Step-1

Consider the following matrix:

$$N = \begin{bmatrix} 0 & a & b \\ b & 0 & a \\ a & b & 0 \end{bmatrix}$$

Then

$$N^{\mathrm{H}} = \begin{bmatrix} 0 & b & a \\ a & 0 & b \\ b & a & 0 \end{bmatrix}$$

Here, a and b are any complex numbers. Note that N is neither of Hermitian, Skew Hermitian, Unitary, or Diagonal.

Step-2

Consider the following:

$$\begin{aligned} N^{\mathrm{H}}N &= \begin{bmatrix} 0 & b & a \\ a & 0 & b \\ b & a & 0 \end{bmatrix}\begin{bmatrix} 0 & a & b \\ b & 0 & a \\ a & b & 0 \end{bmatrix} \\ &= \begin{bmatrix} b^2+a^2 & ab & ba \\ ba & a^2+b^2 & ab \\ ab & ba & b^2+a^2 \end{bmatrix} \\ NN^{\mathrm{H}} &= \begin{bmatrix} 0 & a & b \\ b & 0 & a \\ a & b & 0 \end{bmatrix}\begin{bmatrix} 0 & b & a \\ a & 0 & b \\ b & a & 0 \end{bmatrix} \\ &= \begin{bmatrix} a^2+b^2 & ba & ab \\ ab & b^2+a^2 & ba \\ ba & ab & a^2+b^2 \end{bmatrix} \end{aligned}$$

Step-3

From the properties of complex numbers, we know that the addition and multiplication of complex numbers are commutative. That is, $z_1 + z_2 = z_2 + z_1$ and $z_1 z_2 = z_2 z_1$, for any $z_1, z_2 \in \mathbb{C}$.

Therefore, $N^{\mathrm{H}}N = NN^{\mathrm{H}}$. Choose $a \neq b$ and we have got the required matrix.

Step-4

Every permutation matrix is an orthogonal matrix. Therefore, the transpose of a projection matrix is its inverse.

Therefore, if P is a permutation matrix, then $P^{-1} = P^{\mathrm{T}}$. Since, the entries in a permutation matrix are real numbers, we can say that $P^{\mathrm{T}} = P^{\mathrm{H}}$.

Thus, we have $P^{-1} = P^{\mathrm{H}}$.

Step-5

For any invertible matrix A, we know that $AA^{-1} = A^{-1}A = I$.

Thus, if P is any permutation matrix, we get $PP^{\mathrm{H}} = P^{\mathrm{H}}P = I$.

Thus, we have shown that every permutation matrix is normal.