

Step-1

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Given system of equations is

$$u + v + w = 0$$

$$u + 2v + 3w = 0$$

$$3u + 5v + 7w = 1$$

We have to solve the given system by elimination or show that there is no solution.

Step-2

The augmented matrix of the given system is

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 2 & 3 & 0 \\ 3 & 5 & 7 & 1 \end{bmatrix}$$

Subtracting row 1 from row 2 and 3 times row 1 from row 3 gives

$$\rightarrow \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 2 & 4 & 1 \end{bmatrix}$$

Subtracting 2 times row 2 from row 3 gives

$$\rightarrow \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The last row shows that $0 = 1$ which has no solution.

Hence the given system has no solution.

Step-3

Given system of equations is

$$u + v + w = 0$$

$$u + v + 3w = 0$$

$$3u + 5v + 7w = 1$$

Converting into matrix form

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 3 & 0 \\ 3 & 5 & 7 & 1 \end{bmatrix}$$

Subtracting row 1 from row 2 and 3 times row 1 from row 3 gives

$$\rightarrow \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 2 & 4 & 1 \end{bmatrix}$$

Step-4

Subtracting 2 times row 2 from row 3 gives

$$\rightarrow \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 2 & 0 & 1 \end{bmatrix}$$

Then the equations are

$$u + v + w = 0$$

$$2v = 1$$

$$2w = 0 \Rightarrow w = 0$$

By back substitution gives $u = -\frac{1}{2}, v = \frac{1}{2}, w = 0$

Hence the solution to the given system is $\boxed{u = -\frac{1}{2}, v = \frac{1}{2}, w = 0}$.