

Step-1

Suppose $x = (x_1, x_2, x_3)$ is the orthogonal vector to the plane spanned by the vectors;

$$y = (1, 1, 2) \text{ And}$$

$$z = (1, 2, 3)$$

Then, $x^T y = 0$ and $x^T z = 0$

This implies;

$$(x_1 \quad x_2 \quad x_3) \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = 0 \quad \text{And}$$

$$(x_1 \quad x_2 \quad x_3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = 0$$

And get;

$$x_1 + x_2 + 2x_3 = 0$$

$$x_1 + 2x_2 + 3x_3 = 0$$

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

Write this as $Ax = 0$ where and made it a square matrix.

Step-2

$$\text{Apply } R_2 \rightarrow R_2 - R_1 \text{ on this, and get } \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

This is the echelon matrix and so, we rewrite the homogeneous equations as;

$$x_1 + x_2 + 2x_3 = 0$$

$$x_2 + x_3 = 0$$

Thus;

$$x_2 = -x_3$$

$$x_1 = -x_2 - 2x_3$$

$$= x_3 - 2x_3$$

$$= -x_3$$

Put $x_3 = -1$, and get; $(x_1 \ x_2 \ x_3) = (1, 1, -1)$

Therefore $\boxed{x = (1, 1, -1)}$ is the vector orthogonal to the plane spanned by y , and z .