Step-1

Given that $A^{-1} = \begin{bmatrix} x & t \\ y & z \end{bmatrix}$

We have to solve for the columns of A^{-1}.

Step-2

Given system is $\begin{bmatrix} 10 & 20 \\ 20 & 50 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Subtracting 2 times row 1 from row 2 gives

$\begin{bmatrix} 10 & 20 \\ 0 & 10 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

Subtracting 2 times row 2 from row 1 gives

$\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$

Step-3

Dividing row 1 and row 2 by 10 gives

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.5 \\ -0.2 \end{bmatrix}$

By row column multiplication of matrices, we get $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.5 \\ -0.2 \end{bmatrix}$

From this, we get $x = 0.5$, $y = -0.2$

Step-4

Given system is $\begin{bmatrix} 10 & 20 \\ 20 & 50 \end{bmatrix}\begin{bmatrix} t \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Subtracting 2 times row 1 from row 2 gives

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$$\begin{bmatrix} 10 & 20 \\ 0 & 10 \end{bmatrix} \begin{bmatrix} t \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Subtracting 2 times row 2 from row 1 gives

$$\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} \begin{bmatrix} t \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

Dividing row 1 and row 2 by 10 gives

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} t \\ z \end{bmatrix} = \begin{bmatrix} -0.2 \\ 0.1 \end{bmatrix}$$

By row column multiplication of matrices, we get $\begin{bmatrix} t \\ z \end{bmatrix} = \begin{bmatrix} -0.2 \\ 0.1 \end{bmatrix}$

From this, we get $t = -0.2$, $z = 0.1$

Hence the columns of A^{-1} are $\begin{bmatrix} 0.5 \\ -0.2 \end{bmatrix}$ and $\begin{bmatrix} -0.2 \\ 0.1 \end{bmatrix}$.