Step-1

We have to confirm the statement that the product of two lower triangular matrices is again a lower triangular matrix by giving a matrix of order 3 by 3.

Step-2

Let the two lower triangular matrices (all the entries above the main diagonal are zero)

be $A=\begin{pmatrix}1&0&0\\1&2&0\\1&1&3\end{pmatrix}$ and $B=\begin{pmatrix}2&0&0\\3&2&0\\2&4&3\end{pmatrix}$

The product of A and B is

$$AB=\begin{pmatrix}1&0&0\\1&2&0\\1&1&3\end{pmatrix}\begin{pmatrix}2&0&0\\3&2&0\\2&4&3\end{pmatrix}$$

$$=\begin{pmatrix}1.2+0.3+0.2 & 1.0+0.2+0.4 & 1.0+0.0+0.3\\1.2+2.3+0.2 & 1.2+2.2+0.4 & 1.0+1.0+3.0\\1.2+1.3+3.2 & 1.0+1.2+3.4 & 1.0+1.0+3.3\end{pmatrix}$$

$$=\begin{pmatrix}2&0&0\\8&6&0\\11&14&9\end{pmatrix}$$

Therefore AB is also a lower triangular matrix.

Step-3

The product matrix (by the laws matrix multiplication) is obtained by the multiplication of rows of A with the columns of B .

Since all the entries above the main diagonal are zeros both in A and B we have that the entries above the main diagonal are zeros in AB i.e. AB is a lower triangular matrix.