Step-1

Given $b=(0,8,8,20)$, we have to check that $e=b-p=(-1,3-5,3)$ is perpendicular to both the columns of *A* and we have to find the shortest distance $\|e\|$ from *b* to the column space of *A*.

Step-2

We have

$$
\begin{aligned}
e &= b-p \\
&= \begin{bmatrix} 0 \\ 8 \\ 8 \\ 20 \end{bmatrix} - \begin{bmatrix} 1 \\ 5 \\ 13 \\ 17 \end{bmatrix} \\
&= \begin{bmatrix} -1 \\ 3 \\ -5 \\ 3 \end{bmatrix}
\end{aligned}
$$

Step-3

Let a_1, a_2 are the columns of *A,* where

$$
a_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix},\ a_2 = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 4 \end{bmatrix}
$$

$$
\begin{aligned}
a_1^T e &= \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \\ -5 \\ 3 \end{bmatrix} \\
&= -1+3-5+3 \\
&= 0
\end{aligned}
$$

Therefore *e* is perpendicular to a_1

Step-4

And

$$a_2^T e = \begin{bmatrix} 0 & 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \\ -5 \\ 3 \end{bmatrix}$$

$$= 0 + 3 - 15 + 12$$

$$= 0$$

Therefore e is perpendicular to a_2

Hence e is perpendicular to both columns of A

Step-5

$\|e\|$ is the shortest distance of vector b to the column space of A.

$$\|e\|^2 = e^T e$$

$$= \begin{bmatrix} -1 & 3 & -5 & 3 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \\ -5 \\ 3 \end{bmatrix}$$

$$= 1 + 9 + 25 + 9$$

$$= 44$$

Hence the required shortest distance $= \sqrt{44} = 2\sqrt{11}$