Step-1

Given $A=\begin{bmatrix}1&0\\0&1\\1&1\end{bmatrix}, x=\begin{bmatrix}u\\v\end{bmatrix}, b=\begin{bmatrix}1\\3\\4\end{bmatrix}$

We have to write $E^2=\|Ax-b\|^2$

Step-2

We know that the least-square solution to a problem is $A^T A\hat{x}=A^T b$.

Now

$$\begin{aligned}E^2&=\|Ax-b\|^2\\&=(u+0(v)-1)^2+(u(0)+1(v)-3)^2+(u(1)+1(v)-4)^2\\&=(u-1)^2+(v-3)^2+(u+v-4)^2\end{aligned}$$

Now we have to find the solution $\hat{x}$.

$$A^T A\hat{x}=A^T b$$

$$\Rightarrow\begin{bmatrix}1&0&1\\0&1&1\end{bmatrix}\begin{bmatrix}1&0\\0&1\\1&1\end{bmatrix}\begin{bmatrix}\hat{u}\\\hat{v}\end{bmatrix}=\begin{bmatrix}1&0&1\\0&1&1\end{bmatrix}\begin{bmatrix}1\\3\\4\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}1(1)+0(0)+1(1)&1(0)+0(1)+1(1)\\0(1)+1(0)+1(1)&0(1)+1(1)+1(1)\end{bmatrix}\begin{bmatrix}\hat{u}\\\hat{v}\end{bmatrix}=\begin{bmatrix}1(1)+0(3)+1(4)\\1(0)+1(3)+1(4)\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}2&1\\1&2\end{bmatrix}\begin{bmatrix}\hat{u}\\\hat{v}\end{bmatrix}=\begin{bmatrix}5\\7\end{bmatrix}$$

Step-3

Normal equations are

$2\hat{u}+\hat{v}=5$ â€¦â€¦Â (1)

$\hat{u}+2\hat{v}=7$ â€¦â€¦Â (2)

$$\begin{aligned}(2)\times2-(1)&\Rightarrow 3\hat{v}=9\\&\Rightarrow\hat{v}=3\end{aligned}$$

Substituting $\hat{v} = 3$ in (2), we get

$$\begin{aligned}\hat{u} &= 7 - 2\hat{v} \\ &= 7 - 2(3) \\ &= 7 - 6 \\ &= 1\end{aligned}$$

Hence $\boxed{\hat{x} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}}$

Step-4

Now the projection

$$\begin{aligned}p &= A\hat{x} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix} \\ &= \begin{bmatrix} 1(1)+0(3) \\ 0(1)+1(3) \\ 1(1)+1(3) \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}\end{aligned}$$

Therefore $\boxed{p = b = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}}$