

Step-1

Given that $\mathbf{V} = \mathbf{R}^2$ and $\mathbf{W} = \mathbf{R}^2$

a) Given $T : \mathbf{V} \rightarrow \mathbf{W}$ such that $T(v) = -v$

We have to find $T(T(v))$

Step-2

Now $T(v) = -v$

So

$$\begin{aligned} T(T(v)) &= T(-v) \\ &= -(-v) \\ &= v \end{aligned}$$

Hence $\boxed{T(T(v)) = v}$

Step-3

b) Given $T : \mathbf{V} \rightarrow \mathbf{W}$ such that $T(v) = v + (1, 1)$

We have to find $T(T(v))$

Step-4

Let $v = (v_1, v_2)$

Then

$$\begin{aligned} T(T(v)) &= T(v_1 + 1, v_2 + 1) \\ &= (v_1 + 1, v_2 + 1) + (1, 1) \\ &= (v_1 + 2, v_2 + 2) \\ &= (v_1, v_2) + (2, 2) \\ &= v + (2, 2) \end{aligned}$$

Hence $\boxed{T(T(v)) = v + (2, 2)}$

Step-5

c) Given $T : \mathbf{V} \rightarrow \mathbf{W}$ such that $T(v) = 90^\circ \text{ rotation} = (-v_2, v_1)$

We have to find $T(T(v))$

Step-6

Let $v = (v_1, v_2)$

Then

$$T(v_1, v_2) = (-v_2, v_1)$$

$$\begin{aligned} T(T(v)) &= T(T(v_1, v_2)) \\ &= T(-v_2, v_1) \\ &= (-v_1, -v_2) \\ &= -(v_1, v_2) \\ &= -v \end{aligned}$$

Hence $\boxed{T(T(v)) = -v}$

Step-7

d) Given $T : \mathbf{V} \rightarrow \mathbf{W}$ such that $T(v) = \text{projection} = \left(\frac{v_1 + v_2}{2}, \frac{v_1 + v_2}{2} \right)$.

We have to find $T(T(v))$

Step-8

Let $v = (v_1, v_2)$

Then

$$\begin{aligned} T(T(v_1, v_2)) &= T\left(\frac{v_1 + v_2}{2}, \frac{v_1 + v_2}{2}\right) \\ &= \left(\frac{v_1 + v_2 + v_1 + v_2}{4}, \frac{v_1 + v_2 + v_1 + v_2}{4}\right) \end{aligned}$$

$$= \left(\frac{v_1 + v_2}{2}, \frac{v_1 + v_2}{2} \right)$$

Hence $\boxed{T(T(v)) = T(v)}$