Step-1

Consider the equation $Ax = \lambda Mx$. We need to show that if the matrices A and M are symmetric indefinite, then the eigenvalues might not be real.

Consider the following:

$$\begin{aligned} 0 &= \det(A - \lambda M) \\ &= \det\left(\begin{bmatrix} a & b \\ b & d \end{bmatrix} - \lambda \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\right) \\ &= \det\left(\begin{bmatrix} a - \lambda m_1 & b \\ b & d - \lambda m_2 \end{bmatrix}\right) \end{aligned}$$

$$\begin{aligned} 0 &= (a - \lambda m_1)(d - \lambda m_2) - b^2 \\ &= ad - (am_2 + dm_1)\lambda + m_1 m_2 \lambda^2 - b^2 \\ &= m_1 m_2 \lambda^2 - (am_2 + dm_1)\lambda + (ad - b^2) \end{aligned}$$

Step-2

Consider the quadratic equation $m_1 m_2 \lambda^2 - (am_2 + dm_1)\lambda + (ad - b^2) = 0$. Since, we want complex eigenvalues, the roots of this equation should be complex numbers. We know that a quadratic equation $ax^2 + bx + c = 0$ doesnâ€™t have real roots if $b^2 - 4ac < 0$, that is, if $b^2 < 4ac$.

Consider the following:

$$\begin{aligned} (am_2 + dm_1)^2 &< 4m_1 m_2 (ad - b^2) \\ a^2 m_2^{\ 2} + 2adm_1 m_2 + d^2 m_1^{\ 2} &< 4m_1 m_2 ad - 4m_1 m_2 b^2 \\ a^2 m_2^{\ 2} - 2adm_1 m_2 + d^2 m_1^{\ 2} &< -4m_1 m_2 b^2 \\ (am_2 - dm_1)^2 &< -4m_1 m_2 b^2 \end{aligned}$$

Step-3

Therefore, if we have $A = \begin{bmatrix} a & b \\ b & d \end{bmatrix}$ and $M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$ such that $(am_2 - dm_1)^2 < -4m_1 m_2 b^2$, then the equation $Ax = \lambda Mx$ will not have real eigenvalues. Consider the following example $A = \begin{bmatrix} 3 & 1 \\ 1 & -2 \end{bmatrix}$, $M = \begin{bmatrix} -3 & 0 \\ 0 & 2 \end{bmatrix}$.

Here we get

$$\begin{aligned} (am_2 - dm_1)^2 &= (3 \times 2 - (-3) \times (-2))^2 \\ &= (6 - 6)^2 \\ &= 0 \end{aligned}$$

$$\begin{aligned} -4m_1m_2b^2 &= -4(-3)(2)(1)^2 \\ &= 24 \end{aligned}$$

Note that $(am_2 - dm_1)^2 < -4m_1m_2b^2$.

Thus, a required example is $\boxed{A = \begin{bmatrix} 3 & 1 \\ 1 & -2 \end{bmatrix}}$ and $\boxed{M = \begin{bmatrix} -3 & 0 \\ 0 & 2 \end{bmatrix}}$. Of course, we can create any number of such examples.