

Step-1

Let $P = (1, 0, -1), Q = (1, 1, 1), R = (2, 2, 1)$ and

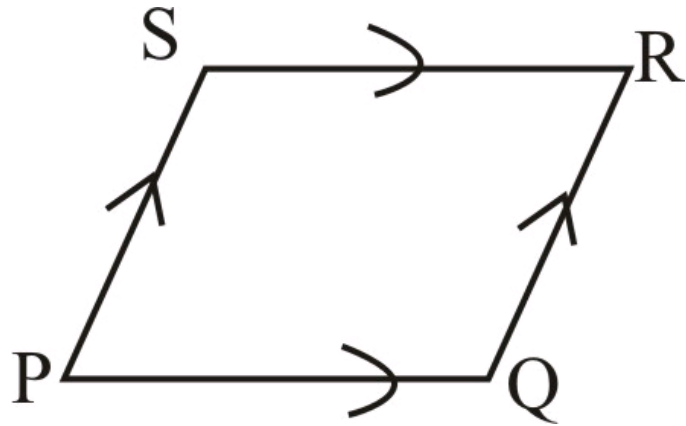
Let $S = (x, y, z)$ be the fourth vertex so that PQRS is a parallelogram. Then we have

$$\left. \begin{array}{l} 1+2 = x+1 \\ 0+2 = y+1 \\ -1+1 = z+1 \end{array} \right\} \Rightarrow (x, y, z) = (2, 1, -1)$$

So,

Step-2

Coordinates of $S = (2, 1, -1)$



$$\overrightarrow{PQ} = (0, 1, 2), \quad \overrightarrow{PS} = (1, 1, 0)$$

Step-3

$$\text{Area of parallelogram PQRS} = \text{length of vector } \begin{vmatrix} i & j & k \\ 0 & 1 & 2 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= |-2i + 2j - k|$$

$$= \sqrt{4 + 4 + 1}$$

$$= 3 \text{ sq.units}$$

Step-4

We can choose

$T = (0, 1, 2), U = (1, 1, 0), V = (1, 2, 2)$ to get a parallelepiped OPQRSTUV and its volume is given by

$$[\overrightarrow{OP} \ \overrightarrow{OT} \ \overrightarrow{OU}] = \begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= |1(0-2) - 1(0-1)|$$

$$= |-2 + 1|$$

$$= 1$$

$$\boxed{= 1 \text{ cubic units}}$$