

Step-1

Consider the parabola,

$$Y = A + Bx + Cx^2$$

By substituting $Y = 4$ and $x = a$, we get,

$$A + Ba + Ca^2 = 4$$

By substituting $Y = 5$ and $x = b$, we get,

$$A + Bb + Cb^2 = 5$$

By substituting $Y = 6$ and $x = c$, we get,

$$A + Bc + Cc^2 = 6$$

Therefore, we get,

$$\begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

The determinant is given by,

$$\begin{aligned} D &= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \\ &= \begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix} \\ &= -(a-b) \begin{vmatrix} 0 & b^2-c^2 \\ 1 & c^2 \end{vmatrix} + (a^2-b^2) \begin{vmatrix} 0 & b-c \\ 1 & c \end{vmatrix} \\ &= -(a-b)[0-(b^2-c^2)] + (a^2-b^2)[0-(b-c)] \\ D &= -(a-b)(c-b)(c+b) + (a^2-b^2)(c-b) \end{aligned}$$

$$\begin{aligned}
&= (c-b) \left[a^2 - b^2 - (a-b)(c+b) \right] \\
&= (c-b) \left[a^2 - b^2 - ac - ab + bc + b^2 \right] \\
&= (c-b) \left[a(a-c) - b(a-c) \right] \\
D &= (c-b)(a-c)(a-b)
\end{aligned}$$

The parabola $Y = A + Bx + Cx^2$ will be possible for all values of a, b, and c except $a \neq b \neq c$.

Step-2

Thus, the parabola $Y = A + Bx + Cx^2$ will be possible for all values of a, b, and c except $a \neq b \neq c$.