Step-1

Consider the following linear programming problem

Minimize: $x+3y$

Subject to

$x+2y \geq 6$
$2x+y \geq 6$

Step-2

Now, consider the problem with four unknown (x, y, and two slack variables)

Therefore, we get the following vectors.

$$A=\begin{bmatrix}1 & 2 & -1 & 0\\ 2 & 1 & 0 & -1\end{bmatrix}$$

$$b=\begin{bmatrix}6\\ 6\end{bmatrix}$$

$$c=\begin{bmatrix}1 & 3 & 0 & 0\end{bmatrix}$$

Step-3

Thus, the original cost and the constraints gives

$$\begin{bmatrix}A & b\\ c & 0\end{bmatrix}=\begin{bmatrix}1 & 2 & -1 & 0 & 6\\ 2 & 1 & 0 & -1 & 6\\ 1 & 3 & 0 & 0 & 0\end{bmatrix}$$

Now, we exchange column 1 and 3 to put basic variable before free variables.

Tableau at point P is shown below

$$T=\begin{bmatrix}-1 & 2 & 1 & 0 & 6\\ 0 & 1 & 2 & -1 & 6\\ 0 & 3 & 1 & 0 & 0\end{bmatrix}$$

Perform the row transformation to get a fully reduced from

Perform multiplication of first row with 1, to give a unit pivot and use the second row to produce zeros in the second column as shown below

$$R=\begin{bmatrix}1 & 0 & 3 & -2 & 6\\ 0 & 1 & 2 & -1 & 6\\ 0 & 0 & -5 & 3 & -18\end{bmatrix}$$

Step-4

Now, consider $r=\begin{bmatrix}-5 & 3\end{bmatrix}$ at the bottom row.

It is observed that it has negative entry in the third column. Therefore, third variable will enter the basis.

And the first slack variable is pushed out of the basis.

Step-5

Let us exchange column 1 and 3 of new tableau

$$T_1=\begin{bmatrix}3 & 0 & 1 & -2 & 6\\ 2 & 1 & 0 & -1 & 6\\ -5 & 0 & 0 & 3 & -18\end{bmatrix}$$

Perform the row transformation to get a fully reduced from

Perform division of first row with 3, to give a unit pivot and use the first row to produce zeros in the second row as shown below

$$R_1=\begin{bmatrix}1 & 0 & \frac{1}{3} & -\frac{2}{3} & 2\\ 0 & 1 & -\frac{2}{3} & \frac{1}{3} & 2\\ 0 & 0 & \frac{5}{3} & -\frac{1}{3} & -8\end{bmatrix}$$

Now, consider $r=\begin{bmatrix}\frac{5}{3} & -\frac{1}{3}\end{bmatrix}$ at the bottom row.

It is observed that it has negative entry in the fourth column. Therefore, fourth variable will enter the basis.

And the second slack variable is pushed out of the basis.

Step-6

Let us exchange column 2 and 4 of new tableau

$$T_2 = \begin{bmatrix} 1 & -\frac{2}{3} & \frac{1}{3} & 0 & 2 \\ 0 & \frac{1}{3} & -\frac{2}{3} & 1 & 2 \\ 0 & -\frac{1}{3} & \frac{5}{3} & 0 & 8 \end{bmatrix}$$

Perform the row transformation to get a fully reduced from

Perform multiplication of first row with $-\frac{3}{2}$, to give a unit pivot and use the first row to produce zeros in the second row as shown below

$$R_2 = \begin{bmatrix} 1 & 0 & -1 & 2 & 6 \\ 0 & 1 & -2 & 3 & 6 \\ 0 & 0 & 1 & 1 & -6 \end{bmatrix}$$

Now, consider $r = [1 \quad 1]$ at the bottom row.

It is observed that it has all positive entries. Therefore, the stopping test is passed.

Thus, point *R* is optimum. And the optimum cost is 6