

Step-1

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$$A = \begin{bmatrix} 1 & v_1 & 0 & 0 \\ 0 & v_2 & 0 & 0 \\ 0 & v_3 & 1 & 0 \\ 0 & v_4 & 0 & 1 \end{bmatrix}$$

Given matrix is

a) We have to factor A into LU , assuming $v_2 \neq 0$

$$A = \begin{bmatrix} 1 & v_1 & 0 & 0 \\ 0 & v_2 & 0 & 0 \\ 0 & v_3 & 1 & 0 \\ 0 & v_4 & 0 & 1 \end{bmatrix}$$

We have

Dividing row 2 by $\left(\frac{1}{v_2}\right)$ gives

$$\rightarrow \begin{bmatrix} 1 & v_1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & v_3 & 1 & 0 \\ 0 & v_4 & 0 & 1 \end{bmatrix}$$

Step-2

Subtracting v_3 times row 2 from row 3, v_4 times row 2 from row 4 gives

$$\rightarrow \begin{bmatrix} 1 & v_1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Subtracting v_1 times row 2 from row 1 gives

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Therefore, the upper triangular matrix is

Step-3

To get the matrix L , we have to do the reverse operations on I .

That is, multiplying row 2 by v_2 gives

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & v_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Adding v_3 times row 2 to row 3, v_4 times row 2 to row 4, v_1 times row 2 to row 3 gives

$$\begin{bmatrix} 1 & v_1 & 0 & 0 \\ 0 & v_2 & 0 & 0 \\ 0 & v_3 & 1 & 0 \\ 0 & v_4 & 0 & 1 \end{bmatrix}$$

Step-4

$$L = \begin{bmatrix} 1 & v_1 & 0 & 0 \\ 0 & v_2 & 0 & 0 \\ 0 & v_3 & 1 & 0 \\ 0 & v_4 & 0 & 1 \end{bmatrix}$$

Therefore, the lower triangular matrix is

$$A = \begin{bmatrix} 1 & v_1 & 0 & 0 \\ 0 & v_2 & 0 & 0 \\ 0 & v_3 & 1 & 0 \\ 0 & v_4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Hence the LU factorization of the given matrix A is

Step-5

b) We have to find A^{-1} , which has the same form as A .

Consider

$$[A \quad I] = \begin{bmatrix} 1 & v_1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & v_2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & v_3 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & v_4 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Dividing row 2 by $\left(\frac{1}{v_2}\right)$ gives

$$\rightarrow \begin{bmatrix} 1 & v_1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{v_2} & 0 & 0 \\ 0 & v_3 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & v_4 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Step-6

Subtracting v_3 times row 2 from row 3, v_4 times row 2 from row 4 gives

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & -\frac{v_1}{v_2} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{v_2} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{v_3}{v_2} & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -\frac{v_4}{v_2} & 0 & 1 \end{bmatrix}$$

By gauss elimination method

$$A^{-1} = \begin{bmatrix} 1 & -\frac{v_1}{v_2} & 0 & 0 \\ 0 & \frac{1}{v_2} & 0 & 0 \\ 0 & -\frac{v_3}{v_2} & 1 & 0 \\ 0 & -\frac{v_4}{v_2} & 0 & 1 \end{bmatrix}$$