Step-1

Consider the following matrices,

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & i^2 & i^3 \\ 1 & i^2 & i^4 & i^6 \\ 1 & i^3 & i^6 & i^9 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \lambda_0 \\ & \lambda_1 \\ & & \lambda_3 \\ & & & \lambda_4 \end{bmatrix}$$

$$\begin{bmatrix} 1+1 & i+i^3 & i^2+i^6 & i^3+i^9 \\ 1 & i^2 & i^4 & i^6 \\ 1 & i^3 & i^6 & i^9 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \lambda_0 \\ & \lambda_1 \\ & & \lambda_3 \\ & & & \lambda_4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & -2 & 0 \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \lambda_0 \\ & \lambda_1 \\ & & \lambda_3 \\ & & & \lambda_4 \end{bmatrix}$$

Therefore, we get,

$\lambda_0 = 2$
$\lambda_1 = -1$
$\lambda_2 = -1$
$\lambda_4 = 1$

Step-2

Thus, the eigenvalues are $\boxed{\lambda_0 = 2,\ \lambda_1 = -1,\ \lambda_2 = -1,\ \text{and } \lambda_4 = 1}$.