Step-1

We have to solve by elimination, exchanging row if necessary:

$$\begin{aligned} u+4v+2w&=-2 \\ -2u-8v+3w&=32 \\ v+w&=1 \end{aligned} \quad , \text{ and } \quad \begin{aligned} v+w&=0 \\ u+v&=0 \\ u+v+w&=1 \end{aligned}$$

And we have to find that which permutation matrices are required.

Step-2

Converting into matrix form gives

$$\begin{pmatrix} 1 & 4 & 2 \\ -2 & -8 & 3 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix}=\begin{pmatrix} -2 \\ 32 \\ 1 \end{pmatrix}$$

Here the first pivot is 1 so adding 2 times of row1 to row2 gives

$$\begin{pmatrix} 1 & 4 & 2 \\ 0 & 0 & 7 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix}=\begin{pmatrix} -2 \\ 28 \\ 1 \end{pmatrix}$$

Step-3

Dividing row 2 by 7 gives

$$\begin{pmatrix} 1 & 4 & 2 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix}=\begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix}$$

It requires row changing so by changing row 2 and row 3 gives

$$\begin{pmatrix} 1 & 4 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix}=\begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$$

Step-4

Now here the pivot is 1, subtracting 4 times row2 from row 1 gives

$$\begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix}=\begin{pmatrix} -6 \\ 1 \\ 4 \end{pmatrix}$$

Subtracting row 3 from row 2 and adding 2 times row 3 to row 1 gives

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix}=\begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$$

Step-5

Hence the solution is $\boxed{x=(2,-3,4)}$

The permutation matrix has same rows as in identity matrix (in some order)

So here the required permutation matrix is $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

Step-6

Now consider the second system.

Converting into matrix form gives

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Here the first row first element is 0 so it needs row exchange so exchanging row 1 and row 2 gives

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Step-7

Subtracting row 1 from row 3 gives

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Now here the pivot is 1, subtracting row2 from row 3 gives

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix}=\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

Step-8

Subtracting row 2 from row 1

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix}=\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

Hence the solution is $\boxed{x=(1,-1,1)}$

Here the permutation matrix is $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$