

Step-1

Given $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ $AB = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $BA = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

Now $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$$\begin{aligned} A - \lambda I &= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \\ &= \begin{bmatrix} 1-\lambda & 0 \\ 1 & 1-\lambda \end{bmatrix} \end{aligned}$$

Step-2

Then

$$\begin{aligned} |A - \lambda I| &= \begin{vmatrix} 1-\lambda & 0 \\ 1 & 1-\lambda \end{vmatrix} \\ &= (1-\lambda)^2 \\ &= 1 + \lambda^2 - 2\lambda \end{aligned}$$

Step-3

Becomes

$$\begin{aligned} \lambda^2 - 2\lambda + 1 &= 0 \\ \lambda^2 - \lambda - \lambda + 1 &= 0 \\ \lambda(\lambda - 1) - 1(\lambda - 1) &= 0 \\ (\lambda - 1)(\lambda - 1) &= 0 \\ \lambda &= 1, 1 \end{aligned}$$

Therefore the eigenvalues of A is 1, 1

Step-4

Now $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$$\begin{aligned}
 A - \lambda I &= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \\
 &= \begin{bmatrix} 1-\lambda & 1 \\ 0 & 1-\lambda \end{bmatrix}
 \end{aligned}$$

Step-5

Then

$$\begin{aligned}
 |A - \lambda I| &= \begin{vmatrix} 1-\lambda & 1 \\ 0 & 1-\lambda \end{vmatrix} \\
 &= (1-\lambda)^2 \\
 &= 1 + \lambda^2 - 2\lambda
 \end{aligned}$$

Step-6

Becomes

$$\begin{aligned}
 \lambda^2 - 2\lambda + 1 &= 0 \\
 \lambda^2 - \lambda - \lambda + 1 &= 0 \\
 \lambda(\lambda - 1) - 1(\lambda - 1) &= 0 \\
 (\lambda - 1)(\lambda - 1) &= 0 \\
 \lambda &= 1, 1
 \end{aligned}$$

Therefore the eigenvalues of B is 1, 1

Step-7

$$AB = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

Now

$$\begin{aligned}
 A - \lambda I &= \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \\
 &= \begin{bmatrix} 1-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix}
 \end{aligned}$$

Step-8

Then

$$\begin{aligned}|A - \lambda I| &= \begin{vmatrix} 1-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} \\ &= (1-\lambda)(2-\lambda) - 1 \\ &= 2 - \lambda - 2\lambda + \lambda^2 - 1 \\ &= \lambda^2 - 3\lambda + 1\end{aligned}$$

$$\lambda^2 - 3\lambda + 1 = 0$$

This is in the form of $ax^2 + bx + c = 0$

Then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Now

$$\begin{aligned}\lambda &= \frac{3 \pm \sqrt{9-4}}{2} \\ &= \frac{3 \pm \sqrt{5}}{2}\end{aligned}$$

Therefore the eigenvalues of AB is $\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}$

Step-9

$$BA = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

Now

$$\begin{aligned}A - \lambda I &= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \\ &= \begin{bmatrix} 2-\lambda & 1 \\ 1 & 1-\lambda \end{bmatrix}\end{aligned}$$

Step-10

Now

$$\begin{aligned}
 |A - \lambda I| &= \begin{vmatrix} 2-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} \\
 &= (2-\lambda)(1-\lambda) - 1 \\
 &= 2 - \lambda - 2\lambda + \lambda^2 - 1 \\
 &= \lambda^2 - 3\lambda + 1
 \end{aligned}$$

$$\lambda^2 - 3\lambda + 1 = 0$$

Step-11

This is in the form of $ax^2 + bx + c = 0$

Then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Now

$$\begin{aligned}
 \lambda &= \frac{3 \pm \sqrt{9 - 4}}{2} \\
 &= \frac{3 \pm \sqrt{5}}{2}
 \end{aligned}$$

Therefore the eigenvalues of BA is $\frac{3 + \sqrt{5}}{2}, \frac{3 - \sqrt{5}}{2}$

Step-12

Eigen values of AB are not equal to eigenvalues of A times Eigen values of B

Eigenvalues of AB are equal to eigenvalues of BA