Step-1

The incidence matrix of some graph is $A = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{bmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \end{array}$

It follows that there are four nodes x_1, x_2, x_3, x_4 and four edges e_1, e_2, e_3, e_4

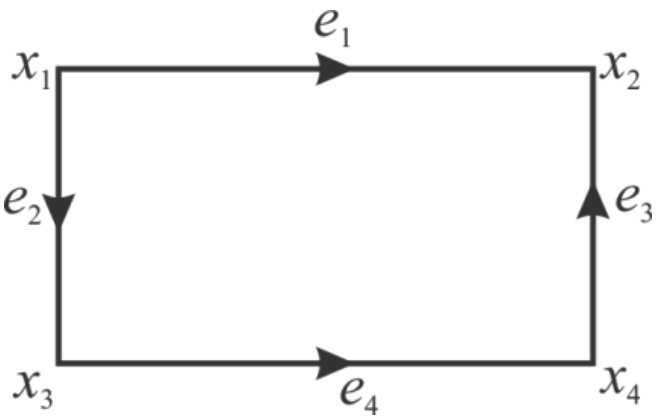
Step-2

The entry in the 1, 1 place is -1.

That means the arrow of the edge e_1 starts at x_1 and reaches x_2

Similarly, with the other edges and vertices.

The graph with number set and directed edges (and numbered nodes) whose incidence matrix is A.



There are two paths from x_1 to x_2

It clears that just by removing an edge; it cannot cause a disconnection in the graph.

Therefore, this graph is $\boxed{\text{not a tree}}$.

On the other hand, to confirm that the given graph is a tree or not, need to check whether the rows of A are linearly independent?

For,

$$\xrightarrow{R_4 \to R_1 + R_2 + R_3 + R_4} \begin{bmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ -2 & 2 & 0 & 0 \end{bmatrix}$$

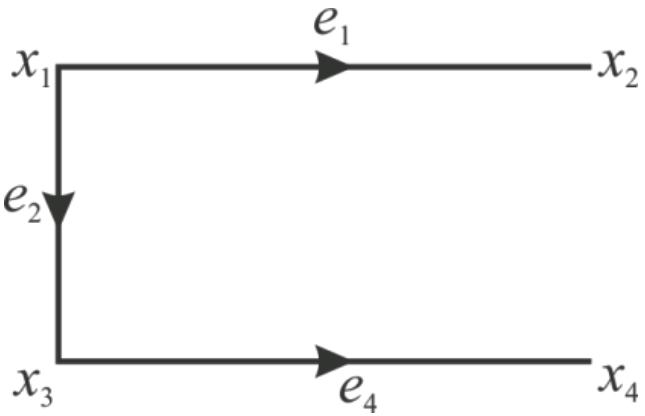
$$\xrightarrow{R_4 \to R_4 - 2R_1} \begin{bmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Observe that one row is made zero using the row operations which shows that one row is dependent of the other rows.

In other words, the graph is $\boxed{\text{not a tree}}$

Step-3

Now, remove the edge e_3 to show that the remaining edges form the spanning tree for the graph.



It can be seen that every vertex has a path from the source vector or root x_1

Therefore, the removable of edge e_3 gives the spanning tree for the graph and this confirms that the presence of e_3 will form a loop or a cycle and thus, the given graph is $\boxed{\text{not a tree}}$.

Step-4

Put the entire discussion together, it is confirm that the remaining rows of the given matrix form the **basis to the spanning tree of the matrix.**