## Step-1

 $_{A \text{ has }} \lambda_{1} = 2_{\text{ with eigen vector}} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ 

 $\lambda_2 = 5$  with eigen vector  $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ 

 $\Lambda = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$  is the diagonal matrix whose diagonal entries are the eigen values of A and  $S = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$  whose columns are the eigen vectors in the order of the eigen values of A

By the process of diagonalizations, we know that  $A = S\Lambda S^{-1}$ 

$$= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 2 & 5 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$$