Step-1

Given that the equations of three planes:

$$x + y + z = 2$$
$$x + 2y + z = 3$$
$$2x + 3y + 2z = 5$$

We have to find that why the three columns form singular case, and we have to find two combinations of the columns that give $b = (2,3,5)$, and finally we have to find the value of c such that $b = (4,6,c)$

Step-2

The three columns of the above equations are

$$\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

The first and third columns are equal, so on column operation $C_3 \to C_3 - C_1$, we will get a zero column, and hence three columns will form the singular case.

Step-3

The given three planes can be written as

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

Apply $R_3 \to R_3 - (R_1 + R_2)$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$

Step-4

Apply $R_2 \to R_2 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

Apply $R_1 \to R_1 - R_2$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

Step-5

$\Rightarrow x + z = 1$

$y = 1$

Put $z = k$

$\Rightarrow x = 1 - k$

Step-6

Therefore the solution is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1-k \\ 1 \\ k \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + k\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

From the above, the two combinations of the columns that give $b = (2,3,5)$ are

$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, k\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, where *k* is any real number

Step-7

If we multiply $b = (2,3,5)$ with 2, then we get $(4,6,10)$, compare this vector with $b = (4,6,c)$, we get $\boxed{c = 10}$