

## Step-1

Given quadratic is  $f(x_1, x_2) = 3(x_1 + 2x_2)^2 + 4x_2^2$

Positive definite is,

$$f(x_1, x_2) = 3x_1^2 + 16x_2^2 + 12x_1x_2$$

$$= 3x_1^2 + 6x_1x_2 + 6x_1x_2 + 16x_2^2$$

So the corresponding matrix is  $A = \begin{pmatrix} 3 & 6 \\ 6 & 16 \end{pmatrix}$ .

## Step-2

Now we have to write  $A$  as  $LDL^T$ .

$$f(x_1, x_2) = 3(x_1 + 2x_2)^2 + 4(0.x_1 + 1.x_2)^2$$

Now the coefficients of the squares are the pivots in  $D$  and the coefficients inside the squares are columns of  $L$ .

Therefore,

$$A = \begin{pmatrix} 3 & 6 \\ 6 & 16 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

$$= LDL^T$$

Therefore,  $A = \begin{pmatrix} 3 & 6 \\ 6 & 16 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ ; the coefficients of the squares are the pivots in  $D$  and the coefficients inside the squares are columns of  $L$ .