Step-1

Let A and B be two matrices. Consider that they have the same full set of Eigen vectors, such that:

Step-2

$$A = S\Lambda_1 S^{-1}$$
$$B = S\Lambda_2 S^{-1}$$

Step-3

Prove that $AB = BA$.

Step-4

Lets calculate first AB

$$\begin{aligned} AB &= \left(S\Lambda_1 S^{-1}\right)\left(S\Lambda_2 S^{-1}\right) \\ &= S\Lambda_1 S^{-1} S\Lambda_2 S^{-1} \\ &= S\Lambda_1 I\Lambda_2 S^{-1} \\ &= S\Lambda_1\Lambda_2 S^{-1} \end{aligned}$$

Diagonal matrix always gives the following result:

$$\Lambda_1\Lambda_2 = \Lambda_2\Lambda_1$$

Step-5

So,

$$\begin{aligned} AB &= S\Lambda_1\Lambda_2 S^{-1} \\ &= S\Lambda_2\Lambda_1 S^{-1} \\ &= S\Lambda_2 I\Lambda_1 S^{-1} \\ &= S\Lambda_2 S^{-1} S\Lambda_1 S^{-1} \end{aligned}$$

$$\begin{aligned} &= \left(S\Lambda_2 S^{-1}\right)\left(S\Lambda_1 S^{-1}\right) \\ AB &= BA \end{aligned}$$

Step-6

Therefore, when A and B have the same full set of Eigen vectors then $\boxed{AB = BA}$.