

Step-1

Note the following;

$$\begin{aligned}P(PA) &= P^2 A \\P(P(PA)) &= P^3 A \\&\vdots \\P(P(\dots(PA))) &= P^n A\end{aligned}$$

When a Permutation Matrix P is multiplied by itself, the resultant matrix is also a Permutation Matrix.

Thus, P, P^2, P^3, \dots, P^n are all Permutation Matrices.

Step-2

When we consider n by n Permutation Matrices, we know that there are only $n!$ distinct Permutation Matrices.

Thus, if we go on multiplying by Permutation Matrices, there are bound to be some P^k and P^r such that $P^k = P^r$. Without loss of generality, let $k > r$. Thus, we have

$$\begin{aligned}P^r &= P^k \\&= P^{r+k-r} \\&= P^r P^{k-r}\end{aligned}$$

This clearly indicates that $P^{k-r} = I$.

Step-3

Thus, when we multiply A by P , as many as $k-r$ times, we get

$$\begin{aligned}P^{k-r} A &= IA \\&= A\end{aligned}$$

Thus, the first row of the matrix $P^{k-r} A$ must be same as that of the matrix A .