

Step-1

We have to factor A into LU , and we have to write down the upper triangular system $Ux = c$ after elimination for the system:

$$Ax = \begin{pmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 6 & 9 & 8 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 5 \end{pmatrix}$$

Step-2

$$A = \begin{pmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 6 & 9 & 8 \end{pmatrix}$$

Here the matrix

Subtracting 3 times row 1 from row 3 gives

$$U = \begin{pmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & -1 \end{pmatrix}$$

Step-3

Applying the reverse rule that is adding three times of row 1 to row 3 gives

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}$$

$$LU = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 6 & 9 & 8 \end{pmatrix}$$

$$\Rightarrow A = LU = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & -1 \end{pmatrix}$$

Step-4

The given system is

$$Ax = \begin{pmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 6 & 9 & 8 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 5 \end{pmatrix}$$

After elimination that is subtracting 3 times row 1 from row 3 both sides of $Ax = b$ gives

$$Ux = \begin{pmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$$