Step-1

(a) Given that A is invertible

i.e. $AA^{-1} = I$

Taking transpose on both sides, we get

$$\left(AA^{-1}\right)^T = I^T$$
$$\left(A^{-1}\right)^T A^T = I$$

Hence inverse of A^T = Transpose of A^{-1}

that is $\left(A^T\right)^{-1} = \left(A^{-1}\right)^T$

Step-2

(b) A is symmetric, then $A = A^T$

Taking inverses on both sides,

$$A^{-1} = \left(A^T\right)^{-1}$$
$$A^{-1} = \left(A^{-1}\right)^T$$

Step-3

(c) Given that $A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$

$$\Rightarrow A^T = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\Rightarrow \left(A^T\right)^{-1} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}^{-1}$$

$$\Rightarrow \left(A^T\right)^{-1} = \frac{1}{2.1-1.1}\begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$
$$\Rightarrow \left(A^T\right)^{-1} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

Step-4

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{2.1 - 1.1} \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

$$\Rightarrow \left(A^{-1}\right)^T = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}^T$$

$$\Rightarrow \left(A^{-1}\right)^T = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}^T$$

$$= \left(A^T\right)^{-1}$$

Step-5

Also A is symmetric and

$$A^{-1} = \left(A^{-1}\right)^T$$

$$= \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}^T$$