

Step-1

If x is in the null space, then $Ax = 0$.

If v is the row space, it is a combination of the rows: $v = A^T z$ for some z . now, in one line:

Null space \perp Row space

Step-2

Given $\alpha = (1, 1, 2)$, $\beta = (1, 2, 3)$

In view of the above notes, if we find the null space of the matrix whose rows are these vectors, then the basis of the null space will be the vector orthogonal to both these vectors.

For, we consider $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

Applying $R_2 \rightarrow R_2 - R_1$ on this, we get $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix}$

This is the row reduced form

So, we rewrite the homogeneous equations from this.

i.e.,

$$\begin{aligned} x + y + 2z &= 0 \\ y + z &= 0 \end{aligned}$$

So, $z = -y = -x$ is the possible solution.

Therefore, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = y \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ spans the null space the given row vectors.

Thus, the required homogeneous equation is $\boxed{x + y - z = 0}$