Step-1

Since **V** is a subspace, it is a vector space in its own right. Therefore, it has a basis. Let the dimension of **V** be *m*. Note that $m < n$.

Let $v_1, v_2, \ldots, v_m$ be a basis of **V**.

Step-2

Form a matrix *A* of order *n* by *n*, such that its first *m* rows are the vectors $v_1, v_2, \ldots, v_m$. Put zero in each entry of the last $n-m$ rows of *A*.

Let us show that the matrix *A* has row space = **V** and null space = **W**.

Step-3

Let $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ be any vector in $\mathbf{R}^n$.

Consider the following:

$$A\alpha = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_m \\ 0\ 0\ 0 \ldots 0 \\ \vdots \\ 0\ 0\ 0 \ldots 0 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix}$$

It is obvious that $A\alpha$ will have all zero entries in the last $n-m$ rows. Therefore, for the matrix *A*, its row space is **V**.

Step-4

If we consider a vector Î² from **W**, then its initial *m* entries will be zeros. The nonzero entries will be from $m+1$ to *n*.

Therefore, $A\beta = 0$.

Thus, null space of the matrix *A* is **W**.

Step-5

As an example, consider the vector space $\mathbf{R}^3$. Let **V** be the *xy*-plane and let **W** be the *z*-axis.

Then $\{(1,0,0),(0,1,0)\}$ is a basis for **V**.

Let $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix}$

Step-6

Let $\alpha=(p,q,r)$ be any vector.

Then

$$A\alpha=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}p\\ q\\ r\end{bmatrix}$$
$$=\begin{bmatrix}p\\ q\\ 0\end{bmatrix}$$

Also, let $\hat{I}^2$ be any vector along the *z*-axis. Then $\beta=(0,0,s)$.

Therefore,

$$A\beta=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}0\\ 0\\ s\end{bmatrix}$$
$$=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$