Southern University of Science and Technology
Advanced Linear Algebra Spring 2023

MA109– Quiz #7

2023/04/09

Name: _____

Student Number: _____

1. 给出一个算子，它关于某个基的矩阵的对角线上全是非零数，但这个算子是不可逆的.

   Proof. $T \in \mathcal{L}(\mathbf{R^2})$, $Te_1 = Te_2 = e_1 + e_2$, where $e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $M(T) = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$. However, T is not invertible.

   □

2. Suppose $T \in \mathcal{L}(\mathbf{R}^3)$ and $-4, 5$, and $\sqrt{7}$ are eigenvalues of T. Prove that there exists $x \in \mathbf{R}^3$ such that $Tx - 9x = (-4, 5, \sqrt{7})$.

   设 $T \in \mathcal{L}(\mathbf{R}^3)$ 且 $-4, 5, \sqrt{7}$ 均为 T 的本征值. 证明存在 $x \in \mathbf{R}^3$ 使得 $Tx - 9x = (-4, 5, \sqrt{7})$.

   Proof. Define $\mathcal{A} \in \mathcal{L}(\mathbf{R}^3)$, $\mathcal{A} = T - 9I$. $\mathcal{A}$ is invertible which is equivalent to $\mathcal{A}$ is injective. If $\mathcal{A}x = 0$, $Tx = 9x$. Since T has three distinct eigenvalues and a linear map in $\mathcal{L}(\mathbf{R}^3)$ has at most 3 distinct eigenvalues, then 9 is not an eigenvalue of T, then $x = 0$, $\mathcal{A}$ is injective.

   □