Step-1

Suppose $A=\begin{bmatrix} a_{11} & a_{12} & - & - & a_{1n} \\ a_{21} & a_{22} & - & - & a_{2n} \\ - & - & - & - & - \\ - & - & - & - & - \\ a_{m1} & a_{m2} & - & - & a_{mn} \end{bmatrix}_{m\times n}$ and $B=\begin{bmatrix} b_{11} & b_{12} & - & - & b_{1k} \\ b_{21} & b_{22} & - & - & a_{2k} \\ - & - & - & - & - \\ - & - & - & - & - \\ b_{n1} & b_{n2} & - & - & b_{nk} \end{bmatrix}_{n\times k}$ are non zero matrices such that *AB* = **0**

This can conveniently be written as $A\begin{bmatrix} b_1 & b_2 & - & - & b_k \end{bmatrix}=\begin{bmatrix} 0 & 0 & - & - & 0 \end{bmatrix}$ where each b_i denotes the column of the matrix *B*. $1\le i\le k$

Or, $\begin{bmatrix} Ab_1 & Ab_2 & - & - & Ab_k \end{bmatrix}=\begin{bmatrix} 0 & 0 & - & - & 0 \end{bmatrix}$

Or, $Ab_i=0,1\le i\le k$ â€¦â€¦ (1)

Step-2

On the other hand, if $Ax=0$ for any column *x*, then we say that *x* is in the null space of *A*.

Using this property on (1), we say that the columns of *B* are in the null space of *A*.

In other words, the column space of *B* is a subspace of the null space of *A*. â€¦â€¦ (2)

Step-3

The equation *AB* = 0 can otherwise be written as

$$\begin{bmatrix} a_1 \\ a_2 \\ - \\ - \\ a_m \end{bmatrix} B=\begin{bmatrix} 0 \\ 0 \\ - \\ - \\ 0 \end{bmatrix}$$

In other words, $a_jB=0,1\le j\le m$, a_j is the j^{th} row of the matrix *A*.

That is the j^{th} row of *A* multiplied with every column of *B* gives the zero row.

In other words, every row of *A* is in the left null space of *B*

That is the row space of *A* is in the left null space of *B*.