Step-1

Consider matrices of (2×2) with 1â€™s and -1â€™s. Determine how many are invertible.

Recall that a matrix is invertible if its determinant is non-zero.

Step-2

Determinants of all the matrices (2×2) containing 1â€™s and 0â€™s are as follows:

Consider the matrices with only one 1â€™s.

$$\begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$$

Determinant of all these matrices are nonzero. So, each of them are invertible.

Step-3

Consider the matrices with two 1â€™s.

$$\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}$$

Determinant of all these matrices are zero. So, none of them are invertible.

Step-4

Consider the matrices with three 1â€™s.

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

Determinant of all these matrices are nonzero. So, each of them are invertible.

Step-5

Consider the matrix with 1â€™s positioned on the diagonal and anti-diagonal.

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

Determinant of all these matrices are zero. So, none of them are invertible.

Step-6

Consider the matrix with all 1â€™s and -1â€™s.

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Determinant of all these matrices are zero. So, none of them are invertible.

Step-7

Therefore, from sixteen (2×2) matrices with -1â€™s and 1â€™s only 8 are invertible.