

Step-1

Consider the orthonormal vectors,

$$\mathbf{a}_1 = \left(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3} \right) \text{ and } \mathbf{a}_2 = \left(\frac{-1}{3}, \frac{2}{3}, \frac{2}{3} \right).$$

Here, $\mathbf{b} = (0, 3, 0)$.

Recall that, the projection of \mathbf{b} onto the vectors \mathbf{a}_1 and \mathbf{a}_2 are given by

$$\mathbf{p}_1 = \frac{\mathbf{b} \cdot \mathbf{a}_1}{\mathbf{a}_1 \cdot \mathbf{a}_1} \mathbf{a}_1 \text{ and } \mathbf{p}_2 = \frac{\mathbf{b} \cdot \mathbf{a}_2}{\mathbf{a}_2 \cdot \mathbf{a}_2} \mathbf{a}_2$$

The projection of \mathbf{b} on \mathbf{a}_1 is calculated as,

$$\begin{aligned} \mathbf{p}_1 &= \frac{\mathbf{b} \cdot \mathbf{a}_1}{\mathbf{a}_1 \cdot \mathbf{a}_1} \mathbf{a}_1 \\ &= \frac{(0, 3, 0) \cdot \left(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3} \right)}{\left(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3} \right) \cdot \left(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3} \right)} \left(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3} \right) \\ &= \frac{2}{\frac{4}{9} + \frac{4}{9} + \frac{1}{9}} \left(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3} \right) \\ &= 2 \left(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3} \right) \\ &= \left(\frac{4}{3}, \frac{4}{3}, \frac{-2}{3} \right) \end{aligned}$$

The projection of \mathbf{b} on \mathbf{a}_2 is calculated as shown below:

$$\begin{aligned}
\mathbf{p}_1 &= \frac{\mathbf{b} \cdot \mathbf{a}_2}{\mathbf{a}_2 \cdot \mathbf{a}_2} \mathbf{a}_2 \\
&= \frac{(0, 3, 0) \cdot \left(\frac{-1}{3}, \frac{2}{3}, \frac{2}{3}\right)}{\left(\frac{-1}{3}, \frac{2}{3}, \frac{2}{3}\right) \cdot \left(\frac{-1}{3}, \frac{2}{3}, \frac{2}{3}\right)} \left(\frac{-1}{3}, \frac{2}{3}, \frac{2}{3}\right) \\
&= \frac{2}{\frac{4}{9} + \frac{4}{9} + \frac{1}{9}} \left(\frac{-1}{3}, \frac{2}{3}, \frac{2}{3}\right) \\
&= 2 \left(\frac{-1}{3}, \frac{2}{3}, \frac{2}{3}\right) \\
&= \left(\frac{-2}{3}, \frac{4}{3}, \frac{4}{3}\right)
\end{aligned}$$

Since the projection of \mathbf{b} onto the plane \mathbf{a}_1 and \mathbf{a}_2 is the sum of the vectors.

Hence, the projection of \mathbf{b} onto the plane \mathbf{a}_1 and \mathbf{a}_2 is

$$\begin{aligned}
\mathbf{p} &= \mathbf{p}_1 + \mathbf{p}_2 \\
&= \left(\frac{4}{3}, \frac{4}{3}, \frac{-2}{3}\right) + \left(\frac{-2}{3}, \frac{4}{3}, \frac{4}{3}\right) \\
&= \left(\frac{4-2}{3}, \frac{4+4}{3}, \frac{-2+4}{3}\right) \\
&= \boxed{\left(\frac{2}{3}, \frac{8}{3}, \frac{2}{3}\right)}.
\end{aligned}$$