Step-1

Suppose that $T(v)=v$, except that $T(0,v_2)=(0,0)$.

We have to show that the given transformation satisfies $T(cv)=cT(v)$ but not $T(v+w)=T(v)+T(W)$.

Step-2

Let $(v_{1,}v_2)=v$ and $v_1\neq 0$

Let $T(v_1,v_2)=(v_1,v_2),c\neq 0$

Now

$$\begin{aligned}T(cv)&=T(c(v_1,v_2))\\&=T(cv_1,cv_2)\\&=(cv_1,cv_2)\\&=c(v_1,v_2)\\&=cT(v)\end{aligned}$$

Therefore, $T(cv)=cT(v)$

Step-3

Let $c=0$

Then

$$\begin{aligned}T(cv)&=T(0,0)\\&=(0,0)\end{aligned}$$

And

$$\begin{aligned}cT(v)&=0T(v_1,v_2)\\&=0(v_1,v_2)\\&=(0,0)\end{aligned}$$

Therefore, $T(cv)=cT(v)$

Step-4

Let $(2,3),(-2,4)\in\mathbf{R}^2$

Now

$$\begin{aligned}T(2,3)+T(-2,4)&=(2,3)+(-2,4) \quad (\text{Since } T(v)=v)\\&=(0,7)\end{aligned}$$

And

$$\begin{aligned}T((2,3)+(-2,4))&=T(0,7)\\&=(0,0) \quad (\text{Since } T(0,v_2)=(0,0))\end{aligned}$$

Therefore, $T(2,3)+T(-2,4)\neq T((2,3)+(-2,4))$

Hence $\boxed{T(v+w)\neq T(v)+T(w)}$