Step-1

Given statement is that if v_1, v_2, v_3, v_4 is a basis for the vector space $\mathbf{R}^4$, and if $\mathbf{W}$ is a subspace, then some subset of the $v's$ is a basis for $\mathbf{W}$. We have to give a counter example for this statement.

Step-2

Consider $\beta = \{(1,0,0,0,),(0,1,0,0)(0,0,1,0)(0,0,0,1)\}$, a basis for R^4

Because $(x_1, x_2, x_3, x_4) = x_1(1,0,0,0) + x_2(0,1,0,0) + x_3(0,0,1,0) + x_4(0,0,0,1)$ and vectors are linearly independent.

Let $\mathbf{W}$ be the subspace spanned by the vector $(1,2,3,4)$.

But no vector $(1,0,0,0),(0,1,0,0)\ (0,0,1,0),(0,0,0,1)$ is in the subspace $\mathbf{W}$.

Therefore no subset of the $v's$ form a basis for $\mathbf{W}$.