Step-1

The objective is to find the vectors (b_1, b_2, b_3) that are in the column space of A that is given below.

Further objective is to find combinations of the rows of A that give zero.

(a)

The column space contains all linear combinations of the columns of A.

So, $(b_1, b_2, b_3) \in$ Column space of A if,

$$\begin{aligned}(b_1, b_2, b_3) &= c_1(1,2,0) + c_2(2,6,2) + c_3(1,3,5) \\ &= (c_1, 2c_1, 0) + (2c_2, 6c_2, 2c_2) + (c_3, 3c_3, 5c_3) \\ &= (c_1 + 2c_2 + c_3, 2c_1 + 6c_2 + 3c_3, 2c_2 + 5c_3)\end{aligned}$$

Step-2

Assume that,

$$\begin{aligned}a(1,2,1) + b(2,6,3) + c(0,2,5) &= (0,0,0) \\ (a + 2b, 2a + 6b + 2c, a + 3b + 5c) &= (0,0,0)\end{aligned}$$

Rewrite into equation from as follows:

$$\begin{aligned}a + 2b &= 0 \\ 2a + 6b + 2c &= 0 \\ a + 3b + 5c &= 0\end{aligned}$$

$a + 2b = 0$ implies, $a = -2b$.

Substitute $a = -2b$ into $2a + 6b + 2c = 0$ as follows:

$$\begin{aligned}2a + 6b + 2c &= 0 \\ 2b + 2c &= 0\end{aligned}$$

Substitute $a = -2b$ into $a + 3b + 5c = 0$ as follows:

$$\begin{aligned}a + 3b + 5c &= 0 \\ b + 5c &= 0\end{aligned}$$

Solve the equation $2b + 2c = 0$ and $b + 5c = 0$ to get $b = c = 0$.

Hence, $0(1,2,1) + 0(2,6,3) + 0(0,2,5) = (0,0,0)$.

Therefore, the rows of A are linearly independent.

Step-3

(b)

The column space contains all linear combinations of the columns of A.

So, $(b_1,b_2,b_3)\in$ Column space of A if,

$$\begin{aligned}(b_1,b_2,b_3)&=c_1(1,1,2)+c_2(1,2,4)+c_3(1,4,8)\\&=(c_1,c_1,2c_1)+(c_2,2c_2,4c_2)+(c_3,4c_3,8c_3)\\&=(c_1+2c_2+c_3,c_1+2c_2+4c_3,2c_1+4c_2+8c_3)\end{aligned}$$

Step-4

Assume that,

$$\begin{aligned}a(1,1,1)+b(1,2,4)+c(2,4,8)&=(0,0,0)\\(a+b+2c,a+2b+4c,a+4b+8c)&=(0,0,0)\end{aligned}$$

Rewrite into equation from as follows:

$$\begin{aligned}a+b+2c&=0\\a+2b+4c&=0\\a+4b+8c&=0\end{aligned}$$

Solving $a+2b+4c=0$ and $a+4b+8c=0$ to get $2b+4c=0$.

$2b+4c=0$ implies $b=-2c$.

Substitute $b=-2c$ in $a+b+2c=0$ to get $a=0$.

For $b=-2c$, take $c=1$.

This implies, $b=-2$.

Hence, $0(1,1,1)+(-2)(1,2,4)+1(2,4,8)=(0,0,0)$ is the required linear combination.