Step-1

a) We have to describe a subspace of **M** that contains $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}_{\text{but not}} B = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$

Let **M** be the subspace that contains $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}_{\text{but not}} B = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$

$$M = \left\{ \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} | a \in \mathbf{R} \right\}$$
Let

Clearly **M** is a vector space where vector addition is matrix addition, whose scalar multiplication is a constant multiple of a 2 by 2 matrix.

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \in \mathbf{M}$$
And also
$$\begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} \notin \mathbf{M}$$

i.e., the \boldsymbol{M} is the smallest subspace containing $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

Step-2

b) Yes, it **M** contains *I*.

Since the linear combination

$$1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + (-1) \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= I$$

Therefore if a subspace M contains A and B, must b contain I.

Step-3

c) Consider the subspace of all matrices where main diagonal is all zero

$$M = \left\{ \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix} \middle| a, b \in \mathbf{R} \right\}$$
Let

Then M is a subspace of the vector space of 2×2 matrices.