Linear Algebra-A

Assignments - Week 3

Supplementary Problem Set

1. Let $\boldsymbol{B}=\begin{bmatrix}1 & 2 & -3 & -2\\ 0 & 1 & 2 & -3\\ 0 & 0 & 1 & 2\\ 0 & 0 & 0 & 1\end{bmatrix}, \boldsymbol{C}=\begin{bmatrix}1 & 2 & 0 & 1\\ 0 & 1 & 2 & 0\\ 0 & 0 & 1 & 2\\ 0 & 0 & 0 & 1\end{bmatrix}$, and $(2\boldsymbol{I}-\boldsymbol{C}^{-1}\boldsymbol{B})\boldsymbol{A}^{\mathrm{T}}=\boldsymbol{C}^{-1}$.
   Please find $\boldsymbol{A}$.

2. Let $2\boldsymbol{C}\boldsymbol{A}-2\boldsymbol{A}\boldsymbol{B}=\boldsymbol{C}-\boldsymbol{B}$, where $\boldsymbol{A}=\begin{bmatrix}2 & 1 & 0\\ 2 & 2 & 0\\ 0 & 0 & 1\end{bmatrix}$, $\boldsymbol{B}=\begin{bmatrix}1 & & \\ & -1 & \\ & & 2\end{bmatrix}$.
   Please calculate $\boldsymbol{C}^5$.

3. If $\boldsymbol{A}=\begin{bmatrix}1 & 1 & 0 & 0\\ 1 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 1 & 1\end{bmatrix}$, then find $\boldsymbol{A}^n$.

4. Please show that: for an invertible $n\times n$ upper (lower) triangular matrix $\boldsymbol{A}$, its inverse $\boldsymbol{A}^{-1}$ is also an upper (lower) triangular matrix.
   [*Hint*: by the technique of partitioned matrices and method of mathematical induction (用分块矩阵和数学归纳法)]

5. (1) Let $\boldsymbol{u},\boldsymbol{v}\in\mathbf{R}^n$. When $\boldsymbol{I}-\boldsymbol{u}\boldsymbol{v}^{\mathrm{T}}$ is invertible, its inverse is $\boldsymbol{I}+\frac{1}{k}\boldsymbol{u}\boldsymbol{v}^{\mathrm{T}}$. What is k?
   (2) If $\boldsymbol{A}^{-1}$ is known, and $\boldsymbol{M}=\boldsymbol{A}-\boldsymbol{u}\boldsymbol{v}^{\mathrm{T}}$. What is $\boldsymbol{M}^{-1}$?
   (3) Let $\boldsymbol{A}=\begin{bmatrix}2 & 1 & 1 & 1\\ 1 & 2 & 1 & 1\\ 1 & 1 & 2 & 1\\ 1 & 1 & 1 & 2\end{bmatrix}$, please calculate the inverse of $\boldsymbol{A}$.