Step-1

We have the matrix equation $\begin{bmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & -1 & 2 & -1 & \\ & & -1 & 2 & -1 \\ & & & -1 & 2 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix} = h^2\begin{bmatrix} f(h) \\ f(2h) \\ f(3h) \\ f(4h) \\ f(5h) \end{bmatrix}$

This can be interpreted as

$$\begin{aligned} 2u_1 - u_2 &= h^2 f(h) \\ -u_1 + 2u_2 - u_3 &= h^2 f(2h) \\ -u_2 + 2u_3 - u_4 &= h^2 f(3h) \\ -u_3 + 2u_4 - u_5 &= h^2 f(4h) \\ -u_4 + 2u_5 - u_6 &= h^2 f(5h) \end{aligned}$$

Step-2

The second difference equation can be obtained from $-u_{j+1} + 2u_j - u_{j-1} = h^2 f(jh)$

$$\frac{d^2u}{dx^2} \approx \frac{u(x+h) - 2u(x) + u(x-h)}{h^2}$$

The initial conditions are $u_0 = u_1, u_5 = u_6$

The differential equation is

Substituting $j = 1$ gives $-u_2 + 2u_1 - u_0 = h^2 f(h)$

Replacing $u_0 = u_1$, gives $-u_2 + 2u_1 - u_1 = h^2 f(h) \Rightarrow u_1 - u_2 = h^2 f(h)$

Step-3

Substituting $j = 2$ gives $-u_3 + 2u_2 - u_1 = h^2 f(2h) \Rightarrow -u_1 + 2u_2 - u_3 = h^2 f(2h)$

Step-4

Substituting $j = 3$ gives $-u_4 + 2u_3 - u_2 = h^2 f(2h) \Rightarrow -u_2 + 2u_3 - u_4 = h^2 f(3h)$

Step-5

Substituting $j = 4$ gives $-u_5 + 2u_4 - u_3 = h^2 f(3h) \Rightarrow -u_3 + 2u_4 - u_5 = h^2 f(4h)$

Step-6

$$-u_6 + 2u_5 - u_4 = h^2 f(3h) \Rightarrow -u_4 + 2u_5 - u_6 = h^2 f(4h)$$

$$\textit{substituting } u_5 = u_6 \textit{ gives}$$

Substituting $j = 5$ gives $-u_4 + 2u_5 - u_5 = h^2 f(5h) \Rightarrow -u_4 + u_5 = h^2 f(5h)$

Step-7

The matrix A_0 becomes $A_0 = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix}$

Step-8

Each row adds to 1 of the equation of matrix 6 so

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} c \\ c \\ c \\ c \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Since by matrix multiplication