Step-1

The objective is to construct a matrix whose column space contains $(1,1,5)$ and $(0,3,1)$ and whose null space contains $(1,1,2)$.

Step-2

Construct a matrix that first two columns taken as $(1,1,5)$, $(0,3,1)$ and take third column as arbitrary as shown below:

$$A = \begin{bmatrix} 1 & 0 & a \\ 1 & 3 & b \\ 5 & 1 & c \end{bmatrix}$$

The vector $(1,1,2)$ is in null space of the matrix A, so

$$\begin{bmatrix} 1 & 0 & a \\ 1 & 3 & b \\ 5 & 1 & c \end{bmatrix} \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Now solve the matrix equation as shown below:

$$\begin{bmatrix} 1 & 0 & a \\ 1 & 3 & b \\ 5 & 1 & c \end{bmatrix} \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1+0+2a \\ 1+3+2b \\ 5+1+2c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2a+1 \\ 2b+4 \\ 2c+6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Equating on both sides, then

$$2a+1=0 \Rightarrow a = \frac{-1}{2}$$
$$2b+4=0 \Rightarrow b = -2$$
$$2c+6=0 \Rightarrow c = -3$$

Substitute the above values in A, then

$$A = \begin{bmatrix} 1 & 0 & \frac{-1}{2} \\ 1 & 3 & -2 \\ 5 & 1 & -3 \end{bmatrix}$$

Therefore, the matrix A is, $\begin{bmatrix} 1 & 0 & \dfrac{-1}{2} \\ 1 & 3 & -2 \\ 5 & 1 & -3 \end{bmatrix}$