Problem 1. Runtime Analysis

(a) For each iteration, i becomes i^2

So $i_0 = 2^1$, $i_1 = (2^1)^2 = 2^2$, $i_2 = (2^2)^2 = 2^4$, $i_3 = (2^4)^2 = 2^8$

In general, $i_k = 2^{2^k}$, where k is the number of iterations

$$2^{2^k} \geqslant n \Rightarrow 2^k \geq \log n \Rightarrow k \geq \log(\log n)$$

The time complexity of this function is $\Theta(\log(\log n))$

(b) There will be $(int)\sqrt{n}$ numbers from 1 to n that satisfies it is a multiple of $(int)\sqrt{n}$.

Namely, $(int)\sqrt{n}, 2\cdot(int)\sqrt{n}, \dots, (int)\sqrt{n}\cdot(int)\sqrt{n} \leqslant n$

$$T(n) = \sum_{i=1}^{n}(\Theta(1)) + \sum_{k=1}^{\sqrt{n}}(\sqrt{n}\cdot k)^3$$

$$= \sum_{i=1}^{n}(\Theta(1)) + n^{\frac{3}{2}}\sum_{k=1}^{\sqrt{n}} k^3$$

$$= \Theta(n) + n^{\frac{3}{2}}\cdot(1+2+\dots+\sqrt{n})^2$$

$$= \Theta(n) + n^{\frac{3}{2}}\cdot\left(\frac{\sqrt{n}(\sqrt{n}+1)}{2}\right)^2$$

$$= \Theta(n) + n^{\frac{3}{2}}\cdot\frac{n^2+2n^{\frac{3}{2}}+n}{4}$$

$$= \Theta(n) + \Theta(n^{\frac{7}{2}}) = \Theta(n^{\frac{7}{2}})$$

(c) $A[k]$ can at most be one i, so the #. of times $A[k] = i$ is true is at most n.

The inner loop takes $\log(n)$ time since m doubles each time

$$T(n) = \sum_{i=1}^{n} \left(\sum_{k=1}^{n} (\Theta(1)) \right) + O(n) \cdot \Theta(\log n)$$

$$= \Theta(n^2) + \Theta(n \log n)$$

$$= \Theta(n^2)$$

(d) Since the size is multiplied by $\frac{3}{2}$ each time,

#. of times if statement is called $= \log_{\frac{3}{2}}\left(\frac{n}{10}\right)$

$$T(n) = \sum_{i=0}^{n} (\Theta(1)) + \sum_{k=0}^{\log_{\frac{3}{2}}(\frac{n}{10})} (10 \cdot 1.5^k)$$

$$= \Theta(n) + 10 \cdot \frac{1 - r^{m+1}}{1 - r}$$

$$= \Theta(n) + 10 \cdot \frac{1 - 1.5^{(\log_{1.5}(\frac{n}{10}) + 1)}}{1 - 1.5}$$

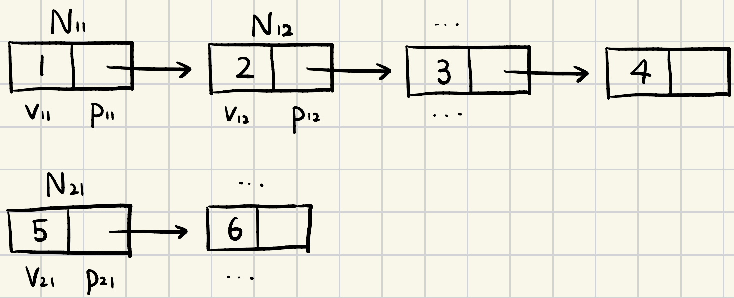
$$= \Theta(n) + 10 \cdot \frac{1 - 1.5 \cdot \frac{n}{10}}{1 - 1.5}$$

$$= \Theta(n) + \Theta(n) = \Theta(n)$$

Problem 2. Linked List Recursion Tracing

(a) List 1:

List 2:



Function	Calls	Returns
llrec(N_{11}, N_{21})	p_{11} = llrec(N_{21}, N_{12})	N_{11}
llrec(N_{21}, N_{12})	p_{21} = llrec(N_{12}, N_{22})	N_{21}
llrec(N_{12}, N_{22})	p_{21} = llrec(N_{22}, N_{13})	N_{12}
llrec(N_{22}, N_{13})	p_{21} = llrec(N_{13}, N_{23})	N_{22}
llrec(N_{13}, N_{23})	N_{23} = nullptr	N_{13}

$N_{11} \rightarrow N_{21} \rightarrow N_{12} \rightarrow N_{22} \rightarrow N_{13} \rightarrow N_{14}$

The new linked list is $1 \rightarrow 5 \rightarrow 2 \rightarrow 6 \rightarrow 3 \rightarrow 4$.

(b) Since in1 = nullptr, llrec(in1, in2) returns in2;

the linked list returned is 2.