

Step-1

When equation 1 is added to the equation 2, then we have to find that which of the following are changed.

The planes in the row picture, the column picture, the coefficient matrix and the solution.

Step-2

For example, a system of four planes is

$$\begin{aligned}x + y + z + t &= 3 \\y + z + t &= 3 \\z + t &= 3 \\t &= 2\end{aligned}$$

The column picture for this system is

$$x \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + y \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + z \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 3 \\ 2 \end{pmatrix}$$

Step-3

The coefficient matrix $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

From the system, we can observe that $0(\text{column1}) + 0(\text{column1}) + 1(\text{column3}) + 2(\text{column4})$ gives the right-hand side vector.

So the solution of the above planes is $(0, 0, 1, 2)$

Step-4

When equation1 is added to equation 2 gives, we have the following system

$$\begin{aligned}x + 2y + 2z + 2t &= 6 \\z + t &= 3 \\t &= 2\end{aligned}$$

Column picture is

$$x \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + y \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} + z \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix}$$

Step-5

The coefficient matrix is $\begin{pmatrix} 1 & 2 & 2 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$

From the column picture, we observe that $0(\text{column1}) + 0(\text{column1}) + 1(\text{column3}) + 2(\text{column4})$ gives the right-hand side vector.

The solution matrix is $(0, 0, 1, 2)$.

Step-6

In the first system we have 4 planes where as in the second system we have 3 planes, so the planes in the row picture changed.

In the first system we have 4 four dimensional columns where as in the second system we have 4 three dimensional columns, so the column picture changed.

The coefficient matrix for the two system is changed

And the solution $(0, 0, 1, 2)$ not changed.