Step-1

Let Î»$_1$ be the smallest eigenvalue of the matrix A and the corresponding eigenvector be x_1. Let Î»$_2$ be the second smallest eigenvalue of the matrix A and let x_2 be the corresponding eigenvector.

Consider the Subspace S_2, which is spanned by the eigenvectors x_1 and x_2.

Consider $\max\limits_{x \in S_2} R(x)$.

Step-2

Then the minimum value of $R(x)$ is Î»$_1$ and its maximum value will be Î»$_2$.

Thus, $\max\limits_{x \in S_2} R(x) = \lambda_2$

This gives the following

$$\begin{aligned}
\min_{S_j}\left[\max_{x \in S_j} R(x)\right] &= \min_{S_2}\left[\max_{x \in S_2} R(x)\right] \\
&= \min_{S_2}\left[\lambda_2\right] \\
&= \lambda_2
\end{aligned}$$

Step-3

Therefore, when the subspace S_2 is spanned by the eigenvectors x_1 and x_2, we get $\min\limits_{S_j}\left[\max\limits_{x \in S_j} R(x)\right] = \lambda_2$.