Step-1

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(a) Given that $\left((AB)^{-1}\right)^T$ comes from $\left(A^{-1}\right)^T$ and $\left(B^{-1}\right)^T$.

We have to find the order.

In $\left((AB)^{-1}\right)^T$ the order of terms are $\left(A^{-1}\right)^T, \left(B^{-1}\right)^T$

Since $\left((AB)^{-1}\right)^T = \left(B^{-1}A^{-1}\right)^T \qquad \left(\text{Since } (AB)^{-1} = B^{-1}A^{-1}\right)$

$= \left(A^{-1}\right)^T \left(B^{-1}\right)^T \qquad \left(\text{Since } (AB)^T = B^T A^T\right)$

Hence the $\left((AB)^{-1}\right)^T$ comes from $\left(A^{-1}\right)^T$ and $\left(B^{-1}\right)^T$ in the order of $\left(A^{-1}\right)^T$ and $\left(B^{-1}\right)^T$.

Step-2

(b) Suppose U is an upper triangular matrix.

We have to find $\left(U^{-1}\right)^T$ is which triangular matrix.

Step-3

Let $U = \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$

Then $U^{-1} = \begin{bmatrix} \frac{1}{a} & -\frac{b}{ad} & \frac{be-cd}{adf} \\ 0 & \frac{1}{d} & -\frac{e}{df} \\ 0 & 0 & \frac{1}{f} \end{bmatrix}$

Now $\left(U^{-1}\right)^{T}=\begin{bmatrix} \frac{1}{a} & 0 & 0 \\ -\frac{b}{ad} & \frac{1}{d} & 0 \\ \frac{be-cd}{adf} & -\frac{e}{df} & \frac{1}{f} \end{bmatrix}$ which is a lower triangular matrix.

Therefore, if U is an upper triangular matrix then $\left(U^{-1}\right)^{T}$ is a lower triangular matrix.