Step-1

Given that the distance from a plane $a^T x = c$ to the origin is $\frac{|c|}{\|a\|}$

We have to find the distance from the origin to the plane $x_1 + x_2 - x_3 - x_4 = 8$, and we have to find the nearest point on the plane from the origin.

Step-2

Given plane is $x_1 + x_2 - x_3 - x_4 = 8$, where $c = 8$

$$\|a\| = \sqrt{1^2 + 1^2 + (-1)^2 + (-1)^2} = 2$$

The distance from a given plane to the origin

$$\frac{|c|}{\|a\|}$$

$$= \frac{8}{2}$$
$$= \boxed{4}$$

Step-3

Let L = Distance from the point $(0,0,0,0)$ to the point (x, y, z, w) on the plane.

Â = $\sqrt{x^2 + y^2 + z^2 + w^2}$

Let $f = L^2 = x^2 + y^2 + z^2 + w^2$

Given plane $\phi(x, y, z, w) \equiv x + y - z - w - 8 = 0$

By Lagrangeâ€™s multipliers, $F = f + \lambda\phi$

Step-4

$$F = x^2 + y^2 + z^2 + w^2 + \lambda(x + y - z - w - 8)$$
$$\frac{\partial F}{\partial x} = 0$$
$$\Rightarrow 2x + \lambda = 0$$
$$\Rightarrow x = \frac{-\lambda}{2}$$

Step-5

$$\frac{\partial F}{\partial y} = 0$$
$$\Rightarrow 2y + \lambda = 0$$
$$\Rightarrow y = \frac{-\lambda}{2}$$

Step-6

$$\frac{\partial F}{\partial z} = 0$$
$$\Rightarrow 2z - \lambda = 0$$
$$\Rightarrow z = \frac{\lambda}{2}$$

Step-7

$$\frac{\partial F}{\partial w} = 0$$
$$\Rightarrow 2w - \lambda = 0$$
$$\Rightarrow w = \frac{\lambda}{2}$$

Step-8

Now substitute the values of x, y, z, w in ϕ, we get

$$\frac{-\lambda}{2} + \frac{-\lambda}{2} - \frac{\lambda}{2} - \frac{\lambda}{2} = 8$$
$$\Rightarrow -2\lambda = 8$$
$$\Rightarrow \lambda = -4$$

Step-9

Therefore

$$x = y$$
$$= \frac{-\lambda}{2}$$
$$= 2$$

Step-10

And

$$z = w$$
$$= \frac{\lambda}{2}$$
$$= -2$$

Hence required closest point $= \boxed{(2, 2, -2, -2)}$