

Step-1

We have to construct a 2 by 2 matrix whose null space equals its column space.

$$\text{Let } A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\text{Column space} = \left\{ a \begin{bmatrix} 0 \\ 0 \end{bmatrix} + b \begin{bmatrix} 1 \\ 0 \end{bmatrix} / a, b \in R \right\}$$

$$= \left\{ \begin{bmatrix} b \\ 0 \end{bmatrix} / b \in R \right\}$$

=The line through $(1,0)$.

Step-2

For finding null space of A , take $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $Ax = 0$

$$\Rightarrow \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow x_2 = 0$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ 0 \end{bmatrix}$$

$$= x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Therefore the null space of A is the line through $(1,0)$

$$\text{Therefore the required matrix is } \boxed{A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}$$