Step-1

Consider the following matrix:

$$A = \begin{bmatrix} a & -0.8 \\ 0.8 & 0.2 \end{bmatrix}$$

To obtain its eigenvalues, solve $\det(A - \lambda I) = 0$. This gives,

$$\begin{aligned} 0 &= \begin{vmatrix} a-\lambda & -0.8 \\ 0.8 & 0.2-\lambda \end{vmatrix} \\ &= (a-\lambda)(0.2-\lambda) + 0.64 \\ &= \lambda^2 - (0.2+a)\lambda + 0.2a + 0.64 \end{aligned}$$

Step-2

Consider the equation, $\lambda^2 - (0.2+a)\lambda + 0.2a + 0.64 = 0$.

Thus,

$$\begin{aligned} \lambda &= \frac{(0.2+a) \pm \sqrt{(0.2+a)^2 - 4(0.2a+0.64)}}{2} \\ &= \frac{(0.2+a) \pm \sqrt{(0.04+0.4a+a^2) - (0.8a+2.56)}}{2} \\ &= \frac{(0.2+a) \pm \sqrt{a^2 - 0.4a - 2.52}}{2} \end{aligned}$$

Step-3

Since $|\lambda_i| < 1$, we get the following:

$$\begin{aligned} -2 < (0.2+a) \pm \sqrt{a^2 - 0.4a - 2.52} &< 2 \\ -2.2 < a \pm \sqrt{a^2 - 0.4a - 2.52} &< 1.98 \\ -2.2 - a < \pm\sqrt{a^2 - 0.4a - 2.52} &< 1.98 - a \\ (-2.2-a)^2 < a^2 - 0.4a - 2.52 &< (1.98-a)^2 \end{aligned}$$

Simplifying, we get

$$a^2+4.4a+4.84<a^2-0.4a-2.52<a^2-3.96a+3.9204$$
$$4.4a+4.84<-0.4a-2.52<-3.96a+3.9204$$
$$4.4a+7.36<-0.4a<-3.96a+6.4404$$
$$-11a-18.4>a>9.9a-16.101$$

Step-4

Consider the two inequalities separately.

$$-11a-18.4>a$$
$$-18.4>12a$$
$$\frac{-18.4}{12}>a$$
$$-1.533>a$$

$$a>9.9a-16.101$$
$$-8.9a>-16.101$$
$$a<\frac{16.101}{8.9}$$
$$a<1.809$$

Step-5

Thus, if matrix A has to be stable, $\boxed{a<-1.533}$.

If the matrix A has to be neutrally stable, some of its eigenvalues satisfy $|\lambda_i|=1$ and other eigenvalues satisfy $|\lambda_i|<1$.

This gives, $\boxed{a=1.809}$ or $\boxed{a=-1.533}$.

Consider the following matrix:

$$B=\begin{bmatrix} b & 0.8 \\ 0 & 0.2 \end{bmatrix}$$

To obtain its eigenvalues, solve $\det(B-\lambda I)=0$. This gives,

$$0=\begin{vmatrix} b-\lambda & 0.8 \\ 0 & 0.2-\lambda \end{vmatrix}$$
$$=(b-\lambda)(0.2-\lambda)+0$$
$$=\lambda^2-(0.2+b)\lambda+0.2b$$

Step-6

Consider the equation $\lambda^2-(0.2+b)\lambda+0.2b=0$.

Thus,

$$\begin{aligned}\lambda &= \frac{(0.2+b)\pm\sqrt{(0.2+b)^2-0.8b}}{2}\\ &= \frac{(0.2+b)\pm\sqrt{b^2+0.4b+0.04-0.8b}}{2}\\ &= \frac{(0.2+b)\pm\sqrt{b^2-0.4b+0.04}}{2}\end{aligned}$$

$$\begin{aligned}\lambda &= \frac{(0.2+b)\pm\sqrt{(b-0.2)^2}}{2}\\ &= \frac{(0.2+b)\pm(b-0.2)}{2}\\ &= b \text{ or } 0.2\end{aligned}$$

Step-7

For the matrix *B* to be stable, we want $|\lambda_i|<1$. Therefore, $\boxed{b<1}$.

For the matrix *B* to be neutrally stable, we want some eigenvalue *Î»* such that $|\lambda_i|=1$ and other Î» such that $|\lambda_i|<1$. Therefore, $\boxed{b=1}$.

Step-8

Consider the following matrix:

$$C=\begin{bmatrix} c & 0.8 \\ 0.2 & c \end{bmatrix}$$

To obtain its eigenvalues, solve $\det(C-\lambda I)=0$. This gives,

$$\begin{aligned}0 &= \begin{vmatrix} c-\lambda & 0.8 \\ 0.2 & c-\lambda \end{vmatrix}\\ &= (c-\lambda)^2-0.16\\ &= \lambda^2-2c\lambda+(c^2-0.16)\end{aligned}$$

Step-9

Consider the equation $\lambda^2-2c\lambda+(c^2-0.16)=0$.

Thus,

$$\begin{aligned}\lambda &= \frac{2c \pm \sqrt{4c^2 - 4\left(c^2 - 0.16\right)}}{2} \\ &= \frac{2c \pm \sqrt{0.64}}{2} \\ &= \frac{2c \pm 0.8}{2} \\ &= c \pm 0.4\end{aligned}$$

Step-10

For the matrix C to be stable, we want $|\lambda_i| < 1$. Therefore, $\boxed{-0.6 < c < 0.6}$.

For the matrix C to be neutrally stable, we want some eigenvalue *Î»* such that $|\lambda_i| = 1$ and other Î» such that $|\lambda_i| < 1$. Therefore, $\boxed{-0.6 \le c \le 0.6}$.