

Step-1

An odd permutation is nothing but the number of interchanges of the immediate entries in that permutation raised to the power of -1 .

i.e., $\det P = (-1)^j$ where j is the number of interchanges odd and so, the permutation is odd.

$P^2 = P \circ P$ is nothing but the composition of functions while a permutation is a bijective function.

If there are j interchanges performed in the permutation P , then there will be $j + j = 2j$ interchanges performed in $P \circ P$

While $2j$ is even, we see that the number of interchanges in P^2 is even and so, $(-1)^{2j} = 1$ which confirms that P^2 is an even permutation.

Step-2

On the other hand, if the permutation P requires j interchanges, then we follow that to reverse these interchanges, we require j interchanges.

So, if j is odd, then we follow that both P and P^{-1} are odd permutations.