## Step-1

Matrix form of given system is Ax = b

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 2 & 3 \\ 3 & 4 & 5 \end{bmatrix}, \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad b = \begin{bmatrix} 5 \\ 5 \\ 9 \end{bmatrix}$$

Considering A and applying  $R_1 + R_2 - R_3$ , we get (1+2-3, 2+2-4, 2+3-5) = (0,0,0)

In other words, multiplying the 1<sup>st</sup> row with 1, 2<sup>nd</sup> row with 1, 3<sup>rd</sup> with -1 and then adding, we get 0.

## Step-2

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \text{ we write}$$
Letting

$$y^{T} A x = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 2 & 3 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x + 2y + 2z \\ 2x + 2y + 3z \\ 3x + 4y + 5z \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x+2y+2z \\ 2x+2y+3z \\ 3x+4y+5z \end{bmatrix}$$

$$= 1(x+2y+2z)+1(2x+2y+3z)-1(3x+4y+5z)$$

$$=0$$
  $\hat{a}\in\hat{a}\in\hat{a}\in\hat{a}$ 

## Step-3

$$y^T b = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 5 \\ 5 \\ 9 \end{bmatrix}$$

On the other hand,

$$=5+5-9$$

Therefore,  $y^T A x = y^T b$  reduces to 0 = 1

This is an absurdity.

Therefore, the given system Ax = b has no solution

## Step-4

We have seen that  $y^T Ax = 0$ 

That means the inner product  $\langle y, Ax \rangle = 0$ 

So, y is perpendicular to Ax

Therefore, the vector y is in the null space of A.