

Step-1

Suppose $\mathbf{v} = (\mathbf{x}, \mathbf{y})$, then we know that the reflection of the point (\mathbf{x}, \mathbf{y}) across the x -axis is given by the following matrix multiplication:

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix} = \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix}$$

So, we get

$$\mathbf{T} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Step-2

or $\mathbf{T}(\mathbf{v}) = (\mathbf{x}, -\mathbf{y})$

Step-3

Let $\mathbf{v} = (\mathbf{x}, \mathbf{y})$, then we know that the reflection of the point (\mathbf{x}, \mathbf{y}) across the y -axis is given by the following matrix multiplication:

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix} = \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix}$$

So, we get

$$\mathbf{S} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

or

$$\mathbf{S}(\mathbf{v}) = (-\mathbf{x}, \mathbf{y})$$

Step-4

To find $\mathbf{S}(\mathbf{T}(\mathbf{v}))$, we have

$$\begin{aligned}
S(T(v)) &= ST(v) \\
&= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \\
&= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \\
&= \begin{bmatrix} -x \\ -y \end{bmatrix}
\end{aligned}$$

Step-5

Therefore,

Step-6

$$S(T(v)) = \boxed{(-x, -y)}$$

Step-7

We know that the reflection of the point (x, y) about the origin is given by the following matrix multiplication:

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Therefore, $\boxed{\text{the product } ST \text{ is the reflection of the point } (x, y) \text{ about the origin}}$.