Step-1

Suppose P is the projection matrix onto the subspace $\mathbf{S}$ and Q is the projection matrix onto the orthogonal complement $\mathbf{S}^{\perp}$

Find $P+Q$, PQ .

Step-2

Since P is a projection onto the subspace $\mathbf{S}$.

That means $P^2 = P$

Since Q is the projection matrix onto the orthogonal complement $\mathbf{S}^{\perp}$

So $Q = I - P$

Now

$$\begin{aligned} P+Q &= P+(I-P) \\ &= I \end{aligned}$$

Step-3

And

$$\begin{aligned} PQ &= P(I-P) \\ &= PI - P^2 \\ &= P - P \qquad \text{since } P^2 = P \\ &= 0 \end{aligned}$$

Therefore, $\boxed{P+Q = I \text{ and } PQ = 0}$, where I is the identity matrix.

Step-4

Show that $P-Q$ is its own inverse.

Now,

$$
\begin{aligned}
(P-Q)(P-Q) &= P^2 - PQ - QP + Q^2 \\
&= P - 0 - 0 + (I-P)(I-P) && \begin{pmatrix} \text{Since } P^2 = P, Q = I - P \\ PQ = 0 \end{pmatrix} \\
&= P + I - IP - PI + P^2 \\
&= P + I - P - P + P && (\text{Since } IP = PI = P) \\
&= I
\end{aligned}
$$

By the well-known property, if $AB = I$ then $A^{-1} = B$.

Here, obtained that $(P-Q)(P-Q) = I$ then $(P-Q)^{-1} = P - Q$.

Hence $P - Q$ is its own inverse.