

Step-1

Now let A be the 3 by 3 identity matrix and let $b = (1, 0, 0)$. We need to obtain the multiple of $V = (1, 1, 1)$, so that $P(y) = \frac{1}{2} y^T y - y_1$ will be minimum?

Step-2

Let the required $y = (y_1, y_2, y_3)^T$.

Therefore, we get

$$\begin{aligned} P(y) &= \frac{1}{2} y^T y - y_1 \\ &= \frac{1}{2} (y_1, y_2, y_3) \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} - y_1 \\ &= \frac{1}{2} (y_1^2 + y_2^2 + y_3^2) - y_1 \\ &= \frac{y_1^2 + y_2^2 + y_3^2 - 2y_1}{2} \end{aligned}$$

Step-3

Whatever be $y = (y_1, y_2, y_3)^T$, the quantities y_2^2 and y_3^2 will be non negative. Therefore, their minimum will be zero.

Thus, $y = (y_1, 0, 0)^T$.

This gives, $P(y) = \frac{y_1^2 - 2y_1}{2}$

Step-4

Let us consider a function f , such that $f(x) = x^2 - 2x$. Differentiating f with respect to x , gives $f'(x) = 2x - 2$.

Equating $f'(x)$ to zero gives $x = 1$.

Therefore, the minimum of f occurs when $x = 1$. Similarly, $P(y) = \frac{y_1^2 - 2y_1}{2}$ is minimum when $y_1 = 1$.

Step-5

Therefore, the required $y = (1, 0, 0)^T$.