Step-1

Let the numbers of trucks in Boston, Los Angeles and Chicago be x, y and z respectively.

Every month half of those in Boston and in Los Angeles go to Chicago, the other 0.5 stay where they are and the trucks in Chicago are split equally between Boston and Los Angeles.

So, after one month, the new numbers of trucks in Los Angeles and Boston are given by

$$X = 0.5x + 0.5z$$
$$Y = 0.5y + 0.5z$$
$$Z = 0.5x + 0.5y$$

The system of equation can be written as follows:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Where the coefficient matrix is $A = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 \end{bmatrix}.$

Step-2

Find one eigenvalue of matrix $A.$

The characteristic equation is as follows:

$$\det(A - \lambda I) = 0$$

$$\begin{vmatrix} 0.5-\lambda & 0 & 0.5 \\ 0 & 0.5-\lambda & 0.5 \\ 0.5 & 0.5 & -\lambda \end{vmatrix} = 0$$
$$-\lambda^3 + \lambda^2 + 0.25\lambda - 0.25 = 0$$
$$(1-\lambda)(0.5-\lambda)(-0.5-\lambda) = 0$$
$$\lambda = 1, 0.5, -0.5$$

So, the one eigenvalue of matrix A is $\lambda = 1.$

Step-3

Find one eigenvector of matrix $A.$

$$(A-\lambda I)\mathbf{x}=\mathbf{0}$$
$$(A-1I)\mathbf{x}=\mathbf{0} \qquad \text{Since } \lambda=1$$
$$\begin{bmatrix} -0.5 & 0 & 0.5 \\ 0 & -0.5 & 0.5 \\ 0.5 & 0.5 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} -0.5 & 0 & 0.5 \\ 0 & -0.5 & 0.5 \\ 0 & 0.5 & -0.5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad R_3 \to R_3+R_1$$
$$\begin{bmatrix} -0.5 & 0 & 0.5 \\ 0 & -0.5 & 0.5 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad R_3 \to R_3+R_2$$

$$\Rightarrow -0.5x+0.5z=0, -0.5y+0.5z=0$$
$$\Rightarrow x=z, y=z$$
$$\Rightarrow x=y=z$$

Therefore, the eigenvector corresponding to eigenvalue $\lambda=1$ is $(1,1,1)$.

Step-4

The initial number of trucks is in Los Angeles, Boston, and Chicago is,

$x=1, y=1, z=1$.

After one month the new numbers of trucks in Los Angeles and Boston are given by

$$\begin{aligned} X &= 0.5x+0.5z \\ &= 0.5(1)+0.5(1) \qquad \text{Since } x=1, z=1 \\ &= 0.5+0.5 \\ &= 1 \end{aligned}$$

$$\begin{aligned} Y &= 0.5y+0.5z \\ &= 0.5(1)+0.5(1) \qquad \text{Since } y=1, z=1 \\ &= 0.5+0.5 \\ &= 1 \end{aligned}$$

$$\begin{aligned} Z &= 0.5x+0.5y \\ &= 0.5(1)+0.5(1) \qquad \text{Since } x=1, y=1 \\ &= 0.5+0.5 \\ &= 1 \end{aligned}$$

Suppose we have the equal numbers of trucks in the three major centers and after every month the numbers of trucks will be same, so this defines a steady state.

Hence, the steady state solution is $u_\infty = \boxed{[1,1,1]^T}$.