

Step-1

Find the projection of $b = (1, 1, 1)$ onto the plane spanned by $(1, 0, 0)$ and $(1, 1, 0)$.

Let $a_1 = (1, 0, 0)$, $a_2 = (1, 1, 0)$.

P = the projection matrix on the plane

$$= A(A^T A)^{-1} A^T$$

Here

$$A = [a_1 \quad a_2] \\ = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Then $A^T = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}.$

Step-2

Now,

$$A^T A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \\ = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

Then

$$(A^T A)^{-1} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}.$$

Step-3

So,

$$\begin{aligned}
 P &= A(A^T A)^{-1} A^T \\
 &= \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}
 \end{aligned}$$

Step-4

The projection of b onto the plane spanned by a_1, a_2 is,

$$\begin{aligned}
 p &= Pb \\
 &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \\
 &= \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}
 \end{aligned}$$

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

Hence, the required projection is