Step-1

Addition rule: let T, W are linear transformations from R^n to R^n

We define $(T+W):\mathrm{R}^n \to \mathrm{R}^n$

By $(T+W)(x)=T(x)+W(x)$, for all $x \in \mathrm{R}^n$ and c is a scalar in R,

$c\mathrm{T}:\mathrm{R}^n \to \mathrm{R}^n$

By $(cT)(x)=c(T(x))$ for all $x \in \mathrm{R}^n$

Thus the set of all $S=\{T/T:\mathrm{R}^n \to \mathrm{R}^n \text{ is a linear transformation}\}$ is a vector space over the field R

$\dim S = n^2$

Step-2

Basis for this $S=\{T_{ij}/T_{ij}:\mathrm{R}^n \to \mathrm{R}^n, 1 \le i, j \le n\}$

When $T_{ij}(\alpha_k)=\begin{cases} 0 & \text{if } k \ne j \\ \alpha_k & \text{if } k = j \end{cases}$

Hence $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ is a basis for R^n