Step-1

Let following be the difference equation of matrices:

$$u_{k+1} = Au_k$$

Here, matrix A is defined as follows:

$$A = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

Step-2

Find the Eigen values and Eigen vectors of matrix A:

To calculate the Eigen values do the following calculations;

$$A - \lambda I = \begin{bmatrix} -1-\lambda & 1 \\ 1 & -1-\lambda \end{bmatrix}$$
$$\det(A - \lambda I) = 0$$
$$(-1-\lambda)(-1-\lambda) - 1 = 0$$
$$\lambda^2 + 2\lambda = 0$$

After solving following values are obtained:

$$\lambda_1 = 0$$
$$\lambda_2 = -2$$

Therefore, Eigen values are $\boxed{0, -2}$

Step-3

To calculate Eigen vectors do the following calculations:

$$(A - \lambda_1 I)x = 0$$
$$\begin{bmatrix} -1-0 & 1 \\ 1 & -1-0 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of y and z corresponding to $\lambda = 0$ is as follows:

$$x_1 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Step-4

Similarly, Eigen vectors corresponding to Eigen value $\lambda = -2$ is as follows:

$$(A - \lambda_2 I)x = 0$$
$$\begin{bmatrix} -1+2 & 1 \\ 1 & -1+2 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of y and z corresponding to $\lambda = -2$ is as follows:

$$x_2 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Step-5

Therefore Eigen values are:

$$x_1 = \boxed{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$
$$x_2 = \boxed{\begin{bmatrix} 1 \\ -1 \end{bmatrix}}$$

Step-6

Find the value of the exponential matrix e^{At}. Recall the following:

$$e^{At} = Se^{\Lambda t}S^{-1}$$

Here Eigen value matrix is given as follows:

$$\Lambda = \begin{bmatrix} -2 & 0 \\ 0 & 0 \end{bmatrix}$$

Substitute the values in the above equation and solve.

Matrix e^{At} can be written as follows:

$$
\begin{aligned}
e^{At} &= Se^{\Lambda t}S^{-1} \\
&= \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{0t} \end{bmatrix} \begin{bmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{bmatrix} \\
&= \frac{1}{2} \begin{bmatrix} e^{-2t} & 1 \\ -e^{-2t} & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \\
&= \frac{1}{2} \begin{bmatrix} e^{-2t}+1 & -e^{-2t}+1 \\ -e^{-2t}+1 & e^{-2t}+1 \end{bmatrix}
\end{aligned}
$$

Step-7

Therefore, value of the exponential e^{At} matrix is:

$$
\boxed{e^{At} = \frac{1}{2} \begin{bmatrix} e^{-2t}+1 & -e^{-2t}+1 \\ -e^{-2t}+1 & e^{-2t}+1 \end{bmatrix}}
$$