Step-1

Now consider the following:

$$Px = \left(I - A^{\mathsf{T}} \left(AA^{\mathsf{T}}\right)^{-1} A\right) x$$
$$= Ix - A^{\mathsf{T}} \left(AA^{\mathsf{T}}\right)^{-1} Ax$$
$$= x - A^{\mathsf{T}} \left(AA^{\mathsf{T}}\right)^{-1} 0$$
$$= x$$

Step-2

Thus, we have shown that when x is in the nullspace of A, Px = x.