Step-1

(a)

The objective is to decide whether the following statement is true or false, and give an example if it is false.

â€œIf columns 1 and 3 of *B* are the same, so are columns 1 and 3 of *AB*â€.

Let $v_1, v_2, v_3, \ldots, v_n$ are columns of the matrix *B*.

The multiplication of matrices *A* and *B* is,

$$\begin{aligned} AB &= A\begin{bmatrix} v_1 & v_2 & v_3 & \cdots & v_n \end{bmatrix} \\ &= \begin{bmatrix} Av_1 & Av_2 & Av_3 & \cdots & Av_n \end{bmatrix} \end{aligned}$$

That is, each column of matrix AB is, A times the corresponding column of matrix B.

So, the columns v_1, and v_3 are same then Av_1, and Av_3 are also same.

Hence, the statement, â€œIf columns 1 and 3 of *B* are the same, so are columns 1 and 3 of *AB*â€ is, **true**.

Step-2

(b)

The objective is to decide whether the following statement is true or false, and give an example if it is false.

â€œIf rows 1 and 3 of *B* are the same, so are rows 1 and 3 of *AB*â€.

In general the above statement is false, which can be proved by the following example. Ex: Let, $A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 2 & 1 \\ 2 & 1 & 3 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & 5 \\ 1 & 1 & 0 \end{pmatrix}$

$$AB = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 2 & 1 \\ 2 & 1 & 3 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & 5 \\ 1 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 4 & 10 \\ 5 & 3 & 10 \\ 7 & 6 & 5 \end{pmatrix}$$

The first and third rows of the matrix B are equal. But the first and third rows of matrix AB are not equal.

Hence, the statement, â€œIf rows 1 and 3 of *B* are the same, so are rows 1 and 3 of *AB*â€ is, **false**.

Step-3

(c)

The objective is to decide whether the following statement is true or false, and give an example if it is false.

â€œIf rows 1 and 3 of *A* are the same, so are rows 1 and 3 of *AB*â€.

Let $u_1, u_2, u_3, \ldots, u_n$ are rows of matrix *A*.

The multiplication of matrices *A* and *B* is,

$$AB = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \cdots \\ u_n \end{bmatrix} B$$

$$= \begin{bmatrix} u_1 B \\ u_2 B \\ u_3 B \\ \cdots \\ u_n B \end{bmatrix}$$

That is, each row of matrix AB is, B times the corresponding row of matrix A.

So, the rows $u_1,$ and u_3 are same then $u_1 B,$ and $u_3 B$ are also same.

Hence, the statement, â€œIf rows 1 and 3 of *A* are the same, so are rows 1 and 3 of *AB*â€ is, **true**.

Step-4

(d)

The objective is to decide whether the following statement is true or false, and give an example if it is false.

$(AB)^2 = A^2 B^2$

In general the above statement is false, which can be proved by the following example. Ex: Let, $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$

$$AB = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} 4 & 1 \\ 1 & 0 \end{pmatrix}$$

Step-5

The square of the matrix AB is,

$$(AB)^2 = (AB)(AB)$$
$$= \begin{pmatrix} 4 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 4 & 1 \\ 1 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} 17 & 4 \\ 4 & 1 \end{pmatrix}$$

The square of the matrix A is,

$$A^2 = A.A$$
$$= \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$$

The square of the matrix B is,

$$B^2 = B.B$$
$$= \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}$$

Multiply the two matrices, A^2, and B^2, obtained as,

$$A^2B^2 = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} 13 & 6 \\ 2 & 1 \end{pmatrix}$$

Therefore, $(AB)^2 \neq A^2B^2$.

Hence, the statement, $(AB)^2 = A^2B^2$ is, **false**.