Step-1

Given that A commutes with every 2 by 2 matrix $(AB = BA)$

where $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$,

$B_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $B_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ â€¦ (i)

We have to show that $a = d$ and $b = c = 0$

Step-2

Since A commutes with B_1 and B_2, we have $AB_1 = B_1A$ and $AB_2 = B_2A$

$$AB_1 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} a & 0 \\ c & 0 \end{pmatrix}$$

$$B_1A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$
$$= \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix}$$

$$AB_1 = B_1A$$
$$\Rightarrow \begin{pmatrix} a & 0 \\ c & 0 \end{pmatrix} = \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix}$$

$\Rightarrow \boxed{b = 0 \text{ and } c = 0}$ â€¦ (i)

Step-3

$$AB_2 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & a \\ 0 & c \end{pmatrix}$$

$$B_2A=\begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}a&b\\c&d\end{pmatrix}$$

$$=\begin{pmatrix}c&d\\0&0\end{pmatrix}$$

$$AB_2=B_2A$$
$$\Rightarrow\begin{pmatrix}0&a\\0&c\end{pmatrix}=\begin{pmatrix}c&d\\0&0\end{pmatrix}$$

$\Rightarrow\boxed{c=0 \text{ and } a=d}$ â€¦ (ii)

From (i) and (ii), we have $\boxed{a=d \text{ and } b=c=0}$.

Step-4

Suppose that $AB=BA$ for all matrices A and B, then

$$A=\begin{pmatrix}a&b\\c&d\end{pmatrix}$$

$$=\begin{pmatrix}a&0\\0&a\end{pmatrix}$$
$$=a\begin{pmatrix}1&0\\0&1\end{pmatrix}$$
$$=aI$$

Hence A is a multiple of the identity.