

Step-1

Given $W = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, x = (2, 3), y = (1, 1)$

We have to find the W -inner product of $x = (2, 3)$ and $y = (1, 1)$, and W -length of x .

Step-2

W -inner product of x and y

$$\begin{aligned} &= (x, y)_W \\ &= (Wy)^T (Wx) \end{aligned}$$

Step-3

Now

$$\begin{aligned} Wx &= \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} \\ &= \begin{bmatrix} 4 \\ 3 \end{bmatrix} \\ Wy &= \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} 2 \\ 1 \end{bmatrix} \end{aligned}$$

Step-4

And

$$\begin{aligned} (Wy)^T &= [2 \quad 1] \\ \Rightarrow (Wy)^T (Wx) &= [2 \quad 1] \begin{bmatrix} 4 \\ 3 \end{bmatrix} \\ &= 8 + 3 \\ &= 11 \end{aligned}$$

Hence W -inner product of x with y is 11

Step-5

The W -length of x

$$\begin{aligned} &= \|x\|_W \\ &= \|Wx\| \end{aligned}$$

Step-6

Now

$$\begin{aligned} \|Wx\|^2 &= (Wx)^T Wx \\ &= \begin{bmatrix} 4 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} \\ &= 16 + 9 \\ &= 25 \end{aligned}$$

Hence the W -length of x $\|Wx\| = 5$