

Step-1

Suppose T is a linear transformation that transforms $(1,1)$ to $(2,2)$ and $(2,0)$ to $(0,0)$.

We have to find $T(v)$ for the given vectors.

We know that a transformation T is said to be a linear transformation if $T(x+y) = T(x) + T(y)$ and $T(cx) = cT(x)$, where x, y are vectors and a, b are scalars.

We have $T(1,1) = (2,2)$ (1)

$T(2,0) = (0,0)$ (2)

Step-2

a) Given vector is $v = (2,2)$

Now

$$\begin{aligned} T(2,2) &= T(2(1,1)) \\ &= 2T(1,1) && \text{(Since } T(cx) = cT(x) \text{)} \\ &= 2(2,2) && \text{(By (1))} \\ &= (4,4) \end{aligned}$$

Therefore, $T(2,2) = (4,4)$

Step-3

b) Given vector is $v = (3,1)$

Now

$$\begin{aligned} T(3,1) &= T((1,1) + (2,0)) \\ &= T(1,1) + T(2,0) && \text{(Since } T(x+y) = T(x) + T(y) \text{)} \\ &= (2,2) + (0,0) && \text{(By (1) and (2))} \\ &= (2,2) \end{aligned}$$

Therefore, $T(3,1) = (2,2)$

Step-4

c) Given vector is $v = (-1, 1)$

Now

$$\begin{aligned} T(-1, 1) &= T((1, 1) - (2, 0)) \\ &= T(1, 1) - T(2, 0) \quad (\text{Since } T(x + y) = T(x) + T(y)) \\ &= (2, 2) - (0, 0) \quad (\text{By (1) and (2)}) \\ &= (2, 2) \end{aligned}$$

Therefore, $T(-1, 1) = (2, 2)$

Step-5

d) Given vector is $v = (a, b)$.

We have to find $T(v)$.

Step-6

Let $(a, b) = x(1, 1) + y(2, 0)$

$$\Rightarrow (a, b) = (x + 2y, x)$$

$$\Rightarrow x = b, x + 2y = a$$

$$\Rightarrow x = b \quad y = \frac{a - b}{2}$$

Therefore, $(a, b) = b(1, 1) + \frac{(a - b)}{2}(2, 0)$

Step-7

Now

$$\begin{aligned}
T(a,b) &= T\left(b(1,1) + \frac{(a-b)}{2}(2,0)\right) \\
&= b T(1,1) + \frac{(a-b)}{2} T(2,0) \\
&= b(2,2) + \frac{(a-b)}{2}(0,0) \\
&= (2b, 2b)
\end{aligned}$$

Hence $\boxed{T(a,b) = (2b, 2b)}$