Step-1

A be an n by n matrix with every entry equal to 1.

$$A = \begin{bmatrix} 1 & 1 & - & - & 1 \\ 1 & 1 & - & - & 1 \\ 1 & 1 & - & - & 1 \\ - & - & - & - & - \\ 1 & 1 & - & - & 1 \end{bmatrix}$$

By using the *n* â€“ 1 row operations $R_n - R_1, R_{n-1} - R_1, R_{n-2} - R_1, \ldots, R_2 - R_1$ upon this matrix, we get $A \approx \begin{bmatrix} 1 & 1 & - & - & 1 \\ 0 & 0 & - & - & 0 \\ 0 & 0 & - & - & 0 \\ - & - & - & - & - \\ 0 & 0 & - & - & 0 \end{bmatrix}$

Rank of the matrix is the number of non zero rows in the reduced matrix = 1

Step-2

B be the ehexer board matrix with

$$a_{ij} = \begin{cases} 1 \text{ if } i+j \text{ is even} \\ 0 \text{ if } i+j \text{ is odd} \end{cases}$$

If n is even

$$B = \begin{bmatrix} 1 & 0 & 1 & - & 0 \\ 0 & 1 & 0 & - & 1 \\ 1 & 0 & 1 & - & 0 \\ - & - & - & - & - \\ 0 & 1 & 0 & - & 1 \end{bmatrix}$$

Step-3

By using the row operations $R_3 - R_1, R_5 - R_1, R_7 - R_1, \ldots, R_4 - R_2, R_6 - R_2, \ldots$ we get

$$B \approx \begin{bmatrix} 1 & 0 & 1 & - & 0 \\ 0 & 1 & 0 & - & 1 \\ 0 & 0 & 0 & - & 0 \\ - & - & - & - & - \\ 0 & 0 & 0 & - & 0 \end{bmatrix}$$

Further, any operation on R_2 using R_1 does not make R_2 completely zero.

In other words, B has two linearly independent rows.

Therefore rank of $B = 2$.