SUSTC

Midterm I for Calculus II in Spring Semester, 2018

1. (30 pts) Determine which of the following series converges absolutely, converges or diverges. Use any method, and give reasons for your answers.

   (1) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n + 4^n}{3^n + 4^n}$;

   (2) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$;

   (3) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt[n]{n}}$;

   (4) $\displaystyle\sum_{n=1}^{\infty} \frac{n!(n+1)!(n+2)!}{(3n)!}$;

   (5) $\displaystyle\sum_{n=1}^{\infty} (-1)^n(\sqrt{n^2+1} - n)$.

2. (15 pts)

   (1) Find the radius and interval of convergence of the series

   $$\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{\sqrt{n^2+3}};$$

   (2) For what values of x does the series converge absolutely, or conditionally?

3. (10 pts) Find the Maclaurin series of the function

   $$f(x) = (x+1)e^x.$$

4. (10 pts) Use series to evaluate the limit

   $$\lim_{x\to 0} \frac{\ln(1+x^2)}{1-\cos x}.$$

5. (10 pts) Find the length of astroid

   $$x = \cos^3 t, \quad y = \sin^3 t, \quad 0 \le t \le 2\pi.$$

6. (10 pts) Find the area of the region bounded by the circle $r = 2\sin\theta$ for $\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$.

7. (5 pts) Find the first four terms of the binomial series for the function

   $$(1+x)^{1/2}.$$

8. (10 pts) Does the following sequence converge ? If so, to what value?

   $$x_1 = 1, \quad x_{n+1} = \frac{x_n}{2} + \frac{1}{x_n}.$$