Step-1

Suppose the given system is $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

It can be written as $Ax = b$.

We can write this as $\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_2 \end{bmatrix} = \mathbf{b}$ where $\mathbf{a}_i$ is the column of A. **b** is the column vector on the right of the given system.

If $\mathbf{a}_1 x_1 + \mathbf{a}_2 x_2 + \mathbf{a}_3 x_3 = \mathbf{b}$ goes into the 3rd column to produce B_3, then we can write

$|\mathbf{a}_1 \ \ \mathbf{a}_2 \ \ \mathbf{b}| = |\mathbf{a}_1 \ \ \mathbf{a}_2 \ \ \mathbf{a}_1 x_1 + \mathbf{a}_2 x_2 + \mathbf{a}_3 x_3|$

$= |\mathbf{a}_1 \ \ \mathbf{a}_2 \ \ \mathbf{a}_1 x_1| + |\mathbf{a}_1 \ \ \mathbf{a}_2 \ \ \mathbf{a}_2 x_2| + |\mathbf{a}_1 \ \ \mathbf{a}_2 \ \ \mathbf{a}_3 x_3|$ By properties of determinants

$= x_1|\mathbf{a}_1 \ \ \mathbf{a}_2 \ \ \mathbf{a}_1| + x_2|\mathbf{a}_1 \ \ \mathbf{a}_2 \ \ \mathbf{a}_2| + x_3|\mathbf{a}_1 \ \ \mathbf{a}_2 \ \ \mathbf{a}_3|$ By the properties of determinants

$= x_1(0) + x_2(0) + x_3|\mathbf{a}_1 \ \ \mathbf{a}_2 \ \ \mathbf{a}_3|$ If two columns are identical, then the determinant is zero.

$= x_3|\mathbf{a}_1 \ \ \mathbf{a}_2 \ \ \mathbf{a}_3|$

Step-2

So, $|\mathbf{a}_1 \ \ \mathbf{a}_2 \ \ \mathbf{b}| = x_3|\mathbf{a}_1 \ \ \mathbf{a}_2 \ \ \mathbf{a}_3|$

Or, $x_3 = \dfrac{|\mathbf{a}_1 \ \ \mathbf{a}_2 \ \ \mathbf{b}|}{|\mathbf{a}_1 \ \ \mathbf{a}_2 \ \ \mathbf{a}_3|}$

Similarly, $x_2 = \dfrac{|\mathbf{a}_1 \ \ \mathbf{b} \ \ \mathbf{a}_2|}{|\mathbf{a}_1 \ \ \mathbf{a}_2 \ \ \mathbf{a}_3|}$