

## Step-1

The give inequality is,

$$\max_x \min_y yAx = \min_y yAx^* \leq y^* Ax^* \leq \max_x y^* Ax = \min_y \max_x yAx$$

## Step-2

In a payoff matrix A, the total expected payoff of X will be  $\sum_j \sum_i x_j a_{ij} y_i$  where  $a_{ij}$  is the element of the matrix where option  $j$  is chosen by X and option  $i$  is chosen by Y, with the probabilities of  $x_j, y_i$ .

## Step-3

As it is the payoff for X, it wants to maximize this payoff  $yAx$  but the player Y wants to minimize it. Hence, player Y will minimize the value of  $\max_x yAx$ .

Similarly, X will maximize the value of  $\min_y yAx$ .

The maximum of the minimized value of  $\min_y yAx$  will be smaller than or equal to minimum of maximized value of  $\max_x yAx$ .

In between these two values there will a point which is optimal for both X and Y, that is,  $y^* Ax^*$ .

This can be written as  $\min_y yAx^* \leq y^* Ax^* \leq \max_x y^* Ax$ .

Using minimax theorem we get,

$$\max_x \min_y yAx = \min_y \max_x yAx$$

Using these two equations we get,

$$\max_x \min_y yAx = \min_y yAx^* \leq y^* Ax^* \leq \max_x y^* Ax = \min_y \max_x yAx$$

## Step-4

There will be optimal values of  $x$  and  $y$  known as  $x^*$  and  $y^*$ .

The maximum value player X wants to attain is  $yAx^*$ .

The minimum value payer Y wants to attain is  $y^* Ax$ .

Hence, the equation  $\min_y yAx^* \leq y^* Ax^* \leq \max_x y^* Ax$  can be written as

$$y^* Ax \leq y^* Ax^* \leq yAx^*$$

