Step-1

The second order differential equation $y'' = 0$ can be written as a first order differential equation system by introducing the velocity y' as unknown.

$$\frac{d}{dt}\begin{bmatrix} y \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} y \\ y' \end{bmatrix}$$

The solution to $y'' = 0$ is a straight line $y = C + Dt$.

Step-2

Let following be the differential equation of matrices:

$$\frac{du}{dt} = Au$$

Here, matrix A is defined as follows:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Step-3

Matrix A cannot be diagonalize so, find A^2 and compute e^{At} from the series and write the solution of $e^{At}u(0)$.

Step-4

Here, matrix A contains only one Eigen value and corresponding to it one Eigen vector, which is not sufficient to diagonalize A. So, use the following series to calculate e^{At}.

$$e^{At} = I + At + \frac{(At)^2}{2!} + \cdots$$

Next, compute the following:

$$A \cdot A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Step-5

Therefore:

Step-6

$$A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Step-7

Thus,

$$e^{At} = I + At$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & t \\ 0 & 0 \end{bmatrix} + 0$$

$$= \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$$

Therefore, value of e^{At} is:

$$e^{At} = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$$

Step-8

So, the solution for differential equation can be written as follows:

$$u(t) = e^{At} u(0)$$

$$= \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y(0) \\ y'(0) \end{bmatrix}$$

$$\begin{bmatrix} y(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} y'(0)t + y(0) \\ y'(0) \end{bmatrix}$$

Step-9

Therefore, solution of the differential equation is:

$$y(t) = y(0) + y'(0)t$$

This represents a straight line.