Step-1

Consider the 5 by 5 Jordan block defined as below:

$$J=\begin{bmatrix}0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\\0&0&0&0&0\end{bmatrix}$$

Here, Eigen values are zero, $\lambda=0$. Find value of J^2 and its Eigen vectors. Also find its Jordan form in two blocks.

Step-2

Jordan block powers can be given as follows:

$$\begin{aligned}(J_i)^k&=\begin{bmatrix}\lambda&1&0\\0&\lambda&1\\0&0&\lambda\end{bmatrix}^k\\&=\begin{bmatrix}\lambda^k&k\lambda^{k-1}&\frac{1}{2}k(k-1)\lambda^{k-2}\\0&\lambda^k&k\lambda^{k-1}\\0&0&\lambda^k\end{bmatrix}\end{aligned}$$

Step-3

To find the value of J^2 do the following calculations:

$$J^2=\begin{bmatrix}\mathbf{0}&1&0&0&0\\0&\mathbf{0}&1&0&0\\0&0&\mathbf{0}&1&0\\0&0&0&\mathbf{0}&1\\0&0&0&0&\mathbf{0}\end{bmatrix}^2$$

$$=\begin{bmatrix}\mathbf{0}&2\cdot\mathbf{0}^{2-1}&\frac{1}{2}2(2-1)\mathbf{0}^{2-2}&0&0\\0&\mathbf{0}&2\cdot\mathbf{0}^{2-1}&\frac{1}{2}2(2-1)\mathbf{0}^{2-2}&0\\0&0&\mathbf{0}&2\cdot\mathbf{0}^{2-1}&\frac{1}{2}2(2-1)\mathbf{0}^{2-2}\\0&0&0&\mathbf{0}&2\cdot\mathbf{0}^{2-1}\\0&0&0&0&\mathbf{0}\end{bmatrix}$$

$$=\begin{bmatrix}\mathbf{0}&\mathbf{0}&1&0&0\\0&\mathbf{0}&\mathbf{0}&1&0\\0&0&\mathbf{0}&\mathbf{0}&1\\0&0&0&\mathbf{0}&\mathbf{0}\\0&0&0&0&\mathbf{0}\end{bmatrix}$$

Step-4

Therefore,

$$\boxed{J^2=\begin{bmatrix}\mathbf{0}&\mathbf{0}&1&0&0\\0&\mathbf{0}&\mathbf{0}&1&0\\0&0&\mathbf{0}&\mathbf{0}&1\\0&0&0&\mathbf{0}&\mathbf{0}\\0&0&0&0&\mathbf{0}\end{bmatrix}}$$

Step-5

For every missing Eigen vector Jordan form will have 1 in the matrix. Here in J^2 matrix three 1 can be seen. This implies that three Eigen vectors are missing. So, from 5 Eigen vectors corresponding to 5 repeated Eigen values 3 are missing.

Therefore, J^2 have 2 independent Eigen vectors corresponding to Eigen value $\lambda=0$.

Step-6

Jordan form of J^2 can be written as follows in two blocks:

$$\begin{bmatrix} J_3 & \\ & J_2 \end{bmatrix}$$

Here, both blocks are defined as:

$$J_3 = \boxed{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}}$$

$$J_2 = \boxed{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}$$