Step-1

Let $c_1v_1 + c_2v_2 + c_3v_3 = 0$

$$\Rightarrow c_1(w_2 - w_3) + c_2(w_1 - w_3) + c_3(w_1 - w_2) = 0$$
$$\Rightarrow (c_2 + c_3)w_1 + (c_1 - c_3)w_2 + (-c_1 - c_2)w_3 = 0$$

Step-2

So,

$$\Rightarrow c_2 + c_3 = 0$$
$$c_1 - c_3 = 0$$
$$-c_1 - c_2 = 0 \qquad (\text{since } w_1, w_2, w_3 \text{ are linearly independent})$$

But,

$$-c_1 - c_2 = 0$$
$$\Rightarrow c_1 = -c_2$$

And,

$$c_1 - c_3 = 0$$
$$\Rightarrow c_3 = c_1$$

Therefore, $c_3 = c_1 = -c_2$

Step-3

So,

$$c_1v_1 + c_2v_2 + c_3v_3 = 0$$

$$c_1v_1 - c_1v_2 + c_1v_3 = 0$$

Let $c_1 = 1, v_1 - v_2 + v_3 = 0,$ therefore v_1, v_2, v_3 are linear dependent

Therefore, the sum $\boxed{v_1 - v_2 + v_3 = 0}$