Step-1

Consider $Ax = b$ has at least one solution.

The objective is to show that the only solution to $A^T y = 0$ is $y = 0$.

If $Ax = b$ has at least one solution, to show that the only solution to $A^T y = 0$ is $y = 0$.

Let A be an n by n matrix.

Suppose that $Ax = b$ has at least one solution x for every b if and only if the columns span $\mathbf{R}^n$.

Step-2

The vector b can be expressed as a combination of the columns of A if and only if the system $Ax = b$ is solvable.

More over all columns of A are linearly independent,

The number of columns in matrix A is n.

Hence, the dimension of column A is n.

$$\dim(\mathbf{C}(A)) = n$$

If columns of A are linearly independent then the rank of A is n.

$$\operatorname{rank}(A) = n$$

Step-3

By a known result,

$$\dim(\mathbf{N}(A)) + \dim(\mathbf{C}(A)) = \text{ Number of columns of } A$$

$$\dim(\mathbf{N}(A)) + n = n$$

$$\dim(\mathbf{N}(A)) = 0$$

Step-4

Let A be an n by n matrix.

Then the transpose matrix A^T is also n by n matrix.

Rank of $A = n$

rank of $A^T = n$

$\dim\left(\mathbf{N}\left(A^T\right)\right) = n$

Step-5

Therefore, the dimension of nullspace A^T is equal to the dimension of nullspace A.

$\dim\left(\mathbf{N}\left(A^T\right)\right) = \dim\left(\mathbf{N}\left(A\right)\right)$

$\dim\left(\mathbf{N}\left(A^T\right)\right) = 0$ Since $\dim\left(\mathbf{N}\left(A\right)\right) = 0$

Therefore, $A^T y = 0$ has only solution $y = 0$.