Step-1

Given that to solve a rectangular system $Ax = b$, we replace A^{-1}(which does not exist) by $\left(A^T A\right)^{-1} A^T$ (which exists if A has independent columns). We have to show that this is a left inverse of A but not right inverse. On the left of A it gives the identity; on the right it gives the projection P.

Step-2

Since A has independent columns, that is, $A^T A = I$, we have

$$\left(A^T A\right)^{-1} A^T A$$
$$= I^{-1} I$$
$$= I \quad \text{â€¦â€¦} (1)$$

Step-3

And

$$A\left(A^T A\right)^{-1} A^T$$
$$= AA^{-1}\left(A^T\right)^{-1} A^T$$
$$\neq I \quad \text{â€¦â€¦} (2)$$

Because A^{-1} does not exist, that is, $AA^{-1} \neq I$

Step-4

Hence from (1) and (2), $\left(A^T A\right)^{-1} A^T$ is a left inverse of A but not right inverse.

Since $A\left(A^T A\right)^{-1} A^T$ is a projection matrix, by (2), on the right of A, $\left(A^T A\right)^{-1} A^T$ gives the projection P.

Hence the required result is proved.