Step-1

The matrix of Fourier Transform is $F_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & i^2 & i^3 \\ 1 & i^2 & i^4 & i^6 \\ 1 & i^3 & i^6 & i^9 \end{bmatrix}$

Given circulant matrix $C = \begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix}$

That is,Â $c_0 = 2, c_1 = -1, c_2 = 0, c_3 = -1$

$$e_0 = c_0 + c_1 + c_2 + c_3 = 2 - 1 + 0 - 1 = 0$$
$$e_1 = c_0 + i\, c_1 + i^2 c_2 + i^3 c_3$$
$$= 2 + i(-1) - 1(0) - i(-1) = 2$$

$$e_2 = c_0 + i^2\, c_1 + i^4 c_2 + i^6 c_3$$
$$= 2 - 1(-1) + 1(0) - 1(-1) = 4$$
$$e_3 = c_0 + i^3\, c_1 + i^6 c_2 + i^9 c_3$$
$$= 2 - i(-1) - 1(0) + i(-1) = 2$$

Therefore, the required eigen values of C are $\boxed{e_0 = 0, e_1 = 2, e_2 = 4, e_3 = 2}$

Check: sum of Eigen values $= 0 + 2 + 4 + 2 = 8$

Trace C $= 2 + 2 + 2 + 2 = 8$

Therefore sum of Eigen values $=$Trace C