Estimating Dynamic Models of Imperfect Competition

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Takeaways

- New 2-step estimation procedure for dynamic discrete choice models.
- Provide both point and set identification estimation techniques.
- Limitations:
  - Method relies on single-dimensional agent uncertainty (monotonicity requirement)
  - Discrete choice relies on additive seperability
  - Perturbation techniques for finding new moment conditions is very local

Primitives

Notation:

- i is firm
- t is time
- s_{it} is state of firm i at time t
- a_{it} is action of firm i at time t
- v_{it} is private shock observable to only firm i at time t

Exogenous transition equations:

- $\mathcal{P}(s_{t+1}|s_t, a_t; \theta_1)$ - state transition (by assumption)
- $\mathcal{G}(\cdot|s_t; \theta_1)$ - private shocks

Parameters to estimate:

- $\theta = (\theta_1, \theta_2)$
- θ_1 are parameters estimated in step 1
- θ_2 are parameters estimated in step 2

Profit function:

- $\pi(a_{it}, s, v_{it}; \theta_2)$

Policy function (Markovian):

- $\sigma_i(s, v)$

2-Step Estimation

Step 1 Estimate policy and transition functions (possibly θ_1) using data

- Estimate $\mathcal{P}(s_{t+1}|s_t, a_t)$ - can do this with observed data
- Estimate policy function (depends on setup):
  - Discrete choice setting - can use Hotz-Miller inversion
  - Continous choice setting - rely on monotonicity assumptions (and invertibility of $\mathcal{G}$)

Step 2 Estimate θ_2 using forward simulation + minimum distance estimator

- Estimate value function $V_i(s; \sigma; \theta)$ by drawing shocks and state transitions and using σ from Step 1
- Find θ_2 by using equilibrium conditions

Assumptions

ES Data are generated by a single markov perfect eq profile σ

H1 Transition functions are markovian and depend on states and action

H2 Profit function linear in parameters:

$$\pi(a_{it}, s, v_{it}; \theta_2) = \theta_2 \pi(a_{it}, s, v_{it})$$

- For Discrete Choice model

  DC Choice specific error terms $v(a_i)$ and add-seperability of profits:

  $$\pi(a_{it}, s, v_{it}) = \tilde{\pi}(a_{it}, s) + v(a_{it})$$

- For Continuous Choice model

Monotone For each agent i, A_i, V_i, $\pi(a_{it}, s, v_{it})$ has increasing differences in (a_i, v_i) ($\partial^2 \pi / (\partial a_i \partial v_i) \geq 0$ for continuously differentiable profit fcns)

Estimating θ_2 with minimum distance

Use equilibrium conditions and perturbation of policy function to estimate parameters:

$$g(x;\theta,\alpha) = V_i(s;\theta_i,\sigma'_{-i};\theta_2) - V_i(s;\sigma'_i,\sigma_{-i};\theta_2)$$

Eq condition is violated if:

$$g < 0$$

Objective function is simply:

$$Q(\theta) = \int \big(\min\{g(x;\theta_2),0\}^2\big) dH(x)$$

where, $H(x)$ is the set of potential combos of (i, s, θ'_i)
An example of to form $H(x)$ is to perturb the policy function (Ellison, Snyder and Zhang 2016):

$$\sigma'(s, v_i) = \sigma_i(s, v_i) + \epsilon$$

Research Going Forward

- Multi-dimensional uncertainty (weakening monotonicity assumption)
- Good applications with multiple equilibria techniques
- Different specifications of errors (correlated across agents/time)
- More efficient construction of moments