Step-1

Consider the matrix $U = \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$

If $a = 0$, then

$$U = \begin{bmatrix} 0 & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$$

If,

$$v_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
$$v_2 = \begin{bmatrix} b \\ d \\ 0 \end{bmatrix}$$
$$v_3 = \begin{bmatrix} c \\ e \\ f \end{bmatrix}$$

And then $2v_1 + 0v_2 + 0v_3 = 0$

Thus, $\boxed{\text{the columns are dependent}}$

Step-2

If $d = 0$,

Then $U = \begin{bmatrix} a & b & c \\ 0 & 0 & e \\ 0 & 0 & f \end{bmatrix}$

So,

$$v_1 = \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix}$$

$$v_3 = \begin{bmatrix} c \\ e \\ f \end{bmatrix}$$

This implies;

$$\left(\frac{b}{a}\right)v_1 + (-1)v_2 + (0)v_3 = 0$$

$$bv_1 - av_2 = 0$$

Therefore, $\boxed{\text{colums } v_1, v_2, v_3 \text{are dependent}}$

Step-3

If $f = 0$, then $U = \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & 0 \end{bmatrix}$

So,

$$v_1 = \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} b \\ d \\ 0 \end{bmatrix}$$

$$v_3 = \begin{bmatrix} c \\ e \\ 0 \end{bmatrix}$$

Let $c_1v_1 + c_2v_2 + c_3v_3 = 0$

This implies,

$$c_1\begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix} + c_2\begin{bmatrix} b \\ d \\ 0 \end{bmatrix} + c_3\begin{bmatrix} c \\ e \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

That is,

$$\begin{aligned} c_1 a + c_2 b + c_3 c &= 0 \\ c_2 d + c_3 e &= 0 \end{aligned}$$

From $c_2 d + c_3 e = 0$

$$c_2 = \left(\frac{-e}{d}\right)c_3$$

Plug this value in the following equation;

$$c_1 a + c_2 b + c_3 c = 0$$

And obtain;

Step-4

$$c_1 a + c_3\left(\frac{-e}{d}\right)b + c_3 c = 0$$

$$c_1 a + c_3\left(\frac{-eb}{d} + c\right) = 0$$

$$c_1 a + c_2\left(\frac{-eb + dc}{d}\right) = 0$$

Therefore, $\boxed{\text{the columns are linearly dependent}}$