Step-1

Given data is $b=0$ at $t=0$

$b=0$ at $t=1$

$b=12$ at $t=3$

We have to best fit this data with a straight line.

Step-2

Let $C+Dt=b$ be the required straight line that best fits the given data, then

$C+0.D=0$

$C+1.D=0$

$C+3.D=12$

In matrix form,

$$\begin{bmatrix}1 & 0\\1 & 1\\1 & 3\end{bmatrix}\begin{bmatrix}C\\D\end{bmatrix}=\begin{bmatrix}0\\0\\12\end{bmatrix}$$

$\Rightarrow Ax=b$

where $A=\begin{bmatrix}1 & 0\\1 & 1\\1 & 3\end{bmatrix}$, $x=\begin{bmatrix}C\\D\end{bmatrix}$ and $b=\begin{bmatrix}0\\0\\12\end{bmatrix}$

Step-3

$$A^TA=\begin{pmatrix}1 & 1 & 1\\0 & 1 & 3\end{pmatrix}\begin{bmatrix}1 & 0\\1 & 1\\1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1+1+1 & 0+1+3\\0+1+3 & 0+1+9\end{bmatrix}$$

$$=\begin{bmatrix}3 & 4\\4 & 10\end{bmatrix}$$

$$\left(A^TA\right)^{-1}=\frac{1}{14}\begin{bmatrix}10 & -4\\-4 & 3\end{bmatrix}$$

$$\left(A^T A\right)^{-1} A^T = \frac{1}{14}\begin{bmatrix} 10 & -4 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \end{bmatrix}$$

$$= \frac{1}{14}\begin{bmatrix} 10 & 6 & -2 \\ -4 & -1 & 5 \end{bmatrix}$$

Step-4

Let $\hat{x} = \begin{bmatrix} \hat{C} \\ \hat{D} \end{bmatrix}$

$$\hat{x} = \left(A^T A\right)^{-1} A^T b$$

$$= \frac{1}{14}\begin{bmatrix} 10 & 6 & -2 \\ -4 & -1 & 5 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$$

$$= \frac{1}{14}\begin{bmatrix} -24 \\ 60 \end{bmatrix}$$

$$= \begin{bmatrix} -12/7 \\ 30/7 \end{bmatrix}$$

Therefore $\hat{C} = \frac{-12}{7}, \hat{D} = \frac{30}{7}$ Â Â Â

Hence required straight line is $\boxed{b = \frac{-12}{7} + \left(\frac{30}{7}\right)t}$