Step-1

Given that $A=\begin{pmatrix}\frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2}\end{pmatrix}$

$$\begin{aligned}A^2 &= A\,A\\ &=\begin{pmatrix}\frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2}\end{pmatrix}\begin{pmatrix}\frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2}\end{pmatrix}\end{aligned}$$

$$\begin{aligned}A^3 &= A^2.\,A\\ &= A.\,A\ (A^2 = A)\\ &= A^2\\ &= A\end{aligned}$$

$$\boxed{A^3 = A}$$

Similarly, we have $A^n = A$

Step-2

Given that $B=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$

$$\begin{aligned}B^2 &= \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\\ &=\begin{pmatrix}1 & 0\\ 0 & (-1)^2\end{pmatrix}\end{aligned}$$

$$\begin{aligned}B^3 &= B^2.\,B\\ &= I.B\\ &= B\\ &=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\\ &=\begin{pmatrix}1 & 0\\ 0 & (-1)^3\end{pmatrix}\end{aligned}$$

Similarly, we have $B^n = \begin{pmatrix} 1 & 0 \\ 0 & (-1)^n \end{pmatrix}$

Step-3

Given that $C = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$

$$
\begin{aligned}
C^2 &= \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix}\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \\
&= \begin{pmatrix} \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 & \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 \\ \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 & \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 \end{pmatrix} \\
&= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\
&= \text{null matrix.}
\end{aligned}
$$

$$
\begin{aligned}
C^3 &= C^2.C \\
&= 0.C \\
&= 0
\end{aligned}
$$

Similarly, we have $C^n = 0$