

Step-1

If A is 5 by 5 matrix with all $|a_{ij}| \leq 1$, then we have to find $|\det A| \leq \underline{\hspace{1cm}}$.

Suppose \vec{v}_i be the i th column of A . Then we have

$$|\vec{v}_i| = \sqrt{a_{1i}^2 + a_{2i}^2 + a_{3i}^2 + a_{4i}^2 + a_{5i}^2}$$

We know that $|a_{ij}| \leq 1$ for all i and j , so $a_{ij}^2 \leq 1$ is true.

Step-2

Since $a_{ij}^2 \leq 1$, so

$$\begin{aligned} \sqrt{a_{1i}^2 + a_{2i}^2 + a_{3i}^2 + a_{4i}^2 + a_{5i}^2} &\leq \sqrt{1+1+1+1+1} \\ &\leq \sqrt{5} \end{aligned}$$

Now consider $|\vec{v}_i| = \sqrt{a_{1i}^2 + a_{2i}^2 + a_{3i}^2 + a_{4i}^2 + a_{5i}^2}$, then we can write

$$|\vec{v}_i| \leq \sqrt{5}$$

Step-3

Now we have $|\vec{v}_i| \leq \sqrt{5}$, hence the edges of a 5-dimensional box spanned by the columns of A is less than $\sqrt{5}$.

The volume of such box, V , can be calculate as follows

$$\begin{aligned} V &\leq (\sqrt{5})^5 \\ &\leq 25\sqrt{5} \\ &\leq 55.091 \end{aligned}$$

Step-4

We know that $|\det A|$ is equivalent to the volume of 5-dimensional box spanned by the columns of A .

So, by using this fact we get

$$|\det A| \leq \boxed{55.091}$$

Step-5

Now, to find an upper bound on the determinant, we use the Big formula for the determinant

$$\det A = \sum_{\text{all } p's} (a_{1\alpha} a_{2\beta} \dots a_{ny}) \det P$$

The sum determinant of the whole matrix is the sum of $n!$.

Step-6

We have considered A is 5 by 5 matrix with all $|a_{ij}| \leq 1$, so the number of terms in the sum is

$$5! = 120$$

Since $|a_{ij}| \leq 1$, so that all of the 120 terms in the sum is smaller than 1 .

Therefore, $|\det A| \leq \boxed{120}$.