

Step-1

Suppose $A = \begin{bmatrix} 0 & 1 \\ a & b \end{bmatrix}$

The characteristic equation of this matrix is $\det(A - \lambda I) = 0$

$$\Rightarrow \begin{vmatrix} -\lambda & 1 \\ a & b - \lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^2 - b\lambda - a = 0$$

$$\Rightarrow \lambda_1 = \frac{b + \sqrt{b^2 + 4a}}{2}, \lambda_2 = \frac{b - \sqrt{b^2 + 4a}}{2} \text{ are the eigen values of } A.$$

Step-2

But given eigen values are 4 and 7

So, we follow that $\frac{b + \sqrt{b^2 + 4a}}{2} = 7, \frac{b - \sqrt{b^2 + 4a}}{2} = 4$

Adding the respective sides of these equations, we get $b = 11$

Further, substitution gives $11 + \sqrt{121 + 4a} = 14, 11 - \sqrt{121 + 4a} = 8$

$$\Rightarrow \sqrt{121 + 4a} = 3,$$

$$\Rightarrow 121 + 4a = 9$$

$$\Rightarrow 4a = -112$$

$$\Rightarrow a = -28$$

Therefore, the required matrix is $\begin{bmatrix} 0 & 1 \\ -28 & 11 \end{bmatrix}$