Step-1

Since with every ordering of the rows of A leaves at least one zero on the diagonal, there must be a column of all zeros in the original square matrix A.

Let i^{th} column be the column of zeros in the matrix A.

Consider again, $\det A = \sum_{\text{all } P's} \left(a_{1\alpha} a_{2\beta} \ldots a_{n\nu}\right) \det P$

Step-2

While carrying out the summation in the right hand side of $\det A = \sum_{\text{all } P's} \left(a_{1\alpha} a_{2\beta} \ldots a_{n\nu}\right) \det P$, every term contains one and only one term from the ith column.

That is, every term in the summation must be equal to zero, since zero multiplied by any number is zero.

Therefore, $\sum_{\text{all } P's} \left(a_{1\alpha} a_{2\beta} \ldots a_{n\nu}\right) \det P = 0$

Step-3

This implies that $\det A = 0$. But this is a contradiction, since we have assumed that $\det A \neq 0$.

Therefore, our assumption that every ordering of A leaves at least one zero on the diagonal is wrong.

Thus, there exists some ordering of A, which leaves no zero on the diagonal.