

Step-1

a) The system of equations is

$$ax + by = 1$$

$$cx + dy = 0$$

Cramer's rule:

The j th component of $x = A^{-1}b$ is the ratio

$$x_j = \frac{\det B_j}{\det A}, \text{ where } B_j = \begin{bmatrix} a_{11} & a_{12} & b_1 & a_{1n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & b_n & a_{nn} \end{bmatrix} \text{ has } b \text{ in column } j.$$

Step-2

We need to solve the given system by Cramer's rule

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad x = \begin{bmatrix} x \\ y \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 1 & b \\ 0 & d \end{bmatrix} \quad A_2 = \begin{bmatrix} a & 1 \\ c & 0 \end{bmatrix}$$

Now

$$\det(A) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \\ = ad - bc$$

Step-3

And

$$\det(A_1) = \begin{vmatrix} 1 & b \\ 0 & d \end{vmatrix} \\ = d$$

$$\det(A_2) = \begin{vmatrix} a & 1 \\ c & 0 \end{vmatrix} \\ = -c$$

Step-4

Thus, by crammers rule, we have

$$\begin{aligned}x &= \frac{\det(A_1)}{\det(A)} \\&= \frac{\begin{vmatrix} 1 & b \\ 0 & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} \\&= \frac{d}{ad - bc}\end{aligned}$$

Step-5

And

$$\begin{aligned}y &= \frac{\det(A_2)}{\det(A)} \\&= \frac{\begin{vmatrix} a & 1 \\ c & 0 \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} \\&= \frac{-c}{ad - bc}\end{aligned}$$

Step-6

Thus, the solution for the given systems

$$\boxed{\begin{aligned}x &= \frac{d}{ad - bc} \\y &= \frac{-c}{ad - bc}\end{aligned}}$$

Step-7

b) The given system is

$$\begin{aligned}x + 4y - z &= 1 \\x + y + z &= 0 \\2x + 3z &= 0\end{aligned}$$

We need to solve the given system by cramers rule

$$A - \begin{bmatrix} 1 & 4 & -1 \\ 1 & 1 & 1 \\ 2 & 0 & 3 \end{bmatrix} X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Step-8

Replacing the first, second and third columns of A with b we get the matrices A_1, A_2 and A_3

$$A_1 = \begin{bmatrix} 1 & 4 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 0 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 4 & 1 \\ 1 & 1 & 0 \\ 2 & 0 & 0 \end{bmatrix}$$

Now

$$\det(A) = |A| = \begin{vmatrix} 1 & 4 & -1 \\ 1 & 1 & 1 \\ 2 & 0 & 3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} - 4 \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix}$$

$$= (3) - 4(1) - (-2)$$

$$= 1$$

Step-9

Now

$$\det(A_1) = |A_1| = \begin{vmatrix} 1 & 4 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} - 4 \begin{vmatrix} 0 & 1 \\ 0 & 3 \end{vmatrix} - \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}$$

$$= (3) - 4(0) - (0)$$

$$= 3$$

Step-10

And

$$\det(A_2) = |A_2| = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 0 & 1 \\ 2 & 0 & 3 \end{vmatrix}$$

$$= (0) - (1) - (0)$$

$$= -1$$

$$\det(A_3) = |A_3| = \begin{vmatrix} 1 & 4 & 1 \\ 1 & 1 & 0 \\ 2 & 0 & 0 \end{vmatrix}$$

$$= 1(0) - 4(0) + 1(-2)$$

$$= -2$$

Step-11

Thus, by cramers rule we have

$$x = \frac{\det(A_1)}{\det(A)}$$

$$= \frac{\begin{vmatrix} 1 & 4 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & 4 & -1 \\ 1 & 1 & 1 \\ 2 & 0 & 3 \end{vmatrix}}$$

$$= \frac{3}{1}$$

$$= 3$$

Step-12

And

$$y = \frac{\det(A_2)}{\det(A)} = \frac{\begin{vmatrix} 1 & 1 & -1 \\ 1 & 0 & 1 \\ 2 & 0 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & 4 & -1 \\ 1 & 1 & 1 \\ 2 & 0 & 3 \end{vmatrix}}$$

$$= \frac{-1}{1}$$

$$= -1$$

Step-13

And

$$z = \frac{\det(A_3)}{\det(A)} = \frac{\begin{vmatrix} 1 & 4 & 1 \\ 1 & 1 & 0 \\ 2 & 0 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 4 & -1 \\ 1 & 1 & 1 \\ 2 & 0 & 3 \end{vmatrix}}$$

$$= \frac{-2}{1}$$

$$= -2$$

Thus, the solution for the given system is

$$\boxed{x = 3, y = -1, z = -2}$$