

Step-1

F_6 is the symmetric matrix whose rows and columns are the products of the entries of 6th roots of unity $1, w, w^2, w^3, w^4$, and w^5 .

$$F_6 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & w & w^2 & w^3 & w^4 & w^5 \\ 1 & w^2 & w^4 & w^6 & w^8 & w^{10} \\ 1 & w^3 & w^6 & w^9 & w^{12} & w^{15} \\ 1 & w^4 & w^8 & w^{12} & w^{16} & w^{20} \\ 1 & w^5 & w^{10} & w^{15} & w^{20} & w^{25} \end{bmatrix}$$

Step-2

Intersecting entries of first row with first, third and fifth columns are 1, 1, 1

Intersecting entries of first row with first, third and fifth columns are $1, w^4, w^8$

Intersecting entries of first row with first, third and fifth columns are $1, w^8, w^{16}$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & w^4 & w^8 \\ 1 & w^8 & w^{16} \end{bmatrix}$$

The matrix formed by above entries is

Step-3

In the case of 3 by 3 matrix, we follow that the entries of fourier matrix are nothing but the cube roots of unity. So, we follow $w^3 = 1, w^6 = 1$.

$$\Rightarrow w^4 = w, w^8 = w^6 w^2 = w^2, w^{16} = (w^8)^2 = w^4$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & w & w^2 \\ 1 & w^2 & w^4 \end{bmatrix} = F_3$$

Therefore above matrix is equal to

This is 3 by 3 sub matrix of F_6

Therefore, the required matrix is F_3 .