

Step-1

Let a projection matrix have n rows and n columns. Consider a matrix A , which too has n rows and n columns.

Let the projection matrix be denoted by P_{ij} . This means, in the i^{th} row of the same, we have $\cos \theta$ and $-\sin \theta$ in the i^{th} column and j^{th} column respectively. Also, the matrix has $\sin \theta$ and $\cos \theta$ in the j^{th} row in the i^{th} column and j^{th} column respectively.

Step-2

Consider the matrix $P_{ij}A$.

It is clear that all the entries in this matrix will be same as that of A , except in the i^{th} and j^{th} rows.

Thus, if b_{kl} denotes the entry in the k^{th} row and l^{th} column of the matrix $P_{ij}A$, and if a_{kl} denotes the entry in the k^{th} row and l^{th} column of the matrix A , then we get the following:

$$b_{kl} = a_{kl} \text{ when } k \neq i \text{ and } k \neq j$$

Step-3

Let the inverse projection matrix be denoted by P_{ij}^{-1} . This means, in the i^{th} row of the same, we have $\cos \theta$ and $\sin \theta$ in the i^{th} column and j^{th} column respectively. Also, the matrix has $-\sin \theta$ and $\cos \theta$ in the j^{th} row in the i^{th} column and j^{th} column respectively.

Consider the matrix $P_{ij}AP_{ij}^{-1}$.

It is clear that all the entries in this matrix will be same as that of $P_{ij}A$, except in the i^{th} and j^{th} columns.

Thus, if b_{kl} denotes the entry in the k^{th} row and l^{th} column of the matrix $P_{ij}AP_{ij}^{-1}$, and if a_{kl} denotes the entry in the k^{th} row and l^{th} column of the matrix $P_{ij}A$, then we get the following:

$$b_{kl} = a_{kl} \text{ when } l \neq i \text{ and } l \neq j$$