Step-1

(a) Since X is twice as old as Y so it represents $X = 2Y$

$\Rightarrow X - 2Y = 0$ ... (1)

Sum of their ages is 39 so it represents the equation

$X + Y = 39$ â€¦ (2)

Converting the two linear equations into matrix form $Ax = b$

$$\begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} 0 \\ 39 \end{pmatrix}$$

Step-2

Using Gauss elimination:

Subtract row 1 from row 2 gives

$$\begin{pmatrix} 1 & -2 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} 0 \\ 39 \end{pmatrix}$$

By dividing row 2 by 3

$$\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} 0 \\ 13 \end{pmatrix}$$

Adding 2 times of row2 to row 1 gives

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} 26 \\ 13 \end{pmatrix}$$

From the above system $X = 26$ and $Y = 13$.

Therefore $\boxed{X = 26, Y = 13}$

Step-3

(b) It is given that the point (2, 5) lies on the line y = mix so

$5 = m.2 + c$

$\Rightarrow 2m + c = 5$ â€¦ (1)

It is given that the point (3, 7) lies on the line y = mix so

$$7 = m.3 + c$$

$\Rightarrow 3m + c = 7$ â€¦ (2)

Converting the two linear equations into matrix form $Ax = b$

$$\begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} m \\ c \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

Step-4

Using Gauss elimination:

Subtract row 1 from row2 gives

$$\begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} m \\ c \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

It requires row exchange changing row 1 and row 2

$$\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} m \\ c \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$

Subtracting 2 times of row1 from row 2 gives

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} m \\ c \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

From the above system $m = 2$ and $c = 1$.

Therefore $\boxed{m = 2, c = 1}$