

## Step-1

As referred, if  $P$  is a plane in  $\mathbf{R}^3$ , then it is spanned by 2 linearly independent vectors.

So, its orthogonal complement must be a vector space with dimension  $3 - 2 = 1$

That is the orthogonal complement of  $P$  must be a line  $L$  but not another plane.

## Step-2

In view of this discussion, if  $\mathbf{V}$  is a subspace of  $\mathbf{R}^n$  with dimension  $p$ , and  $\mathbf{W}$  is a subspace of  $\mathbf{R}^n$  with dimension  $q$ , then it follows that there is necessarily a non zero vector in the intersection of  $\mathbf{V}$  and  $\mathbf{W}$  if  $p + q > n$

## Step-3

We know that the zero vector is a vector whose length is 0 and assumes the direction to which it is added or subtracted.

We know that  $\mathbf{V}$  and  $\mathbf{W}$  are the subspaces orthogonal if and only if their intersection is the zero vector alone.

But when  $p + q > n$ , we have a non zero vector in the intersection of  $\mathbf{V}$  and  $\mathbf{W}$ .

Therefore,  $\mathbf{V}$  and  $\mathbf{W}$  cannot be orthogonal.

## Step-4

We also follow that  $p + q < n$  also cannot guarantee that  $\mathbf{V}$  and  $\mathbf{W}$  are orthogonal.