Step-1

Given vectors are $a_1 = (1,-1,0,0)$, $a_2 = (0,1,-1,0), a_3 = (0,0,1,-1)$

The required ortho normal basis is $\{q_1, q_2, q_3\}$

$$q_1 = \frac{a_1}{\|a_1\|} = \frac{1}{\sqrt{2}}(1,-1,0,0)$$

Step-2

$q_2 = \dfrac{\beta}{\|\beta\|}$ where $\beta = a_2 - (q_1^T a_2) q_1$

$$q_1^T a_2 = \frac{1}{\sqrt{2}}(1,-1,0,0)\begin{pmatrix}0\\1\\-1\\0\end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}(0-1+0+0)$$

$$= -\frac{1}{\sqrt{2}}$$

$$(q_1^T a_2) q_1 = -\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}(1,-1,0,0)$$

$$= -\frac{1}{2}(1,-1,0,0)$$

Step-3

Using this, we get $\beta = (0,1,-1,0) + \frac{1}{2}(1,-1,0,0)$

$$= \left(\frac{1}{2}, \frac{1}{2}, -1, 0\right)$$

$$\|\beta\| = \sqrt{\frac{1}{4} + \frac{1}{4} + 1 + 0}$$

$= \sqrt{\frac{6}{4}}$

$= \frac{\sqrt{6}}{2}$

Therefore, $q_2 = \frac{2}{\sqrt{6}}\left(\frac{1}{2}, \frac{1}{2}, -1, 0\right)$

$= \frac{1}{\sqrt{6}}(1, 1, -2, 0)$

Step-4

$q_3 = \frac{\gamma}{\|\gamma\|}$ where $\gamma = a_3 - \left(q_1^T a_3\right) q_1 - \left(q_2^T a_3\right) q_2$

$$q_1^T a_3 = \frac{1}{\sqrt{2}}(1, -1, 0, 0)\begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}$$

$= \frac{1}{\sqrt{2}}(0 - 0 + 0 + 0)$

$= 0$

$\left(q_1^T a_3\right) q_1 = 0.\frac{1}{\sqrt{2}}(1, -1, 0, 0)$

$= 0$

Step-5

$$q_2^T a_3 = \frac{1}{\sqrt{6}}(1, 1, -2, 0)\begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}$$

$= \frac{1}{\sqrt{6}}(0 + 0 - 2 + 0)$

$= \frac{-2}{\sqrt{6}}$

$$\left(q_2^T a_3\right) q_2 = \frac{-2}{\sqrt{6}} \frac{1}{\sqrt{6}} (1,1,-2,0)$$
$$= \frac{-1}{3}(1,1,-2,0)$$

$$\gamma = (0,0,1,-1) - 0 + \frac{1}{3}(1,1,-2,0)$$
$$= \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -1\right)$$

Step-6

$$\|\gamma\| = \sqrt{\frac{1}{9} + \frac{1}{9} + \frac{1}{9} + 1}$$
$$= \sqrt{\frac{4}{3}}$$
$$= \frac{2}{\sqrt{3}}$$

$$q_3 = \frac{\sqrt{3}}{2}\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -1\right)$$
$$= \frac{1}{2\sqrt{3}}(1,1,1,-3)$$

Thus the required ortho normal basis = $\left\{\frac{1}{\sqrt{2}}(1,-1,0,0), \frac{1}{\sqrt{6}}(1,1,-2,0), \frac{1}{2\sqrt{3}}(1,1,1,-3)\right\}$