Step-1

We have to find that which step is not justified in the following:

A has right inverse B

$\Rightarrow AB = I$

$\Rightarrow A^T AB = A^T$ or $B = \left(A^T A\right)^{-1} A^T$

But that satisfies $BA = I$; it is a left-inverse.

Step-2

We do not have $B = \left(A^T A\right)^{-1} A^T$

Because the inverse of $\left(A^T A\right)$ can exists or not

For example

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Step-3

$$\Rightarrow AB = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= I$$

Step-4

And

$$\left(A^T A\right) = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Step-5

Determinant of $\left(A^T A\right)$

$$= 1(2-1) - 1(1-0)$$
$$= 1-1$$
$$= 0$$

Therefore $\left(A^T A\right)$ has no inverse.

Hence $B = \left(A^T A\right)^{-1} A^T$ is not justified