Step-1

If A is a square matrix, λ is an eigen value and x is the eigen vector, then we have $Ax = \lambda x$

Multiplying A on both sides, we see that $A^2 x = A(\lambda x)$

$$= \lambda(Ax)$$
$$= \lambda(\lambda x)$$
$$= \lambda^2 x$$

So, the eigen value of A^2 is λ^2

Step-2

In view of this observation, while the eigen values of A are 1, 2, and 4, the eigen values of A^2 are

1, 4, and 16.

We know that the trace of a matrix is nothing but the sum of the eigen values of that matrix.

So, the trace of A^2 is 1 + 4 + 16 = 21

Step-3

Further, we can write $Ax = \lambda x$ as $\frac{1}{\lambda}x = A^{-1}x$

From this, the eigen value of A^{-1} is $\frac{1}{\lambda}$ â€¦â€¦ (1)

Also, we follow that $|A - \lambda I| = |A^T - \lambda I|$

From this, the eigen values of a matrix and its transpose are one and the same. â€¦â€¦ (2)

Putting (1) and (2) together, we confirm that the eigen values of $\left(A^{-1}\right)^T$ are $\frac{1}{1}, \frac{1}{2}, \frac{1}{4}$

Step-4

We know that the determinant of the matrix is the product of its eigen values

So, the determinant of $\left(A^{-1}\right)^T$ is $\frac{1}{1} \times \frac{1}{2} \times \frac{1}{4} = \boxed{\frac{1}{8}}$