

Step-1

Given that A, B, C are three invertible square matrices and $M = ABC$. To find a formula for B^{-1} that involves M^{-1} and A and C .

See that as A, B, C are three invertible square matrices and $M = ABC$ so M is also invertible and $M^{-1} = C^{-1}B^{-1}A^{-1}$ as $C^{-1}B^{-1}A^{-1}ABC = C^{-1}B^{-1}BC = C^{-1}C = I$.

Step-2

So, $M^{-1} = C^{-1}B^{-1}A^{-1}$ (1)

Now, multiplying the equation (1) from left by C gives, $CM^{-1} = CC^{-1}B^{-1}A^{-1} = IB^{-1}A^{-1} = B^{-1}A^{-1}$.

So, $CM^{-1} = B^{-1}A^{-1}$ (2)

Now, multiplying the equation (2) from right by A gives, $CM^{-1}A = B^{-1}A^{-1}A = B^{-1}I = B^{-1}$.

Hence, $B^{-1} = CM^{-1}A$.