

Step-1

Solve the following equations for u , v , w and z .

$$u + v + w + z = 6$$

$$u + w + z = 4$$

$$u + w = 2$$

$$\left. \begin{array}{l} u + w + z = 4 \\ u + w = 2 \end{array} \right\} \Rightarrow z = 2$$

$$\left. \begin{array}{l} u + v + w + z = 6 \\ u + w = 2 \\ z = 2 \end{array} \right\} \Rightarrow v = 2$$

Hence the obtained equations are

$$u + w = 2$$

$$z = 2$$

$$v = 2$$

These equations contain one dependent variable, one independent variable and two constants, so these three equations represent a line in four-dimensional space.

Step-2

Include the equation $u = -1$ in the given set of equations and solve them for u , v , w and z .

$$u + v + w + z = 6$$

$$u + w + z = 4$$

$$u + w = 2$$

$$u = -1$$

$$\left. \begin{array}{l} u + w = 2 \\ u = -1 \end{array} \right\} \Rightarrow w = 3$$

$$\left. \begin{array}{l} u + w + z = 4 \\ w = 3 \\ u = -1 \end{array} \right\} \Rightarrow z = 2$$

$$\left. \begin{array}{l} u + v + w + z = 6 \\ z = 2 \\ w = 3 \\ u = -1 \end{array} \right\} \Rightarrow v = 2$$

Hence the solution to the set of equations is $(-1, 2, 3, 2)$ which represents a point in a four-dimensional space.

Therefore the intersection of the given four planes is the point $(-1, 2, 3, 2)$.

Step-3

Many equations can be found as fourth equation to make the given system of equations without a solution.

For example include the equation $z = 1$ in the given set of equations.

$$u + v + w + z = 6$$

$$u + w + z = 4$$

$$u + w = 2$$

$$z = 1$$

Now put $z = 1$ into the equation $u + w + z = 4$ then the equation obtained will be $u + w = 3$ and the new system of equations will be

$$u + v + w + z = 6$$

$$u + w = 3$$

$$u + w = 2$$

$$z = 1$$

Obviously the second and third equations are cannot be solved so this system has no solution if the fourth equation $z = 1$ included.