Step-1

Given matrices are $A=\begin{bmatrix}1 & 1 & 3\\ 0 & 4 & 6\\ 1 & 5 & 8\end{bmatrix}$, $B=\begin{bmatrix}1 & 1 & 3\\ 0 & 4 & 6\\ 0 & 0 & 1\end{bmatrix}$ and $C=\begin{bmatrix}1 & 1 & 3\\ 0 & 4 & 6\\ 1 & 5 & 9\end{bmatrix}$.

Step-2

First we need to evaluate $\det A$ by reducing the matrix to triangular form.

We know that the determinant of triangular matrix is the product of diagonal entries.

Now

$$\det(A)=\begin{vmatrix}1 & 1 & 3\\ 0 & 4 & 6\\ 1 & 5 & 8\end{vmatrix}$$

$$=\begin{vmatrix}1 & 1 & 3\\ 0 & 4 & 6\\ 0 & 4 & 6\end{vmatrix} \leftarrow \text{subtracting first row from the third row}$$

$$=\begin{vmatrix}1 & 1 & 3\\ 0 & 4 & 6\\ 0 & 0 & -1\end{vmatrix} \leftarrow \text{subtracting second row from the third row}$$

$$=(1)(4)(-1)$$

$$=-4$$

Therefore, $\boxed{\det A=-4}$

Step-3

Since B is triangular (upper) $\det B$ is the product of the diagonal entries.

Now

$$\det(B)=|B|$$

$$=\begin{vmatrix}1 & 1 & 3\\ 0 & 4 & 6\\ 0 & 0 & 1\end{vmatrix}$$

$=(1)(4)(1)$
$=4$

Therefore, $\boxed{\det B=4}$

Step-4

Now

$\det C=|C|$

$$=\begin{vmatrix}1 & 1 & 3\\ 0 & 4 & 6\\ 1 & 5 & 9\end{vmatrix}$$

We find $\det C$ by cofactor expansion along the first column

$$\det C=\begin{vmatrix}4 & 6\\ 5 & 9\end{vmatrix}+\begin{vmatrix}1 & 3\\ 4 & 6\end{vmatrix}$$

$=(36-30)+(6-12)$
$=6-6$
$=0$

Thus, $\boxed{\det C=0}$

Step-5

We have to find $\det(AB)$.

We know that $\det(AB)=(\det A)(\det B)$

Now

$\det(AB)=(\det A)(\det B)$

$=(-4)(4)$
$=-16$

Thus, $\boxed{\det(AB)=-16}$

Step-6

We have to find $\det\left(A^T A\right)$.

We know that $\det\left(A^T A\right) = \det\left(A^T\right)\det A$ and $\det\left(A^T\right) = \det A$.

Now

$$\begin{aligned}\det\left(A^T A\right) &= \det\left(A^T\right)\det A \\ &= (-4)(-4) \\ &= 16\end{aligned}$$

Thus, $\boxed{\det\left(A^T A\right) = 16}$

Step-7

We have to find $\det\left(C^T\right)$

Now

$\det\left(C^T\right) = \det C$

= 0 since $\det\left(A^T\right) = \det(A)$

Hence $\boxed{\det\left(C^T\right) = 0}$