Step-1

The cosine space F_3 contains all combination $y(x) = A\cos x + B\cos 2x + C\cos 3x$

If $y(0) = 0$ then $A + B + C = 0$

$A = 1, B = -1, C = 0$ then $y(x) = \cos x - \cos 2x$

$A = 1, B = 0, C = -1$ then $y(x) = \cos x - \cos 3x$

$\cos x - \cos 2x, \cos x - \cos 3x$ are linearly independent and spans $(-A)(\cos x - \cos 2x) + (-B)(\cos x - \cos 3x)$

$y(x) = (-A - B)\cos x + A\cos 2x + B\cos 3x$

This satisfy $(-A - B) + A + B = 0$

Therefore the above two functions spans F_3

So, one of the bases is $= \{\cos x - \cos 2x, \cos x - \cos 3x\}$