

## Step-1

Given that  $w_1, w_2, w_3$  are independent and  $v_1 = w_2 + w_3, v_2 = w_1 + w_3$  and  $v_3 = w_1 + w_2$

Let  $c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$

$$\begin{aligned}\text{Thus, } c_1 (w_2 + w_3) + c_2 (w_1 + w_3) + c_3 (w_1 + w_2) &= 0 \\ (c_2 + c_3) w_1 + (c_1 + c_3) w_2 + (c_1 + c_2) w_3 &= 0\end{aligned}$$

i.e.  $c_2 + c_3 = 0$  (since  $w_1, w_2, w_3$  are linearly independent)

$$c_1 + c_3 = 0$$

$$c_1 + c_2 = 0$$

## Step-2

$$c_2 + c_3 = 0 \quad \dots\dots(1)$$

$$c_1 + c_3 = 0 \quad \dots\dots(2)$$

$$c_1 + c_2 = 0 \quad \dots\dots(3)$$

Subtract equation (2) from equation (1) as follows:

$$\text{So, } c_2 + c_3 - (c_1 + c_3) = 0 - 0$$

$$c_2 + c_3 - c_1 - c_3 = 0$$

$$c_2 - c_1 = 0 \quad \dots\dots (4)$$

Add equation (4) and equation (3) as follows:

$$(c_1 + c_2) + (c_2 - c_1) = 0 + 0$$

$$c_1 + c_2 + c_2 - c_1 = 0$$

$$2c_2 = 0$$

$$c_2 = 0$$

Substitute this value in equation (3) and (4) as follows:

$$\text{So, } c_1 = 0 \text{ and } c_3 = 0$$

$$\text{Hence, } \boxed{c_1 = 0, c_2 = 0 \text{ and } c_3 = 0}.$$

Therefore,  $v_1, v_2, v_3$  are linearly independent.