Step-1

Feasible set: A feasible set is composed of the solutions to a family of linear inequalities, and a feasible point maximizes or minimizes a certain cost function.

Step-2

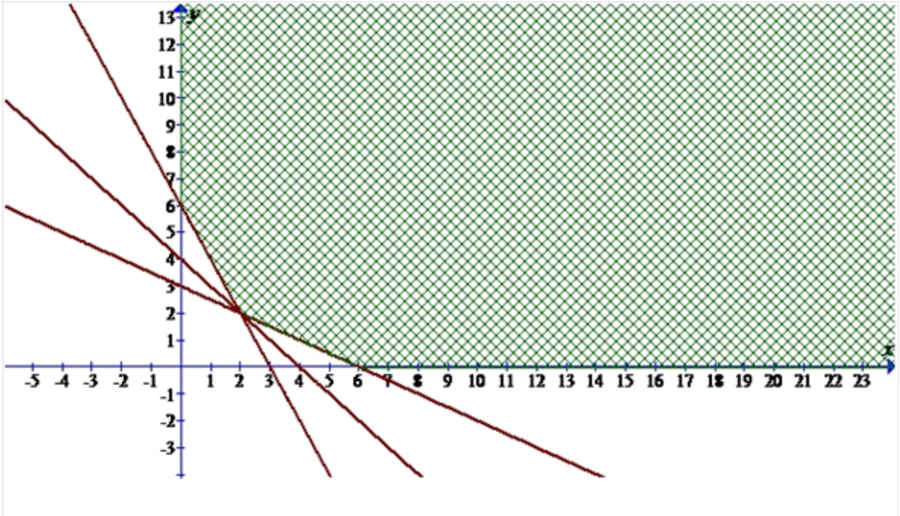
To sketch the feasible set with following constraints:

$$\begin{aligned} x+2y &\geq 6 \\ 2x+y &\geq 6 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

To find the minimum value of the cost function $x+y$, also to sketch the line $x+y=\text{constant}$ in feasible region that touches the first feasible set. Finally, to check which point minimizes the cost functions $3x+y$ and $x-y$.

Step-3

Following sketch gives the feasible set:



Here shaded region denotes the feasible region. The line $x+y=4$ touches the feasible region at $(2,2)$.

Step-4

Three corner sets are as follows:

$(2,2),(0,6),(6,0)$

Step-5

Substitute the following points in the function $x+y$ to get the minimum value:

When point is $(2,2)$

$$\begin{aligned} x+y &= 2+2 \\ &= 4 \end{aligned}$$

When point is $(0,6)$

$$\begin{aligned} x+y &= 0+6 \\ &= 6 \end{aligned}$$

When point is $(6,0)$

$$\begin{aligned} x+y &= 6+0 \\ &= 6 \end{aligned}$$

Above calculations show that at point $(2,2)$ function $x+y$ gets minimum value 4.

Step-6

Substitute the following points in the function $3x+y$ to get the minimum value:

When point is $(2,2)$

$$\begin{aligned} 3x+y &= 3\cdot 2+2 \\ &= 8 \end{aligned}$$

When point is $(0,6)$

$$\begin{aligned} 3x+y &= 3\cdot 0+6 \\ &= 6 \end{aligned}$$

When point is $(6,0)$

$$\begin{aligned} 3x+y &= 3\cdot 6+0 \\ &= 18 \end{aligned}$$

Above calculations show that at point $(0,6)$ function $3x+y$ gets minimum value 6.

Substitute the following points in the function $x-y$ to get the minimum value:

Step-7

When point is $(2,2)$

$$\begin{aligned} x-y &= 2-2 \\ &= 0 \end{aligned}$$

When point is $(0,6)$

$$\begin{aligned} x-y &= 0-(-6) \\ &= 6 \end{aligned}$$

When point is $(6,0)$

$$\begin{aligned} x-y &= 6-0 \\ &= 6 \end{aligned}$$

Above calculations show that at point $(2,2)$ function $x-y$ gets minimum value 0.

Step-8

Therefore, minimum values of the functions are:

$$\boxed{\begin{aligned} x+y &= 4 \\ 3x+y &= 6 \\ x-y &= 0 \end{aligned}}$$