Step-1

Given $A=\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}2 & 0\\ 0 & 5\end{pmatrix}\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$

Now without multiplying we have to determine the information of A.

(a).

The determinant of the matrix is,

$$\begin{aligned}\det A &= (2)(5)\\ &= 10.\end{aligned}$$

Therefore, the determinant of the matrix is $\boxed{10}$.

Step-2

(b).

Clearly, the Eigen values of A are the diagonal elements

i.e. 2 and 5.

Therefore, the Eigen values of A are $\boxed{2 \text{ and } 5}$.

Step-3

(c).

The Eigen vectors of A are $\boxed{\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}^T \text{ and } \begin{pmatrix}-\sin\theta\\ \cos\theta\end{pmatrix}^T}$.

Step-4

(d).

As the Eigen values of A are both positive, A is positive definite.

Therefore, $\boxed{A \text{ is positive definite}}$.