Step-1

The matrix *A* contains *n* eigenvalues Î»$_1$, Î»$_2$, ..., Î»$_n$. Since, the matrix *A* is not a zero matrix, at least one of the eigenvalues of *A* must be nonzero.

Without loss of generality, let Î»$_n$ be a non zero eigenvalue.

Thus, we have $Ax = \lambda_n x$, where $\lambda_n \neq 0$ and $x \neq 0$.

Step-2

Let us apply the product $(A-\lambda_1 I)(A-\lambda_2 I)\cdots(A-\lambda_n I)$ to the vector *x*. Thus, we get

$$\begin{aligned}
\left((A-\lambda_1 I)(A-\lambda_2 I)\cdots(A-\lambda_n I)\right)x &= \left((A-\lambda_1 I)(A-\lambda_2 I)\cdots\right)(A-\lambda_n I)x \\
&= \left((A-\lambda_1 I)(A-\lambda_2 I)\cdots\right)(Ax-\lambda_n Ix) \\
&= \left((A-\lambda_1 I)(A-\lambda_2 I)\cdots\right)(\lambda_n x-\lambda_n x) \\
&= \left((A-\lambda_1 I)(A-\lambda_2 I)\cdots\right)(0)
\end{aligned}$$

$$\left((A-\lambda_1 I)(A-\lambda_2 I)\cdots(A-\lambda_n I)\right)x = 0$$

Step-3

Thus, we have shown that $\boxed{(A-\lambda_1 I)(A-\lambda_2 I)\cdots(A-\lambda_n I)=0}$.