

## Step-1

Let  $\mathbf{V}$  be the subspace spanned by

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ 0 \end{bmatrix}.$$

The objective is to find a matrix  $A$  that has  $\mathbf{V}$  as its row space, and a matrix  $B$  that has  $\mathbf{V}$  as its nullspace.

For  $\mathbf{V}$  to be row space,

The transpose of  $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 0 & 0 & 0 \end{bmatrix}$  is,

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 5 & 0 \end{bmatrix}.$$

## Step-2

For nullspace,

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} 0 & -1 & -4 \\ 1 & 2 & 5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad R_1 \rightarrow R_1 - R_2.$$
$$\begin{bmatrix} 0 & -1 & -4 \\ 1 & 0 & -3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad R_2 \rightarrow R_2 + 2R_1.$$

Then,  $-x_2 - 4x_3 = 0, x_1 - 3x_3 = 0.$

$$\Rightarrow x_2 = -4x_3, x_1 = 3x_3.$$

## Step-3

Let  $x_3$  be free and  $x_1, x_2$  are pivot.

Then,  $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix} x_3.$

Therefore, the required matrix are  $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 5 & 0 \end{bmatrix}$  and  $B = \begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix}.$