

## Step-1

Spherical coordinates  $P, \phi, \theta$  given

$$x = P \sin \phi \cos \theta, y = P \sin \phi \sin \theta, z = P \cos \phi$$

Now

$$x = P \sin \phi \cos \theta$$

$$\Rightarrow \frac{\partial x}{\partial P} = \sin \phi \cos \theta$$

$$\frac{\partial x}{\partial \phi} = P \cos \phi \cos \theta$$

$$\frac{\partial x}{\partial \theta} = -P \sin \phi \sin \theta$$

## Step-2

$$y = P \sin \phi \sin \theta$$

$$\Rightarrow \frac{\partial y}{\partial P} = \sin \phi \sin \theta$$

$$\frac{\partial y}{\partial \theta} = P \sin \phi \cos \theta$$

$$\frac{\partial y}{\partial \phi} = P \cos \phi \sin \theta$$

## Step-3

And

$$Z = P \cos \phi$$

$$\Rightarrow \frac{\partial z}{\partial P} = \cos \phi$$

$$\frac{\partial z}{\partial \phi} = -P \sin \phi$$

$$\frac{\partial z}{\partial \theta} = 0$$

## Step-4

So, the Jacobian matrix of partial derivatives is

$$\det J = \begin{vmatrix} \frac{\partial x}{\partial P} & \frac{\partial x}{\partial \phi} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial P} & \frac{\partial y}{\partial \phi} & \frac{\partial y}{\partial \theta} \\ \frac{\partial z}{\partial P} & \frac{\partial z}{\partial \phi} & \frac{\partial z}{\partial \theta} \end{vmatrix}$$

$$= \begin{vmatrix} \sin \phi \cos \theta & P \cos \phi \cos \theta & -P \sin \phi \sin \theta \\ \sin \phi \sin \theta & P \cos \phi \sin \theta & P \sin \phi \cos \theta \\ \cos \phi & -P \sin \phi & 0 \end{vmatrix}$$

## Step-5

The columns are mutually orthogonal and length of columns are given by

$$\sqrt{\sin^2 \phi \cos^2 \theta + \sin^2 \phi \sin^2 \theta + \cos^2 \phi} = 1$$

$$\sqrt{P^2 (\cos^2 \phi \cos^2 \theta + \cos^2 \phi \sin^2 \theta + \sin^2 \phi)} = P$$

$$\sqrt{P^2 \sin^2 \phi \sin^2 \theta + P^2 \sin^2 \phi \cos^2 \theta} = P \sin \phi$$

$$\therefore \det J = 1 \cdot P \cdot P \cdot \sin \phi$$

$$= P^2 \sin \phi$$

And hence  $\boxed{dV = P^2 \sin \phi dP d\phi d\theta}$