Step-1

Given that
$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$$
 and $B = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$

We have

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$$
$$\Rightarrow A^{T} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$$
$$\Rightarrow B^{T} = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$$

Step-2

$$A.B = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} . \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix}$$

$$A.B = \begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix}$$
$$\Rightarrow (A.B)^{T} = \begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix}$$

Step-3

$$B^T.A^T = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix}$$

$$= (A.B)^T$$

$$A^{T}.B^{T} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}.\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} 7 & 2 \\ 3 & 1 \end{pmatrix}$$
$$\neq (A.B)^{T}$$

Step-4

$$B^{T}.A^{T} = (A.B)^{T}$$

$$= (B.A)^{T}$$

$$= A^{T}.B^{T} \quad (\because AB = BA)$$