Step-1

Let us consider the following vectors

$$\boldsymbol{A}=\begin{bmatrix}0&0&1&0\\0&1&0&0\\1&1&1&1\\1&0&0&1\end{bmatrix},\quad \boldsymbol{b}=\begin{bmatrix}1\\1\\1\\1\end{bmatrix},\text{ and }\quad \boldsymbol{c}=\begin{bmatrix}1\\1\\1\\3\end{bmatrix}$$

Therefore, the corresponding primal of the LPP is as follows

Minimize:

$x_1+x_2+x_3+3x_4$ â€¦â€¦ (1)

Subject to following constraints, along with non-negativity constraints

$$x_3\geq 1$$
$$x_2\geq 1$$
$$x_1+x_2+x_3+x_4\geq 1$$
$$x_1+x_4\geq 1$$

Step-2

Solving the above constraints, we get the following possible vectors

$$x_1=1$$
$$x_2=1$$
$$x_3=1$$
$$x_4=0$$

OR

$$x_1=0$$
$$x_2=1$$
$$x_3=1$$
$$x_4=1$$

Step-3

Since, it is asked to find the minimum value of the objective function (1), the feasible vector is

$$\boxed{\boldsymbol{x}=\begin{bmatrix}1&1&1&0\end{bmatrix}}$$

Step-4

Now, the corresponding dual of the LPP is as follows

Maximize:

$y_1 + y_2 + y_3 + y_4$ â€¦â€¦ (2)

Subject to following constraints, along with non-negativity constraints

$$y_3 + y_4 \le 1$$
$$y_2 + y_3 \le 1$$
$$y_1 + y_3 \le 1$$
$$y_3 + y_4 \le 3$$

Step-5

Solving the above constraints, we get the following possible vector

$$y_1 = 1$$
$$y_2 = 1$$
$$y_3 = 0$$
$$y_4 = 1$$

Thus, the feasible vector is

$$\boxed{y = \begin{bmatrix} 1 & 1 & 0 & 1 \end{bmatrix}}$$

Step-6

Let us calculate the following terms,

$$cx = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 0 \end{bmatrix}$$
$$= Det \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 3 & 3 & 3 & 0 \end{bmatrix}$$
$$= 3$$

Step-7

And

$$\mathbf{yb} = \begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{1} \end{bmatrix} \begin{bmatrix} \mathbf{1} \\ \mathbf{1} \\ \mathbf{1} \\ \mathbf{1} \end{bmatrix}$$

$$= \mathbf{1+1+1}$$

$$= \mathbf{3}$$

Now, according to the property, if the vectors *x* and *y* are feasible and $\mathbf{cx} = \mathbf{yb}$, then *x* and *y* are optimal.

Thus, the value of vectors *x* and *y* are optimal.