

## Step-1

Given lines are  $3x + y = b_1$ ,  $6x + 2y = b_2$ . We have to answer the following questions:

- 1) We have to say that how the two lines are related.
- 2) When do the two lines represent the same line.
- 3) In that case we have to find a vector which is perpendicular to  $(b_1, b_2)$
- 4) We have to find the null space of the matrix.
- 5) We have to find one particular vector in that null space.

## Step-2

- 1) For the lines the ratios of the coefficients are equal.

$$\text{Since } \frac{3}{6} = \frac{1}{2}$$

Therefore  $3x + y = b_1$ ,  $6x + 2y = b_2$  are parallel.

## Step-3

- 2) They are the same line if

$$\frac{3}{6} = \frac{1}{2} = \frac{b_1}{b_2}$$
$$\Rightarrow \boxed{b_2 = 2b_1}$$

## Step-4

And this represents  $(b_1, b_2) = (b_1, 2b_1) = b_1(1, 2)$  is perpendicular to a vector  $(2, -1)$

For the finding the null space we consider the system

$$3x + y = 0$$
$$6x + 2y = 0$$

From this, the null space of the matrix is the line  $3x + y = 0$

## Step-5

Now

$$3x + y = 0$$

$$\Rightarrow y = -3x$$

$$\text{If } x = 1$$

$$\Rightarrow y = -3$$

One particular vector in that nullspace is  $(1, -3)$ .