

Step-1

Given that adding row 1 of A to row 2 produces B . Adding column 1 to column 2 produces C .

So the matrices are $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$.

A combination of columns of matrix C is also a combination of the columns of A .

We have to find which two matrices have the same column space.

Step-2

Let $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \in \mathbf{C}(A)$

Then

$$\begin{aligned} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} &= c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 4 \end{bmatrix} \\ &= c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} - c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 4 \end{bmatrix} \\ &= [c_1 - c_2] \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ 6 \end{bmatrix} \end{aligned}$$

Therefore $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ is linear combination of $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 6 \end{bmatrix}$, the column of C .

Step-3

If $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \in \mathbf{C}(C)$

Then $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = d_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + d_2 \begin{bmatrix} 3 \\ 6 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = d_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + d_2 \left[\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ 4 \end{bmatrix} \right]$$

$$= d_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + d_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + d_2 \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$= (d_1 + d_2) \begin{bmatrix} 1 \\ 2 \end{bmatrix} + d_2 \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

Therefore $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \in \mathbf{C}(A)$

Hence the matrices $\boxed{A \text{ and } C}$ have same column spaces.