Step-1

a) Given system is

$$\begin{bmatrix} 1 & 4 & 2 \\ 2 & 8 & 4 \\ -1 & -4 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

We have to find the values of b_1, b_2 and b_3 or the condition on b_1, b_2 and b_3 for which the given system is solvable.

Step-2

Consider the matrix $[A, B]$ as follows:

$$\begin{bmatrix} 1 & 4 & 2 & b_1 \\ 2 & 8 & 4 & b_2 \\ -1 & -4 & -2 & b_3 \end{bmatrix}$$

$$\xrightarrow[R_3 + R_1]{R_2 - 2R_1} \begin{bmatrix} 1 & 4 & 2 & b_1 \\ 0 & 0 & 0 & b_2 - 2b_1 \\ 0 & 0 & 0 & b_3 + b_1 \end{bmatrix}$$

Therefore the above system is solvable if $b_3 + b_1 = 0$ and $b_2 - 2b_1 = 0$

Step-3

b) Given system is

$$\begin{bmatrix} 1 & 4 \\ 2 & 9 \\ -1 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

We have to find the values of b_1, b_2 and b_3 or the condition b_1, b_2 and b_3 for which the given system is solvable.

Step-4

Consider the following matrix

$$[A, B] = \begin{bmatrix} 1 & 4 & b_1 \\ 2 & 9 & b_2 \\ -1 & -4 & b_3 \end{bmatrix}$$

$$\xrightarrow[R_3+R_1]{R_2-2R_1}\begin{bmatrix}1 & 4 & b_1\\ 0 & 1 & b_2-2b_1\\ 0 & 0 & b_3+b_1\end{bmatrix}$$

Therefore the given system is solvable if $b_3+b_1=0$