Step-1

The symmetric factorization $A = LDL^T$ means that $x^T Ax = x^T LDL^T x$

$$\Rightarrow \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} a & b \\ b & c \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{b}{a} & 1 \end{pmatrix} \begin{pmatrix} a & 0 \\ 0 & \frac{ac-b^2}{a} \end{pmatrix} \begin{pmatrix} 1 & \frac{b}{a} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\Rightarrow ax^2 + 2bxy + cy^2$$

$$\Rightarrow \begin{pmatrix} x + \frac{b}{a} y & y \end{pmatrix} \begin{pmatrix} a & 0 \\ 0 & \frac{ac-b^2}{a} \end{pmatrix} \begin{pmatrix} 1 & \frac{b}{a} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Step-2

So,

$$\Rightarrow \begin{pmatrix} a\left(x + \frac{b}{a} y\right) & \left(\frac{ac-b^2}{a}\right) y \end{pmatrix} \begin{pmatrix} \left(x + \frac{b}{a} y\right) \\ y \end{pmatrix}$$

$$\Rightarrow a\left(x + \frac{b}{a} y\right)^2 + \left(\frac{ac-b^2}{a}\right) y^2$$

Therefore, $\boxed{ax^2 + 2bxy + cy^2 = a\left(x + \frac{b}{a} y\right)^2 + \left(\frac{ac-b^2}{a}\right) y^2}$.

Step-3

Now taking $a = 2,\ b = 4,\ c = 10.$

Therefore,

$$2x^2 + 8xy + 10y^2$$

$$\Rightarrow a\left(x+\frac{b}{a}y\right)^2+\left(\frac{ac-b^2}{a}\right)y^2$$

$$\Rightarrow 2\left(x+\frac{4}{2}y\right)^2+\left(\frac{(2)(10)-(4)^2}{2}\right)y^2$$

$$\Rightarrow 2(x+2y)^2+2y^2.$$

Therefore, $\boxed{2x^2+8xy+10y^2=2(x+2y)^2+2y^2}$.