Step-1

a) We have to describe the three types of subspaces of $\mathbf{R}^2$.

The three types of subspaces of $\mathbf{R}^2$ are

1. itself $\mathbf{R}^2$

2. the lines in $\mathbf{R}^2$ containing $(0,0)$

3. The subspace $\{(0,0,)\}$

For (2), the lines in $\mathbf{R}^2$ does not containing $(0,0)$ are not subspace of $\mathbf{R}^2$

Example:

$2x+3y=1$ is a line does not containing $(0,0)$,

Let L be the line $2x+3y=1$

$(-1,1),(-4,3)\in L$

But $(-1,1)+(-4,3)=(-5,4)\notin L$

Hence L is not a subspace of $\mathbf{R}^2$.

Step-2

b) We have to describe the five types of subspaces of $\mathbf{R}^4$.

The five types of subspaces of $\mathbf{R}^4$ are

1. The space $\mathbf{R}^4$ itself

2. There dimensional planes $n\cdot v=0$

3. Two dimensional subspaces $n_1\cdot v=0$ and $n_2\cdot v=0$

4. One dimensional subspace lines through $(0,0,0,0)$

5. $\{(0,0,0,0)\}$

Step-3

For (2),

$\{(a,b,c,0) \mid a,b,c \in \mathbf{R}\}$ is a subspace where dimension is three.

$\{(0,b,c,d) \mid b,c,d \in \mathbf{R}\}$ is a subspace whose dimension is three etc.,

For (3),

$\{(a,b,0,0) \mid a,b \in \mathbf{R}\}, \{(0,b,c,0) \mid b,c \in \mathbf{R}\}$, the dimension of three subspaces are two

For (4),

$$\{(a,0,0,0) \mid a \in \mathbf{R}\}, \{(0,b,0,0) \mid b \in \mathbf{R}\}$$
$$\{(0,0,c,0) \mid c \in \mathbf{R}\}, \{(0,0,0,d) \mid d \in \mathbf{R}\}$$

are some subspaces of $\mathbf{R}^4$ whose dimension is one.