Step-1

A has eigen values 0, 1, 2

Since the eigen values of A are distinct, the corresponding eigen vectors are linearly independent, which are the columns of the matrix Q and thus, Q is invertible such that

$$A = Q\Lambda Q^{-1}$$

Further, $A^n = Q\Lambda^n Q^{-1}$ â€¦â€¦ (1) where Λ^n stand for the diagonal matrix whose entries are the n^{th} powers of the diagonal entries of Λ.

Step-2

Let us consider $A(A-I)(A-2I) = A^3 - 3A^2 + 2A$

In view of (1), we can write this as $A^3 - 3A^2 + 2A = Q\Lambda^3 Q^{-1} - 3Q\Lambda^2 Q^{-1} + 2Q\Lambda Q^{-1}$

$= Q\left(\Lambda^3 - 3\Lambda^2 + 2\Lambda\right)Q^{-1}$ â€¦â€¦ (2) where $\Lambda = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

So, $\Lambda^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ and $\Lambda^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 8 \end{bmatrix}$

Step-3

Using these in (2) to give

$$A(A-I)(A-2I) = Q\left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 8 \end{bmatrix} - 3\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix} + 2\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}\right)Q^{-1}$$

$$= Q\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}Q^{-1}$$

The diagonal entries of the diagonal matrix are nothing but the eigen values of the matrix on the left hand side.

Therefore, the eigen values of $A(A-I)(A-2I)$ is 0, 0, and 0.