

Step-1

Given that every invertible linear transformation can have I as its matrix.

We have to choose $w_i = T(v_i)$ for the output basis.

We have to verify why T must be invertible.

Step-2

Suppose T is not invertible.

Then $T(v_1), T(v_2), \dots, T(v_n)$ will not be a basis.

Then we could not choose $w_i = T(v_i)$ as output basis.

Hence T must be invertible.