Step-1

We need not out any zero to get 4×4 matrix A such that $\det A = 0$

For example consider

$$A=\begin{bmatrix}1&2&3&4\\2&4&6&8\\3&5&7&9\\5&7&9&11\end{bmatrix}$$

Here $\det A = 0$ since first two rows of A are dependent.

Step-2

Consider

$$A=\begin{bmatrix}1&0&0&0\\0&0&3&0\\0&4&0&0\\0&0&0&1\end{bmatrix}$$

$$\det A=\begin{vmatrix}0&3&0\\4&0&0\\0&0&1\end{vmatrix}$$

$$=-3\begin{vmatrix}4&0\\0&1\end{vmatrix}$$

$$=-12$$

$$\neq 0$$

Step-3

And a diagonal matrix or matrixes that can be got into such form by row interchanges are the only non singular matrices with maximum number of zero entries.