Step-1

$w =$ 6th root of one in complex plane.

$w = \sqrt[6]{1} \Rightarrow w = 1^{1/6}$

$\Rightarrow w = \left(\cos 2k\pi + i \sin 2k\pi\right)^{1/6}$ Â Â Â Â Â ,$k = 0,1,2,3,4,5.$ Â Â Â Â Â

$\Rightarrow w = \left(e^{i2k\pi}\right)^{\frac{1}{6}}$

$= \left(e^{\frac{2i\pi}{6}}\right)^{k}$

$= e^{\frac{2ik\pi}{6}}, k = 0,1,2,3,4,5.$

$w = e^{\frac{2i\pi}{6}}, w^2 = e^{\frac{4i\pi}{6}}, w^3 = e^{\frac{6i\pi}{6}}, w^4 = e^{\frac{8i\pi}{6}}, w^5 = e^{\frac{10i\pi}{6}}, w^6 = e^{\frac{12i\pi}{6}} = 1$ are the six roots of 1.

Step-2

Using $w = e^{\frac{2i\pi}{6}}$, we get $D = \begin{bmatrix} 1 & & \\ & e^{\frac{2i\pi}{6}} & \\ & & e^{\frac{4i\pi}{6}} \end{bmatrix}$

$= \begin{bmatrix} 1 & & \\ & w & \\ & & w^2 \end{bmatrix}$

Step-3

$$
\begin{aligned}
F_3 &= \begin{bmatrix} 1 & 1 & 1 \\ 1 & w & w^2 \\ 1 & w^2 & w^4 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 1 & 1 \\ 1 & w & w^2 \\ 1 & w^2 & w \end{bmatrix} \\
&= \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{\frac{2i\pi}{6}} & e^{\frac{4i\pi}{6}} \\ 1 & e^{\frac{4i\pi}{6}} & e^{\frac{2i\pi}{6}} \end{bmatrix}
\end{aligned}
$$