

Step-1

Consider the system:

$$Ax = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(a)

Objective is to verify the description that the solutions x of form a plane is correct or not.

The system can be written as below:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x_1 + x_2 + x_3 = 0$$

$$x_1 + 2x_3 = 0$$

$$x_1 = -2x_3$$

Step-2

Substitute $x_1 = -2x_3$ in $x_1 + x_2 + x_3 = 0$.

$$-2x_3 + x_2 + x_3 = 0$$

$$x_2 = x_3$$

Hence, the solutions of x :

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} x_3$$

Hence, the solutions x of form a plane is not correct.

Step-3

(b)

Objective is to verify the description that the solutions x of form a line is correct or not.

The system can be written as below:

$$x_1 + x_2 + x_3 = 0$$

$$-2x_3 + x_3 + x_3 = 0$$

$$0 = 0$$

Hence, the solutions x of form a line is not correct.

Step-4

(c)

Objective is to verify the description that the solutions x of form a point is correct or not.

The system can be written as below:

$$x_1 + x_2 + x_3 = 0$$

$$x_1 + 2x_3 = 0$$

$$x_1 = -2x_3$$

$$x_2 = x_3$$

Here $(x_1, x_2, x_3) = x_3(-2, 1, 1)$

Hence, the system has so many solutions.

Hence, the solutions x of form a point is not correct.

Step-5

(d)

Objective is to verify the description that the solutions x of form a subspace is correct or not.

The system can be written as below:

$$x_1 + x_2 + x_3 = 0$$

$$x_1 + 2x_3 = 0$$

The set $\{x \mid Ax = 0\}$ is the null space of A .

$$Ax = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Hence, the solutions x of the system form a nullspace of A .

This is also a subspace \mathbb{R}^3 .

Hence, the solutions x of form a subspace is correct.

Step-6

(e)

Objective is to verify the description that the solutions x of form the nullspace of A is correct or not.

The system can be written as below:

$$x_1 + x_2 + x_3 = 0$$

$$x_1 + 2x_3 = 0$$

The set $\{x \mid Ax = 0\}$ is the null space of A .

$$Ax = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Hence, the solutions x of the system form a nullspace of A .

(f)

Objective is to verify the description that the solutions x of form the column space of A is correct or not.

Step-7

If the vector b can be expressed as a combination of the columns of A then the system $Ax = b$ is solvable.

But here $Ax = 0$.

Hence, the solutions x of form the column space of A is not correct.

$$Ax = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Hence, the solutions x of the system form a column space of A is not correct.