

Step-1

Consider the following vectors:

$$\mathbf{x} = \begin{bmatrix} 1 \\ 4 \\ 0 \\ 2 \end{bmatrix}, \text{ and } \mathbf{y} = \begin{bmatrix} 2 \\ -2 \\ 1 \\ 3 \end{bmatrix}$$

The objective is to find the length of each vector and check that whether they are orthogonal.

Step-2

Length of the vector $\mathbf{x} = \|\mathbf{x}\|$

$$\begin{aligned} &= \sqrt{\mathbf{x}^T \mathbf{x}} \\ &= \sqrt{\begin{pmatrix} 1 & 4 & 0 & 2 \end{pmatrix} \begin{bmatrix} 1 \\ 4 \\ 0 \\ 2 \end{bmatrix}} \\ &= \sqrt{1^2 + 4^2 + 0^2 + 2^2} \\ &= \sqrt{21} \end{aligned}$$

Step-3

Length of the vector $\mathbf{y} = \|\mathbf{y}\|$

$$\begin{aligned} &= \sqrt{\mathbf{y}^T \mathbf{y}} \\ &= \sqrt{\begin{pmatrix} 2 & -2 & 1 & 3 \end{pmatrix} \begin{bmatrix} 2 \\ -2 \\ 1 \\ 3 \end{bmatrix}} \\ &= \sqrt{2^2 + (-2)^2 + 1^2 + 3^2} \\ &= \sqrt{18} \\ &= 3\sqrt{2} \end{aligned}$$

Step-4

Inner product of the vectors \mathbf{x} and $\mathbf{y} = \mathbf{x}^T \mathbf{y}$

$$= (1 \ 4 \ 0 \ 2) \begin{bmatrix} 2 \\ -2 \\ 1 \\ 3 \end{bmatrix}$$

$$= 1(2) + 4(-2) + 0(1) + 2(3)$$

$$= 2 - 8 + 0 + 6$$

$$= 0$$

Therefore the given vectors are orthogonal vectors.