Step-1

Given points are;

$y = 2$ at $t = -1$
$y = 0$ at $t = 0$
$y = -3$ at $t = 1$
$y = -5$ at $t = 2$

To find the best straight line fit to the given measurements.

Consider the equation of line $y = C + Dt$

Step-2

First write the equations that would hold if a line could go through all four points.

Then every $C + Dt$ would agree exactly with *y*.

Now $Ax = b$ is;

$$C + D(-1) = 2$$
$$C + D(0) = 0$$
$$C + D(1) = -3$$
$$C + D(2) = -5$$

Step-3

The matrix from of the given system is;

$$\begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -3 \\ -5 \end{bmatrix}$$

Here,

$$A=\begin{bmatrix}1 & -1\\ 1 & 0\\ 1 & 1\\ 1 & 2\end{bmatrix}$$

$x=\begin{bmatrix}C\\ D\end{bmatrix}$ and

$$b=\begin{bmatrix}2\\ 0\\ -3\\ -5\end{bmatrix}$$

Step-4

Since, the least-squares solution is given by;

$$A^T A\hat{x}=A^T b$$

This implies;

$$\begin{bmatrix}1 & 1 & 1 & 1\\ -1 & 0 & 1 & 2\end{bmatrix}\begin{bmatrix}1 & -1\\ 1 & 0\\ 1 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}\widehat{C}\\ \widehat{D}\end{bmatrix}=\begin{bmatrix}1 & 1 & 1 & 1\\ -1 & 0 & 1 & 2\end{bmatrix}\begin{bmatrix}2\\ 0\\ -3\\ -5\end{bmatrix}$$

$$\begin{bmatrix}\begin{pmatrix}1(1)+1(1)\\ +1(1)+1(1)\end{pmatrix} & \begin{pmatrix}1(-1)+1(0)\\ +1(1)+1(2)\end{pmatrix}\\ \begin{pmatrix}-1(1)+0(1)\\ +1(1)+2(1)\end{pmatrix} & \begin{pmatrix}-1(-1)+0(0)\\ +1(1)+2(2)\end{pmatrix}\end{bmatrix}\begin{bmatrix}\widehat{C}\\ \widehat{D}\end{bmatrix}=\begin{bmatrix}\begin{pmatrix}1(2)+1(0)\\ +1(-3)+1(-5)\end{pmatrix}\\ \begin{pmatrix}-1(2)+0(0)\\ +1(-3)+2(-5)\end{pmatrix}\end{bmatrix}$$

$$\begin{bmatrix}4 & 2\\ 2 & 6\end{bmatrix}\begin{bmatrix}\widehat{C}\\ \widehat{D}\end{bmatrix}=\begin{bmatrix}-6\\ -15\end{bmatrix}$$

Step-5

Now, the required system is:

$$4\widehat{C}+2\widehat{D}=-6$$
$$2\widehat{C}+6\widehat{D}=-15$$

Solving these two equations and get:

$\widehat{D}=-\frac{24}{10}$ and $\widehat{C}=-\frac{3}{10}$

Therefore $\hat{x} = \begin{bmatrix} -\frac{24}{10} \\ -\frac{3}{10} \end{bmatrix}$

Hence, the best line is $\boxed{y = \frac{-24}{10} - \frac{3}{10}t}$.

Step-6

The graph of the solution can be found by calculating the intercept points as below,

Step-7

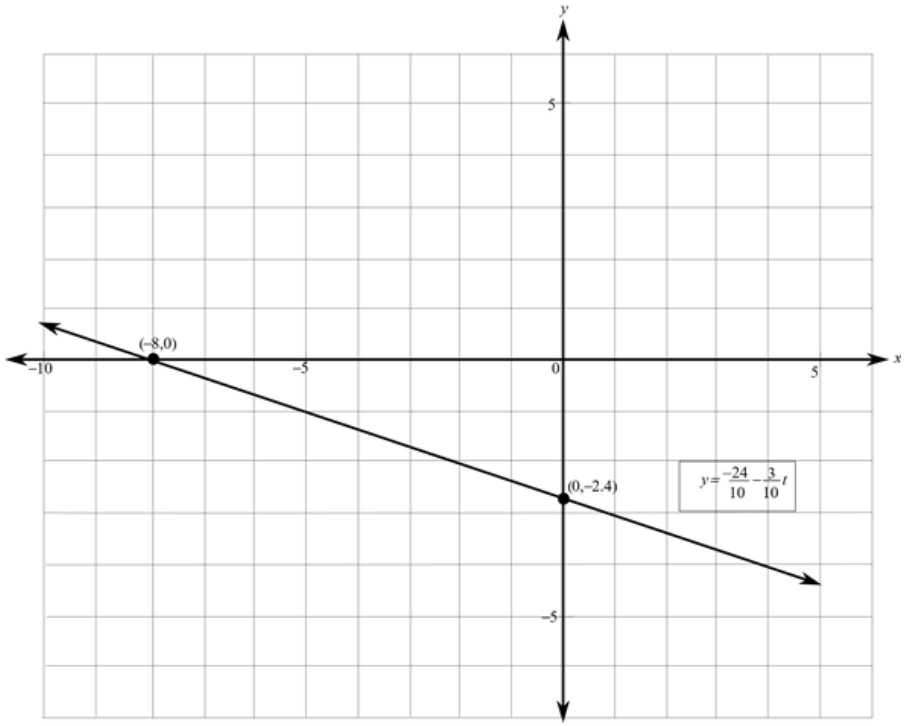
For *t* intercept, put $y = 0$ in the equation $y = \frac{-24}{10} - \frac{3}{10}t$,

$$\begin{aligned} 0 &= \frac{-24}{10} - \frac{3}{10}t \\ \frac{3}{10}t &= \frac{-24}{10} \\ t &= \frac{-24}{10} \times \frac{10}{3} \\ &= -8 \end{aligned}$$

For *y* intercept, put $t = 0$ in the equation $y = \frac{-24}{10} - \frac{3}{10}t$,

$$\begin{aligned} y &= \frac{-24}{10} - \frac{3}{10} \times 0 \\ &= \frac{-24}{10} \\ &= -2.4 \end{aligned}$$

The sketch of the best straight line fit to the given measurements is shown below.


y
x
5
−5
−10
0
(−8,0)
(0,−2.4)
$y = \frac{-24}{10} - \frac{3}{10}t$