Step-1

Def: suppose A is n by n matrix has n linearly independent eigen vectors. If these eigen vectors are the columns of a matrix S, then $S^{-1}AS$ is a diagonal matrix Λ.

The eigen values of A are on the diagonal of Λ.

Step-2

Given that A has eigen values are 1 and 4, and whose eigenvectors are $\begin{bmatrix}3\\1\end{bmatrix}$ and $\begin{bmatrix}2\\1\end{bmatrix}$.

In view of the above definition, we have

$S=\begin{bmatrix}3 & 2\\1 & 1\end{bmatrix}$,

So, $S^{-1}=\frac{1}{3-2}\begin{bmatrix}1 & -2\\-1 & 3\end{bmatrix}$

Also, the diagonal matrix is $\Lambda=\begin{bmatrix}1 & 0\\0 & 4\end{bmatrix}$

The required matrix is $A=S\Lambda S^{-1}$

$$=\begin{bmatrix}3 & 2\\1 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 4\end{bmatrix}\begin{bmatrix}1 & -2\\-1 & 3\end{bmatrix}$$
$$=\begin{bmatrix}3 & 8\\1 & 4\end{bmatrix}\begin{bmatrix}1 & -2\\-1 & 3\end{bmatrix}$$
$$=\begin{bmatrix}-5 & 18\\-3 & 10\end{bmatrix}$$