

Step-1

The objective is to reduce the following matrices to their ordinary echelon forms U :

The objective is to find a special solution for each free variable and describe every solution to $Ax = 0$ and $Bx = 0$.

To reduce echelon forms U to R and draw a box around the identity matrix in the pivot rows and pivot columns.

Step-2

(a)

Given

$$A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix}$$

$$\underline{R_2 - R_1} \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix}$$

$$\underline{R_3 - R_2} \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\underline{R_1 - 2R_2} \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

This is an ordinary echelon form of A

Step-3

Now to find special solution for $Ax = 0$

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_4 + 2x_2 = 0$$

$$x_3 + 2x_4 + 3x_5 = 0$$

x_1, x_3 are pivot variables remaining are free variables.

$$x_1 = -2x_2$$

$$x_3 = -2x_4 - 3x_5$$

Step-4

Therefore

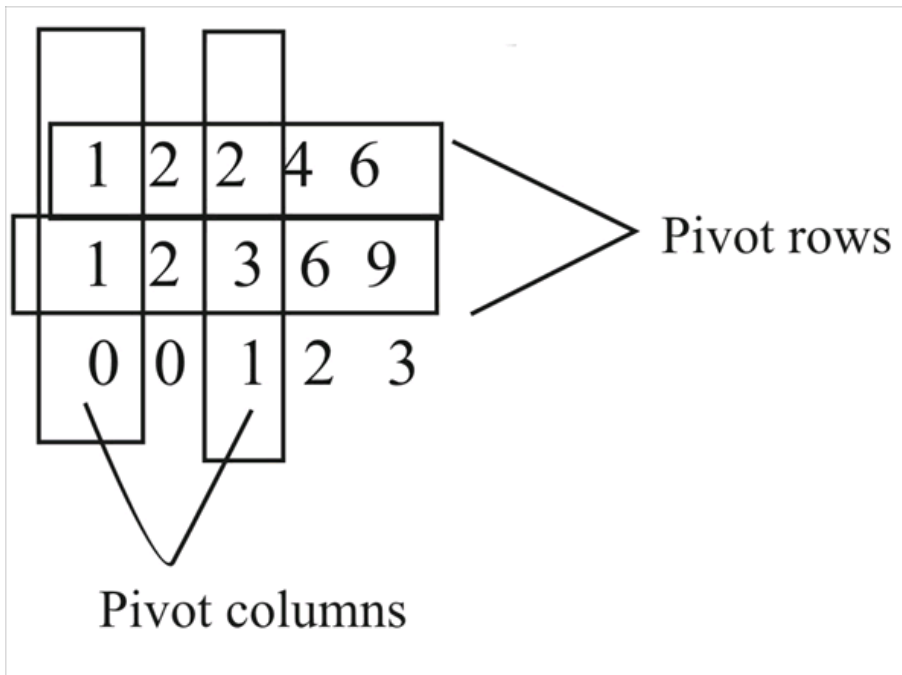
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -2x_2 \\ x_2 \\ -2x_4 - 3x_5 \\ x_4 \\ x_5 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ 0 \\ -3 \\ 0 \\ 1 \end{bmatrix}$$

Are the special solutions of $Ax=0$ and the identity matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Step-5

The pivot rows, pivot columns:



Step-6

b)

Given

$$B = \begin{bmatrix} 2 & 4 & 2 \\ 0 & 4 & 4 \\ 0 & 8 & 8 \end{bmatrix}$$

$$\underline{R_3 - 2R_1} \begin{bmatrix} 2 & 4 & 2 \\ 0 & 4 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\underline{\frac{1}{2}R_1} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 4 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\underline{\frac{1}{4}R_2} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\underline{R_1 - 2R_2} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

This is row reduced echelon form to find special solution of $Bx = 0$

Step-7

Therefore

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

x_1, x_2 are pivots, x_3 is free variables

$$x_1 - x_3 = 0,$$

$$x_2 - x_3 = 0$$

$$x_1 = x_3,$$

$$x_2 = x_3$$

Step-8

Special solutions are $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Therefore, the required identity matrix is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Step-9

The pivot rows, pivot columns:

