

Step-1

Given that A is an n by n matrix and is invertible, that is, $AA^{-1} = I$. Then we have to find that the first column of A^{-1} is orthogonal to the spanned by which rows of A .

Step-2

$$A = \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_n \end{bmatrix}, A^{-1} = [C_1 \ C_2 \ \dots \ C_n]$$

Let

Where R_1, R_2, \dots, R_n are n rows and C_1, C_2, \dots, C_n are n columns of A

Step-3

Then

$$AA^{-1} = I$$

$$\Rightarrow \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_n \end{bmatrix} [C_1 \ C_2 \ \dots \ C_n] = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} R_1 C_1 & R_1 C_2 & R_1 C_3 & \dots & R_1 C_n \\ R_2 C_1 & R_2 C_2 & R_2 C_3 & \dots & R_2 C_n \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ R_n C_1 & R_n C_2 & R_n C_3 & \dots & R_n C_n \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

By equating the first columns of the matrices, we get $R_1 C_1 = 1, R_2 C_1 = 0, R_3 C_1 = 0, \dots, R_n C_1 = 0$

From the second equation onwards, we have the column 1 of A^{-1} is orthogonal to the space spanned by 2nd, 3rdâ€¦ n th rows of A .