Step-1

We have to find the following determinants by using Gaussian elimination.

$$\det\begin{bmatrix} 11 & 12 & 13 & 14 \\ 21 & 22 & 23 & 24 \\ 31 & 32 & 33 & 34 \\ 41 & 42 & 43 & 44 \end{bmatrix}, \text{ and } \det\begin{bmatrix} 1 & t & t^2 & t^3 \\ t & 1 & t & t^2 \\ t^2 & t & 1 & t \\ t^3 & t^2 & t & 1 \end{bmatrix}$$

Step-2

First, we consider

$$\det\begin{bmatrix} 11 & 12 & 13 & 14 \\ 21 & 22 & 23 & 24 \\ 31 & 32 & 33 & 34 \\ 41 & 42 & 43 & 44 \end{bmatrix}$$

$$=\begin{bmatrix} 11 & 12 & 13 & 14 \\ 21 & 22 & 23 & 24 \\ 31 & 32 & 33 & 34 \\ 10 & 10 & 10 & 10 \end{bmatrix}(\text{by adding } -1 \text{ times the third row to the fourth row})$$

Step-3

$$=\begin{bmatrix} 11 & 12 & 13 & 14 \\ 21 & 22 & 23 & 24 \\ 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 \end{bmatrix}(\text{ by adding } -1 \text{ times the second row to the third row})$$

$=\boxed{0}$, since the third and fourth rows are identical.

Step-4

Next, we consider

$$\det\begin{bmatrix} 1 & t & t^2 & t^3 \\ t & 1 & t & t^2 \\ t^2 & t & 1 & t \\ t^3 & t^2 & t & 1 \end{bmatrix}$$

$$= \det \begin{bmatrix} 1 & t & t^2 & t^3 \\ t & 1 & t & t^2 \\ t^2 & t & 1 & t \\ 0 & 0 & 0 & 1-t^2 \end{bmatrix} (\text{by adding } -t \text{ times the third row to the fourth row})$$

Step-5

$$= \det \begin{bmatrix} 1 & t & t^2 & t^3 \\ 0 & 1-t^2 & 1-t^3 & t^2-t^4 \\ 0 & 0 & 1-t^2 & t-t^3 \\ 0 & 0 & 0 & 1-t^2 \end{bmatrix} (\text{by adding } -t \text{ times the second row to the third row})$$

$$= (1-t^2)(1-t^2)(1-t^2)$$

$$= \boxed{(1-t^2)^3}$$

(Since the matrix is triangular, determinant is the product of diagonal elements.)