Step-1

Given that the solutions to the linear differential equation $\frac{d^2u}{dt^2}=u$ form a vector space.

We have to find two independent solutions to give a basis for that solution space.

Step-2

We have $\frac{d^2u}{dt^2}=u$

$\Rightarrow u''-u=0$

The auxiliary equation is $m^2-1=0$

$\Rightarrow m=\pm 1$

Therefore, $u=e^t, u=e^{-t}$ are solutions of $\frac{d^2u}{dt^2}=u$.

And e^t, e^{-t} are independent solutions of the basis for the solution space is $\boxed{\{e^t, e^{-t}\}}$.

The general solution of the given differential equation is $u=c_1e^t+c_2e^{-t}$, where c_1, c_2 are constants.