Step-1

Consider the following matrix:

$$A=\begin{pmatrix} s & -4 & -4 \\ -4 & s & -4 \\ -4 & -4 & s \end{pmatrix}$$

The objective is to find the values of s for which the matrix A is positive definite (or) all eigenvalues of A are $\lambda > 0$.

Step-2

The determinant of the matrix A is,

$$\begin{aligned} \det A &= \det\begin{pmatrix} s & -4 & -4 \\ -4 & s & -4 \\ -4 & -4 & s \end{pmatrix} \\ &= s\left(s^2-16\right)+4\left(-4s-16\right)-4\left(16+4s\right) \\ &= s^3-48s-128 \\ &= \left(s-8\right)\left(s+4\right)^2 \end{aligned}$$

Step-3

The matrix A is positive definite if

$$\begin{aligned} \det A &> 0 \\ \left(s-8\right)\left(s+4\right)^2 &> 0 \\ s-8 &> 0 \quad \left(\text{Since } \left(s+4\right)^2 \geq 0 \text{ for all } s\right) \\ s &> 8 \end{aligned}$$

Therefore, the matrix A is positive definite for $\boxed{s>8}$.

Step-4

Consider the following matrix:

$$B=\begin{pmatrix} t & 3 & 0 \\ 3 & t & 4 \\ 0 & 4 & t \end{pmatrix}$$

The objective is to find the values of t for which the matrix B is positive definite (or) all eigenvalues of B are $\lambda > 0$.

The determinant of the matrix B is,

$$\begin{aligned}\det B &= \det\begin{pmatrix} t & 3 & 0 \\ 3 & t & 4 \\ 0 & 4 & t \end{pmatrix} \\ &= t\left(t^2-16\right)-3(3t)+0 \\ &= t^3-25t \\ &= t\left(t^2-25\right)\end{aligned}$$

Step-5

The matrix B is positive definite if

$$\det B > 0$$

$$t\left(t^2-25\right) > 0$$

$$\begin{aligned} &t>0, t^2-25>0 \ \text{(or)}\ t<0, t^2-25<0 \\ &t>0, (t-5)(t+5)>0 \ \text{(or)}\ t<0, (t-5)(t+5)<0 \\ &t>0, t\in(-\infty,-5)\cup(5,\infty) \ \text{(or)}\ t<0, t\in(-5,5)\end{aligned}$$

$$t\in(5,\infty) \ \text{(or)}\ t\in(-5,0)$$

Therefore, the matrix B is positive definite for $\boxed{t\in(-5,0)\ \text{(or)}\ t\in(5,\infty)}$.