Step-1

Given

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step-2

First let us find the cofactors of 1, the elements in first row of A, that is

$$a_{11} = A_{11} = \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}$$

Cofactor of

$= 0$

$$a_{12} = A_{12} = -\begin{vmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}$$

Cofactor of

$= -(0-1)$

$= 1$

Step-3

$$a_{13} = A_{13} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{vmatrix}$$

Cofactor of

$= 0$

$$a_{14} = A_{14} = -\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Cofactor of

$= \boxed{-1}$

Step-4

Now determinant of A is

$$\det A = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} + a_{14}A_{14}$$

$$= 0(0) + 1(1) + 0(0) + 0(-1)$$

$$= \boxed{1}$$

Step-5

(B)

The elements in first row of B,

$$b_{11} = B_{11} = \begin{vmatrix} 3 & 4 & 5 \\ 7 & 8 & 9 \\ 0 & 0 & 1 \end{vmatrix}$$

Cofactor of

$$= 3(8) - 4(7) + 5(0)$$

$$= 24 - 28$$

$$= -4$$

$$b_{12} = B_{12} = -\begin{vmatrix} 0 & 4 & 5 \\ 6 & 8 & 9 \\ 0 & 0 & 1 \end{vmatrix}$$

Cofactor of

$$= -\left[0(8-0) - 4(6-0) + 5(0) \right]$$

$$= 24$$

Step-6

$$b_{13} = B_{13} = \begin{vmatrix} 0 & 3 & 5 \\ 6 & 7 & 9 \\ 0 & 0 & 1 \end{vmatrix}$$

Cofactor of

$$= 0(7-0) - 3(6-0) + 5(0-0)$$

$$= -18$$

$$b_{14} = B_{14} = -\begin{vmatrix} 0 & 3 & 4 \\ 6 & 7 & 8 \\ 0 & 0 & 0 \end{vmatrix}$$

Cofactor of

$$= 0$$

2

Step-7

Now, determinant of B is

$$\det B = b_{11}B_{11} + b_{12}B_{12} + b_{13}B_{13} + b_{14}B_{14}$$

$$= 0(-4) + 0(24) + 1(-18) + 2(0)$$

$$= \boxed{-18}$$