Step-1

(a).

Given function is $F(x,y) = -1 + 4(e^x - x) - 5x\sin y + 6y^2$ and given stationary point is $(x,y) = (0,0)$.

We need to decide between a minimum, maximum, or saddle point.

We need to find the first and second derivatives now.

$F_x = 4e^x - 4 - 5\sin y.$

$F_{xx} = 4e^x$
$\Rightarrow (F_{xx})_{(0,0)}$
$\Rightarrow 4 > 0$

$F_y = -5x\cos y + 12y$

$F_{yy} = 5x\sin y + 12$

$F_{xy} = -5\cos y$

Step-2

Given the stationary point is $(x,y) = (0,0)$.

Now $(F_{xy})_{(0,0)} = -5$

And $(F_{yy})_{(0,0)} = 12$

And,

$$\begin{aligned}(F_{xx})(F_{yy}) &= 4(12)\\ &= 48\end{aligned}$$

$(F_{xy})^2 = 25$

Clearly $(F_{xx})(F_{yy}) > (F_{xy})^2$

So $F(x,y)$ has minimum at $(0,0)$.

Therefore, $\boxed{F(x,y) \text{ has minimum at } (0,0).}$

Step-3

(b).

Given function is $F(x,y)=(x^2-2x)\cos y$.

We need to decide between a minimum, maximum, or saddle point.

We need to find the first and second derivatives now.

$F_x=(2x-2)\cos y$

$F_{xx}=2\cos y$

$F_y=-(x^2-2x)\sin y$

$F_{yy}=-(x^2-2x)\cos y$

$F_{xy}=-(2x-2)\sin y$

Step-4

Given stationary point is $(x,y)=(1,\pi)$.

Now,

$$\begin{aligned}(F_{xx})_{(1,\pi)} &= 2\cos\pi \\ &= -2<0\end{aligned}$$

And,

$$\begin{aligned}(F_{xx})(F_{yy}) &= (-2)(-1) \\ &= 2\end{aligned}$$

$(F_{xy})^2=0$

Clearly $(F_{xx})(F_{yy})>(F_{xy})^2$

So $F(x,y)$ has maximum at $(1,\pi)$.

Therefore, $\boxed{F(x,y) \text{ has maximum at } (1,\pi).}$