

Step-1

$$a) \quad 2x_1 + 3x_2 = 1$$

$$4x_1 + 6x_2 = 1$$

$$Ax = \begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} = b$$

$$|A| = 12 - 12 = 0$$

So, Cramer's rule cannot confirm the solution of the given system.

But by multiplying the 1st equation with 2 and if 2nd is subtracted from it, we get $0 = 1$.

This is an absurdity.

In other words, the given system is called inconsistent.

Or, there is not solution to it.

Step-2

b) As in the case (a), $\det A = 0$ and so, Cramer's rule is not useful to solve the system.

$$2x_1 + 3x_2 = 1$$

$$4x_1 + 6x_2 = 2$$

On the other hand, we can easily see that the 2nd equation is a multiple of the 1st equation and so, the 2nd equation can be neglected and thus, we are left with $2x_1 + 3x_2 = 1$ only.

Step-3

It can be written as $x_1 = \frac{1-3x_2}{2}$

In other words, we can write $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -3/2 \\ 1 \end{bmatrix}$

Putting $x_2 = k$ a parameter, we get $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 0 \end{bmatrix} + k \begin{bmatrix} -3/2 \\ 1 \end{bmatrix}$

For infinite real values of k , there will be infinite solutions to $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

