Step-1

Consider the matrices:

$$A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}, B=\begin{bmatrix} q & r \\ s & t \end{bmatrix}$$

The objective is to determine AB and BA have the same trace and also deduce that $AB-BA=I$ is impossible.

Step-2

First calculate AB and BA matrices are as follows:

$$\begin{aligned} AB &= \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} q & r \\ s & t \end{bmatrix} \\ &= \begin{bmatrix} aq+bs & ar+bt \\ cq+ds & cr+dt \end{bmatrix} \end{aligned}$$

Trace of AB is sum of the diagonals of AB.

That is $\text{Trace } AB = aq+bs+cr+dt$ â€¦â€¦ (1)

And,

$$\begin{aligned} BA &= \begin{bmatrix} q & r \\ s & t \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} \\ &= \begin{bmatrix} aq+rc & ab+rd \\ sa+tc & sb+td \end{bmatrix} \end{aligned}$$

Trace of BA is sum of the diagonals of AB.

That is $\text{Trace } BA = aq+rc+sb+td$ Â Â Â Â Â Â Â Â Â Â Â Â Â ....(2)

From (1) and (2),

$Trace\ AB = Trace\ BA.$

Therefore, AB and BA have the same trace.

Step-3

Now calculate the value of $AB-BA$ is as follows:

Consider,

$$
\begin{aligned}
AB - BA &= \begin{bmatrix} aq+bs & ar+bt \\ cq+ds & cr+dt \end{bmatrix} - \begin{bmatrix} aq+rc & ab+rd \\ sa+tc & sb+td \end{bmatrix} \\
&= \begin{bmatrix} aq+bs-aq-rc & ar+bt-ab-rd \\ cq+ds-sa-tc & cr+dt-sb-td \end{bmatrix} \\
&= \begin{bmatrix} bs-rc & ar+bt-ab-rd \\ cq+ds-sa-tc & cr-sb \end{bmatrix}
\end{aligned}
$$

Clearly $AB - BA \neq I$.

The trace of $AB - BA$ is 0.

But the trace of I is 2.

Therefore, $AB - BA = I$ is impossible for matrices, since I does not have a trace zero.