## Step-1

Let us denote the diagonal matrix, whose diagonal entries are same as that of $A$, by $D$. The Lower triangular part of the matrix $A$ is denoted by $L$ and the upper triangular part of the matrix $A$ is denoted by $U$. Thus, $A = L + D + U$.

If $J$ denotes the Jacobi matrix of the matrix $A$, then we have $J = D^{-1}(-L-U)$.

## Step-2

We get

$$D = \begin{bmatrix} 2 & 0 & & \\ 0 & . & . & \\ & . & . & 0 \\ & & 0 & 2 \end{bmatrix}$$

$$L = \begin{bmatrix} 0 & 0 & & \\ -1 & . & . & \\ & . & . & 0 \\ & & -1 & 0 \end{bmatrix}$$

$$U = \begin{bmatrix} 0 & -1 & & \\ 0 & . & . & \\ & . & . & -1 \\ & & 0 & 0 \end{bmatrix}$$

## Step-3

Therefore,

$$D^{-1}=\begin{bmatrix} \frac{1}{2} & 0 & & \\ 0 & . & . & \\ & . & . & 0 \\ & & 0 & \frac{1}{2} \end{bmatrix}$$

$$-L=\begin{bmatrix} 0 & 0 & & \\ 1 & . & . & \\ & . & . & 0 \\ & & 1 & 0 \end{bmatrix}$$

$$-U=\begin{bmatrix} 0 & 1 & & \\ 0 & . & . & \\ & . & . & 1 \\ & & 0 & 0 \end{bmatrix}$$

## Step-4

Finally, we get

$$J=D^{-1}(-L-U)$$

$$=\begin{bmatrix} \frac{1}{2} & 0 & & \\ 0 & . & . & \\ & . & . & 0 \\ & & 0 & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 0 & 1 & & \\ 1 & . & . & \\ & . & . & 1 \\ & & 1 & 0 \end{bmatrix}$$

$$=\begin{bmatrix} 0 & \frac{1}{2} & & \\ \frac{1}{2} & . & . & \\ & . & . & \frac{1}{2} \\ & & \frac{1}{2} & 0 \end{bmatrix}$$

## Step-5

Thus, in this case, the Jacobi matrix can be described as the matrix having the diagonal entries 0 and having the same element $\frac{1}{2}$ just below and above the main diagonal. Obviously, all other elements of $J$ are zero.

## Step-6

Let $h = \dfrac{1}{n+1}$.

We need to show that $x_1 = (\sin \pi h, \sin 2\pi h, ..., \sin n\pi h)$ is an eigenvector of the Jacobi matrix $J$ for the eigenvalue $\lambda_1 = \cos \pi h = \cos \dfrac{\pi}{n+1}$.

Consider

$$\begin{bmatrix} 0 & \frac{1}{2} & & \\ \frac{1}{2} & . & . & \\ & . & . & \frac{1}{2} \\ & & \frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} \sin \pi h \\ \sin 2\pi h \\ \vdots \\ \sin n\pi h \end{bmatrix}$$

The $i^{\text{th}}$ row of $J$ has $\frac{1}{2}$ in the $i-1^{st}$ and $i+1^{st}$ column. Rest all entries in the row are zero. The $i-1^{st}$ and $i+1^{st}$ entries in the vector $x_1 = (\sin \pi h, \sin 2\pi h, ..., \sin n\pi h)$ are $\sin(i-1)\pi h$ and $\sin(i+1)\pi h$ respectively.

Therefore, the $i^{\text{th}}$ entry in the above product is $\frac{1}{2}(\sin(i-1)\pi h + \sin(i+1)\pi h)$. By Trigonometry, we know that $\sin C + \sin D = 2\sin\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$.

Therefore,

$$\begin{aligned} \frac{1}{2}(\sin(i-1)\pi h + \sin(i+1)\pi h) - 2\sin i\pi h &= \frac{1}{2}(2)\left(\sin\left[\frac{2i\pi h}{2}\right]\right)\left(\cos\left[\frac{-2\pi h}{2}\right]\right) \\ &= \sin i\pi h \cos \pi h \end{aligned}$$

This is same as the $i^{\text{th}}$ entry in the product $\lambda_1 x_1$.

This shows that $x_1 = (\sin \pi h, \sin 2\pi h, ..., \sin n\pi h)$ is an eigenvector of the Jacobi matrix $J$ for the eigenvalue $\lambda_1 = \cos \pi h = \cos \dfrac{\pi}{n+1}$.