Step-1

Consider the system,

$$\begin{aligned} 2x+5y+z&=0 \\ 4x+dy+z&=2 \\ y-z&=3 \end{aligned}$$

The objective is to find the number d forces a row exchange, and what is the triangular system (not singular) for that d. Which d makes this system singular (no third pivot).

Step-2

Write the system in matrix notation as shown:

$$\underbrace{\begin{bmatrix} 2 & 5 & 1 \\ 4 & d & 1 \\ 0 & 1 & -1 \end{bmatrix}}_{\mathbf{A}}\underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{\mathbf{x}}=\underbrace{\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}}_{\mathbf{b}}$$

Augmented matrix associated with the above system is,

$$[\mathbf{A} \mid \mathbf{b}]=\left[\begin{array}{ccc|c} 2 & 5 & 1 & 0 \\ 4 & d & 1 & 2 \\ 0 & 1 & -1 & 3 \end{array}\right]$$

Step-3

To eliminate the element in the position $(2,1)$, multiply the first row with 2 and subtract it from the second row. Then the obtained matrix is,

$$\approx\left[\begin{array}{ccc|c} 2 & 5 & 1 & 0 \\ 0 & d-10 & -1 & 2 \\ 0 & 1 & -1 & 3 \end{array}\right]$$ â€¦â€¦ (i)

For the case $d-10=0$ gives $d=10$. The above matrix becomes,

$$\approx\left[\begin{array}{ccc|c} 2 & 5 & 1 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 1 & -1 & 3 \end{array}\right]$$ [For the case, $d=10$]

Interchange the rows 2 and 3 to obtain the triangular form as shown:

$$\approx\left[\begin{array}{ccc|c}2 & 5 & 1 & 0\\ 0 & 1 & -1 & 3\\ 0 & 0 & -1 & 2\end{array}\right]$$

System associated with the above notation is,

$$\begin{aligned}2x+5y+z&=0\\ y-z&=3\\ -z&=2\end{aligned}$$

This is in triangular (upper triangular) form.

Hence, rows 2 and 3 are interchanged to obtained **triangular** system if $\boxed{d=10}$

Step-4

For the case $d-10=1$ gives $d=11$.

Augmented matrix in (i) becomes,

$$\approx\left[\begin{array}{ccc|c}2 & 5 & 1 & 0\\ 0 & 11-10 & -1 & 2\\ 0 & 1 & -1 & 3\end{array}\right]$$
$$\approx\left[\begin{array}{ccc|c}2 & 5 & 1 & 0\\ 0 & 1 & -1 & 2\\ 0 & 1 & -1 & 3\end{array}\right]$$

Observe that, rows 2 and 3 are identical (for the matrix $\mathbf{A}$).

Hence determinant of matrix $\mathbf{A}$ is zero for $d=11$

Therefore, the system is **singular** for $\boxed{d=11}$.