Calculus II 第十五章 Section 15.1-15.8 quiz 9

1. (2022年期末) $\int_0^1 \int_y^1 \frac{\cos x}{x}\, dx dy = ($ B $)$.

(A) $\cos 1$ (B) $\sin 1$ (C) $1-\cos 1$ (D) $1-\sin 1$

$x=2r\cos\theta$
$y=\sqrt{2}r\sin\theta$

2. (2021年期末) The region is given by $R: x^2+2y^2 \le 4$. Then $\iint\limits_R (4-x^2-2y^2)\, dx dy =$. A

(A) $4\sqrt{2}\pi$ (B) 8π (C) $8\sqrt{2}\pi$ (D) none of the above.

3. (2020年期末) Let $R: (x-1)^2+y^2 \le 1$, then the integral $\iint\limits_R f(x,y)\, dA$ is not equal to D

(A) $\int_0^2 \int_{-\sqrt{2x-x^2}}^{\sqrt{2x-x^2}} f(x,y)\, dy dx$ √ (B) $\int_{-1}^1 \int_{1-\sqrt{1-y^2}}^{1+\sqrt{1-y^2}} f(x,y)\, dx dy$ √

(C) $\int_0^{2\pi} \int_0^1 f(1+r\cos\theta, r\sin\theta)\, r\, dr d\theta$ √ (D) $\int_0^{2\pi} \int_0^{2\cos\theta} f(r\cos\theta, r\sin\theta)\, r\, dr d\theta$

4. (2019年期末) The iterated integral $\int_0^{\frac{\pi}{2}} \int_0^{\cos\theta} f(r\cos\theta, r\sin\theta)\, r\, dr d\theta$ can be written as (D)

(A) $\int_0^1 \int_0^{\sqrt{y-y^2}} f(x,y)\, dx dy$ (B) $\int_0^1 \int_0^{\sqrt{1-y^2}} f(x,y)\, dx dy$

(C) $\int_0^1 \int_0^1 f(x,y)\, dy dx$ (D) $\int_0^1 \int_0^{\sqrt{x-x^2}} f(x,y)\, dy dx$

5. (2022年期末) If the region $D = \{(x,y) | x^2+y^2 \le 1\}$. then $\iint\limits_D e^{-x^2-y^2}\, dx dy = ($ $(1-e^{-1})\pi$ $)$.

$\int_0^{2\pi}\int_0^1 e^{-r^2}\cdot r\,dr d\theta$

6. (2019年期末) $\int_0^1 \int_y^1 \frac{\tan x}{x}\, dx dy = ($ $\ln(\sec 1)$ $)$

7. (2022年期末) Compute $\iint\limits_D xy\, dx dy$, here D is the disk enclosed by the curve $x^2+y^2 = 2x+2y$. (Hint: use substitution.)

$\int_0^{2\pi}\int_0^{\sqrt{2}}(1+r\cos\theta)(1+r\sin\theta)\cdot r\,dr d\theta$ $(x-1)^2+(y-1)^2=2$

$=\int_0^{2\pi}\left(1+\frac{2\sqrt{2}}{3}\cos\theta+\frac{2\sqrt{2}}{3}\sin\theta+\sin\theta\cos\theta\right)d\theta$

$=\left[\theta+\frac{2\sqrt{2}}{3}\sin\theta-\frac{2\sqrt{2}}{3}\cos\theta+\frac{1}{2}\sin^2\theta\right]_0^{2\pi}$

$=2\pi$

8. (2021年期末) Find $J = \int_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} \int_{|x|}^{\sqrt{1-x^2}} \sqrt{1-y^2}\, dy dx$.

$\int_0^{\frac{\sqrt{2}}{2}}\int_{-y}^{y}\sqrt{1-y^2}\,dx dy+\int_{\frac{\sqrt{2}}{2}}^{2}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}\sqrt{1-y^2}\,dx dy$

$4\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\int_0^1 r\cos\theta\, r\,dr d\theta=\frac{4-2\sqrt{2}}{3}$

9. (2019年期末) Use the substitution in double integral (please find the transformation by yourself) to evaluate the integral $\iint\limits_D e^{\frac{y-x}{y+x}}\, dx dy$, here D is the triangular region bounded by the lines $x=0$, $y=0$, and $x+y=2$.

$=\int_0^2\int_{-v}^{v}e^{\frac{u}{v}}\frac{1}{2}\,du dv = e-e^{-1}$

10. (2018年期末) Consider $\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y}\, dy dx$ $=\int_0^4\int_0^{\sqrt{4-y}}\frac{xe^{2y}}{4-y}\,dx dy=\int_0^2\frac{1}{2}e^{2y}dy=\frac{1}{4}e^{2y}\Big]_0^4=\frac{1}{4}(e^8-1)$

(1) Sketch the region of integration.

(2) Reverse the order of integration, and evaluate the integral.

$u=\sqrt{\frac{y}{x}}$

$v=\sqrt{xy}$

11. (2018年期末) Let R be the region in the first quadrant of the xy-plane bounded by the hyperbolas $xy=1, xy=9$ and the lines $y=x, y=4x$. Use the substitution in double integral (please find the transformation by yourself) to evaluate the integral $\iint\limits_R (\sqrt{\frac{y}{x}}+\sqrt{xy})\, dx dy$.

$\Rightarrow y=uv$ $J=\begin{vmatrix}\frac{\partial x}{\partial u}&\frac{\partial x}{\partial v}\\ \frac{\partial y}{\partial u}&\frac{\partial y}{\partial v}\end{vmatrix}=\begin{vmatrix}-\frac{v}{u^2}&\frac{1}{u}\\ v&u\end{vmatrix}=-\frac{2v}{u}$

$x=\frac{v}{u}$

$1\le v\le 3$ $\int_1^3\int_1^2(u+v)\left(\frac{2v}{u}\right)du dv$

$1\le u\le 2$ $=2\int_1^3\int_1^2\left(v+\frac{v^2}{u}\right)du dv=2\int_1^3(v+v^2\ln 2)dv=8+\frac{52\ln 2}{3}$

$\left(0,\ 0,\ \frac{3}{16}(2+\sqrt{2})\right)$

$M=\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^1 \rho^2\sin\varphi\, d\rho\, d\varphi\, d\theta=\frac{2\pi}{3}\left(1-\frac{\sqrt{2}}{2}\right)$

12. (2022年期末) Find the centroid of the region $D=\{(x,y,z)|\sqrt{x^2+y^2}\le z\le\sqrt{1-x^2-y^2}\}$.

$\bar{x}=\frac{1}{M}\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^1 \rho\sin\varphi\cos\theta\ \rho^2\sin\varphi\, d\rho\, d\varphi\, d\theta$ $\quad \bar{y}=\frac{1}{M}\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^1 \rho\sin\varphi\sin\theta\ \rho^2\sin\varphi\, d\rho\, d\varphi\, d\theta$

$\bar{z}=\frac{1}{M}\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^1 \rho\cos\varphi\ \rho^2\sin\varphi\, d\rho\, d\varphi\, d\theta$

13. (2020年期末) Evaluate the integral $\iiint\limits_D z\sqrt{x^2+y^2+z^2}\,dV$, where D is the solid bounded above by $z=1$ and below by $z=\sqrt{x^2+y^2}$.

$z\le 1$ $\quad z\ge\sqrt{x^2+y^2}$

$\int_0^{\frac{\pi}{4}}\int_0^{\frac{1}{\cos\varphi}}$

$\cancel{\int_0^{2\pi}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\int_0^1 \rho\cos\varphi\cdot\rho\cdot\rho^2\sin\varphi\, d\rho\, d\varphi\, d\theta=\frac{\pi}{6}\left(1-\frac{\sqrt{2}}{4}\right)}$

$=\frac{2\pi}{5}\int_0^{\frac{\pi}{4}}\frac{\sin\varphi}{\cos^4\varphi}d\varphi=\frac{2\pi}{15}\left.\frac{1}{\cos^3\varphi}\right|_0^{\pi/4}$

$=\frac{2\pi}{15}(2\sqrt{2}-1)$

14. (2019年期末) The region D is bounded by $z=\sqrt{x^2+y^2}$ and $z=\sqrt{1-x^2-y^2}$. Consider the following integral $\iiint\limits_D(x+z)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$,

$\int_0^{2\pi}\int_0^{\frac{\sqrt{2}}{2}}\int_r^{\sqrt{1-r^2}}(r\cos\theta+z)\,dz\, r\,dr\,d\theta$

(1) Convert the above integral to an equivalent iterated integral in cylindrical coordinates;

(2) Convert the above integral to an equivalent iterated integral in spherical coordinates.

$\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^1(\rho\sin\varphi\cos\theta+\rho\cos\varphi)\,\rho^2\sin\varphi\, d\rho\, d\varphi\, d\theta=\frac{\pi}{2}$

15. (2019年期末) A solid in the first octant is bounded by the planes $y=0$ and $z=0$ and by the surfaces $z=4-x^2$ and $x=y^2$ (see the figure below). Its density function is $\Delta(x,y,z)=xy$. Find the center of the mass for the solid.

$M=\int_0^2\int_0^{\sqrt{x}}\int_0^{4-x^2} xy\, dz\, dy\, dx=\frac{32}{15}$

$\bar{x}=\frac{1}{M}\int_0^2\int_0^{\sqrt{x}}\int_0^{4-x^2} x\cdot xy\, dz\, dy\, dx=\frac{5}{4}$ $\quad \bar{y}=\frac{1}{M}\int_0^2\int_0^{\sqrt{x}}\int_0^{4-x^2} y\cdot xy\, dz\, dy\, dx=\frac{40}{77}\sqrt{2}$ $\quad \bar{z}=\frac{1}{M}\int_0^2\int_0^{\sqrt{x}}\int_0^{4-x^2} z\cdot xy\, dz\, dy\, dx=\frac{8}{7}$

16. (2018年期末) Set up a triple integral in spherical coordinates that gives the volume of the solid bounded below by the xy-plane, on the sides by the sphere $x^2+y^2+z^2=4$, and above by the cone $z=\sqrt{x^2+y^2}$, and then evaluate the integral.

$z=0$

$\int_0^{2\pi}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\int_0^2 \rho^2\sin\varphi\, d\rho\, d\varphi\, d\theta=\cancel{\frac{2\pi}{3}\left(1-\frac{\sqrt{2}}{2}\right)}\ \frac{8\sqrt{2}}{3}\pi$