Step-1

This series $(I + A + A^2 + \dots)$ represent $(I - A)^{-1}$.

Check the following:

$$(I - A)(I + A + A^2 + \dots) = I$$

Solve the left hand side.

$$\begin{aligned}(I - A)(I + A + A^2 + \dots) &= I(I + A + A^2 + \dots) - A(I + A + A^2 + \dots) \\ &= (I + A + A^2 + \dots) - (A + A^2 + A^3 + \dots) \\ &= I\end{aligned}$$

Therefore, left hand side comes out to be equal to right hand side.

Step-2

Consider the following matrix which has $\lambda_{\max} = 0$.

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Step-3

Find the powers of matrix A as follows:

$$\begin{aligned} A \cdot A &= \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \\ A^2 &= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ A^2 \cdot A &= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \\ A^3 &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

Step-4

Here, A^3 results into zero matrix. so other higher powers of matrix A will also be zero.

Step-5

Substitute the values into the following series:

$$
\begin{aligned}
(I-A)^{-1} &= \left(I+A+A^2\right) \\
&= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

Step-6

Therefore, above result shows the following:

$$
\boxed{(I-A)^{-1} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}}
$$