Homework 6 (Due: 4/25)

Show that the Fourier transforms of

(a) $f(ax)$ is $\frac{1}{a}F(\frac{u}{a})$, where a is any nonzero real number.

(b) $f(x-x_0)$ is $F(u)\exp(-j2\pi ux_0)$.

Proof:

(a) $\int_{-\infty}^{\infty} f(ax)\exp(-j2\pi ux)dx$

(Let $y = ax.\ dy = adx$)

$$= \int_{-\infty}^{\infty} f(y)\exp(-j2\pi uy/a)\frac{1}{a}dy$$

$$= \frac{1}{a}\int_{-\infty}^{\infty} f(y)\exp(-j2\pi \frac{u}{a} y)dy$$

$$= \frac{1}{a}F(\frac{u}{a})$$

(b) $\int_{-\infty}^{\infty} f(x-x_0)\exp(-j2\pi ux)dx$

$$= \exp(-j2\pi ux_0)\int_{-\infty}^{\infty} f(x-x_0)\cdot$$

$$\exp(-j2\pi u(x-x_0))dx$$

(Let $y = x - x_0$, $dy = dx$)

$$= \frac{1}{2\pi}\exp(-j2\pi ux_0)\int_{-\infty}^{\infty} f(y)\exp(-j2\pi uy)dy$$

$$= \exp(-j2\pi ux_0)F(u)$$