

Step-1

Let us consider the following Linear Programming Problem

Minimize: $2x_1 + x_2$

Subject to following constraints

$$x_1 + x_2 \geq 4$$

$$x_1 + 3x_2 \geq 12$$

$$x_1 - x_2 \geq 0$$

$$x \geq 0$$

Step-2

Let us solve the problem by graphical method.

Convert the inequality into equations.

$$x_1 + x_2 = 4$$

$$\frac{x_1}{4} + \frac{x_2}{4} = 1$$

And

$$x_1 + 3x_2 = 12$$

$$\frac{x_1}{12} + \frac{x_2}{4} = 1$$

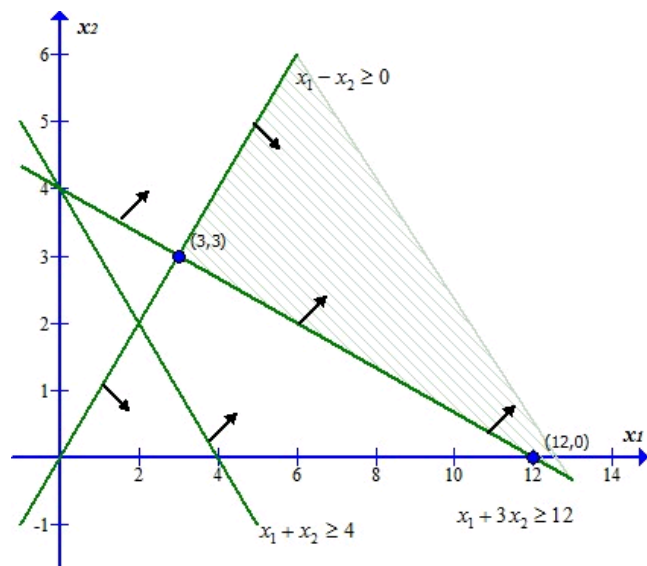
And

$$x_1 - x_2 = 0$$

$$x_1 = x_2$$

Step-3

Let us plot the equations on the co-ordinate axes and mark the region according to inequality.



Step-4

From the graph, the corner points of feasible regions are **$(3,3)$** and **$(12,0)$**

Let us find the value of the function at corner points

$$\begin{aligned} C_{(3,3)} &= 2(3) + 3 \\ &= 9 \end{aligned}$$

And

$$\begin{aligned} C_{(12,0)} &= 2(12) + 0 \\ &= 24 \end{aligned}$$

Thus, the minimum value is 9 at **$(3,3)$**