

Step-1

Given

$$C_1 = |0|$$

$$C_2 = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$$

$$C_3 = \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}$$

$$C_4 = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix}$$

Step-2

(a)

Then determinants are

$$C_1 = |0|$$
$$= \boxed{0}$$

$$C_2 = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$$
$$= 0 - 1$$
$$= \boxed{-1}$$

Step-3

Then

$$\begin{aligned}
 C_3 &= \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} \\
 &= 0 \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} \\
 &= 0 - 0 \\
 &= \boxed{0}
 \end{aligned}$$

Step-4

And

$$\begin{aligned}
 C_4 &= \begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix} \\
 &= - \begin{vmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}
 \end{aligned}$$

$$\begin{aligned}
 &= \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \\
 &= \boxed{1}
 \end{aligned}$$

Step-5

(b)

Now

$$C_5 = \begin{vmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{vmatrix}$$

$$= - \begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix}$$

$$= - \left[\begin{array}{ccc|ccc} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{array} \right]$$

$$= \boxed{0}$$

Step-6

From the above calculations, we observed that

$$C_5 = 0 = -C_{(5-2)}$$

$$= -C_3$$

$$= 0$$

$$C_4 = 1 = -C_{(4-2)}$$

$$= -C_2$$

$$= -(-1)$$

$$= 1$$

$$C_3 = 0 = -(C_{3-2})$$

$$= -C_1$$

$$= 0$$

In general

We can observe

$$C_n = -C_{n-2}$$

Step-7

So

$$C_{10} = -C_8$$

$$= C_6$$

$$= -C_4$$

$$= \boxed{-1}$$

Thus

$$C_{10} = \boxed{-1}$$