Step-1

We have to find L and U for the non-symmetric matrix

$$A=\begin{pmatrix} a & r & r & r \\ a & b & s & s \\ a & b & c & t \\ a & b & c & d \end{pmatrix}$$

And also we have to find the four conditions on a,b,c,d,r,s,t to get $A=LU$ with four pivots.

Step-2

Subtracting row 1 from row 2 and row 3 and row 4 gives

$$\begin{pmatrix} a & r & r & r \\ 0 & b-r & s-r & s-r \\ 0 & b-r & c-r & t-r \\ 0 & b-r & c-r & d-r \end{pmatrix}$$

Subtracting row 2 from row 3 and row 4 gives

$$\begin{pmatrix} a & r & r & r \\ 0 & b-r & s-r & s-r \\ 0 & 0 & c-s & t-s \\ 0 & 0 & c-s & d-s \end{pmatrix}$$

Step-3

Subtracting row 3 from row 4 gives

$$\begin{pmatrix} a & r & r & r \\ 0 & b-r & s-r & s-r \\ 0 & 0 & c-s & t-s \\ 0 & 0 & 0 & d-t \end{pmatrix}=U$$

Step-4

The lower triangular matrix is

$$L=\begin{bmatrix}1 & 0 & 0 & 0\\ l_{21} & 1 & 0 & 0\\ l_{31} & l_{32} & 1 & 0\\ l_{41} & l_{42} & l_{43} & 1\end{bmatrix}$$

Fix $l_{21}=1$ for the row operation, subtracting row 1 from row 2, so from the above row operations we get $l_{21}=l_{31}=l_{41}=1,\ l_{32}=l_{42}=1, l_{43}=1$

Step-5

Therefore

$$L=\begin{bmatrix}1 & 0 & 0 & 0\\ 1 & 1 & 0 & 0\\ 1 & 1 & 1 & 0\\ 1 & 1 & 1 & 1\end{bmatrix}$$

We can observe that

$$A=\begin{pmatrix}a & r & r & r\\ a & b & s & s\\ a & b & c & t\\ a & b & c & d\end{pmatrix}=\begin{bmatrix}1 & 0 & 0 & 0\\ 1 & 1 & 0 & 0\\ 1 & 1 & 1 & 0\\ 1 & 1 & 1 & 1\end{bmatrix}\begin{pmatrix}a & r & r & r\\ 0 & b-r & s-r & s-r\\ 0 & 0 & c-s & t-s\\ 0 & 0 & 0 & d-t\end{pmatrix}$$

The four pivots are $a, b-r, c-s, d-t$ (the diagonal elements of U) and the factorization is possible if $a\neq 0, b\neq r, c\neq s, d\neq t$