Step-1

A is set of 4×4 matrix diagonalized by the eigen vector matrix S.

$\Rightarrow A_i = S\Lambda S^{-1}$ for every A_i matrix in A.

Let $A_1, A_2 \in A$

Then we have $A_1 = S\Lambda_1 S^{-1}$ and $A_2 = S\Lambda_2 S^{-1}$

$$\begin{aligned} A_1 + A_2 &= S\Lambda_1 S^{-1} + S\Lambda_2 S^{-1} \\ &= S(\Lambda_1 + \Lambda_2) S^{-1} \end{aligned}$$

While the sum of diagonal matrices is diagonal, we have $\Lambda_1 + \Lambda_2$ is a diagonal matrix and so, $A_1 + A_2$ is diagonalized by S.

This confirms that $A_1 + A_2$ is in A. â€¦â€¦ (1)

Step-2

Suppose a is any scalar and A_i is any member of A.

Then we have $aA_i = aS\Lambda S^{-1}$

We know that the scalar a commutes with the product of matrices and so, this equation can be written as $aA_i = S(a\Lambda)S^{-1}$

$a\Lambda$ is the product of a with the diagonal entries and allows the resultant matrix is also a diagonal matrix.

So, S diagonalizes aA_i.

In other words, aA_i is also a member of A. â€¦â€¦ (2)

(1), (2) confirms that A is a subspace of all 2×2 matrices.

Step-3

If I diagonalizes A_i, then we write $A_i = IA_iI^{-1} = IA_iI$

But we know that every matrix A_i can be written like this regardless of whether A_i is diagonalizable or not.

In other words, I cannot diagonalize any matrix.

Or, S cannot be replaced by I.

In other words, A_i is a matrix spanned by all the four standard basis matrices.

Thus, the dimension of A is 4.