

## Step-1

Then we consider

$$\begin{aligned}\alpha_1 &= \frac{r_0^\top r_0}{p_0^\top A p_0} \\ &= \frac{r_0^\top r_0}{r_0^\top A r_0}\end{aligned}$$

## Step-2

Then we have

$$\begin{aligned}r_1 &= r_0 - \alpha_1 A p_0 \\ &= r_0 - \frac{r_0^\top r_0}{r_0^\top A r_0} A r_0\end{aligned}$$

## Step-3

Consider

$$\begin{aligned}r_0^\top r_1 &= r_0^\top \left( r_0 - \frac{r_0^\top r_0}{r_0^\top A r_0} A r_0 \right) \\ &= r_0^\top r_0 - r_0^\top \left( \frac{r_0^\top r_0}{r_0^\top A r_0} A r_0 \right) \\ &= r_0^\top r_0 - \left( \frac{r_0^\top r_0}{r_0^\top A r_0} \right) r_0^\top A r_0\end{aligned}$$

$$\begin{aligned}r_0^\top r_1 &= r_0^\top r_0 - r_0^\top r_0 \left( \frac{r_0^\top A r_0}{r_0^\top A r_0} \right) \\ &= r_0^\top r_0 - r_0^\top r_0 \\ &= 0\end{aligned}$$

Therefore,  $r_0$  is orthogonal to  $r_1$ .

## Step-4

Now we show that  $p_1^\top A p_0 = 0$ .

We have  $p_0 = r_0$  and  $p_1 = r_1 + \beta_1 p_0$

Here,  $\beta_1 = \frac{r_1^\top r_1}{r_0^\top r_0}$ .

## Step-5

Consider  $p_1^\top A p_0$

$$\begin{aligned}
 p_1^\top A p_0 &= \left( r_1 + \left( \frac{r_1^\top r_1}{r_0^\top r_0} \right) r_0 \right)^\top A r_0 \\
 &= \left( r_0 - \frac{r_0^\top r_0}{r_0^\top A r_0} A r_0 + \left( \frac{r_1^\top r_1}{r_0^\top r_0} \right) r_0 \right)^\top A r_0 \\
 &= r_0^\top A r_0 - \left( \frac{r_0^\top r_0}{r_0^\top A r_0} \right) (A r_0)^\top A r_0 + \left( \frac{r_1^\top r_1}{r_0^\top r_0} \right) r_0^\top A r_0 \\
 p_1^\top A p_0 &= r_0^\top A r_0 - \left( \frac{r_0^\top r_0}{r_0^\top A r_0} \right) r_0^\top A^\top A r_0 + \left( \frac{r_1^\top r_1}{r_0^\top r_0} \right) r_0^\top A r_0 \\
 &= r_0^\top A r_0 - \left( \frac{r_0^\top r_0}{r_0^\top A r_0} \right) r_0^\top A^2 r_0 + \left( \frac{r_1^\top r_1}{r_0^\top r_0} \right) r_0^\top A r_0 \\
 &= 0
 \end{aligned}$$

## Step-6

Thus, we have shown that  $p_1^\top A p_0 = 0$ .