Step-1

Properties of the determinant

1. The determinant of the identity matrix is 1

2. The determinant changes sign when two rows are exchanged

3. The determinant depends linearly on the first row.

4. If two rows of A are equal, then $\det A = 0$

5. Subtracting a multiple of one row from another row leaves the same determinant.

6. If A has a row of zeros, then $\det A = 0$

7. If A is triangular, then $\det A$ is the product $a_{11}a_{22}a_{33}\ldots a_{nn}$ of the diagonal entries.

8. If A is singular, then $\det A = 0$. If A is invertible, then $\det A \neq 0$.

9. The determinant of AB is product of $\det A$ times $\det B$

10. The transpose of A has the same determinant as A itself; $\det A^T = \det A$

Step-2

We have

$$A = \begin{bmatrix} 3 & 3 & 4 \\ 6 & 8 & 7 \\ -3 & 5 & -9 \end{bmatrix} = LU = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 4 & 1 \end{bmatrix}\begin{bmatrix} 3 & 3 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

Here

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 4 & 1 \end{bmatrix}$$ And $$U = \begin{bmatrix} 3 & 3 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

Step-3

Both L and U are triangular matrices

$$
\begin{aligned}
\det L &= (1)(1)(1) \\
&= 1 \\
\det U &= (3)(2)(-1) \\
&= -6
\end{aligned}
$$

Step-4

Therefore

$$\det A = \det L \times \det U$$

$$
\begin{aligned}
&= (1)(-6) \\
&= -6
\end{aligned}
$$

Step-5

And

$$
\begin{aligned}
A^{-1} &= (LU)^{-1} \\
&= U^{-1}L^{-1}
\end{aligned}
$$

$$
\begin{aligned}
\det U^{-1}L^{-1} &= \det A^{-1} \\
&= \frac{1}{\det A}
\end{aligned}
$$

$$= \frac{-1}{6}$$

Step-6

Further

$$U^{-1}L^{-1}A = A^{-1}A = I$$

$$\det\left(U^{-1}L^{-1}A\right) = \det I = 1$$

Thus

$$
\boxed{
\begin{aligned}
\det L &= 1 \\
\det U &= -6 \\
\det A &= -6 \\
\det U^{-1}L^{-1} &= \frac{-1}{6} \\
\det\left(U^{-1}L^{-1}A\right) &= 1
\end{aligned}
}
$$