Step-1

Let A be any $n \times n$ matrix. Without loss of generality (W.L.O.S) let B be a matrix formed by throwing away last two rows and columns of A. So, B is $(n-2)\times(n-2)$ matrix. Let μ be the smallest eigenvalue of matrix B and λ be the smallest eigenvalue of matrix A. Note that Rayleighâ€™s quotient $\mathrm{R}(x)$ is given by

$$\mathrm{R}(x) = \frac{x^{\mathrm{T}}\mathrm{A}x}{x^{\mathrm{T}}x}$$

Step-2

The Rayleighâ€™s quotient for B agrees with the Rayleighâ€™s quotient for A, whenever $x = (x_1, x_2, x_3, \ldots, x_{n-2}, 0, 0)$. These are the vectors perpendicular to $n-$tuples , $z_1 = (0,0,\ldots,0,0,1)$ and $z_2 = (0,0,\ldots,0,1,0)$ and for them we have $x^{\mathrm{T}}\mathrm{A}x = x^{\mathrm{T}}\mathrm{B}x$.

The minimum over these x's is the smallest eigenvalue μ. It cannot be below the smallest eigenvalue λ of A, because that is the minimum over all x's.

Therefore, $\lambda \le \mu$.