Solution

CSU0049 Analog and Digital Computing Elements, Homework 1

Department of Computer Science and Information Engineering

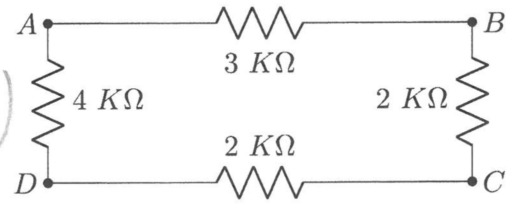
National Taiwan Normal University

September 17, 2024

There are five problem sets, 100 points in total. State clearly how you derived your answer.

Problem 1 (20 points)

For the following circuit, determine the resistance viewed from (1) nodes A and B; (2) nodes A and C. 10 points for each answer.



1.1

It is $3k\Omega // (4k\Omega + 2k\Omega + 2k\Omega) = \frac{3 \cdot 8}{3+8} = \frac{24}{11} k\Omega$ #

1.2

$(4k\Omega + 2k\Omega) // (3k\Omega + 2k\Omega) = \frac{5 \cdot 6}{5+6} = \frac{30}{11} k\Omega$ #

This exercise tells us that for the same circuit, viewing from different pair of nodes we may get different resistance.

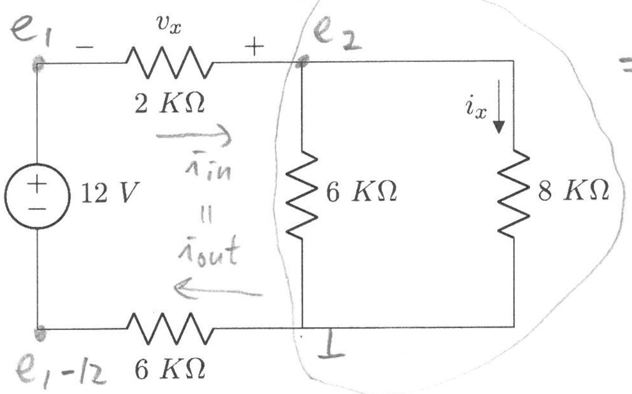
Problem 2 (20 points)

For the following circuit, find v_x and i_x. Watch out for the direction of the given variables. You may use the following strategies in any combination:

1. The basic analysis method.
2. Equivalent resistence.
3. The node analysis method.
4. Voltage divider and current divider patterns.

② I use the node analysis method:

$$\begin{cases} \dfrac{e_1 - e_2}{2k\Omega} = \dfrac{e_2 - 0}{6k\Omega} + \dfrac{e_2 - 0}{8k\Omega} \quad (\text{KCL at } e_2) \\ \dfrac{e_1 - e_2}{2k\Omega} = \dfrac{0 - (e_1 - 12)}{6k\Omega} \end{cases} \Rightarrow \begin{cases} 12e_1 - 12e_2 = 7e_2 \\ 3e_1 - 3e_2 = 12 - e_1 \end{cases}$$



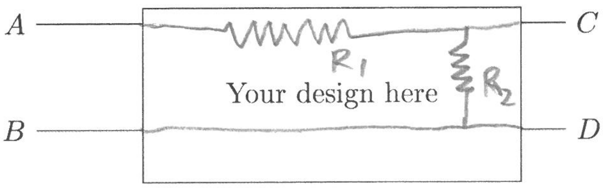
$$\Rightarrow \begin{cases} 12e_1 = 19e_2 \\ 4e_1 = 3e_2 + 12 \end{cases} \Rightarrow \begin{cases} e_2 = \dfrac{36}{10} \\ e_1 = \dfrac{19}{12} \cdot \dfrac{36}{10} = \dfrac{57}{10} \end{cases}$$

$$\Rightarrow \begin{cases} V_x = e_2 - e_1 = \dfrac{-21}{10} V \quad \# \\ i_x = \dfrac{e_2 - 0}{8k\Omega} = \dfrac{36}{10} \cdot \dfrac{1}{8} = \dfrac{9}{20} mA \quad \# \end{cases}$$

Problem 3 (20 points)

Use the voltage divider pattern to design a circuit that performs analog computation function of $v_{CD} = f(v_{AB}) = \frac{2}{3}v_{AB}$. Use only two resistors to complete your design inside the middle box, as illustrated in the schema below. v_{ij} means the branch voltage between nodes i and j (i.e., the electric potential difference between nodes i and j).

③



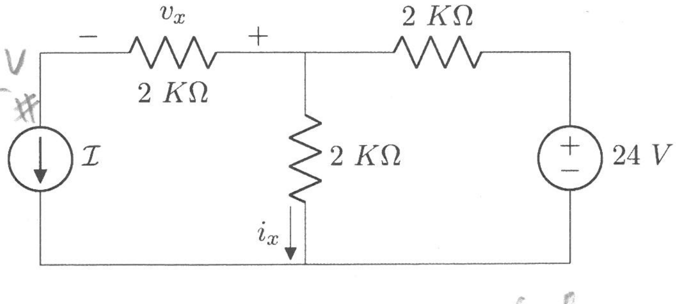
$$\begin{cases} R_1 = 1 k\Omega \\ R_2 = 2 k\Omega \end{cases}$$

or, as long as $\frac{R_1}{R_2} = \frac{1}{2}$ #

Problem 4 (20 points)

For the following circuit, use suposition to find the answer to the following questions:

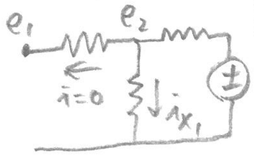
1. (10 points) Given $\mathcal{I} = 8\ mA$, what would be the value of v_x?
2. (10 points) To make $i_x = 3\ mA$, what should be the value of $\mathcal{I}$?



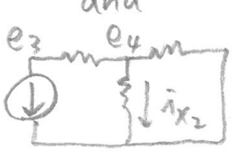
(4.1) First of all, by KCL and Ohm's law we know

$V_x = 8 \cdot 2 = 16V$

Now, to use superposition, consider the following



and



$V_x = (e_2 - e_1) + (e_4 - e_3)$
$= (0) + I \cdot 2k\Omega = 16V$ #

(4.2) $i_x = i_{x_1} + i_{x_2}$

$= \left(\frac{12V}{2k\Omega}\right)$ ← voltage divider + Ohm's law

$+ \left((-I) \cdot \frac{1}{2}\right)$ ← current divider

$= 6 - \frac{I}{2}$ ⇒ For $i_x = 3mA$, we need $I = 6\ mA$ #

Problem 5 (20 points)

In this exercise, we will study a property of *multimeter*, a very handy measuring instrument for electronics[1]: when we switched our multimeter to the mode for voltage measurement, its internal circuit can be regarded as having a very large resistance[2]; in other words, from the view point of the DUT (device under test), the multimeter in this mode can be regarded as a very large resistor.

Now, answer the following question in your own words, and then compare it with the answer from ChatGPT (it is sufficient to use the free version):

Question: Given that when we want to measure the branch voltage of a DUT we would connect the multimeter in parallel to the DUT, use the concepts of equivalent resistance and the current divider pattern to reason about why the multimeter must be designed to have a large internal resistance in this measurement mode.

1. (10 points) Show your own analysis. Use figures to clarify.
2. (5 points) Copy and paste the answer you got from ChatGPT.
3. (5 points) Compare the answer of yours and that of ChatGPT and write down your findings: in which ways does your work looks better, and in which ways does the ChatGPT work looks better? Give your qualitative assessment.

(Hint: as part of the analysis, it may be useful to think of the DUT and the multimeter as one single element).

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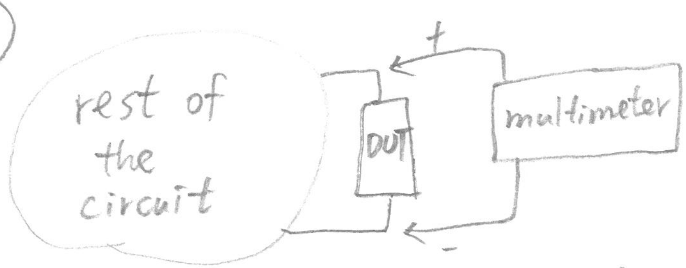
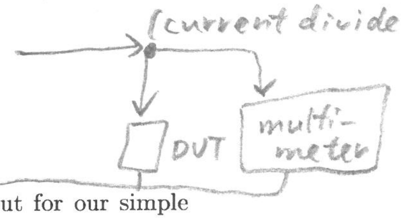


In general, we want to reduce interferences caused by attaching the multimeter. In other words, we don't want the instrument affect the measurement accuracy. This is the key requirement.

Now, there are at least two reasons for using a large internal resistance:

A. less amount of current will branch to the multimeter. (current divider)



B. branch voltage of the DUT will be closer to the case with no multimeter, because the equivalent resistance of the two combined will be closer to that of DUT alone. ※

[1] https://en.wikipedia.org/wiki/Multimeter

[2] In general, it should be called impedance instead of resistance; but for our simple circuit of a set of resistors, we may regard this physical value as resistance.