

Step-1

Given data is $b = 0$ at $t = 0$

$$b = 1 \text{ at } t = 1$$

$$b = 2 \text{ at } t = 3$$

$$b = 5 \text{ at } t = 4$$

We have to best fit this data with a straight line $C + Dt$.

Step-2

Let $C + Dt = b$ be the required straight line that best fits the given data, then

$$C + 0.D = 0$$

$$C + 1.D = 1$$

$$C + 3.D = 2$$

$$C + 4.D = 5$$

In matrix form,

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 3 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} C \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 5 \end{pmatrix}$$

$$\Rightarrow Ax = b$$

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 3 \\ 1 & 4 \end{bmatrix}, x = \begin{bmatrix} C \\ D \end{bmatrix} \text{ and } b = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 5 \end{bmatrix}$$

where

Step-3

$$\hat{x} = \begin{bmatrix} \hat{C} \\ \hat{D} \end{bmatrix}$$

Let

For least squares fit, we have

$$A^T \hat{A} x = A^T b$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} \hat{C} \\ \hat{D} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 8 \\ 8 & 26 \end{bmatrix} \begin{bmatrix} \hat{C} \\ \hat{D} \end{bmatrix} = \begin{bmatrix} 8 \\ 27 \end{bmatrix}$$

Step-4

apply $R_2 \rightarrow R_2 - 2R_1$

$$\Rightarrow \begin{bmatrix} 4 & 8 \\ 0 & 10 \end{bmatrix} \begin{bmatrix} \hat{C} \\ \hat{D} \end{bmatrix} = \begin{bmatrix} 8 \\ 11 \end{bmatrix}$$

$$\Rightarrow 4\hat{C} + 8\hat{D} = 8 \text{ and } 10\hat{D} = 11$$

$$\Rightarrow \hat{D} = \frac{11}{10} \text{ and}$$

$$\hat{C} = \frac{8 - 8\left(\frac{11}{10}\right)}{4}$$

$$= \frac{-8}{40}$$

$$= \frac{-1}{5}$$

$$\hat{x} = \begin{bmatrix} \frac{-1}{5} \\ \frac{11}{10} \end{bmatrix}$$

Therefore

Hence required straight line $\boxed{b = -\frac{1}{5} + \left(\frac{11}{10}\right)t}$