Step-1

Let A be an n by n matrix such that $A^2 = A$ and rank = n

$\text{rank } A = n$ says A is an invertible matrix

Given that $A^2 = A$

Multiplying A^{-1} on both sides, we get

$A^{-1}A^2 = A^{-1}A$

By the associativity of multiplication of matrices, we get

$\left(A^{-1}A\right)A = A^{-1}A$

$\Rightarrow IA = I$ (I is n by n identity matrix)

$\Rightarrow A = I$