Step-1

(a) In the course of discussion, we are needed to distinguish the words orthogonal and orthogonal complement.

$V^{\perp}$ is the orthogonal complement of *V*. but it is not necessary that any two orthogonal spaces are complements of each other.

We follow that dim *V*+ dim $V^{\perp}$ = dim $\mathbf{R}^n$

Step-2

we observe that a straight line in $\mathbf{R}^3$ is a space of dimension 1.

We follow that if *V* is a straight line, and then $V^{\perp}$ is the plane orthogonal to *V*

Now, we consider *V* and *W* are two perpendicular straight lines in $\mathbf{R}^3$.

So, *V* and *W* are subspaces of $\mathbf{R}^3$ whose dimension is 1

Then it follows that $V^{\perp}$ and $W^{\perp}$ are two perpendicular planes whose dimensions are 2.

But $V^{\perp}$ and $W^{\perp}$ are not orthogonal complements while the sum of their dimensions is 4 > dim $\mathbf{R}^3$

Step-3

(b) Suppose ***V*** is orthogonal to ***W*** and ***W*** is orthogonal to ***Z***, then to say ***V*** is not necessarily orthogonal to ***Z***, we give an example.

Suppose ***V= 2x-y+z* = 0, *W= x-2y* = 0, *Z= 4x* â€“ *2y+2z* = 0** are straight lines in $\mathbf{R}^3$

We easily see that $V^{\perp}W = 0$, and $W^{\perp}Z = 0$

But *V* and *W* are parallel while one is a multiple of the other.

This confirms that the statement â€œ***V*** is orthogonal to ***W*** and ***W*** is orthogonal to ***Z*** makes ***V*** is orthogonal to ***Zâ€*** is false.