

Step-1

Given system is $2u - v = 0$

$$-u + 2v - w = 0$$

$$-v + 2w - z = 0$$

$$-w + 2z = 5$$

We have to find the pivots and solve this system.

Step-2

Given system can be written as

$$\begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & 5 \end{bmatrix}$$

apply $R_2 \rightarrow 2R_2 + R_1$

$$\Rightarrow \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ 0 & 3 & -2 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & 5 \end{bmatrix}$$

apply $R_3 \rightarrow 3R_3 + R_2$

$$\Rightarrow \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ 0 & 3 & -2 & 0 & 0 \\ 0 & 0 & 4 & -3 & 0 \\ 0 & 0 & -1 & 2 & 5 \end{bmatrix}$$

Step-3

apply $R_4 \rightarrow 4R_4 + R_3$

$$\Rightarrow \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ 0 & 3 & -2 & 0 & 0 \\ 0 & 0 & 4 & -3 & 0 \\ 0 & 0 & 0 & 5 & 20 \end{bmatrix}$$

which is upper triangular form.

$$\begin{bmatrix} \boxed{2} & -1 & 0 & 0 & 0 \\ 0 & \boxed{3} & -2 & 0 & 0 \\ 0 & 0 & \boxed{4} & -3 & 0 \\ 0 & 0 & 0 & \boxed{5} & 20 \end{bmatrix}$$

The pivots are circled in

That is $\boxed{2, 3, 4, 5}$.

Step-4

From above upper triangular form, we have

$$\begin{aligned} 2u - v &= 0 \\ 3v - 2w &= 0 \\ 4w - 3z &= 0 \\ 5z &= 20 \end{aligned}$$

By back ward substitution,

$$5z = 20$$

$$\Rightarrow \boxed{z = 4}$$

$$4w - 3z = 0$$

$$\Rightarrow 4w - 3(4) = 0$$

$$\Rightarrow \boxed{w = 3}$$

$$3v - 2w = 0$$

$$\Rightarrow 3v - 2(3) = 0$$

$$\Rightarrow \boxed{v = 2}$$

$$2u - v = 0$$

$$\Rightarrow 2u - 2 = 0$$

$$\Rightarrow \boxed{u = 1}$$

Solutions are $\boxed{u = 1, v = 2, w = 3, z = 4}$