

## Step-1

Given equations are

$$x + 4y - 2z = 1$$

$$x + 7y - 6z = 6$$

$$3y + qz = t$$

We have to find that which number  $q$  makes this system singular and which right-hand side  $t$  give it infinitely many solutions. Also we have to find the solution that has  $z = 1$

## Step-2

Performing row 2 - row 1 gives

$$x + 4y - 2z = 1$$

$$3y - 4z = 5$$

$$3y + qz = t$$

Now performing row 3 - row 2 gives

$$x + 4y - 2z = 1$$

$$3y - 4z = 5$$

$$(q + 4)z = t - 5$$

## Step-3

When  $q + 4 = 0$  then  $q = -4$ , the system is singular i.e. no third pivot, then if  $t = 5$ , the third equation is  $0 = 0$ .

So we have two equations in three variables, hence the system has infinite number of solutions.

## Step-4

Choose  $z = 1$ , the equation  $3y - 4z = 5$  gives  $y = 3$  and equation 1 gives  $x = -9$

Therefore one of the solutions is  $(-9, 3, 1)$