## Step-1

Suppose $K$ is a skew-symmeric matrix, and $I$ is an identity matrix, then we have to show

$$Q=(I-K)(I+K)^{-1}$$

is an orthogonal matrix.

Consider $I+K$ is an invertible matrix and $x$ is a vector, then we have

$$x^T(I+K)x=x^Tx$$

We know that $K^T=-K$, if $K$ is skew-Hermitian.

## Step-2

We have show $Q$ is an orthogonal matrix, that is $Q^TQ=I$.

$$\begin{aligned}Q^TQ&=\left[(I+K)(I-K)^{-1}\right]\left[(I-K)(I+K)^{-1}\right]\\&=(I+K)\left[(I-K)^{-1}(I+K)^{-1}\right](I-K)\\&=(I+K)\left(I-K^2\right)^{-1}(I-K)\end{aligned}$$

By multiplying both the sides by $(I-K)$, we get

$$\begin{aligned}(I-K)Q^TQ&=(I-K)(I+K)\left(I-K^2\right)^{-1}(I-K)\\&=\left(I-K^2\right)\left(I-K^2\right)^{-1}(I-K)\\&=(I-K)\end{aligned}$$

## Step-3

Again by multiplying both the sides by $(I-K)^{-1}$, we get

$$\begin{aligned}(I-K)Q^TQ&=(I-K)\\(I-K)^{-1}(I-K)Q^TQ&=(I-K)^{-1}(I-K)\\Q^TQ&=I\end{aligned}$$

Since $Q^TQ=I$, therefore $Q=(I-K)(I+K)^{-1}$ is an orthogonal matrix.

## Step-4

Consider the matrix $K = \begin{bmatrix} 0 & 2 \\ -2 & 2 \end{bmatrix}$.

Now solve the equation $Q = (I - K)(I + K)^{-1}$, to find $Q$.

$$\begin{aligned} Q &= \left( \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 2 \\ -2 & 2 \end{bmatrix} \right) \left( \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 2 \end{bmatrix} \right)^{-1} \\ &= \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}^{-1} \\ &= \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 0.2 & -0.4 \\ 0.4 & 0.2 \end{bmatrix} \\ &= \begin{bmatrix} -0.6 & -0.8 \\ 0.8 & -0.6 \end{bmatrix} \end{aligned}$$

Therefore, $Q = \boxed{\begin{bmatrix} -0.6 & -0.8 \\ 0.8 & -0.6 \end{bmatrix}}$