

Step-1

Given A is an $m \times n$ matrix of rank r . Suppose there are right hand sides b for which $Ax = b$ has no solution.

(a)

We need find at what inequalities must be true between m, n , and r .

A is an $m \times n$ matrix of rank r .

$AX = b$ has no solution means that $r < m$

Since $\dim(C(A)) + \dim(N(A)) =$ number of columns of A . So, $r \leq n$.

We cannot compare m and n .

Step-2

(b)

We need to explain about $A^T y = 0$ has a nonzero solution.

If $m - r > 0$, i.e. $\dim(N(A^T)) > 0$ then $N(A^T)$ has a non zero vector.

Therefore, $A^T y = 0$ has a non zero solution.