Step-1

Given

$$A=\begin{bmatrix}1&2&3\\0&4&5\\0&0&6\end{bmatrix},\quad B=\begin{bmatrix}0&0&1\\0&2&0\\3&0&0\end{bmatrix}\text{ and }\quad C=\begin{bmatrix}2&2&2\\2&2&2\\2&2&2\end{bmatrix}$$

Now, $A=\begin{bmatrix}1&2&3\\0&4&5\\0&0&6\end{bmatrix}$

$$\begin{aligned}A-\lambda I&=\begin{bmatrix}1&2&3\\0&4&5\\0&0&6\end{bmatrix}-\lambda\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\\&=\begin{bmatrix}1&2&3\\0&4&5\\0&0&6\end{bmatrix}-\begin{bmatrix}\lambda&0&0\\0&\lambda&0\\0&0&\lambda\end{bmatrix}\\&=\begin{bmatrix}1-\lambda&2&3\\0&4-\lambda&5\\0&0&6-\lambda\end{bmatrix}\end{aligned}$$

Step-2

Then,

$$A-\lambda I=\begin{vmatrix}1-\lambda&2&3\\0&4-\lambda&5\\0&0&6-\lambda\end{vmatrix}$$

$$\begin{aligned}(1-\lambda)[(4-\lambda)(6-\lambda)]&=(1-\lambda)[24-4\lambda-6\lambda+\lambda^2]\\&=(1-\lambda)[\lambda^2-10\lambda+24]\\&=\lambda^2-10\lambda+24-\lambda^3+10\lambda^2-24\lambda\\&=-\lambda^3+11\lambda^2-34\lambda+24\end{aligned}$$

Step-3

$$A-\lambda I=0$$

Now, $\lambda^3-11\lambda^2+34\lambda-24=0$

$$\begin{array}{r|rrrr} 1 & 1 & -11 & 34 & -24 \\ & 0 & 1 & -10 & 24 \\ \hline 4 & 1 & -10 & 24 & 0 \\ & 0 & 4 & -24 & 0 \\ \hline & 1 & -6 & 0 & \end{array}$$

The eigenvalues are

$$(\lambda-1)(\lambda-4)(\lambda-6)=0$$

Therefore the eigenvalues of A are $\boxed{1,\ 4,\ \text{and}\ 6}$

Step-4

Now, consider $B=\begin{bmatrix}0&0&1\\0&2&0\\3&0&0\end{bmatrix}$

$$\begin{aligned}B-\lambda I &= \begin{bmatrix}0&0&1\\0&2&0\\3&0&0\end{bmatrix}-\lambda\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\\ &= \begin{bmatrix}0&0&1\\0&2&0\\3&0&0\end{bmatrix}-\begin{bmatrix}\lambda&0&0\\0&\lambda&0\\0&0&\lambda\end{bmatrix}\\ &= \begin{bmatrix}-\lambda&0&1\\0&2-\lambda&0\\3&0&-\lambda\end{bmatrix}\end{aligned}$$

Step-5

Now find eigenvalues of B

$$|B-\lambda I|=\begin{vmatrix}-\lambda&0&1\\0&2-\lambda&0\\3&0&-\lambda\end{vmatrix}$$

This implies;

$$= (-\lambda)(2-\lambda)(-\lambda) + 1(-(2-\lambda)3)$$
$$= \lambda^2(2-\lambda) - 6 + 3\lambda$$
$$= 2\lambda^2 - \lambda^3 - 6 + 3\lambda$$
$$= -\lambda^3 + 2\lambda^2 + 3\lambda - 6$$

Step-6

Now,

$B - \lambda I = 0$

This implies;

$$-\lambda^3 + 2\lambda^2 + 3\lambda - 6 = 0$$

$$\begin{array}{c|cccc} 2 & 1 & -2 & -3 & 6 \\ & 0 & 2 & 0 & -6 \\ \hline & 1 & 0 & -3 & 0 \end{array}$$

$$(\lambda - 2)(\lambda^2 - 3) = 0$$

Step-7

The Eigen values are $2, \pm\sqrt{3}$

Therefore the eigenvalues of B are $\boxed{2, \pm\sqrt{3}}$

Step-8

Now, consider

$$C = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

$$|C - \lambda I| = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$
$$= \begin{bmatrix} 2-\lambda & 2 & 2 \\ 2 & 2-\lambda & 2 \\ 2 & 2 & 2-\lambda \end{bmatrix}$$

Step-9

Now,

$$|C - \lambda I| = \begin{vmatrix} 2-\lambda & 2 & 2 \\ 2 & 2-\lambda & 2 \\ 2 & 2 & 2-\lambda \end{vmatrix}$$

This implies;

$$= (2-\lambda)[(2-\lambda)(2-\lambda) - 4] - 2[2(2-\lambda) - 4] + 2[4 - 2(2-\lambda)]$$
$$= (2-\lambda)[4 - 2\lambda - 2\lambda + \lambda^2 - 4] - 2[4 - 2\lambda - 4] + 2[4 - 4 + 2\lambda]$$
$$= (2-\lambda)[\lambda^2 - 4\lambda] - 2[-2\lambda] + 2[2\lambda]$$
$$= 2\lambda^2 - 8\lambda - \lambda^3 + 4\lambda^2 + 4\lambda + 4\lambda$$

Thus,

$$|C - \lambda I| = -\lambda^3 + 6\lambda^2$$

Step-10

Then,

$$\lambda^3 - 6\lambda^2 = 0$$
$$\lambda(\lambda^2 - 6\lambda) = 0$$
$$\lambda = 0$$
$$\lambda^2 - 6\lambda = 0$$

$$\lambda(\lambda - 6) = 0$$
$$\lambda = 0, 6$$

The Eigen values are 0, 0, and 6

Therefore the eigenvalues of C are $\boxed{0,\ 0,\ \text{and}\ 6}$