

Step-1

We know that the matrix exponential for n by n matrix is given by the series

$$e^{At} = I + At + \frac{(At)^2}{2!} + \dots$$

Consider a first-order system $\frac{du}{dt} = Au$ as:

$$\frac{d}{dt} \begin{bmatrix} y \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ y' \end{bmatrix}$$

Step-2

Since we have 2 by 2 matrix so matrix exponential e^{At} is as follows:

$$e^{At} = I + At$$

Substituting $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ for A and $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ for I we get:

$$\begin{aligned} e^{At} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} t \\ &= \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \end{aligned}$$

$$e^{At} = \boxed{\begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}}$$

Therefore,

Step-3

The solution $u(t) = e^{At}u(0)$, starting from $y(0)=3$ and $y'(0)=4$ is as follows:

$$\begin{aligned} u(t) &= \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} \\ &= \boxed{\begin{bmatrix} 3+4t \\ 4 \end{bmatrix}} \end{aligned}$$

Step-4

Since we have $\frac{d}{dt} \begin{bmatrix} y \\ y' \end{bmatrix} = \begin{bmatrix} y' \\ 0 \end{bmatrix}$, so we have

$$\frac{dy}{dt} = y'$$

$$\begin{aligned} \frac{d^2 y}{dt^2} &= y'' \\ &= 0 \end{aligned}$$

Hence, (y, y') , satisfied $\boxed{y'' = 0}$.