Step-1

Given equation is $x_1 + x_2 + x_3 + x_4 = 0$

Suppose S is the subspace of $\mathbf{R}^4$ spanned by this equation.

This can be easily followed that $\mathbf{P} = \{u = (1,1,1,1) : 1 \in \mathbf{R}\}$

By the definition of the orthogonal complement, we have $x \cdot 1 + y \cdot 1 + z \cdot 1 + w \cdot 1 = 0$

$\Rightarrow w = -x - y - z$

$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = t\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} + p\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} + q\begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$ where $x = t$, $y = p$, $z = q$ are the parameters.

Therefore, $\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$ span $\mathbf{P}$ and $\mathbf{P}^{\perp}$ is spanned by $\{(1, 1, 1, 1)\}$.

Observe that dim $S = 1$ and dim $S^{\perp} = 3$

Dim S + dim $S^{\perp} = 4 =$ dim $\mathbf{R}^4$

Step-2

The matrix in which $\mathbf{P}$ is the null space is $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$