Step-1

Consider the following system,

$$\begin{aligned} 3x+2y&=10 \\ 6x+4y&=__ \end{aligned}$$

The objective is to choose a right-hand side which gives no solution, and another right hand-side which gives infinitely many solutions, and find two of those solutions.

Step-2

Write the $6x+4y$ as follows,

$$6x+4y=2(3x+2y)$$

Therefore, the system of equations can be expressed as,

$$\begin{aligned} 3x+2y&=10 \\ 2(3x+2y)&=__ \end{aligned}$$

If the right hand side of the second equation is not $2(10)=20$, then the second equation becomes,

$$3x+2y\neq 10$$

Then, the given system becomes as follows:

$$\begin{aligned} 3x+2y&=10 \\ 3x+2y&\neq 10 \end{aligned}$$

So, this is impossible.

Therefore, the system has no solution if the right hand side of the 2nd equation is a real number which is not 20.

Step-3

If the right hand side of the 2nd equation is 20, then the system reduced as follows,

$$\begin{aligned} 3x+2y&=10 \\ 2(3x+2y)&=20 \end{aligned}$$

This implies,

$$\begin{aligned} 3x+2y&=10 \\ 3x+2y&=10 \end{aligned}$$

So, there is only one equation $3x+2y=10$ to solve. This is a straight line, so it has infinitely many points.

That is, the system has infinitely many solutions if the right side of the 2nd equation is 20.

Step-4

If $x=0$, then from the equation $3x+2y=10$,

$$\begin{aligned} 3(0)+2y&=10 \\ y&=5 \end{aligned}$$

Then, a solution of the system is $(0,5)$.

If $x=4$, then from the equation $3x+2y=10$,

$$\begin{aligned} 3(4)+2y&=10 \\ 2y&=10-12 \\ 2y&=-2 \\ y&=-1 \end{aligned}$$

Then, a solution of the system is $(4,-1)$.

Hence, the two solutions of the system are $\boxed{(0,5)\text{and}(4,-1)}$.