

Step-1

Consider the matrix,

$$A = \begin{bmatrix} 0 & 3 \\ 4 & 6 \end{bmatrix}$$

The objective is to find the inverse of this matrix directly or from the $2 \text{ by } 2$ formula.

Step-2

Use the inverse of matrix formula.

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Compare the matrix $A = \begin{bmatrix} 0 & 3 \\ 4 & 6 \end{bmatrix}$ with $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

$a = 0$, $b = 3$, $c = 4$, and $d = 6$.

Step-3

Inverse of the matrix can be calculated as,

$$A^{-1} = \frac{1}{(0) \cdot (6) - (3) \cdot (4)} \begin{pmatrix} 6 & -3 \\ -4 & 0 \end{pmatrix}$$

$$A^{-1} = \frac{1}{0 - 12} \begin{pmatrix} 6 & -3 \\ -4 & 0 \end{pmatrix}$$

$$= \frac{1}{-12} \begin{pmatrix} 6 & -3 \\ -4 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{6}{-12} & \frac{-3}{-12} \\ \frac{-4}{-12} & \frac{0}{-12} \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{1}{2} & \frac{1}{4} \\ \frac{1}{3} & 0 \end{pmatrix}$$

Hence, the inverse of matrix $A = \begin{bmatrix} 0 & 3 \\ 4 & 6 \end{bmatrix}$ is $A^{-1} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{4} \\ \frac{1}{3} & 0 \end{bmatrix}$.

Step-4

Consider the matrix,

$$B = \begin{bmatrix} a & b \\ b & 0 \end{bmatrix}$$

The objective is to find the inverse of this matrix directly or from the $2 \text{ by } 2$ formula.

Compare the matrix $B = \begin{bmatrix} a & b \\ b & 0 \end{bmatrix}$ with $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

$a = a$, $b = b$, $c = b$, and $d = 0$.

Step-5

Inverse of the matrix can be calculated as,

$$B^{-1} = \frac{1}{a(0) - (b)(b)} \begin{bmatrix} 0 & -b \\ -b & a \end{bmatrix}$$

$$B^{-1} = \frac{1}{0 - b^2} \begin{pmatrix} 0 & -b \\ -b & a \end{pmatrix}$$

$$= \frac{1}{-b^2} \begin{pmatrix} 0 & -b \\ -b & a \end{pmatrix}$$

$$= \begin{pmatrix} \frac{0}{-b^2} & \frac{-b}{-b^2} \\ \frac{-b}{-b^2} & \frac{a}{-b^2} \end{pmatrix}$$

$$B^{-1} = \begin{pmatrix} 0 & \frac{1}{b} \\ \frac{1}{b} & -\frac{a}{b^2} \end{pmatrix}$$

Hence, the inverse of the matrix $B = \begin{bmatrix} a & b \\ b & 0 \end{bmatrix}$ is $B^{-1} = \begin{bmatrix} 0 & \frac{1}{b} \\ \frac{1}{b} & -\frac{a}{b^2} \end{bmatrix}$.

Step-6

Consider the matrix,

$$C = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$$

The objective is to find the inverse of this matrix directly or from the $2 \text{ by } 2$ formula.

Compare the matrix $C = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$ with $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

$a = 3$, $b = 4$, $c = 5$, and $d = 7$.

Step-7

Inverse of the matrix can be calculated as,

$$C^{-1} = \frac{1}{(3) \cdot (7) - (4) \cdot (5)} \begin{pmatrix} 7 & -4 \\ -5 & 3 \end{pmatrix}$$

$$\begin{aligned} C^{-1} &= \frac{1}{21 - 20} \begin{pmatrix} 7 & -4 \\ -5 & 3 \end{pmatrix} \\ &= \frac{1}{1} \begin{pmatrix} 7 & -4 \\ -5 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 7 & -4 \\ -5 & 3 \end{pmatrix} \end{aligned}$$

Hence, the inverse of matrix $C = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$ is $C^{-1} = \begin{bmatrix} 7 & -4 \\ -5 & 3 \end{bmatrix}$.