Step-1

Suppose column 1 + column 3 + column 5 = 0 in a 4 by 5 matrix with four pivots.

i..e column 5 is the combination of column 1 and column 3

Therefore, the column 5 has no pivot since it is a combination of earlier columns.

Therefore,
$$x_5$$
 is free variable.

Step-2

So, x_s is free variable this means that the four pivots in the other columns.

Therefore, the special solution is (1,0,1,0,1). And the null space contains all multiples of (1,0,1,0,1).