

## Step-1

We have to construct a 3 by 3 matrix  $A$  with no zero entries whose columns are mutually perpendicular, and we have to compute  $A^T A$ , and then we have to explain that why is it a diagonal matrix.

## Step-2

Let

$$A = \begin{bmatrix} a & b & c \\ 2 & 2 & -1 \\ -1 & 2 & 2 \\ 2 & -1 & 2 \end{bmatrix}$$

So

$$a = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, b = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}, c = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$$

## Step-3

And

$$\begin{aligned} a^T b &= \begin{bmatrix} 2 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} \\ &= 4 - 2 - 2 \\ &= 0 \end{aligned}$$

## Step-4

$$\begin{aligned} b^T c &= \begin{bmatrix} 2 & 2 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix} \\ &= -2 + 4 - 2 \\ &= 0 \end{aligned}$$

## Step-5

$$\begin{aligned}
 c^T a &= \begin{bmatrix} -1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} \\
 &= -2 - 2 + 4 \\
 &= 0
 \end{aligned}$$

Therefore  $a, b, c$  are mutually perpendicular

## Step-6

And

$$\begin{aligned}
 A^T A &= \begin{bmatrix} 2 & -1 & 2 \\ 2 & 2 & -1 \\ -1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & -1 \\ -1 & 2 & 2 \\ 2 & -1 & 2 \end{bmatrix} \\
 &= \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} \\
 &= \boxed{9I}
 \end{aligned}$$

Hence  $A^T A$  is a diagonal matrix, because its all non-diagonal elements are 0

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That is, column 1 of  $A^{-1}$  is orthogonal to the space spanned by 2<sup>nd</sup>, 3<sup>rd</sup>, ..., n<sup>th</sup> rows of A.