Step-1

$v = \frac{1}{2}[v+w] + \frac{1}{2}[v-w]$ â€¦â€¦ (1)

And

$w = \frac{1}{2}[v+w] - \frac{1}{2}[v-w]$ â€¦â€¦ (2)

The two pairs $\{v, w\}$, $\{v+w, v-w\}$ of vectors span the same space.

Step-2

Since $v+w, v-w$ are linear combinations of v, w .as well as v, w are linear combinations of $v-w, v+w$ as in equations $(1),(2)$.

If v, w are $\boxed{\text{linearly independent}}$. then the sets $\{v, w\}$, $\{v+w, v-w\}$ basis for the same space.

Therefore, they are a basis when v and w are $\boxed{\text{linearly independent}}$.