Step-1

Solve the integral $\int x^2 e^x \, dx.$

The integrand $x^2 e^x$ is a product of two functions, one of which will be designated as *u*, the other as v'. Since applying the integration by parts rule will result in another integral of the form

$\int u'v \, dx,$ We must choose *u* and v' so that uv' is of a simpler form. This suggests the following choices.

$u = x^2$ and $v' = e^x$.

Since $v = e^x$ and $u' = 2x$ the integral $\int u'v \, dx$ is then of the form $\int 2x e^x \, dx.$

Now rewrite the given integral using integration by parts $\int u \, dv = uv - \int v \, du$.

$$\begin{aligned} \int x^2 e^x \, dx &= x^2 e^x - \int 2x e^x \, dx \\ &= x^2 e^x - 2\int x e^x \, dx \qquad \text{......}(2) \end{aligned}$$

Step-2

Again apply the integration by parts to solve the integral $\int x e^x \, dx.$

The integrand $x e^x$ is a product of two functions, one of which will be designated as *u*, the other as v'. Since applying the integration by parts rule will result in another integral of the form

$\int u'v \, dx,$ We must choose *u* and v' so that uv' is of a simpler form. This suggests the following choices.

$u = x$ and $v' = e^x$.

Since $v = e^x$ and $u' = 1$ the integral $\int u'v \, dx$ is then of the form $\int (1) e^x \, dx.$

Now rewrite the given integral using integration by parts $\int u \, dv = uv - \int v \, du$.

$$\begin{aligned} \int x e^x \, dx &= x e^x - \int (1) e^x \, dx \\ &= x e^x - e^x \end{aligned}$$

Hence $\int x e^x \, dx = x e^x - e^x.$

Step-3

Substitute $xe^x - e^x$ for $\int xe^x\,dx$ in (2) and simplify it.

$$\begin{aligned}\int x^2 e^x\,dx &= x^2 e^x - \int 2xe^x\,dx \\ &= x^2 e^x - 2\left[xe^x - e^x\right] + C \\ &= x^2 e^x - 2xe^x + 2e^x + C\end{aligned}$$

Therefore $\int x^2 e^x\,dx = \boxed{x^2 e^x - 2xe^x + 2e^x + C}$. here C is called the constant of integration.