

Step-1

The objective is prove that, every y in $N(A^T)$ is perpendicular to every Ax in the column space.

Let y be a vector/element in the null space of A^T , then $A^T y = 0$.

That is, $y \in N(A^T)$ then $A^T y = 0$.

Let v be a vector in column space, then it can be written as $v = Ax$, for some row vector x .

That is, $v = Ax$ for some row vector x .

Step-2

Find the product of vectors v^T and y .

$$\begin{aligned} v^T y &= (Ax)^T y \\ &= x^T A^T y \\ &= x^T (A^T y) \end{aligned}$$

$$= x^T \mathbf{0}$$

$$= \mathbf{0}$$

Hence, every y in $N(A^T)$ is perpendicular to every Ax in the column space.