Step-1

Consider the following orthogonal matrix:

$$P=\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\1&0&0&0\end{bmatrix}$$

Find the circulant matrix C that satisfies the following relation:

$$C=c_0I+c_1P+c_2P^2+c_3P^3$$

Also, find the four components of product of vector x and circulant matrix C.

Step-2

Circulant matrix C can be calculated as follows:

$$\begin{aligned}C&=c_0I+c_1P+c_2P^2+c_3P^3\\&=c_0\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}+c_1\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\1&0&0&0\end{bmatrix}+c_2\begin{bmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{bmatrix}+c_3\begin{bmatrix}0&0&0&1\\1&0&0&0\\0&1&0&0\\0&0&1&0\end{bmatrix}\\&=\begin{bmatrix}c_0&0&0&0\\0&c_0&0&0\\0&0&c_0&0\\0&0&0&c_0\end{bmatrix}+\begin{bmatrix}0&c_1&0&0\\0&0&c_1&0\\0&0&0&c_1\\c_1&0&0&0\end{bmatrix}+\begin{bmatrix}0&0&c_2&0\\0&0&0&c_2\\c_2&0&0&0\\0&c_2&0&0\end{bmatrix}+\begin{bmatrix}0&0&0&c_3\\c_3&0&0&0\\0&c_3&0&0\\0&0&c_3&0\end{bmatrix}\\&=\begin{bmatrix}c_0&c_1&c_2&c_3\\c_3&c_0&c_1&c_2\\c_2&c_3&c_0&c_1\\c_1&c_2&c_3&c_0\end{bmatrix}\end{aligned}$$

Therefore, circulant matrix is defined as follows:

$$\boxed{C=\begin{bmatrix}c_0&c_1&c_2&c_3\\c_3&c_0&c_1&c_2\\c_2&c_3&c_0&c_1\\c_1&c_2&c_3&c_0\end{bmatrix}}$$

Step-3

Let $x=(x_0,x_1,x_2,x_3)$ be the vector. Product of circulant matrix and vector *x* will be as follows:

$$Cx=\begin{bmatrix} c_0 & c_1 & c_2 & c_3 \\ c_3 & c_0 & c_1 & c_2 \\ c_2 & c_3 & c_0 & c_1 \\ c_1 & c_2 & c_3 & c_0 \end{bmatrix}\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$=\begin{bmatrix} c_0x_0+c_1x_1+c_2x_2+c_3x_3 \\ c_3x_0+c_0x_1+c_1x_2+c_2x_3 \\ c_2x_0+c_3x_1+c_0x_2+c_1x_3 \\ c_1x_0+c_2x_1+c_3x_2+c_0x_3 \end{bmatrix}$$

Step-4

Therefore, the four components of product of vector *x* and circulant matrix *C* is:

$$\boxed{\begin{array}{l} (c_0x_0+c_1x_1+c_2x_2+c_3x_3),(c_3x_0+c_0x_1+c_1x_2+c_2x_3) \\ (c_2x_0+c_3x_1+c_0x_2+c_1x_3),(c_1x_0+c_2x_1+c_3x_2+c_0x_3) \end{array}}$$