

Step-1

Given that A is an m by n matrix of rank r and its reduced echelon form is R .

We have to describe exactly the reduced row echelon form of R^T (not A^T).

Step-2

Given that A is an m by n matrix of rank r and its reduced echelon form is R .

Therefore R has r independent rows (or R has r non zero rows). Then R^T has r independent columns. That is R^T has r columns which are pivot columns.

Therefore R^T has r non-zero columns and $m-r$ non-zero rows.

Thus $R^T R$ is in the form

$$R^T R = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \text{ where } I, \text{ is the identity matrix of } r \text{ by } r.$$