

## Step-1

If  $a, b, c$  are given with  $a \neq 0$ , we have to choose  $d$  so that

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = uv^T \quad \text{has rank 1 and we have to find the pivots.}$$

## Step-2

Suppose  $d$  is a real number such that

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = uv^T \quad \text{has rank 1.}$$

$$\begin{aligned} A &= \begin{bmatrix} a & b \\ c & d \end{bmatrix} \\ \frac{1}{a}R_1 &\begin{bmatrix} 1 & \frac{b}{a} \\ c & d \end{bmatrix} \\ \xrightarrow{R_2 - cR_1} &\begin{bmatrix} 1 & \frac{b}{a} \\ 0 & d - \left(\frac{cb}{a}\right) \end{bmatrix} \end{aligned}$$

## Step-3

$$A \text{ has rank 1 when } d - \left(\frac{cb}{a}\right) = 0$$

$$\Rightarrow \boxed{d = \frac{cb}{a}}$$

## Step-4

And

$$A = \begin{bmatrix} 1 & \frac{b}{a} \\ 0 & 0 \end{bmatrix}$$

This  $A$  can be written as

$$A = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & \frac{b}{a} \end{bmatrix}$$

$$= uv^T$$

## Step-5

Here

$$u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, v^T = \begin{bmatrix} 1 & \frac{b}{a} \end{bmatrix}$$

First column is a pivot column and  $a$  is the only pivot.