Step-1

We need not out any zero to get 4×4 matrix A such that $\det A = 0$

For example consider

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 11 \end{bmatrix}$$

Here $\det A = 0$ since first two rows of A are dependent.

Step-2

Consider

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\det A = \begin{vmatrix} 0 & 3 & 0 \\ 4 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= -3 \begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix}$$

$$= -12$$

$$\neq 0$$

Step-3

And a diagonal matrix or matrixes that can be got into such form by row interchanges are the only non singular matrices with maximum number of zero entries.