

Step-1

Given the 3 by 3 matrix are chosen randomly between 0 and 1.

We need to find the dimensions of the four subspaces.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Let the matrix

We know that the column space of A is represented by $C(A)$ and its dimension is r ,

The row space of A is represented by $C(A^T)$ and its dimension is r , the null space of A is represented by $N(A)$ and its dimension is $n-r$ the left null space of A is represented by $N(A^T)$ and its dimension is $m-r$

Step-2

The rank of the matrix is three. So, the dimensions of four subspaces are,

Dimension of column space of A is less than or equal to 3. i.e. $\dim C(A) \leq 3$

Dimension of row space of A is less than or equal to 3. i.e. $\dim C(A^T) \leq 3$

Dimension of null space of A is less than or equal to 3. i.e. $\dim N(A) \leq 3$

Dimension of left null space of A is less than or equal to 3. i.e. $\dim N(A^T) \leq 3$

Therefore, dimensions are

$$\dim C(A) \leq 3, \dim C(A^T) \leq 3, \dim N(A) \leq 3 \text{ and } \dim N(A^T) \leq 3$$

Step-3

We need to find the dimensions of the four subspaces for 3 by 5 matrix.

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Let the matrix be

The rank of the matrix is three. So, the dimensions of four subspaces are,

Dimension of column space of A is less than or equal to 3. i.e. $\dim C(A) \leq 3$

Dimension of row space of A is less than or equal to 3. i.e. $\dim C(A^T) \leq 3$

Dimension of null space of A is less than or equal to 4 i.e. $\dim N(A) \leq 4$

Dimension of left null space of A is less than or equal to 3 i.e. $\dim N(A^T) \leq 3$

$$\boxed{\dim C(A) \leq 3, \dim C(A^T) \leq 3, \dim N(A) \leq 4 \text{ and } \dim N(A^T) \leq 3}$$