Step-1

Consider $F(x,y)=x^2+10xy+10y^2$.

Compare this with $a^2+2bxy+cy^2$,

So, $a=1,\ 2b=10,\ c=10$

So that $a+c=10+1=11$ and $b=5$

Clearly $a+c>2b$ and $ac=10,\ b^2=25$

Step-2

Thus the required matrix is $\begin{pmatrix} a & b \\ b & c \end{pmatrix}=\begin{pmatrix} 1 & 5 \\ 5 & 10 \end{pmatrix}$

So,

$ac-b^2<0$

$\Rightarrow 10-25<0$

$\Rightarrow -15<0$

The matrix is $\boxed{\text{not positive definite.}}$