Step-1

Since the matrix A is the identity matrix I, $Av = v$, for every v.

Therefore, if $v = (1,2,3)$, we have $A(1,2,3) = (1,2,3)$.

Therefore, for this example, we see that v is in the row space as well as the column space.

Step-2

Again consider the same vector space $\mathbf{R}^3$.

Let $A = \begin{bmatrix} 2 & -1 & 0 \\ 0 & 3 & -2 \\ 3 & 0 & -1 \end{bmatrix}$.

Thus, we get

$$\begin{bmatrix} 2 & -1 & 0 \\ 0 & 3 & -2 \\ 3 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2-2+0 \\ 0+6-6 \\ 3+0-3 \end{bmatrix}$$
$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Step-3

Since $(1,2,3)$ is mapped to the zero vector, $(1,2,3)$ is in the null space.

Note that the three rows (or the three columns) of $A = \begin{bmatrix} 2 & -1 & 0 \\ 0 & 3 & -2 \\ 3 & 0 & -1 \end{bmatrix}$ are linearly independent.

Therefore, $(1,2,3)$ is in the row space of A.

Step-4

When a matrix acts on a vector, nothing goes to the left null space. Therefore, the vector v cannot be simultaneously in the row space and left null space. Similarly, the vector v cannot be simultaneously in column space and left null space.