Step-1

Consider the factored form $A=\begin{bmatrix}1&0&0\\2&1&0\\1&4&1\end{bmatrix}\begin{bmatrix}5&7&8\\0&2&3\\0&0&6\end{bmatrix}$

The objective is to find what multiple l_{32} of row2 of A will elimination subtract from row 3 of A and to find the pivots and to check whether the row exchange is required or not.

Step-2

Given $A=\begin{bmatrix}1&0&0\\2&1&0\\1&4&1\end{bmatrix}\begin{bmatrix}5&7&8\\0&2&3\\0&0&6\end{bmatrix}$

Compare with $A=lu$ where

$$l=\begin{bmatrix}l_{11}&l_{12}&l_{13}\\l_{21}&l_{22}&l_{23}\\l_{31}&l_{32}&l_{33}\end{bmatrix} \text{ and } u=\begin{bmatrix}u_{11}&u_{12}&u_{13}\\u_{21}&u_{22}&u_{23}\\u_{31}&u_{32}&u_{33}\end{bmatrix}$$

Consider l matrix that is $l=\begin{bmatrix}l_{11}&l_{12}&l_{13}\\l_{21}&l_{22}&l_{23}\\l_{31}&\boxed{l_{32}}&l_{33}\end{bmatrix}=\begin{bmatrix}1&0&0\\2&1&0\\1&\boxed{4}&1\end{bmatrix}$

Then $l_{32}=4$

Hence $\boxed{\text{elimination will subtract 4 times row2 from row 3}}$

Step-3

Consider u matrix to find the pivots

Then $u=\begin{bmatrix}u_{11}&u_{12}&u_{13}\\u_{21}&u_{22}&u_{23}\\u_{31}&u_{32}&u_{33}\end{bmatrix}=\begin{bmatrix}5&7&8\\0&2&3\\0&0&6\end{bmatrix}$

The pivots are along the diagonal of the matrix that is $\begin{bmatrix}\boxed{5}&7&8\\0&\boxed{2}&3\\0&0&\boxed{6}\end{bmatrix}$

Hence $\boxed{\boxed{5}, \boxed{2} \text{ and } \boxed{6} \text{ are the pivots of the matrix}}$

Step-4

To find out whether a row exchange will be needed or not, first determine A

Consider

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 4 & 1 \end{bmatrix} \begin{bmatrix} 5 & 7 & 8 \\ 0 & 2 & 3 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 7 & 8 \\ 10 & 16 & 19 \\ 5 & 15 & 26 \end{bmatrix}$$

Carry the first elimination

$$A = \begin{bmatrix} 5 & 7 & 8 \\ 10 & 16 & 19 \\ 5 & 15 & 26 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

Carry the Second elimination

$$A = \begin{bmatrix} 5 & 7 & 8 \\ 0 & 2 & 3 \\ 0 & 8 & 18 \end{bmatrix} \begin{matrix} \\ \\ R_3 \to R_3 - 4R_2 \end{matrix}$$

$$A = \begin{bmatrix} 5 & 7 & 8 \\ 0 & 2 & 3 \\ 0 & 0 & 6 \end{bmatrix} = u$$

Step-5

Hence, $\boxed{\text{there would not be a need for a row exchange}}$