Step-1

If A is an m by n matrix, using row operations, we can reduce A to r non zero rows and

m â€“ r zero rows. Then

1. $C(A)=$ Column space of A; dimension r
2. $N(A)$ = null space of A; dimension n â€“ r
3. $C(A^T)$ = row space of A; dimension r
4. $N(A^T)$ = left null space of A; dimension m â€“ r

Step-2

(a) if $[A]$ is a matrix of size $m\times n$, then $\begin{bmatrix} A \\ A \end{bmatrix}$ is of size $2m\times n$

If rank $A = r$, then there are r linearly independent rows and consequently r linearly independent columns in A

But there are n â€“ r dependent columns in both $[A]$ as well as $\begin{bmatrix} A \\ A \end{bmatrix}$

m â€“ r dependent rows in $[A]$ and $2m$ â€“ r dependent rows in $\begin{bmatrix} A \\ A \end{bmatrix}$

With this information, we can confirm that the row space , column space and null space of both the matrices is one and the same.

But the left null spaces are different.

Step-3

(b) The size of $\begin{bmatrix} A \\ A \end{bmatrix}$ is $2m\times n$ and that of $\begin{bmatrix} A & A \\ A & A \end{bmatrix}$ is $2m\times 2n$

If this number of non zero rows of A are r, then the number of non zero rows of both $\begin{bmatrix} A \\ A \end{bmatrix}$ and $\begin{bmatrix} A & A \\ A & A \end{bmatrix}$ will be r itself as A nullifies any number of *Aâ€™s* either as rows or as columns.

So, number of non zero rows of $\begin{bmatrix} A & A \\ A & A \end{bmatrix}$ and non zero rows of $\begin{bmatrix} A \\ A \end{bmatrix}$ is equal.

Similarly non zero columns are equal.

Further number of zero rows of both these matrices is equal.

The only difference is the number of zero columns in these matrices is different.

From this discussion, we conclude that the row space, column space and left null space of both the matrices $\begin{bmatrix} A \\ A \end{bmatrix}$ and $\begin{bmatrix} A & A \\ A & A \end{bmatrix}$ are one and the same.

The only difference is the null space in this case.