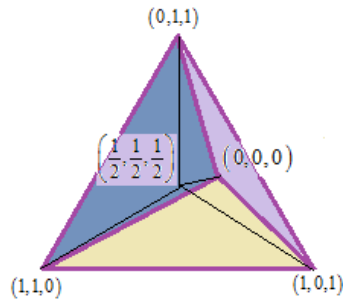


## Step-1



## Step-2

Let  $O = (0, 0, 0)$ ,  $B = (1, 1, 0)$ ,  $C = (1, 0, 1)$ ,  $D = (0, 1, 1)$  are the vertices and  $E = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$  is center of the regular tetrahedron.

We determine the cosine of the angle  $\theta$  between the rays from E to any of above four vertices

Suppose  $u = EA$ ,  $v = EB$

$$\begin{aligned} u &= EA \\ &= OA - OE \\ &= (1, 1, 0) - \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) \\ &= \left(\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right) \end{aligned}$$

## Step-3

$$\begin{aligned} v &= EB \\ &= (1, 0, 1) - \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) \\ &= \left(\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right) \end{aligned}$$

Angle between  $u$  and  $v$  is given by  $\cos \theta = \frac{u^T v}{\|u\| \|v\|}$  (1)

$$\begin{aligned}
 u^T v &= \left( \frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right) \begin{pmatrix} 1/2 \\ -1/2 \\ 1/2 \end{pmatrix} \\
 &= \frac{1}{4} - \frac{1}{4} - \frac{1}{4} \\
 &= -\frac{1}{4}
 \end{aligned}$$

#### Step-4

$$\begin{aligned}
 \|u\| &= \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2} \\
 &= \sqrt{\frac{1}{4} + \frac{1}{4} + \frac{1}{4}} \\
 &= \sqrt{\frac{3}{4}}
 \end{aligned}$$

$$\begin{aligned}
 \|v\| &= \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} \\
 &= \sqrt{\frac{1}{4} + \frac{1}{4} + \frac{1}{4}} \\
 &= \sqrt{\frac{3}{4}}
 \end{aligned}$$

#### Step-5

$$\cos \theta = \frac{-1/4}{\sqrt{\frac{3}{4}}\sqrt{\frac{3}{4}}} = \frac{\left(-\frac{1}{4}\right)}{\frac{3}{4}} = \frac{-1}{3}$$

Using all these results in (1), we get

$$\boxed{\cos^{-1}\left(\pm \frac{1}{3}\right)}$$

Similarly, the angle between any two rays at  $E$  is