

## Step-1

Given that  $a+b=c+d$

$$Ax = \lambda x$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \lambda \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+b \\ c+d \end{bmatrix} = \lambda \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+b \\ a+b \end{bmatrix} = \lambda \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow (a+b) \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \lambda \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Therefore  $\lambda = a+b$  is one eigen value.

Trace of  $A = \lambda_1 + \lambda_2$

$$a+d = (a+b) + \lambda_2$$

$$\begin{aligned} \lambda_2 &= a+d-a-b \\ &= d-b \end{aligned}$$

Therefore the eigen values are  $\lambda_1 = a+b, \lambda_2 = d-b$

## Step-2

To find eigen vector for  $\lambda_1 = a+b$ , we solve  $(A - \lambda_1 I)x = 0$

$$\begin{bmatrix} a-(a+b) & b \\ c & d-(a+b) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -b & b \\ c & d-(c+d) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ by replacing } a+b \text{ with } c+d.$$

$$\begin{bmatrix} -b & b \\ c & -c \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Applying the row operations on the coefficient matrix of this homogeneous system, we get

$$R_2 \rightarrow bR_2 + cR_1, R_1 / -b \Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This is the reduced matrix and so, we rewrite the homogeneous system from this.

$$x_1 - x_2 = 0$$

Putting  $x_1 = 1$ , the solution set  $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$  is the eigen vector corresponding to  $\lambda_1 = a + b$

### Step-3

Similarly, To find eigen vector for  $\lambda_2 = d - b$ , we solve  $(A - \lambda_2 I)x = 0$

$$\begin{bmatrix} a - (d - b) & b \\ c & d - (d - b) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} (a + b) - d & b \\ c & b \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} (c + d) - d & b \\ c & b \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ by replacing } c + d \text{ with } a + b.$$

$$\begin{bmatrix} c & b \\ c & b \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Applying the row operations on the coefficient matrix of this homogeneous system, we get

$$R_2 \rightarrow R_2 - R_1 \Rightarrow \begin{bmatrix} c & b \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This is the reduced matrix and so, we rewrite the homogeneous system from this.

$$cx_1 + bx_2 = 0$$

Putting  $x_1 = 1$ , the solution set  $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -b/c \end{bmatrix}$  is the eigen vector corresponding to  $\lambda_2 = d - b$

### Step-4

Thus the eigen values are  $\lambda_1 = a + b$  and  $\lambda_2 = d - b$  while the respective eigen vectors are  $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$  and  $\begin{bmatrix} 1 \\ -b/c \end{bmatrix}$