Step-1

Given that $(-A)$ is positive definite.

(i).

$(-A)$ is positive definite.

$\Rightarrow x^T(-A)x > 0$ for all $x \neq 0$.

$\Rightarrow x^T Ax < 0$ for all $x \neq 0$.

$\therefore$ A is negative definite if $\boxed{x^T Ax < 0}$ for all non zero vectors x.

Step-2

(ii).

$(-A)$ is positive definite.

$\Rightarrow$ All the Eigen values of $(-A)$ satisfies $\lambda_i > 0$.

So all the Eigen values of A satisfies $\lambda_i < 0$.

Step-3

(iii).

We know that $\det(-A) = (-1)^n \det A$

$(-A)$ is positive definite.

$\Rightarrow$ All the upper left sub matrices of $(-A)$

i.e. A_1, A_2 and A_3 have positive determinants.

That is $\det A_1 > 0, \det A_2 > 0$ and $\det A_3 > 0$.

$$\det A_1^1 = (-1)^1 \det A_1 = -\det A_1$$
$$\Rightarrow \det A_1^1 < 0 \text{ as } \det A_1 > 0$$

$\det A_2^1 = (-1)^2 \det A_2 = \det A_2$

$\Rightarrow \det A_2^1 > 0$ as $\det A_2 > 0$

$\det A_3^1 = (-1)^3 \det A_3 = -\det A_3$

$\Rightarrow \det A_3^1 < 0$ as $\det A_3 > 0$

Thus if A is negative definite then

$\det A_1^1 < 0,\ \det A_2^1 > 0$ and $\det A_3^1 < 0.$

Step-4

(iv).

$(-A)$ is positive definite.

$\Rightarrow$ All the pivots (without row exchanges) satisfies $d_k > 0.$

So if A is negative definite if all the pivots (without row exchanges) satisfies $d_k < 0.$

Step-5

(v).

$(-A)$ is positive definite.

$\Rightarrow$ There is a matrix R with independent columns such that $-A = R^T R$.

There is a matrix R with independent columns such that $A = -R^T R$.