

Step-1

Consider the matrix:

$$A = \begin{bmatrix} .3 & .3 & .2 \\ .3 & .2 & .4 \\ .2 & .2 & .1 \end{bmatrix}$$

For first row,

$$\begin{aligned}\lambda_1 &= |.3| + |.3| + |.2| \\ &= 0.8\end{aligned}$$

For first row,

$$\begin{aligned}\lambda_2 &= |.3| + |.2| + |.4| \\ &= .9\end{aligned}$$

For first row,

$$\begin{aligned}\lambda_3 &= |.2| + |.2| + |.1| \\ &= .5\end{aligned}$$

Therefore, we get,

$$\lambda_{\max} = .9$$

Consider another matrix:

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

For first row,

$$\begin{aligned}\lambda_1 &= |2| + |-1| + |0| \\ &= 3\end{aligned}$$

For first row,

$$\begin{aligned}\lambda_2 &= |-1| + |2| + |-1| \\ &= 4\end{aligned}$$

For first row,

$$\begin{aligned}\lambda_3 &= |0| + |-1| + |2| \\ &= 3\end{aligned}$$

Therefore, we get,

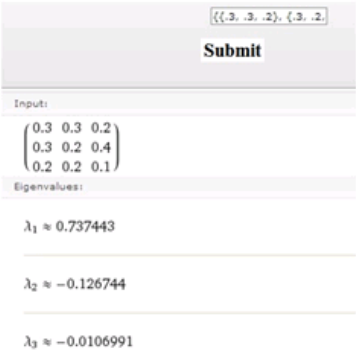
$$\lambda_{\max} = 4$$

Step-2

Consider the matrix:

$$A = \begin{bmatrix} .3 & .3 & .2 \\ .3 & .2 & .4 \\ .2 & .2 & .1 \end{bmatrix}$$

By using matrix calculator (the screenshot is given below), the eigenvalues of A are given by,



The screenshot shows a web-based matrix calculator interface. At the top, there is a text input field containing the matrix definition $\{\{.3, .3, .2\}, \{.3, .2\}\}$. Below this is a prominent "Submit" button. Under the button, the "Input:" section displays the matrix $\begin{pmatrix} 0.3 & 0.3 & 0.2 \\ 0.3 & 0.2 & 0.4 \\ 0.2 & 0.2 & 0.1 \end{pmatrix}$. The "Eigenvalues:" section lists three results: $\lambda_1 \approx 0.737443$, $\lambda_2 \approx -0.126744$, and $\lambda_3 \approx -0.0106991$.

Step-3

The circles that bound the Eigenvalues are C_1 , C_2 , and C_3 .

The center of C_1 is at the point $(.3, 0)$.

Step-4

The radius of C_1 is given by,

$$\begin{aligned} r_1 &= |.3| + |.2| \\ &= .5 \end{aligned}$$

The center of C_2 is at the point $(.2, 0)$.

The radius of C_2 is given by,

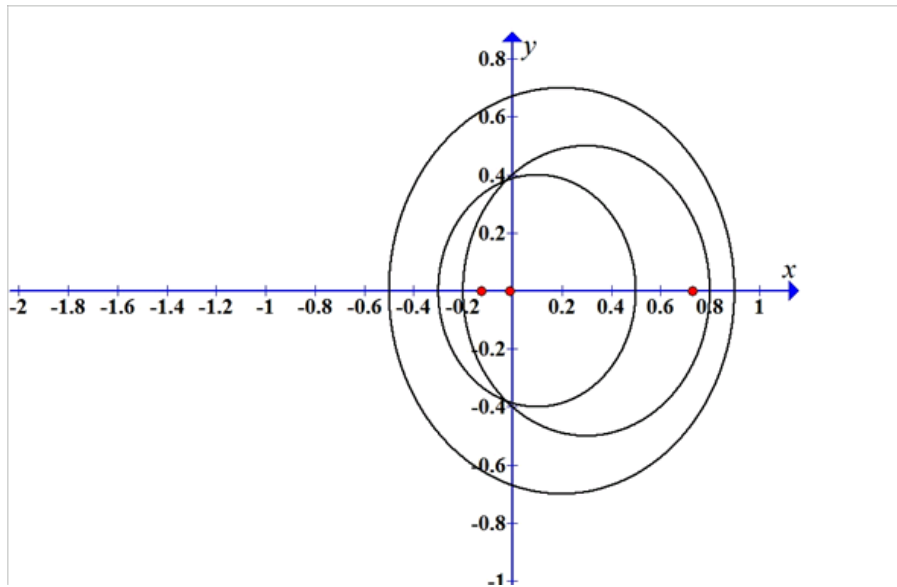
$$\begin{aligned} r_2 &= |.3| + |.4| \\ &= .7 \end{aligned}$$

The center of C_3 is at the point $(.1, 0)$.

The radius of C_3 is given by,

$$\begin{aligned} r_3 &= |.2| + |.2| \\ &= .4 \end{aligned}$$

The graph of circles C_1 , C_2 , and C_3 is given below.

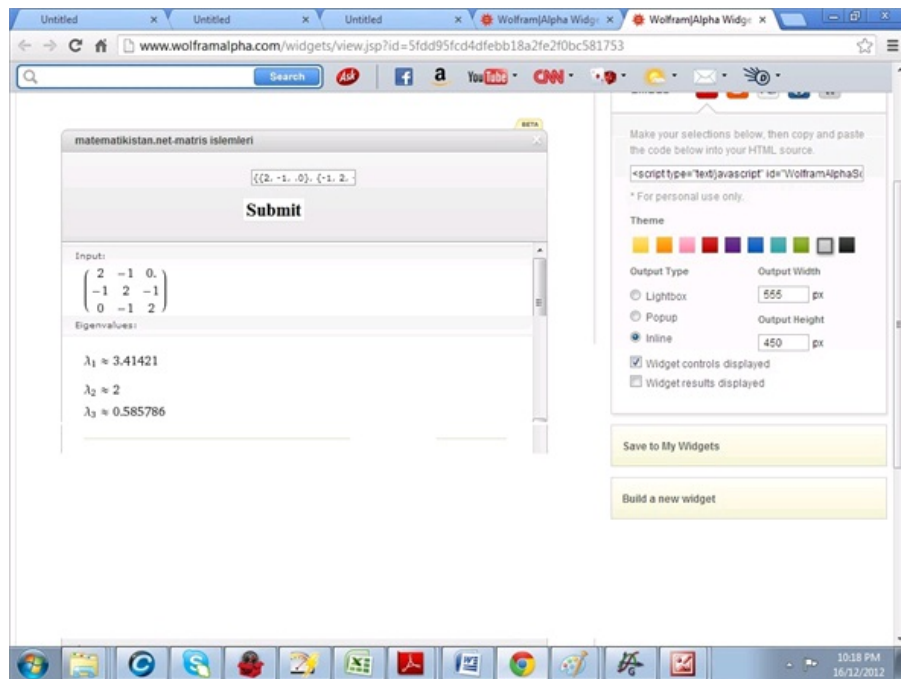


Step-5

Consider the second matrix:

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

By using matrix calculator (the screenshot is given below), the eigenvalues of A are given by,



The Gershgorin circles that bound the Eigenvalues are C_1 , C_2 , and C_3 .

The center of C_1 is at the point (2, 0).

The radius of C_1 is given by,

$$r_1 = |-1| + |0|$$

$$= 1$$

The center of C_2 is at the point (2, 0).

The radius of C_2 is given by,

$$r_2 = |-1| + |-1|$$

$$= 2$$

The center of C_3 is at the point (2, 0).

The radius of C_3 is given by,

$$r_3 = |0| + |1|$$

$$= 1$$

The graph of Gershgorin circles C_1 , C_2 , and C_3 is given below.

