Step-1

We need to find all the odd permutations of $\{1,2,3,4\}$

The odd permutations on four symbols are $\frac{1}{2}(4!)=12$ in number

They are

One inter change $(1,2,4,3),(1,3,4,2)(1,4,3,2),(2,1,3,4),(3,2,1,4),(4,2,3,1)$

Three interchanges $(2,3,4,1),(2,4,1,3),(3,1,4,2),(3,4,2,1),(4,1,2,3),(4,3,1,2)$

Step-2

The permutations that lead to $\det P=-1$ are odd permutations.

The row interchanges are shown by 1â€™s in the respective places of the 4×4 matrix.

If only one interchange has taken place, then $\det P=(-1)^1=-1$ and so, *P* is an odd permutation.

1^{st} and 2^{nd} rows interchange $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

3^{rd} and 4^{th} rows interchange $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

Step-3

2^{nd} and 4^{th} rows interchange $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

1st and 4th rows interchange $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

Step-4

2nd and 3rd rows interchange $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

1st and 3rd rows interchange $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Step-5

Three row interchange:

$$1234 \to 1243 \to 1423 \to 4123 \Rightarrow \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$1234 \to 2134 \to 2143 \to 2413 \Rightarrow \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Step-6

$$1234 \to 1324 \to 3142 \to 3412 \Rightarrow \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$1234 \to 2134 \to 2314 \to 2134 \Rightarrow \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step-7

$$1234 \to 1243 \to 1423 \to 1432 \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$1234 \to 1324 \to 1342 \to 3142 \Rightarrow \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

We easily see that det P = $(-1)^{\text{number of row interchanges}}$

Step-8

$$= (-1)^3$$
$$= -1$$