

## Step-1

The objective is to find the inverse of the matrices.

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 3 & 0 & 0 \\ 4 & 0 & 0 & 0 \end{bmatrix}.$$

The matrix is

Use gauss-Jordan elimination process to find the inverse of the matrices.

Write the echelon form of the matrix as:

$$[A_1 \quad e_1 \quad e_2 \quad e_3 \quad e_4] = \left[ \begin{array}{cccc|cccc} 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 & 1 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right].$$

## Step-2

Swap rows 1 and 4.

$$\left[ \begin{array}{cccc|cccc} 4 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{array} \right].$$

Divide row 1 by 4.

$$\left[ \begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{array} \right].$$

## Step-3

Swap rows 1 and 4.

$$\left[ \begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 3 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{array} \right].$$

Divide row 2 by 3 and row 3 by 2.

$$\left[ \begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 1 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{array} \right].$$

$$A_1^{-1} = \left[ \begin{array}{cccc} 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array} \right].$$

Therefore, the inverse of  $A_1$  is

#### Step-4

$$A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 & 0 \\ 0 & -\frac{2}{3} & 1 & 0 \\ 0 & 0 & -\frac{3}{4} & 1 \end{bmatrix}.$$

The matrix is

Use gauss-Jordan elimination process to find the inverse of the matrices.

Write the echelon form of the matrix as:

$$\begin{bmatrix} A_2 & e_1 & e_2 & e_3 & e_4 \end{bmatrix} = \left[ \begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -\frac{2}{3} & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{3}{4} & 1 & 0 & 0 & 0 & 1 \end{array} \right] \\ = \begin{bmatrix} U & L^{-1} \end{bmatrix}$$

## Step-5

The upper triangular  $U$  appears in the first three columns. The other three columns are the same as  $L^{-1}$ .

The second half will go from  $U$  to  $I$ .

That takes  $L^{-1}$  to  $U^{-1}L^{-1}$  which is  $A^{-1}$ .

## Step-6

Add  $\frac{1}{2}$  times row 1 to row 2.

$$\left[ \begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{2} & 1 & 0 & 0 \\ 0 & -\frac{2}{3} & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{3}{4} & 1 & 0 & 0 & 0 & 1 \end{array} \right]$$

Add  $\frac{2}{3}$  times row 2 to row 3.

$$\left[ \begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{2} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ 0 & 0 & -\frac{3}{4} & 1 & 0 & 0 & 0 & 1 \end{array} \right]$$

## Step-7

Add  $\frac{3}{4}$  times row 3 to row 4.

$$\left[ \begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{2} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ 0 & 0 & 0 & 1 & \frac{1}{4} & \frac{1}{2} & \frac{3}{4} & 1 \end{array} \right]$$

$$A_2^{-1} = \left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{3}{4} & 1 \end{array} \right].$$

Therefore, the inverse of  $A_2$  is

## Step-8

$$A_3 = \begin{bmatrix} a & b & 0 & 0 \\ c & d & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & c & d \end{bmatrix}.$$

The matrix is

Use gauss-Jordan elimination process to find the inverse of the matrices.

Write the echelon form of the matrix as:

$$[A_3 \quad e_1 \quad e_2 \quad e_3 \quad e_4] = \left[ \begin{array}{cccc|cccc} a & b & 0 & 0 & 1 & 0 & 0 & 0 \\ c & d & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & a & b & 0 & 0 & 1 & 0 \\ 0 & 0 & c & d & 0 & 0 & 0 & 1 \end{array} \right].$$

Multiply row 1 by  $\frac{1}{a}$ .

$$\left[ \begin{array}{cccc|cccc} 1 & \frac{b}{a} & 0 & 0 & \frac{1}{a} & 0 & 0 & 0 \\ c & d & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & a & b & 0 & 0 & 1 & 0 \\ 0 & 0 & c & d & 0 & 0 & 0 & 1 \end{array} \right]$$

## Step-9

Add  $-c$  times row 1 to row 2.

$$\left[ \begin{array}{cccc|cccc} 1 & \frac{b}{a} & 0 & 0 & \frac{1}{a} & 0 & 0 & 0 \\ 0 & \frac{da-cb}{a} & 0 & 0 & -\frac{c}{a} & 1 & 0 & 0 \\ 0 & 0 & a & b & 0 & 0 & 1 & 0 \\ 0 & 0 & c & d & 0 & 0 & 0 & 1 \end{array} \right]$$

Multiply row 2 by  $\frac{a}{ad-bc}$ .

$$\left[ \begin{array}{cccc|cccc} 1 & \frac{b}{a} & 0 & 0 & \frac{1}{a} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{c}{ad-bc} & \frac{a}{ad-bc} & 0 & 0 \\ 0 & 0 & a & b & 0 & 0 & 1 & 0 \\ 0 & 0 & c & d & 0 & 0 & 0 & 1 \end{array} \right]$$

## Step-10

Add  $-\frac{1}{ab}$  times row 2 to row 1.

$$\left[ \begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{d}{ad-bc} & -\frac{b}{ad-bc} & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{c}{ad-bc} & \frac{a}{ad-bc} & 0 & 0 \\ 0 & 0 & a & b & 0 & 0 & 1 & 0 \\ 0 & 0 & c & d & 0 & 0 & 0 & 1 \end{array} \right]$$

Multiply row 3 by  $\frac{1}{a}$ .

$$\left[ \begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{d}{ad-bc} & -\frac{b}{ad-bc} & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{c}{ad-bc} & \frac{a}{ad-bc} & 0 & 0 \\ 0 & 0 & 1 & \frac{b}{a} & 0 & 0 & \frac{1}{a} & 0 \\ 0 & 0 & c & d & 0 & 0 & 0 & 1 \end{array} \right]$$

## Step-11

Add  $-c$  times row 3 to row 4.

$$\left[ \begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{d}{ad-bc} & -\frac{b}{ad-bc} & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{c}{ad-bc} & \frac{a}{ad-bc} & 0 & 0 \\ 0 & 0 & 1 & \frac{b}{a} & 0 & 0 & \frac{1}{a} & 0 \\ 0 & 0 & 0 & \frac{ad-bc}{a} & 0 & 0 & -\frac{c}{a} & 1 \end{array} \right]$$

Multiply row 4 by  $\frac{a}{ad-bc}$ .

$$\left[ \begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{d}{ad-bc} & -\frac{b}{ad-bc} & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{c}{ad-bc} & \frac{a}{ad-bc} & 0 & 0 \\ 0 & 0 & 1 & \frac{b}{a} & 0 & 0 & \frac{1}{a} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -\frac{c}{ad-bc} & \frac{a}{ad-bc} \end{array} \right]$$

## Step-12

Add  $-\frac{1}{ab}$  times row 4 to row 3.

$$\left[ \begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{d}{ad-bc} & -\frac{b}{ad-bc} & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{c}{ad-bc} & \frac{a}{ad-bc} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \frac{d}{ad-bc} & -\frac{b}{ad-bc} \\ 0 & 0 & 0 & 1 & 0 & 0 & -\frac{c}{ad-bc} & \frac{a}{ad-bc} \end{array} \right]$$

$$A_3^{-1} = \begin{bmatrix} \frac{d}{ad-bc} & -\frac{b}{ad-bc} & 0 & 0 \\ -\frac{c}{ad-bc} & \frac{a}{ad-bc} & 0 & 0 \\ 0 & 0 & \frac{d}{ad-bc} & -\frac{b}{ad-bc} \\ 0 & 0 & -\frac{c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}.$$

Therefore, the inverse of  $A_3$  is