

## Step-1

Given functions are  $f(x) = x^2$ ,  $g(x) = 5x$  are vectors in the vector space  $\mathbf{F}$  of all real functions.

The combination  $3f(x) - 4g(x)$  is  $h(x)$ .

Therefore,

$$\begin{aligned}h(x) &= 3f(x) - 4g(x) \\&= 3 \cdot x^2 - 4 \cdot (5x) \\&= 3x^2 - 20x\end{aligned}$$

Hence  $h(x) = 3x^2 - 20x$

## Step-2

The scalar multiplication of the vector space  $\mathbf{F}$  of all real function is  $(cf)(x) = cf(x)$

If we define  $(cf)(x) = f(cx)$  which is not equal to  $cf(x)$

For example  $f(x) = x^2$

$$\begin{aligned}(3f)(x) &= 3 \cdot f(x) \\&= 3 \cdot x^2\end{aligned}$$

If we define  $(cf)(x) = f(cx)$

$$\begin{aligned}f(3x) &= (3x)^2 \\&= 9x^2\end{aligned}$$

Then

Therefore,  $3 \cdot f(x) \neq f(3x)$

Therefore scalar multiplication rule is broken.