

## Step-1

Given that  $A = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$  yields shearing transformation which leaves the  $y$ -axis unchanged.

We have to sketch the effect of  $A$  on  $x$ -axis by indicating what happens to  $(1,0)$ ,  $(2,0)$  and  $(-1,0)$  and verify how the whole axis is transformed.

## Step-2

Let  $x_1 = (1,0)$ ,  $x_2 = (2,0)$ ,  $x_3 = (-1,0)$

Then

$$\begin{aligned} Ax_1 &= \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} 1(1) + 0(0) \\ 3(1) + 1(0) \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ 3 \end{bmatrix} \end{aligned}$$

## Step-3

And

$$\begin{aligned} Ax_2 &= \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} 1(2) + 0(0) \\ 3(2) + 1(0) \end{bmatrix} \\ &= \begin{bmatrix} 2 \\ 6 \end{bmatrix} \end{aligned}$$

## Step-4

And

$$\begin{aligned}
 Ax_3 &= \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \end{bmatrix} \\
 &= \begin{bmatrix} 1(-1) + 0(0) \\ 3(-1) + 1(0) \end{bmatrix} \\
 &= \begin{bmatrix} -1 \\ -3 \end{bmatrix}
 \end{aligned}$$

Therefore, the vectors  $(1,0), (2,0), (-1,0)$  transformed to  $(1,3), (2,6), (-1,-3)$ .

The  $x$ -axis turns vertical lines shift up/down but stay vertical.

The sketch is as shown below.

## Step-5

