

Step-1

We have to construct a 3 by 3 matrix whose column space contains $(1,1,0)$ and $(1,0,1)$ but not $(1,1,1)$.

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

If

$$\text{If possible } \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow c_1 + c_2 = 1, c_1 = 1, c_2 = 1$$

Step-2

If $c_1 = 1, c_2 = 1$ then $c_1 + c_2 = 2$

Therefore the also equations has not solution

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Therefore $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is not linear combination of the columns of A

Therefore Column space A does not contain $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Hence the required matrix is

Step-3

We have to construct a 3 by 3 matrix whose column space is only a line.

$$B = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 6 & 0 \\ 4 & 12 & 0 \end{bmatrix}$$

If

$$\text{Let } \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \in \mathbf{C}(B)$$

Then

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ 6 \\ 12 \end{bmatrix} \\ = (c_1 + 3c_2) \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

Step-4

Therefore

$$(b_1, b_2, b_3) = \{(c_1 + 3c_2)(1, 2, 4) / c_1, c_2 \in \mathbf{R}^2\} \\ = \{x(1, 2, 4) / x \in \mathbf{R}\}$$

Therefore the column space of B is only one line that containing the vector $(1, 2, 4)$.

$$B = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 6 & 0 \\ 4 & 12 & 0 \end{bmatrix}$$

Hence the required matrix whose column space is only a line is