

Step-1

Let $b = (b_1, \dots, b_n)$ and $c = (c_1, \dots, c_n)$.

We need to find a matrix M such that $b = Mc$.

Step-2

Let

$$\begin{aligned} b_1 v_1 + \dots + b_n v_n &= Vb \\ &= c_1 w_1 + \dots + c_n w_n \\ &= Wc \end{aligned}$$

Step-3

Therefore, we get

$$\begin{aligned} Vb &= VMc \\ &= Wc \end{aligned}$$

Thus, $VMc = Wc$

Step-4

Thus, we get $VM = W$.

Therefore, $M = V^{-1}W$.