

## Step-1

Given  $\mathbf{V} = \mathbf{R}^4$ , we have to find  $\mathbf{V}^\perp$  and  $(\mathbf{V}^\perp)^\perp$

$$\begin{aligned}\mathbf{V}^\perp &= \left\{ \alpha = (x, y, z, w) \in \mathbf{V} / \alpha^T \beta = 0, \forall \beta \in \mathbf{V} \right\} \\ &= \left\{ \alpha = (x, y, z, w) \in \mathbf{V} / (x, y, z, w)^T (x, y, z, w) = 0 \right\}\end{aligned}$$

$$\text{Now } (x, y, z, w)^T (x, y, z, w) = 0$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} (x, y, z, w) = 0$$

$$\Rightarrow x^2 + y^2 + z^2 + w^2 = 0$$

$$\Rightarrow (x, y, z, w) = (0, 0, 0, 0)$$

$$\Rightarrow \boxed{\mathbf{V}^\perp = \{\mathbf{0}\}}$$

## Step-2

$$\begin{aligned}(\mathbf{V}^\perp)^\perp &= \left\{ \alpha = (x, y, z, w) \in \mathbf{V} / \alpha^T \beta = 0, \forall \beta \in \mathbf{V}^\perp \right\} \\ &= \left\{ \alpha = (x, y, z, w) \in \mathbf{V} / \alpha^T \beta = 0, \beta = \mathbf{0} \right\} \\ &= \left\{ \alpha = (x, y, z, w) \in \mathbf{V} / \alpha^T \mathbf{0} = 0, \text{independent of } (x, y, z, w) \in \mathbf{V} \right\} \\ &= \mathbf{V}\end{aligned}$$

$$\text{Therefore } \boxed{(\mathbf{V}^\perp)^\perp = \mathbf{V}}$$