Step-1

Given that $Ax = \lambda Mx$

$$\Rightarrow \begin{pmatrix} 6 & -3 \\ -3 & 6 \end{pmatrix} x = \frac{\lambda}{18} \begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix} x$$

$$\Rightarrow \begin{pmatrix} 6 - \frac{4\lambda}{18} & -3 - \frac{\lambda}{18} \\ -3 - \frac{\lambda}{18} & 6 - \frac{4\lambda}{18} \end{pmatrix} x = 0$$

For Eigen values,

$$\begin{vmatrix} 6 - \frac{4\lambda}{18} & -3 - \frac{\lambda}{18} \\ -3 - \frac{\lambda}{18} & 6 - \frac{4\lambda}{18} \end{vmatrix} = 0$$

$$\Rightarrow \left(6 - \frac{4\lambda}{18} - 3 - \frac{\lambda}{18}\right) \left(6 - \frac{4\lambda}{18} + 3 + \frac{\lambda}{18}\right) = 0$$

$$\Rightarrow \left(3 - \frac{5\lambda}{18}\right) \left(9 - \frac{3\lambda}{18}\right) = 0$$

$$\Rightarrow \lambda_1 = \frac{54}{5}, \ \lambda_2 = 54$$

Therefore, the Eigen values are $\lambda_1 = \frac{54}{5}$ and $\lambda_2 = 54$.

Step-2

 $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ be the Eigen vector corresponding to $\lambda_1 = \frac{54}{5}$

Now,

$$\Rightarrow \begin{bmatrix} 6 - \frac{4(54)}{18} & -3 - \frac{54}{18} \\ -3 - \frac{54}{18} & 6 - \frac{4(54)}{18} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -6 & -6 \\ -6 & -6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow x_1 + x_2 = 0$$

Step-3

 $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ be the Eigen vector corresponding to $\lambda_1 = \frac{54}{5}$.

$$\Rightarrow \begin{pmatrix} 6 - \frac{4}{18} \cdot \frac{54}{5} & -3 - \frac{1}{18} \cdot \frac{54}{5} \\ -3 - \frac{1}{18} \cdot \frac{54}{5} & 6 - \frac{4}{18} \cdot \frac{54}{5} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \frac{18}{5} & -\frac{18}{5} \\ -\frac{18}{5} & \frac{18}{5} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow x_1 - x_2 = 0$$

Thus $\frac{\binom{x_1}{x_2} = \binom{1}{1}}{\text{is the Eigen vector corresponding to}} \lambda_2 = \frac{54}{5}.$