Step-1

Let A be a 4 by 4 matrix with $\det A = \frac{1}{2}$

The objective is to find $\det(2A), \det(-A), \det(A^2),$ and $\det(A^{-1})$

Step-2

To find $\det(2A)$:

Recall the fact that, â€œif A is $n \times n$ matrix then $\det(kA) = k^n \det(A)$ â€

In this case A is 4×4 matrix and $\det A = \frac{1}{2}$

So, $\det(2A) = 2^4 \det(A)$

$= 2^4 \cdot \frac{1}{2}$
$= 8$

Thus $\det\ (2A) = \boxed{8}$

Step-3

To find $\det(-A)$:

In this case A is 4×4 matrix and $\det A = \frac{1}{2}$

So, $\det(-A) = (-1)^4 \det(A)$

$= (-1)^4 \cdot \frac{1}{2}$
$= \frac{1}{2}$

Thus $\det\ (-A) = \boxed{\frac{1}{2}}$

Step-4

To find $\det\left(A^2\right)$:

Recall the fact that, â€œif A is a matrix then $\det\left(A^n\right)=\left(\det\left(A\right)\right)^n$ â€

From the above fact $\det\left(A^2\right)=\left[\det\left(A\right)\right]^2$

$$=\left(\frac{1}{2}\right)^2$$
$$=\frac{1}{4}$$

Thus $\det\left(A^2\right)=\boxed{\frac{1}{4}}$

Step-5

To find $\det\left(A^{-1}\right)$:

Recall the fact that, â€œif A is a matrix then $\det\left(A^n\right)=\left(\det\left(A\right)\right)^n$ â€

From the above fact $\det\left(A^{-1}\right)=\left[\det\left(A\right)\right]^{-1}$

$$=\left(\frac{1}{2}\right)^{-1}$$
$$=\frac{1}{(1/2)}$$
$$=2$$

Thus $\det\left(A^{-1}\right)=\boxed{2}$