

## Step-1

Given that if every column of  $A$  is a multiple of  $(1,1,1)$ , then  $Ax$  is always a multiple of  $(1,1,1)$ . We have to give an example of 3 by 3 matrix and the number of pivots produced by the elimination.

## Step-2

Since every column of  $A$  is a multiple of  $(1,1,1)$  let  $A = \begin{pmatrix} 2 & 3 & 4 \\ 2 & 3 & 4 \\ 2 & 3 & 4 \end{pmatrix}$

And let  $x = (x_1, x_2, x_3)$  be the column matrix then

$$\begin{aligned} Ax &= \begin{pmatrix} 2 & 3 & 4 \\ 2 & 3 & 4 \\ 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \\ &= \begin{pmatrix} 2x_1 + 3x_2 + 4x_3 \\ 2x_1 + 3x_2 + 4x_3 \\ 2x_1 + 3x_2 + 4x_3 \end{pmatrix} \end{aligned}$$

It shows that every entry in the column of  $Ax$  is multiple of  $(1,1,1)$ .

## Step-3

$$A = \begin{pmatrix} 2 & 3 & 4 \\ 2 & 3 & 4 \\ 2 & 3 & 4 \end{pmatrix}$$

Applying  $R_2 - R_1$  and  $R_3 - R_1$  gives

$$\square \begin{pmatrix} 2 & 3 & 4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Here 2 is the only pivot.

Therefore the number of pivots after elimination is  $\boxed{1}$ .