

Step-1

Given that $(AB)x = A(Bx)$

where $x = (1, 0, \dots, 0)$ is the column vector. We have to show that the first column of AB is equal to A times the first column of B .

Step-2

By definition of product of matrices

$$\begin{aligned}(AB)x &= (AB)(1, 0, \dots, 0) \\ &= (AB, 0, 0, \dots, 0)\end{aligned}$$

$$\begin{aligned}Bx &= B(1, 0, \dots, 0) \\ &= (B, 0, 0, \dots, 0)\end{aligned}$$

$$A(Bx) = A(B, 0, \dots, 0)$$

Step-3

Now $(AB)x = A(Bx)$

$$\Rightarrow (AB, 0, 0, \dots, 0) = A(B, 0, \dots, 0)$$

and by the rule that column j of $AB = A$ times (column j of B) we have the first column of AB is equal to A times the first column of B .