Step-1

The product rule of determinant says that if A and B are square matrices of the same order, then $\det(AB) = (\det A)(\det B)$.

It is also known that $\det I = 1$.

Therefore, we get

$$
\begin{aligned}
1 &= \det I \\
&= \det\left(Q^{T}Q\right) \\
&= \det\left(Q^{T}\right)\det(Q)
\end{aligned}
$$

Step-2

We also know that determinant of a matrix A is equal to the determinant of its transpose, A^T. Therefore, $\det(Q) = \det\left(Q^{T}\right)$.

Thus, $\left(\det(Q)\right)^2 = 1$

Therefore, $\boxed{\det(Q) = \pm 1}$.

By the same logic, $\det\left(Q^{T}\right) = \pm 1$.

Step-3

Now we show that Q^2 is also an Orthogonal matrix. Thus, we need to show that $\left(Q^2\right)^{-1} = \left(Q^2\right)^{T}$.

$$
\begin{aligned}
Q^2\left(Q^2\right)^{T} &= (Q \cdot Q)\left(Q^{T} \cdot Q^{T}\right) \\
&= Q\left(Q \cdot Q^{T}\right)Q^{T} \\
&= Q \cdot Q^{T} \\
&= I
\end{aligned}
$$

Thus, Q^2 is an Orthogonal matrix. Similarly, it can be shown that Q^n is also an Orthogonal matrix.

Step-4

If $\det Q$ is not equal to ± 1, then $\det\left(Q^{n}\right)$ would either tend to zero or would tend to plus or minus infinity.

But, Q^n remains an Orthogonal matrix.

This also shows that $\boxed{\det Q = \pm 1}$.