

Step-1

Suppose that U and V are unitary matrices.

The objective is to show that UV is also unitary matrix by using the criterion $U^H U = I$.

As U and V are unitary matrices,

$$U^H U = I \text{ and } V^H V = I.$$

Step-2

Consider

$$(UV)^H (UV) = (V^H U^H)(UV) \quad (UV)^H = V^H U^H.$$

$$= V^H U^H U V$$

$$= V^H I V \quad U^H U = I.$$

$$= V^H V \quad V^H I = V^H.$$

$$= I \quad V^H V = I.$$

Hence, the matrix UV is also unitary.