Step-1

(a) The projection matrix $P_1 = \frac{aa^T}{a^T a}$

$$aa^T = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 3 & 9 \end{bmatrix}$$

And $a^T a = \begin{bmatrix} 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = 1+9=10$

Therefore, $P_1 = \frac{1}{10} \begin{bmatrix} 1 & 3 \\ 3 & 9 \end{bmatrix} = \begin{bmatrix} \frac{1}{10} & \frac{3}{10} \\ \frac{3}{10} & \frac{9}{10} \end{bmatrix}$

Step-2

While P_1 is the projection onto a subspace, we follow that $I - P_1 = P_2$ is the matrix related to the subspace perpendicular to the projection of P_1

$$P_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{1}{10} & \frac{3}{10} \\ \frac{3}{10} & \frac{9}{10} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{9}{10} & \frac{-3}{10} \\ \frac{-3}{10} & \frac{1}{10} \end{bmatrix}$$

Step-3

(b) while P_1 and P_2 are the orthogonal projection matrices, their sum must be the identity matrix.

$$P_1 + P_2 = \begin{bmatrix} \frac{1}{10} & \frac{3}{10} \\ \frac{3}{10} & \frac{9}{10} \end{bmatrix} + \begin{bmatrix} \frac{9}{10} & -\frac{3}{10} \\ -\frac{3}{10} & \frac{1}{10} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{10} + \frac{9}{10} & \frac{3}{10} - \frac{3}{10} \\ \frac{3}{10} - \frac{3}{10} & \frac{9}{10} + \frac{1}{10} \end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Step-4

The product of the matrices related to the orthogonal projections must be the null matrix.

$$P_1P_2=\begin{bmatrix}\frac{1}{10} & \frac{3}{10}\\ \frac{3}{10} & \frac{9}{10}\end{bmatrix}\begin{bmatrix}\frac{9}{10} & -\frac{3}{10}\\ -\frac{3}{10} & \frac{1}{10}\end{bmatrix}$$

$$=\begin{bmatrix}\frac{9}{100}-\frac{9}{100} & \frac{-3}{100}+\frac{3}{100}\\ \frac{27}{100}-\frac{27}{100} & \frac{-9}{100}+\frac{9}{100}\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$