Step-1

Consider the following matrix:

$$A=\begin{bmatrix}1 & 3\\ 0 & 0\end{bmatrix}$$

Compute A^2 and then fill the following blank:

$$e^{At}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}t & 3t\\ 0 & 0\end{bmatrix}+\frac{1}{2}\begin{bmatrix} & \\ & \end{bmatrix}$$
$$=\begin{bmatrix}e^t & \\ 0 & \end{bmatrix}$$

Step-2

Calculate the following:

$$A\cdot A=\begin{bmatrix}1 & 3\\ 0 & 0\end{bmatrix}.\begin{bmatrix}1 & 3\\ 0 & 0\end{bmatrix}$$
$$A^2=\begin{bmatrix}1 & 3\\ 0 & 0\end{bmatrix}$$

Therefore,

$$\boxed{A^2=\begin{bmatrix}1 & 3\\ 0 & 0\end{bmatrix}}$$

Step-3

Next put the value of A^2 in the expansion of e^{At}.

$$e^{At}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}t & 3t\\ 0 & 0\end{bmatrix}+\frac{1}{2!}\begin{bmatrix}t^2 & 3t^2\\ 0 & 0\end{bmatrix}+\cdots$$
$$=\begin{bmatrix}e^t & 3\left(e^t-1\right)\\ 0 & 1\end{bmatrix}$$

Therefore,

$$\boxed{e^{At}=\begin{bmatrix}e^t & 3\left(e^t-1\right)\\ 0 & 1\end{bmatrix}}$$