Step-1

Consider the following matrix:

$$A=\begin{bmatrix}1&0&1\\1&1&0\\0&1&1\end{bmatrix}$$

Then

$$\begin{aligned}\det(A)&=\begin{vmatrix}1&0&1\\1&1&0\\0&1&1\end{vmatrix}\\&=2\end{aligned}$$

Since $\det(A)\neq 0,$ so the matrix A is non-singular.

Thus, the matrix A has inverse.

Find inverse if it exist, by inspection or by Gauss-Jordan.

Step-2

Using the Gauss-Jordan Method to Find A^{-1}.

$$\begin{bmatrix}A & I\end{bmatrix}=\begin{bmatrix}1&0&1&1&0&0\\1&1&0&0&1&0\\0&1&1&0&0&1\end{bmatrix}$$

$$\sim\begin{bmatrix}1&0&1&1&0&0\\0&1&-1&-1&1&0\\0&1&1&0&0&1\end{bmatrix}\ R_2\to R_2-R_1$$

$$\sim\begin{bmatrix}1&0&1&1&0&0\\0&1&-1&-1&1&0\\0&0&2&1&-1&1\end{bmatrix}\ R_3\to R_3-R_2$$

$$\sim\begin{bmatrix}1&0&1&1&0&0\\0&1&-1&-1&1&0\\0&0&1&1/2&-1/2&1/2\end{bmatrix}\ R_3\to\frac{R_3}{2}$$

Step-3

Continuing the previous steps as follows:

$$\begin{bmatrix} A & I \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1/2 & 1/2 & 1/2 \\ 0 & 0 & 1 & 1/2 & -1/2 & 1/2 \end{bmatrix} R_2 \to R_2 + R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 1/2 & 1/2 & -1/2 \\ 0 & 1 & 0 & -1/2 & 1/2 & 1/2 \\ 0 & 0 & 1 & 1/2 & -1/2 & 1/2 \end{bmatrix} R_1 \to R_1 - R_3$$

$$= \begin{bmatrix} I & A^{-1} \end{bmatrix}$$

Therefore, the inverse of the matrix A is

$$A^{-1} = \boxed{\begin{bmatrix} 1/2 & 1/2 & -1/2 \\ -1/2 & 1/2 & 1/2 \\ 1/2 & -1/2 & 1/2 \end{bmatrix}}.$$

Step-4

Consider the following matrix:

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

Then

$$\det(A) = \begin{vmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix}$$
$$= 4$$

Since $\det(A) \neq 0$, so the matrix A is non-singular.

Thus, the matrix A has inverse.

Find inverse if it exist, by inspection or by Gauss-Jordan.

Step-5

Using the Gauss-Jordan Method to Find A^{-1}.

$$[A \quad I] = \begin{bmatrix} 2 & 1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix} R_2 \leftrightarrow R_1$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & -3 & -2 & 1 & -2 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix} R_2 \to R_2 - 2R_1$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2/3 & -1/3 & 2/3 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix} R_2 \to \frac{R_2}{-3}$$

$$\sim \begin{bmatrix} 1 & 0 & -1/3 & 2/3 & -1/3 & 0 \\ 0 & 1 & 2/3 & -1/3 & 2/3 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix} R_1 \to R_1 - 2R_2$$

Step-6

Continuing the previous steps as follows:

$$[A \quad I] \sim \begin{bmatrix} 1 & 0 & -1/3 & 2/3 & -1/3 & 0 \\ 0 & 1 & 2/3 & -1/3 & 2/3 & 0 \\ 0 & 0 & 4/3 & 1/3 & -2/3 & 1 \end{bmatrix} R_3 \to R_3 - R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 3/4 & -1/2 & 1/4 \\ 0 & 1 & 2/3 & -1/3 & 2/3 & 0 \\ 0 & 0 & 4/3 & 1/3 & -2/3 & 1 \end{bmatrix} R_1 \to R_1 + \frac{1}{4} R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 3/4 & -1/2 & 1/4 \\ 0 & 1 & 0 & -1/2 & 1 & -1/2 \\ 0 & 0 & 4/3 & 1/3 & -2/3 & 1 \end{bmatrix} R_2 \to R_2 - \frac{1}{2} R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 3/4 & -1/2 & 1/4 \\ 0 & 1 & 0 & -1/2 & 1 & -1/2 \\ 0 & 0 & 1 & 1/4 & -1/2 & 3/4 \end{bmatrix} R_3 \to \frac{3}{4} R_3$$

$$=\begin{bmatrix} I & A^{-1} \end{bmatrix}$$

Therefore, the inverse of the matrix A is

$$A^{-1}=\boxed{\begin{bmatrix} 3/4 & -1/2 & 1/4 \\ -1/2 & 1 & -1/2 \\ 1/4 & -1/2 & 3/4 \end{bmatrix}}.$$

Step-7

Consider the following matrix:

$$A=\begin{bmatrix} 1 & 1 & -2 \\ 1 & -2 & 1 \\ -2 & 1 & 1 \end{bmatrix}$$

Then

$$\begin{aligned} \det(A) &= \begin{vmatrix} 1 & 1 & -2 \\ 1 & -2 & 1 \\ -2 & 1 & 1 \end{vmatrix} \\ &= 0 \end{aligned}$$

Since $\det(A)=0,$ so the matrix A is singular.

Thus, the matrix A has no inverse.