Step-1

(a)

Matrix that the null space contains the vector $x=(1,1,2)$

Rewrite from this that the first two components are equal and third is twice the first two.

In other words,

$$x_1 = x_2$$
$$x_3 = 2x_1$$

Hence,

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_1 \\ 2x_1 \end{bmatrix}$$
$$= k\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

Where $x_1 = k$ a parameter

Simply, write this solution set derived from the matrix $\boxed{A = \begin{bmatrix} -1 & 1 & 0 \\ -2 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}}$

Therefore, A is the matrix whose null space is spanned by $x=(1,1,2)$

Step-2

(b)

B be the matrix that contains the vector $y=(1,5)$, $B=\begin{bmatrix} -5 & 1 \\ 0 & 0 \end{bmatrix}$, the null space of B contains $(1,5)$.

$$A = B^T = \begin{bmatrix} 1 & 0 \\ -5 & 0 \end{bmatrix}$$

The left null space of A contains $y=(1,5)$

Thus, $\boxed{B=\begin{bmatrix}-5 & 1\\ 0 & 0\end{bmatrix}}$ is the required matrix.

Step-3

(c)

$C(A)$ is spanned by $(1,1,2)$

$R(A)$ is spanned by $(1,5)$.

$$A=\begin{bmatrix}1 & 5\\ 1 & 5\\ 2 & 10\end{bmatrix}\xrightarrow[R_3-2R_1]{R_2-R_1}\begin{bmatrix}1 & 5\\ 0 & 0\\ 0 & 0\end{bmatrix}$$

Thus, $\boxed{A=\begin{bmatrix}1 & 5\\ 1 & 5\\ 2 & 10\end{bmatrix}}$ is the required matrix.

Step-4

(d)

$\boxed{\text{Yes,}}$ There is a matrix First set of three vectors is the first three columns of the matrix. The second set of three vectors is the rows of the matrix.

$$\boxed{A=\begin{pmatrix}1 & 2 & 3 & 1 & 2\\ 2 & 3 & 4 & 2 & 3\\ 3 & 4 & 5 & 3 & 4\\ 2 & 4 & 6 & 2 & 4\\ 4 & 6 & 8 & 4 & 6\\ 6 & 8 & 10 & 6 & 8\end{pmatrix}}$$

Column space is spanned by $\begin{pmatrix}1\\2\\3\\2\\4\\6\end{pmatrix}\begin{pmatrix}2\\3\\4\\4\\6\\8\end{pmatrix}\begin{pmatrix}3\\4\\5\\6\\8\\10\end{pmatrix}$ and

Row space is spanned by $\begin{pmatrix} 1 & 2 & 3 & 1 & 2 \end{pmatrix}, \begin{pmatrix} 2 & 3 & 4 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 3 & 4 & 5 & 3 & 4 \end{pmatrix}$