## Step-1

Let

$$x = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$z = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Let

$$\sigma = \|x\|$$

Therefore,

$$\sigma = \sqrt{(3)^2 + (4)^2}$$
$$= 5$$

Let

$$v = x + \sigma z$$
$$= \begin{bmatrix} 3 \\ 4 \end{bmatrix} + 5\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} 8 \\ 4 \end{bmatrix}$$

Therefore, Householder matrix is given by,

$$H = I - 2\frac{vv^T}{\|v\|^2}$$
$$= I - 2\frac{\begin{bmatrix} 8 \\ 4 \end{bmatrix}\begin{bmatrix} 8 & 4 \end{bmatrix}}{64 + 16}$$
$$= I - 2\frac{\begin{bmatrix} 64 & 32 \\ 32 & 16 \end{bmatrix}}{80}$$

$$= I - 2\frac{16\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}}{80}$$

$$= I - \frac{2}{5}\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{8}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{2}{5} \end{bmatrix}$$

$$H = \begin{bmatrix} -\frac{3}{5} & -\frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{bmatrix}$$

Therefore,

$$Hx = \begin{bmatrix} -\frac{3}{5} & -\frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{3}{5}\times 3 & -\frac{4}{5}\times 4 \\ -\frac{4}{5}\times 3 + \frac{3}{5}\times 4 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{9}{5} - \frac{16}{5} \\ -\frac{12}{5} + \frac{12}{5} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{25}{5} \\ 0 \end{bmatrix}$$

$$= -5\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$Hx = -\sigma z$$

Thus, $\boxed{Hx = -\sigma z}$