Step-1

Thus, we get $2\lambda x^{H} Ax = -x^{H} x$.

Note that $x^{H} x$ is a positive real number, since x is a non zero vector. Thus, in the above equation, the right hand side is negative. If x is a real vector, then $x^{H} Ax$ too will be a positive real number. Therefore, the real part of the eigenvalue *?* will be negative.

Step-2

Let us show that the real part of the eigenvalue *?* is less than zero. Consider the following:

$$
\begin{aligned}
AM + M^{H} A &= -I \\
x^{H}\left(AM + M^{H} A\right) &= -x^{H} I \\
x^{H}\left(AM + M^{H} A\right)x &= -x^{H} Ix \\
x^{H} AMx + x^{H} M^{H} Ax &= -x^{H} x
\end{aligned}
$$

Step-3

We have $Mx = \lambda x$. Also, we can write $x^{H} M^{H} = (Mx)^{H}$. Therefore, we get

$$
\begin{aligned}
x^{H} AMx + x^{H} M^{H} Ax &= -x^{H} x \\
x^{H} A(\lambda x) + (Mx)^{H} Ax &= -x^{H} x \\
\lambda x^{H} Ax + (\lambda x)^{H} Ax &= -x^{H} x \\
\lambda x^{H} Ax + \lambda x^{H} Ax &= -x^{H} x
\end{aligned}
$$