Step-1

The objective is to find matrix $A^TB, B^TA, AB^T,$ and BA^T if $A=\begin{bmatrix}3\\1\end{bmatrix}$ and $B=\begin{bmatrix}2\\2\end{bmatrix}$.

Step-2

Consider;

$$A=\begin{bmatrix}3\\1\end{bmatrix}$$
$$B=\begin{bmatrix}2\\2\end{bmatrix}$$

So,

$$A=\begin{bmatrix}3\\1\end{bmatrix}$$
$$A^T=\begin{bmatrix}1 & 3\end{bmatrix}$$

And

$$B=\begin{bmatrix}2\\2\end{bmatrix}$$
$$B^T=\begin{bmatrix}2 & 2\end{bmatrix}$$

Step-3

Now, compute;

$$\begin{aligned}A^T.B&=\begin{bmatrix}1 & 3\end{bmatrix}_{1\times 2}.\begin{bmatrix}2\\2\end{bmatrix}_{2\times 1}\\&=(2+6)\\&=(8)_{1\times 1}\end{aligned}$$

Thus, $\boxed{A^TB=8}$

Step-4

$$B^T.A = \begin{bmatrix}2 & 2\end{bmatrix}_{1\times 2}.\begin{bmatrix}3\\1\end{bmatrix}_{2\times 1}$$
$$= (6+2)$$
$$= (8)_{1\times 1}$$

Thus, $\boxed{B^T A = 8}$

Step-5

Now, compute;

$$A.B^T = \begin{bmatrix}3\\1\end{bmatrix}_{2\times 1}.\begin{bmatrix}2 & 2\end{bmatrix}_{1\times 2}$$

Thus, $\boxed{A.B^T = \begin{bmatrix}6 & 6\\2 & 2\end{bmatrix}_{2\times 2}}$

Step-6

$$B.A^T = \begin{bmatrix}2\\2\end{bmatrix}_{2\times 1}.\begin{bmatrix}1 & 3\end{bmatrix}_{1\times 2}$$

Thus, $\boxed{BA^T = \begin{bmatrix}6 & 6\\2 & 2\end{bmatrix}_{2\times 2}}$