Step-1

Suppose a matrix A has s linearly independent eigenvectors, then it is similar to a matrix J that is in Jordan form, with s square blocks on the diagonal:

$$J = M^{-1}AM$$
$$= \begin{bmatrix} J_1 & & \\ & \ddots & \\ & & J_s \end{bmatrix}.$$

Each block has one eigenvalue, one eigenvector, and 1s just above the diagonal:

$$J_i = \begin{bmatrix} \lambda_i & 1 & & \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ & & & \lambda_i \end{bmatrix}$$

Step-2

Consider permutation matrix P with 1s along the cross-diagonal (lower left to upper left).

$$P = \begin{bmatrix} & & & 1 \\ & & 1 & \\ & \cdot^{\cdot^{\cdot}} & & \\ 1 & & & \end{bmatrix}$$
$$P^{-1} = \begin{bmatrix} & & & 1 \\ & & 1 & \\ & \cdot^{\cdot^{\cdot}} & & \\ 1 & & & \end{bmatrix}$$

Suppose we have a Jordan block J_i, then we get

$$P^{-1}J_iP = \begin{bmatrix} & & & 1 \\ & & 1 & \\ & \cdot^{\cdot^{\cdot}} & & \\ 1 & & & \end{bmatrix}\begin{bmatrix} \lambda & 1 & & \\ & \lambda & \ddots & \\ & & \ddots & 1 \\ & & & \lambda \end{bmatrix}\begin{bmatrix} & & & 1 \\ & & 1 & \\ & \cdot^{\cdot^{\cdot}} & & \\ 1 & & & \end{bmatrix}$$
$$= \begin{bmatrix} \lambda & & & \\ 1 & \lambda & & \\ & \ddots & \ddots & \\ & & 1 & \lambda \end{bmatrix}$$
$$= J_i^T$$

Therefore, $\boxed{\text{Jordan block } J_i \text{ is similar to } J_i^T}$.

Step-3

Consider the Jordan form, with A similar to a matrix J, then we have

$$A = P^{-1}JP$$

By taking transpose we have:

$$\begin{aligned}(A)^T &= \left(P^{-1}JP\right)^T \\ &= \left(P^{-1}\right)^T (J)^T (P)^T \\ &= \left(P^T\right)^{-1} (J)^T (P)^T\end{aligned}$$

So, from the above form $A^T = J^T$.

Step-4

We know that Jordan block J is similar to J^T.

So,

$$\begin{aligned}A^T &= J^T \\ &= J\end{aligned}$$

We have already consider A similar to a matrix J, then we have

$$\begin{aligned}A^T &= J^T \\ &= J \\ &= A\end{aligned}$$

Therefore, $\boxed{\text{every matrix } A \text{ is similar to its transpose } A^T}$.