Step-1

4764-1.6-65P AID: 124

RID: 175 | 3/12/12

Suppose $G_1 = \left\{ \begin{bmatrix} 1 & 0 & 0 & 0 \\ l_{21} & 1 & 0 & 0 \\ - & - & 1 & 0 \\ l_{m1} & - & l_{m(m-1)} & 1 \end{bmatrix} \right\}$ the set of lower triangular matrices of square order *m* having 1â€™s on the diagonal.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ l_{21} & 1 & 0 & 0 \\ - & - & 1 & 0 \\ l_{m1} & - & l_{m(m-1)} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ k_{21} & 1 & 0 & 0 \\ - & - & 1 & 0 \\ k_{m1} & - & k_{m(m-1)} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ l_{21}+k_{21} & 1 & 0 & 0 \\ - & - & 1 & 0 \\ l_{m1}+l_{m2}k_{21}+\ldots+k_{m1} & - & l_{m(m-1)}+k_{m(m-1)} & 1 \end{bmatrix}$$

This shows that the product of lower triangular matrices of order *m* is a lower triangular matrix of order *m*.

Inverse of $\begin{bmatrix} 1 & 0 & 0 & 0 \\ l_{21} & 1 & 0 & 0 \\ - & - & 1 & 0 \\ l_{m1} & - & l_{m(m-1)} & 1 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -l_{21} & 1 & 0 & 0 \\ - & - & 1 & 0 \\ (-1)^{m-1} l_{21}l_{32}\ldots l_{m(m-1)}+\ldots & - & -l_{m(m-1)} & 1 \end{bmatrix}$ which is also a lower triangular matrix having 1â€™s on the main diagonal.

Further, the identity matrix is also a lower triangular matrix.

Associativity of product of lower triangular matrices is lower triangular.

Using all these properties, we see that the set of all square lower triangular matrices of order *m* form a group.

We observe that $G_2:$ the diagonal invertible matrices have no diagonal entry 0

So, the product of such matrices is also a diagonal matrix having no diagonal entry 0.

Therefore, closure law holds under multiplication of matrices

Associativity of matrix multiplication holds for any matrices.

Already it is given that G_2 is invertible and the inverse matrix is also a diagonal matrix having no entry zero on the main diagonal.

We easily see that the identity matrix of order *m* is also a diagonal matrix having no entry 0 on the main diagonal.

Therefore, G_2 is a group under multiplication of matrices.

Step-2

Symmetric matrices is not a Group

Let us consider the symmetric matrices $A=\begin{pmatrix}2 & 1\\1 & 3\end{pmatrix}; B=\begin{pmatrix}4 & 2\\2 & 0\end{pmatrix}$

$AB=\begin{pmatrix}2 & 1\\1 & 3\end{pmatrix}\begin{pmatrix}4 & 2\\2 & 0\end{pmatrix}$

$=\begin{pmatrix}10 & 4\\10 & 2\end{pmatrix}$ is not a symmetry matrix.

That is the product of symmetric matrices does not obey closure law.

Step-3

A positive matrix is a square matrix with all eigen values positive.

Their product is also a positive matrix, inverse of a positive matrix is positive and identity matrix is a positive matrix and associativity under multiplication holds.

Therefore, the set of positive matrices form a group.

Step-4

The product of permutation matrix is a square matrix with each row has only one entry 1 and other entries zero, similarly with columns.

The product of permutation matrices is a permutation matrix and the inverse of a permutation matrix is a permutation matrix.

Therefore, the permutation matrices form a group

Step-5

Further, the set of matrices of same order, non singular form a group under multiplication of matrices.