Step-1

Let y be the vector $(6,3,6)$.

Consider the following:

$$\begin{bmatrix} 1 & 5 \\ 3 & 0 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1+5 \\ 3+0 \\ 2+4 \end{bmatrix}$$
$$= \begin{bmatrix} 6 \\ 3 \\ 6 \end{bmatrix}$$
$$= y$$

Step-2

Similarly, note the following;

$$\begin{bmatrix} 3 & 0 \\ 0 & 1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 6+0 \\ 0+3 \\ 0+6 \end{bmatrix}$$
$$= \begin{bmatrix} 6 \\ 3 \\ 6 \end{bmatrix}$$
$$= y$$

Step-3

This gives the following:

$$\begin{bmatrix} A & B \end{bmatrix} x = \begin{bmatrix} 1 & 5 & 3 & 0 \\ 3 & 0 & 0 & 1 \\ 2 & 4 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 2 \\ -3 \end{bmatrix}$$
$$= \begin{bmatrix} 1+5-6 \\ 3-3 \\ 2+4-6 \end{bmatrix}$$
$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Step-4

The column space of each $A=\begin{bmatrix}1 & 5\\ 3 & 0\\ 2 & 4\end{bmatrix}$ and $B=\begin{bmatrix}3 & 0\\ 0 & 1\\ 0 & 2\end{bmatrix}$ has dimension equal to 2. Thus, it is clear that the dimension of $C(A)\cap C(B)$ is equal to 1.