$\ln n \begin{cases} 1、积分 \\ 2、\ln n > 1,\ n \geq 2 \\ 3、\ln n < n^c,\ n \geq N \end{cases}$ $\lim\limits_{n\to\infty} \frac{\ln n}{n^c} = 0$

$\frac{\ln(1+x)}{x} \to 1$

Calculus II quiz 2

考点一: 判断常数项级数 $\sum\limits_{n=1}^{\infty}(-1)^n a_n,\ a_n > 0$ 的收敛性. Section 10.2-10.6

(i) 注意: 收敛包括绝对收敛和条件收敛.

有时候尽管$a_n > 0, a_n \to 0, a_n$ 单调递减, 但不是绝对收敛, 如 $\sum\limits_{n=1}^{\infty}(-1)^n \frac{1}{n^2}$.

而条件收敛需要满足两个得分点: (1) $a_n > 0, a_n \to 0, a_n$ 单调递减; (2)不绝对收敛.

(ii) 比式判别法必须 $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} > 1$ 严格大于1, 若极限等于1, 大部分是 $\lim\limits_{n\to\infty} a_n \neq 0$, 从而级数发散. 如: $\sum\limits_{n=1}^{\infty}(-1)^n(1-\frac{1}{n})^n$.

1. (2018年期末) Which of the following series converge absolutely, which converge conditionally, and which diverge? Give reasons for your answer.

(1) $\sum\limits_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt{n(n+1)}}$ converge conditionally

(2) $\sum\limits_{n=2}^{\infty}(-1)^n \frac{1}{n(\ln n)^3}$ converges absolutely

(3) $\sum\limits_{n=1}^{\infty}(-1)^n \frac{n^2+1}{2n^2+n-1}$ diverges

(4) $\sum\limits_{n=1}^{\infty} \frac{(-3)^n}{n!} = \sum(-1)^n \frac{3^n}{n!}$ converges absolutely

2. (2019年期中) Does the following series absolutely converge, conditionally converge, or diverge ? Give reasons for your answer.

$\lim\limits_{n\to\infty}\sqrt[n]{a_n} = (1-\frac{1}{n})^n \to e^{-1} < 1 \Rightarrow$ converges absolutely

$e^0 = 1$

(1) $\sum\limits_{n=1}^{\infty}(-1)^n(1-\frac{1}{n})^{n^2}$ ~~diverges~~

(2) $\sum\limits_{n=1}^{\infty}(-1)^n e^{-\frac{1}{n}}$ diverges

(3) $\sum\limits_{n=2019}^{\infty}(-1)^n \frac{1}{\sqrt{n^2-2019n+1}}$ Converges conditionally

3.(2019年期末) The sequences $\{a_n\}$ and $\{b_n\}$ satisfy $0 < a_n < \frac{\pi}{2}, 0 < b_n < \frac{\pi}{2}$, and $\cos a_n - a_n = \cos b_n$. The series $\sum\limits_{n=1}^{\infty} b_n$ converges. Show that $\lim\limits_{n\to\infty} a_n = 0$. $\Leftarrow \sum a_n$ converges $\Leftarrow \sum b_n$ converges . $0 < a_n < b_n$?

$\Leftrightarrow a_n = \cos a_n - \cos b_n$

$x \in (0, \frac{\pi}{2})$ $\cos x \downarrow$ $\cos a_n - \cos b_n = a_n > 0$, $\cos a_n > \cos b_n$, $a_n < b_n$ ✓

4.(2021年期中) Determine if the series, $\sum\limits_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^p(\ln n)^2}$ $(p > 0)$, converges absolutely, or converges conditionally, or diverges. Give reasons for your answer.

1°. $\frac{\frac{1}{n^p(\ln n)^2}}{\frac{1}{n^p}} = \frac{1}{(\ln n)^2} \to 0$

2°. $p>1$ converges absolutely; $\frac{1}{n^p(\ln n)^2} < \frac{1}{n^{p+2c}}$, $p+2c<1 \Leftrightarrow 0<c<\frac{1-p}{2}$ $\Rightarrow$ cond

3° $p=1 \Rightarrow$ integral test $\int_2^{\infty}\frac{1}{x(\ln x)^2}dx = \frac{1}{\ln 2}$ converges

$p \geq 1$ converges absolutely

$0 < p < 1$ converges conditionally

考点二: 求幂级数的收敛域, 收敛半径.

注意: 一定要考虑端点的收敛性.

1.(2021年期中) For the power series $f(x) = \sum\limits_{n=1}^{\infty} \frac{n+2}{n(n+1)} x^n = \sum\limits_{n=1}^{\infty} a_n$,

(1) For what values of x does the power series converge? $[-1, 1)$

(2) Find the sum of the series within the interval of convergence.

$S(x) = \sum x^{n-1} = \frac{1}{1-x}$

$p(x) = x\iint S(x)\,dx\,dx = \frac{x^{n+2}}{n(n+1)}$ $\quad \frac{p'(x)}{x} = \frac{n+2}{n(n+1)}x^n$ $\quad = \frac{1-2x}{x}\ln(1-x) + 1$

2.(2020年期末) Find the interval of convergence of the power series $\sum\limits_{n=0}^{\infty} \frac{2^n}{\ln(n+2)} x^n$. $[-\frac{1}{2}, \frac{1}{2})$

$\sum \frac{1}{\ln(n+2)} > \sum \frac{1}{n+2} \Rightarrow$ diverges $\quad$ $*\ln n < n^c$

$\frac{1}{\sqrt{n+9012}\ln n} > \frac{1}{\sqrt{n+9012}\sqrt{n}}$

3.(2019年期末) (1) Find the interval of convergence of the series $\sum\limits_{n=2}^{\infty} \frac{(-1)^n(x-1)^{\boxed{2n+1}}}{\sqrt{n+9012}\ln n}$. $[0, 2]$

(2) For what values of x does the series converge absolutely, or conditionally? $(0, 2)$ $\{0\}$ 0, 2 $x=0, x=2$ 都是交错级数!

$0 < x < 2$ converges absolutely

$x = 0$ converges conditionally

$x = 2$ ~~diverges~~ converges conditionally

4. (2019年期中) (1) Find the radius and interval of convergence of the series $\sum\limits_{n=0}^{\infty} \frac{(-1)^n 2^n x^n}{\sqrt{n^2+n+1}}$. $(-\frac{1}{2}, \frac{1}{2}]$

(2) For what values of x does the series converge absolutely, or conditionally?

$(-\frac{1}{2}, \frac{1}{2})$ $\quad \{\frac{1}{2}\}$