Step-1

Given $b = x_1q_1 + x_2q_2 + ... + x_nq_n$

In matrix notation it is equivalent to $b = Qx$

Here $Q = \begin{bmatrix} q_1 & q_2 & \cdots & q_n \end{bmatrix}, x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

Now we have to prove that $\|b\|^2 = x_1^2 + x_2^2 + ... + x_n^2$, and $\|Qx\|^2 = \|x\|^2$

Step-2

Now

$$\begin{aligned} b^T b &= (Qx)^T Qx \\ &= x^T Q^T Qx \\ &= x^T I \\ &= x^T x \end{aligned}$$

Step-3

$$\begin{aligned} &= \begin{bmatrix} x_1 & x_2 & \ldots . & x_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \\ &= x_1^2 + x_2^2 + ... + x_n^2 \end{aligned}$$

Step-4

Hence

$$\|b\|^2 = b^T b$$
$$\Rightarrow \boxed{\|b\|^2 = x_1^2 + x_2^2 + ... + x_n^2}$$

Step-5

And

$$
\begin{aligned}
\|Qx\|^2 &= (Qx)^T Qx \\
&= x^T Q^T Qx \\
&= x^T Ix \\
&= x^T x \\
&= \|x\|^2
\end{aligned}
$$

Hence $\|Qx\|^2 = \|x\|^2$