Step-1

Given that $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

The objective is to perform row operations, so the given determinant must equals to $-\det B = -\begin{vmatrix} c & d \\ a & b \end{vmatrix}$.

Step-2

We have

$$\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$= \begin{vmatrix} a & b \\ a+c & b+d \end{vmatrix}$ Adding first row to the second row, i.e. $R_2 \to R_2 + R_1$

$= \begin{vmatrix} -c & -d \\ a+c & b+d \end{vmatrix}$ Adding -1 times of second row to the first row

i.e. $R_1 \to R_1 - R_2$

$= \begin{vmatrix} -c & -d \\ a & b \end{vmatrix}$ Adding the first row to the second row, i.e. $R_2 \to R_2 + R_1$

$$= -\begin{vmatrix} c & d \\ a & b \end{vmatrix}$$
$$= -\det B$$

Therefore, the rules $\left(R_2 \to R_2 + R_1, R_1 \to R_1 - R_2\right)$.

$R_2 \to R_2 + R_1, (-1)R_1 \to R_1$ replace the rule that by interchanging two rows of a determinant, the sign of the determinant only changes without changing its value.