

Step-1

We have to find the matrices that are equal to $(A+B)^2$.

$$\text{We have } (A+B)^2 = (A+B).(A+B)$$

$$= A^2 + A.B + B.A + B^2$$

Step-2

Given matrix expression is $A^2 + 2A.B + B^2$

Since $AB \neq BA$, $A^2 + A.B + B.A + B^2 \neq A^2 + 2A.B + B^2$

Therefore $A^2 + 2A.B + B^2 \neq (A+B)^2$.

Step-3

Given matrix expression is $A(A+B) + B(A+B)$

$$A(A+B) + B(A+B) = A.A + A.B + B.A + B^2$$

$$= A^2 + A.B + B.A + B^2$$

$$= (A+B)^2$$

Therefore $A(A+B) + B(A+B) = (A+B)^2$.

Step-4

Given matrix expression is $(A+B)(B+A)$

$$(A+B)(B+A) = A(B+A) + B(B+A)$$

$$= A.B + A.A + B.B + B.A$$

$$= A^2 + A.B + B.A + B^2$$

$$= (A+B)^2$$

Therefore $(A+B)(B+A) = (A+B)^2$.

Step-5

Given matrix expression is $A^2 + AB + BA + B^2$

$$A^2 + AB + BA + B^2 = A(A + B) + B(A + B)$$

$$= (A + B) \cdot (A + B)$$

$$= (A + B)^2$$

Therefore $A^2 + AB + BA + B^2 = (A + B)^2$

Hence we can conclude that $A(A + B) + B(A + B)$, $(A + B)(B + A)$ and $A^2 + AB + BA + B^2$ always equal $(A + B)^2$.