

## Step-1

(a)

Let  $A$  be a triangular matrix.

To find the inverse of triangular matrix is triangular or not.

Let  $A = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$  be a triangular matrix.

Then

$$\begin{aligned} A^{-1} &= \frac{1}{2(1) - 0(3)} \begin{bmatrix} 1 & -3 \\ 0 & 2 \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} 1 & -3 \\ 0 & 2 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{2} & -\frac{3}{2} \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Hence,  $A^{-1}$  is also a triangular matrix

## Step-2

(b)

Let  $A$  be a symmetric matrix.

To find the inverse of symmetric matrix is symmetric or not.

The inverse of symmetric matrices is also a symmetric.

Let  $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ , which a symmetric matrix

Since,

$$\begin{aligned} A^T &= A \\ &= \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \end{aligned}$$

Now

$$\begin{aligned}
 A^{-1} &= \frac{1}{2(3)-0(0)} \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \\
 &= \frac{1}{6} \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \\
 &= \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{bmatrix}
 \end{aligned}$$

Now it is clear that  $(A^{-1})^T = A^{-1}$

Hence,  $A^{-1}$  is also a symmetric matrix

### Step-3

(c)

Let  $A$  be a tridiagonal matrix.

To find the inverse of tridiagonal matrix is tridiagonal or not.

The inverse of tridiagonal matrices is also tridiagonal.

Let  $A = \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}$  be a tridiagonal matrix.

Then,

$$\begin{aligned}
 A^{-1} &= \frac{1}{0(0)-2(3)} \begin{bmatrix} 0 & -2 \\ -3 & 0 \end{bmatrix} \\
 &= \frac{1}{-6} \begin{bmatrix} 0 & -2 \\ -3 & 0 \end{bmatrix} \\
 &= \begin{bmatrix} 0 & \frac{1}{3} \\ \frac{1}{2} & 0 \end{bmatrix}
 \end{aligned}$$

Hence,  $A^{-1}$  is also a tridiagonal matrix since its anti diagonal consists of nonzero elements.

### Step-4

(d)

Let  $A$  be a matrix in which all the entries are whole numbers.

Find all the entries of inverse of  $A$ .

The inverse of the matrix  $A$  in which all entries are whole numbers need not consist of all whole numbers.

For instance let  $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$

## Step-5

Then,

$$\begin{aligned} A^{-1} &= \frac{1}{4(5) - 2(3)} \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix} \\ &= \frac{1}{20 - 6} \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix} \\ &= \frac{1}{14} \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix} \\ &= \begin{bmatrix} \frac{5}{14} & -\frac{3}{14} \\ -\frac{1}{7} & \frac{2}{7} \end{bmatrix} \end{aligned}$$

Here the elements of  $A^{-1}$  are rational numbers.

Hence, all entries of  $A^{-1}$  need not to be whole numbers

## Step-6

(e)

Let  $A$  be a matrix in which all entries are fractions.

The inverse of the matrices  $A$  in which all entries of  $A$  are rational numbers consists of all rational numbers.

Since every whole number is also a rational number.

So it follows from the above example that

Hence,  $A^{-1}$  is a matrix in which all entries are rational numbers