Step-1

Given that the average of four times is $\hat{t} = \frac{1}{4}(0+1+3+4) = 2$.

And the average of the four *b*â€™s is $\hat{b} = \frac{1}{4}(0+8+8+20) = 9$

(a) We have to verify that the best line goes through the center point $(\hat{t}, \hat{b}) = (2,9)$.

First to write the equation that would hold if a line could go through the given point.

Then every $C + Dt$ would agree exactly with *b*.

Now $Ax = b$ is

$C + 2t = 9$

Or $\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = [9]$

Where $A = \begin{bmatrix} 1 & 2 \end{bmatrix}, \quad x = \begin{bmatrix} C \\ D \end{bmatrix}$ and $b = [9]$

Step-2

We know that the least-square solution is $A^T A\hat{x} = A^T b$.

Now

$A^T A\hat{x} = A^T b$

$\Rightarrow \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} \hat{C} \\ \hat{D} \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} [9]$

$\Rightarrow \begin{bmatrix} 1(1) & 1(2) \\ 2(1) & 2(2) \end{bmatrix} \begin{bmatrix} \hat{C} \\ \hat{D} \end{bmatrix} = \begin{bmatrix} 1(9) \\ 2(9) \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} \hat{C} \\ \hat{D} \end{bmatrix} = \begin{bmatrix} 9 \\ 18 \end{bmatrix}$

Step-3

Applying $R_2 \to R_2 - 2R_1$, we get

$$\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \hat{C} \\ \hat{D} \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \end{bmatrix}$$

$\Rightarrow \hat{C} + 2\hat{D} = 9$

We have $\hat{C} = 1, \hat{D} = 4$ satisfies above equation

Therefore $\hat{x} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

Hence the best line $\boxed{b = 1 + 4t}$ passes through the center point $(\hat{t}, \hat{b}) = (2, 9)$.

(b) We have to explain why $C + D\hat{t} = \hat{b}$ comes from the first equation in $A^T A\hat{x} = A^T b$

We know that the normal equation is $Cm + D\sum t_i = \sum b_i$

Â Divided by both sides with *m*, we get

Â Â Â Â Â $C + D\frac{\sum t_i}{m} = \frac{\sum b_i}{m}$ Â â€¦â€¦. (1)

Â Â Â Â Â Here $m = 1, \sum t_i = 2, \ \sum b_i = 9$

Hence (1) is equivalent to $C + D\hat{t} = \hat{b}$, where $\hat{t} = \frac{\sum t_i}{m} = 2, \ \hat{b} = \frac{\sum b_i}{m} = 9$.