Step-1

Given

$A=\begin{bmatrix}4&2\\1&3\end{bmatrix}, A^{-1}=\frac{1}{10}\begin{vmatrix}3&-2\\-1&4\end{vmatrix}$ and $A-\lambda I=\begin{vmatrix}4-\lambda&2\\1&3-\lambda\end{vmatrix}$.

We need to find the determinants of A, A^{-1} and $A-\lambda I$.

And also we need to find the value of λ for which $\mathrm{A}-\lambda\mathrm{I}$ is a singular matrix.

Step-2

a) $\det(A)=\begin{vmatrix}4&2\\1&3\end{vmatrix}$

$=(4)(3)-(1)(2)$ $\left[\text{since }\begin{vmatrix}a&b\\c&d\end{vmatrix}=ad-bc\right]$

$=\boxed{10}$

Hence $\boxed{\det\begin{bmatrix}4&2\\1&3\end{bmatrix}=10}$

Step-3

b) We have to find the determinant of $A^{-1}=\frac{1}{10}\begin{vmatrix}3&-2\\-1&4\end{vmatrix}$.

We know that $\det(tA)=t^n\det A$, where A is an $n\times n$ matrix.

Now

$\det A^{-1}=\det\left(\frac{1}{10}\begin{bmatrix}3&-2\\-1&4\end{bmatrix}\right)$

$=\left(\frac{1}{10}\right)^2\det\begin{bmatrix}3&-2\\-1&4\end{bmatrix}$

$= \left(\frac{1}{10}\right)^2 \left[(3)(4)-(-1)(-2)\right]$

$= \frac{1}{100}(10)$

$\boxed{= \frac{1}{10}}$

Thus, $\boxed{\det A^{-1} = \frac{1}{10}}$

Step-4

c) We have to find the determinant of $A - \lambda I = \begin{vmatrix} 4-\lambda & 2 \\ 1 & 3-\lambda \end{vmatrix}$.

Now

$\det(A-\lambda I) = \begin{vmatrix} 4-\lambda & 2 \\ 1 & 3-\lambda \end{vmatrix}$

$= (4-\lambda)(3-\lambda)-(2)(1)$
$= 12-4\lambda-3\lambda+\lambda^2-2$
$= \lambda^2-7\lambda+10$

Step-5

We need to find the values of λ for which $A-\lambda I$ is a singular matrix.

We know that the determinant of a singular matrix is 0.

$\det(A-\lambda I)=0$

$\Rightarrow \lambda^2-7\lambda+10=0$
$\Rightarrow (\lambda-2)(\lambda-5)=0$
$\Rightarrow \lambda = 2 \text{ or } 5$

Thus, $A-\lambda I$ is a singular matrix if $\boxed{\lambda = 2 \text{ or } 5}$