

Step-1

Let $A = (a_{ij})_{n \times n}$ where

$$a_{ij} = ij \text{ for } 1 \leq i, j \leq n$$

Then

$$A_1 = (1) \text{ so } \det A_1 = 1 \text{ (exceptional case)}$$

$$A_2 = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \text{ so that } \det A_2 = 0$$

$$A_3 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix} \text{ so that}$$

Step-2

$$\det A_3 = \begin{vmatrix} 4 & 6 \\ 6 & 9 \end{vmatrix} - 2 \begin{vmatrix} 2 & 6 \\ 3 & 9 \end{vmatrix} + 3 \begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix}$$

$$= 0 - 2(0) + 3(0)$$

$$= 0$$

Step-3

For any $n \geq 2$ we can note that

$$A_n = \begin{pmatrix} 1 & 2 & 3 & \dots & n \\ 2 & 4 & 6 & \dots & 2n \\ n & 2n & 3n & \dots & n^2 \end{pmatrix}$$

And clearly any two rows of A_n are proportional and hence $\det A_n = 0$

Thus if a_{ij} is i times j , then $\det A = 0$

(exception when $A = [1]$)