Step-1

Let λ be the Eigen value of matrix A and $\lambda \neq c$. Here, c is any constant and not an Eigen value of A. Let

$u = e^{ct} v$

Find v to solve the following:

$du/dt = Au - e^{ct} b$

Also explain how it breaks down when c is an Eigen value.

Step-2

Substitute the $u = e^{ct} v$ in the following and solve:

$$\begin{aligned}
\frac{du}{dt} &= Au - e^{ct} b \\
\frac{d\left(e^{ct} v\right)}{dt} &= A\left(e^{ct} v\right) - e^{ct} b \\
c\left(e^{ct} v\right) &= A\left(e^{ct} v\right) - e^{ct} b \\
cv &= Av - b
\end{aligned}$$

$$\begin{aligned}
(A - cI)v &= b \\
v &= (A - cI)^{-1} b
\end{aligned}$$

Therefore, $\boxed{v = (A - cI)^{-1} b}$. This gives the particular solution.

Step-3

If c is Eigen value then $c = \lambda$ and following must be true:

$\det(A - cI) = 0$

This shows that $(A - cI)$ is not invertible. Therefore, solution for v will not be possible.