Step-1

Consider the function,

$f(x) = \sin 2x.$

The objective is to find the closest function $a\cos x + b\sin x$ to the function f on the interval from $-\pi$ to π.

Step-2

The closest function $a\cos x + b\sin x$ to the function $f(x)$ is $b_1 \sin 2x.$

Here, the coefficient b_1 is the least squares solution to the inconsistent equation $b_1 \sin 2x = f(x)$ and $b_1 = \dfrac{(f(x), \sin x)}{(\sin x, \sin x)}.$

The formula $b_1 = \dfrac{(f(x), \sin x)}{(\sin x, \sin x)}$ brings the $b_1 \sin 2x$ as close as possible to $f(x).$

Now find the coefficient $b_1 = \dfrac{(f(x), \sin x)}{(\sin x, \sin x)}$ as follows:

$$
\begin{aligned}
(f(x), \sin x) &= \int_{-\pi}^{\pi} f(x) \sin x \, dx \\
&= \int_{-\pi}^{\pi} \sin 2x \sin x \, dx \qquad \text{Use } f(x) = \sin 2x \\
&= 2\int_{0}^{\pi} \sin 2x \sin x \, dx \qquad \left(\begin{array}{l} \text{For any even function } f, \text{ i.e. } f(-x) = f(x), \\ \int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx \\ \text{Here, } \sin 2x \cdot \sin x \text{ is an even function.} \end{array} \right) \\
&= \int_{0}^{\pi} 2\sin 2x \sin x \, dx
\end{aligned}
$$

Step-3

Further simplification is as follows:

$$= \int_0^\pi \left(\cos(2x+x) - \cos(2x-x)\right)\, dx$$

$$\left(\text{Use } 2\sin x \sin y = \cos(x+y) - \cos(x-y)\right)$$

$$= \int_0^\pi (\cos 3x - \cos x)\, dx$$

$$= \left[\frac{\sin 3x}{3} - \sin x\right]_0^\pi$$

$$= \frac{\sin 3\pi}{3} - \sin\pi - \frac{\sin(0)}{3} + \sin(0)$$

$= 0$ Use $\sin n\pi = 0$, for all $n = 1, 2, 3, \ldots$

Therefore, $\left(f(x), \sin x\right) = 0.$

Step-4

Now find the inner product $(\sin x, \sin x)$.

$$(\sin x, \sin x) = \int_{-\pi}^{\pi} (\sin x)(\sin x)\, dx$$

$$= \int_{-\pi}^{\pi} \sin^2 x\, dx$$

$$= 2\int_0^\pi \sin^2 x\, dx$$ Here, $\sin^2 x$ is an even function.

$$= 2\int_0^\pi \left(\frac{1 - \cos 2x}{2}\right) dx$$ Use $\cos 2x = 1 - 2\sin^2 x$

$$= \int_0^\pi (1 - \cos 2x)\, dx$$

$$= \left[x - \frac{\sin 2x}{2}\right]_0^\pi$$

$$= \pi - \frac{\sin 2\pi}{2} - 0 + \frac{\sin 2(0)}{2}$$

$= \pi - 0$

$= \pi$

Therefore, $(\sin x, \sin x) = \pi.$

Step-5

Substitute the known values in the formula $b_1 = \dfrac{(f(x), \sin x)}{(\sin x, \sin x)}$.

$$\begin{aligned} b_1 &= \frac{(f(x), \sin x)}{(\sin x, \sin x)} \\ &= \frac{0}{\pi} \\ &= 0 \end{aligned}$$

Therefore, the closest function is $\begin{aligned} b_1 \sin 2x &= 0 \cdot \sin 2x \\ &= 0. \end{aligned}$

Hence, the closest function to the function $f(x)$ is $\boxed{0 \cdot \sin 2x = 0}$.

Step-6

Now the objective is to find the closest straight line $c + dx$ to the function $f(x) = \sin 2x$.

The closest straight line $c + dx$ to the function *f* is $a_1 + b_1 x$.

Here, the coefficients a_1, b_1 are given by the formulas,

$$a_1 = \frac{(f(x), \cos x)}{(\cos x, \cos x)} \text{ and } b_1 = \frac{(f(x), \sin x)}{(\sin x, \sin x)}.$$

From the above calculations, obtained that $b_1 = \dfrac{(f(x), \sin x)}{(\sin x, \sin x)} = 0.$

Step-7

Now find the value of the coefficient a_1.

First compute the inner product $(f(x), \cos x)$.

$$\left(f(x),\cos x\right)=\int_{-\pi}^{\pi} f(x)\cos x\,dx$$

$$=\int_{-\pi}^{\pi}\sin 2x\cos x\,dx \qquad \text{Use } f(x)=\sin 2x$$

$$=0 \qquad \left(\begin{array}{l}\text{For any odd function } f,\ \text{i.e. } f(-x)=-f(x),\\ \int_{-a}^{a} f(x)dx=0.\\ \text{Here, } \sin 2x\cdot\cos x \text{ is an odd function.}\end{array}\right)$$

Therefore, $\left(f(x),\cos x\right)=0$.

Step-8

Now compute the inner product $(\cos x,\cos x)$.

$$(\cos x,\cos x)=\int_{-\pi}^{\pi}(\cos x)(\cos x)dx$$

$$=\int_{-\pi}^{\pi}\cos^2 x\,dx$$

$$=2\int_{0}^{\pi}\cos^2 x\,dx \qquad \text{Here, } \cos^2 x \text{ is an even function.}$$

$$=2\int_{0}^{\pi}\left(\frac{1+\cos 2x}{2}\right)dx \qquad \text{Use } \cos 2x=2\cos^2 x-1$$

$$=\int_{0}^{\pi}(1+\cos 2x)\,dx$$

$$=\left[x+\frac{\sin 2x}{2}\right]_0^{\pi}$$

$$=\pi+\frac{\sin 2\pi}{2}-0-\frac{\sin 2(0)}{2}$$

$$=\pi-0$$
$$=\pi$$

Therefore, $(\cos x,\cos x)=\pi$.

Step-9

Substitute the known values in the formula $a_1 = \frac{(f(x), \cos x)}{(\cos x, \cos x)}$.

$$\begin{aligned} a_1 &= \frac{(f(x), \cos x)}{(\cos x, \cos x)} \\ &= \frac{0}{\pi} \\ &= 0 \end{aligned}$$

Substitute the values of a_1 and b_1 in the function $a_1 + b_1 x$.

$$\begin{aligned} a_1 + b_1 x &= 0 + 0x \\ &= 0 \end{aligned}$$

Hence, the closest straight line to the function *f* is $\boxed{0 + 0x = 0}$.