Step-1

We have to prove that the trace of $P = \dfrac{aa^T}{a^T a}$ is always 1.

Let $a = \left(a_1, a_2, ..., a_n\right)$

$$aa^T = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \begin{bmatrix} a_1 & a_2 & \cdots & a_n \end{bmatrix}$$

$$= \begin{bmatrix} a_1^{\ 2} & a_1a_2 & \cdots & a_1a_n^{\ 2} \\ a_2a_1 & a_2^{\ 2} & \cdots & a_2a_n \\ \cdots & \cdots & \cdots & \cdots \\ a_na_1 & a_na_1 & \cdots & a_n^{\ 2} \end{bmatrix}$$

Step-2

$$a^T a = \begin{bmatrix} a_1 & a_2 & \cdots & a_n \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$$

$$= a_1^2 + a_2^2 + \cdots + a_n^2$$
$$= \sum a_1^2$$

Step-3

The projection matrix

$$P = \frac{aa^T}{a^T a}$$

$$= \frac{1}{\sum a_1^2} \begin{bmatrix} a_1^{\ 2} & a_1a_2 & \cdots & a_1a_n^{\ 2} \\ a_2a_1 & a_2^{\ 2} & \cdots & a_2a_n \\ \cdots & \cdots & \cdots & \cdots \\ a_na_1 & a_na_1 & \cdots & a_n^{\ 2} \end{bmatrix}$$

Step-4

Trace P = Sum of its diagonal elements

$$
\begin{aligned}
&= \frac{a_1 a_1}{\sum a_1^2} + \frac{a_2 a_2}{\sum a_1^2} + \cdots + \frac{a_n a_n}{\sum a_1^2} \\
&= \frac{a_1^2 + a_2^2 + \cdots + a_n^2}{\sum a_1^2} \\
&= \frac{\sum a_1^2}{\sum a_1^2} \\
&= 1
\end{aligned}
$$

Hence trace of P is $\boxed{1}$