

Step-1

If a and b are any two vectors such that the angle between them is \hat{I} , then cosine of the angle \hat{I} is given by the following:

$$\cos \theta = \frac{a^T b}{\|a\| \|b\|}$$

By using this, we get

$$a^T b = \|a\| \|b\| \cos \theta$$

Step-2

Since $\cos \theta \leq 1$ for any \hat{I} , we get the Schwartz's Inequality as follows:

$$|a^T b| \leq \|a\| \|b\|$$

Now suppose a and b are on the same line through the origin. In such case $\theta = 0$ and $\cos \theta = 1$.

Therefore,

$$\begin{aligned} a^T b &= \|a\| \|b\| \cos \theta \\ &= \|a\| \|b\| \end{aligned}$$

There is no other angle for which $\cos \theta = 1$.

Therefore, only when a and b lie on the same line through the origin, the Schwartz's inequality becomes equality.

Step-3

Suppose a and b lie on the opposite sides of the origin, but on the same line. In this case, the angle between a and b is 180° .

We know that $\cos 180^\circ = -1$

Therefore, in such case, we get

$$\begin{aligned} a^T b &= \|a\| \|b\| \cos \theta \\ &= \|a\| \|b\| \cos 180^\circ \\ &= \|a\| \|b\| (-1) \\ &= -\|a\| \|b\| \end{aligned}$$