## Step-1

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$
 Suppose,

Then,

$$I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

## Step-2

Let the initial value of x be  $x_0 = (p_0, q_0)$  and general value of x be given by  $x_k = (p_k, q_k)$ . For the sake of convenience, let  $x_0 = (0, 0)$ . Thus, we have

$$X_{k+1} = (I - A)X_k$$

Therefore,

$$x_{1} = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} p_{0} \\ q_{0} \end{bmatrix}$$

$$= \begin{bmatrix} -p_{0} + q_{0} \\ p_{0} - q_{0} \end{bmatrix}$$

$$x_{2} = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} -p_{0} + q_{0} \\ p_{0} - q_{0} \end{bmatrix}$$

$$= \begin{bmatrix} 2p_{0} - 2q_{0} \\ -2p_{0} + 2q_{0} \end{bmatrix}$$

$$\begin{aligned} x_3 &= \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2p_0 - 2q_0 \\ -2p_0 + 2q_0 \end{bmatrix} \\ &= \begin{bmatrix} -4p_0 + 4q_0 \\ 4p_0 - 4q_0 \end{bmatrix} \\ x_4 &= \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} -4p_0 + 4q_0 \\ 4p_0 - 4q_0 \end{bmatrix} \\ &= \begin{bmatrix} 8p_0 - 8q_0 \\ -8p_0 + 8q_0 \end{bmatrix} \end{aligned}$$

## Step-3

$$x_{k} = \begin{bmatrix} \left(-1\right)^{k} \left(2^{k-1} p_{0} - 2^{k-1} q_{0}\right) \\ \left(-1\right)^{k+1} \left(2^{k-1} p_{0} - 2^{k-1} q_{0}\right) \end{bmatrix}.$$
Thus,

Except when  $(p_0,q_0)=(\alpha,\alpha)$ , the two sequences  $(-1)^k(2^{k-1}p_0-2^{k-1}q_0)$  and  $(-1)^{k+1}(2^{k-1}p_0-2^{k-1}q_0)$  are not converging. Thus, even when we had b=(0,0), the sequences are not converging.

Therefore, when 
$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$
,  $x_{k+1} = (I - A)x_k + b$  does not converge.