

Step-1

Let $x = (x_1, x_2, x_3, \dots, x_n)$

$$x^T = \begin{pmatrix} x_1 \\ x_2 \\ - \\ - \\ - \\ x_n \end{pmatrix}$$

Suppose $x^T y = 0$ for every y

Step-2

Let $y_1 = (1, 0, 0, \dots, 0)$

$$x^T y_1 = 0 \Rightarrow$$

$$\begin{pmatrix} x_1 \\ x_2 \\ - \\ - \\ - \\ x_n \end{pmatrix} (1, 0, 0, \dots, 0) = 0$$

$$\Rightarrow x_1 = 0$$

Step-3

Similarly, $y_2 = (0, 1, \dots, 0)$ is such that $x^T y_2 = 0$ gives $x_2 = 0$

In the same way, $x_3 = x_4 = x_5 = \dots = x_n = 0$

Putting these things together, we can say $x^T y = 0$ for every y , then $x = 0$