Step-1

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Given matrices $A = \begin{bmatrix} 2 & 1 & 4 & 6 \\ 0 & 3 & 8 & 5 \\ 0 & 0 & 0 & 7 \\ 0 & 0 & 0 & 9 \end{bmatrix}$

We have to show that A is not invertible.

Step-2

Subtracting $\frac{9}{7}$ times row 3 from row 4, we get

$$A = \begin{bmatrix} 2 & 1 & 4 & 6 \\ 0 & 3 & 8 & 5 \\ 0 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the last row consists all zeros.

Hence the matrix A has no inverse.

Step-3

Given that the third row of A^{-1}, multiplying A should give the third row $\begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$ of $A^{-1}A = I$

We have to explain why this is impossible.

Step-4

Let the third row of A^{-1} is $\begin{bmatrix} a & b & c & d \end{bmatrix}$

Then $A^{-1}A = I$

$$\begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} 2 & 1 & 4 & 6 \\ 0 & 3 & 8 & 5 \\ 0 & 0 & 0 & 7 \\ 0 & 0 & 0 & 9 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2a & a+3b & 4a+8b & 6a+5b+7c+9d \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$$

Equating the corresponding elements gives

$2a = 0$, $a + 3b = 0$,$4a + 8b = 1$, $6a + 5b + 7c + 9d = 0$

This has no solution.

Hence A is not invertible.