Step-1

Let us consider the following linear programming problem

Minimize the cost: $5x_1+3x_2+4x_3$

Subject to following constraints

$$x_1 \ge 0$$
$$x_2 \ge 0$$
$$x_3 \ge 0$$
$$x_1+x_2+x_3 \ge 1$$

Step-2

(a) To solve the linear programming problem, let us convert the inequality into equation

$$x_1+x_2+x_3 = 1$$

And use the no-negativity constrains on the variables to get the following possible values.

$$x_1^* = 1$$
$$x_2^* = 0$$
$$x_3^* = 0$$

Or

$$x_1^* = 0$$
$$x_2^* = 1$$
$$x_3^* = 0$$

Or

$$x_1^* = 0$$
$$x_2^* = 0$$
$$x_3^* = 1$$

Step-3

Since, it is asked to find the minimum value of the objective function, $5x_1+3x_2+4x_3$

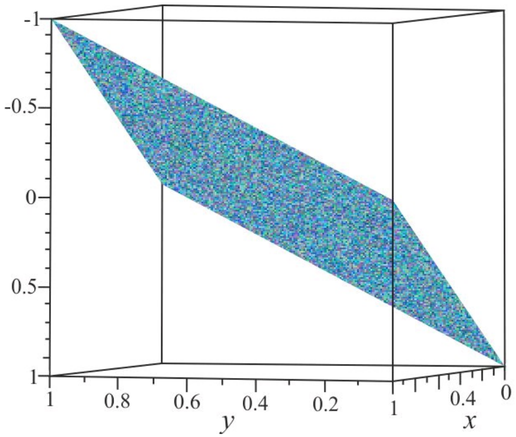
The only possible solution is

$x_1^* = 0$
$x_2^* = 1$
$x_3^* = 0$

And the minimum cost is $\boxed{c_{\min} = 3}$

Step-4

(b) The feasible region for the LPP problem is shown in the following figure.



The feasible region lies in the first quadrant with the tetrahedron in the corner cut off.

Step-5

(c) Let us find the dual of the LPP problem by introducing the dual unknown y_1, y_2 and y_3.

Minimization in the Primal becomes maximization in the dual.

Thus, the dual of the problem is as follows.

Maximize: y_1

Subject to following constraints

$y_1 \geq 0$
$y_2 \geq 0$
$y_3 \geq 0$

$y_1 \le 1$
$y_2 \le 3$
$y_3 \le 4$

Let us solve the dual problem by converting the inequality into equations.

$y_1^* = 1$
$y_2^* = 3$
$y_3^* = 4$

And the maximum value is $\boxed{c_{max} = 1}$

Step-6

Therefore, it is observed that the primal and the corresponding dual have the same solution.