

## Step-1

4764-1.6-17P AID: 124

RID: 175 | 3/12/12

(a) Let  $L_1 D_1 U_1 = L_2 D_2 U_2$

Multiplying left sides with  $L_1^{-1}$  both sides gives

$$L_1^{-1} L_1 D_1 U_1 = L_1^{-1} L_2 D_2 U_2$$

$$I D_1 U_1 = L_1^{-1} L_2 D_2 U_2$$

Since  $L_1^{-1} L_1 = I$ ,  $I D_1 U_1 = D_1 U_1$

$$D_1 U_1 = L_1^{-1} L_2 D_2 U_2$$

Also, by multiplying right side with  $U_2^{-1}$ , we get  $D_1 U_1 = L_1^{-1} L_2 D_2 U_2$

## Step-2

$$D_1 U_1 U_2^{-1} = L_1^{-1} L_2 D_2 U_2 U_2^{-1}$$

$$D_1 U_1 U_2^{-1} = L_1^{-1} L_2 D_2 I$$

Since  $U_2^{-1} U_2 = I$ ,  $I D_2 = D_2$

$$D_1 U_1 U_2^{-1} = L_1^{-1} L_2 D_2$$

We know that the inverse of an upper triangular matrix is upper triangular and the inverse of the lower triangular matrix lower, product of lower triangular matrices is a lower triangular matrix and the product of upper triangular matrices is an upper triangular matrix.

Therefore  $L_1^{-1} L_2 D = D U_1 U_2^{-1}$

## Step-3

(b)  $D_1 U_1 U_2^{-1} = L_1^{-1} L_2 D_2$  is possible if and only if  $L_1^{-1} L_2 = U_1 U_2^{-1} = I$

Consequently,  $D_1 = D_2$

Also,  $L_1^{-1} L_2 = I \Rightarrow L_1 = L_2$  and  $U_1 U_2^{-1} = I \Rightarrow U_1 = U_2$

Therefore, the  $LDU$  factorization is unique for every invertible matrix  $A$ .