin^2\theta\right) \\ -\sin^2\theta & -\sin^2\theta\cos\theta \end{bmatrix} \\
&= \begin{bmatrix} \cos\theta\left(1+\sin^2\theta\right) & -\sin^3\theta \\ -\sin^2\theta & -\sin^2\theta\cos\theta \end{bmatrix}
\end{aligned}
$$

## Step-5

Thus, we see that the off-diagonal entries of the matrix *A* change from $\sin\theta$ to $-\sin^3\theta$.