

## Step-1

Given that the first  $m$  and the last  $m$  components of the vector  $y = F_n c$  are

$$\left. \begin{aligned} y_j &= y_j' + w_n^j y_j'', \quad j = 0, 1, \dots, m-1 \\ y_{j+m} &= y_j' - w_n^j y_j'', \quad j = 0, 1, \dots, m-1 \end{aligned} \right\} \dots (1)$$

For  $n = 2$ , we have to write  $y_0$  from the first line of equation (1) and  $y_1$  from the second line. And also, for  $n = 4$ , use the first line; we have to find  $y_0$  and  $y_1$ , and the second to find  $y_2$  and  $y_3$ , all in terms of  $y'$  and  $y''$ .

## Step-2

For  $n = 2$

First line of equation (1) is  $y_j = y_j' + w_2^j y_j'', j = 0, 1, \dots, m-1$

If  $j = 0$ , then

$$\begin{aligned} y_0 &= y_0' + w_2^0 y_0'' \\ \Rightarrow y_0 &= y_0' + y_0'' \end{aligned}$$

## Step-3

Second line of equation (1) is  $y_{j+m} = y_j' - w_2^j y_j'', j = 0, 1, \dots, m-1$

If  $j = 0, m = 1$ ,

$$\begin{aligned} y_{0+1} &= y_0' - w_2^0 y_0'' \\ \Rightarrow y_1 &= y_0' - y_0'' \end{aligned}$$

## Step-4

For  $n = 4$

First line of equation (1) is  $y_j = y_j' + w_4^j y_j'', j = 0, 1, \dots, m-1$

If  $j = 0$ ,

$$\begin{aligned} \text{then } y_0 &= y_0' + w_4^0 y_0'' \\ \Rightarrow y_0 &= y_0' + y_0'' \end{aligned}$$

## Step-5

If  $j = 1$ ,

then  $y_1 = y_1' + w_4' y_1''$

$$w_4 = e^{\frac{2\pi i}{4}}$$

$$= \cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$$

$$= i$$

Therefore  $y_1 = y_0' + i y_0''$

## Step-6

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Second line of equation (1) is  $y_{j+m} = y_j' - w_4^j y_j''$ ,  $j = 0, 1, \dots, m-1$

If  $j = 0, m = 2$ ,

then  $y_{0+2} = y_0' - w_4^0 y_0''$

$$\Rightarrow y_2 = y_0' - y_0''$$

## Step-7

If  $j = 1, m = 2$ ,

then  $y_{1+2} = y_1' - w_4^1 y_1''$

$$\Rightarrow y_3 = y_1' - i y_1''$$