Step-1

Given that $B = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$

The characteristic equation is $|B - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 3-\lambda & 1 \\ 0 & 2-\lambda \end{vmatrix} = 0$$
$$(3-\lambda)(2-\lambda) = 0$$
$$\Rightarrow \lambda_1 = 2, \lambda_2 = 3$$

Step-2

To get the eigen vector corresponding to $\lambda_1 = 2$, we solve $(\mathrm{B} - \lambda_1 \mathrm{I})x = 0$

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

So, the solution set of $x_1 + x_2 = 0$ is $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is the eigen vector required.

Similarly, we solve $(\mathrm{B} - \lambda_2 \mathrm{I})x = 0$ to get $\begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

The corresponding eigen vector is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Step-3

Using the eigen vectors, $S = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}, \Lambda = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}, S^{-1} = \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$ such that

$$B = S \Lambda S^{-1}$$

Multiplying the respective sides with themselves for *k* times, we get $B^k = S \Lambda^k S^{-1}$

$$= \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 2^k & 0 \\ 0 & 3^k \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2^k & 3^k \\ -2^k & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3^k & 3^k - 2^k \\ 0 & 2^k \end{bmatrix}$$

$$= 3^k \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} + 2^k \begin{bmatrix} 0 & -1 \\ 0 & 1 \end{bmatrix}$$