

Step-1

Given

$$S_1 = |3|, S_2 = \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix}, S_3 = \begin{vmatrix} 3 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{vmatrix}$$

Now changing 3 to 2 in the upper left corner of these matrices, we get

$$A_1 = |2|$$

$$\begin{aligned} A_2 &= \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} \\ &= 6 - 1 \\ &= \boxed{5} \end{aligned}$$

Step-2

Now

$$\begin{aligned} A_3 &= \begin{vmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{vmatrix} \\ &= 2 \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} \\ &= 2 \left[\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 1 & 3 \end{vmatrix} \right] - \begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} \\ &= 2(5) + 2(3) - 3 \\ &= 16 - 3 \\ &= \boxed{13} \end{aligned}$$

Step-3

In general

$$S_n = \begin{vmatrix} 2 & 1 & 0 & \dots & 0 \\ 1 & S_{n-1} & & & \\ \vdots & & & & \end{vmatrix} + \begin{vmatrix} 1 & 0 & \dots & 0 \\ 1 & S_{n-1} & & \\ \vdots & & & \end{vmatrix}$$

(by linearity on row 1)

$$|S_1| = |A_n| + |S_{n-1}|$$

$$|A_n| = |S_n| - |S_{n-1}|$$

$$\begin{aligned} |A_n| &= F_{2n+2} - F_{2n} \\ &= F_{2n+1} \end{aligned}$$

Step-4

We have in Fibonacci series

$$\begin{aligned} F_k &= F_{k-1} + F_{2n-1} \\ &= F_{2n} + F_{2n-1} \end{aligned}$$

So

$$\begin{aligned} F_{2n-1} &= F_{2n} + F_{2n-1} \\ &= F_{2n} + F_{2n} - F_{2n-2} \\ &= 2F_{2n} - F_{2n-2} \end{aligned}$$

Step-5

As in earlier problem we can note that

$$A_n = F_{2n+1} \text{ For each } n$$

So when problem 31 gives all even terms

(Starting from 4th term) of Fibonacci sequence (1, 1, 2, 3, 5, 8, 13, 21,) gives all odd terms (starting from 3rd term) of Fibonacci sequence (1, 1, 2, 3, 5, 8, 13, 21,)