Step-1

Consider the following:

$$Ax = \lambda_1 x$$
$$A^T y = \lambda_2 y$$

To prove that $x^T y = 0$ or Eigen vectors are perpendicular.

Here, all Eigen values, eigenvectors and matrix A are all real.

Step-2

Recall that matrix A has real Eigen values and real orthogonal Eigenvectors if and only if $A = A^T$.

Now do the following calculations:

$$Ax = \lambda_1 x$$

Dot product with vector y will give:

$$\begin{aligned} (\lambda_1 x)^T y &= (Ax)^T y \\ &= x^T A^T y \\ x^T \lambda_1 y &= x^T \lambda_2 y \\ x^T y(\lambda_1 - \lambda_2) &= 0 \end{aligned}$$

Since, $\lambda_1 - \lambda_2 \neq 0$, So, $x^T y = 0$.

Step-3

Therefore, Eigen vectors corresponding to different Eigen values are perpendicular $\boxed{x^T y = 0}$.