Step-1

Singular Value Decomposition (SVD) for any *m* by *n* matrix *A* is as follows

$$\begin{aligned} A &= U\Sigma V^T \\ &= \begin{pmatrix} \text{orthogonal} \\ U \text{ is } m\times m \end{pmatrix} \begin{pmatrix} m\times n \text{ matrix } \Sigma \\ \sigma_1 \cdots \sigma_r \text{ on diagonal} \end{pmatrix} \begin{pmatrix} \text{orthogonal} \\ V \text{ is } n\times n \end{pmatrix} \end{aligned}$$

Here eigenvectors of AA^T are in *U*, eigenvectors of $A^T A$ are in *V*.

The *r* singular-values on the diagonal of Σ are the square roots of the nonzero eigenvalues of both AA^T and $A^T A$.

Step-2

That is $\sigma_r = \sqrt{\lambda_r}$

Step-3

Suppose the SVD for $A+I$ involve $\Sigma + I$, then we have

$$\begin{aligned} U(\Sigma+I)V^T &= U\Sigma V^T + UIV^T \\ &= U\Sigma V^T + UV^T \\ &\neq A+I \end{aligned}$$

Step-4

We know that the *r* singular-values on the diagonal of Σ are the square roots of the nonzero eigenvalues of $A^T A$.

So, singular-values $A+I$ are not $\sigma_r + 1$.

The singular-values $A+I$ come from the eigenvalues of $(A+I)^T(A+I)$.

Therefore, the SVD for $A+I$ just not use $\Sigma + I$.