Step-1

Consider the following equation.

$$\begin{aligned} u_{k+1} &= Au_k \\ &= \begin{bmatrix} a & b \\ 1-a & 1-b \end{bmatrix} u_k \end{aligned}$$

According to Markov process, matrix has no negative entries and each column of the Markov matrix has summation equal to 1.

So, the range of a and b are $\boxed{0 \le a \le 1}$ and $\boxed{0 \le b \le 1}$.

To compute $u_k = S\Lambda S^{-1}u_0$, find the eigenvalues and eigenvector of matrix A.

$$\begin{aligned} \det(A-\lambda I) &= 0 \\ \det\begin{bmatrix} a-\lambda & b \\ 1-a & 1-b-\lambda \end{bmatrix} &= 0 \\ (a-\lambda)(1-b-\lambda)-b(1-a) &= 0 \\ \lambda^2-\lambda(a-b+1)+(a-b) &= 0 \end{aligned}$$

Step-2

Therefore,

$$\begin{aligned} \lambda &= \frac{(a-b+1) \pm \sqrt{(a-b+1)^2-4(1)(a-b)}}{2a} \\ &= \frac{(a-b+1) \pm (b-a+1)}{2} \end{aligned}$$

Consider λ_1 and λ_2 are two values of λ, then

$$\begin{aligned} \lambda_1 &= \frac{(a-b+1)+(b-a+1)}{2} \\ &= 1 \\ \lambda_2 &= \frac{(a-b+1)-(b-a+1)}{2} \\ &= a-b \end{aligned}$$

Step-3

The matrix A has repeated Eigen values for

$$\lambda_1 = \lambda_2$$
$$a - b = 1$$

Since $0 \le a \le 1,\ 0 \le b \le 1$ so, the only possible values of a and b for which $a - b = 1$ holds is,

$$a = 1,\ b = 0$$

Therefore, the matrix A has distinct Eigen values for $a \ne 1$ and $b \ne 0$

Now, the eigenvector corresponding to the eigenvalue $\lambda_1 = 1$ is given by,

$$(A - I)\begin{pmatrix} x \\ y \end{pmatrix} = 0$$
$$\begin{bmatrix} a-1 & b \\ 1-a & -b \end{bmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0$$
$$(a-1)x + by = 0$$

The eigenvector corresponding to the eigenvalue $\lambda_1 = 1$ is

$$\begin{bmatrix} \frac{b}{1-a} \\ 1 \end{bmatrix}$$

Step-4

Now, the eigenvector corresponding to the eigenvalue $\lambda_2 = a - b$ is given by,

$$\begin{aligned} (A-(a-b)I)\begin{pmatrix} x \\ y \end{pmatrix} &= \begin{bmatrix} a-(a-b) & b \\ 1-a & (1-b)-(a-b) \end{bmatrix}\begin{pmatrix} x \\ y \end{pmatrix} \\ &= \begin{bmatrix} b & b \\ 1-a & 1-a \end{bmatrix}\begin{pmatrix} x \\ y \end{pmatrix} \end{aligned}$$
$$bx + by = 0, (1-a)x + (1-a)y = 0$$
$$x + y = 0 \qquad (\text{since } a \ne 1\ b \ne 0)$$

The eigenvector corresponding to the eigenvalue $\lambda_1 = a - b$ is

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Step-5

Therefore,

$$
\begin{aligned}
A &= S\Lambda S^{-1} \\
&= \begin{bmatrix} \frac{b}{1-a} & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & a-b \end{bmatrix} \begin{bmatrix} \frac{a-1}{a-1-b} & \frac{a-1}{a-1-b} \\ \frac{a-1}{a-1-b} & \frac{b}{a-1-b} \end{bmatrix}
\end{aligned}
$$

Now,

$$
\begin{aligned}
A^k &= \left(S\Lambda S^{-1}\right)^k \\
&= S\Lambda S^{-1} \cdot S\Lambda S^{-1} \ldots S\Lambda S^{-1} \\
&= S\Lambda^k S^{-1} \\
&= \begin{bmatrix} \frac{b}{1-a} & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & (a-b)^k \end{bmatrix} \begin{bmatrix} \frac{b}{1-a} & 1 \\ 1 & -1 \end{bmatrix}
\end{aligned}
$$

Step-6

Now $u_k = S\Lambda S^{-1} u_0$ is given by

$$
\begin{aligned}
u_k &= S\Lambda S^{-1} u_0 \\
&= \begin{bmatrix} \frac{b}{1-a} & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & (a-b)^k \end{bmatrix} \begin{bmatrix} \frac{b}{1-a} & 1 \\ 1 & -1 \end{bmatrix} u_0
\end{aligned}
$$

Consider $u_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, then

$$
\begin{aligned}
u_k &= \begin{bmatrix} \frac{b}{1-a} & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & (a-b)^k \end{bmatrix} \begin{bmatrix} \frac{b}{1-a} & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\
&= \begin{bmatrix} \frac{2b}{b-a+1} - \frac{1-a-b}{b-a+1}(a-b)^k \\ \frac{2(1-a)}{b-a+1} - \frac{1-a-b}{b-a+1}(a-b)^k \end{bmatrix}
\end{aligned}
$$

Therefore,

$$
u_k = \boxed{\begin{bmatrix} \frac{2b}{b-a+1} - \frac{1-a-b}{b-a+1}(a-b)^k \\ \frac{2(1-a)}{b-a+1} - \frac{1-a-b}{b-a+1}(a-b)^k \end{bmatrix}}
$$

Step-7

Since $|a-b|<1$, so $(a-b)^k$ approach to zero as $k \to \infty$.

Hence,

$$u_k \to \begin{bmatrix} \frac{2b}{b-a+1} \\ \frac{2(1-a)}{b-a+1} \end{bmatrix}, \text{ As } k \to \infty$$

Step-8

According to Markov process, matrix has no negative entries and each column of the Markov matrix has summation equal to 1.

Thus, the limit values of a and b should be $\boxed{a=\frac{1}{3}}$ and $\boxed{b=-\frac{1}{3}}$.

For these values of a and b,

$$A = \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \\ 1-\frac{1}{3} & 1-\left(-\frac{1}{3}\right) \end{bmatrix}$$
$$= \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \\ \frac{2}{3} & \frac{4}{3} \end{bmatrix}$$

Since matrix A has negative entry, so $\boxed{\text{the matrix } A \text{ is not a markov matrix}}$.