Step-1

We consider $A=\begin{bmatrix} 8 & x \\ y & 2 \end{bmatrix}$

Given $\det A = 25$

$\Rightarrow 16 - xy = 25$

$\Rightarrow xy = -9$ where x and y are integers.

The possible factors are x = - 1, y = 9 or x = -9, y = 1, or x = 1, y = -9, or x = -3, y = 3 or x = 3, y = -3.

Step-2

Given that the eigen value λ = 5 is repeated.

We easily see that $|A-\lambda I| = \begin{vmatrix} 8-\lambda & x \\ y & 2-\lambda \end{vmatrix} = (\lambda-5)^2$ is satisfied by all the possible pairs of x and y obtained above.

(1) when x = -1, y = 9, we have $A=\begin{bmatrix} 2 & -1 \\ 9 & 8 \end{bmatrix}$ or $A=\begin{bmatrix} 8 & -1 \\ 9 & 2 \end{bmatrix}$

So, $Ax=\lambda x$ when λ = 5 is satisfied by $\begin{bmatrix} -3 & -1 \\ 9 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ or $\begin{bmatrix} 3 & -1 \\ 9 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

So, the corresponding eigen vectors are the solution sets of these systems $\begin{bmatrix} 1 \\ -3 \end{bmatrix}$ or $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ respectively.

Step-3

(2) When x = 1, y = - 9, we have $A=\begin{bmatrix} 2 & 1 \\ -9 & 8 \end{bmatrix}$ or $A=\begin{bmatrix} 8 & 1 \\ -9 & 2 \end{bmatrix}$

So, $Ax=\lambda x$ when λ = 5 is satisfied by $\begin{bmatrix} -3 & 1 \\ -9 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ or $\begin{bmatrix} 3 & 1 \\ -9 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

So, the corresponding eigen vectors are the solution sets of these systems $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ or $\begin{bmatrix} 1 \\ -3 \end{bmatrix}$ respectively.

Step-4

(3) When $x = 9$, $y = -1$, we have $A=\begin{bmatrix}2 & 9\\ -1 & 8\end{bmatrix}$ or $A=\begin{bmatrix}8 & 9\\ -1 & 2\end{bmatrix}$

So, $Ax=\lambda x$ when $\lambda = 5$ is satisfied by $\begin{bmatrix}-3 & 9\\ -1 & 3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$ or $\begin{bmatrix}3 & 9\\ -1 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$

So, the corresponding eigen vectors are the solution sets of these systems $\begin{bmatrix}3\\ 1\end{bmatrix}$ or $\begin{bmatrix}3\\ -1\end{bmatrix}$ respectively.

Step-5

(4) When $x = -9$, $y = 1$, we have $A=\begin{bmatrix}2 & -9\\ 1 & 8\end{bmatrix}$ or $A=\begin{bmatrix}8 & -9\\ 1 & 2\end{bmatrix}$

So, $Ax=\lambda x$ when $\lambda = 5$ is satisfied by $\begin{bmatrix}-3 & -9\\ 1 & 3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$ or $\begin{bmatrix}3 & -9\\ 1 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$

So, the corresponding eigen vectors are the solution sets of these systems $\begin{bmatrix}3\\ -1\end{bmatrix}$ or $\begin{bmatrix}3\\ 1\end{bmatrix}$ respectively.

Step-6

(5) When $x = -3$, $y = 3$, we have $A=\begin{bmatrix}2 & -3\\ 3 & 8\end{bmatrix}$ or $A=\begin{bmatrix}8 & -3\\ 3 & 2\end{bmatrix}$

So, $Ax=\lambda x$ when $\lambda = 5$ is satisfied by $\begin{bmatrix}-3 & -3\\ 3 & 3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$ or $\begin{bmatrix}3 & -3\\ 3 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$

So, the corresponding eigen vectors are the solution sets of these systems $\begin{bmatrix}1\\ -1\end{bmatrix}$ or $\begin{bmatrix}1\\ 1\end{bmatrix}$ respectively.

Step-7

(6) When $x = 3$, $y = -3$, we have $A=\begin{bmatrix}2 & 3\\ -3 & 8\end{bmatrix}$ or $A=\begin{bmatrix}8 & 3\\ -3 & 2\end{bmatrix}$

So, $Ax=\lambda x$ when $\lambda = 5$ is satisfied by $\begin{bmatrix}-3 & 3\\ -3 & 3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$ or $\begin{bmatrix}-3 & -3\\ -3 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$

So, the corresponding eigen vectors are the solution sets of these systems $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ or $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ respectively.

Step-8

In all the above cases, we see that for the eigen value 5, there is only one eigen vector available.

But the eigen value repeated two times say that the algebraic multiplicity not equal to the geometric multiplicity.

Therefore, A is not diagonalizable.