# Demand Estimation in Models of Imperfect Competition

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# Outline

# Why don't we just regress price on quantity?

- Consider linear demand

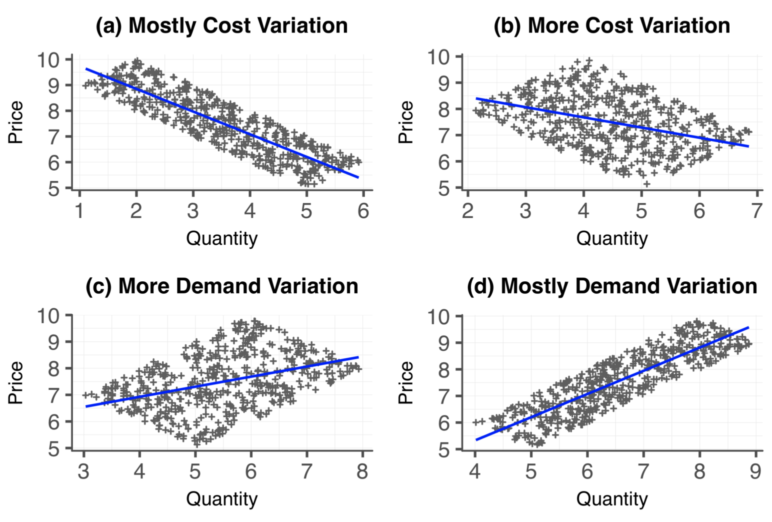
$$q_t = \alpha + \beta p_t + \xi_t, \qquad \mathbb{E}[\xi_t] = 0$$

  and marginal cost

$$c_t = \gamma + \eta_t, \qquad \mathbb{E}[\eta_t] = 0$$

  with a monopolist maximizing profits
- Econometrician observes $(q_t, p_t)_t$, but not $(\xi_t, \eta_t)_t$

# Why don't we just regress price on quantity?




$$\beta = -1$$

# Price is endogenous

- Prices are a function of unobserved demand shocks, so we don't recover the causal $\beta$
- Solution #1: IVs, *e.g.*
  - competing product attributes
  - prices of same good in other markets
  - shifts in equilibrium concept
- Solution #2: Estimate via maximum likelihood if demand and cost shock distributions known
- Solution #3: Bound supply or demand slopes using covariance restrictions in models of perfect competition or monopolistic competition with CES

# Instrument-free demand estimation

- **This paper**: Solution #4: use assumptions in models of imperfect competition of price responses to demand shocks to correct for bias in OLS estimate
    - OLS estimate of price coefficient $\hat{\beta}^{OLS}$ captures a combination of the causal $\beta$ and firms' endogenous responses
    - A supply-side model of firm behavior can provide bounds on possible $\beta$
    - If we impose a covariance restriction on $\xi$ and $\eta$, we can obtain tighter bounds and in some cases consistent point identification of $\beta$
- **Bottom line**: Can consistently and easily estimate demand parameters without requiring instruments (*under certain conditions*)

# Outline

Introduction

Building Intuition: Monopoly pricing

Differentiated-products Bertrand Competition

Conclusion

# Setup

- Markets $t = 1, \ldots, T$
- Linear demand $q_t = \alpha + \beta p_t + \xi_t$, $\beta < 0$
- Marginal cost $c_t = \gamma + \eta_t$
- $\mathbb{E}[\xi_t] = \mathbb{E}[\eta_t] = 0$
- Monopolist maximizes profits:

$$p_t = \underbrace{\gamma + \eta_t}_{\text{marginal cost}} - \underbrace{\left(\frac{\mathrm{d}q}{\mathrm{d}p}\right)^{-1} q_t}_{\text{markup}}$$

# Setup

- Econometrician observes

$$p = [p_1, \ldots, p_T]' \text{ and } q = [q_1, \ldots, q_T]'$$

- Econometrician naively runs OLS

$$\hat{\beta}^{OLS} = \frac{\hat{Cov}(p, q)}{\hat{Var}(p)} \xrightarrow{p} \beta + \underbrace{\frac{Cov(\xi, p)}{Var(p)}}_{\text{bias}}$$

# Proposition 1

- Numerator of the bias can be decomposed into covariance between $\xi$ and marginal cost and covariance between $\xi$ and markup
- **Assume**: $Cov(\xi, \eta) = 0$

**Proposition 1**: Let $(\alpha^{OLS}, \beta^{OLS})$ be the probability limits of $(\hat{\alpha}^{OLS}, \hat{\beta}^{OLS})$ and $\xi_t^{OLS} = q_t - \alpha^{OLS} - \beta^{OLS} p_t$. Then

$$\beta^{OLS} \equiv plim\left(\hat{\beta}^{OLS}\right) = \beta - \frac{1}{\beta + \frac{Cov(p,q)}{Var(p)}} \frac{Cov(\xi^{OLS}, q)}{Var(p)}$$

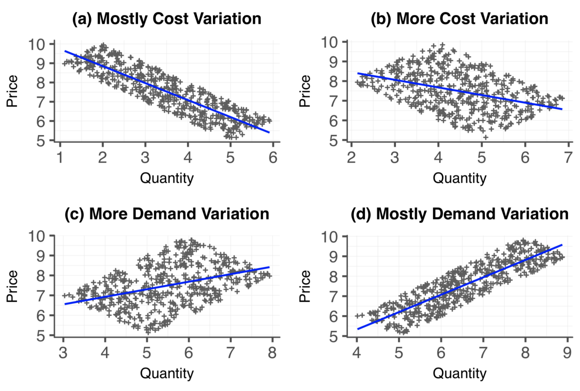
**Proposition 2**: Rearranging,

$$\beta^2 + \beta \left( \frac{Cov(p,q)}{Var(p)} - \beta^{OLS} \right) + \left( - \frac{Cov(\xi^{OLS}, q)}{Var(p)} - \frac{Cov(p,q)}{Var(p)} \beta^{OLS} \right) = 0$$

This has two roots, which are real, one positive, one negative. $\beta < 0$ implies the lower root:

$$\hat{\beta}^{3-stage} = -\sqrt{\left(\hat{\beta}^{OLS}\right)^2 + \frac{Cov(\hat{\xi}^{OLS}, q)}{Var(p)}}$$

# Back to the regressions



# Back to the regressions

| | (1) $Var(\eta) \gg Var(\xi)$ | (2) $Var(\eta) > Var(\xi)$ | (3) $Var(\eta) < Var(\xi)$ | (4) $Var(\eta) \ll Var(\xi)$ |
|---|---|---|---|---|
| $\hat{\beta}^{OLS}$ | -0.87 | -0.42 | 0.42 | 0.88 |
| $Var(q)$ | 1.42 | 1.11 | 1.20 | 1.36 |
| $Var(p)$ | 1.44 | 1.01 | 1.09 | 1.35 |
| $Cov(\hat{\xi}^{OLS}, q)$ | 0.33 | 0.92 | 1.01 | 0.32 |
| $\frac{Cov(\hat{\xi}^{OLS}, q)}{Var(p)}$ | 0.23 | 0.91 | 0.93 | 0.24 |
| $\hat{\beta}^{\textit{3-Stage}}$ | -0.995 | -1.045 | -1.051 | -1.003 |

# Outline

# Demand Setup

- $j = 1, \ldots, J$ products in $t = 1, \ldots, T$ periods
- Econometrician observes prices $p_t = [p_{1t}, \ldots, p_{Jt}]'$, quantities $q_t = [q_{1t}, \ldots, q_{Jt}]'$, and covariates $X_t = [x_{1t}, \ldots, x_{Jt}]'$
  - $\mathbb{E}[X\xi] = \mathbb{E}[X\eta] = 0$
- **Assumption**: The demand schedule for each product is determined by the following semi-linear form:

$$h_{jt} = h(q_{jt}, w_{jt}; \sigma) = \beta p_{jt} + x_{jt}'\alpha + \xi_{jt},$$

  where

  1. $\frac{\partial h_{jt}}{\partial q_{jt}} > 0$
  2. $w_{jt}$ is a vector of observables and $\sigma$ is a parameter vector
  3. total derivative of $h(\cdot)$ w.r.t. $q$ is function of data and $\sigma$

# Semi-linear demand form

$$h_{jt} = h(q_{jt}, w_{jt}; \sigma) = \beta p_{jt} + x'_{jt}\alpha + \xi_{jt}$$

- Additive separability in prices, covariates, shocks after transformation of quantities using observables and non-linear parameters
- May seem like a heroic assumption, but nests some common demand systems, *e.g.*
  - Logit demand system: $h(q_{jt}, w_{jt}; \sigma) = \log s_{jt} - \log s_{0t}$
  - Nested logit: $h(q_{jt}, w_{jt}; \sigma) = \log s_{jt} - \log s_{0t} - \sigma \log \bar{s}_{j|g,t}$
  - BLP random coefficients: $h(q_{jt}, w_{jt}; \sigma)$ is the mean utility $\delta_{jt}$ (calculated using BLP contraction mapping)

# Supply Setup

- Each firm sells a single product and maximizes profits subject to the demand system earlier
- Takes other firms' prices as given
- **Assume**: Each firms' marginal cost schedule is linear and of the form

$$c_{jt} = x_{jt}'\gamma + \eta_{jt}$$

- Firms satisfy their first order conditions

$$p_{jt} = c_{jt} - \frac{1}{\beta}\frac{\mathrm{d}h_{jt}}{\mathrm{d}q_{jt}}q_{jt}$$

# Identification

- Let $p^*$ be vector of residuals from regression of $p$ on $x$

$$\beta^{OLS} = \frac{Cov(p^*, h)}{Var(p^*)} = \beta + \frac{Cov(p^*, \xi)}{Var(p^*)}$$

$$= \beta + \frac{Cov(\eta, \xi)}{Var(p^*)} - \frac{1}{\beta} \frac{Cov\left(\frac{\mathrm{d}h}{\mathrm{d}q} q, \xi\right)}{Var(p^*)}$$

- Rearranging and substituting

$$0 = \beta^2 + \left( \frac{Cov(\eta, \xi)}{Var(p^*)} + \frac{Cov\left(\frac{\mathrm{d}h}{\mathrm{d}q} q, \xi\right)}{Var(p^*)} - \beta^{OLS} \right) \beta$$

$$+ \left( -\beta^{OLS} \frac{Cov\left(\frac{\mathrm{d}h}{\mathrm{d}q} q, \xi\right)}{Var(p^*)} - \frac{Cov\left(\xi^{OLS}, \frac{\mathrm{d}h}{\mathrm{d}q} q\right)}{Var(p^*)} \right)$$

# Point Identification

- If we know $Cov(\xi, \eta)$, maximum of two such $\beta$
- $\beta$ is the lower root if

$$0 \leq \beta^{OLS} \frac{Cov\left(p^*, \frac{\mathrm{d}h}{\mathrm{d}q} q\right)}{Var(p^*)} + \frac{Cov\left(\xi^{OLS}, \frac{\mathrm{d}h}{\mathrm{d}q} q\right)}{Var(p^*)}$$

and $\beta$ is the lower root if and only if

$$-\frac{1}{\beta} \frac{Cov(\eta, \xi)}{Var(p^*)} \leq \frac{Cov\left(p^*, -\frac{1}{\beta}\xi\right)}{Var(p^*)} + \frac{Cov(p^*, \eta)}{Var(p^*)}$$

- We have two separate conditions, if either holds then $\beta$ is point identified; otherwise, set identified with two elements

# Bounds

- Now assume prior is $m \leq Cov(\xi, \eta) \leq n$
- Each $Cov(\xi, \eta)$ maps to one or two roots
- If one of the two previous conditions is met, then a convex prior corresponds to a convex posterior, *e.g.*
  - $Cov(\xi, \eta) \geq m \Rightarrow \beta \in (-\infty, r(m)]$
  - $Cov(\xi, \eta) \leq m \Rightarrow \beta \in [r(m), 0)$

  where $r(m)$ is lower root evaluated at $Cov(\xi, \eta) = m$
- Even if no prior, the quadratic structure places bounds on valid $\beta$

# Estimation

- Assume we know *Cov*$(\xi, \eta)$ and the conditions for point-identification hold
- Two methods for determining $\hat{\beta}$
  1. Apply the quadratic formula to find the appropriate root (three-stage estimator)
  2. Recast covariance restriction as a moment restriction and proceed with method of moments

## Estimation

- Assume we know $Cov(\xi, \eta)$ and the conditions for point-identification hold
- Two methods for determining $\hat{\beta}$
  1. Apply the quadratic formula to find the appropriate root (three-stage estimator)
     - (i) regress $h(q)$ on $p$ and $x$, (ii) regress $p$ on $x$ to obtain $p^*$, (iii) apply quadratic formula
  2. Recast covariance restriction as a moment restriction and proceed with method of moments

# Estimation

- Assume we know $Cov(\xi, \eta)$ and the conditions for point-identification hold
- Two methods for determining $\hat{\beta}$
    1. Apply the quadratic formula to find the appropriate root (three-stage estimator)
    2. Recast covariance restriction as a moment restriction and proceed with method of moments
        - restrict the search for $\beta$ to the appropriate root

# Estimation

- Assume we know $Cov(\xi, \eta)$ and the conditions for point-identification hold
- Two methods for determining $\hat{\beta}$
  1. Apply the quadratic formula to find the appropriate root (three-stage estimator)
  2. Recast covariance restriction as a moment restriction and proceed with method of moments
     - restrict the search for $\beta$ to the appropriate root
     - good for (i) $\frac{\mathrm{d}h}{\mathrm{d}q}$ hard to solve analytically, (ii) can combine with instruments, (iii) can allow more flexible covariance structure between covariates in supply and demand shocks

# Monte Carlo Simulations

(a) Three-Stage: Mean Price Coefficient and Standard Errors

| Observations | (1) $Var(\eta) \gg Var(\xi)$ | | (2) $Var(\eta) > Var(\xi)$ | | (3) $Var(\eta) < Var(\xi)$ | | (4) $Var(\eta) \ll Var(\xi)$ | |
|---|---|---|---|---|---|---|---|---|
| 25 | -1.004 | (0.098) | -1.017 | (0.201) | -1.018 | (0.206) | -1.005 | (0.099) |
| 50 | -1.001 | (0.068) | -1.008 | (0.136) | -1.007 | (0.135) | -1.001 | (0.068) |
| 100 | -1.001 | (0.047) | -1.003 | (0.094) | -1.004 | (0.093) | -1.001 | (0.047) |
| 500 | -1.000 | (0.021) | -1.001 | (0.041) | -1.001 | (0.042) | -1.000 | (0.021) |

(b) Instrumental Variables: Mean Price Coefficient and Standard Errors

| Observations | (1) $Var(\eta) \gg Var(\xi)$ | | (2) $Var(\eta) > Var(\xi)$ | | (3) $Var(\eta) < Var(\xi)$ | | (4) $Var(\eta) \ll Var(\xi)$ | |
|---|---|---|---|---|---|---|---|---|
| 25 | -1.004 | (0.105) | -1.039 | (0.303) | -1.294 | (3.721) | -5.481 | (263.202) |
| 50 | -1.001 | (0.072) | -1.018 | (0.201) | -1.120 | (1.829) | -2.285 | (333.019) |
| 100 | -1.001 | (0.050) | -1.008 | (0.138) | -1.048 | (0.332) | -1.473 | (12.630) |
| 500 | -1.000 | (0.022) | -1.001 | (0.060) | -1.009 | (0.138) | -1.061 | (0.411) |

# Generalizations

- Non-constant marginal costs
  - need to have some knowledge about $g(\cdot;\cdot)$

$$c_{jt} = x_{jt}'\gamma + g(q_{jt};\lambda) + \eta_{jt}$$

- Multi-product firms
  - just changes the firms' first order conditions to be a little more complicated
- Alternative models of competition
  - Cournot
  - multiplicative markups (need demand to be semi-linear in *log* prices

# Outline

# Conclusion

- **Advantages**
    - does not require us to find instruments
    - can complement instruments when they are available
- **Disadvantages**
    - requires strict (but common) assumptions on structure of demand and supply
    - requires us to be very confident about institutional details in order to make an assumption as strong as $Cov(\xi, \eta) = 0$ (how often can we do this?)
    - how useful is putting bounds on $\beta$?