Step-1

By choosing the correct vector in *b* in the Schwarz inequality, we have to prove that

$\left(a_1+a_2+\cdots+a_n\right)^2 \le n\left(a_1{}^2+a_2{}^2+\cdots+a_n{}^2\right)$, and also we have to find that, when does the equality holds.

Step-2

Let $a=\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$

Choose $b=(1,1,\ldots,1)$

Schwarz inequality is $\left|a^T b\right| \le \|a\|\,\|b\|$ â€¦â€¦ (1)

Step-3

$$a^T b=\left(a_1,a_2,\ldots,a_n\right)\begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$$

$$=a_1+a_2+\cdots+a_n$$

$$\Rightarrow \left|a^T b\right|=\left|a_1+a_2+\cdots+a_n\right|$$

$$\Rightarrow \left|a^T b\right|^2=\left(a_1+a_2+\cdots+a_n\right)^2$$

Step-4

And $\|a\|=\sqrt{a_1^2+a_2^2+\cdots+a_n^2}$

$$\|b\|=\sqrt{1^2+1^2+\cdots+1^2\left(n \text{ times}\right)}$$

$$=\sqrt{n.1}$$

Step-5

$$\Rightarrow \|a\|^2=a_1^2+a_2^2+\cdots+a_n^2 \text{ and } \|b\|^2=n$$

By (1), $\left|a^T b\right|^2 \le \|a\|^2 \|b\|^2$

Hence $\boxed{\left(a_1 + a_2 + \cdots + a_n\right)^2 \le n\left(a_1^{\ 2} + a_2^{\ 2} + \cdots + a_n^{\ 2}\right)}$

Step-6

If $a_1 = a_2 = \cdots = a_n$ then

$$
\begin{aligned}
&\left(a_1 + a_2 + \cdots + a_n\right)^2 \\
&= \left(n a_1\right)^2 \\
&= n^2 a_1^{\ 2}
\end{aligned}
$$

Step-7

And

$$
\begin{aligned}
&n\left(a_1^{\ 2} + a_2^{\ 2} + \cdots + a_n^{\ 2}\right) \\
&= n\left(n a_1^{\ 2}\right) \\
&= n^2 a_1^{\ 2}
\end{aligned}
$$

Therefore $\left(a_1 + a_2 + \cdots + a_n\right)^2 = n\left(a_1^{\ 2} + a_2^{\ 2} + \cdots + a_n^{\ 2}\right)$

Hence the equality holds if $a_1 = a_2 = \cdots = a_n$