## Step-1

(i) For the matrix $A=\begin{bmatrix}3 & 5\\ 6 & 9\end{bmatrix}$

Cofactor of 3 = 9

Cofactor of 5 = â€“6

Cofactor of $6=-5$

Cofactor of 9 = 3

So that the matrix of cofactors is

$$C_A=\begin{bmatrix}9 & -6\\ -5 & 3\end{bmatrix}$$

## Step-2

And

$$\begin{aligned}\det A &= 27-30\\ &= -3\end{aligned}$$

$\neq 0$

## Step-3

Hence inverse of A exists and

$$A^{-1}=\frac{1}{\det A}.C_A{}^T$$

$$=\frac{1}{-3}\begin{bmatrix}9 & -5\\ -6 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-3 & \frac{5}{3}\\ 2 & -1\end{bmatrix}$$

And null space of $A=\left\{\begin{bmatrix}0\\ 0\end{bmatrix}\right\}$

## Step-4

(ii) For the matrix

$$A=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$$

Cofactor of $\cos\theta=\cos\theta$

Cofactor of $-\sin\theta=-\sin\theta$

Cofactor of $\sin\theta=\sin\theta$

Cofactor of $\cos\theta=\cos\theta$

## Step-5

So that the matrix of cofactors is

$$C_A=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$$

And

$$\begin{aligned}\det A&=\cos^2\theta+\sin^2\theta\\ &=1\\ &\neq 0\end{aligned}$$

## Step-6

Hence inverse of A exists and

$$A^{-1}=\frac{1}{\det A}C_A^{\,T}$$

$$=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$$

Note: observe that $A^{-1}$ is the transpose of A.

Null space of $A=\left\{\begin{bmatrix}0\\0\end{bmatrix}\right\}$

## Step-7

(iii) $A=\begin{bmatrix} a & b \\ a & b \end{bmatrix}$

Row1: cofactor of $a$ = $b$

Cofactor of $b$ = â€“$a$

Row 2: cofactor of $a=-b$

Cofactor of $b=a$

## Step-8

So that the matrix of cofactors is

$$C_A=\begin{bmatrix} b & -a \\ -b & a \end{bmatrix}$$

Here $\det A=\left\{\begin{bmatrix} x \\ y \end{bmatrix} / \, ax+by=0\right\}$

Here A is singular. So, $A^{-1}$ do not exist.