Step-1

Consider a matrix $A=\begin{bmatrix}0 & -1\\ i & 0\end{bmatrix}$ and the complex number $c=a+ib$.

To show the matrix $A+cI$ is invertible, we have to show $\det(A+cI)\neq 0$.

$$\begin{aligned}\det(A+cI)&=\det\left(\begin{bmatrix}0 & -1\\ i & 0\end{bmatrix}+c\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right)\\&=\det\begin{bmatrix}c & -1\\ i & c\end{bmatrix}\\&=c^2+i\end{aligned}$$

Step-2

Substitute $c=a+bi$ in the equation $\det(A+cI)=c^2+i$.

$$\begin{aligned}\det(A+cI)&=(a+ib)^2+i\\&=\left(a^2+2abi-b^2\right)+i\\&=\left(a^2-b^2\right)+(2ab+1)i\end{aligned}$$

Since $\left(a^2-b^2\right)+(2ab+1)i$ cannot be zero for any real numbers *a* and *b*.

So,

$$\begin{aligned}\det(A+cI)&=\left(a^2-b^2\right)+(2ab+1)i\\&\neq 0\end{aligned}$$

Thus, the matrix $A+cI$ is $\boxed{\text{invertible}}$ for all complex numbers $c=a+ib$.

Step-3

Consider a matrix $A=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$ and $r\in R$.

To show the matrix $A+rI$ is invertible, we have to show $\det(A+rI)\neq 0$.

$$\det(A+rI) = \det\left(\begin{bmatrix} 0 & -1 \\ i & 0 \end{bmatrix} + r\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$$
$$= \det\begin{bmatrix} r & -1 \\ 1 & r \end{bmatrix}$$
$$= r^2 + 1$$

Step-4

Since r^2+1 cannot be zero for any real numbers *r*.

So,

$$\det(A+cI) = r^2+1$$
$$\neq 0$$

Thus, the matrix $A+rI$ is $\boxed{\text{invertible}}$ for all real numbers *r*.