Step-1

The objective is to determine matrix due to the changes the basis.

Step-2

(a)

Obtain the matrix that transforms $(1,0)$ into $(2,5)$ and transforms $(0,1)$ into $(1,3)$.

$$T(1,0)=(2,5)$$
$$T(0,1)=(1,3)$$

The transition matrix is $\begin{bmatrix} x & y \\ z & w \end{bmatrix}$, use to the provided vectors.

Then,

$$\begin{bmatrix} x & y \\ z & w \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}=\begin{bmatrix} 2 \\ 5 \end{bmatrix}$$
$$\begin{bmatrix} x \\ z \end{bmatrix}=\begin{bmatrix} 2 \\ 5 \end{bmatrix}$$
$$x=2$$
$$z=5$$

And

$$\begin{bmatrix} x & y \\ z & w \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}=\begin{bmatrix} 1 \\ 3 \end{bmatrix}$$
$$\begin{bmatrix} y \\ w \end{bmatrix}=\begin{bmatrix} 1 \\ 3 \end{bmatrix}$$
$$y=1$$
$$w=3$$

Now, the matrix is:

$$\begin{bmatrix} x & y \\ z & w \end{bmatrix}=\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$$

Hence, the matrix of the linear transformation under the basis $(1,0),(0,1)$ is $\boxed{\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}}$.

Step-3

(b)

Matrix that transforms $(2,5)$ into $(1,0)$ and transforms $(1,3)$ into $(0,1)$.

Since, the linear transformation of the matrix is the inverse of the previous part.

So, take the inverse of the matrix.

$$\begin{aligned}\begin{bmatrix}2 & 1\\5 & 3\end{bmatrix} &= \frac{1}{1}\begin{bmatrix}3 & -5\\-1 & 2\end{bmatrix}\\ &= \begin{bmatrix}3 & -1\\-5 & 2\end{bmatrix}\end{aligned}$$

Thus, there is no another matrix is founded by the provided statement. Itâ€™s only defined by the inverse of the result matrix of previous part.

The matrix of the linear transformation under the basis $(2,5),(1,3)$ is $\boxed{\begin{bmatrix}3 & -1\\-5 & 2\end{bmatrix}}$.

Step-4

(c)

Objective is to explain why, no matrix is formed by the $(2,6)$ into $(1,0)$ and transforms $(1,3)$ into $(0,1)$.

The matrix transformation is not linear.

$$\begin{aligned}T(2,6) &= (1,0)\\ T(1,3) &= (0,1)\\ T(2,6) &= T(2(1,3))\\ &= (1,0)\end{aligned}$$

So,

$$\begin{bmatrix}2\\6\end{bmatrix} = 2\times\begin{bmatrix}1\\3\end{bmatrix}$$

This is linear transformations of the linear operators. That is:

$$T(\vec{a}) = \vec{b}$$

Then,

$T(\alpha\vec{a}) = \alpha\vec{b}$

So,

$$T\left(\begin{bmatrix}1\\3\end{bmatrix}\right) = \begin{bmatrix}0\\1\end{bmatrix}$$

$$\begin{aligned}T\left(\begin{bmatrix}2\\6\end{bmatrix}\right) &= T\left(2\begin{bmatrix}1\\3\end{bmatrix}\right)\\ &= 2T\left(\begin{bmatrix}1\\3\end{bmatrix}\right)\\ &= \begin{bmatrix}0\\2\end{bmatrix}\end{aligned}$$

Thus, itâ€™s not true. It wants $T\left(\begin{bmatrix}2\\6\end{bmatrix}\right) = \begin{bmatrix}1\\0\end{bmatrix}$.

Hence, there is **no linear transformation of the provided statements**.