Step-1

Therefore, if A is an n by n real matrix, such that $A^2 = -I$, then n must be even.

Step-2

Simplifying we get

$$\begin{aligned} A^2x &= \lambda^2 x \\ -Ix &= \lambda^2 x \\ -x &= \lambda^2 x \end{aligned}$$

This shows that $\lambda^2 = -1$.

Therefore, $\lambda = i$ or $\lambda = -i$

Thus, the eigenvalues of A are i and â€“i.

Step-3

Let A be an n by n real matrix.

If $n = 1$, then $A = [a]$, where a is a real number.

In such case, $A^2 = [a^2]$ and it is clear that $a^2 \neq -1$.

Step-4

Thus, the order of A cannot be 1.

Let A be a 2 by 2 real matrix.

Consider an example, $A = \begin{bmatrix} \sqrt{5} & 2 \\ -3 & -\sqrt{5} \end{bmatrix}$.

Then,

$$
\begin{aligned}
A^2 &= AA \\
&= \begin{bmatrix} \sqrt{5} & 2 \\ -3 & -\sqrt{5} \end{bmatrix} \begin{bmatrix} \sqrt{5} & 2 \\ -3 & -\sqrt{5} \end{bmatrix} \\
&= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \\
&= -I
\end{aligned}
$$

Step-5

Now suppose it were possible to have A to be a 3 by 3 matrix.

Consider the following separation of A.

$$
A = \begin{bmatrix} - & | & - & - \\ & | & & \\ & | & & \end{bmatrix}
$$

This indicates that for the 1 by 1 matrix at the top left corner too should have the property that is square is equal to â€“I. But we have shown that it is not possible.

Therefore, for a 3 by 3 matrix A, $A^2 \neq I$. Similarly, for any odd ordered matrix A, we cannot get $A = -I$.