Step-1

Given that $v=(v_1,v_2)$

We have to verify whether the given transformations are linear or not.

Step-2

We know that a transformation *T* is said to be a linear transformation if $T(ax+by)=aT(x)+bT(y)$, where *x, y* are vectors and *a, b* are scalars.

Step-3

a) Given transformation is $T(v)=(v_2,v_1)$

Let $v=(v_1,v_2), w=(w_1,w_2)$

Now

$$\begin{aligned} T(v+w) &= T(v_1+w_1, v_2+w_2) \\ &= (v_2+w_2, v_1+w_1) \\ &= (v_2,v_1)+(w_2,w_1) \\ &= T(v)+T(w) \end{aligned}$$

Step-4

Let *c* be any scalar.

Now

$$\begin{aligned} T(cV) &= T(c(v_1,v_2)) \\ &= T(cv_1,cv_2) \\ &= (cv_2,cv_1) \\ &= c(v_2,v_1) \\ &= c\,T(v) \end{aligned}$$

Hence $T(v)=(v_2,v_1)$ is a linear transformation.

Step-5

b) Given transformation is $T(v)=(v_1,v_1)$

Let $v=(v_1,v_2), w=(w_1,w_2)$

Now

$$\begin{aligned}T(v+w)&=T(v_1+w_1,v_2+w_2)\\&=(v_1+w_1,v_1+w_1)\\&=(v_1,v_1)+(w_1,w_1)\\&=T(v)+T(w)\end{aligned}$$

Step-6

Let c be any scalar.

Now

$$\begin{aligned}T(cv)&=T(c(v_1,v_2))\\&=T(cv_1,cv_2)\\&=(cv_1,cv_1)\\&=c(v_1,v_1)\\&=c\,T(v)\end{aligned}$$

Hence $T(v)=(v_1,v_1)$ is a linear transformation.

Step-7

c) Given transformation is $T(v)=(0,v_1)$

Let $v=(v_1,v_2), w=(w_1,w_2)$

Now

$$\begin{aligned}T(v+w)&=T(v_1+w_1,v_2+w_2)\\&=(0,v_1+w_1)\\&=(0,v_1)+(0,w_1)\\&=T(v)+T(w)\end{aligned}$$

Step-8

Let c be any scalar.

Now

$$\begin{aligned} T(cv) &= T(c(v_1, v_2)) \\ &= T(cv_1, cv_2) \\ &= (0, cv_1) \\ &= c(0, v_1) \\ &= c\ T(v) \end{aligned}$$

Hence $T(v) = (0, v_1)$ is a linear transformation.

Step-9

d) Given transformation is $T(v) = (0,1)$

Let $v = (v_1, v_2), w = (w_1, w_2)$

Now

Step-10

$$\begin{aligned} T(v+w) &= T(v_1 + w_1, v_2 + w_2) \\ &= (0,1) \end{aligned}$$

$$\begin{aligned} T(v) + T(w) &= T(v_1, v_2) + T(w_1, w_2) \\ &= (0,1) + (0,1) \\ &= (0,2) \end{aligned}$$

Since $T(v+w) \neq T(v) + T(w)$

So T is not linear.

Hence $T(v) = (0,1)$ is not a linear transformation.