Step-1

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}-\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}+\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}+\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}-\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}$$

$p_1 \;\; p_2 \;\; p_3 \;\; p_4 \;\; p_5$

Suppose

$$a\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}+b\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}+c\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}+d\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}+e\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

On scalar multiplication and vector addition, we get

$$\begin{bmatrix}0&a&0\\a&0&0\\0&0&a\end{bmatrix}+\begin{bmatrix}0&b&0\\0&0&b\\b&0&0\end{bmatrix}+\begin{bmatrix}0&0&c\\0&c&0\\c&0&0\end{bmatrix}+\begin{bmatrix}d&0&0\\0&0&d\\0&d&0\end{bmatrix}+\begin{bmatrix}0&0&e\\e&0&0\\0&e&0\end{bmatrix}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}d&a+b&c+e\\a+e&c&b+d\\b+c&d+e&a\end{bmatrix}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$\Rightarrow a=b=c=d=e=0$

Step-2

Thus, we have shown that whenever

$$a\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}+b\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}+c\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}+d\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}+e\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$, we get $a=b=c=d=e=0$

Therefore, the matrices $p_1, p_2, p_3, p_{4,} p_5$ are linearly independent

We see that the row sums of the linear combination $\begin{bmatrix}0&a&0\\a&0&0\\0&0&a\end{bmatrix}+\begin{bmatrix}0&b&0\\0&0&b\\b&0&0\end{bmatrix}+\begin{bmatrix}0&0&c\\0&c&0\\c&0&0\end{bmatrix}+\begin{bmatrix}d&0&0\\0&0&d\\0&d&0\end{bmatrix}+\begin{bmatrix}0&0&e\\e&0&0\\0&e&0\end{bmatrix}=\begin{bmatrix}d&a+b&c+e\\a+e&c&b+d\\b+c&d+e&a\end{bmatrix}$

$a+b+c+d+e$

$a+b+c+d+e$

$a+b+c+d+e$

Similarly, the column sums are

$a+b+c+d+e$

$a+b+c+d+e$

$a+b+c+d+e$

We easily see that all the row sums are equal to $a+b+c+d+e$

Similarly, column sums are equal to $a+b+c+d+e$

Therefore, the matrices $\{p_1, p_2, p_3, p_4, p_5\}$ is a basis for the subspace of 3 by 3 matrices with row and column sum are all equal.