Step-1

Given $A=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix} B=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ and $A+B=\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}$

Now $A=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$

$$\begin{aligned}
A-\lambda I &= \begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}-\lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\\
&= \begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}-\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}\\
&= \begin{bmatrix}1-\lambda & 0\\ 1 & 1-\lambda\end{bmatrix}
\end{aligned}$$

Step-2

Then

$$\begin{aligned}
|A-\lambda I| &= \begin{vmatrix}1-\lambda & 0\\ 1 & 1-\lambda\end{vmatrix}\\
&= (1-\lambda)^2\\
&= 1+\lambda^2-2\lambda
\end{aligned}$$

Step-3

Becomes

$$\begin{aligned}
\lambda^2-2\lambda+1 &= 0\\
\lambda^2-\lambda-\lambda+1 &= 0\\
\lambda(\lambda-1)-1(\lambda-1) &= 0\\
(\lambda-1)(\lambda-1) &= 0\\
\lambda &= 1,1
\end{aligned}$$

That is $\lambda_1=1, \lambda_2=1$

Therefore the Eigen values of A is 1, 1

Step-4

Now $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$
$$= \begin{bmatrix} 1-\lambda & 1 \\ 0 & 1-\lambda \end{bmatrix}$$

Step-5

Then

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 1 \\ 0 & 1-\lambda \end{vmatrix}$$
$$= (1-\lambda)^2$$
$$= 1 + \lambda^2 - 2\lambda$$

Step-6

Becomes

$$\lambda^2 - 2\lambda + 1 = 0$$
$$\lambda^2 - \lambda - \lambda + 1 = 0$$
$$\lambda(\lambda - 1) - 1(\lambda - 1) = 0$$
$$(\lambda - 1)(\lambda - 1) = 0$$
$$\lambda = 1, 1$$

That is $\lambda_1 = 1, \lambda_2 = 1$

Therefore the Eigen values of *B* is 1, 1

Step-7

Now $A + B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$$A-\lambda I=\begin{bmatrix}2&1\\1&2\end{bmatrix}-\lambda\begin{bmatrix}1&0\\0&1\end{bmatrix}$$
$$=\begin{bmatrix}2&1\\1&2\end{bmatrix}-\begin{bmatrix}\lambda&0\\0&\lambda\end{bmatrix}$$
$$=\begin{bmatrix}2-\lambda&1\\1&2-\lambda\end{bmatrix}$$

Step-8

Then

$$|A-\lambda I|=\begin{vmatrix}2-\lambda&1\\1&2-\lambda\end{vmatrix}$$
$$=(2-\lambda)(2-\lambda)-1$$
$$=4-2\lambda-2\lambda+\lambda^2-1$$
$$=\lambda^2-4\lambda+3$$

$$\lambda^2-4\lambda+3=0$$
$$\lambda^2-3\lambda-\lambda+3=0$$
$$\lambda(\lambda-3)-1(\lambda-3)=0$$
$$(\lambda-3)(\lambda-1)=0$$

That is $\lambda_1=3,\lambda_2=1$

Therefore the Eigen values of $A+B$ is 3, 1

Eigen values of $A+B$ are not equal to Eigen values of A plus Eigen values of B.

Because here the Eigen values of $A+B$ is 3, 1

Step-9

Which are not equal to the Eigen values of A is 1, 1 plus the Eigen values of B is 1, 1

Here Eigen values of A is equal to the Eigen values of B

But Eigen values of $A+B$ are not equal to Eigen values of A plus Eigen values of B.