

Step-1

Given that $A = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}$

$$\begin{aligned} A - 3I &= \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

The determinant of this 2×2 matrix is zero.

So, $\text{rank } A - 3I < 2$

But we know that rank of the matrix with at least one non zero entry is greater than or equal to 1.

So, putting the above statements together, we get $\text{rank } A - 3I = 1$.

Step-2

Changing any entry 0 in the given matrix, it can be diagonalizable.

For instance, $A = \begin{bmatrix} 3 & 1 \\ 16 & 3 \end{bmatrix}$ has the eigen values $\neq 1$ and 7 which are distinct and so, the corresponding eigen vectors will be linearly independent and so, it is diagonalizable.