Step-1

If the columns of A are orthogonal to each other, then we have to say about $A^T A$, and also we have to say about $A^T A$ if the columns of A are orthonormal to each other.

Step-2

Suppose $a=\begin{pmatrix}1\\2\end{pmatrix}, b=\begin{pmatrix}-2\\1\end{pmatrix}$ are the columns of A

Now

$$a^T b=(1,2)\begin{pmatrix}-2\\1\end{pmatrix}$$
$$=-2+2$$
$$=0$$

Therefore a and b are orthogonal columns of A.

Step-3

$$A=[a,b]$$
$$=\begin{bmatrix}1 & -2\\2 & 1\end{bmatrix}$$
$$A^T A=\begin{bmatrix}1 & 2\\-2 & 1\end{bmatrix}\begin{bmatrix}1 & -2\\2 & 1\end{bmatrix}$$
$$=\begin{bmatrix}5 & 0\\0 & 5\end{bmatrix}$$

Hence $A^T A$ is a diagonal matrix.

Since diagonal elements are equal, $A^T A$ is also a scalar matrix.

Step-4

Suppose $a=\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix}, b=\begin{pmatrix}-\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix}$ are the columns of A

Now

$$a^T b = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$= 0$$

Step-5

$$\|a\| = \|b\| = 1$$

$$A = [a, b]$$
$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Step-6

$$A^T A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$= I$, which is the identity matrix

Therefore $A^T A$ is the identity matrix.