

Step-1

Suppose, we know the values of $c_0 + c_2$, $c_0 - c_2$, $c_1 + c_3$, and $c_1 - c_3$. In order to obtain Fc , we need not calculate c_0 , c_1 , c_2 , and c_3 .

We know the following properties of the imaginary number $i = \sqrt{-1}$.

$$i^1 = i$$

$$i^2 = -1$$

$$i^3 = -i$$

$$i^4 = 1$$

We also know that if $0 \leq k \leq 4$, then $i^{4n+k} = i^k$

Step-2

Therefore, we have the following:

$$i^6 = i^2$$

$$= -1$$

$$i^9 = i^1$$

$$= i$$

Step-3

Consider the following:

$$\begin{aligned} Fc &= \begin{bmatrix} c_0 + c_1 + c_2 + c_3 \\ c_0 + ic_1 + i^2c_2 + i^3c_3 \\ c_0 + i^2c_1 + i^4c_2 + i^6c_3 \\ c_0 + i^3c_1 + i^6c_2 + i^9c_3 \end{bmatrix} \\ &= \begin{bmatrix} c_0 + c_1 + c_2 + c_3 \\ c_0 + ic_1 - 1c_2 - ic_3 \\ c_0 - 1c_1 + c_2 - 1c_3 \\ c_0 - ic_1 - c_2 + ic_3 \end{bmatrix} \\ &= \begin{bmatrix} (c_0 + c_2) + (c_1 + c_3) \\ (c_0 - c_2) + i(c_1 - c_3) \\ (c_0 + c_2) - (c_1 + c_3) \\ (c_0 - c_2) - i(c_1 - c_3) \end{bmatrix} \end{aligned}$$

Step-4

Thus, suppose we have the following relations:

$$c_0 + c_2 = p$$

$$c_0 - c_2 = q$$

$$c_1 + c_3 = r$$

$$c_1 - c_3 = s$$

Then,
$$Fc = \begin{bmatrix} p+r \\ q+is \\ p-r \\ q-is \end{bmatrix}.$$