Step-1

That is;

$$\begin{aligned} c_1+c_2+c_3&=0\\ -c_1&=0\\ -c_2&=0\\ -c_3&=0 \end{aligned}$$

Therefore, v_1, v_2, v_3 are linearly independent.

Step-2

Let,

$$c_1v_1+c_2v_2+c_3v_3+c_4v_4=0$$

This implies;

$$c_1\begin{bmatrix}1\\-1\\0\\0\end{bmatrix}+c_2\begin{bmatrix}1\\0\\-1\\0\end{bmatrix}+c_3\begin{bmatrix}1\\0\\0\\-1\end{bmatrix}+c_4\begin{bmatrix}0\\1\\-1\\0\end{bmatrix}=0$$

$$\begin{bmatrix}c_1+c_2+c_3\\-c_1+c_4\\-c_2-c_4\\-c_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

Step-3

This implies,

$$\begin{aligned} c_1+c_2+c_3&=0\\ -c_1+c_4&=0\\ -c_2-c_4&=0\\ -c_4&=0 \end{aligned}$$

Thus,

$$\begin{aligned} c_4&=0\\ c_2&=0\\ c_1&=0\\ c_3&=0 \end{aligned}$$

Therefore, v_1, v_2, v_3, v_4 are linearly independent.

Step-4

Now,

Let $c_1v_1 + c_2v_2 + c_3v_3 + c_4v_4 + c_5v_5 = 0$

$$c_1\begin{bmatrix}1\\-1\\0\\0\end{bmatrix}+c_2\begin{bmatrix}1\\0\\-1\\0\end{bmatrix}+c_3\begin{bmatrix}1\\0\\0\\-1\end{bmatrix}+c_4\begin{bmatrix}0\\1\\-1\\0\end{bmatrix}+c_5\begin{bmatrix}0\\1\\0\\-1\end{bmatrix}=0$$

$$\begin{bmatrix}c_1+c_2+c_3\\-c_1+c_4+c_5\\-c_2-c_4\\-c_3-c_5\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

Step-5

That is;

$$\begin{aligned}c_1+c_2+c_3&=0\\-c_1+c_4+c_5&=0\\-c_2-c_4&=0\\-c_3-c_5&=0\end{aligned}$$

This implies,

$$\begin{aligned}c_3&=-c_5\\c_2&=-c_4\\c_1&=-c_2-c_3\\&=c_4+c_5\end{aligned}$$

Thus,

$$(c_4+c_5)v_1+(-c_4)v_2+(-c_5)v_3+c_4v_4+c_5v_5=0$$

Therefore v_1, v_2, v_3, v_4, v_5 are linearly dependent.

Step-6

Similarly, $v_1, v_2, v_3, v_4, v_5, v_6$ are linearly dependent. Here the largest possible number is 4 of independent vectors. This number four of the space spared by vâ€™s is the dimension of the space spanned by the vâ€™s.

Step-7

Therefore, This number $\boxed{\text{four}}$ of the space spared by $v's$