

Step-1

In the parallelogram with corners at $0, v, w$ and $v + w$, we have to show that the sum of the squared lengths of the four sides equals the sum of the squared lengths of the two diagonals.

Step-2

The four corners of parallelogram are $0, v, w, v + w$.

That is, in vw -plane, the four points are

$$O = (0, 0), A = (1, 0), C = (0, 1) \text{ and } B = (1, 1)$$

Since A is a point on v -axis and C is a point on w -axis.

The four sides of parallelogram are OA, AB, BC, CO and the two diagonals are AC .

Step-3

$$\begin{aligned} OA &= \sqrt{(1-0)^2 + (0-0)^2} \\ &= 1 \end{aligned}$$

$$\begin{aligned} AB &= \sqrt{(1-1)^2 + (1-0)^2} \\ &= 1 \end{aligned}$$

Step-4

$$\begin{aligned} BC &= \sqrt{(0-1)^2 + (1-1)^2} \\ &= 1 \end{aligned}$$

$$\begin{aligned} CO &= \sqrt{(0-0)^2 + (0-1)^2} \\ &= 1 \end{aligned}$$

Step-5

And

$$\begin{aligned} OB &= \sqrt{(1-0)^2 + (1-0)^2} \\ &= \sqrt{2} \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{(0-1)^2 + (1-0)^2} \\ \hat{A} &= \sqrt{2} \end{aligned}$$

Step-6

The sum of the squared lengths of the four sides

$$\begin{aligned} &= OA^2 + AB^2 + BC^2 + CO^2 \\ &= 4 \end{aligned}$$

The sum of the squared lengths of the two diagonals

$$\begin{aligned} &= OB^2 + AC^2 \\ &= 2 + 2 \\ &= 4 \end{aligned}$$

Hence the sum of the squared lengths of the four sides is equals to the sum of the squared lengths of the two diagonals.