Step-1

Consider the equation:

$$Ax = b$$
$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} C \\ D \\ E \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$$

The objective is to find the minimum length least squares solution $x^+ = A^+ b$ to the above equation.

Step-2

First find the general solution to $A^{\mathrm{T}} A\hat{x} = A^{\mathrm{T}} b$.

Obtain, $A^{\mathrm{T}} A$ as follows:

$$A^{\mathrm{T}} A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 3 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Step-3

Assume that,

$\hat{x} = (C, D, E)$.

So,

$$A^{\mathrm{T}} A\hat{x} = A^{\mathrm{T}} b$$
$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} C \\ D \\ E \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$$
$$\begin{bmatrix} 3C + D + E \\ C + D + E \\ C + D + E \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix}$$

Step-4

Consider the following three equations:

$$\begin{aligned} 3C+D+E&=4\\ C+D+E&=2\\ C+D+E&=2 \end{aligned}$$

Subtracting second (or third) equation from the first equation, obtained as $2C=2$.

This gives $C=1$.

Substituting this value in the second (or third) equation, obtained as, $D+E=1$.

That is, $E=1-D$.

Thus, the general solution is, $\hat{x}=(1,D,1-D)$.

Step-5

Assume that, $\hat{x}=(1,D,1-D)$ lies in the row space of the matrix A.

So, $\hat{x}$ can be expressed as a linear combination of the rows of the matrix A

Consider,

$$\begin{aligned} (1,D,1-D)&=a(1,0,0)+b(1,0,0)+c(1,1,1)\\ &=(a+b+c,c,c) \end{aligned}$$

Thus, $D=1-D$

$$D=\frac{1}{2}.$$

This gives, $\boxed{\hat{x}=\left(1,\frac{1}{2},\frac{1}{2}\right)}$.

This solution fits for the plane $C+Dt+Ez=2$ for $t=z=1$.