Step-1

Singular Value Decomposition (SVD) for any *m* by *n* matrix *A* is as follows

$$
\begin{aligned}
A &= U\Sigma V^T \\
&= \begin{pmatrix} \text{orthogonal} \\ U \text{ is } m\times m \end{pmatrix}\begin{pmatrix} m\times n \text{ matrix } \Sigma \\ \sigma_1\cdots\sigma_r \text{ on diagonal} \end{pmatrix}\begin{pmatrix} \text{orthogonal} \\ V \text{ is } n\times n \end{pmatrix}
\end{aligned}
$$

Here eigenvectors of AA^T are in *U*, eigenvectors of A^TA are in *V*, and the $\sigma_i = \sqrt{\lambda_i}$.

Step-2

Let *A* be an *m* by *n* zero matrix.

So, AA^T be *m* by *m* zero matrix, whose eigenvalues are $\lambda = 0$.

We know that eigenvectors of AA^T are in *U*.

Since eigenvalues are zero, *U* will be *m* by *m* identity matrix.

$$
\begin{aligned}
U &= [u_1 \cdots u_m] \\
&= \begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix} \\
&= I_{m\times m}
\end{aligned}
$$

Step-3

Now A^TA be *n* by *n* zero matrix, whose eigenvalues are $\lambda = 0$.

We know that eigenvectors of A^TA are in *V*.

Since eigenvalues are zero, *V* will be *n* by *n* identity matrix.

$$
\begin{aligned}
V &= [v_1 \cdots v_n] \\
&= \begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix} \\
&= I_{n\times n}
\end{aligned}
$$

Step-4

The diagonal of Σ are the square roots of the nonzero eigenvalues of both AA^T.

So, Σ is *m* by *n* zero matrix.

$$\Sigma = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

The SVD for *m* by *n* zero matrix is as follows:

$$\begin{aligned} A &= U\Sigma V^T \\ &= I_{m\times m}\Sigma_{m\times n} I_{n\times n} \\ &= \Sigma_{m\times n} I_{n\times n} \\ &= \Sigma_{m\times n} \end{aligned}$$

Therefore, SVD for *m* by *n* zero matrix is $\boxed{m \text{ by } n \text{ zero matrix}}$.

Step-5

If SVD of *m* by *n* matrix *A* is $A = U\Sigma V^T$, then the pseudoinverse of *A* is

$$A^+ = V\Sigma^+ U^T$$

For *m* by *n* zero matrix, we have

$$\begin{aligned} U &= I_{m\times m} \\ V &= I_{n\times n} \end{aligned}$$

Since Σ is *m* by *n* zero matrix, so Σ^+ is *n* by *m* zero matrix.

Hence, pseudoinverse of *m* by *n* zero matrix *A* is

$$\begin{aligned} A^+ &= V\Sigma^+ U^T \\ &= I_{n\times n}\Sigma^+{}_{n\times m} I_{m\times m} \\ &= \Sigma^+{}_{n\times m} \\ &= A^T \end{aligned}$$

Therefore, the pseudoinverse of an zero *n* by *m* matrix is $\boxed{\text{equal to its transpose}}$.