## Step-1

The objective is to show that if $u$ has unit length, then the rank one matrix $P = uu^T$ is a projection matrix.

## Step-2

Consider $P = uu^T$.

$$P^2 = \left(uu^T\right)\left(uu^T\right)$$
$$= u\left(u^T u\right)u^T$$

$= uu^T$ [ since $u^T u = 1$ ]

So, $P^2 = P$.

$$P^T = \left(uu^T\right)^T$$

$= \left(u^T\right)^T u^T$ [ since $(CD)^T = D^T C^T$ ]

$= uu^T$ [ since $\left(C^T\right)^T = C$ ]

So, $P^T = P$.

Since, $P^2 = P, P^T = P, P = uu^T$ is a projection matrix.