

Step-1

We have

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

The cofactors of the elements of A are:

$$\begin{aligned} C_{11} &= \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} \\ &= 4 - 1 \\ &= 3 \end{aligned}$$

Step-2

And

$$\begin{aligned} C_{12} &= - \begin{vmatrix} -1 & -1 \\ 0 & 2 \end{vmatrix} \\ &= -(-2 - 0) \\ &= 2 \end{aligned}$$

$$\begin{aligned} C_{13} &= \begin{vmatrix} -1 & 2 \\ 0 & -1 \end{vmatrix} \\ &= 1 - 0 \\ &= 1 \end{aligned}$$

Step-3

Then

$$\begin{aligned} C_{21} &= - \begin{vmatrix} -1 & 0 \\ -1 & 2 \end{vmatrix} \\ &= -(-2 - 0) \\ &= 2 \end{aligned}$$

$$\begin{aligned} C_{22} &= \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} \\ &= 4 - 0 \\ &= 4 \end{aligned}$$

Step-4

And

$$\begin{aligned}C_{23} &= - \begin{vmatrix} 2 & -1 \\ 0 & -1 \end{vmatrix} \\&= -(-2 - 0) \\&= 2\end{aligned}$$

$$\begin{aligned}C_{31} &= \begin{vmatrix} -1 & 0 \\ 2 & -1 \end{vmatrix} \\&= 1 - 0 \\&= 1\end{aligned}$$

Step-5

Now

$$\begin{aligned}C_{32} &= - \begin{vmatrix} 2 & 0 \\ -1 & -1 \end{vmatrix} \\&= -(-2 - 0) \\&= 2\end{aligned}$$

$$\begin{aligned}C_{33} &= \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} \\&= 4 - 1 \\&= 3\end{aligned}$$

Step-6

$$C = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

So, the matrix of cofactors is

$$C^T = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

Then

Step-7

And

$$\begin{aligned}
AC^T &= \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix} \\
&= \begin{bmatrix} 6-2+0 & 4-4+0 & 2-2+0 \\ -3+4-1 & -2+8-2 & -1+4-3 \\ 0-2+2 & 0-4+4 & 0-2+6 \end{bmatrix} \\
&= \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \\
&= 4I
\end{aligned}$$

Step-8

We can also see $\det A$

$$= 2 \cdot \text{cofactor of } 2 + (-1) \cdot \text{cofactor of } (-1) \quad (\text{cofactor expansion along first row})$$

$$= 2(3) + (-1)2$$

$$= 6 - 2$$

$$= 4$$

Step-9

So, we observe that $\frac{1}{\det A} \cdot C^T = A^{-1}$

Hence

$$A^{-1} = \frac{1}{4} \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$