Step-1

Consider the system $Ax = b$ given by $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 4 \end{pmatrix}$

$$x^T Ax = \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix}\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$= \begin{pmatrix} 2x_1 - x_2 & -x_1 + 2x_2 - x_3 & -x_2 + 2x_3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$= (2x_1 - x_2)x_1 + (-x_1 + 2x_2 - x_3)x_2 + (-x_2 + 2x_3)x_3$$

$$= 2x_1^2 - 2x_1x_2 - 2x_2x_3 + 2x_2^2 + 2x_3^2$$

$$= 2\left(x_1^2 + x_2^2 + x_3^2 - x_1x_2 - x_2x_3\right)$$

Step-2

Now we need to calculate the determinants of upper left sub matrices.

$A_1 = 2 > 0$,

$$A_2 = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}$$

$$= 3 > 0,$$

$A_3 = |A| = 4 > 0$

Thus A is symmetric and positive definite.

Step-3

So the quadratic is

$$P(x_1 \quad x_2 \quad x_3)$$

$$= \frac{1}{2}x^T Ax - x^T b$$

$$= x_1^2 + x_2^2 + x_3^2 - x_1x_2 - x_2x_3 - 4x_1 - 4x_3$$

$$\frac{\partial P}{\partial x_1} = 2x_1 - x_2 - 4$$

$$\frac{\partial P}{\partial x_2} = -x_1 + 2x_2 - x_3$$

$$\frac{\partial P}{\partial x_3} = -x_2 + 2x_3 - 4$$

Step-4

Solving $\frac{\partial P}{\partial x_1} = 0,\ \frac{\partial P}{\partial x_2} = 0,\ \frac{\partial P}{\partial x_3} = 0,$

We get

$$\Rightarrow 2x_1 - x_2 = 4$$

$$-x_1 + 2x_2 - x_3 = 0$$

$$-x_2 + 2x_3 = 4$$

$$\Rightarrow \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 4 \end{pmatrix}$$

Thus we see that $\frac{\partial P}{\partial x_i}$ vanish exactly at the desired solution.

Therefore, $\boxed{P(x) = x_1^2 + x_2^2 + x_3^2 - x_1x_2 - x_2x_3 - 4x_1 - 4x_3}$ has $\boxed{\frac{\partial P}{\partial x_1} = 2x_1 - x_2 - 4, \frac{\partial P}{\partial x_2} = -x_1 + 2x_2 - x_3 \text{ and } \frac{\partial P}{\partial x_3} = -x_2 + 2x_3 - 4}$.