Step-1

If we take powers of a permutation, then we have to explain that why is some P^k eventually equal to *I,* and we have to find a 5 by 5 permutation *P* so that the smallest power to equal *I* is P^6

Step-2

If the order of matrix is *n* then, there exist $n!$ permutation matrices

Eventually two powers of permutation matrix *P* must be the same.

If $P^r = P^s$ then $P^{r-s} = I$

$\Rightarrow r - s \leq n!$

Step-3

Let $P = \begin{pmatrix} P_2 & 0 \\ 0 & P_3 \end{pmatrix}$ in 5 by 5 matrices with

$P_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $P_3 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

Step-4

Then

$$P = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Then by using CAS, we get $P^6 = I$

Step-5

So the required *P* is

$$P=\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$