Step-1

Consider the following matrix:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

The new matrix $\begin{pmatrix} a-mc & b-md \\ c-la & d-lb \end{pmatrix}$ is obtained by applying the two row operations

$R_1 - mR_2 \to R_1$ and $R_2 - lR_1 \to R_2$ on $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Step-2

Now, to find the determinant of the new matrix directly:

$$\begin{aligned} \begin{vmatrix} a-mc & b-md \\ c-la & d-lb \end{vmatrix} &= (a-mc)\cdot(d-lb)-(c-la)\cdot(b-md) \\ &= ad-alb-mcd+mclb-cb+cmd+lab-lamd \\ &= ad-cb+mclb-lamd \\ &= ad(1-lm)-cb(1-lm) \\ &= (ad-cb)\cdot(1-lm) \end{aligned}$$

Therefore, the determinant of the new matrix comes out to be $\boxed{(ad-cb)\cdot(1-lm)}$.