

Step-1

a) We have to find a matrix that transforms $(1,0)$ and $(0,1)$ to (r,t) and (s,u) .

We have

$$T(1,0) = (r,t)$$

$$T(0,1) = (s,u)$$

Therefore, the matrix M of the linear transformation T under the basis $(1,0), (0,1)$ is $\begin{bmatrix} r & s \\ t & u \end{bmatrix}$.

Step-2

b) We have to find a matrix that transforms (a,c) and (b,d) to $(1,0)$ and $(0,1)$.

We have

$$T(a,c) = (1,0)$$

$$T(b,d) = (0,1)$$

The matrix of the transform T under the elements (a,c) and (b,d) (if $\{(a,c), (b,d)\}$ is a basis of R^2) is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Step-3

c) We have to find the condition on a, b, c, d that will make part (b) impossible.

If the vectors $(a,b), (c,d)$ are linearly dependent,

That is there exist not all zero scalars x, y such that $x(a,b) + y(c,d) = 0$

Then part (b) is impossible.

Hence part (b) is impossible if the vectors $(a,b), (c,d)$ are linearly dependent.