Step-1

(a) A real symmetric matrix is given by

$$A^H = A$$

Consider an *n* by *n* symmetric matrix, then there are *n* entries on the diagonal and $\binom{(n-1)+\cdots+1}{n}$ entries above the diagonal that can be chosen arbitrary.

Since, other $1+\cdots+(n-1)$ entries below the diagonal are determined by the symmetry of the matrix.

Therefore, there are only $n + (n-1) + \cdots + 1 = \frac{1}{2}n(n+1)$ degrees of freedom in the selection of the n^2 entries in a n by n real symmetric matrix.

Therefore, degrees of freedom of an *n* by *n* symmetric matrix is $\left|\frac{1}{2}n(n+1)\right|$.

Step-2

Consider an n by n diagonal matrix, then there are n entries on the diagonal and it can be chosen arbitrary.

Therefore, degrees of freedom of an n by n diagonal matrix is \boxed{n} .

Step-3

Consider the entries of ith and jth columns of an orthogonal matrix, as a_{ki} and a_{ij} respectively.

Then by using the condition of orthogonal we have

$$(a_{ki}, a_{lj}) = 0$$
, here $i \neq j$

This condition show a symmetric, so the degree of freedom is given by $\frac{n(n-1)}{2}$.

And

$$(a_{ki}, a_{li}) = 1$$
, for $i = 1, \dots, n$

This condition show the degree of freedom is given by n.

We know that an *n* by *n* matrix has n^2 degrees of freedom.

To find the total degrees of freedom of an n by orthogonal matrix, do the following calculation:

$$n^{2} - \frac{1}{2}(n)(n-1) - n = \frac{2n^{2} - n^{2} + n - 2n}{2}$$
$$= \frac{n^{2} - n}{2}$$
$$= \frac{(n)(n-1)}{2}$$

Step-4

Therefore, degrees of freedom of an *n* by *n* orthogonal matrix is $\frac{(n)(n-1)}{2}$

Step-5

(b) We know that Hermitian matrix A is equal to their conjugate transpose.

$$A^{H} = \overline{A}^{T}$$

Every entry in the transposed matrix is same as the complex conjugate of the corresponding entry in the original matrix.

$$\left(A^H\right)_{ij} = \overline{A_{ji}}$$

Step-6

Consider a 3 by 3 matrix A as follows:

$$A = \begin{bmatrix} a & b+ic & e+if \\ b-ic & d & h+ik \\ e-if & h-ik & g \end{bmatrix}$$

Here a,b,c,d,e,f,g,h,k all are real numbers.

Now to find the transpose matrix, change A_{ij} by A_{ji} .

$$A^{T} = \begin{bmatrix} a & b-ic & e-if \\ b+ic & d & h-ik \\ e+if & h+ik & g \end{bmatrix}$$

Taking the complex conjugate of entry, we get

$$\overline{A}^{T} = \begin{bmatrix} a & b+ic & e+if \\ b-ic & d & h+ik \\ e-if & h-ik & g \end{bmatrix}$$

Hence we get $A = \overline{A}^T$.

Here nine real numbers determine the Hermitian matrices *A*, so the real degrees of freedom are 9.

Therefore, 3 by 3 Hermitian matrices A have $\boxed{9}$ real degrees of freedom.

Step-7

We know that a complex n by n matrix has $2n^2$ degrees of freedom.

Since the unitary matrix U satisfied that

$$UU^H = I$$

So, we get n^2 terms in a unitary matrix.

Hence, n by n unitary matrix has n^2 real degrees of freedom.

Therefore, 3 by 3 unitary matrices A have $\boxed{9}$ real degrees of freedom.