Step-1

Matrices A and B are similar if there exists an invertible matrix M such that $A = M^{-1}BM$.

Similar matrices have same Eigen values. That is if two matrices have different Eigen values, then they are not similar.

The objective is to state the reason for the given true or false statements.

Step-2

(a)

An invertible matrix can not be similar to a singular matrix.

Singular matrices have determinant value zero. This means that only Eigen value that can satisfy $Ax = 0$ is $\lambda = 0$.

However invertible matrix cannot have zero, $\lambda \neq 0$, as an Eigen value.

Therefore, the statement- an invertible matrix cannot be similar to a singular matrix is $\boxed{\text{true}}$.

Step-3

(b)

Consider the statement, â€˜â€™A symmetric matrix canâ€™t be similar to a non-symmetric matrixâ€™â€™.

Consider the following non-symmetric matrix:

$$A = \begin{bmatrix} 0 & -i \\ 0 & 1 \end{bmatrix}$$
$$\neq A^H$$

Now, perform the following calculations:

$$\begin{aligned} M^{-1}AM &= \begin{bmatrix} 1 & i \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -i \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow \text{symmetric} \\ &= J \end{aligned}$$

Thus, a non-symmetric matrix A is similar to symmetric matrix J.

Therefore, a symmetric matrix can be similar to a non-symmetric matrix and the given statement is $\boxed{\text{False}}$.

Step-4

(c)

Consider the statement, â€˜â€™Matrix A canâ€™t be similar to matrix â€“A, unless $A=0$.

Matrix A can be similar to matrix â€“A from the following example:

$$A=\begin{bmatrix}0 & i\\ i & 0\end{bmatrix}$$
$$B=\begin{bmatrix}0 & -i\\ -i & 0\end{bmatrix}$$

Take, $=-A$

Now, perform the following calculations:

$$\begin{aligned}M^{-1}AM &= \frac{1}{2}\begin{bmatrix}-1 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}0 & i\\ i & 0\end{bmatrix}\begin{bmatrix}-1 & 1\\ 1 & 1\end{bmatrix}\\ &= \begin{bmatrix}-i & 0\\ 0 & i\end{bmatrix}\\ &= J_1\end{aligned}$$

$$\begin{aligned}M^{-1}BM &= \frac{1}{2}\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}\begin{bmatrix}0 & -i\\ -i & 0\end{bmatrix}\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}\\ &= \begin{bmatrix}-i & 0\\ 0 & i\end{bmatrix}\\ &= J_2\\ &= J_1\end{aligned}$$

Therefore, matrix A can be similar to matrix â€“A without being a zero matrix and the given statement is $\boxed{\text{False}}$.

Step-5

(d)

Consider the statement, â€˜â€™Matrix $A-I$ canâ€™t be similar to $A+I$ â€™â€™.

As all Eigen values of $A+I$ is increased by 1 and all Eigen values of $A-I$ are decreased by 1. So the Eigen values of $A-I$ and $A+I$ are different.

Therefore, Matrix $A-I$ canâ€™t be similar to $A+I$ and the statement is $\boxed{\text{true}}$.