Step-1

Given matrix is $A = \begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}$

We have to show that A has no inverse by solving $Ax = 0$.

Step-2

The matrix form of $Ax = 0$ is

$$\begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Subtracting 3 times row 1 from row 3 gives

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This shows that $x + y = 0$ which has no solution

Hence the given matrix A has no inverse.

Step-3

Now we have to show that A has no inverse by solving $\begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Given system is

$$\begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By matrix multiplication

$$\begin{bmatrix} a+c & b+d \\ 3a+3c & 3b+3d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Equating the corresponding elements

$a + c = 1 \qquad b + d = 0$
$3a + 3c = 0 \qquad 3b + 3d = 1$

But by try to solve $a + c = 1, 3a + 3b = 0$, it has no solution

Because they are parallel lines and leads to $0 = 1$ case.

Similarly, the other two equations also lead to $0 = 1$ condition.

Hence the given matrix A has no solution.