Step-1

Let following be the differential equation of matrices:

$$\frac{du}{dt} = Au$$

Here, matrix A is defined as follows:

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

Step-2

To show that the infinite series produces the following, when $A^2 = A$:

$$e^{At} = I + \left(e^t - 1\right)A$$

Use the above ration to find e^{At}.

Step-3

Infinite series for e^{At} is given as follows:

$$e^{At} = I + At + \frac{(At)^2}{2!} + \frac{(At)^3}{3!} + \frac{(At)^4}{4!} + \ldots$$

To get the relation put $A^2 = A$ and do the following calculations:

$$\begin{aligned} e^{At} &= I + At + \frac{(At)^2}{2!} + \frac{(At)^3}{3!} + \frac{(At)^4}{4!} + \ldots \\ &= I + At + \frac{A(t)^2}{2!} + \frac{A(t)^3}{3!} + \frac{A(t)^4}{4!} + \ldots \\ &= I + A\left(t + \frac{(t)^2}{2!} + \frac{(t)^3}{3!} + \frac{(t)^4}{4!} + \ldots\right) \\ &= I + A\left(e^t - 1\right) \end{aligned}$$

Step-4

Therefore, $\boxed{e^{At} = I + \left(e^t - 1\right)A}$.

Step-5

Now, find e^{At} with the help of above relation and matrix A.

$$
\begin{aligned}
e^{At} &= I+\left(e^{t}-1\right)A \\
&= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \left(e^{t}-1\right)\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} \left(e^{t}-1\right) & \left(e^{t}-1\right) \\ 0 & 0 \end{bmatrix} \\
&= \begin{bmatrix} e^{t} & \left(e^{t}-1\right) \\ 0 & 1 \end{bmatrix}
\end{aligned}
$$

Step-6

Therefore,

$$
e^{At} = \boxed{\begin{bmatrix} e^{t} & \left(e^{t}-1\right) \\ 0 & 1 \end{bmatrix}}.
$$