Step-1

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Step-2

Given matrix is $A=\begin{pmatrix}0.001 & 0\\ 1 & 1000\end{pmatrix}$

We write this in the form *LDU*.

$$L=\begin{bmatrix}1 & 0\\ l_{21} & 1\end{bmatrix}, DU=\begin{bmatrix}x_{11} & x_{12}\\ 0 & x_{22}\end{bmatrix}$$

$$LDU=\begin{bmatrix}x_{11} & x_{12}\\ l_{21}x_{11} & l_{21}x_{12}+x_{22}\end{bmatrix}=\begin{bmatrix}0.001 & 0\\ 1 & 1000\end{bmatrix}$$

$$\Rightarrow x_{11}=0.001, x_{12}=0,$$

$$l_{21}x_{11}=1\Rightarrow l_{21}=1000, x_{22}=1000$$

Step-3

$$LDU=\begin{bmatrix}1 & 0\\ 1000 & 1\end{bmatrix}\begin{bmatrix}0.001 & 0\\ 0 & 1000\end{bmatrix}$$

$$DU=\begin{bmatrix}d_1 & 0\\ 0 & d_2\end{bmatrix}\begin{bmatrix}1 & u_{12}\\ 0 & 1\end{bmatrix}=\begin{bmatrix}0.001 & 0\\ 0 & 1000\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}d_1 & d_1u_{12}\\ 0 & d_2\end{bmatrix}=\begin{bmatrix}0.001 & 0\\ 0 & 1000\end{bmatrix}$$

$$\Rightarrow d_1=0.001, u_{12}=0, d_2=1000$$

$$D=\begin{bmatrix}0.001 & 0\\ 0 & 1000\end{bmatrix}, U=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

The pivots are 0.001 and 1000. â€¦â€¦ (1)

Step-4

On the other hand, we apply row operations on the pre multiple and column operations on post multiple identity matrices that were applied first upon *A*.

For suppose *A = IAI*

$$A=\begin{bmatrix}0.001 & 0\\ 1 & 1000\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}A\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$R_2 \to R_2 - 1000R_1 \Rightarrow \begin{bmatrix}0.001 & 0\\ 0 & 1000\end{bmatrix}=\begin{bmatrix}1 & 0\\ -1000 & 1\end{bmatrix}A\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Observe that the given matrix is reduced to the diagonal matrix on the left hand side and so, the pivots are 0.001 and 1000. â€¦â€¦ (2)

Comparing (1) and (2), we say that on partial pivoting in two ways by direct elimination, the pivots remain the same.