Step-1

Given the quadratic form $f(x,y)=x^2+4xy+3y^2$.

We need to show that f does not have a minimum at $(0,0)$ even though it has positive coefficients, need to write f as a difference of squares and find a point (x,y) where f is negative.

Step-2

The matrix of the quadratic is,

$$A=\begin{pmatrix}1 & 2\\ 2 & 3\end{pmatrix}$$
$$=\begin{pmatrix}a & b\\ b & c\end{pmatrix}$$

Comparing above two matrices,

So, $a=1,\ b=2,$ and $c=3.$

Now,

$ac-b^2=-1<0.$

Therefore, $\boxed{f(x,y)\text{is not positive definite,}}$ so f $\boxed{\text{does not have a minimum at}(0,0)}$ even though it has positive coefficients,

Step-3

$$f(x,y)=x^2+4xy+3y^2$$

$$=x^2+2.x.(2y)+4y^2-y^2$$

$$=(x+2y)^2-y^2$$

Therefore, $\boxed{f(x,y)\text{ is a difference of squares.}}$

Step-4

For $(x,y)=(2,-1),$

$$f(x,y)=f(2,-1)$$

$$=0-1$$

$= -1 < 0$

Thus at $(2,-1)$, $f(x,y)$ is negative.

Therefore, the point is $\boxed{(2,-1)}$. $f(x,y)$ is negative.