Step-1

Consider the matrix: $A=\begin{bmatrix}1&0&0&1\\0&1&1&1\\1&1&0&1\\1&0&0&1\end{bmatrix}$.

Let the matrix be $A=\begin{bmatrix}a_{11}&a_{12}&a_{13}&a_{14}\\a_{21}&a_{22}&a_{23}&a_{24}\\a_{31}&a_{32}&a_{33}&a_{34}\\a_{41}&a_{42}&a_{43}&a_{44}\end{bmatrix}$.

$$\det A=\sum_{all\ P's}\left(a_{1\alpha}a_{2\beta}\ldots a_{n\upsilon}\right)\det P$$

This can be expanded as follows.

$$\begin{vmatrix}a_{11}&a_{12}&a_{13}&a_{14}\\a_{21}&a_{22}&a_{23}&a_{24}\\a_{31}&a_{32}&a_{33}&a_{34}\\a_{41}&a_{42}&a_{43}&a_{44}\end{vmatrix}=a_{11}\begin{vmatrix}a_{22}&a_{23}&a_{24}\\a_{32}&a_{33}&a_{34}\\a_{42}&a_{43}&a_{44}\end{vmatrix}-a_{12}\begin{vmatrix}a_{21}&a_{23}&a_{24}\\a_{31}&a_{33}&a_{34}\\a_{41}&a_{43}&a_{44}\end{vmatrix}$$
$$+a_{13}\begin{vmatrix}a_{21}&a_{22}&a_{24}\\a_{31}&a_{32}&a_{34}\\a_{41}&a_{42}&a_{44}\end{vmatrix}-a_{14}\begin{vmatrix}a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\\a_{41}&a_{42}&a_{43}\end{vmatrix}$$

Step-2

Now, apply the formula on the determinant of the matrix A.

$$\begin{aligned}\begin{vmatrix}1&0&0&1\\0&1&1&1\\1&1&0&1\\1&0&0&1\end{vmatrix}&=(1)\begin{vmatrix}1&1&1\\1&0&1\\0&0&1\end{vmatrix}-(0)\begin{vmatrix}0&1&1\\1&0&1\\1&0&1\end{vmatrix}+(0)\begin{vmatrix}0&1&1\\1&1&1\\1&0&1\end{vmatrix}-(1)\begin{vmatrix}0&1&1\\1&1&0\\1&0&0\end{vmatrix}\\&=(1)(-1)-0+0-(1)(-1)\\&=-1+1\\&=0\end{aligned}$$

So, the non-zero term is $-a_{11}a_{23}a_{32}a_{44}+a_{44}a_{23}a_{32}a_{41}=(1)\begin{vmatrix}1&1&1\\1&0&1\\0&0&1\end{vmatrix}-(1)\begin{vmatrix}0&1&1\\1&1&0\\1&0&0\end{vmatrix}$.

Also note that last column of A is sum of 1st and 3rd columns of A and 1st row, last row

of A are identical.

Step-3

Let $B = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 3 & 4 & 5 \\ 5 & 4 & 0 & 3 \\ 2 & 0 & 0 & 1 \end{bmatrix}$

Observe that the matrix B also have zero in exactly same places as A and hence exactly two none zero terms occur in the big formula.

They are $-b_{11}b_{23}b_{32}b_{44} + b_{44}b_{23}b_{32}b_{41}$

Hence, the required determinant of the matrix is

$$\begin{aligned} \det B &= -1.4.4.1 + 2.4.4.2 \\ &= -16 + 64 \\ &= 48 \end{aligned}$$

From the above results, the determinant of A is $-1+1$ and also $\boxed{\det B = 48}$.