Rust v. Hotz-Miller

Variables:

- x_t is state, $x_t \in \mathbb{R}^+$
- i_t is action, $i_t \in \{0,1\}$
- ε_t is choice specific shock
- $\bar{\theta} = (\theta_1, \theta_2, \theta_3)$
- θ_1 is params in payoff
- (θ_2, θ_3) is params in transition prob.

Objective Function

$$u(i_t, x_t, \theta_1) = \begin{cases} -c(x_t, \theta_1) + \varepsilon_t(0) & \text{if } i_t = 0 \\ -(RC - c(0, \theta_1)) + \varepsilon_t(1) & \text{if } i_t = 1 \end{cases}$$

Value Function

$$V_\theta(x, \varepsilon) = \max_i \left[u(i, x, \theta) + \beta EV_\theta(x, \varepsilon, i) \right]$$

where,

$$EV_\theta = \int_{|X| \times |\varepsilon|} V_\theta(y, \eta)\, p(dy, d\eta \mid x, \varepsilon, i, \theta_2, \theta_3)$$

Assumption: Conditional independence

$$p(x', \varepsilon' \mid x, \varepsilon, i, \theta_2, \theta_3) = q(\varepsilon' \mid x', \theta_2)\, p(x' \mid x, i, \theta_3)$$

Before assumption		After Assumption
$(x, \varepsilon, i, \theta_2, \theta_3) \rightarrow (x', \varepsilon')$	$\Rightarrow$	$(x, i, \theta_3, \theta_2) \rightarrow x' \rightarrow \varepsilon'$
$EV_\theta(x, \varepsilon, i)$	$\Rightarrow$	$EV_\theta(x, i)$

Assume TEV-1, we get

$$(A) \qquad P(i|x,\theta) = \frac{\exp(v_\theta(x,1))}{1+\exp(v_\theta(x,1))}$$

Rust's Approach:

(1) Use data to find $\hat{p}(x'|x,i,\theta_3)$ + $\hat{q}(\varepsilon'|x')$

(2) Fix guess $\tilde{\theta}_1$

By value function iteration (3):

$$V_\theta(x,\varepsilon_i) = \max_i u(i) + \varepsilon(i) + \beta E_{V_\theta}(i,x)$$

(3) Using (A) evaluate:

$$p(i,x|i',x') = \underbrace{p(i|x,\theta)}_{\text{from (A)}} \underbrace{p(x|x',i',\theta_3)}_{\text{from step (1)}}$$

Hotz-Miller

(1) use data to find $\hat{p}(x'|x,i,\theta_3)$

(2) By inversion + normalization

$$V_\theta(x,1) - \underbrace{V_\theta(x,0)}_{=0} = \hat{P}(1|x,\theta) - \hat{p}(0|x,\theta)$$

(3) Go back to value function:

$$\underbrace{v_\theta(x,i)}_{\text{find in step (2)}} = \underbrace{u(x,i,\theta_1)}_{\substack{\text{Left to}\\ \text{find estimate}\\ \text{of } \theta_1}} + \underbrace{\beta E V_\theta(x,i)}_{\substack{\text{Ex-ante value}\\ \text{function known}\\ \text{with logit errors}\\ \text{(using } v_\theta(x,i) \text{ found in (2))}}}$$

Form guesses of probabilities:

$$\tilde{p}(x,i,\theta) = \frac{\exp\left(u(x,1,\theta_2) + \beta \tilde{E} V_\theta\right)}{1 + \exp\left(u(x,1,\theta_2) + \beta \tilde{E} V_\theta\right)}$$