Step-1

(a) Consider the corner, which satisfies the following:

$$x = 0$$
$$w = b$$

The corners of the feasible set are the basic feasible solutions of the simplex problem $Ax = b$.

A solution is called â€˜basic solutionâ€™ if out of its $m+n$ components, some *n* components are zero.

Similarly, a solution is called â€˜feasible solutionâ€™ if it satisfies the inequality $x \geq 0$.

Step-2

We observe that some components of the solution are zero $(x = 0)$. For the remaining components, we know that $w = b$. We also know that $b \geq 0$.

This gives $w \geq 0$.

Thus, we get

$$x = 0$$
$$w \geq 0$$

Step-3

Thus, some components are zero and other components are positive of the solution. Therefore, the corner is both basic and feasible. Hence, the simplex method can proceed from this point to find the optimal solution.

Step-4

(b) Let $A = \begin{bmatrix} 1 & -1 \end{bmatrix}$ and $b = \begin{bmatrix} 3 \end{bmatrix}$.

The auxiliary problem consists of minimizing $w_1 + \ldots + w_m$, subject to the following constraints:

$$x \geq 0$$
$$w \geq 0$$
$$Ax + w = 3$$

Step-5

In this particular case, the auxiliary problem is as follows:

Minimize $w_1 + w_2$,

subject to the following:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \geq 0$$

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \geq 0$$

$$(1,-1)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = 3$$

Step-6

To obtain the corner of the feasible set, put $x_1 = 0$, $x_2 = 0$. This gives $w_1 + w_2 = 3$. Therefore, we can start with $w_1 = 0$ and $w_2 = 3$. Therefore, the corner of the feasible set is $\boxed{(0,0,0,3)}$. The picture of the set is as follows: