Step-1

Given that the matrix is $C=\begin{pmatrix}3 & 0\\ 1 & 2\end{pmatrix}$,

We need to find the matrix A.

We can determine A by using Cholesky factorization. i.e. $A=CC^T$.

So,

$$\begin{aligned}A&=\begin{pmatrix}3 & 0\\ 1 & 2\end{pmatrix}\begin{pmatrix}3 & 1\\ 0 & 2\end{pmatrix}\\&=\begin{pmatrix}9 & 3\\ 3 & 5\end{pmatrix}\end{aligned}$$

Therefore, the matrix $\boxed{A=\begin{pmatrix}9 & 3\\ 3 & 5\end{pmatrix}}$.

Step-2

Given that $A=\begin{pmatrix}4 & 8\\ 8 & 25\end{pmatrix}$,

We need to find C.

We know that if $A=\begin{pmatrix}a & b\\ b & c\end{pmatrix}$ then a and $\dfrac{ac-b^2}{a}$ are the pivots.

So,

$$\begin{aligned}\begin{pmatrix}a & b\\ b & c\end{pmatrix}&=LDL^T\\&=\begin{pmatrix}1 & 0\\ \frac{b}{a} & 1\end{pmatrix}\begin{pmatrix}a & 0\\ 0 & \frac{ac-b^2}{a}\end{pmatrix}\begin{pmatrix}1 & \frac{b}{a}\\ 0 & 1\end{pmatrix}\\&=\begin{pmatrix}1 & 0\\ 2 & 1\end{pmatrix}\begin{pmatrix}4 & 0\\ 0 & 9\end{pmatrix}\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}\end{aligned}$$

Step-3

Here $a = 4,\ b = 8$ and $c = 25$.

Now $L = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix},\ D = \begin{pmatrix} 4 & 0 \\ 0 & 9 \end{pmatrix}$

Now,

$$C = L\sqrt{D}$$
$$C = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$$
$$= \begin{pmatrix} 2 & 0 \\ 4 & 3 \end{pmatrix}$$

Thus, the matrix $\boxed{C = \begin{pmatrix} 2 & 0 \\ 4 & 3 \end{pmatrix}}$.

Therefore, the matrices are $\boxed{A = \begin{pmatrix} 9 & 3 \\ 3 & 5 \end{pmatrix} \text{ and } C = \begin{pmatrix} 2 & 0 \\ 4 & 3 \end{pmatrix}}$.