Step-1

T is a 3 by 3 upper triangular matrix.

The entries of T are t_{ij}.

Therefore $$T = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ 0 & t_{22} & t_{23} \\ 0 & 0 & t_{33} \end{bmatrix}$$

Then $$T^H = \begin{bmatrix} \overline{t}_{11} & 0 & 0 \\ \overline{t}_{12} & \overline{t}_{22} & 0 \\ \overline{t}_{13} & \overline{t}_{23} & \overline{t}_{33} \end{bmatrix}$$

$$TT^H = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ 0 & t_{22} & t_{23} \\ 0 & 0 & t_{33} \end{bmatrix} \begin{bmatrix} \overline{t}_{11} & 0 & 0 \\ \overline{t}_{12} & \overline{t}_{22} & 0 \\ \overline{t}_{13} & \overline{t}_{23} & \overline{t}_{33} \end{bmatrix}$$

$$= \begin{bmatrix} |t_{11}|^2 + |t_{12}|^2 + |t_{13}|^2 & t_{11}\overline{t}_{22} + t_{13}\overline{t}_{23} & t_{13}\overline{t}_{33} \\ t_{22}\overline{t}_{12} + t_{33}\overline{t}_{13} & |t_{22}|^2 + t_{33}\overline{t}_{23} & t_{23}\overline{t}_{33} \\ t_{33}\overline{t}_{13} & t_{33}\overline{t}_{23} & |t_{33}|^2 \end{bmatrix}$$ â€¦â€¦ (1)

Also, $$T^H T = \begin{bmatrix} \overline{t}_{11} & 0 & 0 \\ \overline{t}_{12} & \overline{t}_{22} & 0 \\ \overline{t}_{13} & \overline{t}_{23} & \overline{t}_{33} \end{bmatrix} \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ 0 & t_{22} & t_{23} \\ 0 & 0 & t_{33} \end{bmatrix}$$

$$= \begin{bmatrix} |t_{11}|^2 & \overline{t}_{11}t_{12} & \overline{t}_{11}t_{13} \\ \overline{t}_{12}t_{11} & |t_{12}|^2 + |t_{22}|^2 & \overline{t}_{12}t_{13} + \overline{t}_{22}t_{23} \\ \overline{t}_{13}t_{11} & \overline{t}_{13}t_{12} + \overline{t}_{23}t_{22} & |t_{13}|^2 + |t_{23}|^2 + |t_{33}|^2 \end{bmatrix}$$ â€¦â€¦ (2)

Step-2

Comparing the ij^{th} entries of (1) and (2) for every i and j, we get

$$|t_{11}|^2 + |t_{12}|^2 + |t_{13}|^2 = |t_{11}|^2$$

$$\Rightarrow t_{12} = t_{13} = 0$$ â€¦â€¦ (3)

$$|t_{22}|^2 + t_{33}\overline{t}_{23} = |t_{12}|^2 + |t_{22}|^2$$

From this, we get $t_{33}\overline{t}_{23} = |t_{12}|^2$

(3) $\Rightarrow t_{33}\overline{t}_{23} = 0$ â€¦â€¦ (4)

$|t_{33}|^2 = |t_{13}|^2 + |t_{23}|^2 + |t_{33}|^2$

$\Rightarrow t_{13} = t_{23} = 0$ â€¦â€¦ (5)

Step-3

In view of (3),(4), and (5), we can see that t_{11}, t_{22}, t_{33} entries can take either real or complex values and $t_{12} = t_{13} = t_{23} = 0$ to satisfy the equality of the products (1) and (2).

Thus, $T = \begin{bmatrix} t_{11} & 0 & 0 \\ 0 & t_{22} & 0 \\ 0 & 0 & t_{33} \end{bmatrix}$ which is a diagonal matrix.

Further, such a matrix T is called the normal triangular matrix.

Therefore, every normal triangular matrix T is diagonal.