**Calculus II    第十一章quiz 4**

考点一:曲线的参数方程求弧长, 旋转体表面积, 二阶导数.

1. (2019年期中) Let $x = \cos^3 t, y = \sin^3 t$, where $0 \leq t \leq \frac{\pi}{2}$, be a parametrization of a curve.

(1) Find the length of the curve.

(2) Find the area of the surface generated by revolving the curve about the x-axis.

2. (2018年期末) Find the length of the astroid $x = \cos^3 t, y = \sin^3 t, 0 \leq t \leq 2\pi$.

$4\int_0^{\frac{\pi}{2}} 3\sin t \cos t \, dt = 6$

3. If $x = \cos^3 t, y = \sin^3 t$, find $\frac{d^2y}{dx^2}$. $= \dfrac{1}{3\cos^4 t \sin^3 t}$

考点二:用极坐标求所围成的面积.

1. (2020年期末) Determine whether the following statements are true or false? No justification is necessary.

True

Equation $r = 2\sin(\theta), 0 \leq \theta \leq \pi$ in polar form is a circle of radius 1 centered at (0,1).

2. (2021年期末) The area of the region enclosed by $r^2 = \cos 2\theta$ is __2__.

$8\int_0^{\frac{\pi}{4}} \frac{1}{2} r^2 d\theta = 4\left(\frac{1}{2}\sin 2\theta\right)\Big|_0^{\frac{\pi}{4}} = 2$

3. (2021年期中) Given a cardioid $r = a(1 + \cos\theta), a > 0$ and a circle $r = a$.

(1) Find the area of the region that lies inside the cardioid and outside the circle.

(2) Find the area of the region that lies inside the cardioid and inside the circle.

4. (2020年期末) Find the area of region that lies inside the circle $r = 3\sin\theta$ and outside the cardioid $r = 1 + \sin\theta$.

$S = 2\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1}{2}\left((3\sin\theta)^2 - (1+\sin\theta)^2\right) d\theta$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (3 - 4\cos 2\theta - 2\sin\theta) d\theta$

$= 3\theta - 2\sin 2\theta + 2\cos\theta\Big]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \pi$