Step-1

a) We have to find A so that the only solution to $Ax=\begin{bmatrix}1\\2\\3\end{bmatrix}$ is $x=\begin{bmatrix}0\\1\end{bmatrix}$

Let $x=\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}x_1\\x_2\end{bmatrix}$

$\Rightarrow x_1=0, x_2=1$

Step-2

For these values, we can take the system as

$$\begin{bmatrix}1&1\\0&2\\0&3\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$$

$$\xrightarrow{\frac{1}{2}R_2,\frac{1}{3}R_3}\begin{bmatrix}1&1\\0&1\\0&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}1\\1\\1\end{bmatrix}$$

Step-3

$$\xrightarrow{R_3-R_2}\begin{bmatrix}1&1\\0&1\\0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}1\\1\\0\end{bmatrix}$$

$\Rightarrow x_1+x_2=1$
$x_2=1$
$\Rightarrow x_1=0$

Step-4

Therefore the matrix A satisfying the given condition is $\begin{bmatrix}1&1\\0&2\\0&3\end{bmatrix}$

Step-5

b) We have to find B so that the only solution to $Bx = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is $x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Given system is $Bx = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, and $x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

From this we can observe that there is a system with two equations and three unknowns, so it has infinite number of solutions, but given that system has only one solution.

$\boxed{\text{Therefore the matrix}}$ B $\boxed{\text{does not exist for the given system}}$.