

Step-1

Consider that $A = S \begin{bmatrix} \lambda_1 & 2 \\ 0 & \lambda_2 \end{bmatrix} S^{-1}$.

The objective is to find the determinant of A and A^{-1} .

Step-2

Find the determinant of matrix A as,

Recall: $\det(A_1 \cdot A_2 \cdots A_n) = \det(A_1) \cdot \det(A_2) \cdots \det(A_n)$

Therefore,

$$\begin{aligned} \det(A) &= \det\left(S \begin{bmatrix} \lambda_1 & 2 \\ 0 & \lambda_2 \end{bmatrix} S^{-1}\right) \\ &= \det(S) \cdot \det\left(\begin{bmatrix} \lambda_1 & 2 \\ 0 & \lambda_2 \end{bmatrix}\right) \cdot \det(S^{-1}) \\ &= \det(S) \cdot \det\left(\begin{bmatrix} \lambda_1 & 2 \\ 0 & \lambda_2 \end{bmatrix}\right) \cdot \left(\frac{1}{\det(S)}\right) \quad \text{Since } \det(A^{-1}) = \frac{1}{\det(A)} \\ &= \left(\cancel{\det(S)} \cdot \frac{1}{\cancel{\det(S)}}\right) (\lambda_1 \cdot \lambda_2 - 0) \\ &= \lambda_1 \lambda_2 \end{aligned}$$

Therefore, the value of $\det(A) = \lambda_1 \lambda_2$ (1)

Step-3

Find the determinant of matrix A^{-1} as,

$$\begin{aligned} \det(A^{-1}) &= \frac{1}{\det(A)} \\ &= \frac{1}{\lambda_1 \lambda_2} \quad \text{From equation (1)} \end{aligned}$$

Therefore, the value of $\det(A^{-1}) = \frac{1}{\lambda_1 \lambda_2}$.