

Step-1

Given that $A = \begin{pmatrix} 3 & -6 & 0 \\ 0 & 2 & -2 \\ 1 & -1 & -1 \end{pmatrix}, x = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$

We have to find Ax to find the solution x to the system $Ax = O$ and try to find more solutions to this system.

Step-2

$$\begin{aligned} Ax &= \begin{pmatrix} 3 & -6 & 0 \\ 0 & 2 & -2 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \\ &= 2 \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} + 1 \begin{bmatrix} -6 \\ 2 \\ -1 \end{bmatrix} + 1 \begin{bmatrix} 0 \\ -2 \\ -1 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \\ &= O \end{aligned}$$

Therefore $Ax = O$

Step-3

Hence $x = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ is the solution of the equation $Ax = O$

And the remaining solutions are $cx = c \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ where $c \in \mathbb{R}$ is arbitrary constant.