

Step-1

Consider a 2×2 matrix $A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

The characteristic equation of A is $|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & 2 \\ 2 & 2-\lambda \end{vmatrix} = 0$$
$$(2-\lambda)^2 - 4 = 0$$
$$4 + \lambda^2 - 4\lambda - 4 = 0$$
$$\lambda = 0, 4$$

Step-2

On the other hand, apply the row transformation $R_2 \rightarrow R_2 - R_1$, and get $A \sim \begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix}$

Its characteristic equation is;

$$\begin{vmatrix} 2-\lambda & 2 \\ 0 & 0-\lambda \end{vmatrix} = 0$$
$$(2-\lambda)(-\lambda) = 0$$
$$\lambda = 0, 2$$

Observe that the elementary row operation on the given matrix $A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ has changed the Eigen values from 0, 4 to 0, and 2.

However, the zero Eigen value remained in the transformed matrix also.

The reason for this is; if the given matrix has dependent rows, then at least one of the Eigen values is 0. Also, when an elementary row or column operation is applied on the given matrix, the dependence of the columns cannot be changed.