Step-1

Explain two reasons that why the exponential matrix e^{At} can never be singular.

(a) *Inverse*: A square matrix that is not invertible is called as singular matrix. However, determinant of e^{At} exists, so inverse will also exist. Inverse of e^{At} is given as follows:

Inverse: $\boxed{e^{-At}}$.

Determinant of the exponential is given as follows:

$$\begin{aligned}\det\left(e^{At}\right) &= e^{\lambda_1 t}\cdot e^{\lambda_2 t}\cdot\ldots\cdot e^{\lambda_n t}\\ &= e^{(\lambda_1+\lambda_2+\ldots+\lambda_n)t}\\ &= e^{\text{trace}(At)}\end{aligned}$$

Step-2

(b) Eigen values: If $Ax = \lambda x$ shows that λ is an Eigen values of A, then $e^{\lambda t}$ is an Eigen value matrix of e^{At}. That means $e^{\lambda t}$ can never be zero.

$e^{At}x = \boxed{e^{\lambda t}x}$.

Step-3

Therefore, exponential matrix e^{At} can never be singular.