Southern University of Science and Technology
Advanced Linear Algebra Spring 2023

MA109– Quiz #1

2023/02/26

Name: ______________________________
Student Number: ______________________________

1. Prove that the intersection of every collection of subspaces of V is a subspace of V.

   Let $U_i, i \in I$ be subspaces of V, where I is the index set, and let $U = \bigcap_{i \in I} U_i$. Then

   1. U contains the zero vector. Since $0 \in U_i$ for all $i \in I$, then $0 \in U$, U is nonempty.
   2. U is closed under addition: Pick $v, w \in U$, then $v, w \in U_i$ for all $i \in I$. And since U_i are subspaces, so $v + w \in U_i$ for all $i \in I$, hence $u + w \in U$.
   3. U is closed under scalar multiplication: Pick $v \in U$, $a \in \mathbf{F}$, since $v \in U$, v is in each U_i. Since each U_i is closed under scalar multiplication, $av \in U_i$, so $av \in U$.

   Thus U is a subspace of V.

2. Prove or gives a counterexample: if U_1, U_2, W are subspaces of V such that

$$V = U_1 \oplus W \quad \text{and} \quad V = U_2 \oplus W,$$

then $U_1 = U_2$.

False!

Let $V = \mathbf{R}^2$, $U_1 = \{(0, y) : y \in \mathbf{R}\}$, $U_2 = \{(x, x) : x \in \mathbf{R}\}$, $W = \{(z, 0) : z \in \mathbf{R}\}$.
Clearly, $U_1 + W = U_2 + W = \mathbf{R}^2$. Moreover $U_1 \cap W = \{0\}$, $U_2 \cap W = \{0\}$, so $\mathbf{R}^2 = U_1 \oplus W$ and $\mathbf{R}^2 = U_2 \oplus W$, but $U_1 \neq U_2$.