Step-1

A matrix is Hermitian matrix if $A = A^H$.

If U is a unitary matrix then following is true:

$UU^H = I$

Columns of the unitary matrix are formed by orthonormal vectors.

Step-2

If $u^H u = 1$ then to show that following is Hermitian and unitary:

$I - 2uu^H$

Also, find that rank-1 matrix uu^H is the projection onto what line in $\mathbf{C}^n$.

Step-3

Do the following calculations:

$$\begin{aligned}\left(uu^H\right)^H &= \left(u^H\right)^H u^H \\ &= uu^H\end{aligned}$$

Now, check for the $I - 2uu^H$ matrix:

$$\begin{aligned}\left(I - 2uu^H\right)^H &= \left(I - 2\left(uu^H\right)^H\right) \\ &= I - 2uu^H\end{aligned}$$

Above calculations shows that $I - 2uu^H$ is Hermitian matrix.

Step-4

Matrix $I - 2uu^H$ will be unitary if:

$$\left(I - 2uu^H\right)^H \cdot \left(I - 2uu^H\right) = I.$$

Do the following calculations:

$$
\begin{aligned}
\left(I-2uu^H\right)^H \cdot \left(I-2uu^H\right) &= \left(I-2uu^H\right)\cdot\left(I-2uu^H\right) \\
&= \left(I-2uu^H\right)^2 \\
&= I+4\left(uu^H\right)^2-4\left(I\cdot uu^H\right) \\
&= I-4\left(uu^H\right)+4\left(uu^H\cdot uu^H\right) \\
&= I-4\left(uu^H\right)+4\left(u\cdot 1\cdot u^H\right) \\
&= I-4\left(uu^H\right)+4\left(uu^H\right)
\end{aligned}
$$

$$\left(I-2uu^H\right)^H \cdot \left(I-2uu^H\right) = I$$

Above calculations shows that $I-2uu^H$ is unitary matrix.

Step-5

Therefore, matrix $\boxed{I-2uu^H}$ is Hermitian and unitary. Rank-1 matrix uu^H is the projection onto the line in $\mathbf{C}^n$ through u.