Step-1

a)

Consider that a statement is â€œIf A and B are identical except that $b_{11} = 2a_{11}$, then $\det B = 2\det A$â€.

The objective is to find that whether the statement is true or false.

Step-2

Clearly the above statement is **false**.

To prove this consider an example as,

Example:

Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} B = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ where $b_{11} = 2a_{11}$.

Therefore,

$$\begin{aligned} \det A &= (1)(1) - (1)(1) \\ &= 0 \end{aligned}$$

$$\begin{aligned} \det B &= (2)(1) - (1)(1) \\ &= 1 \end{aligned}$$

Clearly $\det B \neq 2\det A$

Therefore, the given statement is $\boxed{\text{false}}$.

Step-3

b)

Consider that a statement is â€œThe determinant is the product of pivotsâ€™â€™.

The objective is to find that whether the statement is true or false.

Step-4

Clearly the above statement is **false**.

To prove this, consider an example as,

Example:

$$A=\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\1&0&0&0\end{bmatrix}$$

Three row exchanges (Row 3, Row 4) and (Row 2 , Row3) result in identity matrix which is upper triangular here product of pivots is 1 but $\det A=(-1)^3.1=-1$

The given statement is $\boxed{\text{false}}$

Step-5

c)

The given statement â€œIf A is invertible B is singular then $A+B$ is invertibleâ€™â€™

The objective is to find that whether the statement is true or false.

Step-6

Clearly the above statement is **false**.

To prove this, consider an example as,

Example:

Consider

$A=\begin{bmatrix}2&1\\-1&-1\end{bmatrix}$ and $B=\begin{bmatrix}1&1\\1&1\end{bmatrix}$

Now

$$\begin{aligned}\det A&=-2+1\\&=-1\\&\neq 0\end{aligned}$$

And

$$\begin{aligned}\det B&=1-1\\&=0\end{aligned}$$

So that A is invertible and B is singular, also

$$A+B=\begin{bmatrix}3&2\\0&0\end{bmatrix}$$

This is singular, as $\det(A+B)=0$ and is not invertible.

Hence. the given statement is $\boxed{\text{false}}$.

Step-7

d)

The given statement â€œ If A is invertible and B is singular, then AB is singularâ€.

The objective is to find that whether the statement is true or false

Step-8

If A is invertible and B is singular then $\det(A)\neq 0, \det(B)=0$

We know that $\det(AB)=\det A.\det B$

Consider,

$$\begin{aligned}\det(AB)&=\det A.\det B\\&=(\det A)(0) \qquad \text{since } \det B=0\\&=0\end{aligned}$$

Since $\det(AB)$ then the matrix AB is singular

Hence, the given statement is $\boxed{\text{true}}$.

Step-9

e)

The given statement â€œThe determinant of $AB-BA$ is zeroâ€.

The objective is to find that whether the statement is true or false.

Step-10

Clearly the above statement is **false**.

To prove this, consider an example as,

Example:

Consider

$$A=\begin{bmatrix}2 & 1\\1 & 1\end{bmatrix} B=\begin{bmatrix}1 & 1\\2 & 1\end{bmatrix}$$

Then

$$AB=\begin{bmatrix}4 & 3\\3 & 2\end{bmatrix}, BA=\begin{bmatrix}3 & 2\\5 & 3\end{bmatrix}$$

Step-11

Now,

$$AB-BA=\begin{bmatrix}1 & 1\\-2 & -1\end{bmatrix}$$

$$\begin{aligned}\det(AB-BA)&=-1+2\\&=1\\&\neq 0\end{aligned}$$

Hence, the given statement is $\boxed{\text{false}}$.