

Step-1

Consider volume of a box of length l , breadth b , height h is given by lbh .

Suppose if each side is made t times, then the volume is clearly $tl.tb.th = t^3lbh$

Now $\det A$ is volume of a box of n sides in an n - dimensional space.

The objective is to show that $\det 3A = 3^n \det A$ by volume.

Step-2

Now, multiply A by 3, each row vector is made 3 times.

So, every side of the given box A is made 3 times its original length.

While the case is of 3 dimensions,

So, Volume of the new box = $\det 3A$

= $3^n \cdot \det A$

Hence, $\boxed{\det 3A = 3^n \det A}$