## Step-1

Given that $A = \begin{bmatrix} .6 & .4 \\ .4 & .6 \end{bmatrix}$

To find $\lambda$ value take $|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} .6-\lambda & .4 \\ .4 & .6-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (.6-\lambda)^2 - .16 = 0$$
$$\Rightarrow \lambda^2 - 1.2\lambda + .36 - .16 = 0$$
$$\Rightarrow \lambda^2 - \frac{12}{10}\lambda + \frac{2}{10} = 0$$
$$\Rightarrow 10\lambda^2 - 12\lambda + 2 = 0$$
$$\Rightarrow 10\lambda^2 - 10\lambda - 2\lambda + 2 = 0$$
$$\Rightarrow 10\lambda(\lambda-1) - 2(\lambda-1) = 0$$
$$\Rightarrow (10\lambda-2)(\lambda-1) = 0$$
$$\Rightarrow \lambda = \frac{1}{5}, 1$$

To find eigen vector $\lambda = \frac{1}{5}$ take $\left(A - \frac{1}{5}I\right)x = 0$

$$\Rightarrow \begin{bmatrix} \frac{6}{10} - \frac{1}{5} & \frac{4}{10} \\ \frac{4}{10} & \frac{6}{10} - \frac{1}{5} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 0.4 & 0.4 \\ 0.4 & 0.4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Applying the row operations on the coefficient matrix, $R_2 \to R_2 - R_1, R_1 / 0.4 \Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

This is the reduced matrix, so, the homogeneous equation from this is $x_1 + x_2 = 0$

Putting $x_1 = 1$, the solution set is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is the eigen vector corresponding to the eigen value $\lambda = \frac{1}{5}$

## Step-2

Similarly, when $\lambda = 1$, $(A-\lambda I)x=0$ is $\begin{bmatrix} -0.4 & 0.4 \\ 0.4 & -0.4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

So, proceeding as above, the eigen vector is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

While the eigen values are distinct, the corresponding eigen vectors are linearly independent and so, matrix *S* whose columns are eigen vectors is non singular and thus $S^{-1}=\frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

$A=S\Lambda S^{-1}$ where $\Lambda=\begin{bmatrix} 0.2 & 0 \\ 0 & 1 \end{bmatrix}$ whose diagonal entries are the eigen values of *A*.

So, $A^k = S\Lambda^k S^{-1}$

$$=\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 0.2^k & 0 \\ 0 & 1 \end{bmatrix}\frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

As *k* approaches $\infty$, we get $A^k=\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

$$=\frac{1}{2}\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

This shows that $A^k$ does not approach 0 as *k* approaches $\infty$

## Step-3

The limiting matrix $\Lambda^k=\begin{bmatrix} 0.2^k & 0 \\ 0 & 1^k \end{bmatrix}$, we see that the columns become 0 if the absolute value of the eigen value less than 1, one of the entries is 1 if the eigen value is 1 and one of the entries tend to infinity if the eigen value is greater than 1.