Step-1

The inverse of a diagonal matrix is obtained by replacing all diagonal elements by their reciprocals and keeping other zeros as they are.

Therefore, $C^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Thus, $C^{-1}y + Ax = 0$ can be written as

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} + \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$= \begin{bmatrix} y_1 \\ \frac{y_2}{2} \\ \frac{y_3}{2} \\ y_4 \end{bmatrix} + \begin{bmatrix} -x_1 + x_2 \\ -x_1 + x_3 \\ x_2 \\ -x_3 \end{bmatrix}$$

$$= \begin{bmatrix} y_1 - x_1 + x_2 \\ \frac{y_2}{2} - x_1 + x_3 \\ \frac{y_3}{2} + x_2 \\ y_4 - x_3 \end{bmatrix}$$

Step-2

Now consider the system $A^{\mathrm{T}} y = f$.

$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

$$= \begin{bmatrix} -y_1 - y_2 \\ y_1 + y_3 \\ y_2 - y_4 \end{bmatrix}$$

Step-3

Consider the original system of equations:

$$C^{-1} y + Ax = 0$$
$$A^{\mathrm{T}} y = f$$

The first equation is same as $A^{\mathrm{T}} y + A^{\mathrm{T}} CAx = 0$. Subtracting the second equation from this, we get

$$A^{\mathrm{T}} CAx = -f$$

Step-4

We have the following:

$$A^{\mathrm{T}} CA = \begin{bmatrix} -1 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 & -2 \\ -1 & 3 & 0 \\ -2 & 0 & 3 \end{bmatrix}$$

Let $f = (1,1,6)$. Thus, $A^{\mathrm{T}} CAx = -f$ gives

$$\begin{bmatrix} 3 & -1 & -2 \\ -1 & 3 & 0 \\ -2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -6 \end{bmatrix}$$

Solving this system of equations, we get $(x_1, x_2, x_3) = \left(-4, \frac{-5}{3}, \frac{-14}{3}\right)$.

Note that â€“4, $\frac{-5}{3}$, and $\frac{-14}{3}$ represent the potentials at the three nodes.

Step-5

To find the currents along the edges, we should solve the system $C^{-1}y + Ax = 0$ for *y*. Thus, we have $y = -CAx$.

Therefore, we get

$$y = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} -4 \\ \frac{-5}{3} \\ \frac{-14}{3} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-7}{3} \\ \frac{4}{3} \\ \frac{10}{3} \\ \frac{-14}{3} \end{bmatrix}$$

Step-6

Thus, the potentials at the nodes are â€“4, $\frac{-5}{3}$, $\frac{-14}{3}$ and the currents along the edges are

$\boxed{\frac{-7}{3}, \frac{4}{3}, \frac{10}{3}, \frac{-14}{3}}$.