Step-1

We have to find the reduced row echelon form R and the rank of the following matrices

a) The 3by 4 matrix of all 1s.

So we can take A as

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\xrightarrow[R_3 - R_1]{R_2 - R_1,} \boxed{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = R}$$

R is row reduced echelon form of A the rank of A = number of non zero rows therefore rank of $A = \boxed{1}$.

Step-2

b) The 4by 4 matrix with $a_{ij} = (-1)^{ij}$.

So we can take A as

$$A = \begin{bmatrix} -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\xrightarrow[R_4 - R_2]{R_3 - R_1,} \begin{bmatrix} -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Step-3

$$\xrightarrow{R_1 + R_2} \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R_2 - R_1} \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R_{12}} \boxed{\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = R}$$

R is a reduced row echelon form of A rank of A= number of non zero rows in R.

Therefore rank of $A = \boxed{2}$.

Step-4

c) The 3 by 4 matrix with $a_{ij} = (-1)^j$.

So we can take A as

$$A = \begin{bmatrix} -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & 1 \end{bmatrix}$$

$$\xrightarrow[R_2 - R_1]{R_3 - R_1,} \begin{bmatrix} -1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{-R_1} \boxed{\begin{bmatrix} 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = R}$$

R is reduced row echelon form of A therefore rank of A= number of non zero rows in R.

Thus rank of $A = \boxed{1}$