

Step-1

Given that

$$f(a, b, c, d) = \ln(ad - bc)$$

$$\Rightarrow \frac{\partial f}{\partial a} = \frac{d}{ad - bc}$$

$$\frac{\partial f}{\partial b} = \frac{1}{ad - bc}(-c)$$

Step-2

$$\frac{\partial f}{\partial c} = \frac{-b}{ad - bc}$$

$$\frac{\partial f}{\partial d} = \frac{a}{ad - bc}$$

Step-3

$$\text{Therefore } \begin{bmatrix} \frac{\partial f}{\partial a} & \frac{\partial f}{\partial c} \\ \frac{\partial f}{\partial b} & \frac{\partial f}{\partial d} \end{bmatrix} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \boxed{A^{-1}}$$