

Step-1

Let $a_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, a_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Writing $A = [a_1 \ a_2]$

$= \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$, the required projection matrix is $P = A(A^T A)^{-1} A^T$

$$A^T A = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 1 \\ 1 & 5 \end{bmatrix}$$

$$(A^T A)^{-1} = \frac{1}{4} \begin{bmatrix} 5 & -1 \\ -1 & 1 \end{bmatrix}$$

Step-2

$$A(A^T A)^{-1} = \frac{1}{4} \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ -1 & 1 \end{bmatrix}$$
$$= \frac{1}{4} \begin{bmatrix} 4 & 0 \\ -2 & 2 \end{bmatrix}$$

$$A(A^T A)^{-1} A^T = \frac{1}{4} \begin{bmatrix} 4 & 0 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$$
$$= \frac{1}{4} \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

Therefore, $P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$