

Step-1

The first column of A is the vector $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$. Its length is $\sqrt{5}$.

Therefore, the first column of the matrix Q is $\begin{bmatrix} \frac{2}{\sqrt{5}} \\ \frac{-1}{\sqrt{5}} \end{bmatrix}$. The second column of the matrix Q must be of the length 1 and it should be orthogonal to the first column. Thus, the second column of Q must be $\begin{bmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{bmatrix}$.

$$Q = \begin{bmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ \frac{-1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{bmatrix}$$

Thus,

Step-2

If we assume that the column of the matrix A are a and b , then the matrix R is given by $R = \begin{bmatrix} q_1^T a & q_1^T b \\ 0 & q_2^T b \end{bmatrix}$.

Thus, we get

$$\begin{aligned} R &= \begin{bmatrix} 2 \times \frac{2}{\sqrt{5}} + \frac{1}{\sqrt{5}} & \frac{-2}{\sqrt{5}} - \frac{2}{\sqrt{5}} \\ 0 & \frac{-1}{\sqrt{5}} + \frac{4}{\sqrt{5}} \end{bmatrix} \\ &= \begin{bmatrix} \sqrt{5} & \frac{-4}{\sqrt{5}} \\ 0 & \frac{3}{\sqrt{5}} \end{bmatrix} \end{aligned}$$

Step-3

Thus, we have

$$\begin{aligned} A &= Q_0 R_0 \\ &= \begin{bmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ \frac{-1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{bmatrix} \begin{bmatrix} \sqrt{5} & \frac{-4}{\sqrt{5}} \\ 0 & \frac{3}{\sqrt{5}} \end{bmatrix} \end{aligned}$$

Step-4

Let $A_1 = R_0 Q_0$

Thus, we get

$$\begin{aligned} A_1 &= R_0 Q_0 \\ &= \begin{bmatrix} \sqrt{5} & \frac{-4}{\sqrt{5}} \\ 0 & \frac{3}{\sqrt{5}} \end{bmatrix} \begin{bmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ \frac{-1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{bmatrix} \\ &= \begin{bmatrix} 2 + \frac{4}{5} & 1 - \frac{8}{5} \\ \frac{-3}{5} & \frac{6}{5} \end{bmatrix} \end{aligned}$$

$$\begin{aligned} A_1 &= \begin{bmatrix} \frac{14}{5} & \frac{-3}{5} \\ \frac{-3}{5} & \frac{6}{5} \end{bmatrix} \\ &= \frac{1}{5} \begin{bmatrix} 14 & -3 \\ -3 & 6 \end{bmatrix} \end{aligned}$$

Step-5

Thus, we have shown that $A_1 = \frac{1}{5} \begin{bmatrix} 14 & -3 \\ -3 & 6 \end{bmatrix}$.