Step-1

(1)

Consider the given matrix,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}.$$

The objective is to reduce A to U and find $\det A = \text{produnct of the pivots}$.

Step-2

Reduce A to U as follows:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \qquad R_2 \to R_2 - R_1$$

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix} \qquad R_3 \to R_3 - R_1$$

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad R_3 \to R_3 - R_2$$

$$\begin{aligned} \text{Therefore, } \det A &= \det U \\ &= \text{product of pivots} \\ &= 1 \cdot 1 \cdot 1 \\ &= \boxed{1} \end{aligned}$$

Step-3

(2)

Consider the given matrix,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 3 \\ 3 & 3 & 3 \end{bmatrix}.$$

Reduce A to U as follows:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 3 \\ 3 & 3 & 3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 3 & 3 & 3 \\ 2 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} \qquad R_1 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} 3 & 3 & 3 \\ 0 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix} \qquad R_2 \to R_2 - \frac{2}{3} R_1$$

$$\sim \begin{bmatrix} 3 & 3 & 3 \\ 0 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} \qquad R_3 \to R_3 - \frac{1}{3} R_1$$

$$\sim \begin{bmatrix} 3 & 3 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \qquad R_2 \leftrightarrow R_3$$

Therefore, $\det A = \det U$

$= \text{product of pivots}$

$= 3 \cdot (1) \cdot (1)$

$= \boxed{3}$