

## Step-1

(a)

Consider the following matrix.

$$A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

Real Eigen values of the matrix A is  $0$  only.

Eigen vector corresponding to the Eigen value  $0$  is  $(1, 0, 0)$ .

Therefore, the only Eigen vectors of a matrix A are multiples of the vectors  $x = (1, 0, 0)$ .

Note that matrix A is singular matrix.

That is, matrix A is **not invertible**.

## Step-2

Now consider the following matrix.

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

The only Real Eigen values of the matrix A is  $1$ .

Eigen vector corresponding to the Eigen value  $1$  is  $(1, 0, 0)$ .

Therefore, the only Eigen vectors of a matrix A are multiples of the vectors  $x = (1, 0, 0)$ .

Note that matrix A is non-singular matrix.

That is, matrix A is **invertible**.

Therefore, there are **invertible** as well as **not invertible** matrices so that the only Eigen vectors of the matrix are multiple of the vector  $x = (1, 0, 0)$ .

So, given statement is False.

## Step-3

(b)

Consider the following matrix.

$$A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

The Eigen values of the matrix A are  $0, \pm i$ .

Eigen vector corresponding to the Eigen value 0 is  $(1, 0, 0)$ .

Therefore, the only Eigen vectors of a matrix A are multiples of the vectors  $x = (1, 0, 0)$ .

Note that matrix A has **no repeated Eigen value**.

## Step-4

Now consider the following matrix.

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

The Eigen values of the matrix A are 1, 1, 1.

Eigen vector corresponding to the Eigen value 1 is  $(1, 0, 0)$ .

Therefore, the only Eigen vectors of a matrix A are multiples of the vectors  $x = (1, 0, 0)$ .

Note that matrix A has **a repeated Eigen value**.

Therefore, there are matrices which have **no repeated Eigen value** as well as **a repeated Eigen value** with the property that the only Eigen vectors of a matrix are multiples of the vectors  $x = (1, 0, 0)$ .

So, given statement is False.

## Step-5

(c)

Given that the only Eigen vectors of a matrix A are multiples of the vectors  $x = (1, 0, 0)$ , so Eigen space of the matrix A is

$$\{c(1, 0, 0) : c \in \mathbb{R}\}$$

Therefore, dimension of Eigen space of the matrix A is 1.

That means the Eigen space of the matrix A does not span  $\mathbb{R}^3$ .

Hence, matrix A is not diagonalizable.

Therefore, the given statement is true.