Step-1

We have to find that is there a 3 by 3 matrix with no zero entries for which $U = R = I$

Consider $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\xrightarrow{R_1 + R_2, R_2 + R_3} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{R_1 + R_2} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{R_2 + R_1, R_3 + R_1} \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$$

Step-2

Now let $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$

$$\xrightarrow{R_2 - R_1, R_3 - R_1} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{R_1 - 2R_2} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{R_1 + R_3, R_2 - R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = U = R$$

Therefore, there is a 3 by 3 matrix with all non zero entries such that $U = R = I$.