Step-1

a) We have to construct a subset of the *x-y* plane $\mathbf{R}^2$ that is closed under vector addition and subtraction, but not scalar multiplication.

Consider the set $A=\{(u,v)\,|\,u,v \text{ are rational numbers}\}$ â€œa rational number is in the

for $\frac{p}{q}$ where p,q are integers and $q\neq 0$ â€

It is known that the sum of the two rational numbers is again a rational number.

The product of a rational number and irrational number is irrational number.

For example, â€˜2â€™ is rational, $\sqrt{2}$ is irrational from the product $2\sqrt{2}$ is irrational.

Clearly *A* is a subset of $\mathbf{R}^2$.

Step-2

If $(u_1,v_1),(u_2,v_2)\in A$

Then $(u_1,v_1)+(u_2,v_2)=(u_1+u_2,v_1+v_2)\in A$

Therefore *A* is closed under vector addition.

But it is not closed under scalar multiplication

$\left(\frac{2}{3},7\right)\in A$ and $\sqrt{2}$ is a scalar

Now $\sqrt{2}\left(\frac{2}{3},7\right)=\left(\frac{2\sqrt{2}}{3},7\right)\notin A$

Therefore, *A* is closed under vector addition but not scalar multiplication.

Step-3

b) We have to construct a subset of the *x-y* plane $\mathbf{R}^2$ that is closed under scalar multiplication, but not under vector addition.

Consider $B=\{(u,v)\,|\,u,v\in R \text{ such that } u=0 \text{ or } v=0\}$

Then *B* is closed under scalar multiplication.

$a \in \mathbf{R}, (u,v) \in B$ i.e., $u = 0$ or $v = 0$

Now

$a(u,v) = (au, av) \in B$

Since $au, av \in \mathbf{B}$ and easier $au = 0$ or $av = 0$

So B is closed under scalar multiplication.

Step-4

But B is not closed under vector addition

For, let $(3,0),(0,4) \in B$

But $(3,0)+(0,4) = (3+0, 0+4) = (3,4) \notin B$

Since $3 \neq 0$ and $4 \neq 0$