Step-1

(a)

Let us consider T is a triangular matrix of order 3 by 3.

$$T = \begin{bmatrix} a & x & y \\ 0 & b & z \\ 0 & 0 & c \end{bmatrix}$$

The characteristic equation $|T - \lambda I| = 0$

This implies;

$$\begin{vmatrix} a-\lambda & x & y \\ 0 & b-\lambda & z \\ 0 & 0 & c-\lambda \end{vmatrix} = 0$$
$$(a-\lambda)(b-\lambda)(c-\lambda) = 0$$

This implies

$$\lambda_1 = a$$
$$\lambda_2 = b$$
$$\lambda_3 = c$$

Are the Eigen values of T

Step-2

Now,

$$T - \lambda_1 I = \begin{bmatrix} 0 & 0 & 0 \\ 0 & b-a & 0 \\ 0 & 0 & c-a \end{bmatrix}$$
$$T - \lambda_2 I = \begin{bmatrix} a-b & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & c-a \end{bmatrix} \text{ and}$$
$$T - \lambda_3 I = \begin{bmatrix} a-c & 0 & 0 \\ 0 & b-c & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Step-3

Let us consider $(T-\lambda_1 I)(T-\lambda_2 I)(T-\lambda_3 I)$

$$=\begin{bmatrix}0 & 0 & 0\\ 0 & b-a & 0\\ 0 & 0 & c-a\end{bmatrix}\begin{bmatrix}a-b & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & c-b\end{bmatrix}\begin{bmatrix}a-c & 0 & 0\\ 0 & b-c & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & (c-a)(c-b)\end{bmatrix}\begin{bmatrix}a-c & 0 & 0\\ 0 & b-c & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$
$$=0$$

Step-4

Thus, it is shown that

$$\boxed{\begin{aligned}(T-\lambda_1 I)(T-\lambda_2 I)(T-\lambda_3 I)&=0\\ &=|T-\lambda I|\end{aligned}}$$

Therefore, the products of the factors directly $\boxed{\text{satisfy}}$ the characteristic equation of the triangular matrix T

(b)

Substitute $T=U^{-1}AU$ and get;

$$T^2=U^{-1}A^2U$$
$$T^3=U^{-1}A^3U$$

$$(T-\lambda_1 I)(T-\lambda_2 I)(T-\lambda_3 I)=0$$

Solve and get;

$$\begin{aligned}&=T^3-(\lambda_1+\lambda_2+\lambda_3)T^2+(\lambda_1\lambda_2+\lambda_2\lambda_3+\lambda_3\lambda_1)T-\lambda_1\lambda_2\lambda_3 I\\ &=U^{-1}A^3U-(\lambda_1+\lambda_2+\lambda_3)U^{-1}A^2U+(\lambda_1\lambda_2+\lambda_2\lambda_3+\lambda_3\lambda_1)U^{-1}AU-\lambda_1\lambda_2\lambda_3 I\\ &=U^{-1}\left(A^3-(\lambda_1+\lambda_2+\lambda_3)A^2+(\lambda_1\lambda_2+\lambda_2\lambda_3+\lambda_3\lambda_1)A-\lambda_1\lambda_2\lambda_3 I\right)U\\ &=U^{-1}(A-\lambda_1 I)(A-\lambda_2 I)(A-\lambda_3 I)U\end{aligned}$$

Thus, it is shown that $(A-\lambda_1 I)(A-\lambda_2 I)(A-\lambda_3 I)=0$ while $U\neq 0$

Therefore, A $\boxed{\text{satisfies its own}}$ characteristic equation