

Step-1

Two vectors x and y are orthogonal if $x^T y = 0$

Let $\alpha = (1, 4)$, $\beta = (2, -2)$

$$\begin{vmatrix} 1 & 2 \\ 4 & -2 \end{vmatrix} = -2 - 8 \\ = -10 \\ \neq 0$$

Therefore the vectors α, β are linearly independent.

$$\text{Also, } \alpha^T \beta = [1 \quad 4] \begin{bmatrix} 2 \\ -2 \end{bmatrix} \\ = 2 - 8 \\ = -6 \\ \neq 0$$

So, $\alpha = (1, 4)$, $\beta = (2, -2)$ are not orthogonal.

Therefore, $\alpha = (1, 4)$, $\beta = (2, -2)$ are linearly independent vectors in R^2 are not orthogonal.

Step-2

On the other hand, suppose $a = (1, 0)$, $b = (0, 0)$

$$a^T b = [1 \quad 0] \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ = 0 + 0 \\ = 0$$

Therefore the vectors a, b are orthogonal.

$$\text{Also, } \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} \\ = 0 - 0 \\ = 0$$

Therefore, the set of orthogonal vectors in \mathbb{R}^2 are not linearly independent.