Step-1

Given that $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & b & c \end{bmatrix}$ and the eigen values are -3, 0, and 3

The characteristic equation of A is $\det(A - \lambda I) = \begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ a & b & c-\lambda \end{vmatrix} = 0$

$$\Rightarrow -\lambda \begin{vmatrix} -\lambda & 1 \\ b & c-\lambda \end{vmatrix} - 1 \begin{vmatrix} 0 & 1 \\ a & c-\lambda \end{vmatrix} = 0$$
$$\Rightarrow -\lambda\left(\lambda^2 - c\lambda - b\right) + a = 0$$
$$\Rightarrow -\lambda^3 + c\lambda^2 + b\lambda + a = 0$$

Step-2

On the other hand, the given characteristic equation is $9\lambda - \lambda^3$

Comparing both the above equations, we follow that $a = 0,\ b = 9$ and $c = 0$

Therefore $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 9 & 0 \end{bmatrix}$