Step-1

In case of a diagonal matrix, its inverse is obtained by keeping the off diagonal elements as zeros only and then writing reciprocals of the diagonal elements.

Since D is a diagonal matrix, which has either 1 or â€“1 along the diagonal, its inverse D^{-1} has to be same as D, since reciprocal of 1 is 1 and reciprocal of â€“1 is â€“1 only.

Thus, we want $DAD = B$.

Step-2

Let $D = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix}$, where each of a, b, c, and d is either 1 or â€“1.

Consider the following:

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix}\begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix}$$

$$= \begin{bmatrix} 2a & a & 0 & 0 \\ b & 2b & b & 0 \\ 0 & c & 2c & c \\ 0 & 0 & d & 2d \end{bmatrix}\begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix}$$

$$= \begin{bmatrix} 2a^2 & ab & 0 & 0 \\ ab & 2b^2 & bc & 0 \\ 0 & bc & 2c^2 & cd \\ 0 & 0 & cd & 2d^2 \end{bmatrix}$$

Step-3

Thus, we have $\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 2a^2 & ab & 0 & 0 \\ ab & 2b^2 & bc & 0 \\ 0 & bc & 2c^2 & cd \\ 0 & 0 & cd & 2d^2 \end{bmatrix}$.

Therefore, we get the following three equations:

$$
\begin{aligned}
ab &= -1 \\
bc &= -1 \\
cd &= -1
\end{aligned}
$$

Hence, if $a=1$, then $b=-1$. This gives $c=1$ and $d=-1$. Or, if $a=-1$, then $b=1$. This implies that $c=-1$ and thus, $d=1$.

Step-4

Thus, we get the two following diagonal matrices so that *A* will be symmetric to *B*:

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$ or $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.