

Step-1

Write down second order differential equation $my'' + by' + ky = 0$ in the following way:

$$Mu' = Au$$

Step-2

Second order differential equation can be written as follows:

$$my'' + by' + ky = 0$$

$$m \frac{d^2 y}{dt^2} = -b \frac{dy}{dt} - ky$$

$$m \frac{d^2 y}{dt^2} + \frac{dy}{dt} = -b \frac{dy}{dt} - ky + \frac{dy}{dt}$$

In matrix form, it can be written as:

$$\begin{bmatrix} m & 0 \\ 0 & 1 \end{bmatrix} \cdot \frac{d}{dt} \begin{bmatrix} y' \\ y \end{bmatrix} = \begin{bmatrix} -b & -k \\ 1 & 0 \end{bmatrix} \begin{bmatrix} y' \\ y \end{bmatrix}$$

$$\begin{bmatrix} m & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} y' \\ y \end{bmatrix}' = \begin{bmatrix} -b & -k \\ 1 & 0 \end{bmatrix} \begin{bmatrix} y' \\ y \end{bmatrix}$$

Step-3

Therefore, $my'' + by' + ky = 0$ is written in matrix form as:

$$\boxed{Mu' = Au}.$$