Step-1

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Given that $A^T = A$ and $B^T = B$

a) We have to verify whether $A^2 - B^2$ is symmetric or not.

Now

$$\begin{aligned}
\left(A^2 - B^2\right)^T &= \left(A^2 + \left(-B^2\right)\right)^T \\
&= \left(A^2\right)^T + \left(-B^2\right)^T \quad \left(\text{Since } (A+B)^T = A^T + B^T\right) \\
&= \left(A^2\right)^T - \left(B^2\right)^T \\
&= \left(A^T\right)^2 - \left(B^T\right)^2 \quad \left(\begin{array}{l}\text{Since } \left(A^m\right)^T = \left(A^T\right)^m, \text{ where } m \\ \text{is any scalar}\end{array}\right) \\
&= A^2 - B^2 \quad \left(\text{Since } (A)^T = A\right)
\end{aligned}$$

Therefore, $A^2 - B^2$ is symmetric.

Step-2

b) We have to verify whether $(A+B)(A-B)$ is symmetric or not.

Now

$$\begin{aligned}
\left[(A+B)(A-B)\right]^T &= \left[\left(A^2 - AB + BA - B^2\right)\right]^T \\
&= \left(A^2\right)^T - (AB)^T + (BA)^T - \left(B^2\right)^T \\
&= \left(A^T\right)^2 - B^T A^T + A^T B^T - \left(B^T\right)^2 \\
&= A^2 - BA + AB - B^2
\end{aligned}$$

Since $\left[(A+B)(A-B)\right]^T \neq (A+B)(A-B)$

Hence $(A+B)(A-B)$ is not symmetric.

Step-3

c) We have to verify whether ABA is symmetric or not.

Now

$$\begin{aligned}(ABA)^T &= A^T B^T A^T && \left(\text{Since } (AB)^T = B^T A^T\right)\\ &= ABA && \left(\text{Since } A^T = A \text{ and } B^T = B\right)\end{aligned}$$

Since $(ABA)^T = ABA$

Hence ABA is symmetric.

Step-4

d) We have to verify whether $ABAB$ is symmetric or not.

Now

$$\begin{aligned}(ABAB)^T &= B^T A^T B^T A^T && \left(\text{Since } (AB)^T = B^T A^T\right)\\ &= BABA && \left(\text{Since } A^T = A \text{ and } B^T = B\right)\end{aligned}$$

Since $(ABAB)^T \neq ABAB$

Hence $ABAB$ is not symmetric.