Step-1

If a,d,f are all zero thus $U=\begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$

Then the columns of $v_1=\begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix}, v_2=\begin{bmatrix} b \\ d \\ 0 \end{bmatrix}$ and $v_3=\begin{bmatrix} c \\ e \\ f \end{bmatrix}$

Let $c_1v_1+c_2v_2+c_3v_3=0$

$$\Rightarrow c_1\begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix}+c_2\begin{bmatrix} b \\ d \\ 0 \end{bmatrix}+c_3\begin{bmatrix} c \\ e \\ f \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} \text{i.e.}\; c_1a+c_2b+c_3c&=0 \\ c_2d+c_3e&=0 \\ c_3f&=0 \end{aligned}$$

$$\Rightarrow c_3=0\left(\text{since } f\neq 0\right)$$

Plug this value in the following equation.

$$c_2d+c_3e=0$$

$$\Rightarrow c_2=\left(\frac{-e}{d}\right)c_3\left(\text{since } c_3=0\right)$$
$$\Rightarrow c_2=0$$

Plug these values in the following equation.

$$c_1a+c_2b+c_3c=0$$

$$\Rightarrow c_1a=0 \quad \left(\text{sine } c_3=0, c_2=0\right)$$

$$\Rightarrow c_1=0\left(\text{since } a\neq 0\right)$$

Therefore $\boxed{c_1=c_2=c_3=0}$

Therefore, v_1, v_2, v_3 are linearly Independent.