Step-1

Consider the step function; $y(x)=\begin{cases}1, 0\le x\le \pi\\ 0, \pi<x<2\pi\end{cases}$

And Fourier coefficients;

$$a_0=\frac{(y,1)}{(1,1)}$$
$$a_1=\frac{(y,\cos x)}{(\cos x,\cos x)}$$
$$b_1=\frac{(y,\sin x)}{(\sin x,\sin x)}$$

To find the Fourier coefficients a_0, a_1, b_1 of the above step function.

Step-2

Now, calculate inner product as follows;

$$(y,1)=\int_0^{2\pi} y(x).1dx$$

$$=\int_0^{\pi}1.1dx+\int_{\pi}^{2\pi}0.1dx$$
$$=[x]_0^{\pi}+0$$
$$=\pi-0$$
$$=\pi$$

Also,

$$(1,1)=\int_0^{2\pi}1.1dx$$
$$=[x]_0^{2\pi}$$
$$=2\pi-0$$
$$=2\pi$$

Step-3

Thus, obtain Fourier coefficient by substitute values;

$$a_0 = \frac{(y,1)}{(1,1)}$$
$$= \frac{\pi}{2\pi}$$
$$= \frac{1}{2}$$

Therefore, $\boxed{a_0 = \frac{1}{2}}$

Step-4

Now, again calculate inner product;

$$(y, \cos x) = \int_0^{2\pi} y(x) \cos x dx$$

$$= \int_0^{\pi} 1.\cos x dx + \int_{\pi}^{2\pi} 0.\cos x dx$$
$$= [\sin x]_0^{\pi} + 0$$
$$= \sin \pi - \sin 0$$
$$= 0$$

Step-5

And,

$$(\cos x, \cos x) = \int_0^{2\pi} \cos x \cos x dx$$

$$= \int_0^{2\pi} \frac{1+\cos 2x}{2} dx$$
$$= \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right]_0^{2\pi}$$
$$= \frac{1}{2}\left[2\pi + \frac{\sin 4\pi}{2}\right] - \frac{1}{2}[0+0]$$
$$= \pi$$

Step-6

Therefore, substitute values obtained above to get;

$$a_1 = \frac{(y, \cos x)}{(\cos x, \cos x)}$$
$$= \frac{0}{\pi}$$
$$= 0$$

Hence, $\boxed{a_1 = 0}$

Step-7

Now,

$$(y, \sin x) = \int_0^{2\pi} y(x) \sin x dx$$

$$= \int_0^{\pi} 1.\sin x dx + \int_{\pi}^{2\pi} 0.\sin x dx$$
$$= [-\cos x]_0^{\pi} + 0$$
$$= -\cos \pi + \cos 0$$
$$= 2$$

Step-8

And,

$$(\sin x, \sin x) = \int_0^{2\pi} \sin x \sin x dx$$

$$= \int_0^{2\pi} \frac{1 - \cos 2x}{2} dx$$
$$= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{2\pi}$$
$$= \frac{1}{2}\left[2\pi - \frac{\sin 4\pi}{2}\right] - \frac{1}{2}[0 + 0]$$
$$= \pi$$

Step-9

Therefore,

$$b_1 = \frac{(y, \sin x)}{(\sin x, \sin x)}$$

$$= \frac{2}{\pi}$$

Hence, $\boxed{b_1 = \frac{2}{\pi}}$