Step-1

This tells us that k must be equal to 1. There is a condition $z(0) = -2$.

Thus,

$$\begin{aligned} z &= e^{kt} + C_1 \\ -2 &= e^{k(0)} + C_1 \\ &= 1 + C_1 \\ C_1 &= -3 \end{aligned}$$

Step-2

Now consider the differential equation $\frac{dy}{dt} = 4y + 3z$, where $y(0) = -5$.

Let $y = e^{mt} + e^{kt} + A$ be the solution of the above differential equation.

Then,

$$\begin{aligned} 4\left(e^{mt} + e^{kt} + A\right) + 3\left(e^{kt} + C_1\right) &= \frac{d}{dt}\left(e^{mt} + e^{kt} + A\right) \\ 4e^{mt} + 7e^{kt} + 4A + 3C_1 &= me^{mt} + ke^{kt} \end{aligned}$$

Therefore, $m = 4$.

We have $y(0) = -5$. Also, we have $C_1 = -3$. This gives $A = 1$.

Step-3

Thus, the solution of the differential equation is $\boxed{y = e^{4t} + e^{t}}$.