

Step-1

Consider a pure exponential equation:

$$u = xe^{i\omega t}$$

Substitute this equation in the following equation and find a generalised Eigen value problem for the frequency ω .

$$Mu'' + Ku = 0$$

Here, M is a mass matrix.

Step-2

Do the following calculations:

$$u = xe^{i\omega t}$$

$$u' = i\omega xe^{i\omega t}$$

$$u'' = -\omega^2 xe^{i\omega t}$$

Substitute these values in the following equation and solve:

$$Mu'' + Ku = 0$$

$$-\omega^2 xe^{i\omega t} M + Kxe^{i\omega t} = 0$$

$$-\omega^2 xM + Kx = 0$$

$$\omega^2 x = KM^{-1}x$$

Step-3

Assume the following:

$$KM^{-1} = A$$

Above equation can be written as follows:

$$Ax = \omega^2 x$$

$$(A - \omega^2 I)x = 0$$

Step-4

Therefore, generalised Eigen value problem that must be solved for the frequency ω and the vector x is:

$$\boxed{Ax = \omega^2 x}$$