

Step-1

A 3 dimensional space is understood by 3 axes x , y , and z . Each axis can be divided into two types – positive and negative.

Consider the two vectors $(1,0,0)$ and $(-1,0,0)$, which are along the x -axis. Any vector along the x -axis can be expressed as $\alpha(1,0,0)$ or $\alpha(-1,0,0)$, where \hat{I}_{\pm} is a nonnegative number.

Step-2

Similarly, any vector along the y -axis can be expressed as $\beta(0,1,0)$ or $\beta(0,-1,0)$, where \hat{I}^2 is a non negative number.

Finally, any vector along the z -axis can be expressed as $\gamma(0,0,1)$ or $\gamma(0,0,-1)$, where \hat{I}^3 is a non negative number.

Step-3

Thus, any vector in the 3 dimensional space can be expressed as a nonnegative combination of the following six vectors: $(1,0,0)$, $(-1,0,0)$, $(0,1,0)$, $(0,-1,0)$, $(0,0,1)$, and $(0,0,-1)$.

Thus, a required set of six vectors is as follows: $\{(1,0,0), (-1,0,0), (0,1,0), (0,-1,0), (0,0,1), (0,0,-1)\}$

It may be noted that this set is not unique. If a is any positive number, then the following set of six vectors will certainly serve the purpose: $\{(a,0,0), (-a,0,0), (0,a,0), (0,-a,0), (0,0,a), (0,0,-a)\}$.

Step-4

Now a nonzero component of a vector is either positive or negative. If p is a positive number, then we can write $p = p \times 1$ and if p is negative, (that is – p is positive), we can write $p = -p \times (-1)$.

This gives us the idea that any vector in the 3 dimensional space can be expressed as a nonnegative combination of the following vectors: $(1,0,0)$, $(0,1,0)$, $(0,0,1)$, and $(-1,-1,-1)$.

Thus, a required set of four vectors is as follows: $\{(1,0,0), (0,1,0), (0,0,1), (-1,-1,-1)\}$. As before, this set is not unique. The set $\{(a,0,0), (0,a,0), (0,0,a), (-a,-a,-a)\}$ also serves the required purpose.