

Step-1

Given that A is symmetric which means that

$$\Rightarrow A^T = A.$$

Given that A is positive definite which means that

$$\Rightarrow x^T A x > 0 \text{ for } x \neq 0.$$

So,

$$\begin{aligned} B^T &= (C^T A C)^T \\ &= C^T A^T (C^T)^T \quad \left(\text{since } (AB)^T = B^T A^T \right) \\ &= C^T A^T C \quad \left(\text{since } (C^T)^T = C \right) \\ &= B \end{aligned}$$

Therefore, B is symmetric matrix.

Step-2

Now we need to prove that B is positive definite.

$$\begin{aligned} x^T B x &= x^T (C^T A C) x \\ &= (x^T C^T) A (C x) \\ &= (C x)^T A (C x) \\ &> 0 \quad \quad \quad (\text{Since } A \text{ is positive definite}) \end{aligned}$$

Therefore, B is also positive definite.

Therefore, B is symmetric positive definite.