Step-1

Consider A is a square matrix with order $m \times n$, and rank r.

Step-2

(a)

The objective is to determine the conditions for A under which $AA^{-1} = A^{-1}A = I$

Step-3

If A is square matrix of order $m \times n$ with rank r then

The matrix A has 2 sided inverse if $\boxed{m = n = r}$

Step-4

(b)

The objective is to determine the condition for the square matrix A with order $m \times n$ such $Ax = b$ has infinite solutions for every b.

Step-5

If for the system $Ax = b$, A has linearly independent rows then the system has infinitely many solutions for every b.

In that case $r < m, n$

So, the system $Ax = b$ has infinitely many solutions for every b if $r < m, n$