Step-1

A) determinant of 4 by 4 matrix whose entries are a_{ij} = smaller of i and j

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

And

$$A=LU\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The pivots are $\boxed{1,1,1,1}$ and $\det A=\boxed{1}$

Step-2

b) Given $n_1=2, n_2=6, n_3=8, n_4=10$

a_{ij} = smaller of n_i nad n_j

Here $$A=\begin{bmatrix} 2 & 2 & 2 & 2 \\ 2 & 6 & 6 & 6 \\ 2 & 6 & 8 & 8 \\ 2 & 6 & 8 & 10 \end{bmatrix}$$

$$\det A=\begin{vmatrix} 2 & 2 & 2 & 2 \\ 2 & 6 & 6 & 6 \\ 2 & 6 & 8 & 8 \\ 2 & 6 & 8 & 10 \end{vmatrix}$$

Step-3

Now

$$=\begin{vmatrix}2&2&2&2\\2&6&6&6\\2&6&8&8\\0&0&0&2\end{vmatrix} \text{ adding } -1 \text{ time the third row to the fourth}$$

$$=\begin{vmatrix}2&2&2&2\\2&6&6&6\\0&0&2&2\\0&0&0&2\end{vmatrix} \text{ adding } -1 \text{ time the second row to the third}$$

Step-4

Then

$$=\begin{vmatrix}2&2&2&2\\0&4&4&4\\0&0&2&2\\0&0&0&2\end{vmatrix} \text{ adding } -1 \text{ time the first row to the second}$$

$$=(2)(4)(2)(2)$$

$$=\boxed{32}$$

Step-5

We can give a general rule for any $n_1 \le n_2 \le n_3 \le n_4$ as $\det A$ for A defined as above I s given by

$$\det A = n_1(n_2-n_1)(n_3-n_2)(n_4-n_3)$$