Southern University of Science and Technology
Advanced Linear Algebra Spring 2023

MA109– Quiz #4

2023/03/16

Name: ______________________________

Student Number: ______________________________

1. Suppose V and W are 2-dimensional vector spaces, try to construct $T \in \mathcal{L}(V, W)$ such that the matrix of T with respect to a basis of V and a basis of W satisfies $(\mathcal{M}(T))^2 = 0$ and $\mathcal{M}(T) \neq 0$.

   设 V 和 W 均是 2 维向量空间，构造 $T \in \mathcal{L}(V, W)$，使得 T 在 V 的一组基和 W 的一组基下的矩阵满足 $(\mathcal{M}(T))^2 = 0$ 且 $\mathcal{M}(T) \neq 0$.

   Proof. Let v_1, v_2 be a basis of V, w_1, w_2 be a basis of W. Suppose a map $T : V \to W$ satisfies $Tv_1 = w_2$, $Tv_2 = 0$. It's easy to check $T \in \mathcal{L}(V, W)$.

   Then $\forall v \in V$, $\exists a_1, a_2 \in \mathcal{F}$, s.t. $v = a_1 v_1 + a_2 v_2$, then $Tv = a_1 w_2$, the matrix of T w.r.t. bases v_1, v_2 and w_1, w_2 is

$$\mathcal{M}(T) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

   then $\mathcal{M}(T)$ satisfies the condition above. □

2. Are $\mathbf{R}^2$ and $\mathbf{C}^2$ isomorphic as vector spaces? If they are isomorphic, please give the proof; if not, please give the reason.

   请问 $\mathbf{R}^2$ 和 $\mathbf{C}^2$ 作为向量空间是否同构？如果是，请给出证明；如果不是，请说明理由.

   Proof. They may not be isomorphic. Since $\mathbf{R}^2$ is a vector space over $\mathbf{R}$, C^2 is a vector space over $\mathbf{C}$, their are not isomorphic. $\Rightarrow$ not isomorphic $\square$