

## Step-1

We have to describe all 2 by 3 matrices  $A_1, A_2$  with row echelon forms  $R_1$  and  $R_2$  such that  $R_1 + R_2$  is the row echelon form of  $A_1 + A_2$ . We have to find that is it true that  $R_1 = A_1$  and  $R_2 = A_2$  in this case.

## Step-2

Let  $A_1, A_2$  be 2 by 3 matrices  $A_1$  and  $A_2$  with row echelon forms  $R_1$  and  $R_2$  such that  $R_1 + R_2$  is the row echelon form of  $A_1 + A_2$ .

If possible  $R_1 \neq A_1$

Then  $R_1$  is  $2 \times 3$  matrix with rank either 1 or 2.

$R_2$  is  $2 \times 3$  matrix with rank either 1 or 2.

## Step-3

Then  $R_1 + R_2$  is not in the form of row echelon form of  $A_1 + A_2$ .

This is a contradiction.

Therefore  $R_1 = A_1$ , similarly  $R_2 = A_2$ .

Therefore the given statement is true.