

Step-1

Consider the function,

$$P_1 = \frac{1}{2}x^2 + xy + y^2 - 3y$$

Find the partial derivatives with respect to the variables x and y .

$$\frac{\partial P_1}{\partial x} = x + y,$$

$$\frac{\partial P_1}{\partial y} = x + 2y - 3$$

Step-2

For the extremum values,

$$\frac{\partial P_1}{\partial x} = 0$$

$$x + y = 0 \quad \text{--- (1)}$$

And

$$\frac{\partial P_1}{\partial y} = 0$$

$$x + 2y - 3 = 0$$

$$x + 2y = 3 \quad \text{--- (2)}$$

Solve equations (1) and (2) to obtain,

$$x = -3 \text{ and } y = 3.$$

Now find the value of the expression,

$$\begin{aligned} P_{1xx}P_{1yy} - P_{1xy}^2 &= (1)(2) - 1 \\ &= 1 \\ &> 0 \end{aligned}$$

Thus, P_1 has minimum at $x = -3, y = 3$.

Step-3

Consider the function,

$$P_2 = \frac{1}{2}x^2 - 3y$$

Find the partial derivatives with respect to the variables x and y .

$$\frac{\partial P_2}{\partial x} = x,$$

$$\frac{\partial P_2}{\partial y} = -3 \neq 0$$

So, P_2 has no minimum.

Step-4

Now the matrix associated with P_2 is given,

$$A = \begin{pmatrix} \frac{\partial^2 P_2}{\partial x^2} & \frac{\partial^2 P_2}{\partial x \partial y} \\ \frac{\partial^2 P_2}{\partial y \partial x} & \frac{\partial^2 P_2}{\partial y^2} \end{pmatrix} \\ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}.$$

Thus, the semi definite matrix associated with P_2 is,

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}.$$