

Step-1

Consider the following matrices,

$$A = \begin{bmatrix} 1 & & 1 & \\ & 1 & & i \\ 1 & & -1 & \\ & 1 & & -i \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & & 1 & \\ 1 & -1 & & \\ & & 1 & 1 \\ & & 1 & -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

On multiplication, we get,

$$\begin{aligned} ABC &= \begin{bmatrix} 1 & & 1 & \\ & 1 & & i \\ 1 & & -1 & \\ & 1 & & -i \end{bmatrix} \begin{bmatrix} 1 & 1 & & \\ 1 & -1 & & \\ & & 1 & 1 \\ & & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & i & -i \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -i & i \end{bmatrix} \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & i & -i \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -i & i \end{bmatrix} \end{aligned}$$

The matrix F is given by,

$$F = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & i^2 & i^3 \\ 1 & i^2 & i^4 & i^6 \\ 1 & i^3 & i^6 & i^9 \end{bmatrix}$$

By comparing F with ABC , you need to know that $i^2 = -1$ in the following entries,

$$\begin{aligned}
 i^9 &= i^8 i \\
 &= (i^2)^4 i \\
 &= (-1)^4 i \\
 &= i
 \end{aligned}$$

$$\begin{aligned}
 i^3 &= i^2 i \\
 &= (-1)i \\
 &= -i
 \end{aligned}$$

$$\begin{aligned}
 i^6 &= (i^2)^3 \\
 &= (-1)^3 \\
 &= -1
 \end{aligned}$$