Step-1

Suppose A is a linear transformation from the x-y plane to itself.

We have to verify why $A^{-1}(x+y) = A^{-1}x + A^{-1}y$.

Step-2

Let $u, v \in \mathrm{R}^2$ and $Au = x,\ Av = y$

$\Rightarrow\ u = A^{-1}x,\ v = A^{-1}y$

And $A^{-1}x,\ A^{-1}y$ are also in x-y plane.

Since A is linear transformation.

So

$A\left(A^{-1}x + A^{-1}y\right) = A\left(A^{-1}x\right) + A\left(A^{-1}\right)y$

$= \left(AA^{-1}\right)x + \left(AA^{-1}\right)y$

$= x + y \quad \left(\text{Since } AA^{-1} = A^{-1}A = I\right)$

Thus $\boxed{A^{-1}(x+y) = A^{-1}x + A^{-1}y}$

Suppose A is represented by matrix M.

If A^{-1} exists, then A^{-1} is represented by M^{-1}

Hence A^{-1} is represented by M^{-1}.