Step-1

Consider that $AB = 0$, the columns of B are in the null space of A, and the vectors in R^n.

The objective is to prove that $\operatorname{rank}(B) + \operatorname{rank}(A) \le n$.

Step-2

We know that the dimension of column space of the matrix is the rank of that particular matrix.

Given the columns of B are in the null space of A.

i.e. $\dim(C(B)) \le \dim(N(A))$ â€¦â€¦ (1)

Therefore, $\dim(N(A))$ is rank of the matrix B.

i.e. $\operatorname{rank}(B) \le \dim(N(A))$ â€¦â€¦ (2)

Step-3

Again, we know that,

dimension of $C(A)$ + dimension of $N(A)$ = number of columns.

$\dim C(A) + \dim N(A) = n \quad (\because n \text{ is number of columns})$ â€¦â€¦ (3)

From the equations (1), (2) and (3),

$$\begin{aligned} \dim C(A) + \dim N(A) &= n \\ \operatorname{rank}(A) + \dim N(A) &= n \\ \operatorname{rank}(A) + \operatorname{rank}(B) &\le n \end{aligned}$$

Therefore, it is proved that $\boxed{\operatorname{rank}(B) + \operatorname{rank}(A) \le n}$.