Step-1

Consider the matrix A of order is m by n.

$$A_{m \times n} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}.$$

Consider n dimensional vector x as shown below:

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

The objective is to find how many separate multiplications needed when the matrix A is multiply with n dimensional vector x.

Step-2

Find the product of A and $\mathbf{x}$.

Consider $A\mathbf{x} = C$

If A has $m \times n$ order and $\mathbf{x}$ has $n \times 1$ order then $A\mathbf{x}$ has $m \times 1$ order.

Therefore, the matrix C has $m \times 1$ order.

$$A\mathbf{x} = C = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$C_{m \times 1} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n \end{bmatrix}$$

From the above matrix $C_{m \times 1}$, in each row there are n number of multiplications involved and there are m number of rows in total.

Therefore, there are $\boxed{mn}$ number of separate multiplications involved.

Step-3

Consider the matrix
$$B_{n\times p} = \begin{bmatrix} b_{11} & \cdots & b_{1p} \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & b_{np} \end{bmatrix}$$

Now find the multiplication of A and B.

$$AB = E$$

Here, A is a $m\times n$ matrix and B is a $n\times p$ matrix then the multiplication AB should be $m\times p$ matrix.

$$E_{m\times p} = AB$$

$$= \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}\begin{bmatrix} b_{11} & \cdots & b_{1p} \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & b_{np} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}b_{11}+\cdots+a_{1n}b_{n1} & \cdots & a_{11}b_{1p}+\cdots+a_{1n}b_{np} \\ \vdots & \ddots & \vdots \\ a_{m1}b_{11}+\cdots+a_{mn}b_{n1} & \cdots & a_{m1}b_{1p}+\cdots+a_{mn}b_{np} \end{bmatrix}$$

From the matrix $E_{m\times p}$, notice that there are n number of multiplications involved.

Since the order of E is $m\times p$, there are $\boxed{nmp}$ separate multiplications involved.