Step-1

The objective is to find the complete solutions $x = x_p + x_n$ for the provided systems.

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 4 & 5 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & 2 & 2 \\ 2 & 4 & 4 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

Step-2

Now, change the matrix of order 2×3 in reduced row echelon form which is calculated below,

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 4 & 5 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$\xrightarrow{R_2 - 2R_1}\begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\xrightarrow{R_1 - 2R_2}\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$$

Step-3

So, the equation in term v, w and u is written below,

$$\begin{aligned} u + 2v &= -3 \\ w &= 2 \\ u &= -3 - 2v \end{aligned}$$

Now, write the value of v, w and u in the form of $x = x_p + x_n$ which is shown below,

$$\begin{aligned} \begin{bmatrix} u \\ v \\ w \end{bmatrix} &= \begin{bmatrix} -3-2v \\ v \\ 2 \end{bmatrix} \\ &= \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix} + v\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \end{aligned}$$

Step-4

Thus, the value of null space is $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$

And the complete solution is given by;

$$
\begin{aligned}
x &= x_p + x_n \\
&= x_{\text{particular}} + x_{\text{nullspace}}
\end{aligned}
$$

$$x = \boxed{\begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix} + v\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}}$$

Step-5

Consider the second system, change the matrix of order 2×3 in reduced row echelon form which is calculated below,

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 4 & 4 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$\underline{R_2 - 2R_1}\begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$u + 2v + 2w = 1,$$

$$0 = 2, \text{which is impossible.}$$

Therefore the system has $\boxed{\text{no solution}}$.