Step-1

Consider the matrix equation $AB = C$

The objective is to find a formula for A^{-1}.

Step-2

Multiplying $AB = C$ with A^{-1} both sides on left side gives,

$$A^{-1}AB = A^{-1}C$$
$$A^{-1}AB = A^{-1}C \qquad (\text{Since } A^{-1}A = I)$$
$$IB = A^{-1}C$$
$$B = A^{-1}C \qquad (\text{Since } IB = B)$$

Again, multiplying with C^{-1} on right sides gives

$$BC^{-1} = A^{-1}CC^{-1}$$
$$BC^{-1} = A^{-1}(I) \qquad (\text{Since} CC^{-1} = I)$$
$$BC^{-1} = A^{-1}$$

Hence the formula for A^{-1} is $\boxed{A^{-1} = BC^{-1}}$

Step-3

Consider the matrix equation $PA = LU$

The objective is to find a formula for A^{-1} .

Step-4

Multiplying right side of $PA = LU$ with A^{-1} on both sides obtain,

$$PAA^{-1} = LUA^{-1}$$
$$PI = LUA^{-1} \qquad \text{Since } AA^{-1} = I$$
$$P = LUA^{-1}$$

Multiplying left side of $P = LUA^{-1}$ with $U^{-1}L^{-1}$ on both sides obtain,

$$U^{-1}L^{-1}P = U^{-1}L^{-1}LUA^{-1}$$
$$U^{-1}L^{-1}P = U^{-1}\left(L^{-1}L\right)UA^{-1}$$
$$U^{-1}L^{-1}P = U^{-1}IUA^{-1} \qquad \text{Since } L^{-1}L = I$$
$$U^{-1}L^{-1}P = U^{-1}UA^{-1} \qquad \text{Since } IU = U$$
$$U^{-1}L^{-1}P = IA^{-1} \qquad \text{Since } U^{-1}U = I$$
$$U^{-1}L^{-1}P = A^{-1}$$
$$A^{-1} = U^{-1}L^{-1}P$$

Hence, the formula for A^{-1} is $\boxed{A^{-1} = U^{-1}L^{-1}P}$.