Step-1

Given m independent measurements of pulse rate are $b_1, \ldots, b_m$ are weighted by $w_1, \ldots, w_m$.

We have to find the weighted average that replaces $\bar{x}_w = \dfrac{w_1^2 b_1 + w_2^2 b_2}{w_1^2 + w_2^2}$.

Step-2

We have the weighted error is $E^2 = w_1^2 (x - b_1)^2 + w_2^2 (x - b_2)^2 + \ldots + w_m^2 (x - b_m)^2$

And the minimizing process is $\dfrac{dE^2}{dx} = 0$

$$\Rightarrow 2w_1^2 (x - b_1) + 2w_2^2 (x - b_2) + \ldots + 2w_m^2 (x - b_m) = 0$$

$$\Rightarrow 2\left[w_1^2 (x - b_1) + w_2^2 (x - b_2) + \ldots + w_m^2 (x - b_m) \right] = 0$$

$$\Rightarrow w_1^2 (x - b_1) + w_2^2 (x - b_2) + \ldots + w_m^2 (x - b_m) = 0$$

Step-3

Continuation to the above

$$\Rightarrow w_1^2 x - w_1^2 b_1 + w_2^2 x - w_2^2 b_2 + \ldots + w_m^2 x - w_m^2 b_m = 0$$

$$\Rightarrow w_1^2 x + w_2^2 x + \ldots + w_m^2 x - \left(w_1^2 b_1 + w_2^2 b_2 + \ldots + w_m^2 b_m \right) = 0$$

$$\Rightarrow x\left(w_1^2 + w_2^2 + \ldots\ldots\ldots\ldots + w_m^2 \right) = w_1^2 b_1 + w_2^2 b_2 + \ldots\ldots\ldots\ldots + w_m^2 b_m$$

$$\Rightarrow x = \frac{w_1^2 b_1 + w_2^2 b_2 + \ldots + w_m^2 b_m}{w_1^2 + w_2^2 + \ldots + w_m^2}$$

Hence the weighted average is $\boxed{x = \bar{x}_w = \dfrac{w_1^2 b_1 + w_2^2 b_2 + \ldots + w_m^2 b_m}{w_1^2 + w_2^2 + \ldots + w_m^2}}$.