Step-1

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Given that producing x_1 trucks and x_2 planes requires x_1+50x_2 tons of steel, $40x_1+1000x_2$ pounds of rubber, and $2x_1+50x_2$ months of labor.

Suppose the unit costs y_1, y_2, y_3 are $\$700$ per ton, \$3 perpound and \$3000 per month.

We have to find the value of one truck and one plane.

Step-2

Given that the truck and the plane are the components of $A^T y$.

From the given data, we get

$$Ax = \begin{bmatrix} x_1+50x_2 \\ 40x_1+1000x_2 \\ 2x_1+50x_2 \end{bmatrix} = \begin{bmatrix} 1 & 50 \\ 40 & 1000 \\ 2 & 50 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Step-3

And $$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{pmatrix} 700 \\ 3 \\ 3000 \end{pmatrix}$$

We have the matrix A is $$A = \begin{bmatrix} 1 & 50 \\ 40 & 1000 \\ 2 & 50 \end{bmatrix}$$

The transpose of A is $$A^T = \begin{bmatrix} 1 & 40 & 2 \\ 50 & 1000 & 50 \end{bmatrix}.$$

Step-4

The values of one truck and one plane is $A^T y$.

Therefore,

$$A^T y = \begin{bmatrix} 1 & 40 & 2 \\ 50 & 1000 & 50 \end{bmatrix} \begin{bmatrix} 700 \\ 3 \\ 3000 \end{bmatrix}$$
$$= \begin{bmatrix} 1(700)+40(3)+2(3000) \\ 50(700)+1000(3)+50(3000) \end{bmatrix}$$
$$= \begin{bmatrix} 700+120+6000 \\ 35000+3000+150000 \end{bmatrix}$$
$$= \begin{bmatrix} 6820 \\ 188000 \end{bmatrix}$$

Hence the value od one truck is $\boxed{\$6,820}$ and the value of one plane is $\boxed{\$188,000}$.