## Step-1

If every row of $A$ adds to zero, we have to prove that $\det A = 0$, and if every row adds to 1 , we have to prove that $\det(A - I) = 0$. Also we have to show by an example that this does not imply $\det A = 1$.

## Step-2

Given that every row of $A$ adds to zero.

(That is sum of entries in every row is zero)

We shall prove that $\det A = 0$

Now doing the row operation of adding all remaining rows to 1st row of $A^T$ we get a matrix $B$ consisting of all zeros in 1st row.

## Step-3

We know that addition or subtraction of row to another row donâ€™t alter the value of determinant.

Hence $\det A^T = \det B = 0$

And we know that $\det A = \det A^T$

Therefore $\det A = 0$.

## Step-4

Next we suppose that every row of $A$ adds to 1.

Write $A - I = B$ then

$$B = \begin{bmatrix} a_{11}-1 & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22}-1 & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn}-1 \end{bmatrix}$$

We can observe that every row of $B$ adds to zero. From the first part, we have $\det B = 0$ Hence $\det(A - I) = 0$

## Step-5

Now consider

$A = \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$, then

$$A - I = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}$$

## Step-6

Now

$$\det(A)$$
$$= 1 - 4$$
$$= -3$$

## Step-7

And

$$\det(A - I)$$
$$= 4 - 4$$
$$= 0$$

So $\det(A - I) = 0$ does not imply $\det A = 1$