Step-1

Consider a function $F = x^2y^2 - 2x - 2y$. Objective is to determine whether the function has a minimum at the point $(x, y) = (1, 1)$ or not.

For the extreme points, the partial derivatives of function F with respect to x and y should vanish at $(1, 1)$. So, calculate the following:

$$\begin{aligned}\frac{\partial F}{\partial x} &= \frac{\partial}{\partial x}\left(x^2y^2 - 2x - 2y\right) \\ &= 2xy^2 - 2\end{aligned}$$

$$\begin{aligned}\frac{\partial F}{\partial y} &= \frac{\partial}{\partial y}\left(x^2y^2 - 2x - 2y\right) \\ &= 2x^2y - 2\end{aligned}$$

At the point $(x, y) = (1, 1)$,

$$\begin{aligned}\frac{\partial F}{\partial x} &= 2\cdot 1\cdot 1^2 - 2 \\ &= 2 - 2 \\ &= 0\end{aligned}$$

$$\begin{aligned}\frac{\partial F}{\partial y} &= 2 - 2 \\ &= 0\end{aligned}$$

Step-2

It shows that $(1, 1)$ is a stationary point of F. Now, calculate the second partial derivatives of F at the point $(1, 1)$ and get,

$$\begin{aligned}\frac{\partial^2 F}{\partial x^2} &= \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial x}\right) \\ &= \frac{\partial}{\partial x}\left(2xy^2 - 2\right) \\ &= 2y^2\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 F}{\partial y^2} &= \frac{\partial}{\partial y}\left(\frac{\partial F}{\partial y}\right) \\ &= \frac{\partial}{\partial y}\left(2x^2y - 2\right) \\ &= 2x^2\end{aligned}$$

$$\frac{\partial^2 F}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial y}\right)$$
$$= \frac{\partial}{\partial x}\left(2x^2 y - 2\right)$$
$$= 4xy$$

And then

$$\frac{\partial^2 F}{\partial x^2}(1,1) = 2, \frac{\partial^2 F}{\partial y^2}(1,1) = 2, \frac{\partial^2 F}{\partial x \partial y}(1,1) = 4.$$

Observe that

$\left[\frac{\partial^2 F}{\partial x^2}\right] > 0$ and $\left[\frac{\partial^2 F}{\partial x^2}\right]\left[\frac{\partial^2 F}{\partial y^2}\right] = \left[\frac{\partial^2 F}{\partial x \partial y}\right]^2$.

That is, function F satisfy the properties for being the positive semi-definite. Therefore, function F does not attain its minimum at the point $(1,1)$.

Step-3

Hence, function F has no minimum at $(1,1)$.