Step-1

Given that $A=\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$

Also given that $\det(A-\lambda I)=(\lambda-a)(\lambda-b)$

We have to check that given matrix satisfies the Cayley-Hamilton theorem.

Step-2

By Cayley-Hamilton theorem, every square matrix satisfies its characteristic polynomial.

Since $\det(A-\lambda I)=(\lambda-a)(\lambda-b)$

So the characteristic polynomial of A is $(\lambda-a)(\lambda-b)$.

Therefore, by Cayley-Hamilton theorem, we have to verify that $(A-aI)(A-dI)=0$

Step-3

Now

$$\begin{aligned} A-aI &= \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} - a\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} - \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \\ &= \begin{bmatrix} a-a & b-0 \\ 0-0 & d-a \end{bmatrix} \\ &= \begin{bmatrix} 0 & b \\ 0 & d-a \end{bmatrix} \end{aligned}$$

Step-4

And

$$
\begin{aligned}
A-dI &= \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} - d\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} - \begin{bmatrix} d & 0 \\ 0 & d \end{bmatrix} \\
&= \begin{bmatrix} a-d & b-0 \\ 0-0 & d-d \end{bmatrix} \\
&= \begin{bmatrix} a-d & b \\ 0 & 0 \end{bmatrix}
\end{aligned}
$$

Step-5

Therefore,

$$
\begin{aligned}
(A-aI)(A-dI) &= \begin{bmatrix} 0 & b \\ 0 & d-a \end{bmatrix}\begin{bmatrix} a-d & b \\ 0 & 0 \end{bmatrix} \\
&= \begin{bmatrix} 0(a-d)+b(0) & 0(b)+b(0) \\ 0(a-d)+(d-a)(0) & 0(b)+(d-a)(0) \end{bmatrix} \\
&= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\
&= 0
\end{aligned}
$$

Therefore, the given matrix A satisfies its characteristic polynomial $(A-aI)(A-dI)$.

Hence the given matrix satisfies Cayleyâ€“Hamilton theorem.