Step-1

(a)

The objective is to find the multiplications required to find an n by n determinant from the big formula.

Big Formula is;

$$\det A = \sum_{all\ P's} \left(a_{1\alpha} a_{2\beta} \ldots a_{n\nu}\right) \det P$$

Each term require $(n-1)$ multiplications.

So, the maximum number of multiplication required in the big formula is $n!(n-1)$ as it is permutation of order n.

Hence, the total number of multiplications required is $\boxed{n!(n-1)}$

Step-2

(b)

The objective is to find the multiplications required to find an n by n determinant from the cofactor formula.

$$\det A = a_{i1}C_{i1} + a_{i2}C_{i2} + \ldots + a_{in}C_{in}$$

For n by n matrix, consider sub matrix of order $n-1$. Then multiplications required for cofactors of sub matrix is $(n-1)!$

Then, multiply these cofactors of a row with elements of the respective row to obtain determinant of the original matrix.

For 1 by 1 matrix, divide by $1!$

For 2 by 2 matrix, divide by $2!$

Since it is n by n matrix so obtain the result and multiply by n and get;

$$= n\left[(n-1)! + \frac{(n-1)!}{2!} + \frac{(n-1)!}{3!} + \ldots 1\right]$$
$$= n! + \frac{n!}{2!} + \frac{n!}{3!} + \ldots \frac{n!}{(n-1)!}$$
$$= \left(1 + \frac{1}{2!} + \frac{1}{3!} + \ldots \frac{1}{(n-1)!}\right) n!$$

Hence, the total number of multiplications required is $\boxed{\left(1+\frac{1}{2!}+\frac{1}{3!}+\ldots\frac{1}{(n-1)!}\right)n!}$

Step-3

(c)

Reduce a given matrix to upper triangular form;

For this number of multiplications required are $(n-1)+(n-2)+\ldots+1$ steps that is $\frac{n(n-1)}{2}$ and then get determinant by multiplication of pivots.

Each elimination step while reducing first row require n multiplications.

So, total $n(n-1)$ multiplications and for second row $(n-1)(n-2)$ steps and finally in last step 2 multiplications.

So, total steps required;

$$=\sum_{k=2}^{n}k(k-1)$$
$$=\sum_{k=2}^{n}\left(k^2-k\right)$$
$$=\sum_{k=1}^{n}k^2-\sum_{k=1}^{n}k$$
$$=\frac{n(n+1)(2n+1)}{6}-\frac{n(n+1)}{2}$$
$$=\frac{n(n+1)}{2}\left(\frac{2n+1}{3}-1\right)$$
$$=\frac{n(n+1)(2n-2)}{6}$$
$$=\frac{n\left(n^2-1\right)}{3}$$ â€¦â€¦ (1)

Now, multiply pivots to calculate determinant;

For this number of multiplications required are $n-1$ â€¦â€¦ (2)

So, add equation (1) and (2) and obtain total number of multiplications required;

$$= \frac{n\left(n^2-1\right)}{3}+\left(n-1\right)$$

$$= \frac{n^3-n+3n-3}{3}$$

$$= \frac{n^3+2n-3}{3}$$

$$= \frac{1}{3}\left(n^3+2n-3\right)$$

Hence, the total number of multiplications required is $\boxed{\frac{1}{3}\left(n^3+2n-3\right)}$