Step-1

A linear transformation from V to W has an inverse from W to V when the range is all of W and the kernel contains only $V=0$.

We have to verify which of these transformations are invertible.

Step-2

(a) Given transformation is $T(v_1,v_2)=(v_2,v_2)$

Now

$$\begin{aligned}\ker T &= \{(v_1,v_2)/T(v_1,v_2)=0\}\\ &= \{(v_1,v_2)/(v_2,v_2)=0\}\\ &= \{(v_1,v_2)/v_2=0\}\\ &= \{(v_1,0)/v_1\in\mathbf{R}\}\end{aligned}$$

Since $\text{kernel of } T\neq 0$

So T is not invertible.

Hence the given transformation is not invertible.

Step-3

(b) Given transformation is $T:\mathbf{R}^2\to\mathbf{R}^3$ by $T(v_1,v_2)=(v_1,v_2,v_1+v_2)$

Consider $(1,1,5)\in\mathbf{R}^3$

But there is no $(v_1,v_2)\in\mathbf{R}^2$ such that $T(v_1,v_2)=(1,1,5)$

Since $(v_1,v_2,v_1+v_2)=(1,1,5)$

$v_1=1,\ v_2=1,\ v_1+v_2=5$

Which is impossible

Thus range is not all of $\mathbf{R}^3$

Hence the given transformation T is not invertible.

Step-4

(c) Given transformation is $T(v_1, v_2) = v_1$

Now

$$\begin{aligned}\ker T &= \{(v_1, v_2) / T(v_1, v_2) = 0\} \\ &= \{(v_1, v_2) / v_2 = 0\} \\ &= \{(0, v_2) / v_2 \in \mathbf{R}\}\end{aligned}$$

$$T(v_1, v_2) = 0$$

$$\Rightarrow v_1 = 0$$

Since $\text{kernel of } T \neq 0$

So T is not invertible.

Hence the given transformation is not invertible.