Step-1

We have to find the ranks of *AB* and *AM:*

$A=\begin{bmatrix}1 & 2\\ 2 & 4\end{bmatrix}$ and $B=\begin{bmatrix}2 & 1 & 4\\ 3 & 1.5 & 6\end{bmatrix}$, and $M=\begin{bmatrix}1 & b\\ c & bc\end{bmatrix}$

Step-2

Given

$$AB=\begin{bmatrix}8 & 4 & 16\\ 16 & 8 & 32\end{bmatrix}$$
$$\xrightarrow{R_2-2R_1}\begin{bmatrix}8 & 4 & 16\\ 0 & 0 & 0\end{bmatrix}$$
$$=R$$

R is row reduced echelon form of *AB*. Therefore rank of *AB* = number of non- zero rows in *R*.

Therefore rank of *AB* $=\boxed{1}$

Step-3

Now

$$AM=\begin{bmatrix}1+2c & b+2bc\\ 2+4c & 2b+4bc\end{bmatrix}$$
$$\xrightarrow{R_2-2R_1}\begin{bmatrix}1+2c & b+2bc\\ 0 & 0\end{bmatrix}$$

If $1+2c=0$ then

$$b+2bc=b(1+2c)$$
$$=b.0$$
$$=0$$

Step-4

Therefore $AM=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

So if $1+2c=0$ then the rank of $AM=0$

That is if $c = -\frac{1}{2}$ then the rank of $AM = 0$

Step-5

Let $C \neq \frac{-1}{2}$

Since the row reduced echelon form is $\left(\frac{1}{1+2c}R_1\right)$

$AM \square \begin{bmatrix} 1 & b \\ 0 & 0 \end{bmatrix}$, the rank of $AM = 1$