Step-1

Given that the null space of 3 by 4 matrix A is a line through $(2,3,1,0)$.

a) We have to find the rank of A and the complete solution of $Ax=0$

From the given condition, the dimension of null space is 1.

The number of columns of A is 4, there is only one non pivot column.

Therefore the number of pivot columns is 3.

Hence the rank of A is $\boxed{3}$.

Step-2

b) We have to find the exact row reduced echelon form R of A.

Since the null space of A is the line through $(2,3,1,0)$, the complete solution to $Ax=0$ is $x=(2,3,1,0)$

Therefore x_3 is a free variable $x_4=0$ and x_1, x_2 are pivot variables.

So $x_1=2x_3, x_2=3x_3$

Thus the reduced form of A is

$$\begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step-3

Now

$$Ax=0$$
$$\Rightarrow A\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow x_1-2x_3=0$$
$$x_2-3x_3=0$$
$$x_4=0$$

Step-4

$$\Rightarrow x_1 = 2x_3$$
$$x_2 = 3x_3$$
$$x_4 = 0$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2x_3 \\ 3x_3 \\ x_3 \\ 0 \end{bmatrix}$$
$$= x_3 \begin{bmatrix} 2 \\ 3 \\ 1 \\ 0 \end{bmatrix}$$

Step-5

Therefore the null space of A is line through $(2,3,1,0)$.

Thus reduced row echelon form of A is $\boxed{R = \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$