Step-1

If we let C_n to denote the n by n matrices, with the same properties, then it is obviously clear that no exchange of rows can produce 1â€™s along the diagonal of C_1.

Consider $C_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Exchanging first and second row gives all 1â€™s along the main diagonal.

Consider $C_3 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. In order to have 1 in the first row and first column, we need to exchange first row and second row. This gives $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$. Thus, we cannot have a 1 in the third row and third column. Thus, in case of C_3 also, no exchange of rows can produce only 1â€™s along the main diagonal.

Consider $C_4 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$. Exchange row 1 and row 2. Then exchange row 3 and row 4. This will make all 1â€™s along the main diagonal.

Step-2

Let us show that if n is even, it is always possible to exchange rows to obtain all 1â€™s along the main diagonal. Exchange first row and second row. Then exchange third row and fourth row. Then exchange fifth row and sixth row. Continue this process and finally exchange $n-1^{st}$ row and n^{th} row. This makes all 1â€™s along the main diagonal.

Step-3

When n is even, either it is divisible by 4 or it is not divisible by 4. Thus, we can write $n = 4k$ or $n = 4k + 2$, where k is some integer.

Consider the following:

$$\begin{aligned} n &= 4k \\ \frac{n}{2} &= \frac{4k}{2} \\ &= 2k \end{aligned}$$

Thus, in such case, row exchange is done $2k$ number of times. That is, row exchange is done even number of times. Every row exchange changes the sign of the determinant. When row exchange is done even number of times, this is same as applying proper permutation to C_n. Since the value of the determinant will remain the same along with the sign (due to even number of exchanges), this is same as even permutation.

Step-4

Consider the following:

$$n = 4k + 2$$
$$\frac{n}{2} = \frac{4k+2}{2}$$
$$= 2k + 1$$

Thus, in such case, row exchange is done $2k+1$ number of times. That is, row exchange is done odd number of times. Every row exchange changes the sign of the determinant. When row exchange is done odd number of times, this is same as applying proper permutation to C_n. Since the value of the determinant will remain the same but the sign will change (due to odd number of exchanges), this is same as odd permutation.

Step-5

Thus, for $n = 4, 8, 12, \ldots$ we get even permutation and when $n = 2, 6, 10, \ldots$ we get odd permutation.

It may be noted that when n is an odd number, there is no exchange of rows possible, due to which we can get all 1â€™s along the main diagonal.