

## Step-1

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Let the matrix  $A$  be such that row 1 + row 2 = row 3.

That is

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ a+d & b+e & c+f \end{bmatrix}$$

We have to show that  $A$  has no inverse.

## Step-2

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ a+d & b+e & c+f \end{bmatrix}$$

We have

Subtracting row 1 from row 3 gives

$$A \approx \begin{bmatrix} a & b & c \\ d & e & f \\ d & e & f \end{bmatrix}$$

Subtracting row 2 from row 3 gives

$$A \approx \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 0 \end{bmatrix}$$

The last row of  $A$  is zero.

Therefore, its determinant becomes zero.

So that the matrix  $A$  does not possess inverse.

## Step-3

(a) The given system is  $Ax = (1, 0, 0)$ .

We have to show that the given system has no solution.

Now the system becomes

$$\begin{bmatrix} a & b & c \\ d & e & f \\ a+d & b+e & c+f \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Now performing (row 1 + row 2)  $\leftarrow$  row3 gives

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

The last row shows that  $0 = -1$  condition.

So the system  $Ax = (1, 0, 0)$  has no solution.

## Step-4

(b) We have to explain which right hand sides  $(b_1, b_2, b_3)$  might allow a solution to  $Ax = b$ .

The right hand sided must satisfy  $b_1 + b_2 = b_3$

Then the system  $Ax = b$  has a solution.

## Step-5

(c) We have to tell what happens to row 3 of  $A$  in elimination.

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ a+d & b+e & c+f \end{bmatrix}$$

We have

Subtracting row 1 from row 3 gives

$$A \approx \begin{bmatrix} a & b & c \\ d & e & f \\ d & e & f \end{bmatrix}$$

Subtracting row 2 from row 3 gives

$$A \approx \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 0 \end{bmatrix}$$

Row 3 becomes a row of zeros and that has no third pivot.

Hence the row 3 becomes a zero row in elimination.