Step-1

Let B be a symmetric positive definite matrix. Then for any vector y, we have $y^{\mathrm{T}}By > 0$. Suppose such a matrix B is added to the matrix A.

$$\begin{aligned}
\lambda_2(A+B) &= \min_{x^{\mathrm{T}}x_1=0} R(x) \\
&= \min_{x^{\mathrm{T}}x_1=0} \frac{x^{\mathrm{T}}(A+B)x}{x^{\mathrm{T}}x} \\
&= \min_{x^{\mathrm{T}}x_1=0} \frac{x^{\mathrm{T}}Ax}{x^{\mathrm{T}}x} + \min_{x^{\mathrm{T}}x_1=0} \frac{x^{\mathrm{T}}Bx}{x^{\mathrm{T}}x} \\
&= \lambda_2(A) + \min_{x^{\mathrm{T}}x_1=0} \frac{x^{\mathrm{T}}Bx}{x^{\mathrm{T}}x}
\end{aligned}$$

But it is clear that $\min_{x^{\mathrm{T}}x_1=0} \frac{x^{\mathrm{T}}Bx}{x^{\mathrm{T}}x} > 0$, for any x.

Step-2

Therefore, it is proved that $\boxed{\lambda_2(A+B) > \lambda_2(A)}$.