Step-1

By apply Gram-Schmidt to $(1,-1,0),(0,1,-1),(1,0,-1)$, we have to find an orthonormal basis on the plane $x_1+x_2+x_3=0$. We have to find the dimension of this subspace, and we have to find that how many nonzero vectors come out of Gram-Schmidt.

Step-2

Let $S=\{(1,-1,0),(0,1,-1),(1,0,-1)\}$ is a given set.

Given equation of the plane is $x_1+x_2+x_3=0$

Put $x_2=k, x_3=r,$
then $x_1=-x_2-x_3$
$=-k-r$

Step-3

Therefore

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} -k-r \\ k \\ r \end{bmatrix}$$
$$=-k\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}-r\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

Step-4

Therefore $a_1=\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, a_2=\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ are two vectors in the plane $x-y+z=0$

Let $S_1=\{a_1,a_2\}$ be a subset of *S*.

Step-5

Then

$$q_1 = \frac{a_1}{\|a_1\|}$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{bmatrix}$$

Step-6

And

$$q_2 = \frac{\beta}{\|\beta\|} \text{ where}$$

$$\beta = a_2 - \left(q_1^T a_2\right) q_1$$

Step-7

Now

$$q_1^T a_2 = \frac{1}{\sqrt{2}}(1,-1,0)\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}(1+0+0)$$

$$= \frac{1}{\sqrt{2}}$$

Step-8

$$\left(q_1^T a_2\right) q_1 = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

Step-9

$$\beta = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} - \frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} 1/2 \\ 1/2 \\ -1 \end{bmatrix}$$

$$\|\beta\| = \sqrt{\frac{1}{4} + \frac{1}{4} + 1}$$
$$= \sqrt{\frac{6}{4}}$$

$$q_2 = \frac{\beta}{\|\beta\|}$$
$$= \frac{2}{\sqrt{6}}\begin{bmatrix} 1/2 \\ 1/2 \\ -1 \end{bmatrix}$$
$$= \begin{bmatrix} 1/\sqrt{6} \\ 1/\sqrt{6} \\ -2/\sqrt{6} \end{bmatrix}$$

Step-10

Hence the required orthonormal basis $\left\{ \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{bmatrix}, \begin{bmatrix} 1/\sqrt{6} \\ 1/\sqrt{6} \\ -2/\sqrt{6} \end{bmatrix} \right\}$

Dimension of the subspace = $\boxed{2}$

Two nonzero vectors come out of Gram-Schmidt process.