## Step-1

Given that the sum of the vectors $f(x)$ and $g(x)$ in **F** is defined to be $f(g(x))$.

Then the zero vector is $g(x) = x$

Keeping the usual scalar multiplication,

Then $(f+g)(x)$ is the usual $f(g(x))$

And $(g+f)(x)$ is $g(f(x))$

But $g(f(x)) \neq f(g(x))$

For example $f(x) = x^2$, $g(x) = x+3$

## Step-2

Now

$$
\begin{aligned}
f(g(x)) &= f(x+3) \\
&= (x+3)^2 \\
&= x^2 + 3x + 9 \\
g(f(x)) &= g(x^2) \\
&= x^2 + 3
\end{aligned}
$$

Therefore, $f(g(x)) \neq g(f(x))$

So the rule $f+g=g+f$ is broken.

Rule 4 is also broken, because there must be no inverse function $f^{-1}(x)$ such that $f(f^{-1}(x)) = x$.

If the inverse function exists, it will be the vector $-f$

For example:

Suppose $f(x) = x^2 + 3$, there is no function $f^{-1}$

$f(f^{-1}(x)) = x$

Therefore rule 4 is also broken.