# Linear Algebra-A

## *Assignments - Week 6*

### Supplementary Problem Set

1. Prove that:
   $\operatorname{rank}(\boldsymbol{A}+\boldsymbol{B}) \leq \operatorname{rank}(\boldsymbol{A})+\operatorname{rank}(\boldsymbol{B})$, where $\boldsymbol{A}$ and $\boldsymbol{B}$ are matrices of same size.

2. If $\boldsymbol{A}$ is a square matrix of order $n$, and $\boldsymbol{A}^2-\boldsymbol{I}=\boldsymbol{0}$. Prove that
   $$\operatorname{rank}(\boldsymbol{A}-\boldsymbol{I})+\operatorname{rank}(\boldsymbol{A}+\boldsymbol{I})=n.$$
   [*Hint*: ~~Apply the results of Problem 1 above, and Problem 38 in Section 2.4.~~]

3. Prove the following properties for the rank of block matrices:
   (a) $\operatorname{rank}\left(\begin{bmatrix}\boldsymbol{A} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{B}\end{bmatrix}\right)=\operatorname{rank}(\boldsymbol{A})+\operatorname{rank}(\boldsymbol{B})$.
   (b) $\operatorname{rank}\left(\begin{bmatrix}\boldsymbol{A} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{B}\end{bmatrix}\right)\leq \operatorname{rank}\left(\begin{bmatrix}\boldsymbol{A} & \boldsymbol{0}\\ \boldsymbol{C} & \boldsymbol{B}\end{bmatrix}\right)$.

4. Prove that: If $\boldsymbol{P}$ and $\boldsymbol{Q}$ are $m\times m$ and $n\times n$ *invertible* matrices respectively, and $\boldsymbol{A}$ is an $m\times n$ matrix, then $\operatorname{rank}(\boldsymbol{PAQ})=\operatorname{rank}(\boldsymbol{A})$.

5. Let $\boldsymbol{A}=\begin{bmatrix}1 & 2 & 1 & 0 & 0\\ 1 & 2 & 2 & 2 & 3\\ -1 & -2 & 0 & 2 & 3\end{bmatrix}$.

(a) Find the general solution to $\boldsymbol{Ax}=\begin{bmatrix}0\\0\\0\end{bmatrix}$.

(b) Find the complete solution to $\boldsymbol{Ax}=\begin{bmatrix}1\\2\\0\end{bmatrix}$.

(c) Find the rank of $\boldsymbol{A}$ and dimensions of the four fundamental subspaces of $\boldsymbol{A}$.
(d) Find bases of the four fundamental subspaces of $\boldsymbol{A}$.