Step-1

Factorization of *A* into triangular times symmetric:

Let *A* be factorized into *LDU*.

$$A = LDU$$

Then matrix *A* can be written as $L\left(U^T\right)^{-1}$ times $U^T DU$. Here, $L\left(U^T\right)^{-1}$ is a matrix with diagonal all 1â€™s. Explain why $L\left(U^T\right)^{-1}$ is triangular and why $U^T DU$ is symmetric.

Step-2

Recall that if matrix *A* is factored into *LDU* with no row exchanges then matrix *U* is exactly L^T.

Now consider the product of lower triangular matrix and inverse transpose of upper triangular matrix:

$$\begin{aligned} L\left(U^T\right)^{-1} &= L\left(\left(L^T\right)^T\right)^{-1} \\ &= L(L)^{-1} \\ &= I \end{aligned}$$

In other way it can also be said that product of two triangular matrices results into a triangular matrix. As here *L* and *U* are both triangular matrices.

Therefore, $L\left(U^T\right)^{-1}$ is triangular.

Step-3

Recall that $A = A^T$ then matrix *A* is symmetric. Now, do the following calculations:

$$\begin{aligned} \left(U^T DU\right)^T &= U^T D^T \left(U^T\right)^T \\ &= U^T D^T U \\ &= U^T DU \end{aligned}$$

Here, $D = D^T$ is taken as it is a pivot matrix. Therefore, above calculations shows that $U^T DU$ is symmetric.