Step-1

Given that the equation $x-3y-z=0$ determines a plane $\mathbf{R}^3$, we have to find the matrix A in this equation, and we have to find that which are the free variables and the special solutions. Also we have to fill in the first components of the points on this plane having the following form by taking the particular point $(12,0,0)$:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \\ 0 \\ 0 \end{bmatrix} + y \begin{bmatrix} \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} \\ 0 \\ 1 \end{bmatrix}$$

Step-2

$x-3y-z=0$

$\Rightarrow x=3y+z$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3y+z \\ y \\ z \end{bmatrix}$$

$$= y\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Therefore y, z are free variables

Step-3

And x is pivot variable, matrix

$$A = \begin{bmatrix} 1 & -3 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The special solutions are $(3,1,0)$ and $(1,0,1)$

Therefore the complete solution is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12+3y+z \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix} + y\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$