Step-1

We have to check whether the following statements in (a) and (b) are true or false with an example.

(a) Q^{-1} is an orthogonal matrix when Q is an orthogonal matrix.

For example, suppose $Q=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$

Step-2

$$QQ^T=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$$
$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$
$$=I$$

Therefore Q is an orthogonal matrix.

Step-3

$$Q^{-1}=\frac{1}{0-1}\begin{bmatrix}0 & -1\\ -1 & 0\end{bmatrix}$$
$$=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$$

$$\left(Q^{-1}\right)^T Q^{-1}=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$$
$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$
$$=I$$

Hence Q^{-1} is also an orthogonal matrix.

Therefore the given statement is $\boxed{\text{true}}$.

(b) If Q (3 by 2) has orthonormal columns then $\|Qx\|=\|x\|$

For example, suppose $Q=\begin{bmatrix}q_1 & q_2\end{bmatrix}$ is a 3×2 matrix, where q_1, q_2 are orthonormal columns of Q

That is,

$q_1^T q_1 = 1, q_2^T q_2 = 1$, and $q_1^T q_2 = 0, q_2^T q_1 = 0$ Â â€¦â€¦ (1)

Step-4

Let $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, then

$$\|Qx\|^2 = (Qx)^T (Qx) = (q_1 x_1 + q_2 x_2)^T (q_1 x_1 + q_2 x_2)$$
$$= \left[(q_1 x_1)^T + (q_2 x_2)^T \right] (q_1 x_1 + q_2 x_2)$$
$$= \left(x_1^T q_1^T + x_2^T q_2^T \right) (q_1 x_1 + q_2 x_2)$$
$$= x_1 q_1^T q_1 x_1 + x_2 q_2^T q_1 x_1 + x_1 q_1^T q_2 x_2 + x_2 q_2^T q_2 x_2$$
$$\left(\text{Since are real numbers}, x_1^T = x_1, x_2^T = x_2 \right)$$

Step-5

$$= x_1 1 x_1 + x_2 .0 x_1 + x_1 .0 x_2 + x_2 1 x_2$$

$$= x_1^2 + x_2^2$$

Step-6

Therefore

$$\|Qx\|^2 = x_1^2 + x_2^2$$
$$= \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$
$$= x^T x$$
$$= \|x\|^2$$ Â Â Â

$$\Rightarrow \|Qx\| = \|x\|$$ Â

Therefore the given statement is $\boxed{\text{true}}$.Â

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