Step-1

The objective is to determine the dimensions of the four subspaces of $A, B,$ and C matrices.

The matrix A is $A = [I \quad 0]$ here I is a 3×3 identity matrix and 0 is a 3×2 zero matrix.

That is,
$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

The dimension of matrix A is $m\times n = 3\times 5.$

As there are three linearly independent rows in matrix $A,$ so rank of A is $r = 3.$

Therefore, $\dim(C(A)) = \dim(C(A^T)) = 3.$

Step-2

By rank and nullity theorem, the dimension of null space is $n - r.$

So, the dimension of null space is

$\dim(N(A)) = 5 - 3 = 2.$

Also $\dim(N(A^T)) = m - r = 3 - 3 = 0.$

Hence, the dimensions of the four subspaces of A is $\boxed{\dim(C(A)) = \dim(C(A^T)) = 3}, \boxed{\dim(N(A)) = 2},$ and $\boxed{\dim(N(A^T)) = 0}.$

Step-3

The matrix B is $B = \begin{bmatrix} I & I \\ 0^T & 0^T \end{bmatrix}.$

That is,
$$B = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The dimension of matrix B is $m\times n = 5\times 6.$

As there are three linearly independent rows in matrix B, so rank of B is $r = 3$.

Therefore, $\dim(C(B)) = \dim(C(B^T)) = 3$.

Step-4

By rank and nullity theorem, the dimension of null space is $n - r$.

So, the dimension of null space is

$\dim(N(B)) = 6 - 3 = 3$.

Also $\dim(N(B^T)) = m - r = 5 - 3 = 2$.

Hence, the dimensions of the four subspaces of B is $\boxed{\dim(C(B)) = \dim(C(B^T)) = 3}$, $\boxed{\dim(N(B)) = 3}$, and $\boxed{\dim(N(B^T)) = 2}$.

Step-5

The matrix C is $C = [0]$.

That is, $C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

The dimension of matrix C is $m \times n = 2 \times 3$.

As all rows are zero, so rank of C is $r = 0$.

Therefore, $\dim(C(C)) = \dim(C(C^T)) = 0$.

Step-6

By rank and nullity theorem, the dimension of null space is $n - r$.

So, the dimension of null space is

$\dim(N(C)) = 2 - 0 = 2$.

Also $\dim(N(C^T)) = m - r = 3 - 0 = 3$.

Hence, the dimensions of the four subspaces of C is $\boxed{\dim(C(C)) = \dim(C(C^T)) = 0}$, $\boxed{\dim(N(C)) = 2}$, and $\boxed{\dim(N(C^T)) = 3}$.