Step-1

Given $a=\begin{bmatrix}0\\0\\1\end{bmatrix}, b=\begin{bmatrix}0\\1\\1\end{bmatrix}, c=\begin{bmatrix}1\\1\\1\end{bmatrix}$

We have to apply the Gram-Schmidt process to a,b,c and we have to write the result in the form $A=QR$

Step-2

By Gram-Schmidt process,

$$q_1=\frac{a}{\|a\|}$$
$$=\frac{1}{\sqrt{0^2+0^2+1^2}}\begin{bmatrix}0\\0\\1\end{bmatrix}$$
$$=\begin{bmatrix}0\\0\\1\end{bmatrix}$$

Step-3

And

$$q_2=\frac{\beta}{\|\beta\|} \text{ where}$$
$$\beta=b-\left(q_1^Tb\right)q_1$$

Step-4

Now

$$q_1^Tb=\begin{bmatrix}0 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\1\\1\end{bmatrix}$$
$$=0+0+1$$
$$=1$$

Step-5

$$\left(q_1^T b\right) q_1 = 1\begin{bmatrix}0\\0\\1\end{bmatrix}$$
$$= \begin{bmatrix}0\\0\\1\end{bmatrix}$$

Step-6

$$\beta = \begin{bmatrix}0\\1\\1\end{bmatrix} - \begin{bmatrix}0\\0\\1\end{bmatrix}$$
$$= \begin{bmatrix}0\\1\\0\end{bmatrix}$$

$$\|\beta\| = \sqrt{0+1+0}$$
$$=1$$

Step-7

Therefore

$$q_2 = \frac{1}{1}\begin{bmatrix}0\\1\\0\end{bmatrix}$$
$$= \begin{bmatrix}0\\1\\0\end{bmatrix}$$

Step-8

$q_3 = \dfrac{\gamma}{\|\gamma\|}$ where

$$\gamma = c - \left(q_1^T c\right) q_1 - \left(q_2^T c\right) q_2$$

Step-9

$$q_1^T c = (0,0,1)\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$= 0 + 0 + 1$$

$$= 1$$

$$\left(q_1^T c\right) q_1 = 1.\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$= \begin{bmatrix}0\\0\\1\end{bmatrix}$$

Step-10

$$q_2^T c = (0,1,0)\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$= 0 + 1 + 0$$

$$= 1$$

Step-11

$$\left(q_2^T c\right) q_2 = 1\begin{bmatrix}0\\1\\0\end{bmatrix}$$

$$= \begin{bmatrix}0\\1\\0\end{bmatrix}$$

Step-12

$$\gamma = \begin{bmatrix}1\\1\\1\end{bmatrix} - 1\begin{bmatrix}0\\0\\1\end{bmatrix} - 1\begin{bmatrix}0\\1\\0\end{bmatrix}$$
$$= \begin{bmatrix}1\\1\\1\end{bmatrix} - \begin{bmatrix}0\\0\\1\end{bmatrix} - \begin{bmatrix}0\\1\\0\end{bmatrix}$$
$$= \begin{bmatrix}1\\0\\0\end{bmatrix}$$

Step-13

$$\|\gamma\| = \sqrt{1+0+0}$$
$$= 1$$

Therefore

$$q_3 = \frac{1}{1}\begin{bmatrix}1\\0\\0\end{bmatrix}$$
$$= \begin{bmatrix}1\\0\\0\end{bmatrix}$$

Step-14

We have

$$A = \begin{bmatrix}a\ b\ c\end{bmatrix}$$
$$= \begin{bmatrix}q_1\, q_2\ \ q_3\end{bmatrix}\begin{bmatrix}q_1^T a & q_1^T b & q_1^T c\\ 0 & q_2^T b & q_2^T c\\ 0 & 0 & q_3^T c\end{bmatrix}$$

Step-15

$$q_1^T a = \begin{bmatrix}0 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix}$$
$$= 0+0+1$$
$$= 1$$

Step-16

$$q_1^T b = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$
$$= 0 + 0 + 1$$
$$= 1$$

Step-17

$$q_1^T c = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$
$$= 0 + 0 + 1$$
$$= 1$$

Step-18

$$q_2^T b = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$
$$= 0 + 1 + 0$$
Â $= 1$

$$q_2^T c = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$
$$= 0 + 1 + 0$$
$$= 1$$

Step-19

$$q_3^T c = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$
$$= 1 + 0 + 0$$
$$= 1$$

Hence $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = QR$