

## Step-1

Consider any two different vectors of the same length,  $\|x\| = \|y\|$ .

Let

$$v = x - y$$

A Householder matrix  $H$  is given by,

$$H = I - 2 \frac{vv^T}{\|v\|^2}$$

On substitution, we get,

$$H = I - 2 \frac{(x-y)(x-y)^T}{\|x-y\|^2}$$

By multiplying by  $x$ , we get,

$$\begin{aligned} Hx &= Ix - 2 \frac{(x-y)(x-y)^T}{\|x-y\|^2} x \\ &= x - (x-y) \frac{2(x-y)^T}{(x-y)(x-y)^T} x \\ &= x - (x-y) \\ &= y \end{aligned}$$

Therefore, we get,

$$Hx = y$$

Therefore,

$$\begin{aligned} H(Hx) &= H(y) \\ x &= Hy \end{aligned}$$

Thus,  $\boxed{Hx = y}$  and  $\boxed{x = Hy}$ .