Step-1

We have to find the rank of the following matrices, and we have to express A as $A = uv^T$:

$A = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 6 \end{bmatrix}$, and $A = \begin{bmatrix} 2 & -2 \\ 6 & -6 \end{bmatrix}$

Step-2

Consider the first matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 6 \end{bmatrix}$$

$$\xrightarrow{R_3 - 2R_1} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Step-3

Therefore there is only one pivot column (first column)

So rank of $A = 1$.

Step-4

Now consider the product of the first column which is a linear combination the remaining columns and the first row which also a linear combination of the remaining rows then we have

$$\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 6 \end{bmatrix} = A$$

Step-5

Therefore

$A = uv^T$

$$\Rightarrow \boxed{A = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 3 \end{bmatrix}}$$

Here $u = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, v = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 3 \end{bmatrix}$

Step-6

Next we consider the second matrix

$$A = \begin{bmatrix} 2 & -2 \\ 6 & -6 \end{bmatrix}$$

$$\xrightarrow{R_2 - 3R_1, \frac{1}{2}R_1} \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

Therefore A has one pivot column (first column)

$\boxed{\text{So rank } A = 1}$

Step-7

And consider the product of the first column which is a linear combination the remaining columns and the first row which also a linear combination of the remaining rows then we have

$$\begin{bmatrix} 2 \\ 6 \end{bmatrix} \begin{bmatrix} 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 6 & -6 \end{bmatrix} = A$$

Step-8

Therefore

$A = uv^T$

$$\Rightarrow \boxed{A = \begin{bmatrix} 2 \\ 6 \end{bmatrix} \begin{bmatrix} 1 & -1 \end{bmatrix}}$$

Here $u = \begin{bmatrix} 2 \\ 6 \end{bmatrix}, v = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$