

Step-1

When a matrix P has left inverse, it is given by,

$$P_{\text{left}}^{-1} = (P^T P)^{-1} P^T$$

In this case, A is a 3 by 5 matrix and therefore, A^T is a 5 by 3 matrix. This gives $A^T A$ to be a 5 by 5 matrix. Therefore, $(A^T A)^{-1}$ too will be a 5 by 5 matrix. Finally, $(A^T A)^{-1} A^T$ will be a 5 by 3 matrix. Thus, $A_{\text{left}}^{-1} A$ will be a 5 by 5 matrix and thus, it cannot be an identity matrix.

Therefore, A cannot have a left inverse.

Step-2

When a matrix P has right inverse, it is given by,

$$P_{\text{right}}^{-1} = P^T (P P^T)^{-1}$$

In this case, A is a 3 by 5 matrix and therefore, A^T is a 5 by 3 matrix. This gives $A A^T$ to be a 3 by 3 matrix. Therefore, $(A A^T)^{-1}$ too will be a 3 by 3 matrix. Finally, $(A^T A)^{-1} A^T$ will be a 5 by 3 matrix. Thus, $A A_{\text{right}}^{-1}$ will be a 3 by 3 matrix. Since e_1 , e_2 , and e_3 are in the column space of A , it is clear that A can have a right inverse.