

## Step-1

Consider the following matrices equation:

$$\mathbf{M} = \mathbf{I} - \mathbf{UV}$$

$$\mathbf{M}^{-1} = \mathbf{I}_{\mathbf{n}} + \mathbf{U}(\mathbf{I}_{\mathbf{m}} - \mathbf{VU})^{-1}\mathbf{V}$$

Multiply  $\mathbf{M}$  and  $\mathbf{M}^{-1}$  to get  $\mathbf{I}$  ( $\mathbf{MM}^{-1} = \mathbf{I}$ ). This  $\mathbf{M}^{-1}$  shows the change in  $\mathbf{A}^{-1}$  when a matrix is subtracted from  $\mathbf{A}$ .

## Step-2

Do the following calculations:

$$\begin{aligned}\mathbf{MM}^{-1} &= (\mathbf{I}_{\mathbf{n}} - \mathbf{UV})(\mathbf{I}_{\mathbf{n}} + \mathbf{U}(\mathbf{I}_{\mathbf{m}} - \mathbf{VU})^{-1}\mathbf{V}) \\ &= \mathbf{I}_{\mathbf{n}} - \mathbf{UV} + \mathbf{U}(\mathbf{I}_{\mathbf{m}} - \mathbf{VU})^{-1}\mathbf{V} - \mathbf{UVU}(\mathbf{I}_{\mathbf{m}} - \mathbf{VU})^{-1}\mathbf{V} \\ &= \mathbf{I}_{\mathbf{n}} - \mathbf{UV} + \mathbf{U}(\mathbf{I}_{\mathbf{m}} - \mathbf{VU})^{-1}\mathbf{V}(\mathbf{I}_{\mathbf{m}} - \mathbf{VU}) \\ &= \mathbf{I}_{\mathbf{n}} - \mathbf{UV} + \mathbf{U}(\mathbf{I}_{\mathbf{m}} - \mathbf{VU})^{-1}(\mathbf{I}_{\mathbf{m}} - \mathbf{VU})\mathbf{V} \\ &= \mathbf{I}_{\mathbf{n}} - \mathbf{UV} + \mathbf{UV} \\ \mathbf{MM}^{-1} &= \mathbf{I}_{\mathbf{n}}\end{aligned}$$

## Step-3

Therefore,  $\boxed{\mathbf{MM}^{-1} = \mathbf{I}_{\mathbf{n}}}$ .