Step-1

Given that $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

The characteristic equation is $|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} a - \lambda & b \\ c & d - \lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^2 - (a + d)\lambda + (ad - bc) = 0$$

Step-2

Cayley – Hamilton theorem states that every square matrix satisfies its characteristic equation.

i.e., when the eigen value λ is replaced by the matrix A in $|A - \lambda I| = 0$, it has to get satisfied.

i.e., $A^2 - (a + d)A + (ad - bc)I$:

$$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix}$$

Step-3

$A^2 - (a + d)A + (ad - bc)I =$

$$= \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} - (a + d)\begin{bmatrix} a & b \\ c & d \end{bmatrix} + (ad - bc)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} a^2 + bc - a^2 - ad + ad - bc & ab + bd - ab - db + 0 \\ ac + cd - ac - cd + 0 & bc + d^2 - ad - d^2 + ad - bc \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$= 0$

Therefore the matrix A satisfied its characteristic equation.

Thus, Cayley – Hamilton theorem is verified.