Step-1

If P is the projection matrix onto a k-dimensional subspace $\boldsymbol{S}$ of the whole space $\mathbf{R}^n$, then we have to find the column space of P and its rank.

If $\bar{x} \in R^n$, then $P\bar{x} \in \mathbf{S}$, since P projects $\bar{x}$ to **S**.

Therefore column space P contained **S**, that is, $\text{col}(P) \subset \mathbf{S}$ â€¦â€¦ (1)

Step-2

On the other hand, if $\bar{b} \in \mathbf{S}$, then $P\bar{b} = \bar{b}$

So $S \subset \text{col}(P)$ â€¦â€¦ (2)

From (1) and (2), $\text{col}(P) = \boxed{\mathbf{S}}$

Therefore the rank of P is equal to the dimension of col (P)

That is, since **S** is k-dimensional the rank of P is k