Step-1

Given that $A=\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}$

The characteristic equation of this matrix is $|A-\lambda I|=0$

$$\begin{aligned}|A-\lambda I| &= \begin{vmatrix}1-\lambda & 2\\ 0 & 3-\lambda\end{vmatrix}\\ &=(1-\lambda)(3-\lambda)\\ &=3-\lambda-3\lambda+\lambda^2\\ &=\lambda^2-4\lambda+3\end{aligned}$$

$\Rightarrow(\lambda-1)(\lambda-3)=0$

$\Rightarrow \lambda_1=1,\lambda_2=3$ are the eigen values of A.

Step-2

For $\lambda_1=1$, suppose x_1 is the vector such that $\left(A-\lambda_1 I\right)x_1=0$

$$\Rightarrow\begin{bmatrix}0 & 2\\ 0 & 2\end{bmatrix}\begin{bmatrix}t_1\\ t_2\end{bmatrix}=0$$

Using the operation $R_2\to R_2-R_1$, we get $\begin{bmatrix}0 & 2\\ 0 & 0\end{bmatrix}\begin{bmatrix}t_1\\ t_2\end{bmatrix}=0$

Rewriting the homogeneous equations, we get $2t_2=0$

So, $t_2=0$

But any value of t_1 satisfies the system.

So, putting $t_1=k$ a parameter and conveniently putting k = 1, the eigen vector corresponding to $\lambda_1=1$ is $x_1=\begin{bmatrix}t_1\\ t_2\end{bmatrix}=\begin{bmatrix}1\\ 0\end{bmatrix}$

Step-3

For $\lambda_2=3$, suppose x_2 is the vector such that $\left(A-\lambda_2 I\right)x_2=0$

$\Rightarrow \begin{bmatrix} -2 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = 0$

Using the operation $R_1 / -2$, we get $\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = 0$

Rewriting the homogeneous equations, we get $t_1 - t_2 = 0$

So, $t_2 = t_1$

Putting $t_1 = 1$, the eigen vector corresponding to $\lambda_2 = 3$ is $x_2 = \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Step-4

Using the eigen vectors as the columns of *S*, we get $S = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and so, $S^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

Consequently, $A = S \Lambda S^{-1}$

i.e., $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

Step-5

Given that $A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$

The characteristic equation of this matrix is $|A - \lambda I| = 0$

$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 1 \\ 2 & 2-\lambda \end{vmatrix}$

$\Rightarrow \lambda^2 - 3\lambda = 0$

$\Rightarrow \lambda_1 = 0, \lambda_2 = 3$ are the eigen values of *A*.

Step-6

For $\lambda_1 = 0$, suppose x_1 is the vector such that $(A - \lambda_1 I) x_1 = 0$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = 0$$

Using the operation $R_2 \to R_2 - 2R_1$, we get $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = 0$

Rewriting the homogeneous equations, we get $t_1 + t_2 = 0$

Putting $t_1 = 1$, the eigen vector corresponding to $\lambda_1 = 0$ is $x_1 = \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Step-7

For $\lambda_2 = 3$, suppose x_2 is the vector such that $(A - \lambda_2 I)x_2 = 0$

$$\Rightarrow \begin{bmatrix} -2 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = 0$$

Using the operation $R_2 \to R_2 + R_1$, we get $\begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = 0$

Rewriting the homogeneous equations, we get $2t_1 - t_2 = 0$

So, $t_2 = 2t_1$

Putting $t_1 = 1$, the eigen vector corresponding to $\lambda_2 = 3$ is $x_2 = \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Step-8

Using the eigen vectors as the columns of *S*, we get $S = \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}$ and so, $S^{-1} = \frac{1}{3}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$

Consequently, $A = S\Lambda S^{-1}$

i.e., $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$