Step-1

If $A = R^T R$, then we need to prove the generalized Schwarz inequality.

i.e. $\left|x^T Ay\right|^2 \le \left(x^T Ax\right)\left(y^T Ay\right)$.

So,

$$\begin{aligned} x^T Ay &= x^T \left(R^T R\right) y \\ &= \left(x^T R^T\right)\left(Ry\right) \\ &= \left(Rx\right)^T \left(Ry\right) \end{aligned}$$

Also, $\left\|\left(Rx\right)\right\|^T = \left\|\left(Rx\right)\right\|$.

Step-2

Therefore,

$$\begin{aligned} \left|x^T Ay\right|^2 &= \left|\left(Rx\right)^T \left(Ry\right)\right|^2 \\ &\le \left\|\left(Rx\right)\right\|^2 \left\|\left(Ry\right)\right\|^2 \qquad (\text{ by Schwarz inequality}) \\ &= \left(x^T R^T Rx\right)\left(y^T R^T Ry\right) \\ &= \left(x^T Ax\right)\left(y^T Ay\right) \end{aligned}$$

Thus, $\boxed{\left|x^T Ay\right|^2 = \left(x^T Ax\right)\left(y^T Ay\right)}$.