Step-1

Consider the system,

$$\begin{aligned} kx + 3y &= 6 \\ 3x + ky &= -6 \end{aligned}$$

For $k = 3$, the system becomes,

$$\begin{aligned} 3x + 3y &= 6 \\ 3x + 3y &= -6 \end{aligned}$$

This is impossible because $6 \neq -6$.

Hence the given system has no solution (zero solutions) for $\boxed{k = 3}$.

Step-2

For $k = 0$, the system becomes,

$$\begin{cases} 0 \cdot x + 3y = 6 \\ 3x + 0 \cdot y = -6 \end{cases}$$

This gives us,

$$\begin{cases} 3y = 6 \\ 3x = -6 \end{cases}$$

This implies that $x = -2, y = 2$

Hence the given system has **unique solution** (one solution) for $\boxed{k = 0}$.

Step-3

For $k = -3$, the system becomes,

$$\begin{cases} -3x + 3y = 6 \\ 3x - 3y = -6 \end{cases}$$

This gives us,

$$3x - 3y = -6$$

This implies that,

$x - y = -2$

Note that there are two variables (namely x, y) and one equation. So, solution of it must contain $2-1=1$ variable as free variables.

Suppose that y be the free variable. That is, $y = r,\ r \in \mathbb{R}$

From the equation $x - y = -2$ get $x = r - 2$

Since r is arbitrary, for $\boxed{k=-3}$ the given system has **infinitely many solutions**.