Step-1

The three steps of the Fast Fourier Transform are

$$c=\begin{bmatrix}c_0\\c_1\\c_2\\c_3\\c_4\\c_5\\c_6\\c_7\end{bmatrix}\rightarrow\begin{bmatrix}c_0\\c_2\\c_4\\c_6\\c_1\\c_3\\c_5\\c_7\end{bmatrix}\rightarrow\begin{bmatrix}F_4c^{1}\\F_4c^{11}\end{bmatrix}\rightarrow\ y$$

â€¦â€¦ (1)

Step-2

Given $c=(1,0,1,0,1,0,1,0)$

That is,Â $c_0=1, c_1=0, c_2=1, c_3=0, c_4=1, c_5=0, c_6=1, c_7=0.$

$$c=\begin{bmatrix}c_0\\c_1\\c_2\\c_3\\c_4\\c_5\\c_6\\c_7\end{bmatrix}=\begin{bmatrix}1\\0\\1\\0\\1\\0\\1\\0\end{bmatrix}$$

â€¦â€¦ (2)

Step-3

$$\begin{bmatrix}c_{even}\\c_{odd}\end{bmatrix}=\begin{bmatrix}c_0\\c_2\\c_4\\c_6\\c_1\\c_3\\c_5\\c_7\end{bmatrix}=\begin{bmatrix}1\\1\\1\\1\\0\\0\\0\\0\end{bmatrix}$$

â€¦â€¦ (3)

Step-4

$$y^{1} = F_4 c^{1} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & i^2 & i^3 \\ 1 & i^2 & i^4 & i^6 \\ 1 & i^3 & i^6 & i^9 \end{bmatrix} \begin{bmatrix} c_0 \\ c_2 \\ c_4 \\ c_6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & i^2 & i^3 \\ 1 & i^2 & i^4 & i^6 \\ 1 & i^3 & i^6 & i^9 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Step-5

$$y^{11} = F_4 c^{11} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix} \begin{bmatrix} c_1 \\ c_3 \\ c_5 \\ c_7 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Step-6

Therefore $\begin{bmatrix} F_4 c^1 \\ F_4 c^{11} \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ â€¦â€¦ (4)

Step-7

$$y = F_8 c = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & w & w^2 & w^3 & w^4 & w^5 & w^6 & w^7 \\ 1 & w^2 & w^4 & w^6 & w^8 & w^{10} & w^{12} & w^{14} \\ 1 & w^3 & w^6 & w^9 & w^{12} & w^{15} & w^{18} & w^{21} \\ 1 & w^4 & w^8 & w^{12} & w^{16} & w^{20} & w^{24} & w^{28} \\ 1 & w^5 & w^{10} & w^{15} & w^{20} & w^{25} & w^{30} & w^{35} \\ 1 & w^6 & w^{12} & w^{18} & w^{24} & w^{30} & w^{36} & w^{42} \\ 1 & w^7 & w^{14} & w^{21} & w^{28} & w^{35} & w^{42} & w^{49} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Step-8

Â Â Â Â $= \begin{bmatrix} 1+0+1+0+1+0+1+0 \\ 1+0+w^2+0+w^4+0+w^6+0 \\ 1+0+w^4+0+w^8+0+w^{12}+0 \\ 1+0+w^6+0+w^{12}+0+w^{18}+0 \\ 1+0+w^8+0+w^{16}+0+w^{24}+0 \\ 1+0+w^{10}+0+w^{20}+0+w^{30}+0 \\ 1+0+w^{12}+0+w^{24}+0+w^{36}+0 \\ 1+0+w^{14}+0+w^{28}+0+w^{42}+0 \end{bmatrix}$ â€¦â€¦ (5)

Step-9

$$=\begin{bmatrix} 4 \\ 1+w^2+w^4+w^6 \\ 2+2w^4 \\ 1+w^2+w^4+w^6 \\ 4 \\ 1+w^2+w^4+w^6 \\ 2+2w^4 \\ 1+w^2+w^4+w^6 \end{bmatrix}$$

Step-10

Substituting $(2),(3),(4)$ and (5) in (1), we get

$$c=\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \rightarrow \begin{matrix} c_{even} = \\ c_{odd} = \end{matrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\rightarrow \begin{matrix} y^{1} = \\ y^{11} = \end{matrix} \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow y = \begin{bmatrix} 4 \\ 1+w^2+w^4+w^6 \\ 2+2w^4 \\ 1+w^2+w^4+w^6 \\ 4 \\ 1+w^2+w^4+w^6 \\ 2+2w^4 \\ 1+w^2+w^4+w^6 \end{bmatrix}$$ while $w^8=1$.

The three steps of the Fast Fourier Transform are

$$c = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{bmatrix} \to \begin{bmatrix} c_0 \\ c_2 \\ c_4 \\ c_6 \\ c_1 \\ c_3 \\ c_5 \\ c_7 \end{bmatrix} \to \begin{bmatrix} F_4 c^{1} \\ F_4 c^{11} \end{bmatrix} \to y$$

â€¦â€¦ (1)

Given $c = (0,1,0,1,0,1,0,1)$

That is,Â $c_0 = 0, c_1 = 1, c_2 = 0, c_3 = 1, c_4 = 0, c_5 = 1, c_6 = 0, c_7 = 1.$

$$c = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

â€¦â€¦. (2)

$$\begin{bmatrix} c_{even} \\ c_{odd} \end{bmatrix} = \begin{bmatrix} c_0 \\ c_2 \\ c_4 \\ c_6 \\ c_1 \\ c_3 \\ c_5 \\ c_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

â€¦â€¦ (3)

$$y^1 = F_4c^1 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix}\begin{bmatrix} c_1 \\ c_3 \\ c_5 \\ c_7 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$y^{11} = F_4c^{11} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & i^2 & i^3 \\ 1 & i^2 & i^4 & i^6 \\ 1 & i^3 & i^6 & i^9 \end{bmatrix}\begin{bmatrix} c_0 \\ c_2 \\ c_4 \\ c_6 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & i^2 & i^3 \\ 1 & i^2 & i^4 & i^6 \\ 1 & i^3 & i^6 & i^9 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Therefore $\begin{bmatrix} F_4c^1 \\ F_4c^{11} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 4 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ â€¦â€¦ (4)

$$y = F_8c = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & w & w^2 & w^3 & w^4 & w^5 & w^6 & w^7 \\ 1 & w^2 & w^4 & w^6 & w^8 & w^{10} & w^{12} & w^{14} \\ 1 & w^3 & w^6 & w^9 & w^{12} & w^{15} & w^{18} & w^{21} \\ 1 & w^4 & w^8 & w^{12} & w^{16} & w^{20} & w^{24} & w^{28} \\ 1 & w^5 & w^{10} & w^{15} & w^{20} & w^{25} & w^{30} & w^{35} \\ 1 & w^6 & w^{12} & w^{18} & w^{24} & w^{30} & w^{36} & w^{42} \\ 1 & w^7 & w^{14} & w^{21} & w^{28} & w^{35} & w^{42} & w^{49} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

Â Â Â Â $= \begin{bmatrix} 0+1+0+1`+0+1+0+1 \\ 0+w+0+w^3+0+w^5+0+w^7 \\ 0+w^2+0+w^6+0+w^{10}+0+w^{14} \\ 0+w^3+0+w^9+0+w^{15}+0+w^{21} \\ 0+w^4+0+w^{12}+0+w^{20}+0+w^{28} \\ 0+w^5+0+w^{15}+0+w^{25}+0+w^{35} \\ 0+w^6+0+w^{18}+0+w^{30}+0+w^{42} \\ 0+w^7+0+w^{21}+0+w^{35}+0+w^{49} \end{bmatrix} = \begin{bmatrix} 4 \\ w+w^3+w^5+w^7 \\ 2w^2+2w^6 \\ w+w^3+w^5+w^7 \\ 4w^4 \\ w+w^3+w^5+w^7 \\ 2w^2+2w^6 \\ w+w^3+w^5+w^7 \end{bmatrix}$ â€¦â€¦â€¦. (5)

$(2),(3),(4)$ *and* (5) values substitute in(1)

$$c=\begin{bmatrix}0\\1\\0\\1\\0\\1\\0\\1\end{bmatrix}\rightarrow\begin{bmatrix}c_{even}\\c_{odd}\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\\1\\1\\1\\1\end{bmatrix}\rightarrow\begin{matrix}y^{1}=\\y^{11}=\end{matrix}\begin{bmatrix}0\\0\\0\\0\\4\\0\\0\\0\end{bmatrix}\rightarrow y=\begin{bmatrix}4\\w+w^{3}+w^{5}+w^{7}\\2w^{2}+2w^{6}\\w+w^{3}+w^{5}+w^{7}\\4w^{4}\\w+w^{3}+w^{5}+w^{7}\\2w^{2}+2w^{6}\\w+w^{3}+w^{5}+w^{7}\end{bmatrix}\text{ since } w^{8}=1$$