

Step-1

Given that $x = (1, 1, 1, 1, 1)$ and $x = (0.1, 0.7, 0.3, 0.4, 0.5)$

We have to compute $\|x\|$, $\|x\|_1$ and $\|x\|_\infty$ for the given vectors.

Step-2

The ℓ^1 norm is defined by $\|x\|_1 = |x_1| + |x_2| + \dots + |x_n|$ and the ℓ^∞ norm is defined by $\|x\|_\infty = \max \{|x_i| : 1 \leq i \leq n, x = (x_1, x_2, \dots, x_n)\}$

Also, we know that the hilbert norm is $\|x\| = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$

Step-3

Now we have $x = (1, 1, 1, 1, 1)$

Then

$$\begin{aligned}\|x\|_1 &= 1 + 1 + 1 + 1 + 1 \\ &= 5\end{aligned}$$

And

$$\begin{aligned}\|x\|_\infty &= \max \{1, 1, 1, 1, 1\} \\ &= 1\end{aligned}$$

And

$$\begin{aligned}\|x\| &= \sqrt{1^2 + 1^2 + \dots + 1^2} \\ &= \sqrt{5}\end{aligned}$$

Hence for $x = (1, 1, 1, 1, 1)$, we get $\boxed{\|x\| = \sqrt{5}, \|x\|_1 = 5 \text{ and } \|x\|_\infty = 1}$

Step-4

Now we have $x = (0.1, 0.7, 0.3, 0.4, 0.5)$

Then

$$\begin{aligned}\|x\|_1 &= 0.1 + 0.7 + 0.3 + 0.4 + 0.5 \\ &= 2\end{aligned}$$

And

$$\begin{aligned}\|x\|_{\infty} &= \max \{0.1, 0.7, 0.3, 0.4, 0.5\} \\ &= 0.7\end{aligned}$$

Step-5

And

$$\begin{aligned}\|x\| &= \sqrt{0.1^2 + 0.7^2 + 0.3^2 + 0.4^2 + 0.5^2} \\ &= \sqrt{0.01 + 0.49 + 0.09 + 0.16 + 0.25} \\ &= \sqrt{1} \\ &= 1\end{aligned}$$

Hence for $x = (0.1, 0.7, 0.3, 0.4, 0.5)$, we get $\boxed{\|x\| = 1, \|x\|_1 = 2 \text{ and } \|x\|_{\infty} = 0.7}$.