

Step-1

We get

$$\begin{aligned}(y, 1-y)A &= (y, 1-y) \begin{bmatrix} 3 & 4 & 1 \\ 2 & 0 & 3 \end{bmatrix} \\ &= (3y+2(1-y), 4y, y+3(1-y)) \\ &= (y+2, 4y, 3-2y)\end{aligned}$$

Step-2

Equating $y+2$ and $4y$, we get $y = \frac{2}{3}$. For this value of y , we get

$$\begin{aligned}(y+2, 4y, 3-2y) &= \left(\frac{2}{3}+2, 4\left(\frac{2}{3}\right), 3-2\left(\frac{2}{3}\right)\right) \\ &= \left(\frac{8}{3}, \frac{8}{3}, \frac{5}{3}\right)\end{aligned}$$

The maximum value is $\frac{8}{3}$.

Step-3

Equating $y+2$ and $3-2y$, we get $y = \frac{1}{3}$. For this value of y , we get

$$\begin{aligned}(y+2, 4y, 3-2y) &= \left(\frac{1}{3}+2, 4\left(\frac{1}{3}\right), 3-2\left(\frac{1}{3}\right)\right) \\ &= \left(\frac{7}{3}, \frac{4}{3}, \frac{7}{3}\right)\end{aligned}$$

The maximum value is $\frac{7}{3}$.

Step-4

Equating $3-2y$ and $4y$, we get $y = \frac{1}{2}$. For this value of y , we get

$$\begin{aligned}(y+2, 4y, 3-2y) &= \left(\frac{1}{2}+2, 4\left(\frac{1}{2}\right), 3-2\left(\frac{1}{2}\right)\right) \\ &= \left(\frac{5}{2}, 2, 2\right)\end{aligned}$$

The maximum value is $\frac{5}{2}$.

Step-5

Out of $\frac{8}{3}$, $\frac{7}{3}$, and $\frac{5}{2}$, the least value is $\frac{7}{3}$. Therefore, the best strategy of Y will have $y = \frac{1}{3}$.

Thus, we have $y^* = \frac{1}{3}$. Naturally, for the best strategy of X , X should get the amount $\frac{7}{3}$.

Step-6

Thus, X will combine the columns of A to obtain the value $\frac{7}{3}$. Consider the following:

$$\begin{aligned}\frac{2}{3}(y+2) + 0(4y) + \frac{1}{3}(3-2y) &= \frac{2y}{3} + \frac{4}{3} + 1 - \frac{2y}{3} \\ &= \frac{7}{3}\end{aligned}$$

Therefore, X chooses the three columns in the frequencies $\frac{2}{3}, 0, \frac{1}{3}$.