Step-1

Similar matrices: Matrices A and B are similar if $A = M^{-1}BM$ for invertible matrix M.

Consider the following matrices:

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$
$$B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Step-2

Eigen values of A and B matrices are $\lambda = (0,0,0)$.

Step-3

One Eigen vector of matrix A is $(1,0,0)$ only. Its Jordan form will be as follows.

$$J_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Matrix A must be similar to J_1.

Step-4

Matrix B has additional Eigen vector $(0,1,0)$. Its Jordan form will contain two blocks:

$$J_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Matrix B must be similar to J_2.

Step-5

Another three 1 by 1 block Jordan matrix is:

$$J_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The only matrix similar to J_3 is $M^{-1}0M = 0$.

Step-6

Show that no two of these Jordan forms are similar.

$$J_1 \neq M^{-1}J_2M$$
$$J_1 \neq M^{-1}J_3M$$
$$J_2 \neq M^{-1}J_3M$$

Step-7

As mentioned earlier that only matrix similar to J_3 is $M^{-1}0M = 0$. So, J_3 cannot be similar to J_1 and J_2.

$$\boxed{\begin{array}{l} J_1 \neq M^{-1}J_3M \\ J_2 \neq M^{-1}J_3M \end{array}}$$

If matrices J_1 and J_2 are similar than following must be true:

$$\begin{aligned} J_1 &= M^{-1}J_2M \\ MJ_1 &= J_2M \end{aligned}$$

Step-8

Consider matrix M containing generalized Eigen vector defined as follows and do the calculations:

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} MJ_1 &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

$$J_2 M = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

This shows that $MJ_1 \neq J_2 M$.

Step-9

Therefore, $\boxed{J_1 \neq M^{-1} J_2 M}$ matrices J_1 and J_2 are also not similar. So, no two of these Jordan forms are similar.