Step-1

Consider the following;

$$(y, 1-y)A = (y, 1-y)\begin{bmatrix} 2 & 8 \\ 6 & -6 \end{bmatrix}$$
$$= (2y + 6(1-y), 8y - 6(1-y))$$
$$= (6 - 4y, 14y - 6)$$

Step-2

Equate $6 - 4y$ and $14y - 6$. This gives,

$$6 - 4y = 14y - 6$$
$$6 + 6 = 14y + 4y$$
$$12 = 18y$$
$$y = \frac{2}{3}$$

Step-3

Thus, the optimal strategy involves $y = \frac{2}{3}$.

When $y = \frac{2}{3}$, we have $6 - 4y = \frac{10}{3}$.

Thus, the average gain on each play is $\frac{10}{3}$.