Step-1

Writ1e the quadratic form,

$$\begin{aligned}P &= \frac{1}{2}x^T Ax - x^T b \\ &= \frac{1}{2}\left(x - A^{-1}b\right)^T A\left(x - A^{-1}b\right) + \text{costant}\end{aligned}$$

If A is symmetric positive definite matrix, then $P = \frac{1}{2}x^T Ax - x^T b$ reaches its minimum at the point where $Ax = b$

The objective is to complete the square in P.

Step-2

Rewrite the quadratic form as follows:

$$\begin{aligned}P &= \frac{1}{2}\left(x - A^{-1}b\right)^T A\left(x - A^{-1}b\right) + \text{costant} \\ &= \frac{1}{2}\left(x - A^{-1}b\right)^T \left(Ax - AA^{-1}b\right) + \text{costant} \\ &= \frac{1}{2}\left(x^T - A^{-1}b^T\right)\left(Ax - b\right) + \text{costant}\end{aligned}$$

$$\begin{aligned}P &= \frac{1}{2}\left(x^T\right)\left(Ax - b\right) - \frac{1}{2}\left(A^{-1}b^T\right)\left(Ax - b\right) + \text{costant} \\ &= \frac{1}{2}\left(x^T Ax - x^T b\right) - \frac{1}{2}\left(A^{-1}b^T Ax - A^{-1}b^T b\right) + \text{costant} \\ &= \frac{1}{2}x^T Ax - \frac{1}{2}x^T b - \frac{1}{2}b^T x + \frac{1}{2}b^T A^{-1}b + \text{costant}\end{aligned}$$

Step-3

Note that the term,

$$\frac{1}{2}x^T Ax - \frac{1}{2}x^T b - \frac{1}{2}b^T x + \frac{1}{2}b^T A^{-1}b \geq 0$$

The minimum is at $x = A^{-1}b$

$$\begin{aligned}P_{\min} &= \frac{1}{2}\left(x - A^{-1}b\right)^T A\left(x - A^{-1}b\right) + \text{costant} \\ &= \frac{1}{2}\left(A^{-1}b - A^{-1}b\right)^T A\left(A^{-1}b - A^{-1}b\right) + \text{costant} \\ &= \text{costant}\end{aligned}$$