

Step-1

Consider the vectors $(1, 4, 5), (x, y, z)$ as the matrix multiplication Ax

The objective is to write the inner product and complete the blanks in the statement “The solutions to $Ax = 0$ lie on a _____ perpendicular to the vector _____. The columns of A are only in ____ space.”

Step-2

Assume the matrix $A = \begin{bmatrix} 1 & 4 & 5 \end{bmatrix}$

The inner product of matrix A and the vector $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is given as the dot product of $A \cdot x$ where each element is multiplied element wise.

$$Ax = \begin{bmatrix} 1 & 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$Ax = x + 4y + 5z$$

Hence, $Ax = x + 4y + 5z$

Now, $Ax = 0$ implies that $x + 4y + 5z = 0$

Step-3

Recall that an equation $ax + by + cz + d = 0$ where a, b, c, d are constants represents the equation of plane.

Thus, the equation $x + 4y + 5z = 0$ represents the equation of plane.

Also, for an equation of the form $ax + by + cz + d = 0$, the normal or perpendicular to the plane is (a, b, c)

Here, the solutions to equation $Ax = 0$ lie on the plane $x + 4y + 5z = 0$ and the vector perpendicular to the plane is $\begin{bmatrix} 1 & 4 & 5 \end{bmatrix}$

Each column of matrix A is a non-zero singleton set which is one-dimensional.

Thus, the columns of A are in one-dimensional space.

Hence, the blanks are filled with plane, $\begin{bmatrix} 1 & 4 & 5 \end{bmatrix}$ and one-dimensional