Step-1

To prove that $\mathbf{V}^{\perp} \subset \mathbf{S}^{\perp}$ if $\mathbf{S} \subset \mathbf{V}$

Orthogonal complement of **S** is defined as;

$$\mathbf{S}^{\perp} = \left\{ \alpha \in \mathbf{V} / \alpha^{T} \beta = 0, \forall \beta \in \mathbf{S} \right\}$$

In the same way, orthogonal complement of $\mathbf{V}$ is defined as;

$$\mathbf{V}^{\perp} = \left\{ \alpha \in \mathbf{V} / \alpha^{T} \beta = 0, \forall \beta \in \mathbf{V} \right\}$$

Step-2

Suppose,

$\alpha \in \mathbf{V}^{\perp}$

this implies $\alpha^{T} \beta = 0, \forall \beta \in \mathbf{V}$

this implies $\alpha^{T} \beta = 0, \forall \beta \in \mathbf{S}$ (since $\mathbf{S} \subset \mathbf{V}$)

this implies $\alpha \in \mathbf{S}^{\perp}$

Therefore, $\boxed{\mathbf{V}^{\perp} \subset \mathbf{S}^{\perp}}$