

## Step-1

Let  $\lambda_1, \lambda_2, \dots, \lambda_n$  be Eigen values of  $A$ .

It is given that  $A$  Is positive definite.

This implies;

$$\lambda_1 > 0, \lambda_2 > 0, \dots, \lambda_n > 0.$$

It is known that if  $\lambda_1, \lambda_2, \dots, \lambda_n$  are Eigen values of  $A$  then  $\lambda_1^2, \lambda_2^2, \dots, \lambda_n^2$  are Eigen values of  $A^2$ .

Clearly  $\lambda_1^2 > 0, \lambda_2^2 > 0, \dots, \lambda_n^2 > 0$ .

So the Eigen values of  $A^2$  are all positive.

Thus,  $A^2$  is also positive definite.

## Step-2

It is known that if  $\lambda_1, \lambda_2, \dots, \lambda_n$  are Eigen values of  $A$ , then  $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \dots, \frac{1}{\lambda_n}$  are Eigen values of  $A^{-1}$ .

Clearly,  $\frac{1}{\lambda_1} > 0, \frac{1}{\lambda_2} > 0, \dots, \frac{1}{\lambda_n} > 0$ .

So the Eigen values of  $A^{-1}$  are also positive.

Hence, the matrix  $A^{-1}$  is also positive definite.