

Step-1

Given

The matrix B_n is the $-1, 2, -1$ matrix A_n except that $b_{11} = 1$ instead of $a_{11} = 2$

So that

$$B_2 = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}, B_3 = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$B_4 = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

Step-2

Then

$$\begin{aligned} \det(B_2) &= 2 - 1 \\ &= 2 \end{aligned}$$

$$\det(B_3) = (4 - 1) + (-2) = 2 \det(B_2) + \begin{vmatrix} 1 & 0 \\ -1 & -1 \end{vmatrix}$$

$$= 2 \det(B_3) - \det(B_2) \quad (\text{Expanding determinant by least columns})$$

$$= 3 - 2$$

$$= 1$$

Step-3

Expanding any B_n by last column use get

$$\det(B_n) = \begin{bmatrix} 1 & -1 & 0 & 0 \dots \dots 0 \\ -1 & 2 & -1 & 0 \dots \dots \dots \\ 0 & -1 & 2 & -1 \dots \dots \dots \\ 0 & 0 & 0 & \dots \dots \dots -1 \end{bmatrix} + 2 \det(B_{n-1})$$

$$= -\det(B_{n-2}) + 2 \det(B_{n-1})$$

(expanding the earlier determinant. 1 term in R.H.S by last column)

Step-4

So, the recursion formula is as of A_n 's

Here we have

$$|B_1| = |B_2| = |B_3| = 1$$

And we notice that all pivots are equal to 1