Step-1

To complete the diagonalization of Fibonacci matrix by completing S^{-1}:

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \lambda_1 & \lambda_2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

Also, find the second component of the matrix by multiplying $S\Lambda^k S^{-1}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

Step-2

From the following matrices, matrix S can be given as follows:

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \lambda_1 & \lambda_2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

Thus, matrix S is :

$$S = \begin{bmatrix} \lambda_1 & \lambda_2 \\ 1 & 1 \end{bmatrix}$$

Inverse of S is given as follows:

$$S^{-1} = \frac{1}{\lambda_1 - \lambda_2} \begin{bmatrix} 1 & -\lambda_2 \\ -1 & \lambda_1 \end{bmatrix}$$

Step-3

Multiplying $S\Lambda^k S^{-1}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ gives the following:

$$\begin{aligned} S\Lambda^k S^{-1}\begin{bmatrix} 1 \\ 0 \end{bmatrix} &= \frac{1}{\lambda_1 - \lambda_2} \begin{bmatrix} \lambda_1 & \lambda_2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1^k & 0 \\ 0 & \lambda_2^k \end{bmatrix} \begin{bmatrix} 1 & -\lambda_2 \\ -1 & \lambda_1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ &= \frac{1}{\lambda_1 - \lambda_2} \begin{pmatrix} \lambda_1^{k+1} - \lambda_2^{k+1} \\ \lambda_1^k - \lambda_2^k \end{pmatrix} \end{aligned}$$

Step-4

Therefore, the second component is $\boxed{\dfrac{\lambda_1^k - \lambda_2^k}{\lambda_1 - \lambda_2}}$. This is the kth Fibonacci number.