Step-1

Let x_1, y_1, and z_1 be the eigenvectors of A, B, and $A+B$ respectively, corresponding to the smallest eigenvalues.

Thus, we have

$$\begin{aligned}
Ax_1 &= \lambda_1 x_1 \\
By_1 &= \mu_1 y_1 \\
Cz_1 &= \theta_1 z_1
\end{aligned}$$

Step-2

Suppose w is any vector. From the properties of Rayleigh quotient, we can say the following:

$$\begin{aligned}
\frac{w^{\mathrm{T}} A w}{w^{\mathrm{T}} w} &\geq \lambda_1 \\
\frac{w^{\mathrm{T}} B w}{w^{\mathrm{T}} w} &\geq \mu_1
\end{aligned}$$

Step-3

Now consider the following;

$$\begin{aligned}
\theta_1 &= R(z_1) \\
&= \frac{z_1^{\mathrm{T}}(A+B)z_1}{z_1^{\mathrm{T}} z_1} \\
&= \frac{z_1^{\mathrm{T}} A z_1 + z_1^{\mathrm{T}} B z_1}{z_1^{\mathrm{T}} z_1}
\end{aligned}$$

$$\begin{aligned}
\theta_1 &= \frac{z_1^{\mathrm{T}} A z_1}{z_1^{\mathrm{T}} z_1} + \frac{z_1^{\mathrm{T}} B z_1}{z_1^{\mathrm{T}} z_1} \\
&\geq \lambda_1 + \mu_1
\end{aligned}$$

Step-4

Thus, we have shown that $\boxed{\theta_1 \geq \lambda_1 + \mu_1}$.