Step-1

Consider the following problem:

Minimize cost: $x_1 + x_2 - x_3$

Subject to:

$$\begin{aligned} 2x_1 - 4x_2 + x_3 + x_4 \quad &= 4 \\ 3x_1 + 5x_2 + x_3 \quad + x_5 &= 2 \end{aligned}$$

Determine which x_i will enter the basis and which x_i will leave. Compute new pair of basic variables and cost at new corner. Decide and prepare for next step.

Step-2

Start from the corner, $x_4 = 4$ and $x_5 = 2$ are basic variables. At that corner x_1, x_2, x_3 will be zero. Entering variable will be x_3 as it has negative cost coefficient. As x_3 will enter x_4 or x_5 must leave.

In first equation when $x_3 + x_4 = 4$, x_4 can get maximum value 4 and in second equation when $x_3 + x_5 = 2$, x_5 can get value 2. So choose minimum value, this means that x_5 will be the leaving variable.

Step-3

So, the new corner is:

$$x = (0, 0, 2, 4, 0)$$

New cost at the corner:

$$\begin{aligned} x_1 + x_2 - x_3 &= 0 + 0 - 2 \\ &= -2 \end{aligned}$$

New basic variables will be x_3, x_4.

Step-4

Now, x_3, x_4 should stand by themselves, so put the following into the cost function and in the first equation.

$$x_3 = 2 - 3x_1 - 5x_2 - x_5$$

Cost function:

$$\begin{aligned} x_1 + x_2 - x_3 &= x_1 + x_2 - (2 - 3x_1 - 5x_2 - x_5) \\ &= 4x_1 + 6x_2 + x_5 - 2 \end{aligned}$$

Equation:

$$\begin{aligned} 2x_1 - 4x_2 + \left(2 - 3x_1 - 5x_2 - x_5\right) + x_4 \quad &= 4 \\ -x_1 - 9x_2 + x_4 - x_5 &= 2 \\ 3x_1 + 5x_2 + x_3 \qquad + x_5 &= 2 \end{aligned}$$

Step-5

Therefore, new problem from new corner is:

Minimize cost: $4x_1 + 6x_2 + x_5 - 2$

Subject to:

$$\begin{aligned} -x_1 - 9x_2 + x_4 - x_5 &= 2 \\ 3x_1 + 5x_2 + x_3 \qquad + x_5 &= 2 \end{aligned}$$

Here, cost coefficients are positive so the corner gives the feasible value.

Step-6

Therefore, following can be said:

$$\boxed{x^* = (0,0,2,4,0)}$$

The new cost is:

$$\boxed{x_1 + x_2 - x_3 = -2}$$