Step-1

Given vectors $(1,1,1),(0,1,3)$.

We have to construct the projection matrix P onto the space spanned by these vectors.

Step-2

Let $a_1 = (1,1,1), a_1 = (0,1,3)$

Write $A = \begin{bmatrix} a_1 & a_2 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 3 \end{bmatrix}$$

Required projection matrix $P = A\left(A^T A\right)^{-1} A^T$

$$A^T A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 4 \\ 4 & 10 \end{bmatrix}$$

$$\left(A^T A\right)^{-1} = \frac{1}{14} \begin{bmatrix} 10 & -4 \\ -4 & 3 \end{bmatrix}$$

$$A\left(A^T A\right)^{-1} = \frac{1}{14} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 10 & -4 \\ -4 & 3 \end{bmatrix}$$

$$= \frac{1}{14} \begin{bmatrix} 10 & -4 \\ 6 & -1 \\ -2 & 5 \end{bmatrix}$$

Step-3

$$P = A\left(A^T A\right)^{-1} A^T$$

$$=\frac{1}{14}\begin{bmatrix}10 & -4\\ 6 & -1\\ -2 & 5\end{bmatrix}\begin{bmatrix}1 & 1 & 1\\ 0 & 1 & 3\end{bmatrix}$$

$$=\frac{1}{14}\begin{bmatrix}10 & 6 & -2\\ 6 & 5 & 3\\ -2 & 3 & 13\end{bmatrix}$$

Hence $\boxed{P=\frac{1}{14}\begin{bmatrix}10 & 6 & -2\\ 6 & 5 & 3\\ -2 & 3 & 13\end{bmatrix}}$

Step-4

Verification: $P^2=\frac{1}{196}\begin{bmatrix}10 & 6 & -2\\ 6 & 5 & 3\\ -2 & 3 & 13\end{bmatrix}\begin{bmatrix}10 & 6 & -2\\ 6 & 5 & 3\\ -2 & 3 & 13\end{bmatrix}$

$$=\frac{1}{196}\begin{bmatrix}140 & 84 & -28\\ 84 & 70 & 42\\ -28 & 42 & 182\end{bmatrix}$$

$$=P$$