Step-1

Complex inner product: $u^H v = \overline{u}_1 v_1 + \cdots + \overline{u}_n v_n$.

Orthogonality: $u^H v = 0$

Step-2

Consider the following vectors:

$$v = (1, i, 1)$$
$$w = (i, 1, 0)$$
$$z = (a, b, c)$$

These vectors form an orthogonal basis. Find vector *z*.

Step-3

If these vectors form an orthogonal basis then their inner product will be zero. Calculate the inner product of these vectors:

$$\begin{aligned} v \cdot w &= 1 \cdot i + (-i) \cdot 1 + 1 \cdot 0 \\ &= i - i + 0 \\ &= 0 \end{aligned}$$

Step-4

Similarly, inner product of other two vectors must also be zero.

$$w \cdot z = 0$$
$$v \cdot z = 0$$

Calculate the following:

$$\begin{aligned} w \cdot z &= (-i) \cdot a + 1 \cdot b + 0 \cdot c \\ &= -ia + b \\ v \cdot z &= 1 \cdot a + (-i) \cdot b + 1 \cdot c \\ &= a - ib + c \end{aligned}$$

Step-5

On solving following results are obtained:

$$
\begin{aligned}
w\cdot z &= 0 \\
-ia+b &= 0 \\
b &= ia
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
v\cdot z &= 0 \\
a-ib+c &= 0 \\
a+a+c &= 0 \\
2a+c &= 0
\end{aligned}
$$

Put $a=1$ in the above results. Following values are obtained:

$$
\begin{aligned}
b &= i \\
c &= -2
\end{aligned}
$$

Step-6

Therefore, vector is $\boxed{z=(1,i,-2)}$. All these three vectors *v*, *w*, and *z* are an orthogonal basis for $\boxed{\mathbf{C}^3}$.