

Step-1

To expand and write atleast five terms of the infinite series for e^{At} . After taking derivative of each term with respect to t , show that there are four terms of Ae^{At} .

Step-2

Expansion of infinite series for e^{At} is as follows:

$$e^{At} = I + At + \frac{(At)^2}{2!} + \frac{(At)^3}{3!} + \frac{(At)^4}{4!} + \dots$$

Differentiate each term of infinite series with respect to t .

$$\begin{aligned}\frac{d}{dt} e^{At} &= A + \frac{2(A^2t)}{2!} + \frac{3(A^3t^2)}{3!} + \frac{4(A^4t^3)}{4!} + \dots \\ &= A + A^2t + \frac{(A^3t^2)}{2!} + \frac{(A^4t^3)}{3!} + \dots \\ &= A \left(I + At + \frac{(A^2t^2)}{2!} + \frac{(A^3t^3)}{3!} + \dots \right) \\ &= Ae^{At}\end{aligned}$$

Step-3

Therefore, after taking derivative of each five terms of e^{At} , four terms of Ae^{At} results.