Step-1

(a) The set $\{(1,2,0),(0,1,-1)\}$ is linearly independent as one vector is not a scalar multiple of the other.

But the dimension of R^3 is 3 while there are only two vectors present in the given set.

So, these vectors cannot span R^3

In other words, there are vectors in R^3 which cannot be written as linear combinations of $\{(1,2,0),(0,1,-1)\}$

For example, (1,2,3) = *a*(1,2,0) + *b*(0,1,-1) is not satisfied by any scalars *a* and *b*.

Therefore the set is not a basis for R^3

Step-2

(b) We know that the maximum number of linearly independent vectors that can span R^3 is 3 and forms a basis.

But in our case, there are four vectors in the set $\{(1,1,-1),(2,3,4),(-4,1,-1),(0,1,-1)\}$

So, this is not linearly independent and thus cannot form a basis to R^3

Step-3

(c) Let $a(1,2,2)+b(-1,2,1)+c(0,8,0)=(0,0,0)$

$a-b=0$ â€¦â€¦ (1)

$2a+2b+8c=0$ â€¦â€¦ (2)

$2a+b=0$ â€¦â€¦ (3)

By (1) + (3), we have $3a=0$

$\Rightarrow a=0$

Using this in (1), we get $b=0$

Using these in (2), we get $c = 0$

Hence the vector $\{(1,2,2),(-1,2,1)(0,8,0)\}$ is a linearly independent set.

Also, the number of vectors in this set = 3 = dimension of R^3

So, these vectors can span R^3

Therefore, $\{(1,2,2),(-1,2,1)(0,8,0)\}$ forms a basis to R^3

Step-4

(d) Let a $a(1,2,2)+b(-1,2,1)+c(0,8,6)=0$

$\Rightarrow a-b=0$

$2a+2b+8c=0$

$2a+b+6c=0$

$\Rightarrow a-b=0$ â€¦â€¦ (1)

$a+b+4c=0$ â€¦â€¦ (2)

$2a+b+6c=0$ â€¦â€¦ (3)

Using $a = b$ from (1) in (2) and (3), we get 2b + 4c = 0

Consequently, $a = b = -2c$ such that the equation is satisfied.

That means one vector is linearly dependent of the other two.

Therefore, they cannot form a basis to R^3