

## Step-1

Suppose one or more eigenvalues of  $\hat{L}_1$ ,  $\hat{L}_2$ , and  $\hat{L}_3$  are zero.

Without loss of generality, let  $\lambda_3 = 0$  and other eigenvalues be positive.

Then we get  $\lambda_1 y_1^2 + \lambda_2 y_2^2 = 1$

This represents an elliptical cylinder in 3 dimensions. If we consider the plane, which contains the vectors  $y_1$  and  $y_2$ , then the cross section of this cylinder with the plane is the ellipse, whose equation is given by  $\lambda_1 y_1^2 + \lambda_2 y_2^2 = 1$ .

## Step-2

Consider the case of two eigenvalues being zero and the third one being positive. Without loss of generality, let  $\lambda_2 = 0$ ,  $\lambda_3 = 0$ , and  $\lambda_1 > 0$ .

This gives,  $\lambda_1^2 = 1$ .

Therefore,  $\lambda_1 = \pm 1$ .

This represents pair of two planes, perpendicular to the vector  $y_1$ , passing through it at a distance of  $\pm 1$  from the origin.

## Step-3

Suppose we consider the case where all the three eigenvalues are zero. Then we get  $0 = 1$ , which is impossible.