Step-1

A graph consists of a set of vertices or nodes, and a set of edges that connect them. The edge goes from node j to node k, then that row has -1 in column j and + 1 in column k.

The incidence matrix A is,

$$A=\begin{bmatrix}1 & -1 & 0\\0 & 1 & -1\\1 & 0 & -1\end{bmatrix}$$

Step-2

We need to show directly from the columns that every vector b in the column space will satisfy $f_1+f_2+f_3=0$.

Let $f=(f_1,f_2,f_3)$ is in the row space of A.

We know that the row space of A is equal to the column space of A^T.

If $A=\begin{bmatrix}1 & -1 & 0\\0 & 1 & -1\\1 & 0 & -1\end{bmatrix}$ then $A^T=\begin{bmatrix}1 & 0 & 1\\-1 & 1 & 0\\0 & -1 & -1\end{bmatrix}$

Step-3

Let $\begin{bmatrix}A^T & f\end{bmatrix}=\begin{bmatrix}1 & 0 & 1 & f_1\\-1 & 1 & 0 & f_2\\0 & -1 & -1 & f_3\end{bmatrix}$

Apply $R_2\to R_2+R_1$ $=\begin{bmatrix}1 & 0 & 1 & f_1\\0 & 1 & 1 & f_2+f_1\\0 & -1 & -1 & f_3\end{bmatrix}$

Apply $R_3\to R_3+R_2$

$$=\begin{bmatrix}1 & 0 & 1 & f_1\\0 & 1 & 1 & f_2+f_1\\0 & 0 & 0 & f_3+f_2+f_1\end{bmatrix}$$

Therefore, the first two columns are independent and every vector $f = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}$ in the row space will satisfy $f_1 + f_2 + f_3 = 0$.

Step-4

Let $y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

Then $A^T Y = f$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}$$

$$\begin{aligned} \Rightarrow y_1 + y_3 &= f_1 \\ -y_1 + y_2 &= f_2 \\ -y_2 - y_3 &= f_3 \end{aligned}$$

So, add the three equations then

$$\Rightarrow f_1 + f_2 + f_3 = 0$$

Therefore, this means that $\boxed{\text{the total current entering from outside is zero.}}$