Step-1

We have to describe all the 3 by 3 matrices that are simultaneously Hermitian, unitary and diagonal.

Step-2

Let A be any 3×3 matrix.

Suppose A is Hermitian, unitary and diagonal.

Since A is diagonal.

So all the elements are zeros except the diagonal elements.

Step-3

Since A is Hermitian

$\Rightarrow \mathrm{A}^H = A$

$\Rightarrow$ The diagonal elements must be real since any eigenvalue is real.

Since A is unitary

So every eigenvalue of A has absolute value $|\lambda| = \pm 1$.

Since the absolute value of the eigenvalue is $|\lambda| = \pm 1$

So this is possible for the matrices having diagonal elements ± 1 or -1 and the remaining elements are zeros.

Step-4

The possible matrices with the above property are 8.

They are

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Step-5

Hence the matrices those are simultaneously Hermitian, unitary and diagonal are

$$\boxed{\begin{matrix}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix},\begin{bmatrix}0&0&0\\0&-1&0\\0&0&1\end{bmatrix},\begin{bmatrix}1&0&0\\0&1&0\\0&0&-1\end{bmatrix},\begin{bmatrix}1&0&0\\0&-1&0\\0&0&-1\end{bmatrix}\\ \begin{bmatrix}-1&0&0\\0&1&0\\0&0&1\end{bmatrix},\begin{bmatrix}-1&0&0\\0&-1&0\\0&0&1\end{bmatrix},\begin{bmatrix}-1&0&0\\0&1&0\\0&0&-1\end{bmatrix},\begin{bmatrix}-1&0&0\\0&-1&0\\0&0&-1\end{bmatrix}\end{matrix}}$$