Step-1

Let $c_1v_1 + c_2v_2 + c_3v_3 = 0$

$$\Rightarrow c_1\begin{bmatrix}1\\0\\0\end{bmatrix} + c_2\begin{bmatrix}1\\1\\0\end{bmatrix} + c_3\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}c_1 + c_2 + c_3\\c_2 + c_3\\c_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

Step-2

i.e. $c_3 = 0$

Plug this in the following equation.

$$c_2 + c_3 = 0$$
$$\Rightarrow c_2 = 0$$

Plug these values in the following equation.

$$c_1 + c_2 + c_3 = 0$$
$$\Rightarrow c_1 = 0$$

Therefore, $\boxed{c_1 = c_2 = c_3 = 0}$

Therefore v_1, v_2, v_3 are linearly independent.

Step-3

Now,

let $c_1v_1 + c_2v_2 + c_3v_3 + c_4v_4 = 0$

$$\Rightarrow c_1\begin{bmatrix}1\\0\\0\end{bmatrix} + c_2\begin{bmatrix}1\\1\\0\end{bmatrix} + c_3\begin{bmatrix}1\\1\\1\end{bmatrix} + c_4\begin{bmatrix}2\\3\\4\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}c_1 + c_2 + c_3 + 2c_4\\c_2 + c_3 + 3c_4\\c_3 + 4c_4\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

Step-4

i.e. $c_3 + 4c_4 = 0$

$\Rightarrow c_3 = -4c_4$

Plug this value in the following equation.

$$\begin{aligned}
c_2 + c_3 + 3c_4 &= 0 \\
c_2 &= -c_3 - 3c_4 \\
&= +4c_4 - 3c_4 \\
&= c_4
\end{aligned}$$

Plug this value in the following equation.

$$\begin{aligned}
&c_1 + c_2 + c_3 + 2c_4 \\
&c_1 = -c_2 - c_3 - 2c_4 \\
&\quad = -c_4 + 4c_4 - 2c_4 \\
&c_1 = c_4
\end{aligned}$$

If $c_4 = 1$, then $c_1 = 1, c_2 = 1, c_3 = -4$

$v_1 + v_2 - 4v_3 + v_4 = 0$

Therefore, $\boxed{v_1 + v_2 - 4v_3 + v_4 = 0}$

$\Rightarrow v_1, v_2, v_3, v_4$ are linearly dependent