

Step-1

Given that A and B are of n by n matrices with all entries equals to 1 and C is n by n matrix with entries $c_{jl} = 2$. We have to find $(AB)_{ij}$ and $(AB)C, A(BC)$.

Step-2

By definition of product of matrices

$$(AB)_{ij} = \sum_k a_{ik} b_{kj}$$

The elements of AB are the sum of products of corresponding elements of rows and columns so

$$\begin{aligned}\sum_k a_{ik} b_{kj} &= 1.1 + 1.1 + 1.1 + \dots + 1.1 \\ &= 1 + 1 + 1 + \dots + 1 \text{ (n times)} \\ &= n\end{aligned}$$

Hence the elements of AB of order n by n consists all elements as n .

Step-3

Since C is n by n matrix with entries $c_{jl} = 2$, by definition

$$\begin{aligned}\sum_k b_{kj} c_{jl} &= 1(2) + 1(2) + 1(2) + \dots + 1(2) \\ &= 2 + 2 + 2 + \dots + 2 \text{ (n times)} \\ &= 2n\end{aligned}$$

$$\begin{aligned}\sum_k a_{ik} \left(\sum_l b_{kj} c_{jl} \right) &= 1.2n + 1.2n + 1.2n + \dots + 1.2n \\ &= 2n + 2n + 2n + \dots + 2n \text{ (n times)} \\ &= n.(2n) \\ &= 2n^2\end{aligned}$$

Hence all the elements of $A(BC)$ are $2n^2$.

Step-4

We have $\sum_k a_{ik} b_{kj} = n$ and $c_{jl} = 2$

$$\begin{aligned}
\text{Now } \sum_j \left(\sum_k a_{ik} b_{kj} \right) c_{jl} &= n.2 + n.2 + \dots + n.2 \\
&= 2n + 2n + 2n + \dots + 2n \text{ (n times)} \\
&= n.(2n) \\
&= 2n^2
\end{aligned}$$

Hence all the elements of $(AB)C$ are $2n^2$.