

Step-1

If Q is an orthogonal matrix, so that $Q^T Q = I$, we have to prove that $\det Q$ equals to $+1$ or -1 and we have to find that what kind of box is formed from the rows (or columns) of Q .

Step-2

We have $Q^T A = I$ (since Q is an orthogonal matrix)

$$\Rightarrow \det(Q^T Q) = \det I$$

$$\Rightarrow \det Q^T \cdot \det Q = 1 \quad (\text{since } \det I = 1)$$

(since $\det(AB) = \det A \det B$ for any 2 matrices A,B)

Step-3

$$\Rightarrow \det Q \cdot \det Q = 1$$

(since $\det(A^T) = \det A$ for any matrix A)

$$\Rightarrow (\det Q)^2 = 1$$

$$\Rightarrow \det Q = \sqrt{1} = \boxed{\pm 1}$$

Thus, $\det Q = \pm 1$, where Q is an orthogonal matrix.

A box of volume 1 is formed from rows of Q .