Step-1

Let P be the point on the plane and on the x_1 axis. Then $P \equiv (3,0,0)$.

Let Q be the point on the plane and on the x_2 axis. Then $Q \equiv (0,3,0)$.

Let R be the point on the plane and on the x_3 axis. Then $R \equiv (0,0,3)$.

Step-2

Therefore, at $P \equiv (3,0,0)$, we get

$$\begin{aligned} c^{\mathrm{T}} x &= 5x_1 + 4x_2 + 8x_3 \\ &= 5(3) + 4(0) + 8(0) \\ &= 15 \end{aligned}$$

Therefore, at $Q \equiv (0,3,0)$, we get

$$\begin{aligned} c^{\mathrm{T}} x &= 5x_1 + 4x_2 + 8x_3 \\ &= 5(0) + 4(3) + 8(0) \\ &= 12 \end{aligned}$$

Therefore, at $R \equiv (0,0,3)$, we get

$$\begin{aligned} c^{\mathrm{T}} x &= 5x_1 + 4x_2 + 8x_3 \\ &= 5(0) + 4(0) + 8(3) \\ &= 24 \end{aligned}$$

Thus, the least cost is $\boxed{12}$.

Step-3

(b) Let us project $c = (5,4,8)$ onto the nullspace of $A = [1\ 1\ 1]$.

The projection matrix is give by, $P = I - A^{\mathrm{T}} \left(AA^{\mathrm{T}}\right)^{-1} A$. Therefore,

$$P = I - \begin{bmatrix}1\\1\\1\end{bmatrix}\left(\begin{bmatrix}1 & 1 & 1\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}\right)^{-1}\begin{bmatrix}1 & 1 & 1\end{bmatrix}$$

$$= I - \begin{bmatrix}1\\1\\1\end{bmatrix}([3])^{-1}\begin{bmatrix}1 & 1 & 1\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} - \begin{bmatrix}\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\end{bmatrix}$$

$$= \begin{bmatrix}\frac{2}{3} & -\frac{1}{3} & -\frac{1}{3}\\-\frac{1}{3} & \frac{2}{3} & -\frac{1}{3}\\-\frac{1}{3} & -\frac{1}{3} & \frac{2}{3}\end{bmatrix}$$

Step-4

Thus, we get

$$Pc = \begin{bmatrix}\frac{2}{3} & -\frac{1}{3} & -\frac{1}{3}\\-\frac{1}{3} & \frac{2}{3} & -\frac{1}{3}\\-\frac{1}{3} & -\frac{1}{3} & \frac{2}{3}\end{bmatrix}\begin{bmatrix}5\\4\\8\end{bmatrix}$$

$$= \begin{bmatrix}-\frac{2}{3}\\-\frac{5}{3}\\\frac{7}{3}\end{bmatrix}$$

The minimum cost obtained was 12.

Let $s = (s_1, s_2, s_3)$. Thus we have

$$0 \le 12 - (s_1, s_2, s_3)\begin{bmatrix} -\frac{2}{3} \\ -\frac{5}{3} \\ \frac{7}{3} \end{bmatrix}$$

$$= 12 - \left(-\frac{2s_1}{3} - \frac{5s_2}{3} + \frac{7s_3}{3}\right)$$

$$= 12 - \left(\frac{7s_3 - 2s_1 - 5s_2}{3}\right)$$

Therefore, $7s_3 - 2s_1 - 5s_2 \le 36$.