

Step-1

We have to find that which number b leads later to a row exchange, which number b leads to a missing pivot, and in that singular case we have to find a non-zero solution x, y, z :

$$\begin{aligned}x + by &= 0 \\x - 2y - z &= 0 \\y + z &= 0\end{aligned}$$

Step-2

After eliminating first pivot when applying (row2) \leftarrow (row 1) the second pivot position will contain $-2 - b$

If $b = -2$, we exchange with row 3 since the system is

$$\begin{aligned}x - 2y &= 0 \\x - 2y - z &= 0 \\y + z &= 0\end{aligned}$$

So we can exchange row 3 if $b = -2$

Step-3

If $b = -1$, the second equation is $-y - z = 0$. And the system is

$$\begin{aligned}x - y &= 0 \\x - 2y - z &= 0 \\y + z &= 0\end{aligned}$$

Performing (row 2) \leftarrow (row1) gives

$$\begin{aligned}x - y &= 0 \\-y - z &= 0 \\y + z &= 0\end{aligned}$$

Step-4

Adding (row 3) + (row 2) give

$$\begin{aligned}x - y &= 0 \\-y - z &= 0\end{aligned}$$

To solving these equations by letting $x = y = 1$, one of the solutions is $(1, 1, -1)$