Step-1

Consider that there are six vectors $v_1, v_2 \ldots v_6 \in \mathbb{R}^4$

(a)

The objective is to determine whether the vectors(do) (do not) or (might not) span $\mathbb{R}^4$

Step-2

Assume there are six vectors $v_1, v_2 \ldots v_6 \in \mathbb{R}^4$

It is given that there are six vectors, but it is not mentioned that the vectors are linearly dependent or independent in the 4-dimensional space.

The provided information does not guarantee that there are four independent vectors that span $\mathbb{R}^4$

Hence, the vectors $\boxed{\text{might not}}$ span $\mathbb{R}^4$.

Step-3

(b)

The objective is to determine whether the vectors(are) (are not) or (might not) be linearly independent.

Step-4

Assume there are six vectors $v_1, v_2 \ldots v_6 \in \mathbb{R}^4$

Recall the statement that in a *m*-dimensional space $\mathbb{R}^m$, set of *n* vectors must be linearly dependent if $n > m$.

Here, there are six vectors in a 4-dimensional space $\mathbb{R}^4$ that is there are two more vectors $(6 > 4)$.

This implies that the set of 6-vectors must be linearly dependent.

Hence, the vectors $\boxed{\text{are not}}$ linearly independent.

Step-5

(c)

The objective is to determine whether any four vectors (are) (are not) or (might be) a basis of $\mathbb{R}^4$.

Step-6

Assume there are six vectors $v_1, v_2 \dots v_6 \in \mathbb{R}^4$

Recall that basis for a vector space is a sequence of vectors which satisfy the below properties:

(1) The vectors are Linearly Independent

(2) The vectors span the vector space.

Here, there is a possibility that the four vectors are linearly independent and span $\mathbb{R}^4$.

If this is true, then the vectors are basis of $\mathbb{R}^4$.

But since there is no enough information therefore there is no guarantee that the vectors are basis of $\mathbb{R}^4$.

Hence, the vectors $\boxed{\text{might be}}$ a basis of $\mathbb{R}^4$.

Step-7

(d)

The objective is to determine if the vectors are columns of matrix A, then the system $Ax = b$ (has)(does not have)(might not have) a solution.

Step-8

From the last part it is not guaranteed that the vectors are linearly independent vectors

Case1:

Assume that the vectors are linearly dependent and span space of dimension say one or two but the vector b doesnâ€™t belong in the dimension.

In this case there is no solution of the system $Ax = b$.

Case 2:

Assume that the four vectors out of six vectors are linearly independent and span space and b belongs in the dimension.

In this case there is solution of the system $Ax = b$.

Hence, the vectors $\boxed{\text{might not have}}$ a solution.