

Step-1

(a) $BA = 4A$

Multiply with A^{-1} on both side

We have

$$\Rightarrow (BA).A^{-1} = (4A).A^{-1}$$

$$\Rightarrow B(A.A^{-1}) = 4(A.A^{-1})$$

Since by associative property

$$\Rightarrow B.I = 4.I$$

$$\Rightarrow \boxed{B = 4I}$$

Step-2

(b) $BA = 4B$

$$B.A = 4B, \text{ if } \boxed{B=0}$$

Step-3

(c) BA has row 1 and 3 of A are reversed and row 2 unchanged.

$$B \text{ must be } \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Example:-

$$\begin{aligned} BA &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{pmatrix} \\ &= \begin{pmatrix} 5 & 6 & 7 \\ 3 & 4 & 5 \\ 1 & 2 & 3 \end{pmatrix} \end{aligned}$$

Step-4

(d) all rows of BA are the same as row 1 of A

B must be $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

Let $A = \begin{pmatrix} 5 & 6 & 7 \\ 3 & 4 & 5 \\ 1 & 2 & 3 \end{pmatrix}$

$$BA = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 5 & 6 & 7 \\ 3 & 4 & 5 \\ 1 & 2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$

Hence all the rows of BA same as row 1 of A.