Step-1

Consider the following matrix:

$$A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Then

$$\det(A_1) = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$
$$= 1$$

Since $\det(A_1) \neq 0,$ so the matrix A_1 is non-singular.

Thus, the matrix A_1 has inverse.

The object is to find the inverse of the given matrix using Gauss-Jordan method.

Step-2

Using the Gauss-Jordan Method to Find A_1^{-1}.

$$[A_1 \quad I] = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} R_2 \to R_2 - R_1$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} R_2 \to R_2 - R_3$$

$$= \begin{bmatrix} I & A_1^{-1} \end{bmatrix}$$

Therefore, the inverse of the matrix A_1 is,

$$A_1^{-1} = \boxed{\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}}.$$

Step-3

Consider the following matrix:

$$A_2 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

Then

$$\det(A_2) = \begin{vmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{vmatrix}$$
$$= 4$$

Since $\det(A_2) \neq 0,$ so the matrix A_2 is non-singular.

Thus, the matrix A_2 has inverse.

The object is to find the inverse of the given matrix using Gauss-Jordan method.

Step-4

Using the Gauss-Jordan Method to Find A_2^{-1}.

$$[A_2 \quad I] = \begin{bmatrix} 2 & -1 & 0 & 1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 1 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1/2 & 0 & 1/2 & 0 & 0 \\ -1 & 2 & -1 & 0 & 1 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{bmatrix} R_1 \to \frac{1}{2} R_1$$

$$\sim \begin{bmatrix} 1 & -1/2 & 0 & 1/2 & 0 & 0 \\ 0 & 3/2 & -1 & 1/2 & 1 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{bmatrix} R_2 \to R_2 + R_1$$

$$\sim\begin{bmatrix} 1 & -1/2 & 0 & 1/2 & 0 & 0 \\ 0 & 1 & -2/3 & 1/3 & 2/3 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{bmatrix} R_2 \to \frac{2}{3}R_2$$

Step-5

Continuing the previous steps as follows:

$$[A_2 \quad I] \sim \begin{bmatrix} 1 & 0 & -1/3 & 2/3 & 1/3 & 0 \\ 0 & 1 & -2/3 & 1/3 & 2/3 & 0 \\ 0 & 0 & 4/3 & 1/3 & 2/3 & 1 \end{bmatrix} R_1 \to R_1 + \frac{1}{2}R_2, R_3 \to R_3 + R_2$$

$$\sim\begin{bmatrix} 1 & 0 & -1/3 & 2/3 & 1/3 & 0 \\ 0 & 1 & -2/3 & 1/3 & 2/3 & 0 \\ 0 & 0 & 1 & 1/4 & 1/2 & 3/4 \end{bmatrix} R_3 \to \frac{3}{4}R_3$$

$$\sim\begin{bmatrix} 1 & 0 & 0 & 3/4 & 1/2 & 1/4 \\ 0 & 1 & 0 & 1/2 & 1 & 1/2 \\ 0 & 0 & 1 & 1/4 & 1/2 & 3/4 \end{bmatrix} R_1 \to R_1 + \frac{1}{3}R_3, R_2 \to R_2 + \frac{2}{3}R_3$$

$$= \begin{bmatrix} I & A_2^{-1} \end{bmatrix}$$

Step-6

Therefore, the inverse of the matrix A_2 is,

$$A_2^{-1} = \boxed{\begin{bmatrix} 3/4 & 1/2 & 1/4 \\ 1/2 & 1 & 1/2 \\ 1/4 & 1/2 & 3/4 \end{bmatrix}}.$$

Step-7

Consider the following matrix:

$$A_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Then

$$\det(A_3) = \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$
$$= -1$$

Since $\det(A_3) \neq 0,$ so the matrix A_3 is non-singular.

Thus, the matrix A_3 has inverse.

The object is to find the inverse of the given matrix using Gauss-Jordan method.

Step-8

Using the Gauss-Jordan Method to Find A_3^{-1}.

$$[A_3 \quad I] = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} R_3 \leftrightarrow R_1$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} R_1 \to R_1 - R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} R_2 \to R_2 - R_3$$

$$= \begin{bmatrix} I & A_1^{-1} \end{bmatrix}$$

Therefore, the inverse of the matrix A_3 is,

$$A_3^{-1} = \boxed{\begin{bmatrix} 0 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}}.$$