Step-1

Given $a = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \quad c = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$

We have to find the orthogonal vectors A, B, C by Gram-Schmidt from a, b, c

Step-2

We know that

$$A = a = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

Step-3

And $B = b - P_A$, where

$$P_A = A\frac{A^T b}{A^T A}$$

Now

$$A^T b = \begin{bmatrix} 1 & -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}$$

$$= 0 - 1 + 0 + 0$$
$$= -1$$

Step-4

$$A^T A = \begin{bmatrix} 1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$$= 1 + 1 + 0 + 0$$

$$= 2$$

Step-5

$$P_A = \frac{-1}{2} \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1/2 \\ 1/2 \\ 0 \\ 0 \end{bmatrix}$$

Step-6

Therefore

$$B = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} - \begin{bmatrix} -1/2 \\ 1/2 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 \\ 1/2 \\ -1 \\ 0 \end{bmatrix}$$

Step-7

$C = c - P_A - P_B$, where

$$P_A = A \frac{A^T c}{A^T A}, P_B = B \frac{B^T c}{B^T B}$$

Step-8

Now

$$A^T c = \begin{bmatrix} 1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$
$$= 0 - 0 + 0 - 0$$
$$= 0$$

Step-9

$$A^T A = \begin{bmatrix} 1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$
$$= 1 + 1 + 0 + 0$$
$$= 2$$

$$B^T c = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$
$$= 0 + 0 - 1 - 0$$
$$= -1$$

Step-10

$$B^T B = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -1 & 0 \end{bmatrix} \begin{bmatrix} 1/2 \\ 1/2 \\ -1 \\ 0 \end{bmatrix}$$
$$= \frac{1}{4} + \frac{1}{4} + 1 + 0$$
$$= \frac{3}{2}$$

Step-11

$$P_A = \frac{0}{2}\begin{bmatrix}1\\-1\\0\\0\end{bmatrix}$$

$$= \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

Step-12

$$P_B = \frac{-1}{3/2}\begin{bmatrix}1/2\\1/2\\-1\\0\end{bmatrix}$$

$$= -\frac{2}{3}\begin{bmatrix}1/2\\1/2\\-1\\0\end{bmatrix}$$

$$= \begin{bmatrix}-1/3\\-1/3\\2/3\\0\end{bmatrix}$$

Step-13

Therefore

$$C = \begin{bmatrix}0\\0\\1\\-1\end{bmatrix} - \begin{bmatrix}0\\0\\0\\0\end{bmatrix} - \begin{bmatrix}-1/3\\-1/3\\2/3\\0\end{bmatrix}$$

$$= \begin{bmatrix}1/3\\1/3\\1/3\\-1\end{bmatrix}$$

Hence $A = (1,-1,0,0),\ B = \left(\frac{1}{2},\frac{1}{2},-1,0\right),\ C = \left(\frac{1}{3},\frac{1}{3},\frac{1}{3},-1\right)$ Â