Step-1

We have to verify which of the given transformations satisfy $T(v+w)=T(v)+T(w)$ and which satisfy $T(cv)=cT(v)$

(a) Given transformation is $T(v)=\frac{v}{\|v\|}$.

Now

$$T(v+w)=\frac{v+w}{\|v+w\|}$$

$$T(v)+T(w)=\frac{v}{\|v\|}+\frac{w}{\|w\|}$$

Therefore, $T(v+w)\neq T(v)+T(w)$

Step-2

And also $T(cv)\neq cT(v)$

Since let $v=(1,4)$

Then $2v=(2,8)$

$$T(2v)=\frac{(2,8)}{\sqrt{4+64}}$$

$$=\frac{(2,8)}{2\sqrt{17}}$$

$$=\frac{(1,4)}{\sqrt{17}}$$

Step-3

$$2T(v)=2.\frac{(1,4)}{\sqrt{1+16}}$$

$$=\frac{2}{\sqrt{17}}(1,4)$$

Therefore, $T(2v) \neq 2T(v)$

Hence the given transformation does not satisfy $T(v+w) = T(v) + T(w)$ and $T(cv) = cT(v)$.

Step-4

(b) Given transformation is $T(v) = v_1 + v_2 + v_3$.

Let $v = (v_1, v_2, v_3)$ and $w = (w_1, w_2, w_3)$

Then $v + w = (v_1 + w_1, v_2 + w_2, v_3 + w_3)$

Now

$$\begin{aligned} T(v+w) &= (v_1 + w_1, v_2 + w_2, v_3 + w_3) \\ &= (v_1 + w_1) + (v_2 + w_2) + (v_3 + w_3) \\ &= (v_1 + v_2 + v_3) + (w_1 + w_2 + w_3) \\ &= T(v) + T(w) \end{aligned}$$

Therefore, $T(v+w) = T(v) + T(w)$

Step-5

And

$$T(cv) = T(cv_1, cv_2, cv_3)$$

$$= cv_1 + cv_2 + cv_3$$

$$= cT(v)$$

Hence the given transformation T satisfies both $T(v+w) = T(v) + T(w)$ and $T(cv) = cT(v)$.

Step-6

(c) Given transformation is $T(v) = (v_1, 2v_2, 3v_3)$.

Let $v = (v_1, v_2, v_3)$ and $w = (w_1, w_2, w_3)$

Now

$$T(v+w)=T\left(v_1+w_1,(v_1+w_2),(v_3+w_3)\right)$$

$$=\left(v_1+w_1,\,2(v_2+w_2),3(v_3+w_3)\right)$$

$$=T(v)+T(w)$$

$$T(cv)=(cv_1,cv_2,cv_3)$$

$$=c(v_1,v_2,v_3)$$

$$=cT(v)$$

Hence the given transformation T satisfies both $T(v+w)=T(v)+T(w)$ and $T(cv)=cT(v)$.

Step-7

(d) Given transformation is $T(v)=$ largest component of v

Let $T(1,2,3)=3$ and $T(1,2,-3)=2$

Then

Step-8

$$T(1,2,3)+T(1,2,-3)=3+2$$
$$=5$$

And

$$T\left((1,2,3)+(1,2,-3)\right)=T(2,4,0)$$

$$=4$$

Therefore, $T\left((1,2,3)+(1,2,-3)\right)\neq T(1,2,3)+T(1,2,-3)$

Hence the given transformation does not satisfy $T(v+w)=T(v)+T(w)$ and $T(cv)=cT(v)$.