

## Step-1

a) Given that a skew symmetric matrix satisfies  $K^T = -K$ , as in

$$K = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}.$$

In the 3 by 3 case, we have to find why is  $\det(-K) = (-1)^3 \det(K)$  and on the other hand,  $\det(K^T) = \det(K)$ , and we have to deduce that the determinant must be zero.

## Step-2

$$\begin{aligned} K^T &= \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix} \\ &= -K \\ &= (-1)^3 K \end{aligned}$$

## Step-3

So we get

$$\begin{aligned} \det K^T &= \det(-K) \\ &= \det K \end{aligned}$$

## Step-4

For any  $n \times n$  matrix  $A$ , we have

$$\det(tA) = t^n \det A$$

So for a  $3 \times 3$  matrix  $K$  we have

$$\begin{aligned} \det(-K) &= (-1)^3 \det K \\ &= -\det K \end{aligned}$$

## Step-5

Therefore for skew symmetric matrix  $K$  we get

$$\det K = -\det K \text{ and hence } 2 \det K = 0$$

Giving that  $\det K = 0$

## Step-6

b) We have to write down a 4 by 4 skew symmetric matrix with  $\det K \neq 0$

Consider

$$K = \begin{bmatrix} 0 & 1 & 0 & 2 \\ -1 & 0 & 3 & -1 \\ 0 & -3 & 0 & 2 \\ -2 & 1 & -2 & 0 \end{bmatrix}, \text{ where } K \text{ is a skew symmetric matrix of order } 4 \times 4.$$

## Step-7

$$\det K = - \begin{vmatrix} -1 & 3 & -1 \\ 0 & 0 & 2 \\ -2 & -2 & 0 \end{vmatrix} - 2 \begin{vmatrix} -1 & 0 & 3 \\ 0 & -3 & 0 \\ -2 & 1 & -2 \end{vmatrix}$$

$$= -[-2(2+6)] - 2[(-3)(2+6)]$$

$$= 64$$

$$\neq 0$$

## Step-8

$$k = \begin{bmatrix} 0 & 1 & 0 & 2 \\ -1 & 0 & 3 & -1 \\ 0 & -3 & 0 & 2 \\ -2 & 1 & -2 & 0 \end{bmatrix}$$

Thus  $k$  is an example of a 4 by skew symmetric matrix with  $\det K \neq 0$ .