Linear Algebra-A

Assignments - Week 1

Supplementary Problem Set

1. Apply elementary row operations to transform the following matrices into reduced echelon form:

(1) $\begin{bmatrix} 0 & 2 & -3 & 1 \\ 0 & 3 & -4 & 3 \\ 0 & 4 & -7 & -1 \end{bmatrix}$; (2) $\begin{bmatrix} 1 & -1 & 3 & -4 & 3 \\ 3 & -3 & 5 & -4 & 1 \\ 2 & -2 & 3 & -2 & 0 \\ 3 & -3 & 4 & -2 & -1 \end{bmatrix}$.

2. Show that the system of linear equations

$$\begin{cases} x_1 - x_2 = b_1 \\ x_2 - x_3 = b_2 \\ x_3 - x_4 = b_3 \\ x_4 - x_1 = b_4 \end{cases}$$

is consistent (solvable) if and only if $b_1 + b_2 + b_3 + b_4 = 0$.

3. Suppose that the curve of a quadratic function $y = f(x)$ goes through three points: $(1,1), (2,2), (3,0)$. Find $f(4)$.

4. Find the value of a to make the following system of linear equations inconsistent:

$$\begin{cases} x_1 + 2x_2 - x_3 + 3x_4 = 1 \\ 2x_1 + x_2 + 4x_3 + 3x_4 = 5 \\ ax_2 + 2x_3 - x_4 = -6 \end{cases}$$

5. The following system of linear equations has infinitely many solutions. Find the value of a.

$$\begin{cases} ax_1 + x_2 + x_3 = a - 3 \\ x_1 + ax_2 + x_3 = -2 \\ x_1 + x_2 + ax_3 = -2 \end{cases}$$