Linear IV Regression Estimators for Structural Dynamic Discrete Choice Models

Kalouptsidi, Scott, and Souza-Rodrigues, WP 2018

Presented by Tomás Domínguez-Iino

New York University

Motivation

Standard dynamic discrete choice estimators may be biased due to

1. Unobservable state variables that are serially correlated, or correlated with observable state variables (endogeneity)[1]
2. Measurement error in observable state variables

In a reduced form world we would use IV to deal with these issues. How can we use these IV's in a structural model?

[1] e.g. in Rust (1987) or Hotz and Miller (1993), the only unobservables are iid shocks.

This paper

- Euler equation in CCP (ECCP) method for estimating dynamic discrete choice models $\rightarrow$ we know how to do this for continuous choices (Hansen and Singleton, 1982).
- ECCP does not require stating how unobservable state variables evolve, and is robust to bias from measurement error or endogeneity.
- Recent papers have used this approach (Scott 2013, Diamond et al. 2018) but there hasn't been a comprehensive econometric treatment of the methodology.

Setup

- Agent i in market m at time t chooses an action a_{imt} to maximize discounted payoffs given the state

$$s_{imt} = \left(\underbrace{k_{imt}, w_{mt}}_{\text{observed}}, \underbrace{\eta_{mt}, \varepsilon_{imt}}_{\text{unobserved}} \right)$$

- Agents are small: i's actions affect k_{imt} but not $\omega_{mt} = (w_{mt}, \eta_{mt})$

$$F(s_{imt+1}|a, s_{imt}) = F^k(k_{imt+1}|a, k_{imt}, w_{mt})F^{\omega}(\omega_{mt+1}|\omega_{mt})F^{\varepsilon}(\varepsilon_{imt+1})$$

- Per period payoff includes unobservables other than iid errors

$$\Pi(a, s_{imt}) = \bar{\pi}(a, k_{imt}, w_{mt}) + \xi(a, k_{imt}, \omega_{mt}) + \varepsilon_{aimt}$$

Value functions

$$V(s_{imt}) = \max_{a \in \mathcal{A}} \quad \Pi(a, s_{imt}) + \beta E[V(s_{imt}) | a, s_{imt}] \tag{VF}$$

$$V(k_{imt}, \omega_{mt}) \equiv \int V(k_{imt}, \omega_{mt}, \varepsilon_{imt}) dF^{\varepsilon}(\varepsilon_{imt}) \tag{EA}$$

$$v_a(k_{imt}, \omega_{mt}) = \pi(a, k_{imt}, \omega_{mt}) + \beta E[V(k_{imt+1}, \omega_{mt+1}) | a, k_{imt}, \omega_{mt}] \tag{C}$$

Conditional choice probabilities

$$p_a(k, \omega) = \int 1\{v_a(k, \omega) + \varepsilon_a \geq v_j(k, \omega) + \varepsilon_j \quad, \forall j \in \mathcal{A}\} dF^{\varepsilon}(\varepsilon) \tag{CCP}$$

Important relationships

$$v_a(k, \omega) - v_j(k, \omega) = \psi_a(p(k, \omega)) - \psi_j(p(k, \omega)) \tag{HM Inversion}$$

$$V(k, \omega) - v_a(k, \omega) = \psi_a(p(k, \omega)) \tag{AM Lemma}$$

With no unobservable states ($\xi = 0$) we know how to estimate this:

1. Estimate CCP's directly from the data.
2. Choose a reference action j_r and set $\pi(j_r, k, w) = 0 \ \forall (k, w)$ so that you can solve for $v_{j_r}(k, w)$.
3. Back out $v_j(k, w) \ \forall j \neq j_r$ using the HM inversion.
4. Use conditional value functions to back out $\pi(a, k, w; \theta)$.
5. Estimate θ using $\pi(a, k, w; \theta)$.

If we apply this method to a DGP where $\xi \neq 0$ our estimates will be biased.

Key assumptions

1. Rational expectations (RE): allows us to write expected values as their realization + a forecast error that is serially uncorrelated

2. Finite dependence (FD): allows future value functions to cancel out.

3. Instruments (IV) : $E[\tilde{\xi}_{ajmt}|z_{mt}] = 0$ and $E[\tilde{e}^V_{ajmt}|z_{mt}] = 0$

Obtaining the ECCP equations

From Conditional VF + AM Lemma

$$\pi(a, k_{imt}, \omega_{mt}) = V(k_{imt}, \omega_{mt}) - \beta \underbrace{E\left[V(k_{imt+1}, \omega_{imt+1}) | a, k_{imt}, \omega_{imt}\right]}_{\text{realization + a forecast error under RE}} - \psi_a(k_{imt}, \omega_{mt})$$

In matrix form,

$$\pi_{amt} = V_{mt} - \psi_{amt} - \beta F^k_{amt} V_{mt+1} - \beta e^V_{am,t,t+1} \qquad \text{(MF)}$$

Taking differences wrt to another action j eliminates V_{mt}

$$\underbrace{\psi_{jmt} - \psi_{amt}}_{observable} = \underbrace{\bar{\pi}_{amt} - \bar{\pi}_{jmt}}_{observable} + \underbrace{\xi_{amt} - \xi_{jmt} + \beta(e^V_{am,t,t+1} - e^V_{jm,t,t+1})}_{unobservable} - \underbrace{\beta(F^k_{jmt} - F^k_{amt})}_{observable} \underbrace{V_{mt+1}}_{\text{remove with FD}}$$

- Work recursively on MF using a renewal action J.

$$\begin{aligned}\pi_{amt} =& V_{mt} - \psi_{amt} - \beta e^{V}_{am,t,t+1} \\ & - \beta F^k_{amt}\left[\pi_{Jmt+1} + \beta F^k_{amt} E_{t+1}[V_{mt+2}] + \psi_{Jmt+1}\right]\end{aligned}$$

- Do the same for j and substract from $a \rightarrow V_{mt}$ and $E_{t+1}[V_{mt+2}]$ will cancel since $F^k_{amt}F^k_{Jmt} = F^k_{jmt}F^k_{Jmt}$ (due to FD). The ECCP is then,

$$\begin{aligned}\pi_{jmt} - \pi_{amt} + \beta(F^k_{jmt} - F^k_{amt})\psi_{Jmt+1} =& \bar{\pi}_{amt} - \bar{\pi}_{jmt} - \beta(F^k_{jmt} - F^k_{amt})\bar{\pi}_{Jmt+1} \\ & + u_{ajmt}\end{aligned}$$

$$\begin{aligned}\text{where} \quad & u_{ajmt} \equiv \tilde{\xi}_{ajmt} + \tilde{e}^V_{ajmt} \\ & \tilde{\xi}_{ajmt} \equiv (\xi_{amt} - \xi_{jmt}) + \beta(F^k_{amt} - F^k_{jmt})\xi_{Jmt+1} \\ & \tilde{e}^V_{ajmt} = \beta(e^V_{am,t,t+1} - e^V_{jm,t,t+1})\end{aligned}$$

Estimation

Two step implementation

1. Estimate $\delta_{amt} = \left\{ p_{amt}(k), F^k_{amt}(k'|k) \right\}$ nonparametrically
2. Use Step 1 results to construct $u_{mt}\left(\theta, \delta_{mt}\right)$ residually and use instruments z_{mt} to construct moment conditions.

$$E[h(z_{mt})u_{mt}\left(\theta, \delta_{mt}\right)] = 0$$

Durable goods application

- $a \in \{\text{b, nb}\}$, $k \in \{0, 1\}$, w_{mt} = price, ξ_{mt} = unobserved quality
- $\pi(a, k_{imt}, \omega_{mt}) = \begin{cases} \theta_0 + \theta_1 w_{mt} + \xi_{mt} & \text{if } a = b \\ \theta_0 & \text{if } a = nb, k_{imt} = 1 \\ 0 & k_{imt} = 0 \end{cases}$
- ECCP equation

$$\ln\left(\frac{p_b(0, \omega_{mt})}{p_{nb}(0, \omega_{mt})}\right) + \beta \ln\left(\frac{p_b(1, \omega_{mt+1})}{p_{nb}(0, \omega_{mt+1})}\right) = \theta_0 + \theta_1 w_{mt} + u_{mt}$$

$$\text{where } u_{mt} \equiv \xi_{mt} + \beta\left(e^V(1, \omega_{mt}, \omega_{mt+1}) - e^V(0, \omega_{mt}, \omega_{mt+1})\right)$$

- Instrument for w_{mt} with a cost-shifter.
- Monte Carlo results show Standard CCP and ECCP OLS are biased, ECCP IV is not.

Advantages and Limitations of ECCP

Advantages

- Estimates are robust to endogeneity, as evident from MC results.
- Do not need to take a stand on how market level states evolve, and thus computationally light.
- The method can also leverage quasi-experimental data: instead of deriving a linear IV regression equation from the structural model, Diamond et al (2018) derive a diff-in-diff style equation to exploit a quasi-experiment.

Limitations

- Agents must be atomistic $\rightarrow$ rules out precisely the questions that define IO as a field, but it may be useful for other fields.
- Since we do not take a stand on how market level states evolve, the type of counterfactuals we may carry out are limited.