Step-1

(a) $\frac{dy}{dx}-2y=0$

$\frac{dy}{dx}=2y$

$\Rightarrow \frac{dy}{2y}=dx$

Integrating both sides

$\int \frac{1}{2y}dy=\int dx$

$\Rightarrow \log y=2x+C$

$\Rightarrow y=e^{2x+C}=e^{C}\cdot e^{2x}$

Therefore $\Rightarrow y=ke^{2x}$ where $e^{C}=K$

The basis for functions $\{e^{2x}\}$

Dimension of the space is infinite while $e^{2x}=\sum_{0}^{\infty}\frac{(2x)^{n}}{n!}$ and each x in the summand is spanned by one dimension.

Step-2

(b) $\frac{dy}{dx}-\frac{y}{x}=0$

$\Rightarrow xdy-ydx=0$

$\Rightarrow \frac{dy}{y}-\frac{dx}{x}=0$

Integrating both sides

$\log y-\log x=\log c$ where $\log c$ is constant

$\Rightarrow \log\frac{y}{x}=\log c$

$\Rightarrow \frac{y}{x}=c$

$\Rightarrow y = cx$

The basis for the functions $\{x\}$

Dimension of the space $=1$.