Step-1

Given that $A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$

We need to compute the Eigen values and eigenvectors of the matrix A

Now $A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$

$$\begin{aligned} A - \lambda I &= \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \\ &= \begin{bmatrix} 1-\lambda & -1 \\ 2 & 4-\lambda \end{bmatrix} \end{aligned}$$

Step-2

Then

$$\begin{aligned} |A - \lambda I| &= \begin{vmatrix} 1-\lambda & -1 \\ 2 & 4-\lambda \end{vmatrix} \\ &= (1-\lambda)(4-\lambda) + 2 \\ &= 4 - \lambda - 4\lambda + \lambda^2 + 2 \\ &= \lambda^2 - 5\lambda + 6 \end{aligned}$$

Step-3

We know that

$$\begin{aligned} |A - \lambda I| &= 0 \\ \lambda^2 - 5\lambda + 6 &= \lambda^2 - 3\lambda - 2\lambda + 6 \\ &= \lambda(\lambda - 3) - 2(\lambda - 3) \\ &= (\lambda - 3)(\lambda - 2) \\ (\lambda - 3)(\lambda - 2) &= 0 \end{aligned}$$

Now $\lambda = 3, 2$

Hence the Eigen values of A are 3, 2

Step-4

Case (1)

Let $\lambda = 3$

Eigen vectors X corresponding to the Eigen value 3 are given by

$(A - 3I)X = 0$

That is

$$\begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Step-5

Add +1 times first row to the second row

$$\begin{bmatrix} -2 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-2x_1 - x_2 = 0$$

Let $x_2 = k$

Therefore

$x_1 = -k/2$

Therefore eigenvectors corresponding to eigenvalue -3 are given by $k\begin{bmatrix} -1/2 \\ 1 \end{bmatrix}$ where k is a non-zero parameter

Step-6

Case (2):

Let $\lambda = 2$

Eigen vector X corresponding to the Eigen value 2 are given by

$(A - 2I)X = 0$

That is

$$\begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Add -2 times first row to the second row

$$\begin{bmatrix} -1 & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-x_1 - x_2 = 0$$

Step-7

Let

$x_2 = k(\neq 0)$

Therefore

$x_1 = -k$

Therefore Eigen vectors corresponding to Eigen value 2 are given by $k\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

Where k is a non-zero parameter

Step-8

The trace of A equals the sum of the Eigen values, and the determinant equals their product.

Verification: The trace of A is $1+4(\text{diagonal elements}) = \boxed{5}$ is equals the sum of the Eigen values $3+2=\boxed{5}$

Step-9

And the determinant of $A = \begin{vmatrix} 1 & -1 \\ 2 & 4 \end{vmatrix} = 4-(-2) = \boxed{6}$ is equals the product of the Eigen values $3 \times 2 = \boxed{6}$