

## Step-1

Consider the matrix  $A$ ,

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

The Rayleigh quotient is given by,

$$R(x) = \frac{x^T A x}{x^T x}$$

As per Rayleigh's Principle, the minimum value of the Rayleigh quotient is the smallest eigenvalue  $\lambda_1$

Thus, the eigenvalues of  $A$  are given by,

$$\lambda_1 = 1$$

$$\lambda_2 = 3$$

Thus, the minimum value of  $R(x)$  is  $\lambda_1 = 1$ .

## Step-2

Therefore, the minimum value of  $R(x)$  is  $\boxed{\lambda_1 = 1}$ .