Step-1

Let $Ax = b$ has to be solved for three right hand side b. Let the three solutions be as follows:

$x_1 = (1,1,1)$
$x_2 = (0,1,1)$
$x_3 = (0,0,1)$

These solution form a column of matrix X. if matrix $b = (3,5,8)$ solve $Ax = b$. Challenge problem and find matrix A.

Step-2

Let three right hand sides be:

$b_1 = [3]$
$b_2 = [5]$
$b_3 = [8]$

It can be seen that $x_1, x_2, \text{and } x_3$ contains only elements 0 and 1. So to get these right hand sides matrix A must be row matrix defined as follows:

$A = [3 \quad 5 \quad 8]$

Step-3

Now solve $Ax = b$ as follows:

$$Ax = 3x_1 + 5x_2 + 8x_3$$
$$= 3\begin{bmatrix}1\\1\\1\end{bmatrix} + 5\begin{bmatrix}0\\1\\1\end{bmatrix} + 8\begin{bmatrix}0\\0\\1\end{bmatrix}$$
$$= \begin{bmatrix}3\\8\\16\end{bmatrix}$$
$$\neq b$$

This calculation shows that right hand side is not equal to $b = (3,5,8)$. This gives the challenge to the solution in the problem.

Step-4

To calculate matrix A letâ€™s consider the result found above. Consider $x_1, x_2, \text{and } x_3$ be the three solutions, then:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \\ 16 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 3+5 \\ 3+5+8 \end{bmatrix}$$

$$= \begin{bmatrix} b_1 \\ b_1+b_2 \\ b_1+b_2+b_3 \end{bmatrix}$$

Step-5

Write the solution in terms of x as follows:

$$x_1 = 3$$

$$x_2 = 8$$

$$= x_1 + b_2$$

$$-x_1 + x_2 = b_2$$

$$x_3 = 16$$

$$= x_2 + b_3$$

$$-x_2 + x_3 = b_3$$

Step-6

Therefore, solution in the form of matrix will be:

$$A\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ -x_1 + x_2 \\ -x_2 + x_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ -x_1 + x_2 \\ -x_2 + x_3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 8 \\ 16 \end{bmatrix}$$

Therefore, following matrix A gives the solution $x_1, x_2, \text{and } x_3$.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$