## Step-1

The objective is to determine the basis for the space $S^1$ when subspace consists all vectors orthogonal to $S$.

## Step-2

Consider the provided equation is $x_1 + x_2 + x_3 + x_4 = 0$. Suppose $S$ is the subspace of $\mathbf{R}^4$ spanned by this equation.

So,

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 0 \\ x_4 &= -x_1 - x_2 - x_3 \end{aligned}$$

This is a single equation with three unknown variables.

The solution set is $[x_1, x_2, x_3, -x_1, -x_2, -x_3]$.

Consider the vector $V = [P, Q, R, S]$ in subspace $S^1$ which contains all vectors that are orthogonal to subspace.

So,

$$\begin{aligned} Px_1 + Qx_2 + Rx_3 - Sx_1 - Sx_2 - Sx_3 &= 0 \\ x_1(P - S) + x_2(Q - S) + x_3(R - S) &= 0 \end{aligned}$$

For any values,

$$\begin{aligned} P - S &= 0 \\ Q - S &= \\ R - S &= 0 \end{aligned}$$

This implies,

$$\begin{aligned} P &= S \\ Q &= S \\ R &= S \end{aligned}$$

Thus,

$$P = Q = R = S$$

Hence, the basis of the subspace $S^1$ which contains all vectors are orthogonal to subspace $S$ is $\boxed{[1,1,1,1]}$.