

Step-1

Let A be a Hermitian matrix and c be any real scalar.

The objective is to show that cA is also Hermitian.

Step-2

A matrix A is said to be Hermitian if it is equal to the conjugate transpose, that is $A^H = A$.

Now consider the expression,

$$\begin{aligned}(cA)^H &= \bar{c}A^H \\ &= cA^H \quad \text{As } c \text{ is real, so } \bar{c} = c. \\ &= cA \quad \text{As } A \text{ is Hermitian, so } A^H = A.\end{aligned}$$

As $(cA)^H = cA$, for any real scalar c , so cA is **Hermitian**.

Step-3

Suppose $c = i$.

Now show that (iA) is skew-Hermitian.

A matrix A is said to be skew-Hermitian if $A = -A^H$.

Consider the expression,

$$\begin{aligned}(iA)^H &= \bar{i}A^H \\ &= (-i)A^H \quad \text{Use } \bar{i} = -i. \\ &= -iA \quad \text{As } A \text{ is Hermitian, so } A^H = A.\end{aligned}$$

As $(iA)^H = -iA$, so iA is a **skew-Hermitian**.