Step-1

Given that $b = 4, 2, -1, 0, 0$ and $t = -2, -1, 0, 1, 2$

We have to find the best line $C + Dt$ to fit $b = 4, 2, -1, 0, 0$ at times $t = -2, -1, 0, 1, 2$.

Step-2

First we write the equations that would hold if a line could go through four points.

Then every $C + Dt$ would agree exactly with b

So the equations are

$$C + D(-2) = 4$$
$$C + D(-1) = 2$$
$$C + D(0) = -1$$
$$C + D(1) = 0$$
$$C + D(2) = 0$$

Step-3

The matrix form of the above system is

$$\begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

Let $A = \begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}, x = \begin{bmatrix} C \\ D \end{bmatrix}$ and $b = \begin{bmatrix} 4 \\ 2 \\ -1 \\ 0 \\ 0 \end{bmatrix}$

Step-4

We know that the least-square solution to the solution is

$A^T A\hat{x} = A^T b$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -2 & -1 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} \hat{C} \\ \hat{D} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -2 & -1 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5 & 0 \\ 0 & 10 \end{bmatrix} \begin{bmatrix} \hat{C} \\ \hat{D} \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$\Rightarrow 5\hat{C} = 5$ and $10\hat{D} = -10$

$\Rightarrow \hat{C} = 1$ and $\hat{D} = -1$

Therefore $\hat{x} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Hence best line is $\boxed{b = 1 - t}$