Step-1

Then we get $M^{-1}AM$ as follows:

$$M^{-1}AM=\begin{bmatrix}\frac{1}{d} & 0 & 0\\ 0 & \frac{1}{d^2} & 0\\ 0 & 0 & \frac{1}{d^3}\end{bmatrix}\begin{bmatrix}a & b & c\\ e & f & g\\ h & i & j\end{bmatrix}\begin{bmatrix}d & 0 & 0\\ 0 & d^2 & 0\\ 0 & 0 & d^3\end{bmatrix}$$

$$=\begin{bmatrix}\frac{1}{d} & 0 & 0\\ 0 & \frac{1}{d^2} & 0\\ 0 & 0 & \frac{1}{d^3}\end{bmatrix}\begin{bmatrix}ad & bd^2 & cd^3\\ ed & fd^2 & gd^3\\ hd & id^2 & jd^3\end{bmatrix}$$

$$=\begin{bmatrix}a & bd & cd^2\\ \frac{e}{d} & f & gd\\ \frac{h}{d^2} & \frac{i}{d} & j\end{bmatrix}$$

Step-2

The matrix $M^{-1}AM$ is similar to the matrix A. Moreover, the determinant of A is equal to the determinant of $M^{-1}AM$.

We have to obtain the eigenvalues of $M^{-1}AM$, in case $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$.

We know that the similar matrices have the same eigenvalues. Thus, the eigenvalues of $M^{-1}AM$ are same as that of A.

Step-3

Let us obtain the eigenvalues of A. For this, consider $\det(A-\lambda I)=0$.

$$
\begin{aligned}
0 &= \begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix} \\
&= (1-\lambda)^3 + 1 + 1 - 3(1-\lambda) \\
&= 3\lambda^2 - \lambda^3 \\
&= \lambda^2 (3-\lambda)
\end{aligned}
$$

Therefore, the eigenvalues of A are 0 and 3. This gives us that the eigenvalues of $M^{-1}AM$ are 0 and 3.