

## Step-1

a) The given system of equations is

$$2x_1 + 5x_2 = 1$$

$$x_1 + 4x_2 = 2$$

We need to solve the given system by Cramer's rule

$$A = \begin{bmatrix} 2 & 5 \\ 1 & 4 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

## Step-2

Replacing the first and second columns of  $A$  with  $b$ , we get the matrices  $B_1$  and  $B_2$

$$B_1 = \begin{bmatrix} 1 & 5 \\ 2 & 4 \end{bmatrix}, B_2 = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Now

$$\det(A) = |A| = \begin{vmatrix} 2 & 5 \\ 1 & 4 \end{vmatrix}$$

$$= (2)(4) - (5)(1)$$

$$= 3$$

## Step-3

And

$$\det(B_1) = |A_1| = \begin{vmatrix} 1 & 5 \\ 2 & 4 \end{vmatrix}$$

$$= (1)(4) - (2)(5)$$

$$= -6$$

$$\det(B_2) = |A_2| = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$$

$$= (2)(2) - (1)(1)$$

$$= 3$$

## Step-4

Thus, by Cramer's rule we have

$$x_1 = \frac{\det(B_1)}{\det(A)}$$

$$= \frac{-6}{3}$$

$$= -2$$

$$x_2 = \frac{\det(B_2)}{\det(A)}$$

$$= \frac{3}{3}$$

$$= 1$$

Thus, the solution for the given system is  $x_1 = -2$  and  $x_2 = 1$

## Step-5

b) The given system of equations is

$$2x_1 + x_2 = 1$$

$$x_1 + 2x_2 + x_3 = 0$$

$$x_2 + 2x_3 = 0$$

We need to solve the given system by Cramer's rule

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

## Step-6

Replacing the first, second and third columns of  $A$  with  $b$  we get the matrices  $B_1, B_2$  and  $B_3$

$$B_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \quad B_2 = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 2 \end{bmatrix} \quad \text{And} \quad B_3 = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Now

$$\det(A) = |A| = \begin{vmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix}$$

$$= 2(3) - (2)$$

$$= 4$$

## Step-7

And

$$\det(B_1) = |B_1| = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix}$$

$$= (3) - (0)$$

$$= 3$$

$$\det(B_2) = |B_2|$$

## Step-8

$$= \begin{vmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 2 \end{vmatrix}$$

$$= 2(0) - (2)$$

$$= -2$$

$$\det(B_3) = |B_3|$$

$$= \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & 0 \end{vmatrix}$$

$$= 2(0) - (0) + (1)$$

$$= 1$$

## Step-9

Thus, by cramers rule we have

$$\begin{aligned}x_1 &= \frac{\det(B_1)}{\det(A)} \\&= \frac{3}{4}\end{aligned}$$

$$\begin{aligned}x_2 &= \frac{\det(B_2)}{\det(A)} \\&= \frac{-2}{4} \\&= \frac{-1}{2}\end{aligned}$$

$$\begin{aligned}x_3 &= \frac{\det(B_3)}{\det(A)} \\&= \frac{1}{4}\end{aligned}$$

## Step-10

Thus, the solution for the given system is

$$\boxed{x_1 = \frac{3}{4}, x_2 = -\frac{1}{2}, x_3 = \frac{1}{4}}$$