Step-1

(a)

Use row operations to simplify and compute the determinant.

Consider the determinant,

$$A = \begin{vmatrix} 101 & 201 & 301 \\ 102 & 202 & 302 \\ 103 & 203 & 303 \end{vmatrix}$$

Properties of determinants:

(i) If matrix, A has row of zeroâ€™s, then $\det A = 0$

(ii) Adding a multiple of one row to another row leaves same determinant.

(iii) If row of matrix, A is multiplied by constant, k resulting determinant is $k|A|$

Subtract first row from second and third rows.

$$A \sim \begin{vmatrix} 101 & 201 & 301 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{vmatrix}$$

Subtract 2 times of second row from third row.

$$= \begin{vmatrix} 101 & 201 & 301 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{vmatrix}$$

$=0$ Since third row of determinant has zeroâ€™s

Therefore, the determinant value is $\boxed{0}$.

Step-2

(b)

Consider the determinant,

$$\begin{vmatrix} 1 & t & t^2 \\ t & 1 & t \\ t^2 & t & 1 \end{vmatrix}$$

Subtract the first row from second row and third rows.

$$=\begin{vmatrix}1 & t & t^2\\ t-1 & 1-t & t-t^2\\ t^2-1 & 0 & 1-t^2\end{vmatrix}$$

$$=\begin{vmatrix}1 & t & t^2\\ -(1-t) & 1-t & t(1-t)\\ -(1-t^2) & 0 & 1-t^2\end{vmatrix}$$

$$=(1-t)(1-t^2)\begin{vmatrix}1 & t & t^2\\ -1 & 1 & t\\ -1 & 0 & 1\end{vmatrix}$$ Takeout common factors

Add first row to second and third rows.

$$=(1-t)(1-t^2)\begin{vmatrix}1 & t & t^2\\ 0 & 1+t & t+t^2\\ 0 & t & 1+t^2\end{vmatrix}$$

Subtract third row from second row.

$$=(1-t)(1-t^2)\begin{vmatrix}1 & t & t^2\\ 0 & 1 & t-1\\ 0 & t & 1+t^2\end{vmatrix}$$

Step-3

Subtract third row from second row.

$$=(1-t)(1-t^2)\begin{vmatrix}1 & 0 & -1\\ 0 & 1 & t-1\\ 0 & t & 1+t^2\end{vmatrix}$$

$$=(1-t)(1-t^2)\left[1(1+t^2)-t(t-1)\right]$$
$$=(1-t)(1-t^2)(1+t)$$
$$=\boxed{(1-t^2)}.$$