

Step-1

(a)

Given vectors are $b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $a = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

The projection of b onto a is $\hat{x} = \left(\frac{a^T b}{a^T a} \right) a$

$$\begin{aligned} a^T b &= \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \\ &= 1 + 2 + 2 \\ &= 5 \end{aligned}$$

Also,

$$\begin{aligned} a^T a &= \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \\ &= 1 + 1 + 1 \\ &= 3 \end{aligned}$$

Step-2

So, put values in above formula;

$$\hat{x} = \frac{5}{3}$$

Therefore,

$$\begin{aligned} P &= \hat{x} a \\ &= \frac{5}{3} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} 5/3 \\ 5/3 \\ 5/3 \end{bmatrix} \end{aligned}$$

$$\begin{aligned}
 e &= b - P \\
 &= \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} - \begin{bmatrix} 5/3 \\ 5/3 \\ 5/3 \end{bmatrix} \\
 &= \begin{bmatrix} -2/3 \\ 1/3 \\ 1/3 \end{bmatrix}
 \end{aligned}$$

Now, verify e is perpendicular to a or not.

$$\begin{aligned}
 e^T &= (-2/3 \quad 1/3 \quad 1/3) \\
 e^T a &= (-2/3 \quad 1/3 \quad 1/3) \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \\
 &= \frac{-2}{3} + \frac{1}{3} + \frac{1}{3} \\
 &= 0
 \end{aligned}$$

Therefore, e is perpendicular to a .

Step-3

(b)

$$b = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \quad a = \begin{bmatrix} -1 \\ -3 \\ -1 \end{bmatrix}$$

Given vectors are

$$\text{The projection of } b \text{ onto } a = \hat{x} = \frac{a^T b}{a^T a}$$

$$\begin{aligned}
 a^T b &= [-1 \quad -3 \quad -1] \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \\
 &= -1 - 9 - 1 \\
 &= -11
 \end{aligned}$$

$$\begin{aligned}
 a^T a &= [-1 \quad -3 \quad -1] \begin{bmatrix} -1 \\ -3 \\ -1 \end{bmatrix} \\
 &= 1 + 9 + 1 \\
 &= 11
 \end{aligned}$$

Step-4

Put values in formula, and obtain;

$$\begin{aligned}\hat{x} &= \frac{-11}{11} \\ &= -1\end{aligned}$$

Therefore,

Step-5

$$\begin{aligned}P &= \hat{x} a \\ &= -1 \begin{bmatrix} -1 \\ -3 \\ -1 \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \\ &= b\end{aligned}$$

$$\begin{aligned}e &= b - P \\ &= \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\end{aligned}$$

Observe that the zero vectors are perpendicular to every vector.

So, $\boxed{e^T a = 0}$ verifies the projection of b upon a .