

Step-1

By the properties of determinants, we have $|XY| = |X||Y|$ for every square matrices X and Y of size $n \times n$.

Using this, we have $A = SAS^{-1}$ where all the matrices are of same size and Λ is the diagonal matrix whose diagonal entries are nothing but the eigen values of A .

Applying determinant throughout, we get $|A| = |S\Lambda S^{-1}|$

By the above property, we get $|A| = |S||\Lambda||S^{-1}|$

We observe that the determinant is a scalar quantity and so, product of determinants is commutative.

So, the above equation can be written as $|A| = |S||S^{-1}||\Lambda|$

$$|A| = |SS^{-1}||\Lambda|$$

$$\Rightarrow |A| = |I||\Lambda|$$

$$\Rightarrow |A| = |\Lambda|$$

We know that the determinant of the diagonal matrix is nothing but the product of the diagonal entries.

Further, Λ has the diagonal entries nothing but the eigen values of A .

Therefore, $|A|$ is the product of eigen values of A .