Step-1

We can choose C (4, 2) = 6 pairs of vectors from the given 4 vectors.

$$v_1^T v_2 = (1,2,-2,1)\begin{pmatrix}4\\0\\4\\0\end{pmatrix}$$

$$\begin{aligned}
&= 1(4)+2(0)+(-2)(4)+1(0)\\
&= 4+0-8+0\\
&= -4\\
&\neq 0
\end{aligned}$$

Therefore v_1, v_2 are not orthogonal

Step-2

$$v_1^T v_3 = (1,2,-2,1)\begin{pmatrix}1\\-1\\-1\\-1\end{pmatrix}$$

$$\begin{aligned}
&= 1(1)+2(-1)+(-2)(-1)+1(-1)\\
&= 1-2+2-1\\
&= 0
\end{aligned}$$

Therefore v_1, v_3 are orthogonal

Step-3

$$v_1^T v_4 = (1,2,-2,1)\begin{pmatrix}1\\1\\1\\1\end{pmatrix}$$

$$\begin{aligned}
&= 1(1)+2(1)+(-2)(1)+1(1)\\
&= 1+2-2+1\\
&= 2\\
&\neq 0
\end{aligned}$$

Therefore v_1, v_4 are not orthogonal

Step-4

$$v_2^T v_3 = (4,0,4,0)\begin{pmatrix}1\\-1\\-1\\-1\end{pmatrix}$$

$$\begin{aligned}&= 4(1)+0(-1)+4(-1)+0(-1)\\&= 4+0-4+0\\&= 0\end{aligned}$$

Therefore v_2, v_3 are orthogonal

Step-5

$$v_2^T v_4 = (4,0,4,0)\begin{pmatrix}1\\1\\1\\1\end{pmatrix}$$

$$\begin{aligned}&= 4(1)+0(1)+4(1)+0(1)\\&= 4+0+4+0\\&= 8\\&\neq 0\end{aligned}$$

Therefore v_2, v_4 are not orthogonal

Step-6

$$v_3^T v_4 = (4,-1,-1,-1)\begin{pmatrix}1\\1\\1\\1\end{pmatrix}$$

$$\begin{aligned}&= 4(1)-1(1)-1(1)-1(1)\\&= 4-1-1-1\\&= 1\\&\neq 0\end{aligned}$$

Therefore v_3, v_4 are not orthogonal

Step-7

Therefore, v_1, v_3 and v_2, v_3 are orthogonal and others are not.

An alternative method to find those vectors orthogonal among the given vectors, we consider the square matrix and its transpose.

By multiplying these, we observe those zero entries and the respective row and column numbers which confirm the orthogonality of the vectors present in the given set of vectors.