Step-1

Consider that S is the subspace of $\mathbf{R}^n$.

The objective is to explain the meaning of $\left(S^{\perp}\right)^{\perp} = S$ and why it is true.

Step-2

Here, the meaning of $\left(S^{\perp}\right)^{\perp} = S$ is that the orthogonal compliment of any given subspace is uniquely determined.

That is if $S^{\perp}$ is the orthogonal compliment subspace of S and S is the orthogonal compliment subspace of $S^{\perp}$.

Step-3

Recall the following:

If S is the orthogonal compliment subspace of $S^{\perp}$ then, $x \cdot y = 0$ for all $x \in S^{\perp}, y \in S,$.

Step-4

Consider, that, $x \in S^{\perp}, y \in S, z \in \left(S^{\perp}\right)^{\perp}$, none of them is 0. then, observe the following;

As $S \text{ and } S^{\perp}$ are orthogonal compliment of each other.

So, $x \cdot y = 0, \forall x \in S^{\perp}, y \in S, \ \{\text{By the defination of } \textit{orthogonal} \text{ compliment subspace}\}$.

Also, $x \cdot z = 0, x \in S^{\perp}, z \in \left(S^{\perp}\right)^{\perp} \ \{\text{By the defination of } \textit{orthogonal} \text{ compliment subspace}\}$.

Step-5

Note that, orthogonal compliment of any given subspace is uniquely determined.

So, the orthogonal compliment of the element x is also unique.

But here, y and z are both the orthogonal compliment. Which is the contradiction.

So, $y = z$.

As, y and z were chosen arbitrary.

So, it is true for all the elements of $x \in S^{\perp}, y \in S, z \in \left(S^{\perp}\right)^{\perp}$.

Therefore, $\left(S^{\perp}\right)^{\perp} = S$.