

Step-1

$$A = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$$

Given matrix,

$$|A_{11}| = 2 > 0$$

$$|A_{22}| = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}$$

$$= 4 - 1$$

$$= 3 > 0$$

$$|A_{33}| = |A|$$

$$= 2(4 - 1) + 1(-2 - 1) - (1 + 2)$$

$$= 6 - 3 - 3$$

$$= 0$$

So A is not positive definite.

Step-2

$$B = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & 1 \\ -1 & 1 & 2 \end{pmatrix}$$

Given matrix,

$$|B_{11}| = 2 > 0$$

$$|B_{22}| = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}$$

$$= 4 - 1$$

$$= 3 > 0$$

$$|B_{33}| = |B|$$

$$= 2(4 - 1) + 1(-2 + 1) - (-1 + 2)$$

$$= 6 - 1 - 1$$

$$= 4 > 0$$

Thus $|B_{11}| > 0$, $|B_{22}| > 0$, $|B_{33}| > 0$.

Therefore, B is positive definite.

Step-3

$$C = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}^2$$

Given matrix,

$$C = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 2 & 1 \\ 2 & 2 & 2 \\ 1 & 2 & 5 \end{pmatrix}$$

Step-4

Now calculate determinant of left sub matrices

$$|C_{11}| = 5 > 0$$

$$|C_{22}| = \begin{vmatrix} 5 & 2 \\ 2 & 2 \end{vmatrix}$$

$$= 10 - 4$$

$$= 6 > 0$$

$$|C_{33}| = |C|$$

$$= 5(6) - 2(8) + 1(2)$$

$$= 30 - 16 + 2$$

$$= 16 > 0$$

Thus $|C_{11}| > 0$, $|C_{22}| > 0$, $|C_{33}| > 0$.

Therefore C is positive definite.