Step-1

Suppose A is an n by n matrix, which is positive definite.

By using cofactors, we can write the following:

$$|A| = a_{11}A_{11} + a_{12}A_{12} + \ldots + a_{1n}A_{1n}$$

Since, A is positive definite, its determinant should be positive. Thus we get,

$$a_{11}A_{11} + a_{12}A_{12} + \ldots + a_{1n}A_{1n} > 0$$

Therefore,

$$a_{11}A_{11} > -\left(a_{12}A_{12} + \ldots + a_{1n}A_{1n}\right)$$

Step-2

Suppose, the value of a_{11} is increased to b_{11}.

Thus, $a_{11} < b_{11}$.

Let if possible, $a_{11}A_{11} + a_{12}A_{12} + \ldots + a_{1n}A_{1n} > b_{11}A_{11} + a_{12}A_{12} + \ldots + a_{1n}A_{1n}$

This gives, $a_{11}A_{11} > b_{11}A_{11}$.

But since $a_{11} < b_{11}$, we can get $a_{11}A_{11} > b_{11}A_{11}$ only if $A_{11} < 0$.

Step-3

But the matrix A is positive definite. Therefore, it is automatically positive semidefinite. Thus, none of its principal submatrices has negative determinant.

This contradicts with $A_{11} < 0$.

Therefore, our assumption that $a_{11}A_{11} + a_{12}A_{12} + \ldots + a_{1n}A_{1n} > b_{11}A_{11} + a_{12}A_{12} + \ldots + a_{1n}A_{1n}$ is wrong.

Therefore, as a_{11} is increased, the determinant of A must increase.