Calculus II　第十四章　Section 14.1-14.2　quiz 7

考点:二元函数的极限和连续性.　$= \lim\limits_{(x,y)\to(0,0)} \dfrac{\sin(r\cos\theta \, r\sin\theta)}{r^2} = \lim\limits_{r\to 0} \dfrac{\sin(r^2 \sin\theta \cos\theta)}{r^2 \sin\theta\cos\theta} \cdot r \sin^2\theta\cos\theta = 0$

1. (2022年期末) $\lim\limits_{(x,y)\to(0,0)} \dfrac{\sin(xy^2)}{x^2+y^2} = ($ 0 $)$

2. (2021年期中) $\lim\limits_{(x,y)\to(0,0)} (1+xy)^{\frac{1}{x^2+y^2}} = ($ D $)$　　$\dfrac{k}{1+k^2}$

$= \lim\limits_{(x,y)\to(0,0)} \left[(1+xy)^{\frac{1}{xy}} \right]^{\frac{xy}{x^2+y^2}}$　$\boxed{\dfrac{xy}{x^2+y^2}}$ does not exist

(A) 0　　　　(B) 1　　　　(C) e　　　　(D) does not exist

3. (2021年期中) Let $f(x,y) = y \boxed{\sin \frac{1}{x^2+y^2}}^{\leq 1}$, if $(x,y) \neq (0,0)$; $f(x,y) = 0$, if $(x,y) = (0,0)$.

Is $f(x,y)$ continuous at $(0,0)$?　$\lim\limits_{(x,y)\to(0,0)} f(x,y) = \lim\limits_{r\to 0} r\sin\theta \sin\frac{1}{r^2} = 0$

continuous

4. (2021年期中) Find the limit, if it exists, or show that the limit does not exist.

(1) $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy}{\sqrt{x^2+y^2}}$ 　$= \lim\limits_{r\to 0} \dfrac{r^2 \sin\theta\cos\theta}{r} = 0$　　(2) $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy^3 + 2x^2 y^4}{x^2 + y^6}$ 　$x = ky^3$ 　$= \lim\limits_{y\to 0} \dfrac{ky^6 + 2k^2 y^{10}}{(k^2+1)y^6} = \lim\limits_{y\to 0} \dfrac{k+2k^2 y^4}{k^2+1} = \dfrac{k}{k^2+1}$ does not exist

5. (2019年期末) Compute the limit: $\lim\limits_{(x,y)\to(0,0)} \dfrac{\sqrt{x^2+y^2+1}-1}{x^2+y^2} = ($ $\frac{1}{2}$ $)$.

$= \lim\limits_{(x,y)\to(0,0)} \dfrac{1}{\sqrt{x^2+y^2+1}+1}$

6. (2019年期中) Is the following function $f(x,y)$ continuous at $(0,0)$? Give reasons for your answer.　$\lim\limits_{(x,y)\to(0,0)} f(x,y) = \lim\limits_{(x,y)\to(0,0)} \dfrac{\sin(x^3+y^3)}{x^3+y^3} \cdot \dfrac{x^3+y^3}{x^2+y^2} = \lim\limits_{r\to 0} \dfrac{(r\cos\theta)^3 + (r\sin\theta)^3}{r^2} = 0$

$f(x,y) = \dfrac{\sin(x^3+y^3)}{x^2+y^2}$, if $(x,y) \neq (0,0)$; $f(x,y) = 0$, if $(x,y) = (0,0)$

7. (2018年期末) Let $f(x,y) = \dfrac{xy(x^2-y^2)}{x^2+y^2}$, if $(x,y) \neq (0,0)$; $f(x,y) = 0$, if $(x,y) = (0,0)$.

Show that $f(x,y)$ is continuous at $(0,0)$.

$\lim\limits_{(x,y)\to(0,0)} f(x,y) = \lim\limits_{r\to 0} \dfrac{r\cos\theta \cdot r\sin\theta \left((r\cos\theta)^2 - (r\sin\theta)^2 \right)}{r^2}$

$= \lim\limits_{r\to 0} r^2 (\cos^2\theta - \sin^2\theta) \sin\theta\cos\theta = 0$