Step-1

Given that $A=\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}$ and $A+I=\begin{bmatrix}2 & 4\\ 2 & 4\end{bmatrix}$

We need to compute the eigenvalues and the eigenvectors of the above two matrices.

Step-2

Now $A=\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}$

$$\begin{aligned}
A-\lambda I &= \begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}-\lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\\
&= \begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}-\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}\\
&= \begin{bmatrix}1-\lambda & 4\\ 2 & 3-\lambda\end{bmatrix}
\end{aligned}$$

Step-3

$$\begin{aligned}
|A-\lambda I| &= \begin{vmatrix}1-\lambda & 4\\ 2 & 3-\lambda\end{vmatrix}\\
&= (1-\lambda)(3-\lambda)-8\\
&= 3-\lambda-3\lambda+\lambda^2-8\\
&= \lambda^2-4\lambda-5
\end{aligned}$$

Step-4

we know that $|A-\lambda I|=0$

$\lambda^2-4\lambda-5=0$

$\lambda^2-5\lambda+\lambda-5=0$

$\lambda(\lambda-5)+1(\lambda-5)=0$

$(\lambda-5)(\lambda+1)=0$

Step-5

$\lambda = 5, -1$

$\lambda = 5, -1$

Hence the eigenvalues of A are 5,-1

Step-6

Case(i) Let $\lambda = 5$

Eigenvectors X corresponding to the eigenvalue 5 are given by

$(A - 5I)X = 0$

That is $\begin{bmatrix} -4 & 4 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Step-7

By $2R_2 + R_1 = R_2$

$\begin{bmatrix} -4 & 4 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow -4x_1 + 4x_2 = 0$

Let $x_1 = k$(say)

Therefore $x_2 = k$

Therefore eigenvectors corresponding to eigenvalue 5 are given by $k\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ where k is a non-zero parameter.

Step-8

Case(ii) Let $\lambda = -1$

Eigenvectors X corresponding to the eigenvalue -1 are given by

$(A + I)X = 0$

That is $\begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Step-9

$By\ R_2 - R_1 = R_2$

$$\begin{bmatrix} 2 & 4 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow 2x_1 + 4x_2 = 0$

$\Rightarrow x_1 + 2x_2 = 0$

Step-10

Let $x_1 = k$(say)

Therefore $2x_2 = -k$

$x_2 = -k/2$

Therefore eigenvectors corresponding to eigenvalue -1 are given by $k\begin{bmatrix} 1 \\ -1/2 \end{bmatrix}$ where k is a non-zero parameter

Step-11

Now $A + I = \begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}$

$$(A+I) - \lambda I = \begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 2-\lambda & 4 \\ 2 & 4-\lambda \end{bmatrix}$$

$= (2-\lambda)(4-\lambda) - 8$

$= 8 - 2\lambda - 4\lambda + \lambda^2 - 8$

Step-12

we know that $|A-\lambda I|=0$

$$\lambda^2-6\lambda=0$$
$$\lambda(\lambda-6)=0$$
$$\lambda=6,0$$

Step-13

Hence the eigenvalues of $A+I$ are 6,0

Case(i) Let $\lambda=6$

Eigenvectors X corresponding to the eigenvalue 6 are given by

$$\left((A+I)-6I\right)X=0$$

That is $\begin{bmatrix}-4 & 4\\ 2 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$

Step-14

$By\ 2R_2+R_1=R_2$

$$\begin{bmatrix}-4 & 4\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$\Rightarrow -4x_1+4x_2=0$

$\Rightarrow x_1-x_2=0$

Step-15

Let $x_1=k$(say)

Therefore $x_2=k$

Therefore eigenvectors corresponding to eigenvalue 6 are given by $k\begin{bmatrix}1\\ 1\end{bmatrix}$ where k is a non-zero parameter

Step-16

Case(ii) Let $\lambda = 0$

Eigenvectors X corresponding to the eigenvalue 0 are given by

$((A+I)-0I)X=0$

That is $\begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Step-17

By $R_2 - R_1 = R_2$

$\begin{bmatrix} 2 & 4 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow 2x_1 + 4x_2 = 0$

$\Rightarrow x_1 + 2x_2 = 0$

Step-18

Let $x_1 = k(\text{say})$

Therefore $x_2 = -k/2$

Therefore eigenvectors corresponding to eigenvalue 0 are given by $k\begin{bmatrix} 1 \\ -1/2 \end{bmatrix}$ where k is a non-zero parameter.

$A+I$ has the same eigenvectors as A. Its eigenvalues are different by 1.