Step-1

(a) Let us consider the two vectors $(1,0)$ and $(0,1)$. Note that every vector x lies in between these two vectors.

Therefore, every vector Ax will lie in between $A\begin{bmatrix}1\\0\end{bmatrix}$ and $A\begin{bmatrix}0\\1\end{bmatrix}$.

Step-2

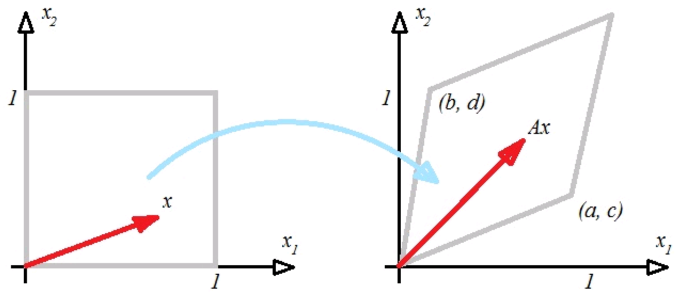
We have assumed that $A=\begin{bmatrix}a & b\\c & d\end{bmatrix}$.

Thus, we get

$$A\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}$$
$$=\begin{bmatrix}a\\c\end{bmatrix}$$
$$A\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}$$
$$=\begin{bmatrix}b\\d\end{bmatrix}$$

Step-3

Thus, every vector Ax lies in between (a,c) and (b,d). Observe the figure drawn below:



The shape of the transformed region will depend on the matrix A. But, it has to be a parallelogram. It might be a rectangle or a square or in the degenerate case, it might be a straight line also. Since every rectangle and every square is a parallelogram and the straight line can be considered as the limiting case of a parallelogram, only, we can say that the shape of the transformed region will be a $\boxed{\text{parallelogram}}$.

Step-4

(b) We have the following:

$$A\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}a\\c\end{bmatrix}$$
$$A\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}b\\d\end{bmatrix}$$

If we want the transformed region to be a square, then the vectors (a,c) and (b,d) should be perpendicular.

Therefore, $ab+cd=0$.

Thus, the transformed region will be a square if the matrix $A=\begin{bmatrix}a & b\\c & d\end{bmatrix}$ is such that $ab+cd=0$.

Step-5

(c) Consider $\begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}ax_1+bx_2\\cx_1+dx_2\end{bmatrix}$.

Now if $b=0$ and $d=0$, then $\begin{bmatrix}ax_1+bx_2\\cx_1+dx_2\end{bmatrix}=\begin{bmatrix}ax_1\\cx_1\end{bmatrix}$, which is equal to $x_1\begin{bmatrix}a\\c\end{bmatrix}$. In this case, every vector x will be transformed to a vector along (a,c). Therefore, in this case the transformed region will be a straight line.

Similarly, if $a=0$ and $c=0$, then $\begin{bmatrix}ax_1+bx_2\\cx_1+dx_2\end{bmatrix}=\begin{bmatrix}bx_2\\dx_2\end{bmatrix}$, which is equal to $x_2\begin{bmatrix}b\\d\end{bmatrix}$. In this case, every vector x will be transformed to a vector along (b,d). Therefore, here also the transformed region will be a straight line.

Step-6

Thus, the transformed region will be a straight line if the matrix A has $a=0$ and $c=0$ or $b=0$ and $d=0$.

Step-7

(d) We have

$$A\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}a\\c\end{bmatrix}$$
$$A\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}b\\d\end{bmatrix}$$

The area of the region due to the transformation will be 1 if the cross product of (a,c) (b,d) has magnitude equal to 1.

Therefore, $ad-bc$ must be equal to 1.

Step-8

Thus, the area of the transformed region will be 1, provided $\boxed{ad-bc=1}$.