Step-1

State true or false:

(a) Differential equation $\dfrac{du}{dt} = Au$ has a solution for every matrix A, starting from $u(0) = (1,1,\ldots,1)$.

This statement is true. The solution to the differential equation is $u(t) = e^{At}u(0)$, this depends on the exponential of A and e^{At} is never singular. If matrix A could not be diagonalize then Jordan form can be used which involves generalised Eigen vectors.

Step-2

(b) Every invertible matrix can be diagonalize.

This statement is false. Diagonalisation depends only on number of Eigen vectors corresponding to the Eigen values.

Step-3

(c) Every diagonalizable matrix can be inverted.

This statement is false. Invertibility is concerned with Eigen values whether it is zero or not. It has no connection with diagonalization of the matrix.

Step-4

(d) Exchanging the rows of a 2 by 2 matrix reverses the signs of its Eigen values.

This statement is false. Exchanging the rows changes the equation $\det(A - \lambda I) = 0$. Thereafter Eigen values are also changed.

Step-5

(e) If Eigen vectors x and y correspond to distinct Eigen values then $x^H y = 0$.

This statement is false. Recall that The Eigen values of a Hermitian matrix are orthogonal provided they correspond to different Eigen values. So, if the matrix is Hermitian matrix then only $x^H y = 0$.