Step-1

$$B = \begin{bmatrix} .6 & .9 \\ .1 & .6 \end{bmatrix}$$

The characteristic equation is $(\lambda - 0.6)^2 - 0.09 = 0$

$$\Rightarrow \lambda^2 - 1.2\lambda + 0.27 = 0$$
$$\Rightarrow \lambda^2 - 0.9\lambda - 0.3\lambda + 0.27 = 0$$
$$\Rightarrow (\lambda - 0.9)(\lambda - 0.3) = 0$$
$$\Rightarrow \lambda_1 = 0.3, \lambda_2 = 0.9$$

Step-2

The eigen vector corresponding to $\lambda_1 = 0.3$ is obtained by solving $(B - \lambda_1 I)x = 0$

$$\Rightarrow \begin{bmatrix} 0.3 & 0.9 \\ 0.1 & 0.3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

After the application of row operations on the coefficient matrix, it reduces to $\begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

So, the eigen vector as above is $\begin{bmatrix} 3 \\ -1 \end{bmatrix}$ by putting $x_2 = -1$

Step-3

Similarly, the eigen vector corresponding to $\lambda_2 = 0.9$ is obtained by solving $(B - \lambda_2 I)x = 0$

$$\Rightarrow \begin{bmatrix} -0.3 & 0.9 \\ 0.1 & -0.3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

So, the solution $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ is the eigen vector corresponding.

Step-4

So, $B = S\Lambda S^{-1}$ where $S = \begin{bmatrix} 3 & 3 \\ -1 & 1 \end{bmatrix}, \Lambda = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.9 \end{bmatrix}, S^{-1} = \frac{1}{6}\begin{bmatrix} 1 & -3 \\ 1 & 3 \end{bmatrix}$

Applying the k^{th} power on both sides, it becomes $B^k = S\begin{bmatrix}(0.3)^k & 0\\ 0 & (0.9)^k\end{bmatrix}S^{-1}$

Putting $k = 10$, we get $B^{10} = \begin{bmatrix}3 & 3\\ -1 & 1\end{bmatrix}\begin{bmatrix}(0.3)^{10} & 0\\ 0 & (0.9)^{10}\end{bmatrix}\frac{1}{6}\begin{bmatrix}1 & -3\\ 1 & 3\end{bmatrix}$

$$= \frac{1}{6}\begin{bmatrix}3\times 0.3^{10} & 3\times 0.9^{10}\\ -1\times 0.3^{10} & 0.9^{10}\end{bmatrix}\begin{bmatrix}1 & -3\\ 1 & 3\end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix}3\left(0.3^{10}+0.9^{10}\right) & 9\left(0.9^{10}-0.3^{10}\right)\\ 0.9^{10}-0.3^{10} & 3\left(0.9^{10}+0.3^{10}\right)\end{bmatrix}$$

$$= \frac{3^{10}}{6}\begin{bmatrix}3 & -9\\ -1 & 3\end{bmatrix} + \frac{0.9^{10}}{6}\begin{bmatrix}3 & 9\\ 1 & 3\end{bmatrix}$$

Step-5

Using B^{10}, we find the products $B^{10}u_0$

(1) $u_0 = \begin{bmatrix}3\\ 1\end{bmatrix}$, then $B^{10}u_0$

$$= \left\{\frac{3^{10}}{6}\begin{bmatrix}3 & -9\\ -1 & 3\end{bmatrix} + \frac{0.9^{10}}{6}\begin{bmatrix}3 & 9\\ 1 & 3\end{bmatrix}\right\}\begin{bmatrix}3\\ 1\end{bmatrix}$$

$$= \frac{3^{10}}{6}\begin{bmatrix}0\\ 0\end{bmatrix} + \frac{0.9^{10}}{6}\begin{bmatrix}18\\ 6\end{bmatrix}$$

$$= 0.9^{10}\begin{bmatrix}3\\ 1\end{bmatrix}$$

Step-6

(2) $u_0 = \begin{bmatrix}3\\ -1\end{bmatrix}$, then $B^{10}u_0$

$$= \left\{ \frac{3^{10}}{6} \begin{bmatrix} 3 & -9 \\ -1 & 3 \end{bmatrix} + \frac{0.9^{10}}{6} \begin{bmatrix} 3 & 9 \\ 1 & 3 \end{bmatrix} \right\} \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$
$$= \frac{3^{10}}{6} \begin{bmatrix} 18 \\ -6 \end{bmatrix} + \frac{0.9^{10}}{6} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$= 0.3^{10} \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

Step-7

(3) $u_0 = \begin{bmatrix} 6 \\ 0 \end{bmatrix}$, then $B^{10} u_0$

$$= \left\{ \frac{3^{10}}{6} \begin{bmatrix} 3 & -9 \\ -1 & 3 \end{bmatrix} + \frac{0.9^{10}}{6} \begin{bmatrix} 3 & 9 \\ 1 & 3 \end{bmatrix} \right\} \begin{bmatrix} 6 \\ 0 \end{bmatrix}$$
$$= \frac{3^{10}}{6} \begin{bmatrix} 18 \\ -6 \end{bmatrix} + \frac{0.9^{10}}{6} \begin{bmatrix} 18 \\ 6 \end{bmatrix}$$
$$= 0.3^{10} \begin{bmatrix} 3 \\ -1 \end{bmatrix} + 0.9^{10} \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$