# Step-1

**(a)**

$A$ is a matrix of $m$ rows and $n$ columns

It is known that the maximum number of linearly independent rows = maximum number of linearly independent columns = min $\{m, n\}$.

Given that the rows of $A$ are linearly independent. So, rank is $m$.

Further, it follows that for $m \leq n$.

The rank of column space is also $m$.

Consequently, the column space is $R^m$

The rank of the left null space is $n$ â€“ $m$.

Therefore, the left null space is $\boxed{R^{n-m}}$

# Step-2

**(b)**

Given that $A$ is a matrix of size $8\times 10$

That means the number of rows is 8 and columns is 10

Given that the null space of $A$ is of dimension 2

So, the dimension of range space of $A$ is $10-2=8$

Let, consider $Ax=b$

Then $x$ is of size $10\times 1$ and $b$ is of size $8\times 1$

Since the rank of $A$ is 8, it can be written the two variables of $x$ as the linear combinations of remaining 8 variables.

Therefore, the solution set has two linearly independent columns and each column has 10 entries in which the bottom two entries are zero denoting $x_9, x_{10}$.

Let them be $b_1, b_2$

# Step-3

Suppose $b(8\times 1)$ is any non zero vector.

( free var )

Then it can be seen that the matrix $[b_1\ b_2\ b]$ is of size $10\times 3$ and ~~bottom two rows are necessarily zero.~~

So, the non zero rows of this matrix are 8 and columns are 3

By the properties of matrices, it can be said that the number of linearly dependent rows is less than 3

In other words, it is not necessary that one column is dependent of other four columns for the infinite possible columns *b*.

More precisely, every column b cannot necessarily be written as a linear combination of the solutions of $Ax = b$.

Therefore, every vector $b$ cannot be in the form $Ax$.

Therefore, the given statement is **false**.