Step-1

Given matrix is $A = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$.

So,

$$\begin{aligned} x^T Ax &= \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} \begin{pmatrix} x \\ x_2 \\ x_3 \end{pmatrix} \\ &= 2x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 - 2x_2x_3 - 2x_3x_1 \\ &= 2\left(x_1^2 + x_2^2 + x_3^2 - x_1x_2 - x_2x_3 - x_3x_1\right) \end{aligned}$$

Step-2

$$A = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}.$$

Apply $R_2 \to R_2 + \frac{1}{2}R_1$ and $R_3 \to R_3 + \frac{1}{2}R_1$

$$= \begin{pmatrix} 2 & -1 & -1 \\ 0 & \frac{3}{2} & -\frac{3}{2} \\ 0 & -\frac{3}{2} & \frac{3}{2} \end{pmatrix}$$

Apply $R_3 \to R_3 + R_2$

$$= \begin{pmatrix} 2 & -1 & -1 \\ 0 & \frac{3}{2} & -\frac{3}{2} \\ 0 & 0 & 0 \end{pmatrix}$$

So we can write it as $x^T Ax = 2\left(x_1 - \frac{x_2}{2} - \frac{x_3}{2}\right)^2 + \frac{3}{2}\left(x_2 - x_3\right)^2$

Therefore, $\boxed{x^T Ax = 2\left(x_1 - \frac{x_2}{2} - \frac{x_3}{2}\right)^2 + \frac{3}{2}(x_2 - x_3)^2}$.

Thus, $x^T Ax$ is sum of two squares and A has two pivots.

Step-3

$$B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\begin{aligned} x^T Bx &= \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \\ &= x_1^2 + x_2^2 + x_3^2 + 2x_1x_2 + 2x_2x_3 + 2x_3x_1 \\ &= (x_1 + x_2 + x_3)^2 \end{aligned}$$

Therefore, $\boxed{x^T Bx = (x_1 + x_2 + x_3)^2}$.

Thus, $x^T Bx$ can be written as one square and B has one pivot.