

Step-1

The objective is to find eigenvalues and eigenvectors of matrix A and sketch the ellipse.

Step-2

Consider an ellipse $u^2 + 4v^2 = 1$, which corresponds to the matrix $A = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$.

The characteristic equation for the matrix is,

$$\begin{aligned} |A - \lambda I| &= 0 \\ \begin{vmatrix} 1-\lambda & 0 \\ 0 & 4-\lambda \end{vmatrix} &= 0 \\ (\lambda-1)(\lambda-4) &= 0 \end{aligned}$$

So, the Eigen values are $\lambda_1 = 1$ and $\lambda_2 = 4$.

The corresponding vectors are,

$$\begin{aligned} (A - \lambda_1 I)s_1 &= \begin{bmatrix} 1-1 & 0 \\ 0 & 4-1 \end{bmatrix} s_1 \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix} s_1 \end{aligned}$$

A eigenvectors is $s_1 = (0, 0)^T$.

$$\begin{aligned} (A - \lambda_2 I)s_1 &= \begin{bmatrix} 1-4 & 0 \\ 0 & 4-4 \end{bmatrix} s_1 \\ &= \begin{bmatrix} -3 & 0 \\ 0 & 0 \end{bmatrix} s_1 \end{aligned}$$

A eigenvectors is $s_2 = (0, 0)^T$.

The given ellipse equation is centered at origin, its major axis has half-length of $\frac{1}{\sqrt{1}} = 1$ and points in the direction of s_1 while its major axis has half-length of $\frac{1}{\sqrt{4}} = \frac{1}{2} = 0.5$ and points in the direction of s_2 .

Step-3

The graph of ellipse is as shown below.

