Step-1

Consider the following 4×4 matrix:

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}.$$

Objective is to determine the elimination matrices E_{21}, E_{32}, E_{43} for the matrix *A*.

The elementary matrix E_{ij} subtracts *l* times row *j* from row *i*, that is,

$$E_{ij} = R_i - lR_j.$$

To convert the matrix *A* into triangular form, there is a need to perform some elementary row operations. Make each entry below the principal diagonal zero.

Step-2

For the first entry of second row, perform $R_2 = R_2 + \frac{R_1}{2}$. Then the reduced matrix *A* will be:

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & \frac{3}{2} & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}.$$

For the second entry of third row, perform $R_3 = R_3 + \frac{2R_2}{3}$. Then the reduced matrix *A* will be:

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & \frac{3}{2} & -1 & 0 \\ 0 & 0 & \frac{4}{3} & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}.$$

For the third entry of fourth row, perform $R_4 = R_4 + \frac{3R_3}{4}$. Then the reduced matrix *A* will be:

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & \frac{3}{2} & -1 & 0 \\ 0 & 0 & \frac{4}{3} & -1 \\ 0 & 0 & 0 & \frac{5}{4} \end{bmatrix}.$$

Thus, the obtained matrix is triangular matrix.

Step-3

The identity matrix is $I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Since $R_2 = R_2 + \frac{R_1}{2}$, therefore $E_{21} = R_2 + \frac{1}{2} R_1$. So, apply $R_2 = R_2 + \frac{1}{2} R_1$ over I and get

$$E_{21} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Since $R_3 = R_3 + \frac{2R_2}{3}$, so apply this over I and get

$$E_{32} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & \frac{2}{3} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Since $R_4 = R_4 + \frac{3R_3}{4}$, therefore

$$E_{43} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{3}{4} & 1 \end{bmatrix}.$$