Step-1

We have $x = \begin{pmatrix} x_1 \\ x_2 \\ - \\ - \end{pmatrix}, y = \begin{pmatrix} y_1 \\ y_2 \\ - \\ - \end{pmatrix}$

$$\begin{aligned} x^T y &= x_1 y_1 + x_2 y_2 + \ldots \\ &= y_1 x_1 + y_2 x_2 + \ldots \\ &= y^T x \end{aligned}$$ â€¦â€¦ (1)

$x - y$ and $x + y$ are orthogonal if and only if $(x-y)^T (x+y) = 0$

If and only if $xx^T + x^T y - y^T x - yy^T = 0$ â€¦â€¦ (2)

Step-2

Using (1), we can write $x - y$ and $x + y$ are orthogonal if and only if $xx^T - yy^T = 0$

If and only if $\|x\|^2 - \|y\|^2 = 0$

While norm is a non negative quantity, we can write that

$x - y$ and $x + y$ are orthogonal if and only if $\|x\| = \|y\|$