## Step-1

Let A, B, and C be matrices with respect to the vector subspaces V, W, and  $V \cap W$  respectively.

Therefore,

$$\operatorname{rank}(A+B) + \operatorname{rank} C = \operatorname{rank} A + \operatorname{rank} B$$

## Step-2

The rank of a matrix cannot be negative.

Therefore, rank  $C \ge 0$ .

Thus, 
$$\operatorname{rank}(A+B) \leq \operatorname{rank} A + \operatorname{rank} B$$

Hence proved