Step-1

The volume of the box with adjacent edges meet at O (0,0,0) and the end vertices (x_1, y_1, z_1), (x_2, y_2, z_2), and (x_3, y_3, z_3) is $\det\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$

Given that the edges of the box are from $(0,0,0)$ to $(3,1,1),(1,3,1),(1,1,3)$ respectively.

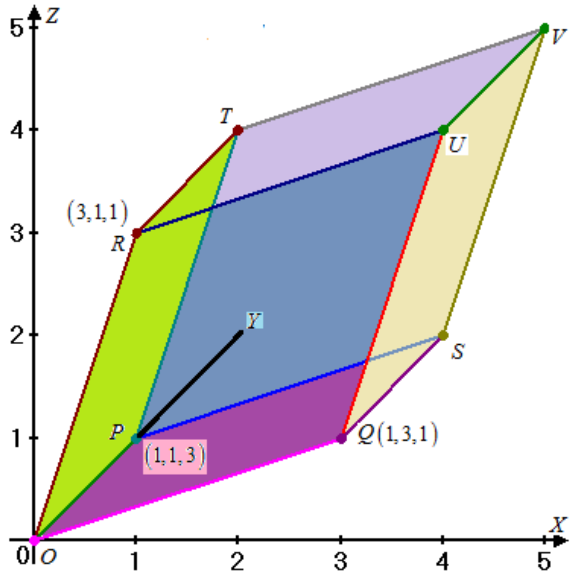
So, its volume $= \begin{vmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{vmatrix}$

$= 3(9-1)-(3-1)+(1-3)$

$= 20$ cubic units

Step-2

A box with three adjacent sides meeting at the origin will have 6 faces.



We see that OPQS, RTUV, OPRT, QSVU, ORUQ and PTVS are the six faces of the box.

We easily see that the opposite sides of this box are parallel and so have the same area.

On the other hand, the length of each given vectors is equal to $\sqrt{1^2+3^2+1^2}=\sqrt{11}$

So, all the parallelograms are having equal areas.

Area of OPQS =

$$=\left|(3\mathbf{i}+\mathbf{j}+\mathbf{k})\times(\mathbf{i}+3\mathbf{j}+\mathbf{k})\right|$$

$$=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 3 & 1 & 1\\ 1 & 3 & 1\end{vmatrix}$$

$$=\left|-2\vec{i}-2\vec{j}+8\vec{k}\right|$$

$$=\sqrt{4+4+64}$$

$$=\sqrt{72}\text{ sq.units}$$

Therefore, each side of the box is of $6\sqrt{2}$ square units area.