

Step-1

Let L, U, Q and R be known.

Now we have to decide which is faster to solve $LUx = b$ or $QRx = b$.

Step-2

If $Ax = b$ is the system of non homogeneous linear equations, then we can solve it in two ways namely LU factorization and QR factorization.

Using the elimination method or row transformations, we reduce A to the row reduced or echelon form namely U and correspondingly, the right side b is reduced to c .

Rewriting the non homogeneous system from this reduced form, by back substitution, we get the solutions for u, v, w .

The steps used for elimination above, give the elementary matrices E, F, G .

So, the procedure above can be written as $GFEA = U$.

We know that the inverse matrix of an elementary matrix is elementary and applying the product of inverse matrices on both sides of the above equation, we get

$$\begin{aligned} A &= E^{-1}F^{-1}G^{-1}U \\ &= LU \end{aligned}$$

Step-3

On the other hand, using the eigenvalues and the respective eigenvectors, we find the matrix whose columns are the eigenvectors as L and then we can write $A = LDL^T$

This can further be written as $A = R^T R$ where $R = L\sqrt{D}$, D is the diagonal matrices whose diagonal entries are nothing but the eigenvalues and other entries are zero.

Now, writing $R^T = Q$, we can see that $A = QR$

Step-4

Procedurally, the QR factorization requires more work than LU factorization.

But the factorization QR is more useful than LU .