

Step-1

Suppose A is $m \times n$ matrix and its columns are c_1, c_2, \dots, c_n which pair wise orthogonal and unit vectors are.

So, it follows that

$$C_i^T C_j = 0 \text{ For every } 1 \leq i \neq j \leq n \text{ and}$$

$$C_i^T C_i = 1 \text{ For every } 1 \leq i \leq n \quad (1)$$

Then,

$$A^T = \begin{bmatrix} c_1^T \\ c_2^T \\ \vdots \\ c_n^T \end{bmatrix} \text{ Is } n \times m \text{ matrix}$$

Step-2

Thus,

$$\begin{aligned} A^T A &= \begin{bmatrix} c_1^T \\ c_2^T \\ \vdots \\ c_n^T \end{bmatrix} [c_1 c_2 \dots c_n] \\ &= \begin{pmatrix} c_1^T c_1 & \dots & c_1^T c_n \\ \vdots & \ddots & \vdots \\ c_n^T c_1 & \dots & c_n^T c_n \end{pmatrix} \text{ Is } n \times n \text{ matrix} \end{aligned}$$

Step-3

$$A^T A = \begin{bmatrix} 1 & 0 & - & - & 0 \\ 0 & 1 & - & - & 0 \\ - & - & - & - & - \\ - & - & - & - & - \\ 0 & 0 & - & - & 1 \end{bmatrix}$$

In view of (1), obtain this matrix

Hence, $A^T A$ is an $n \times n$ identity matrix.

