Step-1

Given that $A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$ and $A^2 = \begin{bmatrix} 7 & -3 \\ -2 & 6 \end{bmatrix}$

Now $A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$

$$\begin{aligned} A - \lambda I &= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \\ &= \begin{bmatrix} -1-\lambda & 3 \\ 2 & -\lambda \end{bmatrix} \end{aligned}$$

Step-2

Now

$$\begin{aligned} |A - \lambda I| &= \begin{vmatrix} -1-\lambda & 3 \\ 2 & -\lambda \end{vmatrix} \\ &= (-1-\lambda)(-\lambda) - 6 \\ &= \lambda + \lambda^2 - 6 \\ &= \lambda^2 + \lambda - 6 \end{aligned}$$

Step-3

We know that

$$\begin{aligned} |A - \lambda I| &= 0 \\ \lambda^2 + \lambda - 6 &= \lambda^2 + 3\lambda - 2\lambda - 6 \\ &= \lambda(\lambda+3) - 2(\lambda+3) \\ &= (\lambda+3)(\lambda-2) \\ (\lambda+3)(\lambda-2) &= 0 \end{aligned}$$

$\lambda = -3, 2$

Hence the Eigen values of A are -3, 2

Step-4

Case (i) Let $\lambda = -3$

Eigen vectors X corresponding to Eigen value -3 are given by

$(A - (-3)I)X = 0$

$$\begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Add -1 times second row to the first row to the second row

$$\begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2x_1 + 3x_2 = 0$$

Step-5

Let $x_2 = k$

Therefore

$x_1 = -3k/2$

Therefore eigenvectors corresponding to eigenvalue -3 are given by $k\begin{bmatrix} -3/2 \\ 1 \end{bmatrix}$ where k is a non-zero parameter.

Step-6

Case (ii) Let $\lambda = 2$

Eigen vectors X corresponding to the Eigen value 2 are given by

$(A - 2I)X = 0$

$$\begin{bmatrix} -3 & 3 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Add +1 times first row to the second row

$$\begin{bmatrix} -3 & 3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Step-7

Add 3 times second row and -1 times first row to second row

$$\begin{bmatrix} -3 & 3 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$-3x_1+3x_2=0$$

Let $x_2=k(\neq 0)$

Therefore $x_1=k$

Therefore Eigen vectors corresponding to Eigen value 2 are given by $k\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Where k is a non-zero parameter

Step-8

Now $A^2=\begin{bmatrix} 7 & -3 \\ -2 & 6 \end{bmatrix}$

$$A-\lambda I=\begin{bmatrix} 7 & -3 \\ -2 & 6 \end{bmatrix}-\lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$=\begin{bmatrix} 7-\lambda & -3 \\ -2 & 6-\lambda \end{bmatrix}$$

Step-9

$$|A-\lambda I|=\begin{vmatrix} 7-\lambda & -3 \\ -2 & 6-\lambda \end{vmatrix}$$
$$=(7-\lambda)(6-\lambda)-6$$
$$=42-7\lambda-6\lambda+\lambda^2-6$$
$$=\lambda^2-13\lambda+36$$

We know that

$$|A-\lambda I|=0$$
$$\lambda^2-13\lambda+36=0$$

Step-10

We have a formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$
\begin{aligned}
&= \frac{13 \pm \sqrt{(13)^2 - 4 \times 1 \times 36}}{2 \times 1} \\
&= \frac{13 \pm \sqrt{169 - 144}}{2} \\
&= \frac{13 \pm \sqrt{25}}{2} \\
&= \frac{13 \pm 5}{2} \\
&= \frac{18}{2}, \frac{8}{2} \\
&= 9, 4
\end{aligned}
$$

$\lambda = 9, 4$

Hence the Eigen value of A^2 are 9, 4

Step-11

Case (i) Let $\lambda = 9$

Eigen vectors X corresponding to Eigen value 9 are given by

$(A - 9I)X = 0$

$$\begin{bmatrix} -2 & -3 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Add -1 times second row and +1 times the first row to the second row

$$\begin{bmatrix} -2 & -3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-2x_1 - 3x_2 = 0$$

Step-12

Let $x_2 = k$

Therefore $x_1 = -3k/2$

Therefore eigenvectors corresponding to eigenvalue 9 are given by $k\begin{bmatrix} -3/2 \\ 1 \end{bmatrix}$ where k

is a non-zero parameter.

Step-13

Case (ii) Let $\lambda = 4$

Eigen vectors X corresponding to Eigen value 4 are given by

$(A - 4I)X = 0$

$$\begin{bmatrix} 3 & -3 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Add +1 times the first row to the second row

$$\begin{bmatrix} 3 & -3 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Step-14

Add 3 times second row and -1 times first row to second row

$$\begin{bmatrix} 3 & -3 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$3x_1 - 3x_2 = 0$$

Let $x_2 = k$

Therefore $x_1 = k$

Therefore Eigen vectors corresponding to Eigen value 4 are given by $k\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Where k is a non-zero parameter

A^2 has the same square as A

When A has an Eigen value λ_1, λ_2 then A^2 has Eigen values λ_1^2, λ_2^2