

Step-1

Given that the equations of two planes are $x + y + 3z = 6$ and $x - y + z = 4$

We have to find a point with $z = 2$ on the intersection line of the given planes; we have to find a point with $z = 0$ on the intersection line of the given planes, and we have to find a third point halfway between these two.

Step-2

When $z = 2$ the equations becomes,

$$x + y = 0$$

$$x - y = 2$$

The column vector of the above equations is

$$x \begin{pmatrix} 1 \\ 1 \end{pmatrix} + y \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

Step-3

The linear combination for the vectors is $1(\text{first column}) + (-1)(\text{second column})$ that gives right hand side vector and for taking $z = 2$, the required point is $\boxed{(1, -1, 2)}$

Step-4

When $z = 0$ the equations becomes,

$$x + y = 6$$

$$x - y = 4$$

The column vector of the above equations is

$$x \begin{pmatrix} 1 \\ 1 \end{pmatrix} + y \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$$

Step-5

The linear combination for the vectors is $5(\text{first column}) + 1(\text{second column})$ that gives right hand side vector and by taking $z = 0$, the required point is $\boxed{(5, 1, 0)}$

Step-6

The point that is half way between $(1, -1, 2)$ and $(5, 1, 0)$ is $\boxed{(3, 0, 1)}$

Since it is the mid point of the points $(1, -1, 2)$ and $(5, 1, 0)$