Step-1

Given symmetric matrices are

$A=\begin{pmatrix}2 & 4\\ 4 & 11\end{pmatrix}$, and $B=\begin{pmatrix}1 & 4 & 0\\ 4 & 12 & 4\\ 0 & 4 & 0\end{pmatrix}$

We have to find their triple factorizations LDU and we have to say that how U and L are related.

Step-2

Given $A=\begin{pmatrix}2 & 4\\ 4 & 11\end{pmatrix}$

Subtracting 2 times row 1 from row 2

$$U=\begin{pmatrix}2 & 4\\ 0 & 3\end{pmatrix}$$

Dividing row 1 with 2 and row 2 with 3 gets

$$U=\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}$$

Step-3

To get L, we have to do reverse operations on the identity matrix I_2 which are held on A;

Adding 2 times row 1 to row 2 gives

$$L=\begin{pmatrix}1 & 0\\ 2 & 1\end{pmatrix}$$

D is the diagonal matrix with pivots 2, 3 on the diagonal so

$$D=\begin{pmatrix}2 & 0\\ 0 & 3\end{pmatrix}$$

Step-4

Now $A=LDU$ is $\begin{pmatrix}2 & 4\\ 4 & 11\end{pmatrix}=\begin{pmatrix}1 & 0\\ 2 & 1\end{pmatrix}\begin{pmatrix}2 & 0\\ 0 & 3\end{pmatrix}\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}$

And we can observe that $L^T = U$ and $U^T = L$.

Step-5

Given $B = \begin{pmatrix} 1 & 4 & 0 \\ 4 & 12 & 4 \\ 0 & 4 & 0 \end{pmatrix}$

Subtracting 4 times row 1 from row 2 gives

$$= \begin{pmatrix} 1 & 4 & 0 \\ 0 & -4 & 4 \\ 0 & 4 & 0 \end{pmatrix}$$

Step-6

Adding row 2 to row 3 gets

$$= \begin{pmatrix} 1 & 4 & 0 \\ 0 & -4 & 4 \\ 0 & 0 & 4 \end{pmatrix}$$

Dividing row 2 with -4 and row 3 with 4 gets U

$$U = \begin{pmatrix} 1 & 4 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

Step-7

To get L, we have to do reverse operations on the identity matrix I_3 which are held on A;

Adding 4 times row 1 to row 2 and subtracting row 2 from row 3 gives

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$$

D is the diagonal matrix with pivots 1,-4, 4 on the diagonal so

$$D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

Step-8

The factorization $A = LDU$ is

$$\begin{pmatrix} 1 & 4 & 0 \\ 4 & 12 & 4 \\ 0 & 4 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} 1 & 4 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

And we can observe that $L^T = U$ and $U^T = L$.