Step-1

From the relation $x_1 + x_2 = x_3 + x_4 = x_5 + x_6$, we see that there may not be any restriction on x_1, x_2, x_3, and x_5. Once, these values are fixed, the remaining values x_4 and x_6 are automatically fixed. That is, we have

$$x_4 = x_1 + x_2 - x_3$$
$$x_6 = x_1 + x_2 - x_5$$

Thus, we get the basis of the subspace as follows:

$$\boxed{\{(x_1, x_2, x_3, x_4, x_5, x_6) \in \mathbf{R}^6 \mid x_4 = x_1 + x_2 - x_3, x_6 = x_1 + x_2 - x_5\}}$$

Step-2

(b) We need to obtain a matrix, whose nullspace contains the above subspace.

Suppose we contain the equality $a = b = c$, then observe the following:

$$a - b = 0$$
$$a - c = 0$$
$$b - c = 0$$

$$a + b - 2c = 0$$
$$a - 2b + c = 0$$
$$-2a + b + c = 0$$

Step-3

The above 6 equations give us the idea of the required matrix:

$$\begin{bmatrix} 1 & 1 & -1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 & -2 & -2 \\ 1 & 1 & -2 & -2 & 1 & 1 \\ -2 & -2 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Step-4

Consider the following:

$$\begin{bmatrix} 1 & 1 & -1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 & -2 & -2 \\ 1 & 1 & -2 & -2 & 1 & 1 \\ -2 & -2 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_1+x_2-x_3 \\ x_5 \\ x_1+x_2-x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Step-5

Therefore, $\boxed{\begin{bmatrix} 1 & 1 & -1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 & -2 & -2 \\ 1 & 1 & -2 & -2 & 1 & 1 \\ -2 & -2 & 1 & 1 & 1 & 1 \end{bmatrix}}$ is a required matrix.

Step-6

(c) We need to find a matrix, whose column space is the above subspace.

Consider the following matrix:

$$\boxed{\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}}$$

Any vector $(x_1, x_2, x_3, x_4, x_5, x_6)$, such that $x_1 + x_2 = x_3 + x_4 = x_5 + x_6$, can be expressed as a linear combination of the columns of the above matrix.