Step-1

a) Given that $A = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 2 \end{bmatrix}$

We have to find the rank of A.

Step-2

Now

$$A = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 1(2) & 1(-1) & 1(2) \\ 4(2) & 4(-1) & 4(2) \\ 2(2) & 2(-1) & 2(2) \end{bmatrix}$$
$$= \begin{bmatrix} 2 & -1 & 2 \\ 8 & -4 & 8 \\ 4 & -2 & 4 \end{bmatrix}$$

Step-3

Since any two rows of A are proportional and we know that if the rows of any

matrix are proportional then the determinant of that matrix is zero.

So $\det A = 0$

Hence the determinant of the given matrix is $\boxed{0}$.

Step-4

b) Given matrix is $U = \begin{bmatrix} 4 & 4 & 8 & 8 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

We have to find the determinant of the given matrix.

Step-5

Since U is a triangular matrix and we know that determinant of a triangular matrix

is the product of diagonal entries.

Therefore,

$$\det U = \begin{vmatrix} 4 & 4 & 8 & 8 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 2 \end{vmatrix}$$

$= 4.1.2.2$

$= \boxed{16}$

Hence $\boxed{\det U = 16}$

Step-6

c) Given upper triangular matrix is $U = \begin{bmatrix} 4 & 4 & 8 & 8 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

Then $U^T = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 \\ 8 & 2 & 2 & 0 \\ 8 & 2 & 6 & 2 \end{bmatrix}$ is a lower triangular matrix.

We have to find the determinant of U^T

Step-7

Since U^T is a triangular matrix and we know that determinant of a triangular

matrix is the product of diagonal entries.

Therefore,

$$\det U^T = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 \\ 8 & 2 & 2 & 0 \\ 8 & 2 & 6 & 2 \end{bmatrix}$$

$= 4.1.2.2$

$= \boxed{16}$

Hence $\boxed{\det U^T = 16}$

Step-8

d) We have to find the determinant of the inverse matrix U^{-1}.

We know that $\det A^{-1} = \dfrac{1}{\det A}$, for any matrix A.

Therefore,

Step-9

$$\det U^{-1} = \frac{1}{\det U}$$
$$= \frac{1}{16}$$

Hence $\boxed{\det U^{-1} = \dfrac{1}{16}}$

Step-10

e) Given the reverse triangular matrix is $M = \begin{bmatrix} 0 & 0 & 0 & 2 \\ 0 & 0 & 2 & 6 \\ 0 & 1 & 2 & 2 \\ 4 & 4 & 8 & 8 \end{bmatrix}$.

We have to find the determinant of the given matrix M.

Step-11

The reverse triangular matrix M results from two exchanges (1) row 1 and row 4

(ii) row 2 and row 3 from the

So M results from U by even number of row exchanges and hence $\det M = \det U$

Since $\det U = 16$ and $\det M = \det U$.

So $\det M = 16$

Hence the determinant of the given reverse triangular matrix is $\boxed{\det M = 16}$.