

## Step-1

Matrix  $A$  can be factorized into product of lower and upper triangular matrices.

$$\mathbf{A} = \mathbf{LU}$$

Here, matrix  $L$  is a lower triangular matrix with 1 at the diagonal position and matrix  $U$  is the upper triangular matrix with pivots at the diagonal position.

## Step-2

Consider the following matrices:

$$\mathbf{A} = \begin{bmatrix} x & x & x & x \\ x & x & x & 0 \\ 0 & x & x & x \\ 0 & 0 & x & x \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} x & x & x & 0 \\ x & x & 0 & x \\ x & 0 & x & x \\ 0 & x & x & x \end{bmatrix}$$

Here, nonzero positions are marked by  $x$ . Determine which zeros will be still zero in their factors  $L$  and  $U$ .

## Step-3

First take matrix  $A$ .

$$\mathbf{A} = \begin{bmatrix} x & x & x & x \\ x & x & x & 0 \\ 0 & x & x & x \\ 0 & 0 & x & x \end{bmatrix}$$

It can be seen that a part  $(3 \times 3)$  of the  $(4 \times 4)$  matrix is already in lower triangular form. Now recall that if a row of matrix  $A$  starts with zeros, so does that row of matrix  $L$  and if a column starts with zero so does that column of matrix  $U$ .

Therefore, matrix  $L$  will contain three zeros at the position  $\boxed{a_{31}, a_{41}, a_{42}}$  however matrix  $U$  may not contain zero at the position  $\boxed{a_{24}}$ .

## Step-4

Similarly, in the case of matrix  $B$ , bottom left side zero will be in matrix  $L$  and top right side zero will be in matrix  $U$ . Rest zeros may be filled in by the non zeros.

Therefore, matrix  $L$  will contain one zero at the position  $\boxed{a_{41}}$  and matrix  $U$  will contain zero at the position  $\boxed{a_{44}}$ .