

Step-1

From the big formula for 6 terms, to have

$$\begin{aligned}|A| &= +a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \\ &= +(1 \cdot 1 \cdot 1) + (2 \cdot 2 \cdot 3) + (3 \cdot 3 \cdot 2) - (1 \cdot 2 \cdot 2) - (2 \cdot 3 \cdot 1) - (3 \cdot 1 \cdot 3) \\ &= 1 + 12 + 18 - 4 - 6 - 9 \\ &= 12\end{aligned}$$

Since $|A| \neq 0$, therefore **rows are independent.**

Step-2

Consider the following matrix B :

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 4 & 4 \\ 5 & 6 & 7 \end{bmatrix}$$

For the given matrix B , $n = 3$ hence no. of terms is:

$$\begin{aligned}n! &= 3! \\ &= 6\end{aligned}$$

Step-3

From the big formula for 6 terms, to have

$$\begin{aligned}|B| &= +b_{11}b_{22}b_{33} + b_{12}b_{23}b_{31} + b_{13}b_{21}b_{32} - b_{11}b_{23}b_{32} - b_{12}b_{21}b_{33} - b_{13}b_{22}b_{31} \\ &= +(1 \cdot 4 \cdot 7) + (2 \cdot 4 \cdot 5) + (3 \cdot 4 \cdot 6) - (1 \cdot 4 \cdot 6) - (2 \cdot 4 \cdot 7) - (3 \cdot 4 \cdot 5) \\ &= 28 + 40 + 72 - 24 - 56 - 60 \\ &= 140 - 140 \\ &= 0\end{aligned}$$

Since $|B| = 0$, therefore **rows are dependent.**

Step-4

Consider the following matrix C :

$$C = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$$

For the matrix C , $n = 3$ hence no. of terms is:

$$\begin{aligned} n! &= 3! \\ &= 6 \end{aligned}$$

Step-5

From the big formula for 6 terms, to have

$$\begin{aligned} |C| &= +c_{11}c_{22}c_{33} + c_{12}c_{23}c_{31} + c_{13}c_{21}c_{32} - c_{11}c_{23}c_{32} - c_{12}c_{21}c_{33} - c_{13}c_{22}c_{31} \\ &= +(1 \cdot 1 \cdot 0) + (1 \cdot 0 \cdot 1) + (1 \cdot 1 \cdot 0) - (1 \cdot 0 \cdot 0) - (1 \cdot 1 \cdot 0) - (1 \cdot 1 \cdot 1) \\ &= 0 + 0 + 0 - 0 - 0 - 1 \\ &= -1 \end{aligned}$$

Since $|C| \neq 0$, therefore **rows are independent.**