Step-1

The objective is to determine when an upper triangular matrix is nonsingular.

Let $U = \begin{bmatrix} a_{11} & a_{12} & \cdots a_{1(n-1)} & a_{1n} \\ 0 & a_{22} & \cdots a_{2(n-1)} & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots 0 & a_{1n} \end{bmatrix}$ be an upper triangular matrix.

Then, all the entries below the main diagonal are zeros.

A matrix A is nonsingular if its determinant is not equal to zero.

For any $n \times n$ triangular matrix, the determinant is the product of the diagonal entries.

Therefore, the determinant of U is the product of all the elements in the main diagonal and it is not equal to zero only if all the diagonal entries are nonzero.

That is, the diagonal entries $a_{11}, a_{22}, \ldots, a_{nn}$ must be all nonzero.

Hence, an upper triangular matrix is nonsingular if all the diagonal entries of the matrix are nonzero.