Southern University of Science and Technology

Advanced Linear Algebra Spring 2023

MA109– Quiz #3

2023/03/09

Name: _____

Student Number: _____

1. Give an example of a function $\varphi : \mathbf{R}^2 \to \mathbf{R}$ such that

$$\varphi(av) = a\varphi(v)$$

for all $a \in \mathbf{R}$ and all $v \in \mathbf{R}^2$ but φ is not linear.

给出一个函数 $\varphi : \mathbf{R}^2 \to \mathbf{R}$, 使得对于任意 $a \in \mathbf{R}, v \in \mathbf{R}^2$ 有

$$\varphi(av) = a\varphi(v)$$

成立，但是 φ 不是线性的.

2. Suppose $b, c \in \mathbf{R}$. Define $T : \mathbf{R} \to \mathbf{R}^2$ by

$$Tp = (3p(4) + 5p'(6) + bp(1)p(2), \int_{-1}^{2} x^3 p(x)dx + c \sin p(0)).$$

Show that T is linear if and only if $b = c = 0$.

设 $b, c \in \mathbf{R}$, 定义 $T : \mathbf{R} \to \mathbf{R}^2$ 如下

$$Tp = (3p(4) + 5p'(6) + bp(1)p(2), \int_{-1}^{2} x^3 p(x)dx + c \sin p(0)).$$

证明 T 是线性的当且仅当 $b = c = 0$.