Step-1

Given that $A=\begin{bmatrix}1 & -1\\ 2 & 4\end{bmatrix}$

To find the Eigen values and Eigen vectors of $B=A-7I$ and their relation with those of A

Step-2

Given;

$$B=A-7I$$
$$=\begin{bmatrix}-6 & -1\\ 2 & -3\end{bmatrix}$$

The number λ is Eigen value of B if and only if $|B-\lambda \mathrm{I}|=0$

This implies;

$$\begin{vmatrix}-6-\lambda & -1\\ 2 & -3-\lambda\end{vmatrix}=0$$

This implies;

$$(-6-\lambda)(-3-\lambda)-(-1)(2)=0$$
$$\lambda^2+9\lambda+20=0$$
$$(\lambda+4)(\lambda+5)=0$$

Thus;

$\lambda=-4$ and $\lambda=-5$

Step-3

If $\lambda=-4$ and $(B-\lambda \mathrm{I})x_1=0$

This implies;

$$\begin{bmatrix}-2 & -1\\ 2 & 1\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$
$$2y+z=0$$
$$\frac{y}{-1}=\frac{z}{2}$$

The eigen vector is any non-zero multiple of x_1

Eigen vector for $\lambda_1 = -4$ is $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$.

Step-4

If $\lambda_2 = -5$ and $(B - \lambda_2 I)x_2 = 0$

This implies;

$$\begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$y + z = 0$$
$$\frac{y}{-1} = \frac{z}{1}$$

The Eigen vector is any non zero multiple of x_2.

Eigen vector for $\lambda_2 = -5$ *is* $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

Step-5

Step-6

Now, calculate Eigen values of $A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$

The number λ is Eigen value of A if and only if $|A - \lambda I| = 0$

This implies;

$$\begin{vmatrix} 1-\lambda & -1 \\ 2 & 4-\lambda \end{vmatrix} = 0$$

This implies;

$$(1-\lambda)(4-\lambda) - (-1)(2) = 0$$
$$\lambda^2 - 5\lambda + 6 = 0$$
$$(\lambda - 2)(\lambda - 3) = 0$$

Thus,

$\lambda = 2$and
λ=3

Step-7

If $\lambda = 2$ and $(A - \lambda \mathrm{I})x_1 = 0$

This implies;

$$\begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2y + 2z = 0$$

$$\frac{y}{-1} = \frac{z}{1}$$

The Eigen vector is any non-zero multiple of x_1

Eigen vector for $\lambda_1 = 2$ is $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

Step-8

If $\lambda = 3$ and $(A - \lambda \mathrm{I})x_1 = 0$

This implies;

Step-9

$$\begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2y + z = 0$$

$$\frac{y}{-1} = \frac{z}{2}$$

The Eigen vector is any non-zero multiple of x_1

Eigen vector for $\lambda_1 = 3$ is $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$.

Step-10

From the above Eigen values of A and B, it is clear that both the Eigen values are $\boxed{\text{reduced by 7}}$ to that of A and there is $\boxed{\text{no change}}$ in their corresponding Eigen vectors.