Calculus II quiz 1 第十章选择填空题

1. Multiple Choice Questions: (only one correct answer for each of the following questions.)

(1) If $\sum\limits_{n=1}^{\infty} a_n$ converges, then( )

A. $\sum\limits_{n=1}^{\infty} |a_n|$ converges. B. $\sum\limits_{n=1}^{\infty} (-1)^n a_n$ converges.

C. $\sum\limits_{n=1}^{\infty} a_n a_{n+1}$ converges. D. $\sum\limits_{n=1}^{\infty} \frac{a_n+a_{n+1}}{2}$ converges.

(2) When $\lim\limits_{n\to\infty} a_n = 0$, then (C)

A. If $\sum\limits_{n=1}^{\infty} b_n$ converges, then $\sum\limits_{n=1}^{\infty} a_n b_n$ converges.

B. If $\sum\limits_{n=1}^{\infty} b_n$ diverges, then $\sum\limits_{n=1}^{\infty} a_n b_n$ diverges.

假设 a_n, b_n 正负号

C. If $\sum\limits_{n=1}^{\infty} |b_n|$ converges, then $\sum\limits_{n=1}^{\infty} a_n^2 b_n^2$ converges.

$a_n^2 b_n^2 \le b_n^2 \le |b_n|$

D. If $\sum\limits_{n=1}^{\infty} |b_n|$ diverges, then $\sum\limits_{n=1}^{\infty} a_n^2 b_n^2$ diverges.

(3) Let $a_n > 0$ for all n. Which of the following statements must be true ? ( )

(A) If $\lim\limits_{n\to\infty} na_n = 0$, then the series $\sum\limits_{n=1}^{\infty} a_n$ converges.

(B) If $\lim\limits_{n\to\infty} na_n = l$ and $l \neq 0$, then the series $\sum\limits_{n=1}^{\infty} a_n$ converges.

(C) If $\lim\limits_{n\to\infty} na_n = l$ and $l \neq 0$, then the series $\sum\limits_{n=1}^{\infty} a_n$ diverges.

(D) None of the above statements is correct.

积分判别法

(4) Let a be a constant, the series $\sum\limits_{n=2}^{\infty} (\frac{\sin(n+a)}{n^{1.01}} - \boxed{\frac{1}{n\ln n}})$( )

(A) converges absolutely. (B) converges conditionally. (C) diverges.

(D) converges or not depending on the value of a.

(5) The interval of the convergence for the power series $\sum\limits_{n=1}^{\infty} \frac{x^n}{n3^n}$ is ( )

(A) $[-\frac{1}{3}, \frac{1}{3}]$. (B) $[-\frac{1}{3}, \frac{1}{3})$. (C) $[-3, 3]$. (D) $[-3, 3)$.

(6) Let a be a constant. Then the series $\sum\limits_{n=1}^{\infty} (\frac{\sin(an)}{n^2} + \frac{(-1)^n}{n+1})$( )

(A) converges absolutely. (B) converges conditionally. (C) diverges. (D) converges or not depending on the value of a.

(7) Which one of the following series diverges? ( )

(A) $\sum\limits_{n=1}^{\infty} \frac{n^2}{2^n}$. (B) $\sum\limits_{n=1}^{\infty} \frac{1}{n^{1+\frac{1}{n}}}$. (C) $\sum\limits_{n=1}^{\infty} \frac{(-1)^n \ln n}{\sqrt{n}}$. (D) $\sum\limits_{n=1}^{\infty} \frac{(-1)^n(3+(-1)^n \cdot 2)^n}{6^n}$.

(8) Suppose $0 \le a_n < \frac{1}{n}, (n = 1.2\cdots)$, then which of the following series converges( )

(A) $\sum\limits_{n=1}^{\infty} a_n$. (B) $\sum\limits_{n=1}^{\infty} (-1)^n a_n$. (C) $\sum\limits_{n=1}^{\infty} \sqrt{a_n}$. (D) $\sum\limits_{n=1}^{\infty} (-1)^n a_n^2$.

$\downarrow$

$\frac{1}{2n} + \frac{1}{2n}\sin$

(oscillating)

2. Fill in the blanks.

(1) If $\sum\limits_{n=1}^{\infty}(\tan\frac{1}{n} - k\ln(1-\frac{1}{n}))$ converges, then $k =$

(2) $\lim\limits_{x\to 0}\frac{\sin x - x}{(\cos x - 1)(e^{2x} - \cos x)} =$

(3) The sum of the series $\frac{1}{2} + \frac{1}{4\cdot 2!} + \frac{1}{8\cdot 3!} + \frac{1}{16\cdot 4!} + \cdots + \frac{1}{2^n\cdot n!} + \cdots$ is

3. Determine whether the following statements are true or false? No justification is necessary.

$\sum a_n$ 绝对收敛 $\Rightarrow \sum a_n^2$ 收敛

(1) If $a_n < 0$ for $n > 1000$, and $\sum\limits_{n=1}^{\infty} a_n$ converges, then $\sum\limits_{n=1}^{\infty} a_n^2$ may diverge. $\sum a_n$ 收敛 $\not\Rightarrow \sum a_n^2$ 收敛

(2) Suppose that the power series $\sum\limits_{n=1}^{\infty} a_n(x-1)^n$ converges at $x = 0$, and diverges at $x = 2$, then the interval of convergence of this series is $[0,\, 2)$.