Step-1

Let P_1 = the projection of b onto the line through $a_1 = \dfrac{a_1^T b}{a_1^T a_1} a_1$

$$a_1^T b = (-1,2,2)\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= -1 + 0 + 0$$

$$= -1$$

$$a_1^T a_1 = \begin{bmatrix} -1 & 2 & 2 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$$

$$= 1 + 4 + 4$$

$$= 9$$

$$P_1 = \frac{-1}{9}\begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1/9 \\ -2/9 \\ -2/9 \end{pmatrix}$$

Step-2

P_2 = The projection of b onto the line through $a_2 = \dfrac{a_2^T b}{a_2^T a_2} a_2$

$$a_2^T b = (2,2,-1)\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$= 2 + 0 + 0$$

$$= 2$$

$$a_2^T a_2 = \begin{bmatrix} 2 & 2 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$$

$$= 4 + 4 + 1$$

$$= 9$$

1

$$p_2 = \frac{2}{9}\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 4/9 \\ 4/9 \\ -2/9 \end{bmatrix}$$

Step-3

p_3 = The projection of b onto the line through $\quad a_3 = \dfrac{a_3^T b}{a_3^T a_3} a_3$

$$a_3^T b = (2,-1,2)\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$= 2 + 0 + 0$$

$$= 2$$

$$a_3^T a_3 = \begin{bmatrix} 2 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$$

$$= 4 + 1 + 4$$

$$= 9$$

$$p_3 = \frac{2}{9}\begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4/9 \\ -2/9 \\ 4/9 \end{bmatrix}$$

Step-4

$$P_1 + P_2 + P_3 = \begin{bmatrix} 1/9 \\ -2/9 \\ -2/9 \end{bmatrix} + \begin{bmatrix} 4/9 \\ 4/9 \\ -2/9 \end{bmatrix} + \begin{bmatrix} 4/9 \\ -2/9 \\ 4/9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

2

Therefore $P_1 + P_2 + P_3 = b$