Step-1

(a) True

If W is a subspace of R^4, then the dimension of W is 0 to 4

If dimension of W is zero, then the subspace is $\{0\}$, the trivial null space of any non singular matrix.

If W is a subspace of dimension less than 4, then it is spanned by the solutions of the homogeneous system $Ax = 0$ and so is the null space of the related matrix A.

If W is the subspace of dimension 4, then the matrix A is the zero matrix.

Therefore, in any case the subspace of R^4 has the basis which is the solution set of the homogeneous system $Ax = 0$.

So, this basis spans the null space of the matrix A

Step-2

(b) True

If A is a matrix of size m by n, with the rank r,

Then the null space of A is of dimension *n* â€“ *r* and the null space of A^T is of dimension $m - r$.

In our case it is given that *m* â€“ *r* = *n* â€“ *r*

Consequently $m = n$.

In other words, A is a square matrix.

Step-3

(c) We can write $mx+b$ as mx^1+bx^0

We call that the x term with power 1 is linear and with power 0 is constant.

In other words, the x term with power other than 1 is not linear.

In other words, $T(x)=mx+b$ is not linear.