

Step-1

The best solution is given by the average of the four readings.

$$\hat{x} = \frac{b_1 + b_2 + b_3 + b_4}{4}$$

Thus,

In such case, the expected error is given by $\sigma^2 (A^T A)^{-1}$.

Step-2

Now suppose the matrix A is a column, containing only 1's.

$$A = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Therefore,

Step-3

Thus, we have

$$\begin{aligned} \sigma^2 (A^T A)^{-1} &= \sigma^2 \left\{ (1, 1, 1, 1) \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}^{-1} \\ &= \sigma^2 \{ [1+1+1+1]^{-1} \} \\ &= \sigma^2 [4]^{-1} \end{aligned}$$

$$\begin{aligned} \sigma^2 (A^T A)^{-1} &= \sigma^2 \left(\frac{1}{4} \right) \\ &= \frac{\sigma^2}{4} \end{aligned}$$

Step-4

Thus, we have $\boxed{\sigma^2 (A^T A)^{-1} = \frac{\sigma^2}{4}}$.