Step-1

When t = 0, A(t) = A and as $t \to \infty$, $A(t) \to I$. Also, for any t, the determinant of A(t) cannot be negative. Thus,

$$A(t) = \begin{bmatrix} a_{11} - \frac{t(a_{11} - 1)}{t + 1} & \frac{a_{12}}{t + 1} & \dots & \frac{a_{1n}}{t + 1} \\ \frac{a_{21}}{t + 1} & a_{22} - \frac{t(a_{22} - 1)}{t + 1} & \dots & \frac{a_{2n}}{t + 1} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{a_{n1}}{t + 1} & \frac{a_{n2}}{t + 1} & \dots & a_{nn} - \frac{t(a_{nn} - 1)}{t + 1} \end{bmatrix}$$

is the required chain of matrices.

Step-2

Its determinant can be written as follows:

$$\det A = \sum_{\text{all } P's} \left(a_{1\alpha} a_{2\beta} \cdots a_{m} \right) \det P$$

Consider the following matrix:

$$A(t) = \begin{bmatrix} a_{11} - \frac{t(a_{11} - 1)}{t+1} & \frac{a_{12}}{t+1} & \dots & \frac{a_{1n}}{t+1} \\ \frac{a_{21}}{t+1} & a_{22} - \frac{t(a_{22} - 1)}{t+1} & \dots & \frac{a_{2n}}{t+1} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{a_{n1}}{t+1} & \frac{a_{n2}}{t+1} & \dots & a_{nn} - \frac{t(a_{nn} - 1)}{t+1} \end{bmatrix}$$