

Step-1

(a) Given that $a_{ij} = i - j$

Let

$$A = [a_{ij}]_{3 \times 3}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$A = \begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix} \quad (\because a_{ij} = i - j)$$

Step-2

Given that $b_{ij} = \frac{i}{j}$

Let

$$B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 2 & 1 & \frac{2}{3} \\ 3 & \frac{3}{2} & 1 \end{pmatrix} \quad \left(\because b_{ij} = \frac{i}{j} \right)$$

Step-3

(b)

$$A.B = \begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 2 & 1 & \frac{2}{3} \\ 3 & \frac{3}{2} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0-2-6 & 0-1-3 & 0-\frac{2}{3}-2 \\ 1+0-3 & \frac{1}{2}+0-\frac{3}{2} & \frac{1}{3}+0-1 \\ 2+2+0 & 1+1+0 & \frac{2}{3}+\frac{2}{3}+0 \end{pmatrix}$$

$$= \boxed{\begin{pmatrix} -8 & -4 & -\frac{8}{3} \\ -2 & -1 & -\frac{2}{3} \\ 4 & 2 & \frac{4}{3} \end{pmatrix}}$$

Step-4

$$B.A = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 2 & 1 & \frac{2}{3} \\ 3 & \frac{3}{2} & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0+\frac{1}{2}+\frac{2}{3} & -1+0+\frac{1}{3} & -2-\frac{1}{2}+0 \\ 0+1+\frac{4}{3} & -2+0+\frac{2}{3} & -4-1+0 \\ 0+\frac{3}{2}+2 & -3+0+1 & -6-\frac{3}{2}+0 \end{pmatrix}$$

$$= \boxed{\begin{pmatrix} \frac{7}{6} & -\frac{2}{3} & -\frac{5}{2} \\ \frac{7}{3} & -\frac{4}{3} & -5 \\ \frac{7}{2} & -2 & -\frac{15}{2} \end{pmatrix}}$$

Step-5

$$A^2 = A.A$$

$$= \begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0-1-4 & 0+0-2 & 0+1+0 \\ 0+0-2 & -1+0-1 & -2+0+0 \\ 0+1+0 & -2+0+0 & -4-1+0 \end{pmatrix}$$

Step-6

$$= \boxed{\begin{pmatrix} -5 & -2 & 1 \\ -2 & -2 & -2 \\ 1 & -2 & -5 \end{pmatrix}}$$