Step-1

Fibonacci rule is given by: $F_{k+2} = F_{k+1} + F_k$.

The numbers λ_1^k and λ_2^k satisfy the Fibonacci rule:

$$\lambda_1^{k+2} = \lambda_1^{k+1} + \lambda_1^k$$
$$\lambda_2^{k+2} = \lambda_2^{k+1} + \lambda_2^k$$

Step-2

Let the Fibonacci matrix be as follows:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

To find the Eigen values determinant can be written as follows:

$$\begin{aligned} \det(A - \lambda I) &= \begin{bmatrix} 1-\lambda & 1 \\ 1 & -\lambda \end{bmatrix} \\ &= \lambda^2 - \lambda - 1 \end{aligned}$$

Step-3

Put the determinant equal to zero.

$$\begin{aligned} \lambda^2 - \lambda - 1 &= 0 \\ \lambda^2 &= \lambda + 1 \end{aligned}$$

Multiply it by λ_1^k and λ_2^k.

$$\lambda_1^{k+2} = \lambda_1^{k+1} + \lambda_1^k$$
$$\lambda_2^{k+2} = \lambda_2^{k+1} + \lambda_2^k$$

Therefore, this shows that the numbers λ_1^k and λ_2^k satisfy the Fibonacci rule:

Step-4

Any combination of λ_1^k and λ_2^k satisfies the rule. One of the combinations is given as follows:

$$F_k = \frac{\left(\lambda_1^k - \lambda_2^k\right)}{\lambda_1 - \lambda_2}$$

By putting different values of k, different value of F_k can be calculated.

Put $k = 0$ we get value of F_0.

$F_0 = 0$

Put $k = 1$ we get value of F_0.

$F_1 = 1$

Similarly, different values of k can give different Fibonacci numbers.