# Southern University of Science and Technology
# Advanced Linear Algebra Spring 2023

## MA109– Quiz #1

2023/02/23

**Name**: ____________________

**Student Number**: ____________________

1. Does the operation of addition on the subspaces of $V$ have an additive identity? Which subspaces have additive inverses?

   Let $V$ be a vector space and $U$ be a subspace, then $U + \{0\} = \{0\} + U = U$. Thus $\{0\}$ is an additive identity for the operation of the sum of subspaces.

   Since the subspace $U + W$ contains both $U$ and $W$, the only way the sum could give $\{0\}$ is if both $U$ and $W$ are $\{0\}$. Hence $\{0\}$ is the only subspace with an additive inverse, namely itself.

2. Let $\infty$ and $-\infty$ denote two distinct objects, neither of which is in $\mathbf{R}$. Define an addition and scalar multiplication on $\mathbf{R} \cup \{\infty\} \cup \{-\infty\}$ as you could guess from the notation. Specifically, the sum and product of two real numbers is as usual, and for $t \in \mathbf{R}$ define

$$t(\infty) = \begin{cases} -\infty & \text{if } t < 0, \\ 0 & \text{if } t = 0, \\ \infty & \text{if } t > 0. \end{cases} \qquad t(-\infty) = \begin{cases} \infty & \text{if } t < 0, \\ 0 & \text{if } t = 0, \\ -\infty & \text{if } t > 0. \end{cases}$$

$$t + \infty = \infty + t = \infty, \quad t + (-\infty) = (-\infty) + t = -\infty,$$

$$\infty + \infty = \infty, \quad (-\infty) + (-\infty) = -\infty, \quad \infty + (-\infty) = 0.$$

Is $\mathbf{R} \cup \{\infty\} \cup \{-\infty\}$ a vector space over $\mathbf{R}$? Explain.

$\mathbf{R} \cup \{\infty\} \cup \{-\infty\}$ is not a vector space over $\mathbf{R}$, since it fails to satisfy associativity:

$$(-\infty + \infty) + \infty = 0 + \infty = \infty, \quad -\infty + (\infty + \infty) = -\infty + \infty = 0.$$