

## Step-1

Suppose,  $(Q_0 Q_1 \dots Q_{k-1})(R_{k-1} \dots R_1 R_0)$  is the  $QR$  factorization of  $A_k$ .

We know that  $A_{k+1} = R_k Q_k$

Therefore,

$$A_{k+1} Q_k^{-1} = R_k Q_k Q_k^{-1}$$

$$A_{k+1} Q_k^T = R_k$$

Thus,  $R_k = (Q_k^T \dots Q_0^T A Q_0 \dots Q_k) Q_k^T$ .

## Step-2

This gives

$$\begin{aligned} R_k (R_{k-1} \dots R_0) &= (Q_k^T \dots Q_0^T A Q_0 \dots Q_k) Q_k^T (R_{k-1} \dots R_0) \\ &= Q_k^T \dots Q_0^T A_{k+1} \end{aligned}$$

Therefore,  $A_{k+1} = (Q_0 Q_1 \dots Q_k)(R_k \dots R_1 R_0)$ .