一、

1~3 F T F

二、

1~5 B D D C D.

三、 (1) $\frac{1}{2}$　　(2) $(0,1)$　　(3) $\ln(\sec 1)$

四、 (1) $\lim\limits_{n\to\infty} \sqrt[n]{|a_n|} = (x-1)^2 < 1$

$x \in (0,2)$

① $x=0$　　$a_n = \frac{(-1)^{n+1}}{\sqrt{n+9012}\,\ln n}$.

$|a_n|$ 单减.

且　$\frac{|a_n|}{\frac{1}{n\ln n}} = \infty$　$\sum \frac{1}{n\ln n}$ 发散

∴ 条件收敛.

② $x=2$　　$a_n = \frac{(-1)^{n}}{\sqrt{n+9012}\,\ln n}$

同上

(2)　绝对收敛 $(0,2)$

条件收敛 $x=0$、$x=2$

五、 (1) $\int_0^{2\pi} \int_0^{\frac{\sqrt{2}}{2}} \int_r^{\sqrt{1-r^2}} (r\cos\theta + z)\, r\, dz\, dr\, d\theta$

(2) $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^1 (\rho\sin\varphi\cos\theta + \rho\cos\varphi)\, \rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta$

六、 $V = 8xyz$

$g(x,y,z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

$\nabla \vec{V} = \lambda \nabla \vec{g}$

$\begin{cases} 8yz = \lambda \frac{2x}{a^2} \\ 8xz = \lambda \frac{2y}{b^2} \\ 8xy = \lambda \frac{2z}{c^2} \end{cases}$　　① $\lambda = 0$ 不成立

② $\lambda \neq 0$

$\frac{x^2}{a^2} = \frac{y^2}{b^2} = \frac{z^2}{c^2}$　$\Rightarrow$　$x = \frac{a}{\sqrt{3}}$　$y = \frac{b}{\sqrt{3}}$　$z = \frac{c}{\sqrt{3}}$

∴ $V = \frac{8abc}{3\sqrt{3}}$

七、 $y = x^2$　$x=1$

$\vec{r}(t) = (t, t^2)$

$\vec{r}'(t) = (1, 2t)$

$|\vec{r}'(t)| = \sqrt{4t^2+1}$

$\vec{T} = (\frac{1}{\sqrt{4t^2+1}}, \frac{2t}{\sqrt{4t^2+1}})$

$\frac{d\vec{T}}{dt} = (-\frac{4t}{(4t^2+1)^{\frac{3}{2}}}, \frac{2}{(1+4t^2)^{\frac{3}{2}}})$

$K(1) = \frac{2}{5\sqrt{5}}$　$r = \frac{5\sqrt{5}}{2}$

$(1,1) + \frac{5\sqrt{5}}{2} \cdot \frac{1}{\sqrt{5}}(-2,1) = (-4, \frac{7}{2})$

∴ $(x+4)^2 + (y-\frac{7}{2})^2 = \frac{125}{4}$

八　$M = \int_0^2 \int_0^{\sqrt{x}} \int_0^{4-x^2} xy \, dz \, dy \, dx = \frac{32}{15}$

$M_{xy} = \frac{256}{105}$
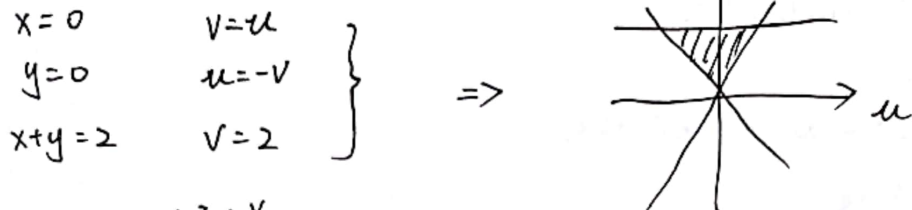
$M_{yz} = \frac{8}{3}$

$M_{xz} = \frac{256\sqrt{2}}{231}$

$\therefore \bar{x} = \frac{M_{yz}}{M} = \frac{5}{4}$ 　　$\bar{y} = \frac{M_{xz}}{M} = \frac{40\sqrt{2}}{77}$ 　　$\bar{z} = \frac{M_{xy}}{M} = \frac{8}{7}$.

九　$u = y - x$ 　　　$J = \begin{Vmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{Vmatrix} = \frac{1}{2}$
　　$v = y + x$

$\begin{aligned} x = 0 & \quad v = u \\ y = 0 & \quad u = -v \\ x+y = 2 & \quad v = 2 \end{aligned} \Bigg\} \Rightarrow$

$I = \int_0^2 \int_{-v}^v e^{\frac{u}{v}} \cdot \frac{1}{2} du \, dv = e - e^{-1}$

十、　(1) $\frac{\partial P}{\partial y} = \frac{\partial N}{\partial z}$; 　$\frac{\partial P}{\partial x} = \frac{\partial M}{\partial z}$; 　$\frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}$

　　　　$\therefore$ exact.

　(2) 　$\frac{\partial f}{\partial x} = e^x \ln y$

　　　　$f = e^x \ln y + g(y, z)$

　　　　$\frac{\partial f}{\partial z} = y \cos z$ 　　$\therefore f = e^x \ln y + y \sin z + h(y)$

　　　　$\frac{\partial f}{\partial y} = \frac{e^x}{y} + \sin z + \frac{\partial h}{\partial z}$ 　　$\therefore \frac{\partial h}{\partial z} = C$

　　　　$\therefore f = e^x \ln y + y \sin z + C$

　　　　$f(1, 3, \pi) - f(1, 1, 1) = e \ln 3 - \sin 1$

十一、　$\nabla \cdot (4x\vec{j}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 4x & 0 \end{vmatrix} = 4\vec{k}$

　　$\therefore \iint_S \nabla \times (4x\vec{j}) \cdot \vec{n} \, d\sigma = \iint 4\vec{k} \cdot \vec{k} \, d\sigma = 4 \iint_S d\sigma = 64\pi$

十二、　$\nabla \cdot \vec{F} = 6 + 4y$ 　　　stocks' 定理.

　　$\iiint_V (6 + 4y) dv = \int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{x^2+y^2}} (6+4y) \, dz \, dy \, dx$

　　　　　　　　　　$= \int_0^{\frac{\pi}{2}} \int_0^1 \int_0^r (6 + 4r\sin\theta) r \, dz \, dr \, d\theta$

　　　　　　　　　　$= \pi + 1$

十二、 $\cos a_n - a_n = \cos b_n$

$\cos a_n - \cos b_n = a_n > 0$

$\Rightarrow 0 < a_n < b_n$

$\sum\limits_{n=1}^{\infty} b_n$ 收敛 $\Rightarrow \lim\limits_{n \to \infty} b_n = 0$

$0 < \lim\limits_{n \to \infty} a_n < b_n$ $\quad \therefore \quad \lim\limits_{n \to \infty} a_n = 0$.