Chapter 1

CSCI-1510

What is a Number?

- An expression of a numerical quantity
- A mathematical quantity
- Many types:
  - Natural Numbers
  - Real Numbers
  - Rational Numbers
  - Irrational Numbers
  - Complex Numbers
  - Etc.

Numerical Representation

- A quantity can be expressed in several ways
  - XIV
  - 1110_2
  - E_{16}
  - 16_8
  - Fourteen
  - Quatorze (French)

Positional Number Representation

Ex. 7,392

$$7 \times 10^3 + 3 \times 10^2 + 9 \times 10^1 + 2 \times 10^0$$

Positional Number Representation

$$10^4a_4 + 10^3a_3 + 10^2a_2 + 10^1a_1 + 10^0a_0.\ 10^{-1}a_{-1} + 10^{-2}a_{-2} + 10^{-3}a_{-3}$$

$$a_4a_3a_2a_1a_0.a_{-1}a_{-2}a_{-3}$$

Positional Number Representation

$$a_n r^n + a_{n-1} r^{n-1} + \cdots + a_2 r^2 + a_1 r^1 + a_0 r^0 . a_{-1} r^{-1} + a_{-2} r^{-2} + \cdots + a_{-m} r^{-m}$$

Where r is the radix or base.

What is the Base?

- The cardinality of the set of symbols in a number system
- The value of the highest symbol is always one less than the base.
- Denoted with a subscript
  - 1110_2
  - E_{16}
  - 16_8

Numerical Representation

- The base determines the set of symbols
  - Base 10: S = { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9}
  - Base 2: S = {0, 1}
  - Base 3: S = { 0, 1, 2}
  - Base 8: S = {0, 1, 2, 3, 4, 5, 6, 7}
  - Base 16: S = ?
    - Borrow the needed digits from the alphabet, so

      S = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F}

Some terminology

- Bits – digits in a binary number
- Byte – 8 bits
- Base 8 – Octal
- Base 10 - Decimal
- Base 16 – Hexadecimal or Hex
- Base 32 – Duotrigesimal

Why do we care about the base?

- Without knowing what base you are working in, there is no way to know what quantity is being enumerated.
  - 11 could mean 11_{10} or 3_{10}
  - 15 could mean 15_{10}, $15_8 = 13_{10}$, or $15_{16} = 21_{10}$

Why use Hexadecimal and Octal?

To avoid long strings of 1's and 0's.

Ex. 1011011001010000

Better written as: 1011 0110 0101 0000

$= B65F_{16}$