

Step-1

(b) Let $Nx = \lambda x$. Then we get the following:

$$\begin{aligned} Nx &= \lambda x \\ N(Nx) &= N(\lambda x) \\ &= \lambda Nx \\ &= \lambda^2 x \end{aligned}$$

Thus, $N^2 x = \lambda^2 x$. Further,

$$\begin{aligned} N^2 x &= \lambda^2 x \\ N(N^2 x) &= N(\lambda^2 x) \\ &= \lambda^2 (Nx) \\ &= \lambda^3 x \end{aligned}$$

Step-2

Thus, we have $N^3 x = \lambda^3 x$.

But $N^3 = 0$. This gives us $N^3 x = 0$, for any x .

Therefore, $\lambda^3 x = 0$, for any x .

This indicates that $\lambda^3 = 0$. This is possible if and only if $\lambda = 0$.

Therefore, $\boxed{\lambda = 0}$.

Step-3

(c) Suppose, if possible, N is a similar matrix of order k by k . Then $n_{ij} = n_{ji}$, for each i, j . Consider the product of i^{th} row and i^{th} column in N^2 .

$$\begin{aligned} [n_{i1}, n_{i2}, \dots, n_{ik}] \begin{bmatrix} n_{1i} \\ n_{2i} \\ \vdots \\ n_{ki} \end{bmatrix} &= n_{i1}n_{1i} + n_{i2}n_{2i} + \dots + n_{ik}n_{ki} \\ &= n_{i1}^2 + n_{i2}^2 + \dots + n_{ik}^2 \end{aligned}$$

This indicates that $n_{ij} = 0$, for each i and j . But N is assumed to be non zero matrix. Thus, our assumption that N could be a similar matrix is false. So N is not a similar matrix.