## Step-1

Let $A$ be the following matrix:

$$A=\begin{bmatrix}3 & 2\\ 2 & 3\end{bmatrix}$$

Diagonalize $A$ and compute $S\Lambda^k S^{-1}$ to get following formula for $A^k$:

$$A^k=\frac{1}{2}\begin{bmatrix}5^k+1 & 5^k-1\\ 5^k-1 & 5^k+1\end{bmatrix}$$

## Step-2

To diagonalize the matrix $A$ follow the following steps:

$$\begin{aligned} A&=\begin{bmatrix}3 & 2\\ 2 & 3\end{bmatrix}\\ A-\lambda I&=\begin{bmatrix}3-\lambda & 2\\ 2 & 3-\lambda\end{bmatrix}\\ \det(A-\lambda I)&=(3-\lambda)^2-4\\ &=\lambda^2-6\lambda+5\end{aligned}$$

Put the determinant value equal to zero, to get following roots as Eigen values:

$$\lambda_1=5$$
$$\lambda_2=1$$

## Step-3

Eigen vectors corresponding to the Eigen values are calculated as follows:

For $\lambda_1=5$

$$(A-\lambda I)x_1=0$$
$$\begin{bmatrix}-2 & 2\\ 2 & -2\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$
$$x_1=\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}1\\ 1\end{bmatrix}$$

## Step-4

For $\lambda_2 = 1$

$$(A - \lambda_2 I)x_2 = 0$$
$$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$x_2 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

## Step-5

Thus, Eigen vector matrix is as follows:

$$S = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$
$$S^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

## Step-6

Therefore, diagonalisation of matrix $A$ is as follows:

$$A = S\Lambda S^{-1}$$
$$= \boxed{\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}}$$

## Step-7

Now, do the following calculations to get $A^k$:

$$A^k = S\Lambda^k S^{-1}$$
$$= \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 5^k & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$
$$= \frac{1}{2}\begin{bmatrix} 5^k + 1 & 5^k - 1 \\ 5^k - 1 & 5^k + 1 \end{bmatrix}$$

## Step-8

Therefore,

$$\boxed{A^k = \frac{1}{2}\begin{bmatrix} 5^k+1 & 5^k-1 \\ 5^k-1 & 5^k+1 \end{bmatrix}}$$