Step-1

Any vector $x = (x_1, x_2, x_3, x_4)$ is perpendicular to $a = (1, 4, 4, 1)$ and $b = (2, 9, 8, 2)$

By definition of perpendicular, we have $x^T y = 0$ and $x^T z = 0$

$$\Rightarrow \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \end{pmatrix}\begin{pmatrix} 1 \\ 4 \\ 4 \\ 1 \end{pmatrix} = 0 \quad \text{and} \quad \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \end{pmatrix}\begin{pmatrix} 2 \\ 9 \\ 8 \\ 2 \end{pmatrix} = 0$$

$\Rightarrow x_1 + 4x_2 + 4x_3 + x_4 = 0$ and $2x_1 + 9x_2 + 8x_3 + 2x_4 = 0$

Writing the system as Ax = 0 where $A = \begin{bmatrix} 1 & 4 & 4 & 1 \\ 2 & 9 & 8 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ and $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$ by making A square.

Step-2

Applying the row operation $R_2 \to R_2 - R_1$, we get $\begin{bmatrix} 1 & 4 & 4 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

This is the row reduced form and so, we rewrite the homogeneous equations from this.

$$\begin{aligned} x_1 + 4x_2 + 4x_3 + x_4 &= 0 \\ x_2 &= 0 \end{aligned}$$

Consequently, we get $x_1 = -4x_3 - x_4$

So, we write the solution as $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -4x_3 - x_4 \\ 0 \\ x_3 \\ x_4 \end{pmatrix}$

Putting $x_3 = k, x_4 = m$ parameters, we get $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = k\begin{pmatrix} -4 \\ 0 \\ 1 \\ 0 \end{pmatrix} + m\begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

Step-3

Using $k = 1$, $m = 0$, we get $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ is a solution and using $k = 0$ and $m = 1$, we get $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ which are the fundamental solutions of the system.

Thus, $\begin{pmatrix} -4 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ are in the orthogonal space or null space of A.

The space spanned by these vectors is perpendicular to the given vectors.