Step-1

Consider the system of equations is

$u+v+w=-2$

$3u+3v-w=6$
$u-v+w=-1$

Find the solution to above system by applying elimination.

Step-2

Write the system in the matrix form

$$\begin{bmatrix} 1 & 1 & 1 & -2 \\ 3 & 3 & -1 & 6 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

Apply $R_2 \to R_2 - 3R_1$,

$R_3 \to R_3 - R_1$

$$\sim \begin{bmatrix} 1 & 1 & 1 & -2 \\ 0 & 0 & -4 & 12 \\ 0 & -2 & 0 & 1 \end{bmatrix}$$

Apply $\mathrm{R}_2 \leftrightarrow \mathrm{R}_3$

$$\sim \begin{bmatrix} 1 & 1 & 1 & -2 \\ 0 & -2 & 0 & 1 \\ 0 & 0 & -4 & 12 \end{bmatrix}$$

This is upper triangular form.

Step-3

From above upper triangular form, we have

$$\begin{aligned} u+v+w &= -2 \\ -2v &= 1 \\ -4w &= 12 \end{aligned}$$

From $-4w=12$

$$\frac{-4w}{-4}=\frac{12}{-4}$$

$$\boxed{w=-3}$$

From $-2v=1$

$$v=\boxed{\frac{-1}{2}}$$

From $u+v+w=-2$

$$u+\left(\frac{-1}{2}\right)-3=-2 \qquad \left(\text{Since } v=-\frac{1}{2}, w=-3\right)$$

$$u=-2+\frac{1}{2}+3$$

$$u=1+\frac{1}{2}$$

$$\boxed{u=\frac{3}{2}}$$

Therefore solutions are $\boxed{u=\frac{3}{2}, v=\frac{-1}{2}, w=-3}$.

Step-4

In the given $\begin{bmatrix} 1 & 1 & 1 & -2 \\ 3 & 3 & -1 & 6 \\ 1 & -1 & 1 & -1 \end{bmatrix}$ if we replace the coefficient -1 of v by1, then we get

$$\begin{bmatrix} 1 & 1 & 1 & -2 \\ 3 & 3 & -1 & 6 \\ 1 & 1 & 1 & -1 \end{bmatrix}$$

Apply $R_2 \to R_2 - 3R_1, R_3 \to R_3 - R_1$

$$\sim \begin{bmatrix} 1 & 1 & 1 & -2 \\ 0 & 0 & -4 & 12 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Therefore the system becomes singular (2 equal columns) and hence it has no solution.

Hence the change of coefficient -1 of v by 1 would make the system impossible to proceed and elimination break down.