Step-1

The objective is to find the submatrix S from the pivot rows and pivot columns of each A.

The first matrix is $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 4 \end{bmatrix}$.

Subtract row 1 from row 2 to get the reduced matrix.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

From the reduced form, observe that the pivot columns are the first and third columns and the pivot rows are the first and second rows.

The rank of A is 2.

Thus, the invertible 2×2 submatrix of A is $\begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix}$.

Step-2

The second matrix is $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$

Subtract 2 times of row 1 from row 2 to get the reduced matrix.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}.$$

From the reduced form, observe that the first column is the only pivot column and the first row is the only pivot row.

Thus, the rank of A is 1 and the invertible 1×1 submatrix consists of the first column of first row.

That is, [1].

Step-3

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The third matrix is

The pivot columns of A are the second and third columns and the pivot rows of A are the first and third rows.

Thus, the rank of A is 2.

The invertible 2×2 submatrix consists of the second and third columns of the first and third rows.