Step-1

Given that the transformation T that transposes every matrix is definitely linear.

We have to determine which of the given statements are true.

Let A be any matrix.

Since T is the linear transformation of transpose of a matrix.

So $T(A) = A^T$

Step-2

a) Given that $T^2 =$ identity transformation.

The given statement is **true**.

Since

$$\begin{aligned} T^2(A) &= T(T(A)) \\ &= T(A^T) \\ &= (A^T)^T \\ &= A \end{aligned}$$

Hence T^2 is an identity transformation.

Step-3

b) Given that â€˜The kernel of T is the zero matrixâ€™.

The given statement is **true**.

Since, let $A = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

We know that the kernel of T is $\ker T = \{A / T(A) = 0\}$

$$\begin{aligned} \ker T &= \{A / T(A) = 0\} \\ &= \{A / A^T = 0\} \end{aligned}$$

Step-4

Now $A^T = 0$

$$\Rightarrow \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow a = 0, c = 0, b = 0, d = 0$$

$$\Rightarrow A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence the kernel of *T* is the zero matrix.

Step-5

c) Given that â€˜Every matrix is in the range of *T* â€™.

The given statement is **true**.

Let *A* be a 2 by 2 matrix then A^T is also 2 by 2 matrix.

So

$$\begin{aligned} T\left(A^T\right) &= \left(A^T\right)^T \\ &= A \end{aligned}$$

Hence $A \in$ range T

Therefore, every matrix is in the range of *T*.

Step-6

d) Given that $T(M) = -M$ is impossible.

The given statement is **false**.

Since, let $M = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$

Then

$$\begin{aligned} T(M) &= M^T \\ &= \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} \\ &= -M \end{aligned}$$

Therefore, $T(M) = -M$ is possible.

Hence the given statement is **false**.