Step-1

4764-1.6-55P AID: 124

RID: 232 | 28/1/2012

We have to explain why the inner product of x and y equals the inner product of Px and Py .

Step-2

Let A be any matrix.

Now

$$\begin{aligned}(Px)^T A(Py) &= x^T P^T APy \\ &= x^T \left(P^T AP\right) y \\ &= x^T Ay \qquad \begin{pmatrix}\text{Since } P^T AP = A, \text{ for} \\ \text{any permutation matrix}\end{pmatrix}\end{aligned}$$

Hence the inner product of x and y equals the inner product of Px and Py .

Step-3

Given $x=(1,2,3)$ and $y=(1,4,2)$

We have to choose the permutation matrix P to show that $(Px)^T y$ is always not equal to $x^T\left(P^T y\right)$

Let the permutation matrix P be $P=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 0 & 0\end{bmatrix}$

Given $x=\begin{bmatrix}1\\2\\3\end{bmatrix}$ and $y=\begin{bmatrix}1\\4\\2\end{bmatrix}$

Then $x^T=\begin{bmatrix}1 & 2 & 3\end{bmatrix}$ and $P^T=\begin{bmatrix}0 & 0 & 1\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}$

Step-4

Now

$$Px = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$
$$= \begin{bmatrix} 0(1)+1(2)+0(3) \\ 0(1)+0(2)+1(3) \\ 1(1)+0(2)+0(3) \end{bmatrix}$$
$$= \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

Therefore, $(Px)^T = [2 \quad 3 \quad 1]$

Step-5

Now

$$(Px)^T y = [2 \quad 3 \quad 1] \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$$
$$= [2(1)+3(4)+1(2)]$$
$$= [2+12+2]$$
$$= [16]$$

Step-6

Now we find $x^T (P^T y)$

So

$$P^T y = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$$
$$= \begin{bmatrix} 0(1)+0(4)+1(2) \\ 1(1)+0(4)+0(2) \\ 0(1)+1(4)+0(2) \end{bmatrix}$$
$$= \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

Step-7

Now

$$x^T \left(P^T y\right) = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$
$$= \left[1(2)+2(1)+3(4)\right]$$
$$= \left[2+2+12\right]$$
$$= \left[16\right]$$

Hence thereexists no P such that $\left(Px\right)^T y \neq x^T \left(P^T y\right)$.