Calculus II　第十三章　quiz 6

考点:空间曲线的参数方程, 求$\mathbf{T}=\frac{\mathbf{v}}{|\mathbf{v}|}, \mathbf{N}=\frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|}, \kappa=\frac{1}{|\mathbf{v}|}\left|\frac{d\mathbf{T}}{dt}\right|=\frac{|f''(x)|}{[1+(f'(x))^2]^{3/2}}$.

1. (2021年期中) The maximum curvature κ of function $y(x)=\sin x$ is (1).

2. (2021年期中) Assume $\mathbf{r}(t)=f(t)\mathbf{i}+g(t)\mathbf{j}+h(t)\mathbf{k}$, where

$$f(t)=\int_0^t \cos(x^2)\,dx, \quad g(t)=-t\cos t, \quad h(t)=\sum_{n=1}^{\infty}\frac{t^n}{n}.$$

Calculate $\mathbf{r}'(0)$. $=\vec{i}+(-1)\vec{j}+\vec{k}$

3. (2020年期末) Find the $\boxed{\text{points}}$ on the curve $\mathbf{r}(t)=(12\sin t)\mathbf{i}-(12\cos t)\mathbf{j}+5t\mathbf{k}$, at a distance 26π units along the curve from the point $(0,-12,0)$.

$$\int_0^m \sqrt{(12\cos t)^2+(12\sin t)^2+5^2}\,dt = \pm 26\pi$$

$m=\pm 2\pi$　$(0,-12,10\pi)$　$(0,12,-10\pi)$

4. (2019年期末) Find the equation of the osculating circle for the parabola $y=x^2$ at $x=1$. $R=\frac{1}{K}=\frac{5\sqrt{5}}{2}$　$(-4, \frac{7}{2})$

5. (2019年期中) Determine whether the following statements are true or false? No justification is necessary.

(4) If a vector function $\mathbf{r}(t)$ is always perpendicular to its derivative $\frac{d\mathbf{r}}{dt}$, then $|\mathbf{r}(t)|$ must be constant. True　$(\vec{r}\cdot\vec{r})'=2\vec{r}\cdot\frac{dr}{dt}=0$　$=$ Const.

(5) The curvature of a unit circle is greater than the curvature of the parabola $y=x^2$ at the origin. False　1　2.

6. (2019年期中) A particle is located at the point $(1,0,-2)$. Its initial speed is $|\mathbf{v}(0)|=3$ at time $t=0$, and the direction of its initial velocity is toward the point $(2,-1,3)$. $(1,-1,5)$ The particle moves with constant acceleration $\mathbf{i}+2\mathbf{j}+\mathbf{k}$. Find its position vector $\mathbf{r}(t)$ at time t.

$\vec{v}=(t+\frac{\sqrt{3}}{3})\vec{i}+(2t-\frac{\sqrt{3}}{3})\vec{j}+(t+\frac{5\sqrt{3}}{3})\vec{k}$

$\vec{r}(t)=(\frac{1}{2}t^2+\frac{\sqrt{3}}{3}t+1)\vec{i}+(t^2-\frac{\sqrt{3}}{3}t)\vec{j}+(\frac{1}{2}t^2+\frac{5\sqrt{3}}{3}t-2)\vec{k}$

7. (2018年期末) If $\mathbf{r}$ is a differentiable vector function of t of constant length, then $\mathbf{r}\cdot\frac{d\mathbf{r}}{dt}=($ 0 $)$.

8. (2018年期末) Find the unit tangent vector $\mathbf{T}$, the principal unit normal vector $\mathbf{N}$, and the curvature κ for the plane curve $\mathbf{r}(t)=(2t+3)\mathbf{i}+(5-t^2)\mathbf{j}$.

$$\vec{v}(t)=2\vec{i}-2t\vec{j} \qquad \vec{T}=\frac{\vec{v}}{|\vec{v}|}=\frac{1}{\sqrt{t^2+1}}\vec{i}-\frac{t}{\sqrt{t^2+1}}\vec{j}$$

$$\frac{d\vec{T}}{dt}=-\frac{t}{(t^2+1)^{\frac{3}{2}}}\vec{i}-\frac{1}{(t^2+1)^{\frac{3}{2}}}\vec{j}$$

$$K=\frac{1}{|\vec{v}|}\left|\frac{d\vec{T}}{dt}\right|=\frac{1}{2\sqrt{t^2+1}}\cdot\frac{(t^2+1)^{\frac{1}{2}}}{(t^2+1)^{3/2}}=\frac{1}{2(t^2+1)^{3/2}}$$

$$\vec{N}=\frac{d\vec{T}/dt}{|d\vec{T}/dt|}=(t^2+1)\left(-\frac{t}{(t^2+1)^{3/2}}\vec{i}-\frac{1}{(t^2+1)^{3/2}}\vec{j}\right)$$

$$=-\frac{t}{\sqrt{t^2+1}}\vec{i}-\frac{1}{\sqrt{t^2+1}}\vec{j}$$