Step-1

Given symmetric is, $A=\begin{pmatrix} a & b \\ b & c \end{pmatrix}$

Now,

$$A-\lambda I=\begin{pmatrix} a & b \\ b & c \end{pmatrix}-\lambda\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$=\begin{pmatrix} a-\lambda & b \\ b & c-\lambda \end{pmatrix}$$

Step-2

The characteristic equation is,

$$\det(A-\lambda I)=0$$

$$\Rightarrow \begin{vmatrix} a-\lambda & b \\ b & c-\lambda \end{vmatrix}=0$$

$$\Rightarrow (a-\lambda)(c-\lambda)-b^2=0$$

$$\Rightarrow ac-(a+c)\lambda+\lambda^2-b^2=0$$

$$\Rightarrow \lambda^2-(a+c)\lambda+(ac-b^2)=0$$

$$\Rightarrow \lambda=\frac{(a+c)\pm\sqrt{(a+c)^2-4(ac-b^2)}}{2}$$

$$=\frac{(a+c)\pm\sqrt{(a-c)^2+4b^2}}{2}$$

Therefore, the Eigen values are $\boxed{\lambda_1=\frac{(a+c)+\sqrt{(a-c)^2+4b^2}}{2}}$ and $\boxed{\lambda_2=\frac{(a+c)-\sqrt{(a-c)^2+4b^2}}{2}}$.

Step-3

Let

$$\lambda_1 = \frac{(a+c)+\sqrt{(a-c)^2+4b^2}}{2} \text{ and } \lambda_2 = \frac{(a+c)-\sqrt{(a-c)^2+4b^2}}{2}$$

Given that $a > 0,\ \ ac > b^2 \ \ \Rightarrow c > 0.$

Therefore,

$a+c > 0,\ (a-c)^2 + 4b^2 > 0$

Step-4

Thus $\lambda_1 > 0$

$ac - b^2 > 0$

$\Rightarrow 4ac - 4b^2 > 0$

$\Rightarrow (a^2 + c^2 + 2ac) > (a^2 + c^2 - 2ac) + 4b^2$

$\Rightarrow (a+c)^2 > (a-c)^2 + 4b^2$

$\Rightarrow (a+c) > \sqrt{(a-c)^2 + 4b^2}$

$\Rightarrow \lambda_2 > 0$

Thus both the Eigen value of $\det(A - \lambda I) = 0$ are $\boxed{\text{positive}}$

Step-5

Moreover, the product of Eigen values.

$$\begin{aligned}\lambda_1\lambda_2 &= \frac{ac-b^2}{1}\\ &= \boxed{ac-b^2}.\end{aligned}$$

Therefore, the product of Eigen values is $\boxed{ac-b^2}$.