Step-1

Consider the solution:

$$u_2 = e^{8t} \left(t x_1 + x_2 \right)$$

By differentiating with respect to t, we get,

$$\frac{du_2}{dt} = e^{8t} \frac{d}{dt} (tx_1 + x_2) + (tx_1 + x_2) \frac{d}{dt} (e^{8t})$$

$$= e^{8t} x_1 + (x_1 + x_2) 8e^{8t}$$

$$= e^{8t} x_1 + 8x_1 e^{8t} + 8x_2 e^{8t}$$

$$= 8x_1 e^{8t} + (8x_2 + x_1) e^{8t}$$

Step-2

Thus,
$$\frac{du_2}{dt} = 8x_1 e^{8t} + (8x_2 + x_1)e^{8t}$$