Step-1

We have to show that set of nonsingular 2 by 2 matrices is not a vector space. Also we have to show that singular 2 by 2 matrices is not a vector space.

Step-2

Let A be the set of all nonsingular 2 by 2 matrices.

i.e., $A=\left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} / ad-bc \neq 0, a,b,c,d \in \mathbf{R}\right\}$

Let $\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 7 & 8 \end{bmatrix} \in A$

Then both are non singular matrices.

Now $\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}+\begin{bmatrix} 0 & -1 \\ 7 & 8 \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 9 & 11 \end{bmatrix}$ is a singular matrix

Therefore vector addition is not closed in A.

Hence A is not a vector space.

Step-3

Let B be the set of all singular 2 by 2 matrices.

i.e., $B=\left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} / a,b,c,d \in \mathbf{R} \text{ and } ad-bc=0\right\}$

Let $\begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 4 \\ 0 & 0 \end{bmatrix} \in B$

Then both are singular matrices.

Step-4

Now

$$\begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix}+\begin{bmatrix} 3 & 4 \\ 0 & 0 \end{bmatrix}=\begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$$

Here,

$$ad - bc = 3.2 - 4.1$$
$$= 2 \neq 0$$

Therefore, $\begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$ is a nonsingular does not belonging to B.

Here B is not closed under vector addition.

Hence, B is not a vector space.