Step-1

Next we consider $v = Aq_1$.

Then we have $h_{1,n} = q_1^{\mathrm{T}} v$

Thus, we have $h_{1,n} = q_1^{\mathrm{T}} Aq_1$.

Step-2

Then we have $v = v - h_{1,n} q_1$

This gives,

$$\begin{aligned} v &= v - h_{1,n} q_1 \\ &= Aq_1 - \left(q_1^{\mathrm{T}} Aq_1\right) q_1 \end{aligned}$$

Step-3

Then we consider $h_{2,1} = \|v\|$. Therefore, $h_{2,1} = \left\| Aq_1 - \left(q_1^{\mathrm{T}} Aq_1\right) q_1 \right\|$.

Finally,

$$\begin{aligned} q_2 &= \frac{v}{h_{2,1}} \\ &= \frac{Aq_1 - \left(q_1^{\mathrm{T}} Aq_1\right) q_1}{\left\| Aq_1 - \left(q_1^{\mathrm{T}} Aq_1\right) q_1 \right\|} \end{aligned}$$

Step-4

Now we need to show that $q_1^{\mathrm{T}} q_2 = 0$.

Consider

$$\begin{aligned} q_1^{\mathrm{T}} \left(Aq_1 - \left(q_1^{\mathrm{T}} Aq_1\right) q_1 \right) &= q_1^{\mathrm{T}} Aq_1 - q_1^{\mathrm{T}} \left(q_1^{\mathrm{T}} Aq_1\right) q_1 \\ &= q_1^{\mathrm{T}} Aq_1 - \left(q_1^{\mathrm{T}} Aq_1\right) q_1^{\mathrm{T}} q_1 \\ &= q_1^{\mathrm{T}} Aq_1 - \left(q_1^{\mathrm{T}} Aq_1\right) \\ &= 0 \end{aligned}$$

Therefore, $q_1^{\mathrm{T}} q_2 = 0$. Thus, q_2 and q_1 are orthogonal vectors.