

Step-1

(a).

Given statement is "Every positive definite matrix is invertible."

It is known that for a positive definite matrix determinant is always positive and hence determinant is non-zero. So the matrix is non singular.

Hence, every positive definite matrix is invertible.

Hence, the given statement is true.

Step-2

(b).

Given statement is "the only positive definite projection matrix is $P = I$."

It is known that all projection matrices except identity matrix I are singular.

Also, it is understood that the positive definite matrix is non-singular.

So, the only positive definite projection matrix is $P = I$.

Hence, the given statement is true.

Step-3

(c).

Given statement is "A diagonal matrix with positive diagonal entries is positive definite."

It is known that diagonal entries of diagonal matrix are the Eigen values of the matrix.

So, all the Eigen values of the matrix are positive. So the given diagonal matrix with positive diagonal entries is positive definite.

Hence, the given statement is true.

Step-4

(d).

Given statement is "A symmetric matrix with a positive determinant might not be positive definite."

It is known that if A is a square matrix of order ' n ' then

$$\det(A) = (-1)^n \det A.$$

So the negative definite matrix $-I$ has determinant $+1$ when n is even.

Thus, negative definite matrix has positive determinant.

Thus a symmetric matrix with a positive determinant might not be positive definite.

Hence the given statement is true.