

## Step-1

We have to fill the following blanks:

If the columns of  $A$  are linearly independent ( $A$  has  $m$  by  $n$ ), then the rank is ———, the null space is ———, the row space is ———, and there exists a ———-inverse.

## Step-2

If the columns of  $A$  are linearly independent ( $A$  has  $m$  by  $n$ ), then the rank is number of linearly independent columns. So we have the rank of  $A$  is  $n$ , and we know that and if full column rank  $r = n$  then  $A$  has a left inverse  $B$  such that  $BA = I_n$  and this is possible only if  $m \geq n$

## Step-3

Therefore we have the following:

If the columns of  $A$  are linearly independent ( $A$  has  $m$  by  $n$ ), then the rank is  $n$ , the null space is  $\{0\}$ , the row space is  $\mathbf{R}^n$ , and there exists a left-inverse.