

## Step-1

Given that  $P$  is a projection onto the column space of  $A$ .

The objective is to find the projection onto the left null space of  $A$ .

## Step-2

If  $P$  is the projection onto the column space of  $A$

It is given a matrix formula for splitting any  $b$  into two perpendicular components.

$Pb$  is in the column space  $C(A)$  and the other component  $(I - P)b$  is in the left null space  $N(A^T)$ , which is orthogonal to the column space.

Hence,  $(I - P)$  is the projection onto the left null space.