Step-1

Suppose that A and B are square matrices of order n; and $AB = I$.

The objective is to prove that the rank of A is n, if $\text{rank}(AB) \le \text{rank}(A)$.

Step-2

Since, A, and B are square matrices of order n.

So,

$\text{rank}(A) \le n$ â€¦â€¦ (1)

Since, I identity matrix of order n, then

$$\begin{aligned} n &= \text{rank}(I_n) \\ &= \text{rank}(AB) \\ &\le \min\{\text{rank}(A), \text{rank}(B)\} \\ &= \text{rank}(A) \end{aligned}$$

Therefore,

$n \le \text{rank}(A)$ â€¦â€¦ (2)

From equations, (1), and (2), $\boxed{\text{rank}(A) = n}$.