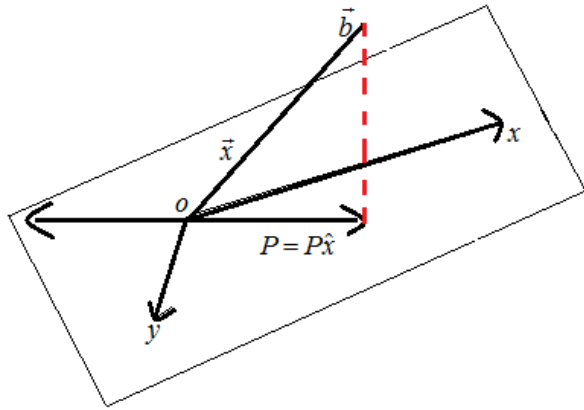


## Step-1

Given that  $P$  is the projection matrix onto a line in the  $x-y$  plane.

The geometrical explain is as follows,



$$P = Px$$

$$\begin{aligned}\vec{b} &= x - P \\ &= (I - P)x\end{aligned}$$

## Step-2

The projection matrix  $P$  has two basic properties.

$$(i) P^2 = P$$

$$(ii) P^T = P$$

Conversely, any symmetric matrix with  $P^2 = P$  represents a projection.

So,

$$\begin{aligned}H^2 &= (I - 2P)(I - 2P) \\ &= I^2 - 2IP - 2PI + 4PP \\ &= I - 2P - 2P + 4P^2 \quad (\text{Since } I^2 = I)\end{aligned}$$

$$= I - 4P + 4P \quad (\text{From the properties})$$

$$\hat{A} \hat{A} \hat{A} \hat{A} \hat{A} = I$$

$$\hat{A} \hat{A} \hat{A} \hat{A} \hat{A} \hat{A} \hat{A} \hat{A} \hat{A}$$

Therefore,  $H^2 = I$  and two reflections (i.e. reflection of the same reflection) gives  $I$ .

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