Step-1

(a) The projection matrix $P_1 = \dfrac{aa^T}{a^T a}$

$$aa^T = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 3 & 9 \end{bmatrix}$$

And

$$a^T a = \begin{bmatrix} 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = 1 + 9 = 10$$

Therefore,

$$P_1 = \frac{1}{10} \begin{bmatrix} 1 & 3 \\ 3 & 9 \end{bmatrix} = \begin{bmatrix} \dfrac{1}{10} & \dfrac{3}{10} \\ \dfrac{3}{10} & \dfrac{9}{10} \end{bmatrix}$$

Step-2

While P_1 is the projection onto a subspace, we follow that $I - P_1 = P_2$ is the matrix related to the subspace perpendicular to the projection of P_1

$$P_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \dfrac{1}{10} & \dfrac{3}{10} \\ \dfrac{3}{10} & \dfrac{9}{10} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{9}{10} & \dfrac{-3}{10} \\ \dfrac{-3}{10} & \dfrac{1}{10} \end{bmatrix}$$

Step-3

(b) while P_1 and P_2 are the orthogonal projection matrices, their sum must be the identity matrix.

$$P_1 + P_2 = \begin{bmatrix} \dfrac{1}{10} & \dfrac{3}{10} \\ \dfrac{3}{10} & \dfrac{9}{10} \end{bmatrix} + \begin{bmatrix} \dfrac{9}{10} & -\dfrac{3}{10} \\ -\dfrac{3}{10} & \dfrac{1}{10} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{1}{10}+\dfrac{9}{10} & \dfrac{3}{10}-\dfrac{3}{10} \\ \dfrac{3}{10}-\dfrac{3}{10} & \dfrac{9}{10}+\dfrac{1}{10} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Step-4

The product of the matrices related to the orthogonal projections must be the null matrix.

$$P_1 P_2 = \begin{bmatrix} \dfrac{1}{10} & \dfrac{3}{10} \\ \dfrac{3}{10} & \dfrac{9}{10} \end{bmatrix} \begin{bmatrix} \dfrac{9}{10} & -\dfrac{3}{10} \\ -\dfrac{3}{10} & \dfrac{1}{10} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{9}{100} - \dfrac{9}{100} & \dfrac{-3}{100} + \dfrac{3}{100} \\ \dfrac{27}{100} - \dfrac{27}{100} & \dfrac{-9}{100} + \dfrac{9}{100} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$