Step-1

We know that if m by n matrix Q has orthonormal columns then

$$QQ^T = I_{m\times m}$$
$$Q^TQ = I_{n\times n}$$

Singular Value Decomposition (SVD for any m by n matrix Q is as follows

$$\begin{aligned} Q &= U\Sigma V^T \\ &= \begin{pmatrix}\text{orthogonal} \\ U \text{ is } m\times m\end{pmatrix}\begin{pmatrix} m\times n \text{ matrix } \Sigma \\ \sigma_1\cdots\sigma_r \text{ on diagonal}\end{pmatrix}\begin{pmatrix}\text{orthogonal} \\ V \text{ is } n\times n\end{pmatrix}\end{aligned}$$

Here eigenvectors of QQ^T are in U, eigenvectors of Q^TQ are in V.

The r singular-values on the diagonal of Σ are the square roots of the nonzero eigenvalues of both QQ^T and Q^TQ.

Step-2

Since Q^TQ be m by m identity matrix, so eigenvalues are $\lambda = 1$.

We know that eigenvectors of QQ^T are in U.

Eigenvectors of QQ^T will make matrix U as m by m identity matrix.

$$\begin{aligned} U &= [u_1\cdots u_m] \\ &= \begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1\end{bmatrix} \\ &= I_{m\times m}\end{aligned}$$

Step-3

Now Q^TQ be n by n identity matrix, so eigenvalues are $\lambda = 1$.

We know that eigenvectors of Q^TQ are in V.

Eigenvectors of Q^TQ will make matrix U as n by n identity matrix.

$$
\begin{aligned}
V &= [v_1 \cdots v_n] \\
&= \begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix} \\
&= I_{n\times n}
\end{aligned}
$$

Step-4

The diagonal of Σ are the square roots of the nonzero eigenvalues of QQ^T.

When A has rank r, Σ is m by n matrix, where

$$\sigma_1, \cdots, \sigma_r = 1$$

And all other entries will be zero.

Let us denote $\Sigma = I_{m\times n}$.

The SVD for m by n matrix Q is as follows:

$$
\begin{aligned}
Q &= U\Sigma V^T \\
&= I_{m\times m} I_{m\times n} \left(I_{n\times n}\right)^T \\
&= I_{m\times m} I_{m\times n} I_{n\times n} \\
&= I_{m\times n}
\end{aligned}
$$

Therefore, SVD for m by n matrix Q with orthonormal column is $\boxed{m \text{ by } n \text{ identity matrix}}$.

Step-5

If SVD of m by n matrix A is $A = U\Sigma V^T$, then the pseudoinverse of A is

$$A^+ = V\Sigma^+ U^T$$

For m by n matrix Q with orthonormal columns, we have

$$
\begin{aligned}
U &= I_{m\times m} \\
U^T &= I_{m\times m} \\
V &= I_{n\times n}
\end{aligned}
$$

When A has rank r, Σ^+ is n by m matrix, where

$$\frac{1}{\sigma_1},\ldots,\frac{1}{\sigma_r}=1$$

And all other entries will be zero.

Let us denote $\Sigma^+ = I_{n\times m}$.

Hence, pseudoinverse of Q is

$$\begin{aligned} Q^+ &= V\,\Sigma^+\,U^T \\ &= I_{n\times n} I_{n\times m} I_{m\times m} \\ &= I_{n\times m} \end{aligned}$$

Therefore, the pseudoinverse of m by n matrix Q with orthonormal columns is $\boxed{n \text{ by } m \text{ identity matrix}}$.