Step-1

Given that $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ and M is any 2 by 2 matrix.

And the linear transformation T is defined as $T(M)=AM$.

We have to verify what rules of matrix multiplication show that T is linear.

Step-2

Let M,N be any 2 by 2 matrices.

Now

$T(M+N)=A(M+N)$

$=AM+AN$

$=T(M)+T(N)$

Step-3

Let c be any scalar.

Now

$T(cM)=A(cM)$

$=c(AM)$

For $A(M+N)=AM+AN$, Distributive law among matrices,

For $A(cM)=c(AM)$, this is one type of associate law, and commutative law and constant multiplication of matrix.