

Step-1

The columns of the matrix A are: $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

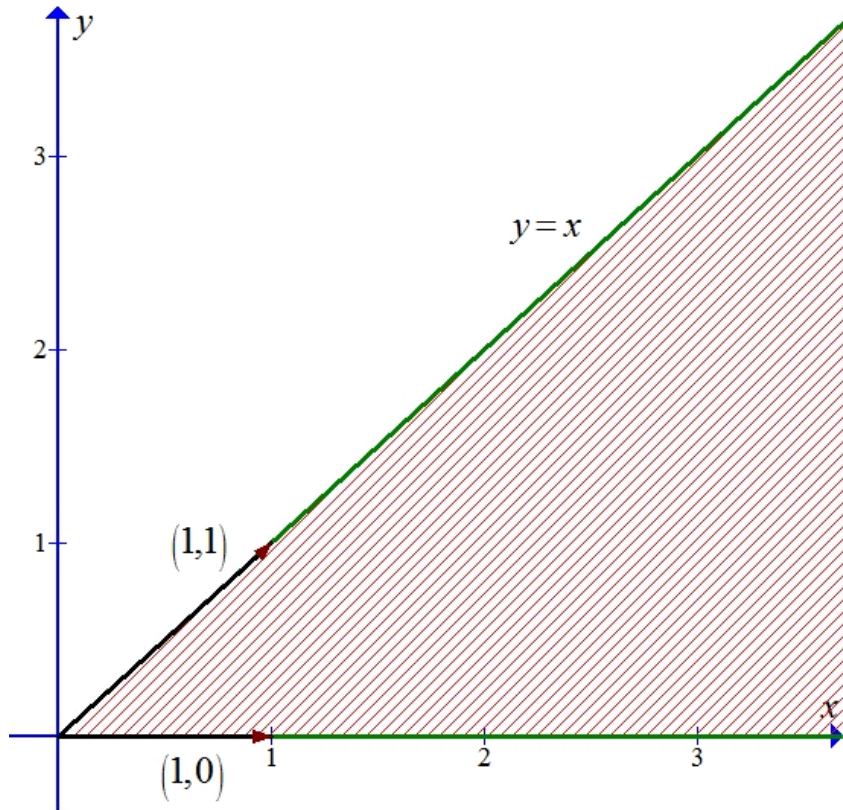
Thus, the nonnegative combinations of the columns of the matrix A is given by

$$\alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Here, $\alpha \geq 0$ and $\beta \geq 0$.

Step-2

The vector $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is along the x -axis and the vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ makes the angle of 45° with the positive direction of x -axis. Thus, the space is the region in the first quadrant between the x -axis and the line $y = x$. The region is as drawn below:



Step-3

Consider the vector $b = (3, 2)$, which lies inside the space.

Let $Ax = b$, where $x = (x_1, x_2)$. Then we have

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$
$$\begin{bmatrix} x_1 + x_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Therefore, $x_2 = 2$ and this implies that $x_1 = 1$.

Therefore, $\boxed{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}}$.

Step-4

Suppose $b = (0, 1)$. Let us write, $Ax = b$. This gives,

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$\begin{bmatrix} y_1 + y_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Therefore, $y_2 = 1$ and this implies that $y_1 = -1$.

Therefore, $\boxed{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}}$.