Step-1

Permutation matrix: Matrix P has single 1 in every row and every column. It has the rows of identity matrix I in any order.

Step-2

Let matrix P is a rotation matrix which multiplies to a vector (x,y,z) to give (z,y,x). Determine P and P^3.

Matrix P if multiplied to the vector (x,y,z) puts last column first by shifting other columns. This gives the permutation matrix P defined as follows. Therefore,

$$\boxed{P=\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}}$$

Step-3

Now do the following calculations:

$$P\cdot P=\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}\cdot\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}$$

$$P^2=\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}$$

$$P^2\cdot P=\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}\cdot\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}$$

$$=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$P^3=I$$

Therefore, $\boxed{P^3=I}$.

Step-4

Next consider the rotation axis $a=(1,1,1)$ that doesnâ€™t move but equals to Pa. determine the angle of rotation from $v=(2,3,-5)$ to $Pv=(-5,2,3)$.

Matrix P if multiplied to the vector $v=(2,3,-5)$ puts last column first by shifting other columns. This gives the permutation matrix P as defined above.

Also, above calculation, $\boldsymbol{P}^3 = \boldsymbol{I}$, shows that after taking three rotations matrix equals to identity matrix. This implies that three rotations equals to $\mathbf{360°}$, so one rotation will make an angle of $\mathbf{360°/3 = 120°}$.

Therefore, angle of rotation from *v* to $\boldsymbol{P} \cdot \boldsymbol{v}$ around $\boldsymbol{a} = (\mathbf{1,1,1})$ is $\boxed{\mathbf{120°}}$.