Step-1

Consider the following matrix:

$$A = \begin{bmatrix} I & B \\ B^{\mathrm{T}} & 0 \end{bmatrix}$$

Here B is a matrix of order $\frac{n}{2}$, such that B is a non singular matrix. Therefore, it is clear that n should be even.

Step-2

The diagonal elements of A are 1, 1,..., 1, 0, 0,..., 0. The I block is symmetric and 0 block of zero matrix is also symmetric. Although B may or may not be symmetric, we are writing B in the upper right triangle and B^{T} in the lower left triangle. Therefore, it should be clear that A is a symmetric matrix.

Thus, the signs of the pivots of A agree with the signs of the eigenvalues of A.

Step-3

Now while obtaining pivots of A, to obtain zeros below the I block, we will be multiplying the initial rows by some non zero constant and we will be subtracting them from the below rows.

This will create negative numbers in the diagonal along the 0 block.

Thus, there should be $\frac{n}{2}$ positive pivots and $\frac{n}{2}$ negative pivots.

Step-4

Let $A = \begin{bmatrix} 1 & 3 \\ 3 & 0 \end{bmatrix}$. Then, $\det(A - \lambda I) = 0$ gives the following,

$$\begin{aligned} 0 &= \begin{vmatrix} 1-\lambda & 3 \\ 3 & -\lambda \end{vmatrix} \\ &= (1-\lambda)(-\lambda) - 9 \\ &= \lambda^2 - \lambda - 9 \end{aligned}$$

Step-5

Therefore, we have

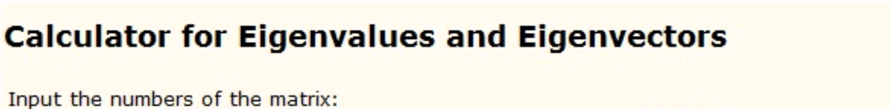
$$\lambda = \frac{1 \pm \sqrt{1+36}}{2}$$

Thus, there is one positive eigenvalue and one negative eigenvalue.

Step-6

Let $A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 4 & 7 \\ 2 & 4 & 0 & 0 \\ 3 & 7 & 0 & 0 \end{bmatrix}$

We obtain its eigenvalues by online eigenvalue calculator. See the screenshot below:



Thus, there are 2 positive eigenvalues and 2 negative eigenvalues.

Let $A = \begin{bmatrix} 1 & 0 & 0 & 2 & 3 & 5 \\ 0 & 1 & 0 & -1 & -1 & 3 \\ 0 & 0 & 1 & 8 & 7 & 2 \\ 2 & -1 & 8 & 0 & 0 & 0 \\ 3 & -1 & 7 & 0 & 0 & 0 \\ 5 & 3 & 2 & 0 & 0 & 0 \end{bmatrix}$

We obtain its eigenvalues by online eigenvalue calculator. See the screenshot below:

Thus, here we find that there are 3 positive eigenvalues and 3 negative eigenvalues.

Step-7

Thus, for an n by n matrix of the form $\begin{bmatrix} I & B \\ B^{\mathrm{T}} & 0 \end{bmatrix}$, we find that it has $\frac{n}{2}$ positive and $\frac{n}{2}$ negative eigenvalues.