Step-1

Let $\mathbf{C}^n$ be the complex vector space. It contains *n* independent unit coordinate vectors. Let $v_1, v_2, \ldots, v_n$ be an orthonormal basis for $\mathbf{C}^n$. If a matrix is formed by substituting orthonormal basis as column vectors then this matrix is called as unitary matrix.

Show that any vector *z* equals to the following:

$\left(v_1^H z\right)v_1 + \cdots + \left(v_n^H z\right)v_n$.

Step-2

Recall that if *U* is a unitary matrix then following is true:

$$UU^H = I$$

Columns of the unitary matrix are formed by orthonormal vectors.

Any vector *z* can be written as follows:

$$\begin{aligned} z &= Iz \\ &= UU^H z \\ &= v_1\left(v_1^H z\right) + v_2\left(v_2^H z\right) + \cdots + v_n\left(v_n^H z\right) \\ &= \left(v_1^H z\right)v_1 + \left(v_2^H z\right)v_2 + \cdots + \left(v_n^H z\right)v_n \end{aligned}$$

Step-3

Therefore, vector *z* can be written as $\boxed{\left(v_1^H z\right)v_1 + \cdots + \left(v_n^H z\right)v_n}$.