## Step-1

We have to solve the system $F_4 c = y$, where

$$c_0 + c_1 + c_2 + c_3 = 2$$
$$c_0 + ic_1 + i^2 c_2 + i^3 c_3 = 0$$
$$c_0 + i^2 c_1 + i^4 c_2 + i^6 c_3 = 2$$
$$c_0 + i^3 c_1 + i^6 c_2 + i^9 c_3 = 0$$

## Step-2

Let $c = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix}$

$$y = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 0 \end{bmatrix}$$

## Step-3

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Fourier matrix

$$F_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & i^2 & i^3 \\ 1 & i^2 & i^4 & i^6 \\ 1 & i^3 & i^6 & i^9 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix}$$

$$\begin{pmatrix} \text{since } i^2 = -1, i^3 = -i, \\ i^4 = 1, i^6 = i^4 i^2 = -1, \\ i^9 = i^6 i^3 = i \end{pmatrix}$$

## Step-4

$F_4 c = y$

$\Rightarrow c = F_4^{-1} y$

But $F_n^{-1} = \frac{\overline{F_n}}{n} y, (n \text{ is the order of } F_n)$

Therefore $c = \frac{\overline{F_4}}{4} y$

## Step-5

$$\Rightarrow \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 2 \\ 0 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 4 \\ 0 \\ 4 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Hence the solution of the system $F_4 c = y$ is $c = (1, 0, 1, 0)$