Step-1

A is diagonalizable and is $A = S\Lambda S^{-1}$

Then we follow that $\left(S\Lambda S^{-1}\right)^T = \left(S^{-1}\right)^T \left(S\Lambda\right)^T$ by the properties of transposition of matrices

$=\left(S^T\right)^T\left(\Lambda^T S^T\right)$ because $S^{-1} = S^T$

$= S\Lambda S^T$
$= S\Lambda S^{-1}$

Thus, we have shown that $\left(S\Lambda S^{-1}\right)^T = S\Lambda S^{-1}$

So, $S\Lambda S^{-1}$ is symmetric.

Step-2

Further, we consider $S^T S =$

$= S^{-1}S$ while $S^T = S^{-1}$

$= I$

So, we follow that $s_i^T s_j = 0 \forall i \neq j$ and $s_i^T s_i = 1 \forall i$ where s_i, s_j are the rows of the matrix *S*.

This says that the rows of the matrix *S* are pair wise orthogonal.

Therefore, *S* is an orthogonal matrix.