

Step-1

$$Ax = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

If , by cramers rule

$$B_1 = \begin{bmatrix} 1 & a_{12}, \dots, a_{1n} \\ 0 & a_{22}, \dots, a_{2n} \\ \vdots & \vdots \\ 0 & a_{n2}, \dots, a_{nn} \end{bmatrix}$$

$$= C_{11}$$

Step-2

$$\Rightarrow x_1 = \frac{C_{11}}{\det A}$$

$$B_2 = \begin{bmatrix} a_{11} & 1 & a_{13}, \dots, a_{1n} \\ a_{21} & 0 & a_{23}, \dots, a_{2n} \\ \vdots & \vdots & \vdots \\ a_{n1} & 0 & a_{n3}, \dots, a_{nn} \end{bmatrix}$$

$$= C_{12}$$

$$\Rightarrow x_2 = \frac{C_{12}}{\det A}$$

Step-3

Containing this process we get

$$B_n = C_{1n} \Rightarrow x_n = \frac{C_{1n}}{\det A} \text{ here } C_{ij} \text{ denotes the cofactor of } i \text{ } j \text{th element of } A.$$

Thus,

$$\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \frac{1}{\det A} \begin{bmatrix} c_{11} \\ c_{12} \\ \vdots \\ c_{1n} \end{bmatrix}$$

is the first column of A^{-1} .