

Step-1

q_1, q_2, \dots, q_n are orthogonal.

By definition $q_i^T q_i = 1$ for every $1 \leq i \leq n$ (1) and

$q_i^T q_j = 0$ for every $1 \leq i \neq j \leq n$ (2)

$$b = c_1 q_1 + c_2 q_2 + \dots + c_n q_n$$

$$\Rightarrow c_1 q_1 = b - c_2 q_2 - c_3 q_3 - \dots - c_n q_n$$

$$\Rightarrow c_1 q_1^T q_1 = b q_1^T - c_2 q_1^T q_2 - c_3 q_1^T q_3 - \dots - c_n q_1^T q_n$$

$$\Rightarrow c_1 \cdot 1 = q_1^T b - c_2 \cdot 0 - \dots - c_n \cdot 0 \text{ while from (1) and (2)}$$

Therefore, $c_1 = q_1^T b$.

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