

Step-1

Given matrix $\begin{vmatrix} 4 & 4 & 4 & 4 \\ 1 & 2 & 0 & 1 \\ 2 & 0 & 1 & 2 \\ 1 & 1 & 0 & 2 \end{vmatrix}$

So, consider

$$\begin{vmatrix} 4 & 4 & 4 & 4 \\ 1 & 2 & 0 & 1 \\ 2 & 0 & 1 & 2 \\ 1 & 1 & 0 & 2 \end{vmatrix} \\ = 4 \begin{vmatrix} 2 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 2 \end{vmatrix} - 4 \begin{vmatrix} 1 & 0 & 1 \\ 2 & 1 & 2 \\ 1 & 0 & 2 \end{vmatrix} + 4 \begin{vmatrix} 1 & 2 & 1 \\ 2 & 0 & 2 \\ 1 & 1 & 2 \end{vmatrix} - 4 \begin{vmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

Step-2

On solving

$$\begin{aligned} &= 4[2(2)+1(-1)] - 4[1(2)+(-1)] + 4[(-2)-2(2)+2] - 4[0-1-2(-1)] \\ &= 4(3) - 4(1) + 4(-4) - 4(1) \\ &= 4(3-1-4-1) \\ &= 4(-3) \\ &= \boxed{-12} \end{aligned}$$

Step-3

b) We recomputed the same determinant by subtracting 1st column from other columns.

$$\begin{vmatrix} 4 & 4 & 4 & 4 \\ 1 & 2 & 0 & 1 \\ 2 & 0 & 1 & 2 \\ 1 & 1 & 0 & 2 \end{vmatrix} = \begin{vmatrix} 4 & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 \\ 2 & -2 & -1 & 0 \\ 1 & 0 & -1 & 1 \end{vmatrix} \text{ subtracting 1st column from remaining columns}$$

$$= 4 \begin{vmatrix} 1 & -1 & 0 \\ -2 & -1 & 0 \\ 0 & -1 & 1 \end{vmatrix}$$

Step-4

On solving

$$= 4 \begin{vmatrix} 1 & -1 \\ -2 & -1 \end{vmatrix} \text{expanding by last columns}$$

$$= 4(-1-2)$$

$$= \boxed{-12}$$