

## Step-1

Given that  $b = \begin{bmatrix} 0.217 \\ 0.254 \end{bmatrix}, A = \begin{bmatrix} 0.780 & 0.563 \\ 0.913 & 0.659 \end{bmatrix}, y = \begin{bmatrix} 0.341 \\ -0.087 \end{bmatrix}, z = \begin{bmatrix} 0.999 \\ -1.0 \end{bmatrix}$

We have to compute  $b - Ay$  and  $b - Az$

## Step-2

Now

$$\begin{aligned} Ay &= \begin{bmatrix} 0.780 & 0.563 \\ 0.913 & 0.659 \end{bmatrix} \begin{bmatrix} 0.341 \\ -0.087 \end{bmatrix} \\ &= \begin{bmatrix} 0.780(0.341) + (0.563)(-0.087) \\ 0.913(0.341) + (0.659)(-0.087) \end{bmatrix} \\ &= \begin{bmatrix} 0.21699 \\ 0.254 \end{bmatrix} \end{aligned}$$

Therefore,

$$\begin{aligned} b - Ay &= \begin{bmatrix} 0.217 \\ 0.254 \end{bmatrix} - \begin{bmatrix} 0.216999 \\ 0.254 \end{bmatrix} \\ &= \begin{bmatrix} 0.000001 \\ 0 \end{bmatrix} \approx (1) \end{aligned}$$

Hence  $b - Ay = \begin{bmatrix} 0.000001 \\ 0 \end{bmatrix}$

## Step-3

Now

$$\begin{aligned} Az &= \begin{bmatrix} 0.780 & 0.563 \\ 0.913 & 0.659 \end{bmatrix} \begin{bmatrix} 0.999 \\ -1.0 \end{bmatrix} \\ &= \begin{bmatrix} 0.780(0.999) + (0.563)(-1.0) \\ 0.913(0.999) + (0.659)(-1.0) \end{bmatrix} \\ &= \begin{bmatrix} 0.216220 \\ 0.253087 \end{bmatrix} \end{aligned}$$

## Step-4

Therefore,

$$\begin{aligned} b - Az &= \begin{bmatrix} 0.217 \\ 0.254 \end{bmatrix} - \begin{bmatrix} 0.21622 \\ 0.253087 \end{bmatrix} \\ &= \begin{bmatrix} 0.00078 \\ 0.000913 \end{bmatrix} \in \mathbb{R}^2 \quad (2) \end{aligned}$$

Hence  $\boxed{b - Az = \begin{bmatrix} 0.00078 \\ 0.000913 \end{bmatrix}}$

## Step-5

If  $Ax = b$  is a non homogeneous system of equations in two variables whose solutions are  $y$  and  $z$ , then we follow that  $Ay = b$ ,  $Az = b$ .

Consequently,  $b - Ay$  and  $b - Az$

In view of (1) and (2), we see that  $y$  is the better solution than  $z$ .

## Step-6

On the other hand, comparing the solutions  $y$  and  $z$  to the true solution  $x = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ ,  $z$  is the better solution than  $y$ .

The corresponding residuals  $x - y = \delta y = \begin{bmatrix} 0.659 \\ -0.913 \end{bmatrix}$  and  $x - z = \delta z = \begin{bmatrix} 0.001 \\ 0 \end{bmatrix}$  confirms this.