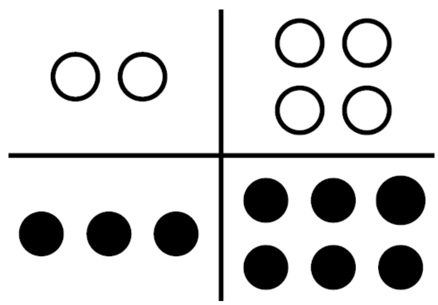
Mathnasium Teaching Constructs: Proportional Thinking



• Introduction to Proportional Thinking •

Proportional Thinking

Reasoning in groups, according to amount.

Tracing the term "proportion" to its Latin roots gives a definition of "according to amount," meaning "…a part, share, or portion, especially in its relation to the whole. The comparative relation between parts or things with respect to size. A portion or part in its relation to the whole or other parts."

Proportional Thinking is introduced to students early in the curriculum, establishing a fundamental concept that will eventually lead to a stronger understanding of critical concepts like ratios, proportions, direct and indirect variation, and algebraic reasoning. Students will develop **Proportional Thinking** through the practice of "reasoning in groups," where students will examine ratios of physical objects or quantities, like the ratio of lemons to glasses of lemonade. This practice is aided by visual representations of the objects and quantities, and Instructors should be ready to use drawings and/or manipulatives to further illustrate the types of quantitative relationships that require **Proportional Thinking**.

This lesson book contains examples of the tools used throughout the Mathnasium curriculum that utilize and reinforce **Proportional Thinking**. As you proceed through this lesson book, work through each of the exercises to become familiar with the thinking and reasoning processes that students will undergo as they encounter this curriculum, as well as the challenges they may face. Additionally, take note of the pages with reflection questions at the bottom, and answer them in the space provided.

When you have completed this lesson book, submit it to your Center Director for review and discussion.

Useful Desktool pages for Proportional Thinking:
Building Up by 10s
Proportional Thinking
X-Y Axis
First Quadrant Grid

How many legs do 3 cats have?

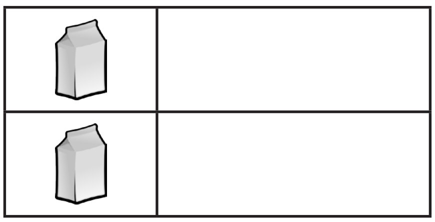
Try using these words in your instruction to reinforce the Proportional Thinking construct by using Socratic Questioning, extending the student's knowledge of the concept beyond the content of the page, and/or other helpful instructional techniques.

• Reasoning in Groups •

A store gives away 1 milk carton for every package of 4 cookies sold.

Draw a Picture

1) If **2 milk cartons** were given away, how many *cookies* were sold?



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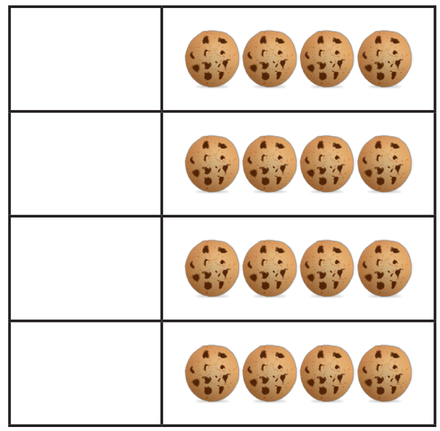
2) If **16 cookies** were sold, how many *milk cartons* were given away?



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PK-3045-00_Reasoning_in_Groups

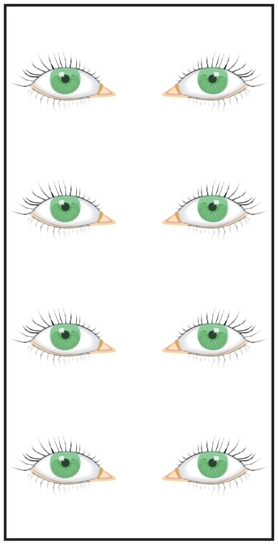
Larry purchases a certain number of cookies and receives 6 cartons of milk from the store. Each package of 4 cookies costs $2.

a) How many cookies did he purchase from the store?

b) How much did he spend?

Include a visual representation of this exercise.

• Picture Questions •



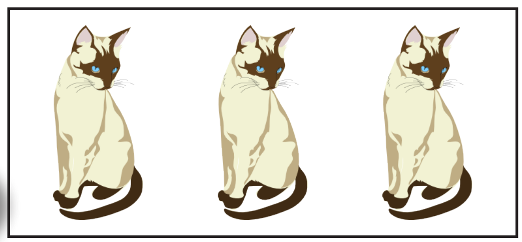
1) If you see 10 eyes, how many people are there? _______

2) If you see 20 eyes, how many people are there? _______

3) If you see 4 people, how many eyes are there? _______

4) If you see 12 people, how many eyes are there? _______

How many legs do 3 cats have?



5) If you see 3 cats, how many legs are there? ___________

6) If you see 8 legs, how many cats are there? __________

7) If you see 10 cats, how many legs are there? ___________

8) If you see 20 legs, how many cats are there? ___________

9) If you see 14 tails, how many cats are there? ___________

• Reasoning in Groups •

If 4 pounds of grapes costs $7.00...

1) How much does **12 pounds of grapes** cost? ______________

2) How many pounds of grapes can you buy for **$35.00**? ______________

3) How much does **28 pounds of grapes** cost? ______________

4) How many pounds of grapes can you buy for **$84.00**? ______________

5) How much does **40 pounds of grapes** cost? ______________

6) How many pounds of grapes can you buy for **$56.00**? ______________

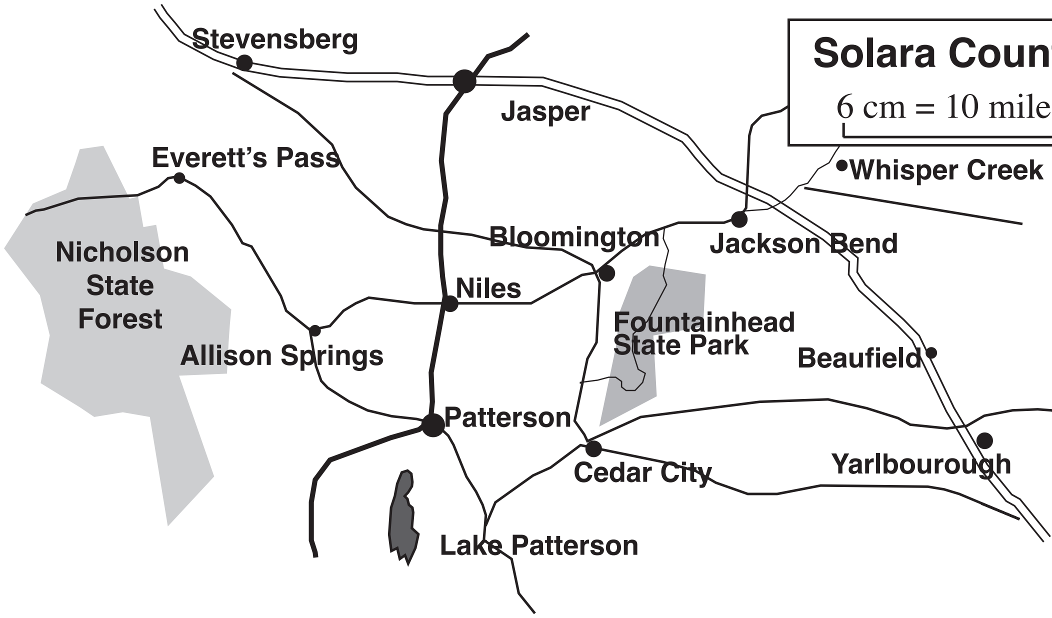
PK-3090-00_Reasoning_in_Groups

When working with a student on Reasoning in Groups "off the page," one potential pitfall to avoid is only using multiples of the given ratio.

The examples shown on this page are valuable, but can be extended further to include fractional parts. Provide three examples that do not use exact multiples of the given ratio, and include an explanation for each example using the Proportional Thinking construct.

• Proportional Reasoning •

In a scale drawing, 6 centimeters represents 10 miles.



1) How many centimeters represents 25 miles?

Exactly how many *groups* of **10 miles** are there *inside of* **25 miles**? $\boxed{2\frac{1}{2}}$

6 cm, $\boxed{2\frac{1}{2}}$ times = ___________ **cm**

2) How many miles is represented by 9 cm?

Exactly how many *groups* of **6 cm** are there *inside of* **9 cm**? $\boxed{}$

10 miles, $\boxed{}$ times = ___________ **miles**

PK-3222-00_Problem_Solving-Proportional_Reasoning

Assuming the student has access to a ruler, what are some extended prompts an Instructor could provide using the scale drawing?

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• Coin Equivalence •

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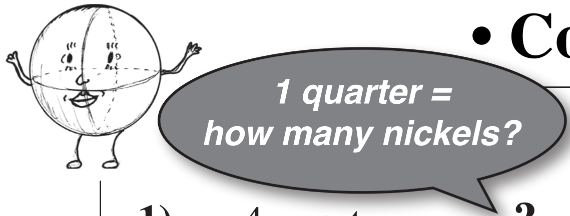


1) 4 quarters = __?__ nickels

☐ ¢ = ______

Value of **4 quarters** — NUMBER OF NICKELS

4 quarters = _____ nickels

2) 5 dimes = __?__ half dollars

☐ ¢ = ______

Value of **5 dimes** — NUMBER OF HALF DOLLARS

5 dimes = _____ half dollar

3) 8 nickels = _______¢ = ______ dimes

7) 9 nickels = _______ pennies

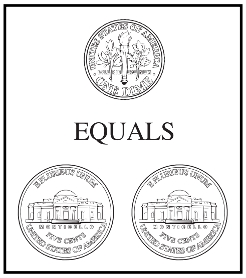
8) 3 quarters = _______ nickels

PK-3091-00_Coin Equivalence

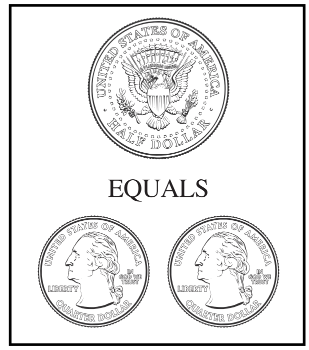
How could an Instructor provide a visual prompt to solve exercise #3?

• Doubling and Halving •

There are two nickels for every dime, so double the number of dimes to find the equivalent number of nickels.



1 dime = 2 nickels



1 half dollar = 2 quarters

Try these:

1) 4 dimes = ______ nickels

2) 3 half dollars = ______ quarters

3) 18 nickels = ______ dimes

4) 12 quarters = ______ half dollars

5) 16 dimes = ______ nickels

6) 9 half dollars = ______ quarters

PK-3091-00_Coin Equivalence

A student successfully completes this page by multiplying the number of coins by the value and then dividing by the value of the coin to which they are converting. Given the intent of this page, how can the Instructor demonstrate a Proportional Thinking approach?

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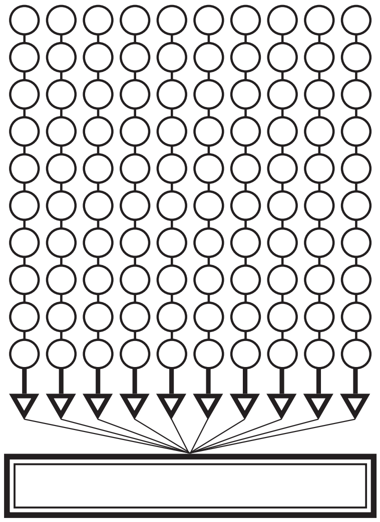
• Building Up by 10s •

In each exercise, fill in the names of the missing symbols.

1)

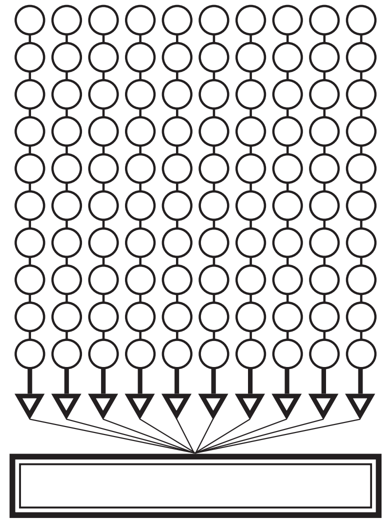


○ = **penny**

▽ = ______________________

▭ = ______________________

2)



○ = **year**

▽ = ______________________

▭ = ______________________

WOB_2_Chapter_2

Besides a penny and a year, identify two other units you can apply to the circle in the diagram, and write two "for each/every" statements that correspond to each unit.

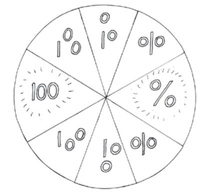
For example: there are 10 pennies for each dime; there are 10 dimes for every dollar.

• Percent – For Each 100 •

Count 5 for each 100.

Percent

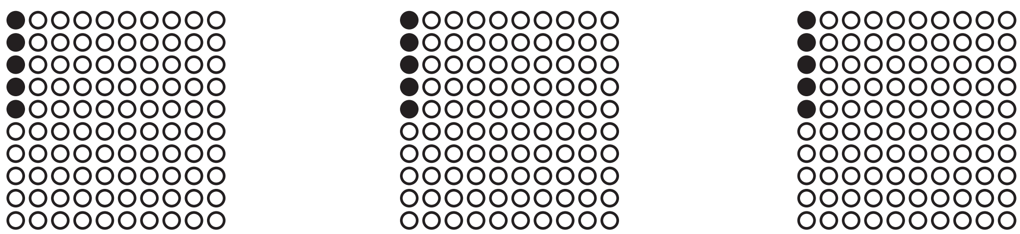
Per means "*for each*" and **cent** means "*100*."
So **percent** means "*for each 100*."



Example: **Find 5% of 300.**

5% means "5 for each 100." **So count...**

5 for the *first* 100, **5** for the *second* 100, ***and*** **5** for the *third* 100.



5 + 5 + 5 = 15

So, **5% of 300 is 15**.

Try these:

1) 5% of 200 = ______________ **2)** 9% of 300 = ______________

3) 15% of 200 = ______________ **4)** 15% of 400 = ______________

5) 6% of 200 = ______________ **6)** 6% of 100 = ______________

7) 16% of 200 = ______________ **8)** 4% of 300 = ______________

PK-3177-00_Percent-For_Each_100

Although they are equivalent, discuss how you can demonstrate why the distinction between 9% of 300 and 3% of 900 is important in practice, using the example of determining the amount of sugar needed so that 300 glasses of lemonade are composed of 9% sugar, and the amount of sugar needed so that 900 glasses of lemonade are composed of 3% sugar.

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• Percent – For Each 100 •

2) 8% of 150 = ______________ **3)** 6% of 50 = ______________

5) 90% of 150 = ______________ **6)** 9% of 250 = ______________

9) 24% of 125 = ______________ **10)** 8% of 25 = ______________

13) A basketball player shot 200 practice shots in one day. 41% of the shots went in the basket. How many shots did the basketball player make?

____

15) A country club has 300 members. 65% of the members are 30 years of age or older. How many members are **younger** than 30 years of age?

____

PK-3201-00_Percent-For_Each_100

"Since 8% of 150 = 12, 16% of 150 = 24."
Provide a similar extension to each numerical exercise above.

• Direct Percent •

In each exercise, estimate the answer. Then find the actual answer.

1) 5% of 497 = **?**

_____ % of _____ = $\boxed{}$

For Each 100 — 497 rounded to the nearest 50 — Estimate

____
Actual Answer

2) 20% of 648 = **?**

_____ % of _____ = $\boxed{}$

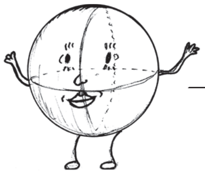
For Each 100 — 648 rounded to the nearest 50 — Estimate

____
Actual Answer

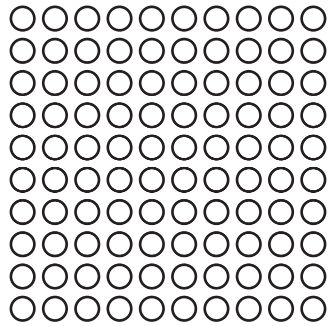
PK-3178-00_Direct_Percent

For exercises #1 and #2, demonstrate how you would both estimate and calculate the "complement" to a student.

Example:
Given that 17% of 602 is approximately 102, 83% of 602 is approximately 500.
Given that 17% of 602 is exactly 102.34, 83% of 602 is exactly 499.66.



• Special Percents •



Key words: **0%** means "**none** of it."

25% means "**a quarter** of it."

50% means "**half** of it."

100% means "**all** of it."

200% means "**all** of it, **twice**."

1) 100% of 20 = ______________ **2)** 50% of 20 = ______________

3) 0% of 20 = ______________ **4)** 200% of 20 = ______________

5) 25% of 20 = ______________ **6)** 100% of 12 = ______________

PK-3176-00_Percent-Special_Percents

1) Fill in each blank with the correct phrase.

• $66\frac{2}{3}\%$	____________________	• a third of it
• 0%	____________________	• all of it, twice
• 150%	____________________	• none of it
• $33\frac{1}{3}\%$	____________________	• two-thirds of it
• 200%	____________________	• all of it and half of it

2) **25%** means "____."

3) **100%** means "____."

5) **300%** means "____."

PK-3200-00_Percent-Special_Percents

• Converting Fractions to Percents •

EXAMPLE 1: Convert $\frac{3}{25}$ to a percent.

Since the denominator is a *factor* of 100, we can use the following steps:

STEP 1. Convert to a decimal.	**STEP 2.** Multiply by 100.
First find an *equivalent fraction* using 100 as the denominator. Then name the decimal. $\frac{3}{25} = \frac{\boxed{12}}{100} = \underline{0.12}$	This moves the *decimal point* two places to the right. $0.12 \rightarrow 0.12. \rightarrow 12\%$ $\frac{3}{25} = \underline{12\%}$

EXAMPLE 2: Convert $\frac{4}{5}$ to a percent.

STEP 1. Convert to a decimal.	**STEP 2.** Multiply by 100.
$\frac{4}{5} = \frac{\boxed{80}}{100} = \underline{0.80}$	$0.80 \rightarrow 0.80. \rightarrow 80\%$ $\frac{4}{5} = \underline{80\%}$

Try these: **Convert each fraction to a percent.**

1) $\frac{11}{20} = \frac{\boxed{}}{100} = \underset{\text{DECIMAL}}{\underline{\qquad\qquad}} = \underset{\text{PERCENT}}{\underline{\qquad\qquad}}$

2) $\frac{19}{50} = \frac{\boxed{}}{100} = \underset{\text{DECIMAL}}{\underline{\qquad\qquad}} = \underset{\text{PERCENT}}{\underline{\qquad\qquad}}$

PK-3207-00_Converting_Fractions_to_Percents

A student incorrectly completes exercise #2 as follows: $\frac{19}{50} = \frac{36}{100} = 0.36 = 36\%$.
Outline the instructional language you would use to correct this computation error.

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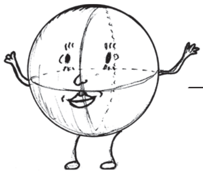

• Converting Percents to Fractions •

When converting a percent that is greater than 100% to a fraction, the value of the fraction is greater than 1 and can be written as a **MIXED NUMBER**.

EXAMPLE: Convert 240% to a MIXED NUMBER in lowest terms.

240% means, "**240 *for each 100***," which is the same as $\frac{240}{100}$.

$240\% = \boxed{240}$ for each $100 = \frac{\boxed{240}}{100}$

240% as a **MIXED NUMBER** in lowest terms = $\underline{2\frac{2}{5}}$

This can also be written as $2\frac{40}{100}$ or $2\frac{2}{5}$.

Try these: **Convert each percent to a MIXED NUMBER in lowest terms.**

1) $125\% = \boxed{}$ for each $100 = \frac{\boxed{}}{100}$

125% as a **MIXED NUMBER** in lowest terms = ______

PK-3212-00_Converting_Fractions_to_Percents

Provide alternative language to describe 125%.

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• Reducing Ratios to Lowest Terms •

Direct Teaching

Ratios can be ***reduced*** to LOWEST TERMS just like fractions.

EXAMPLE: In a preschool classroom, there are 5 teachers for every 15 children.

The ratio of TEACHERS to CHILDREN is **5**:**15**, which ***reduces*** to **1**:**3**.

So, the ratio of **5** TEACHERS for every **15** CHILDREN is the same as:

__1__ TEACHER for every __3__ CHILDREN.

Try these: **Reduce each ratio to lowest terms.**

1) The ratio of **18** CHOCOLATE CHIPS for every **4** COOKIES is the same as:

____ CHOCOLATE CHIPS for every ____ COOKIES.

2) The ratio of **3** LIFEGUARDS for every **60** SWIMMERS is the same as:

____ LIFEGUARD for every ____ SWIMMERS.

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4) **12**:**14** *reduces* to ____:____. IN LOWEST TERMS

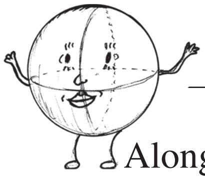
5) **24**:**15** *reduces* to ____:____. IN LOWEST TERMS

6) **40**:**12** *reduces* to ____:____. IN LOWEST TERMS

7) **10**:**25** *reduces* to ____:____. IN LOWEST TERMS

PK-3172-00_Finding_Ratios

If the ratio is 3 lifeguards for every 60 swimmers, how many lifeguards would you need for 70 swimmers? Explain.



• Comparing Ratios •

Along with finding ratios, we can *compare* the values of ratios.

Direct Teaching

EXAMPLE: On Tuesday, Ben's ice cream shop sold 24 cones and 18 cups. On the same day, Jerry's ice cream shop sold 15 cones and 12 cups.

The ratio of CUPS to CONES sold at Ben's shop was 18 : 24 → 3 : 4.

The ratio of CUPS to CONES sold at Jerry's shop was 12 : 15 → 4 : 5.

Whose shop sold the *higher* ratio of cups to cones? **Jerry's shop**.

$\frac{4}{5}$ is *greater than* $\frac{3}{4}$, so 4:5 is the *higher* ratio (Jerry's shop).

Try these: **Write all answers in lowest terms.**

1) Irene has 16 PANTS and 28 SHIRTS in her closet. Harry has 20 PANTS and 25 SHIRTS in his closet.

What is the ratio of PANTS to SHIRTS in Irene's closet? __________

What is the ratio of PANTS to SHIRTS in Harry's closet? __________

Who has the *higher* ratio of PANTS to SHIRTS in their closet? ____________________

PK-3255-00_Finding_and_Comparing_Ratios

How many pants/shirts would Irene need to buy to have the exact same ratio as Harry? Could she donate pants/shirts and accomplish the same ratio? Explain.

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• Applying Ratios • ☆

If we know the ratio between two things we can find a part or a whole.

Example:

The ratio of women to men at a meeting is 3 to 2.

a) There are 18 women. How many men are there? _________

b) How many people are at the meeting? _________

For part **a)**, a **ratio** of 3 women for every 2 men means that for every group of 3 women, there is a group of 2 men.

WWW **MM**

So, if there are **18** women at a meeting, there are **6** groups of **3** ($6 \times 3 =$ **18** women total).
This means that there are **6** groups of **2** men ($6 \times 2 =$ **12** men total).

WWW **WWW** **WWW** **WWW** **WWW** **WWW**

MM **MM** **MM** **MM** **MM** **MM**

So, there are **12** men at the meeting.

For part **b)**, the total number of people at the meeting is **30** (18 women + 12 men).

Try these:

1) The ratio of red to green candies is 5 to 3. If there are 15 red candies, how many groups of red candies and green candies are there?

____

PK-3361-00_Prob_Solving_Ratios

A student reasons that if she has a ratio of 5 red candies to 3 green candies, and green candies cost 3 times as much as red candies, then a cost ratio of the candies that she has could be 5:9. Is this correct? Explain.

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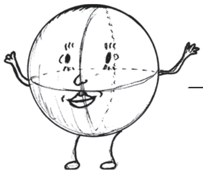

• Portioning – Hidden Ratios •

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Direct Teaching

Some **portioning** word problems will not give the exact amount of parts. Instead, they may give a ratio in the form of "**twice as many**" or "**three times as many**."

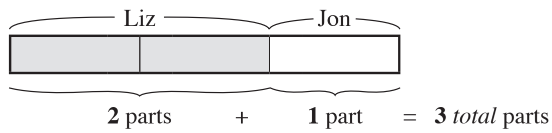
EXAMPLE: At the train show, Liz took twice as many photos as Jon did. Together, they took 51 photos. How many photos did they each take?

Steps to Solve:

STEP 1:

Find the total number of *equal parts* into which the whole is divided.

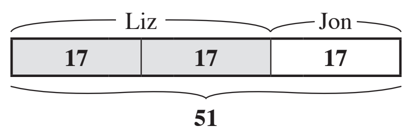
"**Twice as many**" means **2** *for every* **1**.



STEP 2:

Find the *value* of each part by dividing the whole (51) by the total number of parts (3).

51 (whole) ÷ **3** (parts) =
17 photos for each part



STEP 3:

Find the number of photos that each person took.

LIZ: **2** parts × **17** = 34 photos

JON: **1** part × **17** = 17 photos

Try this:

1) Albert and Beth collected shells at the beach. Albert collected three times as many shells as Beth. Together, they gathered 44 shells. How many shells did they collect individually?

ALBERT = ____________ BETH = ____________

PK-3171-00_Problem_Solving_Portioning

• Portioning – Hidden Ratios •

Draw a Picture

1) Chris and Holly take turns mowing their neighbor's lawn. Chris works for twice as many hours as Holly. They are paid a total of $60. How should they divide the money so that each person gets a fair share?

Chris = ____________ **Holly** = ____________

2) Marla has three times as much money as Billy. Together, they have $280. How much money does each person have?

Marla = ____________ **Billy** = ____________

PK-3171-00_Problem_Solving_Portioning

Complete exercise #2. Given the solution, how would you prompt a student to solve mentally if the total that Marla and Billy have together is instead:

a) $2800 *b) $560* *c) $140* *d) $70*

• Solving Proportions by Equivalent Fractions • ☆

A ***proportion*** is a statement that two ratios (fractions) are *equal*.

Direct Teaching

We can solve a proportion by making equivalent fractions.

Example 1: $\frac{3}{y} = \frac{9}{15}$

$\times 3$

$$\frac{3}{y} = \frac{9}{15}$$

$\times 3$

So $y = \mathbf{5}$.

Example 2: $\frac{2}{7} = \frac{12}{x}$

$\times 6$

$$\frac{2}{7} = \frac{12}{x}$$

$\times 6$

So $x = \mathbf{42}$.

We can also solve a proportion by **reducing** *first* and then making equivalent fractions.

Example: $\frac{5}{15} = \frac{n}{6}$

$\frac{5}{15}$ **reduces** to $\frac{\mathbf{1}}{\mathbf{3}}$.

$\times 2$

$$\frac{1}{3} = \frac{n}{6}$$

$\times 2$

So $n = \mathbf{2}$.

Try these: **Solve for the variable by making equivalent fractions.**

1) $\frac{3}{8} = \frac{6}{n}$ $n =$ ________

2) $\frac{y}{21} = \frac{10}{3}$ $y =$ ________

3) $\frac{a}{10} = \frac{20}{25}$ $a =$ ________

PK-3237-00_Solving_Proportions

Explain the importance of not allowing a student to complete this page using cross-multiplication.

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• Setting Up **Proportions** •

Direct Teaching

When a **proportion word problem** is challenging to solve *mentally*, we set up a ***proportion*** (two ratios that are equivalent) using a variable to help us find the "unknown."

Example: If 3 pies can be made from 16 peaches, exactly how many peaches are needed to make 7 pies?

Step 1: Set up the first ratio.

$$\frac{\text{3 pies}}{\text{16 peaches}}$$

Step 2: Complete the proportion by setting up the second ratio. Keep the position of the units consistent with the first ratio ("pies" in the numerator and "peaches" in the denominator) and use a variable to represent the "unknown."

$$\frac{\text{3 pies}}{\text{16 peaches}} = \frac{\textbf{7 pies}}{x\ \textbf{peaches}}$$

Try these: **In each exercise, set up the proportion. Use "*x*" as the variable and include labels. Do NOT solve.**

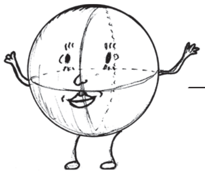
1) If 9 tons of sand is needed to fill 4 sandboxes, exactly how many tons of sand is needed to fill 60 sandboxes?

$$\frac{\text{9 tons of sand}}{\text{4 sandboxes}} = \frac{\quad}{\quad}$$

2) If a snail can travel 2 inches in 7 minutes, exactly how many minutes will it take the snail to travel 11 inches?

$$\frac{\quad}{\quad} = \frac{\quad}{\quad}$$

PK-3223-00_Problem_Solving-Proportions



• Setting Up **Proportions** •

In each exercise, set up the proportion. Use "*x*" as the variable and include labels. Do NOT solve.

1) If 3 cakes serve 35 people, exactly how many cakes would serve 50 people?

$$\frac{3\text{ cakes}}{35\text{ people}} = \frac{\qquad}{\qquad}$$

2) If 6 tomatoes are needed to make 5 cups of salsa, exactly how many cups of salsa can be made with 26 tomatoes?

$$\frac{\qquad}{\qquad} = \frac{\qquad}{\qquad}$$

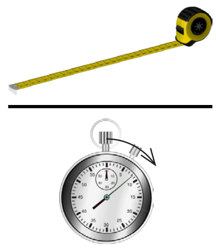
PK-3223-00_Problem_Solving-Proportions

Provide a Proportional Thinking explanation for exercise #2 without cross-multiplication.

• Finding Unit Rates •

1) A sprinter ran 80 meters in 10 seconds. Find the unit rate.

80 meters



10 seconds

$\rightarrow \dfrac{\text{80 meters}}{\text{10 seconds}} = \dfrac{\qquad}{\qquad}$

80 meters in 10 seconds → _____ meters in 1 second.

The unit rate is _______________ per _______________ .

2) A plane traveled 2,550 miles in 5 hours. Find the unit rate.

$\dfrac{\qquad}{\qquad} = \dfrac{\qquad}{\qquad}$

2,550 miles in 5 hours → _____ miles in 1 hour.

The unit rate is _______________ per _______________ .

PK-3184-00_Unit_Rate

A student correctly answers "510 miles per 1 hour" in exercise #2 but that isn't the only unit rate present. Approximate the other unit rate.

• Scale Factor •

Direct Teaching

In some scale factor word problems, the scale factor and *one measurement* are given, and we are asked to solve for the ***other measurement***.

Example: A museum gift shop sells a stuffed dinosaur that is modeled after a dinosaur the museum displays in an exhibit. The stuffed dinosaur was created using a scale factor of 1:6. If the stuffed dinosaur is 3 feet tall, how tall is the dinosaur in the exhibit?

Proportional Thinking

Since the scale factor (1:6) tells us that the heights of the stuffed dinosaur and the dinosaur in the exhibit are in proportion, we can *multiply* the height of the stuffed dinosaur (3 feet) by 6 to find the height of the dinosaur in the exhibit.

3 feet × 6 = **18 feet**

So, the height of the dinosaur in the exhibit is **18 feet**.

Try these: **In each exercise, use the scale factor to determine the length of the object.**

1) A model of a real truck was designed using a scale factor of 1:3. If the model truck is 4 feet long, how long is the real truck?

____

PK-3271-00_Scale_Factor

Provide two methods to solve exercise #1 other than the method presented above.

____

____

____

____

• Writing **Direct Variation** Equations •

In **Direct Variation**, it is always true that

one changing quantity *divided* by another changing quantity is **constant**.

In equation form, we write $\frac{y}{x} = k$,
where x and y are the changing quantities
and k is the *constant of proportionality*.

In the previous example, the total cost (T) divided by the number of CDs purchased (n) always equals 12 $\left(\frac{T}{n} = 12\right)$. (Check this by dividing the entries in the table you created on the last page.)

The equation $\frac{y}{x} = k$ is usually written $y = kx$.

To solve problems involving **Direct Variation**, we must be given two of the elements x, y, and k. Then we solve for the missing element.

Example: Four candy bars cost 80¢. Find the cost of 11 candy bars.
Since

$$\frac{\textbf{total}}{\textbf{number}} = \frac{80}{4} = 20$$

the *constant of proportionality* is 20¢. Now, we can calculate the cost of any number of candy bars by using the equation $T = 20n$.

So, 11 candy bars cost (20¢)(11) = $2.20. Notice that the total cost divided by the number purchased always equals the *constant of proportionality*.

Try these:

2) Phred travelled 200 miles in 4 hours. At the same average speed, how far will she travel in 15 hours? ______________ in h hours? ________________

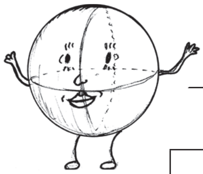
PK-0744-00_Direct_Variation

Explain how to find the constant of proportionality when values that vary directly are given in a table format.

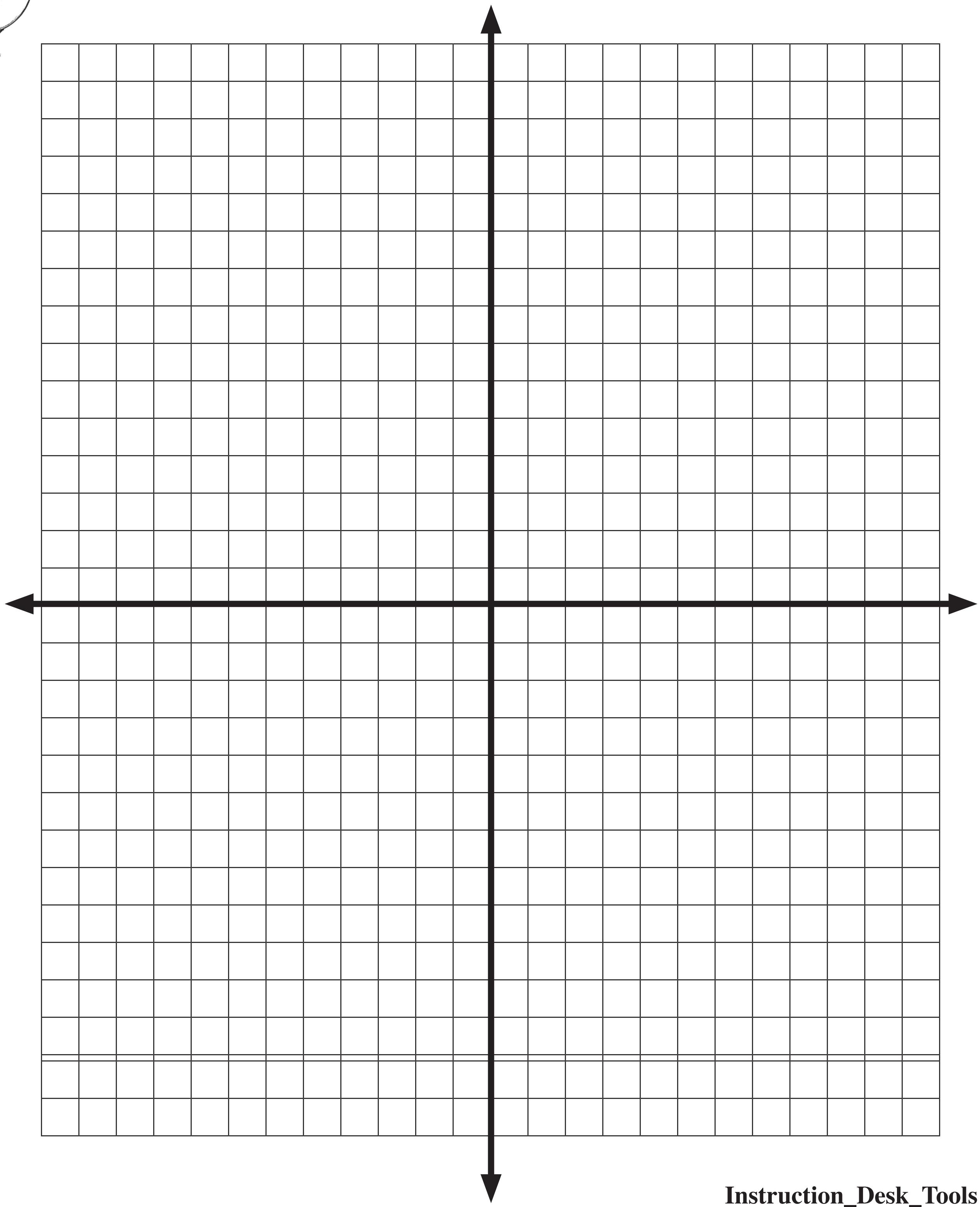
____

____

____



• X-Y Axis (with Grid) •



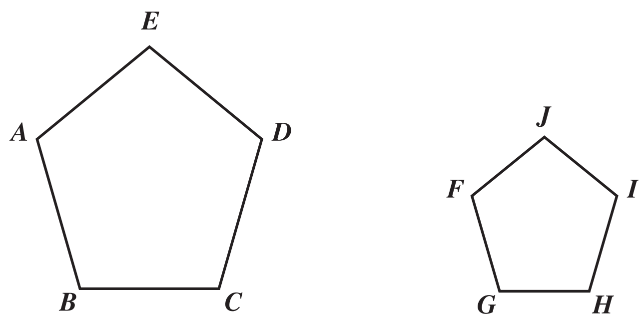
Instruction_Desk_Tools

Use slope to demonstrate that the points ($^-1$, 5), (2, $^-1$) (4, $^-5$) are collinear.

• Similar and Congruent Figures •

When two figures are "*the same shape but not the same size*" they are called **similar** figures.

When figures are **similar**, *corresponding angles* are *equal*, and *corresponding sides* are **proportional**. Sides are **proportional** if the *corresponding sides* have equal ratios.

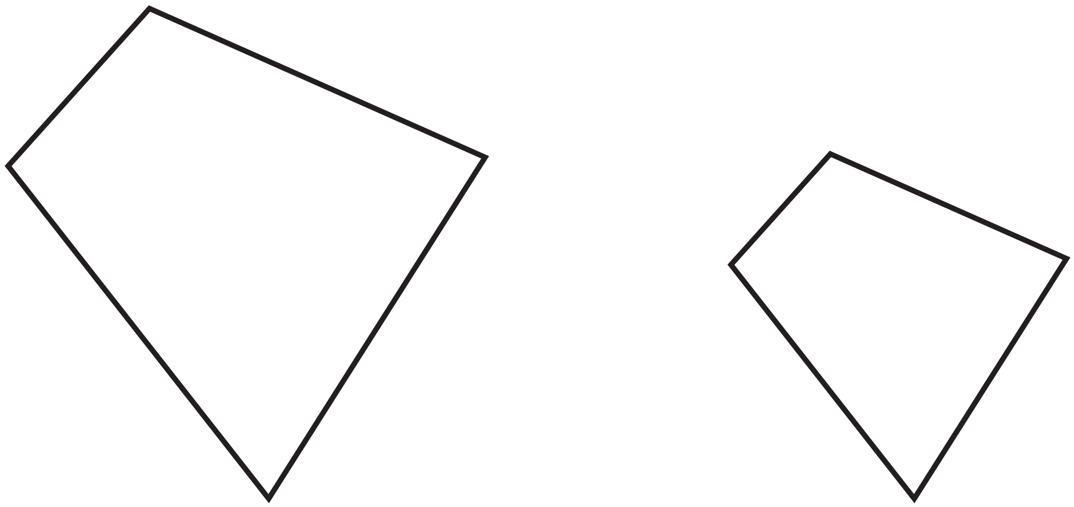


Pentagons ***ABCDE*** and ***FGHIJ*** are **similar**. This is written as ***ABCDE* ~ *FGHIJ***.

Notice that the two pentagons are "*the same shape but* ***not*** *the same size*."

PK-0819-00_Similar_Triangles

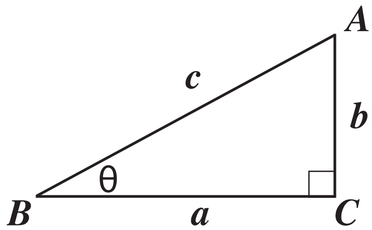
Identify three different proportional relationships between ***ABCDE*** and ***FGHIJ***.

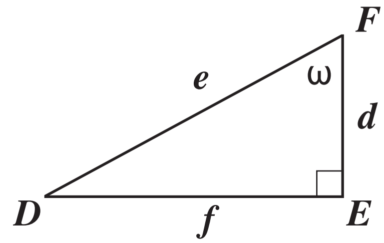


Instruction_Desk_Tools

Create two different Proportional Thinking exercises using this Desktool.

• Sine–Cosine–Tangent Ratios •





$\sin\theta = \dfrac{\textit{opposite}}{\textit{hypotenuse}} = \dfrac{b}{c}$

$\cos\theta = \dfrac{\textit{adjacent}}{\textit{hypotenuse}} = \dfrac{a}{c}$

$\tan\theta = \dfrac{\textit{opposite}}{\textit{adjacent}} = \dfrac{b}{a}$

$\sin\omega = \dfrac{\textit{opposite}}{\textit{hypotenuse}} = \dfrac{f}{e}$

$\cos\omega = \dfrac{\textit{adjacent}}{\textit{hypotenuse}} = \dfrac{d}{e}$

$\tan\omega = \dfrac{\textit{opposite}}{\textit{adjacent}} = \dfrac{f}{d}$

Use the diagrams above.

Given $AC = 6$ and $BC = 8$.

1) $\sin\theta =$ ______.

2) $\cos\theta =$ ______.

3) $\tan\theta =$ ______.

Given $DF = 13$ and $DE = 12$.

4) $\sin\omega =$ ______.

5) $\cos\omega =$ ______.

6) $\tan\omega =$ ______

7) If **tan** $A = \frac{5}{12}$, find **sin** A. ____________

9) If cos $A = \frac{24}{25}$, find **tan** A. ____________

FO-0024-00_Trigonometric_Ratios_Unit_Circle_Trig_Word_Problems

Demonstrate that the trigonometric values have the same values between similar right triangles.

EXAMPLE 2: In ΔDEF, $\overline{DE} = 6$, $\angle D = 40°$, $\angle E = 75°$. Find $\overline{DF}$ and $\overline{EF}$.

Solution: First, we find $\angle F$:

$$\angle F = 180° - (\angle D + \angle E) = 180° - (40° + 75°) = 65°$$

Applying the Law of Sines to ΔDEF we have:

$$\frac{\overline{EF}}{\sin D} = \frac{\overline{DF}}{\sin E} = \frac{\overline{DE}}{\sin F}$$

$$\overline{EF} = \frac{\sin D \cdot \overline{DE}}{\sin F} = \frac{6 \sin 40°}{\sin 65°} \approx \frac{6 \cdot 0.643}{0.906} \approx 4.255$$

$$\overline{DF} = \frac{\sin E \cdot \overline{DE}}{\sin F} = \frac{6 \sin 75°}{\sin 65°} \approx \frac{6 \cdot 0.966}{0.906} \approx 6.395$$

PK-1535-00_Law_of_Sines_for_Area_of_Triangles

When applying the Law of Sines, you must account for the "ambiguous case." What are the criteria for this case?

• Final Reflection: Proportional Thinking •

Share an experience where you have used Proportional Thinking with a student. Are there any particular students at your center who will benefit from this construct as they continue to learn a topic? What opportunities will you seek to use Proportional Thinking with students in the future?