Step-1

4764-1-26RE AID: 124

RID: 232 | 3/2/2012

(a) We have to find a vector x that will make $Ax = \text{column 1 of } A + 2(\text{column 3})$, for a 3 by 3 matrix.

Let $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and $x = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$

Then

$$\begin{aligned} Ax &= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \\ &= \begin{bmatrix} 2(1)+0(0)+0(2) \\ 0(1)+1(0)+0(2) \\ 0(1)+0(0)+3(2) \end{bmatrix} \\ &= \begin{bmatrix} 2+0+0 \\ 0+0+0 \\ 0+0+6 \end{bmatrix} \\ &= \begin{bmatrix} 2 \\ 0 \\ 6 \end{bmatrix} \end{aligned}$$

This shows that $\boxed{Ax = \text{column 1 of } A + 2(\text{column 3})}$

Step-2

(b) We have to construct a matrix that has $\text{column } 1 + 2(\text{column } 3) = 0$.

Let $A = \begin{bmatrix} 2 & 1 & -1 \\ 4 & 1 & -2 \\ 6 & 1 & -3 \end{bmatrix}$

From the matrix it is clear that

$$\text{column } 1 + 2(\text{column } 3) = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} + 2\begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix}$$
$$= \begin{bmatrix} 2-2 \\ 4-4 \\ 6-6 \end{bmatrix}$$
$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Hence the required matrix that has $\text{column } 1 + 2(\text{column } 3) = 0$ is $\boxed{A = \begin{bmatrix} 2 & 1 & -1 \\ 4 & 1 & -2 \\ 6 & 1 & -3 \end{bmatrix}}$.

Step-3

Now we have to check that A is singular.

Now

$$\begin{aligned} \det A &= 2(-3+2) - 1(-12+12) - 1(4-6) \\ &= 2(1) - 1(0) - 1(-2) \\ &= 2 - 2 \\ &= 0 \end{aligned}$$

Since $\det A = 0$

So A $\boxed{\text{is singular}}$.

Step-4

We have $A = \begin{bmatrix} 2 & 1 & -1 \\ 4 & 1 & -2 \\ 6 & 1 & -3 \end{bmatrix}$

Subtracting 2 times row 1 from row 2 and 3 times row 1 from row 3 gives

$$\to \begin{bmatrix} 2 & 1 & -1 \\ 0 & -1 & 0 \\ 0 & -2 & 0 \end{bmatrix}$$

Subtracting 2 times row 2 from row 3

$$\to \begin{bmatrix} 2 & 1 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The last row of the echelon form of A is zero.

So the matrix A is singular.

Hence A has no inverse.