

Step-1

Given system is $u + v + w = 2$

$$u + 3v + 3w = 0$$

$$u + 3v + 5w = 2$$

We have to find the triangular system after forward elimination and the solution.

Step-2

Given system can be written as

$$\begin{pmatrix} 1 & 1 & 1 & 2 \\ 1 & 3 & 3 & 0 \\ 1 & 3 & 5 & 2 \end{pmatrix}$$

Subtract \hat{e}_1^T time the row 1 from the row 2

Subtract \hat{e}_1^T time the row 1 from the row 3

$$\sqcup \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 2 & 2 & -2 \\ 0 & 2 & 4 & 0 \end{pmatrix}$$

Step-3

Subtract \hat{e}_1^T times the row 2 from the row 3.

$$\sqcup \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 2 & 2 & -2 \\ 0 & 0 & 2 & 2 \end{pmatrix}$$

which is upper triangular form.

Hence the triangular system after forward elimination

$$\boxed{\begin{array}{l} u + v + w = 2 \\ 2v + 2w = -2 \\ 2w = 2 \end{array}}$$

Step-4

From above upper triangular form, we have

$$2w = 2$$

$$\Rightarrow w = 1$$

$$2v + 2w = -2$$

$$\Rightarrow 2v + 2(1) = -2$$

$$\Rightarrow v = -2$$

$$u + v + w = 2$$

$$\Rightarrow u - 2 + 1 = 2$$

$$\Rightarrow u = 3$$

Therefore the solution is $\boxed{u = 3, v = -2, w = 1}$