Step-1

The first column of A is the vector $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Its length is 1.

Therefore, the first column of the matrix Q is $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ only. The second column of the matrix Q must be of the length 1 and it should be orthogonal to the first column. Thus, the second column of Q must be $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

Thus, $Q = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

Step-2

If we assume that the column of the matrix A are a and b, then the matrix R is given by $R = \begin{bmatrix} q_1^T a & q_1^T b \\ 0 & q_2^T b \end{bmatrix}$.

Thus, we get

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Step-3

Thus, we have

$$\begin{aligned} A &= Q_0 R_0 \\ &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Step-4

Let $A_1 = R_0 Q_0$

Thus, we get

$$\begin{aligned} A_1 &= R_0 Q_0 \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\ &= A \end{aligned}$$

Step-5

Thus, we have shown that the QR algorithm leaves the matrix $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ unchanged, that is $\boxed{A = A_1}$.