Step-1

Given that (x_1, x_2, x_3) is transformed to $(0, x_1, x_2, x_3)$.

We have to find the 4 by 3 matrix *A* that represents the given right shift.

Step-2

We know that the standard basis for $\mathbf{R}^3$ is $\{(1,0,0),(0,1,0),(0,0,1)\}$ and that of $\mathbf{R}^4$ is $\{(1,0,0,0),(0,1,0,0),(0,0,1,0),(0,0,0,1)\}$

We have $T:\mathbf{R}^3 \to \mathbf{R}^4$ defined as $T(x_1, x_2, x_3) = (0, x_1, x_2, x_3)$

Therefore,

$$\begin{aligned} T(1,0,0) &= (0,1,0,0) \\ &= 0(1,0,0,0) + 1(0,1,0,0) + 0(0,0,1,0) + 0(0,0,0,1) \end{aligned}$$

$$\begin{aligned} T(0,1,0) &= (0,0,1,0) \\ &= 0(1,0,0,0) + 0(0,1,0,0) + 1(0,0,1,0) + 0(0,0,0,1) \end{aligned}$$

Step-3

And

$$\begin{aligned} T(0,0,1) &= (0,0,0,1) \\ &= 0(1,0,0,0) + 0(0,1,0,0) + 0(0,0,1,0) + 1(0,0,0,1) \end{aligned}$$

The matrix *A* represented by standard basis is $A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Hence the matrix of the right shift is $\boxed{A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$

Step-4

Given that (x_1, x_2, x_3, x_4) is transformed to (x_1, x_2, x_3).

We have to find the 3 by 4 matrix A that represents the given right shift.

Step-5

Now let R: $\mathbf{R}^4 \to \mathbf{R}^3$ by $R(x_1, x_2, x_3, x_4) = (x_2, x_3, x_4)$

Let $e_1 = (1,0,0,0)$, $e_2 = (0,1,0,0)$, $e_3 = (0,0,1,0)$, $e_4 = (0,0,0,1)$

$v_1 = (1,0,0)$, $v_2 = (0,1,0)$, $v_3 = (0,0,1)$

$$\begin{aligned} R(e_1) &= (0,0,0) \\ &= 0v_1 + 0v_2 + 0v_3 \end{aligned}$$

$$\begin{aligned} R(e_2) &= (1,0,0) \\ &= 1v_1 + 0v_2 + 0v_3 \end{aligned}$$

Step-6

And

$$\begin{aligned} R(e_3) &= (0,1,0) \\ &= 0v_1 + 1v_2 + 0v_3 \end{aligned}$$

$$\begin{aligned} R(e_4) &= (0,0,1) \\ &= 0v_1 + 0v_2 + 1v_3 \end{aligned}$$

Therefore, the matrix is $B = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Hence the matrix of left shift is $\boxed{B = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$

Step-7

Now $AB = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Therefore, $\boxed{AB = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$

$$BA = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Therefore, $\boxed{BA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$