Step-1

Given complex number is $a+ib$.

We have to find the values of a and b on the unit circle at the angles $\theta = 30°, 60°, 90°$.

Step-2

We can write $a+ib$ as $r(\cos\theta + i\sin\theta)$

Since the complex number on the unit circle

So $r = 1$

Therefore the complex number becomes $a+ib = \cos\theta + i\sin\theta$.

Step-3

Let $\theta = 30°$

Then complex number

$a+ib = \cos 30° + i\sin 30°$

$$= \frac{\sqrt{3}}{2} + \frac{1}{2}i \qquad \left(\begin{array}{l} \text{Since } \cos 30° = \frac{\sqrt{3}}{2} \\ \sin 30° = \frac{1}{2} \end{array}\right)$$

Therefore, $\boxed{a = \frac{\sqrt{3}}{2}, \ b = \frac{1}{2}}$ for $\theta = 30°$

Step-4

Let $\theta = 60°$

Then complex number

$a+ib = \cos 60° + i\sin 60°$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2}i \qquad \left(\begin{array}{l} \text{Since } \sin 60° = \frac{\sqrt{3}}{2} \\ \cos 0° = \frac{1}{2} \end{array}\right)$$

Therefore, $\boxed{a=\frac{1}{2}\ ,\ b=\frac{\sqrt{3}}{2}}$ for $\theta=60^\circ$

Step-5

Let $\theta=90^\circ$

Then complex number

$$\begin{aligned} a+ib &= \cos 90^\circ + i\sin 90^\circ \\ &= 0+1i \qquad \begin{pmatrix} \text{Since } \sin 90^\circ = 1 \\ \cos 90^\circ = 0 \end{pmatrix} \\ &= i \end{aligned}$$

Therefore, $\boxed{a=0\ ,\ b=1}$ for $\theta=90^\circ$

Step-6

Let the first, second and third complex numbers are $\frac{\sqrt{3}}{2}+\frac{1}{2}i, \frac{1}{2}+\frac{\sqrt{3}}{2}i, i$

Square of the first complex number is

$$\begin{aligned} \left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^2 &= \left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right) \\ &= \frac{3}{4}+\frac{\sqrt{3}}{4}i+\frac{\sqrt{3}}{4}i+\frac{1}{4}i^2 \\ &= \left(\frac{3}{4}-\frac{1}{4}\right)+\frac{2\sqrt{3}}{4}i \\ &= \frac{1}{2}+\frac{\sqrt{3}}{2}i \end{aligned}$$

Hence the square of the first complex number is equal to the second number.

Step-7

Now we have to verify the cube of the first complex number is third number.

The cube of the first complex number is

$$
\begin{aligned}
\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^3 &= \left(\frac{\sqrt{3}}{2}\right)^3+3\left(\frac{3}{4}\right)\left(\frac{1}{2}i\right)+3\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}i\right)^2+\left(\frac{1}{2}i\right)^3 \\
&= \frac{3\sqrt{3}}{8}+\frac{9}{8}i-\frac{3\sqrt{3}}{8}+\frac{1}{8}i^3 \\
&= \frac{9}{8}i-\frac{1}{8}i \qquad \left(\text{Since } i^2=-1\right) \\
&= \left(\frac{9-1}{8}\right)i \\
&= \left(\frac{8}{8}\right)i \\
&= i
\end{aligned}
$$

Hence the cube of the first complex number is third number.