Step-1

Let P_1 be an even permutation matrix and P_2 be an odd permutation matrix. Then we have

$$\det(P_1) = 1$$
$$\det(P_2) = -1$$

Step-2

We know that $P_1 + P_2 = P_1\left(P_1^{\mathrm{T}} + P_2^{\mathrm{T}}\right)P_2$

Thus,

$$\begin{aligned}\det(P_1 + P_2) &= \det\left(P_1\left(P_1^{\mathrm{T}} + P_2^{\mathrm{T}}\right)P_2\right)\\ &= \det(P_1)\det\left(P_1^{\mathrm{T}} + P_2^{\mathrm{T}}\right)\det(P_2)\\ &= -1 \times \det\left(P_1^{\mathrm{T}} + P_2^{\mathrm{T}}\right) \times 1\\ &= -\det\left(P_1^{\mathrm{T}} + P_2^{\mathrm{T}}\right)\end{aligned}$$

Step-3

We further know the following two properties:

1. Addition of transpose is equal to the transpose of addition.

2. Determinant of a matrix is equal to the determinant of its transpose.

Thus, we have

$$\begin{aligned}\det(P_1 + P_2) &= -\det\left(P_1^{\mathrm{T}} + P_2^{\mathrm{T}}\right)\\ &= -\det(P_1 + P_2)^{\mathrm{T}}\\ &= -\det(P_1 + P_2)\end{aligned}$$

There is only one number, which is equal to negative of itself. It is the zero.

Thus, $\boxed{\det(P_1 + P_2) = 0}$.