

Step-1

We have to construct a 3 by 3 system that has 9 different coefficients on the left-hand side, but rows 2 and 3 become zero in elimination. And we have to find that how many solutions to our system with $b = (1, 10, 100)$ and how many with $b = (0, 0, 0)$

Step-2

Let the required 3 by 3 system be

$$x + 2y + 3z = 1$$

$$4x + 8y + 12z = 4$$

$$5x + 10y + 15z = 5$$

The above system has 9 different coefficients in the left hand side. By performing row 2 $- 4$ times row 1 and row 3 $- 5$ times row 1, row 2 and row 3 will be eliminated.

Step-3

When $b = (1, 10, 100)$, then the system has no solution because after elimination it leads to $0 = 6$, this is absurd, so there is no solution.

Step-4

When $b = (0, 0, 0)$, then the system has infinite solutions since after elimination of row 2 and row 3, we get a single row as $x + 2y + 3z = 0$.