Step-1

To fit $y = C + D\,t$ leads to four equations in two unknowns

These are $C - 2D = -4$

$C - 1D = -3$

$C + 1D = -1$

$C + 2D = 0$

Or $\begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} -4 \\ -3 \\ -1 \\ 0 \end{bmatrix}$

This is equivalent to the system $Ax = b$ where $A = \begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix},\ x = \begin{bmatrix} C \\ D \end{bmatrix},\ b = \begin{bmatrix} -4 \\ -3 \\ -1 \\ 0 \end{bmatrix}$

Step-2

(a) The columns of A are $a_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix},\ a_2 = \begin{bmatrix} -2 \\ -1 \\ 1 \\ 2 \end{bmatrix}$

$$a_1^T a_2 = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} -2 \\ -1 \\ 1 \\ 2 \end{bmatrix}$$

$$= -2 - 1 + 1 + 2 = 0$$

The columns of coefficient matrix are orthogonal.

Step-3

(b) $\hat{C} = \dfrac{\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} -4 & -3 & -1 & 0 \end{bmatrix}^T}{\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^T\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}}$

$$= \frac{\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -4 \\ -3 \\ -1 \\ 0 \end{bmatrix}}{1+1+1+1}$$
$$= \frac{-4-3-1+0}{4}$$
$$= -2$$

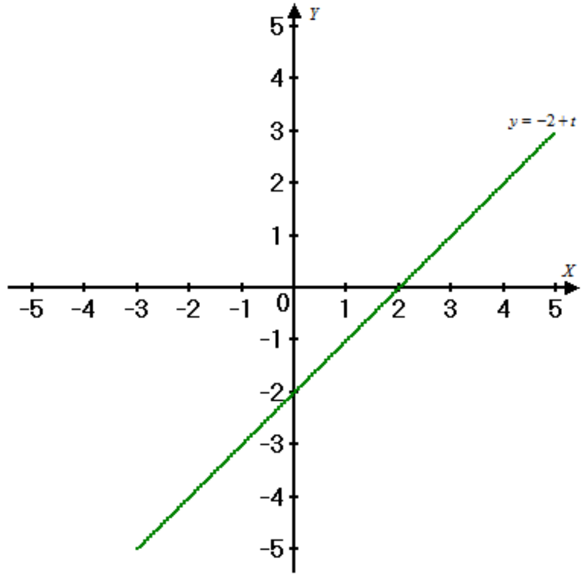
$$\hat{D} = \frac{\begin{bmatrix} -2 & -1 & 1 & -2 \end{bmatrix} \begin{bmatrix} -4 & -3 & -1 & 0 \end{bmatrix}^T}{\begin{bmatrix} -2 & -1 & 1 & -2 \end{bmatrix}^T \begin{bmatrix} -2 & -1 & 1 & -2 \end{bmatrix}}$$

$$= \frac{\begin{bmatrix} -2 & -1 & 1 & -2 \end{bmatrix} \begin{bmatrix} -4 \\ -3 \\ -1 \\ 0 \end{bmatrix}}{4+1+1+4}$$
$$= \frac{8+3-1-0}{10}$$
$$= 1$$

Using $\hat{C} = -2,\ \hat{D} = 1$, the optimal straight line is $\boxed{y = -2 + t}$

Step-4

The graph of the straight line is



$$E^2=(y_1-C-Dt_1)^2+(y_2-C-Dt_2)^2+(y_3-C-Dt_3)^2+(y_4-C-Dt_4)^2$$
$$=(-4+2+2)^2+(-3+2+1)^2+(-1+2-1)^2+(0+2-2)^2=0$$

Therefore, $\boxed{E^2=0}$

Step-5

Interpret the zero error in terms of the original system of equations in two unknowns:

The right hand side $(-4,-3,-1,0)$ is in the column space.