

Step-1

We have to explain that why the pivot rows and pivot columns of A (not R) always give an r by r invertible submatrix of A .

Step-2

Let A be an m by n matrix and suppose A has r pivot columns.

Then the r pivot columns of A form a m by r submatrix of A .

Then A has r pivot rows.

Step-3

Then the \tilde{r} pivot rows of A forms an r by n submatrix of A

Therefore these r pivot rows and r pivot columns forms an r by r matrix, which is sub matrix of A and the determinant is not equal to zero. Hence this r by r matrix is an invertible matrix, which is the submatrix of A .