Step-1

Given that $x=\begin{bmatrix}2-4i\\4i\end{bmatrix}, y=\begin{bmatrix}2+4i\\4i\end{bmatrix}$

We have to find the lengths and inner product of the given numbers.

Step-2

Now the length of x is

$$\begin{aligned}\|x\|^2 &= |2-4i|^2+|4i|^2\\ &= \sqrt{2^2+(-4)^2}+\sqrt{4^2}\\ &= (4+16)+16\\ &= 36\\ \Rightarrow \|x\| &= \sqrt{36}\\ &= 6\end{aligned}$$

Therefore, the length of x is $\boxed{6}$.

Step-3

Now the length of y is

$$\begin{aligned}\|y\|^2 &= |2+4i|^2+|4i|^2\\ &= \sqrt{2^2+4^2}+\sqrt{4^2}\\ &= (4+16)+16\\ &= 36\\ \Rightarrow \|y\| &= \sqrt{36}\\ &= 6\end{aligned}$$

Therefore, the length of y is $\boxed{6}$.

Step-4

Now the inner product of x and y is

$$\bar{x}^T\cdot y=\begin{bmatrix}2+4i & -4i\end{bmatrix}\begin{bmatrix}2+4i\\4i\end{bmatrix}$$

$$
\begin{aligned}
&= (2+4i)^2 + (-4i)(4i) \\
&= 4 + 16i^2 + 16i - 16i^2 \\
&= 4 - 16 + 16i + 16 \qquad (\text{Since } i^2 = -1) \\
&= 4 + 16i
\end{aligned}
$$

Hence the inner product of x and y is $\boxed{4+16i}$.