

Step-1

Given that the columns of a 5 by 5 matrix A are a basis for \mathbf{R}^5 .

Hence the columns are linearly independent and span \mathbf{R}^5 .

Step-2

(a)

Since the columns of A are linearly independent, its determinant is nonzero and hence $Ax = 0$ has only one solution $x = 0$

So the equation $Ax = 0$ has only one solution $x = 0$ because the columns are independent (In fact A is invertible).

Step-3

(b)

If b is in \mathbf{R}^5 then $Ax = b$ is solvable because the columns span \mathbf{R}^5 and b can be expressed as the linear combination of the columns of A .