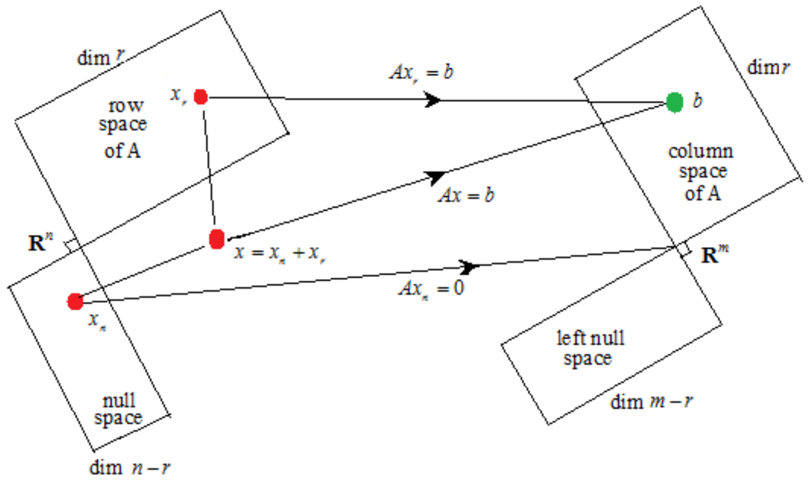
Step-1



We have

Step-2

(a) The simplest way of constructing a vector whose column space contains a given vector in to make that a vector a column of the matrix.

Hence let $A = \begin{bmatrix} 1 & 2 & a_1 \\ 2 & -3 & a_2 \\ -3 & 5 & a_3 \end{bmatrix}$

By definition of null space Ax = 0 $x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 & a_1 \\ 2 & -3 & a_2 \\ -3 & 5 & a_3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3+a_1 \\ -1+a_2 \\ 2+a_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow 3+a_1 = 0, -1+a_2 = 0, 2+a_3 = 0$$
$$\Rightarrow a_1 = -3, a_2 = 1, a_3 = -2$$

Hence the matrix $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & -3 & 1 \\ -3 & 5 & -2 \end{bmatrix}$

Step-3

(b) Let $a = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$, $b = \begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix}$ and $c = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$$a^T c = \begin{bmatrix} 1 & 2 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 1 + 2 - 3 = 0$$

$$b^T c = \begin{bmatrix} 2 & -3 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 2 - 3 + 5 = 4 \neq 0$$

The vector b is not orthogonal to the vector c

Therefore, the suggested matrix is not possible.

Step-4

(c) given that $Ax = b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ has a solution.

Obviously, the solution is $x = A^{-1}b$ and so, A is invertible.

That means A is non singular.

Consequently, A^T is non singular.

That means $A^T x = 0$ has a solution if and only if $x = 0$

But given $x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \neq 0$

This is an absurdity

So, such a matrix A is not possible.

Step-5

(d) Let $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Given that every row is orthogonal to every column. i.e.,

$$R_1^T R_1 = (a\ b)\begin{pmatrix} a \\ b \end{pmatrix} = a^2 + b^2 = 0$$

$$R_1^T R_2 = (a\ b)\begin{pmatrix} c \\ d \end{pmatrix} = ac + bd = 0$$

$$R_2^T R_1 = (c\ d)\begin{pmatrix} a \\ b \end{pmatrix} = ac + bd = 0$$

$$R_2^T R_2 = (c\ d)\begin{pmatrix} c \\ d \end{pmatrix} = c^2 + d^2 = 0$$

So, the only possibility by equating the sum of the squares to zero is $a = b = c = d = 0$

Thus, in the zero matrix only every row is orthogonal to every column.

Step-6

(e) Columns add up to a column of zeroes

So, at least one of the columns of the given matrix is linearly dependent.

Rows add up to a row of 1â€™s

That means no row is linearly dependent.

But we follow that in a square matrix the number of dependent rows = number of dependent columns.

So, the given statement is not possible in the case of square matrices.