Step-1

Given that C is non-singular.

We have to prove that A and $C^T AC$ have the same rank.

Here obviously C is square and the sizes of A and $C^T AC$ are same.

We know that

$\operatorname{rank}\left(C^T AC\right) \leq \operatorname{rank}(A)$ â€¦â€¦ (1)

Since C is non-singular and C^{-1} exists, it is also non-singular.

Step-2

Now applying equation (1) to the matrix $C^T AC$ and the non-singular matrix C^{-1},

$$\begin{aligned}
\operatorname{rank}\left(C^T AC\right) &\geq \operatorname{rank}\left(\left(C^{-1}\right)^T C^T AC\left(C^{-1}\right)\right) \\
&= \operatorname{rank}\left(\left(C^T\right)^{-1} C^T AC\left(C^{-1}\right)\right) \\
&= \operatorname{rank}(IAI) \\
&= \operatorname{rank}(A)
\end{aligned}$$

Thus $\operatorname{rank}\left(C^T AC\right) \geq \operatorname{rank}(A)$ â€¦â€¦ (2)

Step-3

From (1) and (2),

$\operatorname{rank}(A) = \operatorname{rank}\left(C^T AC\right)$.

Since the ranks of A and $C^T AC$ are same, then have the same number of zero Eigen values.