Step-1

Given equations are

$$x + 4y - 2z = 1$$
$$x + 7y - 6z = 6$$
$$3y + qz = t$$

We have to find that which number q makes this system singular and which right-hand side t give it infinitely many solutions. Also we have to find the solution that has $z = 1$

Step-2

Performing row $2 -$ row1 gives

$$x + 4y - 2z = 1$$
$$3y - 4z = 5$$
$$3y + qz = t$$

Now performing row3 $-$ row 2 gives

$$x + 4y - 2z = 1$$
$$3y - 4z = 5$$
$$(q + 4)z = t - 5$$

Step-3

When $q + 4 = 0$ then $q = -4$, the system is singular – no third pivot, then if $\boxed{t = 5}$, the third equation is $0 = 0$.

So we have two equations in three variables, hence the system has infinite number of solutions.

Step-4

Choose $z = 1$, the equation $3y - 4z = 5$ gives $y = 3$ and equation 1 gives $x = -9$

Therefore one of the solutions is $\boxed{(-9, 3, 1)}$