## Step-1

Given that addition and multiplication are required to satisfy the following eight rules.

1. $x+y=y+x$

2. $x+(y+z)=(x+y)+z$

3. There is a unique â€œzero vectorâ€ such that $x+0=x$ for all *x*.

4. For each $x$ there is a unique number $-x$ such that $x+(-x)=0$.

5. $1x=x$

6. $(c_1c_2)x=c_1(c_2x)$

7. $c(x+y)=cx+cy$

8. $(c_1+c_2)x=c_1x+c_2x$

## Step-2

(a)

Suppose addition in $\mathbf{R}^2$ adds an extra 1 to each component,

So that $(3,1)+(5,0)=(9,2)$ instead of $(8,1)$.

The objective is to verify with scalar multiplication unchanged, which rules are broken.

Rule 7 is broken since

If $c=3$, $x=(3,1)$, $y=(5,0)$

Then,

$$\begin{aligned}3(x+y)&=3((3,1)+(5,0))\\&=3(9,2)\\&=(27,6)\\3x+3y&=(9,3)+(15,0)\\&=(25,4)\end{aligned}$$

Therefore, $3(x+y) \neq 3x+3y$

Rule 8 is also broken.

$$\begin{aligned}
&c_1 = 2, c_2 = 3 \\
&(c_1+c_2)x = 5(3,1) = (15,5) \\
&\quad c_1x + c_2x = 2(3,1) + 3(3,1) \\
&\qquad\qquad = (6,2) + (9,3) \\
&\qquad\qquad = (16,6)
\end{aligned}$$

Therefore, $(c_1+c_2)x \neq c_1x + c_2x$

Thus, the rules $\boxed{\text{7 and 8}}$ are broken.

## Step-3

(b)

The objective is to show that the set of all positive real numbers, with $x+y$ and $cx$ redefined to equal the usual $xy$ and $x^c$ is a vector space.

Rules 1 and 2 are not satisfied.

One is a zero vector since $x+1 = x1 = x$.

Therefore, the required zero vector is, $\boxed{1}$

For each $x$ position, real number $\frac{1}{x}$ is the $(-x)$ for $x$.

$$\begin{aligned}
4.\ x+(-x) &= x \cdot \frac{1}{x} \\
&= 1
\end{aligned}$$

$$\begin{aligned}
6.\ (c_1c_2)x &= x^{c_1c_2} \\
&= \left(x^{c_2}\right)^{c_1} \\
&= (c_2x)^{c_1} \\
&= c_1(c_2x)
\end{aligned}$$

7. $(c_1+c_2)=x^{c_1+c_2}$
$$=x^{c_1}\cdot x^{c_2}$$
$$=x^{c_1}+x^{c_2}$$
$$=c_1x+c_2x$$

8. $c(x+y)=(x+y)^c$
$$=(xy)^c$$
$$=x^c y^c$$
$$=x^c+y^c$$
$$=cx+cy$$

For the above defined vector addition and scalar multiplication, the set of positive real numbers is a $\boxed{\text{vector space}}$ with $1$ as a $\boxed{\text{zero vector}}$.

## Step-4

(c)

Suppose $(x_1,x_2)+(y_1,y_2)$ is defined as, $(x_1+y_2,x_2+y_1)$.

The objective is to verify that which of the eight conditions are not satisfied with the usual scalar multiplication defined as, $cx=(cx_1,cx_2)$.

## Step-5

Rule $1$ is not satisfied since

$$\begin{aligned}(1,2)+(3,3)&=(1+3,2+3)\\&=(4,5)\\(3,3)+(1,2)&=(3+2,3+1)\\&=(5,4)\end{aligned}$$

Therefore, $(1,2)+(3,3)\neq(3,3)+(1,2)$

Similarly, Rule $2$ is not satisfied.

$$x+(y+z)\neq(x+y)+z$$

For,

$$\begin{aligned}((1,1)+(2,3))+(4,5)&=(4,3)+(4,5)\\&=(9,7)\\(1,1)+((2,3)+(4,5))&=(1,1)+(7,7)\\&=(8,8)\end{aligned}$$

Therefore, Rule (2) is also not satisfied.

## Step-6

In the same way, Rule $8$ is not satisfied since

$$\begin{aligned}(2+3)(1,2)&=(5,10)\\2(1,2)+3(1,2)&=(2,4)+(3,6)\\&=(2+6,4+3)\\&=(8,7)\end{aligned}$$

$$(2+3)(1,2)\neq 2(1,2)+3(1,2)$$

Thus, the rules $1,2 \text{ and } 8$ are $\boxed{\text{not satisfied}}$.