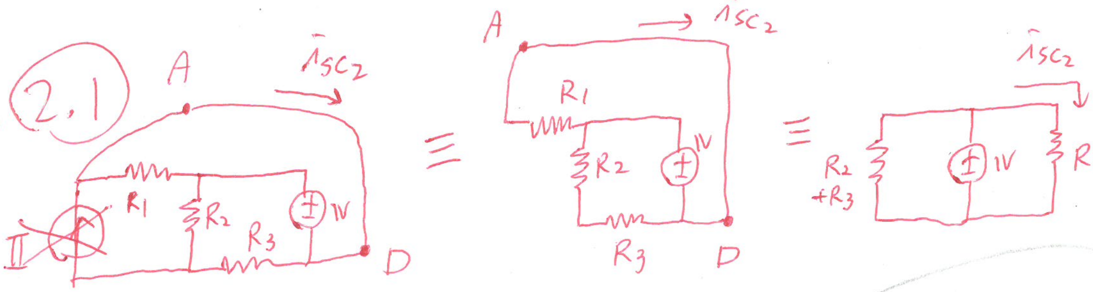


$i_{sc_2} = \frac{1V}{R_1}$

Solution

CSU0049 Analog and Digital Computing Elements, Homework 2

Department of Computer Science and Information Engineering

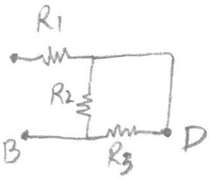
National Taiwan Normal University

October 2, 2024

- 100 points in total. State clearly how you derived your answer.
- For equivalent circuits, label the terminals so that it is clear which maps to which. For example, when constructing the equivalent circuit viewed from terminals A and B, remember to always label A and B to the terminals in your final construction.
- You may use operator $\|$ to represent the equivalent resistance of parallel connection. The $\|$ operator has a higher precedence than operators $+$ and $-$. For example, $(R_A \| R_B + R_C)$ is the same as $((R_A \| R_B) + R_C)$ but is different from $(R_A \| (R_B + R_C))$. When in doubt, use parenthesis.

Problem 1 (20 points)

Often, we learn more when we teach. Try to use plain language to explain Thévenin's Theorem and its usefulness to your friend or family member who does not major in engineering. Write down a question they asked, and also write down how you responded to it. For this problem only, you may use Chinese.

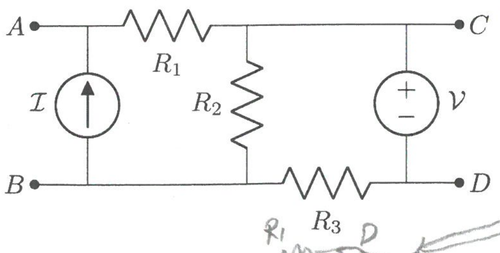


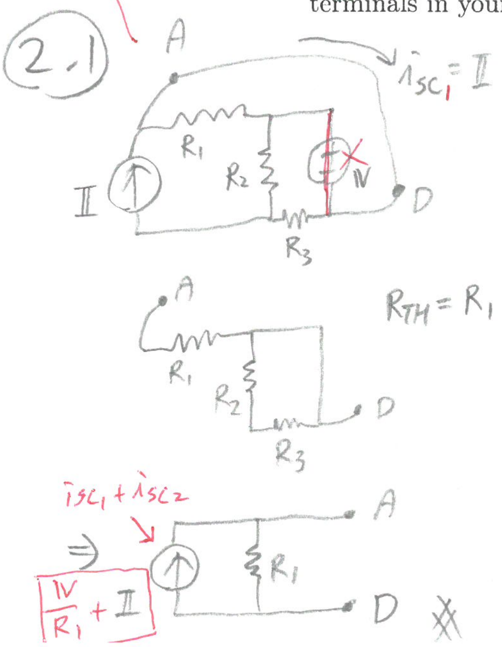
$\Rightarrow R_{TH} = R_2 // R_3$

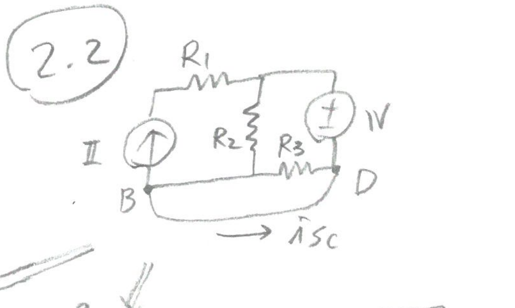
superposition

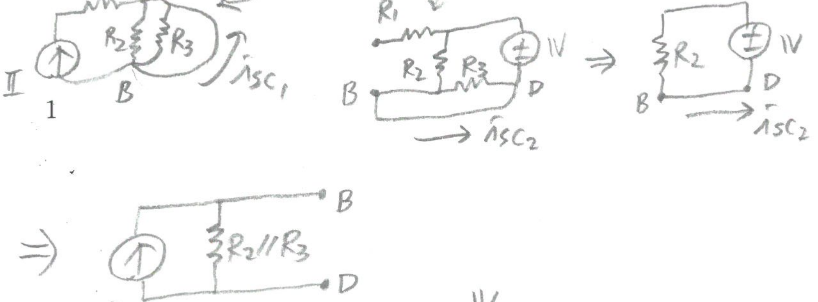
Problem 2 (20 points)

For the following circuit, obtain the Norton equivalent circuit viewed from (1) nodes A and D, and (2) nodes B and D. Ten points each. Label the terminals in your circuits.



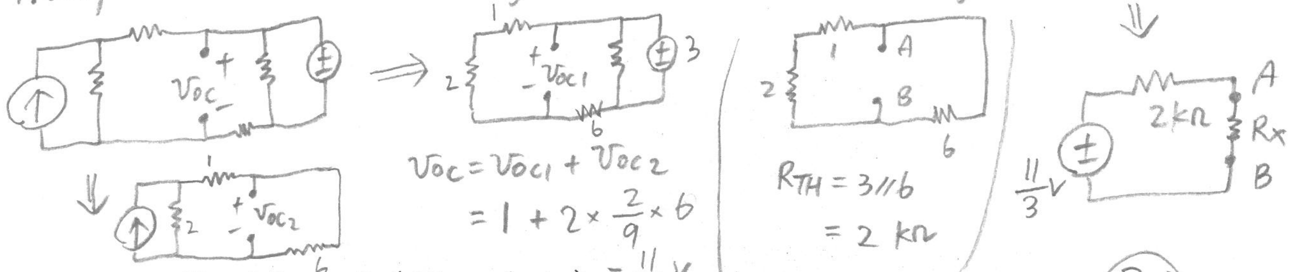






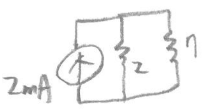
$i_{sc} = i_{sc_1} + i_{sc_2} = -I + \frac{1V}{R_2}$

③ Transform the circuit excluding R_x into its Thévenin equivalence:



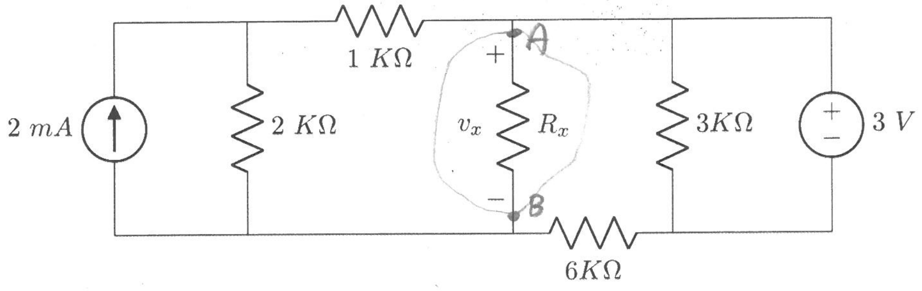
$V_{oc} = V_{oc1} + V_{oc2}$
$= 1 + 2 \times \frac{2}{9} \times 6$
$= \frac{11}{3}$ V

$R_{TH} = 3 // 6$
$= 2$ kΩ



Problem 3 (45 points)

For the following circuit, determine the branch voltage v_x when (1) $R_x = 2\ K\Omega$; (2) $R_x = 4\ K\Omega$; (3) $R_x = 6\ K\Omega$. 15 points each.



(3.1) $V_x = \frac{11}{3} \times \frac{2}{2+2} = \frac{11}{6}$ V #

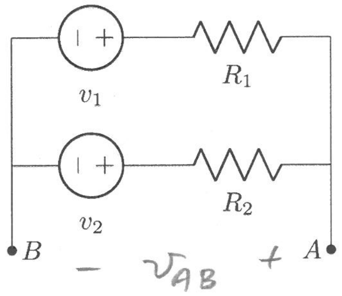
(3.2) $V_x = \frac{11}{3} \times \frac{4}{2+4} = \frac{22}{9}$ V #

(3.3) $V_x = \frac{11}{3} \times \frac{6}{2+6} = \frac{11}{4}$ V #

You might want to try this: first transform the circuit without R_x into a simple equivalence, and then attach R_x back to the corresponding terminals. This way, you may reuse the equivalent circuit to speed up the subsequent analysis that will only change R_x.

Problem 4 (15 points)

In class, we have learned that the following circuit may compute a weighted-sum function $f(v_1, v_2) = w_1 v_1 + w_2 v_2$ for some w_1 and w_2:



④ In class we have derived that

$V_{AB} = \frac{R_2}{R_1+R_2} V_1 + \frac{R_1}{R_1+R_2} V_2$

let $w_1 = \frac{R_2}{R_1+R_2}$, $w_2 = \frac{R_1}{R_1+R_2}$

$\Rightarrow V_{AB} = w_1 V_1 + w_2 V_2$

BUT this circuit will have $w_1 + w_2 = \frac{R_2 + R_1}{R_1 + R_2} = 1$.

Thus, $w_1 + w_2 = 1$ is an invariant.

The statement includes conditions where $w_1 + w_2$ might not equal to one, which cannot be implemented by the given circuit. Therefore, the statement is false. #

Now, validate the following statement. Either prove it or disprove it:

> Given any function $f(v_1, v_2) = w_1 v_1 + w_2 v_2$ with $0 < w_1 \le 1$ and $0 < w_2 \le 1$ and both w_1 and w_2 being rational numbers, the above circuit can always compute this function by some choices of $R_1 \ge 0$ and $R_2 \ge 0$.

This exercise gives us an example of telling the *expressive power*[1] of a hardware module, to study what a module can do and what it can not.

Reading Task

Study Section 4.4 of our first textbook (Agarwal and Lang) in detail, up to and including Example 4.12. No need to return anything for this reading task.

[1]https://en.wikipedia.org/wiki/Expressive_power_(computer_science)