南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

考试科目: 高等数学(下) A　　**开课单位:** 数 学 系

考试时长: 150 分钟　　**命题教师:** 王融 等

题 号	1	2	3	4	5	6	7	8	9	10
分 值	6 分	9 分	12 分	8 分	8 分	8 分	8 分	8 分	8 分	8 分
题 号	11									
分 值	9 分									

本试卷共 11 道大题, 满分 100 分. (考试结束后请将试卷、答题本、草稿纸一起交给监考老师)

注意：本试卷里的中文为直译（即完全按英文字面意思直接翻译），所有数学词汇的定义请参照教材（Thomas' Calculus，13th Edition）中的定义。如果其中有些数学词汇的定义不同于中文书籍（比方说同济大学的高等数学教材）里的定义，以教材（Thomas' Calculus，13th Edition）中的定义为准。

1. (6 pts) Determine whether the following statements are **true** or **false**? No justification is necessary.

   (1) Parametric curves $x(t) = \cos t, y(t) = \sin t$ and $x(t) = \sin t, y(t) = \cos t$ have the same graph.

   True

   (2) If $x(t) = f(t)$ and $y(t) = g(t)$ are twice differentiable, then

   $$\frac{d^2y}{dx^2} = \frac{d^2f(t)/dt^2}{d^2g(t)/dt^2}.$$

   F

2. (9 pts) **Multiple Choice Questions:** (only one correct answer for each of the following questions.)

   (1) If $|\mathbf{u}| = 2$, $|\mathbf{v}| = \sqrt{2}$, and $\mathbf{u}\cdot\mathbf{v} = 2$, then $|\mathbf{u}\times\mathbf{v}|$ is　A

   (A) 2.　(B) $2\sqrt{2}$.　(C) $\frac{\sqrt{2}}{2}$.　(D) 1.

   (2) How many points of intersection do the curves $r = 1/2$ and $r = \cos 2\theta$ have?　B

   (A) 2.　(B) 4.　(C) 6.　(D) 8 .

   (3) If $f(x+y, x-y) = x^2 - y^2$, then $\frac{\partial f(x,y)}{\partial x} + \frac{\partial f(x,y)}{\partial y} =$　D

   (A) $2x - 2y$.　(B) $2x + 2y$.　(C) $x - y$.　(D) $x + y$.

   $f(x,y) = xy$

3. (12 pts) Please fill in the blank for the questions below.

$(3,\lambda,-3)$ $\qquad$ $6x \quad 2y \quad 2z$

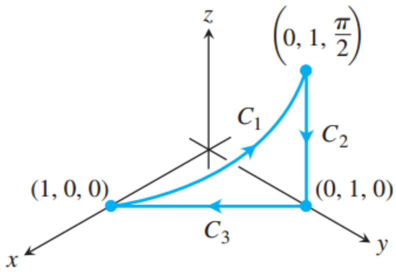
(1) If the plane $3x + \lambda y - 3z + 16 = 0$ is tangent to the surface $3x^2 + y^2 + z^2 = 16$, then $\lambda =$ ___± 2___.

(2) Let $z = \ln\sqrt{x^2+y^2} + \tan^{-1}\frac{x+y}{x-y}$, then $dz =$ ___$f_x dx + f_y dy$___. $= \frac{x-y}{x^2+y^2}dx + \frac{y+x}{x^2+y^2}dy$

(3) The distance from the point $P(1,4,0)$ to the plane through $A(0,0,0)$, $B(2,0,1)$ and $C(2,-1,0)$ is ___3___.

(4) A closed path C consists of three curves:

$$C_1: \quad \mathbf{r}(t) = (\cos t)\,\mathbf{i} + (\sin t)\,\mathbf{j} + t\,\mathbf{k}, \quad 0 \le t \le \pi/2 \qquad \pi - 2$$
$$C_2: \quad \mathbf{r}(t) = \mathbf{i} + (\pi/2)(1-t)\,\mathbf{k}, \quad \le t \le 1$$
$$C_3: \quad \mathbf{r}(t) = t\,\mathbf{i} + (1-t)\,\mathbf{j}, \quad 0 \le t \le 1. \qquad 1$$



Then the circulation of $\mathbf{F} = 2x\,\mathbf{i} + 2z\,\mathbf{j} + 2y\,\mathbf{k}$ around path C traversed in the direction of increasing t is ___$\pi - 1$___.

4. (8 pts) Determine the length of polar curve $r = \sin^3(\frac{\theta}{3})$, $0 \le \theta \le \pi/4$. $\int_0^{\frac{\pi}{4}} \sqrt{r^2 + (\frac{dr}{d\theta})^2}\, d\theta = \frac{\pi - 3}{8}$

5. (8 pts) Given a curve $r(t) = (\cos^3 t, \sin^3 t, 0), 0 < t < \frac{\pi}{2}$ in $\mathbb{R}^3$, find its curvature and principal unit normal. $K = \frac{1}{27 \sin t \cos t}$

$v(t) = -3\cos^2 t \sin t\, \vec{i} + 3\sin^2 t \cos t\, \vec{j}$ $\quad$ $\vec{T} = -\frac{1}{3}\cos t\, \vec{i} + \frac{1}{3}\sin t\, \vec{j}$ $\quad$ $N = \sin t\, \vec{i} + \cos t\, \vec{j}$

$|v| = 9 \sin t \cos t$ $\quad$ $\frac{d\vec{T}}{dt} = \frac{1}{3}\sin t\, \vec{i} + \frac{1}{3}\cos t\, \vec{j}$

6. (8 pts) The sequence $\{a_n\}$ is defined by $a_{2k-1} = \frac{1}{k}$, $a_{2k} = -\frac{1}{k+2}$ (k can be any positive integer). Is the series $\sum_{n=1} \infty a_n$ convergent or divergent ? Prove your conclusion. convergent

7. (8 pts) Find the Maclaurin series for $f(x) = \frac{1}{(1+x)^3}$. $= \frac{1}{2}\sum_{n=2}^{\infty} n(n-1)(-1)^n x^{n-2}$

$4 \quad -81$ $\qquad$ $f_x = 4y^2 - 2xy^2 - y^3$

8. (8 pts) Find the absolute maximum and minimum values of $f(x, y) = 4xy^2 - x^2y^2 - xy^3$ on the close triangular region in the xy-plane with vertices $(0, 0)$, $(0, 6)$ and $(6, 0)$. $(0,0)$ $(1,2)$ $f_y = 8xy - 2x^2y - 3xy^2$

$0, 0, \; y = 6 - x$ $\quad -81 \quad 4$

9. (8 pts) Find the centroid of the solid bounded above by the surface $z = \sqrt{r}$, on the sides by the cylinder $r = 4$, and below by the xy -plane. $M = \int_0^{2\pi}\int_0^4\int_0^{\sqrt{r}} dz\, r\,dr\,d\theta = \frac{128\pi}{5}$

$\bar{z} = \frac{1}{M}\int_0^{2\pi}\int_0^4\int_0^{\sqrt{r}} z\,dz\,r\,dr\,d\theta = \frac{1}{2}$ $\quad$ $(0, 0, \frac{1}{2})$

10. (8 pts) Use the Stokes' Theorem to compute the surface integral $\iint_S \nabla \times \mathbf{F} \cdot \mathbf{n} d\sigma$, here $\mathbf{F} = xz\,\mathbf{i} + yz\,\mathbf{j} + xy\,\mathbf{k}$, and S is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 1$ and above the xy-plane (the boundary is counterclockwise when viewed from above) .

$$\iint_S \nabla \times \vec{F} \cdot n\, d\sigma = \oint_C \vec{F}\, \frac{d\vec{r}}{dt} \cdot dt = 0$$

$$\vec{r}(\theta) = r\cos\theta\, \vec{i} + r\sin\theta\, \vec{j} + \sqrt{3}\, \vec{k}$$

$$\frac{d\vec{r}}{dt} = (-r\sin\theta)\vec{i} + (r\cos\theta)\vec{j}$$

11. (9 pts) Use the Divergence Theorem to find the outward flux of $\mathbf{F}$ across the boundary of the region D, here $\mathbf{F} = xy\,\mathbf{i} + \left(y^2 + e^{xz^2}\right)\mathbf{j} + \sin(xy)\,\mathbf{k}$; and D is the region bounded by the parabolic cylinder $z = 1 - x^2$, and the planes $z = 0$, $y = 0$, and $y + z = 2$.

$$\text{Flux} = \iiint 3y\,dV$$

$$= \int_{-1}^{1}\int_{0}^{1-x^2}\int_{0}^{2-z} 3y\,dy\,dz\,dx = \frac{157}{35}$$

$0 \le z \le 1-x^2$