Step-1

Consider the following proof:

A projection matrix P is given by $P = A\left(A^{\mathrm{T}}A\right)^{-1}A^{\mathrm{T}}$

Therefore, $\det(P) = \det\left(A\left(A^{\mathrm{T}}A\right)^{-1}A^{\mathrm{T}}\right)$.

This is same as $|A|\dfrac{1}{\left|A^{\mathrm{T}}\right||A|}\left|A^{\mathrm{T}}\right|$.

Now, cancel $|A|$ and $|A^{\mathrm{T}}|$ and thus get $\det P = 1$.

Step-2

Although this proof seems to be correct, the matrix A may be rectangular. In such case A^{T} is also rectangular.

Determinant can be defined of a square matrix only. Determinant of a rectangular matrix is not defined.

So, if A is rectangular then, $\boxed{\det\left(A^{T}A\right) \neq \det\left(A^{T}\right)\det(A)}$

Thus, the expression $|A|\dfrac{1}{\left|A^{\mathrm{T}}\right||A|}\left|A^{\mathrm{T}}\right|$ is meaningless.

This is the mistake in the above proof.