

Step-1

Let $A = S\Lambda S^{-1}$. Matrix B is given as follows:

$$B = \begin{bmatrix} A & 0 \\ 0 & 2A \end{bmatrix}$$

Then to diagonalize the block matrix and to determine its Eigen values and Eigen vector matrices.

Step-2

To diagonalize the block matrix B means to write the matrix in the following form:

$$B = S_B \Lambda_B S_B^{-1}$$

Here, S_B , and Λ_B represents Eigen vector matrices and Eigen value matrices.

Step-3

Let $A = S\Lambda S^{-1}$, then:

$$2A = S(2\Lambda)S^{-1}$$

Substitute the values in matrix B and do the following calculations:

$$\begin{aligned} B &= \begin{bmatrix} S\Lambda S^{-1} & 0 \\ 0 & S(2\Lambda)S^{-1} \end{bmatrix} \\ &= \begin{bmatrix} S & 0 \\ 0 & S^{-1} \end{bmatrix} \begin{bmatrix} \Lambda & 0 \\ 0 & 2\Lambda \end{bmatrix} \begin{bmatrix} S^{-1} & 0 \\ 0 & S^{-1} \end{bmatrix} \\ &= S_B \Lambda_B S_B^{-1} \end{aligned}$$

Step-4

Therefore, $\boxed{\Lambda, 2\Lambda}$ are Eigen values and S_B is Eigen vector matrices.