Step-1

Consider the equations,

$$-u'' = x,$$
$$u(0) = 0,$$
$$u(1) = 1$$

By using four intervals and two and an extra half- hat functions, with $h = \dfrac{1}{3}$, the matrix A(2 by 2) is given by,

$$A = 3 \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

Let

$$f(x) = x$$

Therefore, we get,

$$b = hf(x)$$
$$= \left(\frac{1}{3}\right)x$$
$$= \frac{x}{3}$$
$$= \frac{1}{3}\begin{bmatrix} x \\ x \end{bmatrix}$$

Step-2

By substituting A, and b into $Ay = b$, we get,

$$3\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} y = \frac{1}{3}\begin{bmatrix} x \\ x \end{bmatrix}$$
$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} y = \frac{1}{9}\begin{bmatrix} x \\ x \end{bmatrix}$$
$$y = \frac{1}{9}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}^{-1}\begin{bmatrix} x \\ x \end{bmatrix}$$

The inverse matrix A is given by,

$$A^{-1} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}^{-1}$$

$$= \frac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Step-3

On substitution, we get,

$$y = \frac{1}{27}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ x \end{bmatrix}$$

$$= \frac{1}{27}\begin{bmatrix} 3x \\ 3x \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} x \\ x \end{bmatrix}$$

The linear finite element is given by,

$$U(x) = \frac{1}{9}V_1 + \frac{1}{9}V_2$$

Step-4

Thus, the linear finite element is $\boxed{U(x) = \frac{1}{9}V_1 + \frac{1}{9}V_2}$.