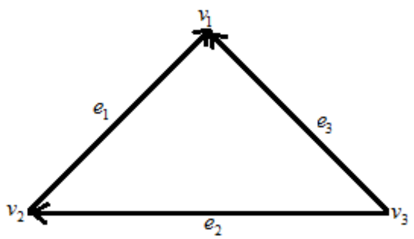
Step-1



We observe that edge 1 is from node 2 to node 1

So, entry corresponding to 1,2 is -1 and the entry in 1,1 is 1

The edge 1 is nothing to do with the node 3 and so, the 1,3 entry is 0.

We proceed in the same way to get

$$A = \begin{array}{c} \\ edge1 \\ edge2 \\ edge3 \end{array} \begin{array}{c} \begin{array}{ccc} node1 & node2 & node3 \end{array} \\ \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix} \end{array}$$

Step-2

To find the solution for $Ax = 0$, we use the row operations to reduce A to the echelon form and then by rewriting the homogeneous equations, we get the solution set.

$$\overset{R_3+R_1}{\sim} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

$$\overset{R_3-R_2}{\sim} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$Ax = 0$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow x_1 - x_2 = 0$

$x_2 - x_3 = 0$

$\Rightarrow x_1 = x_2$ and

$x_2 = x_3$

Step-3

Using $x_1 = k$, we get $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = k \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and for infinite real values of k, we get infinite solutions.

Putting $k = 1$, the solution set forms the basis to the null space of A.

Basis for null space is $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

Step-4

$$A^T = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & -1 \end{bmatrix}$$

$$\overset{R_2+R_1}{\sim} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix}$$

$$\overset{R_3+R_2}{\sim} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Step-5

$A^T y = 0$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow y_1 + y_3 = 0$

$y_2 + y_3 = 0$

$\Rightarrow y_1 = -y_3$

$y_2 = -y_3$

Putting $y_3 = -m$, we get $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = m \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ and thus the basis for null space of A^T is

$\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \right\}$