## Step-1

(a) Consider matrices of $(2\times2)$ with 1â€™s and 0â€™s. Determine how many are invertible.

Recall that a matrix is invertible if its determinant is non-zero.

## Step-2

Determinants of all the matrices $(2\times2)$ containing 1â€™s and 0â€™s are as follows:

Consider the matrices with only one 1â€™s.

$$\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix},\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix},\begin{bmatrix}0 & 0\\1 & 0\end{bmatrix},\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}$$

Determinant of all these matrices are zero. So, none of them are invertible.

## Step-3

Consider the matrices with two 1â€™s.

$$\begin{bmatrix}1 & 1\\0 & 0\end{bmatrix},\begin{bmatrix}0 & 1\\0 & 1\end{bmatrix},\begin{bmatrix}1 & 0\\1 & 0\end{bmatrix},\begin{bmatrix}0 & 0\\1 & 1\end{bmatrix}$$

Determinant of all these matrices are zero. So, none of them are invertible.

## Step-4

Consider the matrices with three 1â€™s.

$$\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix},\begin{bmatrix}0 & 1\\1 & 1\end{bmatrix},\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix},\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$$

Determinant of all these matrices are nonzero. So, each one of them are invertible.

## Step-5

Consider the matrix with 1â€™s positioned on the diagonal and anti-diagonal.

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix},\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$$

Determinant of all these matrices are nonzero. So, each one of them are invertible.

## Step-6

Consider the matrix with all 1â€™s and 0â€™s.

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Determinant of all these matrices are zero. So, none of them are invertible.

## Step-7

Therefore, from sixteen $(2\times 2)$ matrices with 0â€™s and 1â€™s only 6 are invertible.

## Step-8

(b) Consider matrices of $(10\times 10)$ with 1â€™s and 0â€™s. These entries are done at random. Determine the matrix is more likely to be singular or invertible.

If entries 1â€™s and 0â€™s are filled at random in $(10\times 10)$ matrix then it is likely to be more singular. As the effect of the element 1 can be cancelled out when multiplied by element 0. Therefore, matrix is more likely to be singular.