

## Step-1

The derivative of  $a + bx + cx^2$  is  $b + 2cx + 0x^2$

$$D \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} a \\ 2c \\ 0 \end{bmatrix}$$

a)

$$D = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \text{ such that } \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} b \\ 2c \\ 0 \end{bmatrix}$$

The 3 by 3 matrix

$$D = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

So, the required matrix is

## Step-2

$$D^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

c)

$$= \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$D^3 = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= 0$$

## Step-3

This means  $D$  stands for the derivative and the third derivative of a second degree polynomial is zero.

The first derivative  $= b + 2cx$

The second derivative =  $2c$

The derivative = 0

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$

The third derivative of

## Step-4

c) The characteristic equation of  $D = |D - \lambda I| = 0$

$$\begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 2 \\ 0 & 0 & -\lambda \end{vmatrix} = 0$$

$$= -\lambda(\lambda^2) - 1(0) + 0 = 0$$

$$\Rightarrow \lambda^3 = 0$$

$\Rightarrow \lambda = 0, 0, 0$  are the eigen values of the derivative matrix.

## Step-5

To find eigen vector corresponding to  $\lambda = 0$ , we solve the homogeneous system  $(D - 0I)x = 0$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$x_2 = 0, x_3 = 0$$

We further, observe that  $x_1$  is satisfied by any real number  $x_1 = k$  and not necessarily zero.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} k \\ 0 \\ 0 \end{bmatrix}$$

So, the solution set is

Putting  $k = 1$ , we get the eigen vector  $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$  corresponding to  $\lambda = 0$ .

Observe that the eigen values are repeated 3 times and the corresponding eigen vector is only one.