Derivatives

COMP0164 Lecture 5 (Week 10)

Geoff Goodell (University College London)

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Agenda

Readings

- Hull chapters **2–3, 5–7**[1]
- Bodie Kane Marcus chapters **22–23**

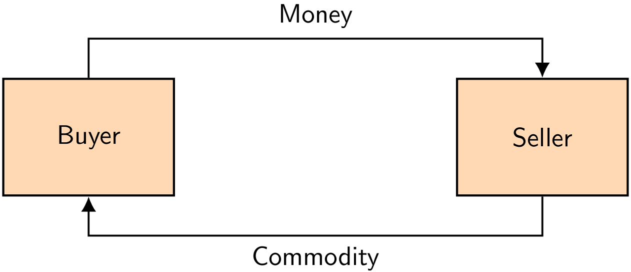
Topics

- Forward contracts
- Margin
- Futures markets
  - Equities
  - Rates
  - Commodities
  - Currencies
- Cost of carry
- Investment versus consumption assets
- Interest rate swaps
- Currency swaps

[1] Direct source of examples in Slides 30-35.

Forward contracts

forward contract: sale of some asset with deferred delivery and a price agreed in advance (the **forward price**).



The contract is an **obligation** of both parties to transact, designed to protect both the buyer and the seller from price fluctuations in the future.

Investment assets and consumption assets

investment asset: asset normally held for investment purposes

- financial assets (e.g. stocks, bonds)
- precious metals (e.g. gold, silver)

consumption asset: asset <u>not</u> normally held for investment purposes

- industrial metals (e.g. copper, aluminium)
- agricultural products (e.g. orange juice, pork bellies)
- energy products (e.g. natural gas, heating oil)

Forward price of an investment asset

The forward price of an investment asset can be given as:

$$F_0 = S_0 e^{rT} \tag{1}$$

- F_0 = initial forward price
- S_0 = initial spot price
- r = risk-free rate
- T = time to maturity

Forward price of an investment asset with known income

If the investment asset shall generate a predictable **income** I, then:

$$F_0 = (S_0 - I)e^{rT} \tag{2}$$

- $F_0 =$ initial forward price
- $S_0 =$ initial spot price
- $r =$ risk-free rate
- $T =$ time to maturity

Forward price of an investment asset with known yield

If the investment asset shall generate income according with a predictable **yield** q over the time of the forward contract, then:

$$F_0 e^{qT} = S_0 e^{rT} \tag{3}$$

$$F_0 = S_0 e^{(r-q)T} \tag{4}$$

- $F_0 =$ initial forward price
- $S_0 =$ initial spot price
- $r =$ risk-free rate
- $T =$ time to maturity

Value of a forward contract

The present value of a forward contract can be given by:

$$PV = (F_0 - K)e^{-rT} \tag{5}$$

- PV = present value of the forward contract
- F_0 = initial forward price
- K = agreed-upon **delivery price**
- r = risk-free rate
- T = time to maturity

This equation can be combined with the equations for the forward value of an investment asset to express the present value of the forward contract in terms of the current price of the asset.

Forward prices of equity indices

Given a predicted **dividend yield** q of an equity index over time period T, then the forward price of the index is given by:

$$F_0 = S_0 e^{(r-q)T} \tag{6}$$

- F_0 = initial forward price
- S_0 = initial spot price
- r = risk-free rate
- T = time to maturity

Index arbitrage involves either:

- buying the stocks and shorting forward contracts if $F_0 > S_0 e^{(r-q)T}$
- buying forward contracts and shorting the stocks if $F_0 < S_0 e^{(r-q)T}$

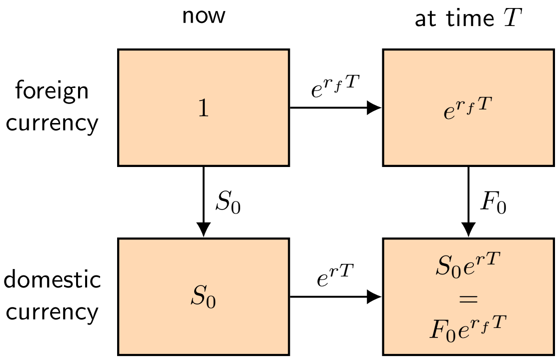
Forward prices of currencies (1/2)

Given **domestic** and **foreign** risk-free rates r and r_f, respectively:

$$F_0 = S_0 e^{(r - r_f)T} \tag{7}$$

- $F_0 =$ initial forward price
- $S_0 =$ initial spot price
- $r =$ risk-free rate for the **domestic** currency
- $r_f =$ risk-free rate for the **foreign** currency
- $T =$ time to maturity

Forward prices of currencies (2/2)



- F_0 = initial forward price
- S_0 = initial spot price
- r = risk-free rate for the domestic currency
- r_f = risk-free rate for the foreign currency
- T = time to maturity

Forward prices of commodities

Physical commodities have other features, such as:

- **storage costs**
- **income**, e.g. to lenders of investment commodities

$$F_0 = (S_0 + U)e^{rT} \tag{8}$$

$$F_0 = S_0 e^{(r+u)T} \tag{9}$$

- $U =$ PV of storage costs minus income, over the life of the contract
- $u =$ storage costs minus income, as a constant proportion of S
- $F_0 =$ initial forward price
- $S_0 =$ initial spot price
- $r =$ risk-free rate
- $T =$ time to maturity

Convenience yield

convenience yield: specific value y of holding a physical asset, e.g. to keep a production process running or to profit from temporary local shortages

$$F_0 e^{yT} = (S_0 + U)e^{rT} \tag{10}$$
$$F_0 e^{yT} = S_0 e^{(r+u)T} \tag{11}$$
$$F_0 = S_0 e^{(r+u-y)T} \tag{12}$$

- $U =$ PV of storage costs minus income, over the life of the contract
- $u =$ storage costs minus income, as a constant proportion of S
- $F_0 =$ initial forward price
- $S_0 =$ initial spot price
- $r =$ risk-free rate
- $T =$ time to maturity

Cost of carry

cost of carry: the storage cost plus the interest paid to finance the asset minus the income earned on the asset

$$\text{for \textbf{investment} assets:} \quad F_0 = S_0 e^{cT} \tag{13}$$

$$\text{for \textbf{consumption} assets:} \quad F_0 = S_0 e^{(c-y)T} \tag{14}$$

- $c =$ **cost of carry**, e.g.
  - $r - q$ for an equity index, where q is the dividend yield
  - $r - r_f$ for a foreign currency, where r_f is the foreign risk-free rate
  - $r - q + u$ for a commodity with income q and storage cost u
- $F_0 =$ initial forward price
- $S_0 =$ initial spot price
- $r =$ risk-free rate
- $T =$ time to maturity
- $y =$ convenience yield

Futures markets

futures contract: **fungible**, **standardised** contract for delivery of a specific **commodity** at a specific **delivery** or **maturity** date for an agreed-upon **price** (the **futures price**), to be paid at contract maturity

futures market: market for trading **futures contracts** wherein buyers and sellers trade in a centralised futures exchange (wherein some **flexibility** is sacrificed for **liquidity**)

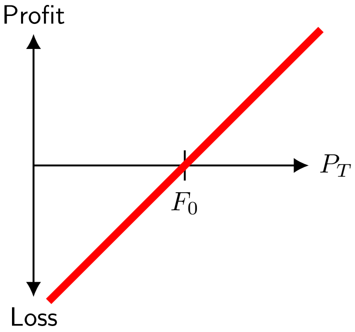
The **futures exchange** establishes features of the contract:

- **size** of the contract (e.g. mass, volume, number of units)
- acceptable **grade** of the commodity
- contract **delivery dates**
- nature of **settlement** (e.g. cash, warehouse receipts)

The trader with the **long position** (the buyer) commits to purchasing the commodity on the delivery date.

The trader with the **short position** (the seller) commits to delivering the commodity on the delivery date.

Payoff function for futures contracts



profit to **long** $= P_T - F_0$

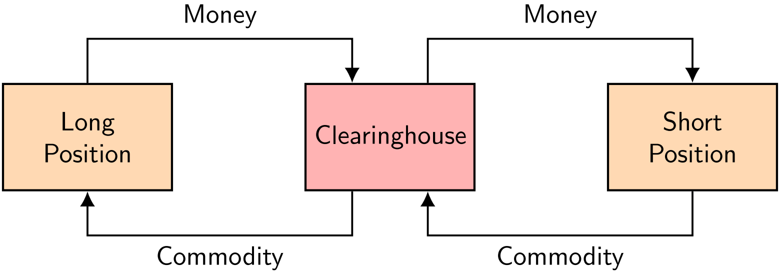
profit to **short** $= F_0 - P_T$

- $F_0 =$ initial futures contract price
- $P_T =$ spot price at maturity

convergence property: at maturity, the futures price and spot price must converge; $F_T = P_T$

The clearinghouse

clearinghouse: designated intermediary between a buyer and seller in a financial market [that] validates and finalises the transaction, ensuring that both the buyer and the seller honour their contractual obligations
`https://www.investopedia.com/terms/c/clearinghouse.asp`



Traders on both sides face the **clearinghouse** rather than each other.

The **clearinghouse** bears the risk of non-performance by any trader.

Contracts are therefore **fungible**: traders can **reverse** a position by entering the countervailing position with the clearinghouse.

Margin and open interest

margin: a good-faith deposit to guarantee contract performance

marking to market: daily settling (**realising**) of gains and losses with the clearinghouse [note special tax treatment]

maintenance margin: minimum quantity that a trader must hold in reserve with the clearinghouse

- Margin safeguards the position of the clearinghouse.
- If the mark-to-market value of the trader's account falls below the maintenance margin, the trader receieves a **margin call**.
- A **margin call** requires the trader to replenish the margin account to the maintenance margin. Otherwise, the position is reduced to a size commensurate with the remaining funds (the trader is **"bought in"** by the exchange).

open interest: the number of contracts outstanding

- The position of the clearinghouse always nets out to zero.

Examples of futures contracts

Commodities:

- agricultural (e.g. oats)
- metals (e.g. copper, gold)
- energy (e.g. crude oil)

Financial assets:

- foreign currencies (e.g. euro)
- interest rates (e.g. gilts)
- equity indices (e.g. Nikkei 225)
- single stocks

Delivery can be specified as **physical** or **cash-settled**.

- Cash settlement provides the trader with the same profit that would result from directly purchasing the units in the spot market.

Hedging and speculation

Hedgers use futures to insulate themselves (**hedge**) against price movements in the underlying asset.

- Example: both airplane manufacturers and bauxite miners might seek to hedge their exposure to the price of aluminium.

Speculators use futures to profit from movements in futures prices.

- Speculators take a **long** position if they expect an increase in price and a **short** position if they expect a decrease in price.
- Usually, **transaction costs** in futures markets are considerably smaller than in markets for the underlying asset.
- Speculators also gain the advantage of **leverage** because margin requirements are generally much less than the value of the underlying asset.

Cross hedging

hedge ratio h: ratio of the size of the position taken in futures contracts to the size of the exposure

- it is natural to use $h = 1$ when the asset underlying the futures contract is the same as the asset being hedged

Assuming a linear relationship between spot price S and futures price F over the life of the hedge, then:

$$\Delta S - h\Delta F = a + (b - h)\Delta F + \epsilon \tag{15}$$

- a, b are constants; ϵ is an error term

The **minimum variance hedge ratio** $h^* = b$:

$$h^* = \rho \frac{\sigma_S}{\sigma_F} \tag{16}$$

- $\sigma_S, \sigma_F =$ standard deviation of ΔS and ΔF, respectively
- $\rho =$ correlation coefficient between ΔS and ΔF

Optimal number of contracts

Using the hedge ratio, we can determine the optimal number of contracts to use:

$$N^* = \frac{h^* Q_A}{Q_F} \tag{17}$$

- N^* = optimal number of futures contracts
- h^* = minimum variance hedge ratio
- Q_A = number of units of position being hedged
- Q_F = number of units in one futures contract

Basis risk and hedging

basis: difference between the futures price and the spot price.

- The convergence property states that $F_T = P_T$ at maturity T, but prior to maturity, the two prices may differ.

basis risk: risk of gains or losses resulting from inconsistent price movement between the futures price and the spot price prior to maturity

- Some speculators seek to profit from changes in the basis.

calendar spread: strategy wherein an investor takes a long position in a futures contract of one maturity and a short position in a futures contract of a different maturity

- The investor profits if the futures price of the contract held long increases relative to the futures price of the contract held short.

Risk of a futures position

Ignoring daily settlement, if an investor's required return on a futures investment is k, then:

$$F_0 = E(S_T)e^{(r-k)T} \tag{18}$$

- F_0 = initial forward price
- r = risk-free rate
- T = time to maturity
- $E(S_T)$ = expected spot price at maturity

The relationship between k and r depends upon the **systemic risk** β of the underlying asset:

$\beta = 0$	$\Rightarrow$	$k = r$	$F_0 = E(S_T)$
$\beta > 0$	$\Rightarrow$	$k > r$	$F_0 < E(S_T)$
$\beta < 0$	$\Rightarrow$	$k < r$	$F_0 > E(S_T)$

Normal backwardation and contango

The terms **normal backwardation** and **contango** describe the state of a futures market:

- In **normal backwardation**, the futures price is less than the expected future spot price, $F_0 < E(S_T)$
- In **contango**, the futures price is greater than the expected future spot price, $F_0 > E(S_T)$

(In practice, these terms are sometimes used with reference to the current spot price rather than the expected future spot price.)

Interest rate futures: convexity adjustment

For **interest rate futures** longer than about two years, factors related to settlement influence the value of the contracts:

- Traders will tend to invest the proceeds from **daily settlement** at prevailing rates, so futures contracts for which there is a mark-to-market gain from an increase in rates (or loss from a decrease in rates) will outperform similar forward contracts.
- **Eurobond** futures are settled at the beginning of the period to which the rate applies, calculated as the present value of what the settlement would be if it were made at the end of the period, and the adverse impact of delaying settlement on holders of forward contracts is greater when there is a gain.

So:

$$\text{forward rate} = \text{futures rate} - c \tag{19}$$

- $c =$ convexity adjustment (note: $c > 0$)

Interest rate futures: estimating forward interest rates

Interest rate futures can be used to bootstrap zero curves (e.g. SONIA):

$$R_F = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1} \tag{20}$$

$$R_2 = \frac{R_F(T_2 - T_1) + R_1 T_1}{T_2} \tag{21}$$

- $R_1, R_2 =$ zero rate for maturities T_1, T_2 respectively
- R_F forward rate applicable to the period from T_1 to T_2

Interest rate futures: duration-based hedging

If we assume that the change in forward yield Δy is the same for all maturities, then:

$$\Delta P = -PD_P \Delta y \tag{22}$$

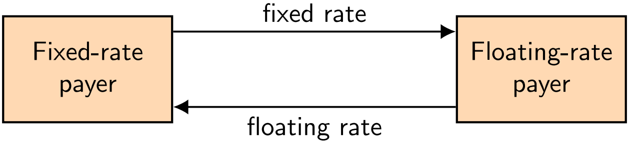
$$\Delta V_F = -V_F D_F \Delta y \tag{23}$$

$$N^* = \frac{PD_P}{V_F D_F} \tag{24}$$

- V_F = contract price for one interest rate futures contract
- D_F = duration of underlying asset at maturity of the futures contract
- P = forward value of the portfolio being hedged at maturity of the hedge (often assumed to be the same as the initial value of the portfolio)
- D_P = duration of the portfolio at the maturity of the hedge
- N^* = **duration-based hedge ratio**, the optimal number of contracts to hedge against uncertain Δy

Interest rate swaps

interest rate swap: arrangement, applied to some notional principal, wherein interest at a **predetermined fixed rate** is exchanged for interest at a **floating reference rate**, with one or more regular exchanges being made for an agreed period of time

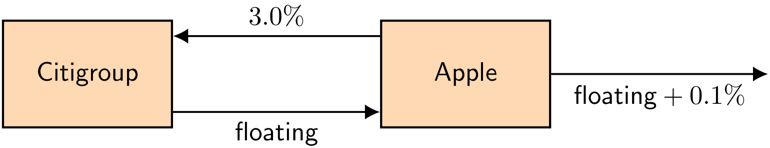


overnight indexed swap: interest rate swap wherein a fixed rate of interest (the **OIS rate**) is exchanged for a reference rate of interest calculated from a realised overnight rate (e.g. SONIA, SOFR).[2]

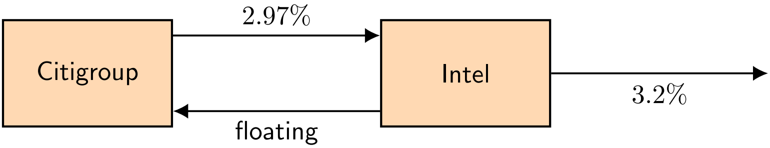
- If there is only one exchange, the OIS rate is a **risk-free zero rate** equivalent to the underlying overnight rate.
- Otherwise, the OIS rates define a risk-free bond worth par.

[2] An OIS can be contrasted with a LIBOR swap, wherein the LIBOR rate for a period is known at the start of the period, so the floating rate of the first exchange is known.

Apple uses a swap to convert floating-rate borrowings to fixed-rate borrowings:

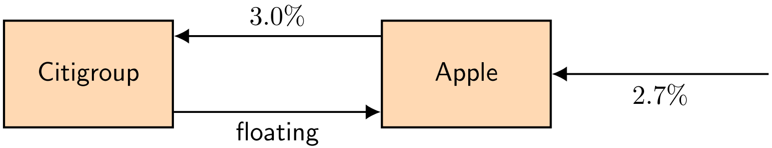


Intel uses a swap to convert fixed-rate borrowings to floating-rate borrowings:

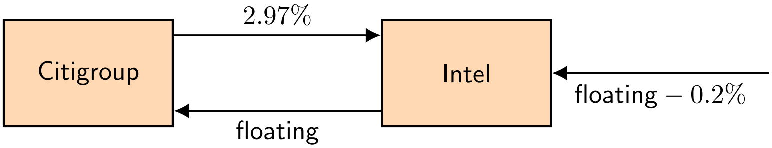


Interest rate swaps: transforming assets

Apple converts a fixed-rate investment to a floating-rate investment:



Intel converts a floating-rate investment to a fixed-rate investment:

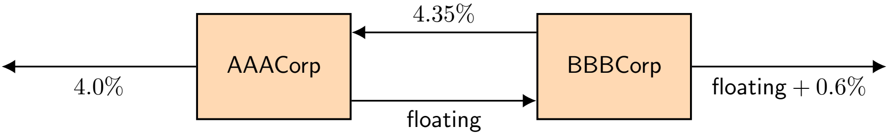


Comparative advantage (1/2)

Why use swaps? One reason might be **comparative advantage**: a company might have a relative advantage to borrowing in either fixed-rate markets or floating-rate markets.

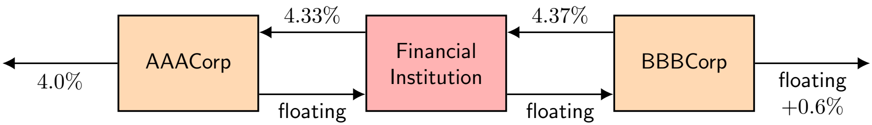
	fixed rate	**floating rate**
AAACorp	4.0%	floating $- 0.1\%$
BBBCorp	5.2%	floating $+ 0.6\%$

Here, AAACorp and BBBCorp might seek to collaborate:



Comparative advantage (2/2)

In practice, the swap might be brokered by a **financial institution**:



Why has comparative advantage not been arbitraged away?

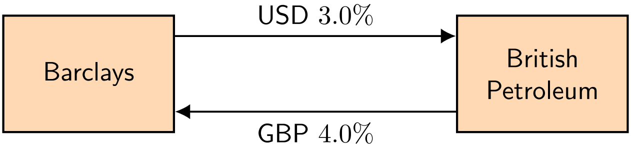
Note: The **maturities** of contracts available via fixed-rate financing are generally different than those available via floating-rate financing:

- Fixed-rate contracts are often longer than floating-rate contracts.
- The spread over the reference rate can effectively be adjusted by floating-rate lenders.
- Fixed-rate lenders often lack this option.

Currency swaps

fixed-for-fixed currency swap: arrangement wherein principal and interest payments in one currency are exchanged for principal and interest payments in another currency

Example:

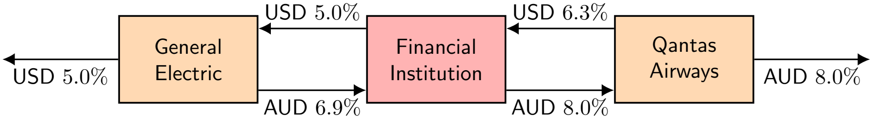


Variations:

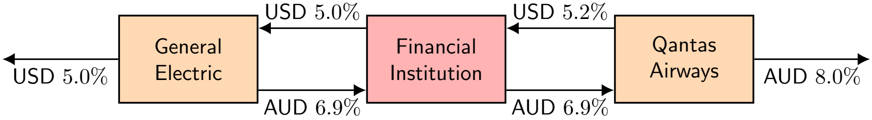
- **fixed-for-floating currency swap**
- **floating-for-floating currency swap**
- **quanto** (or **diff swap**): arrangement wherein a rate observed in one currency is applied to a principal amount in another currency

Currency swaps: example with comparative advantage

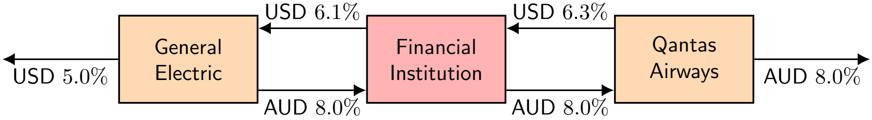
Suppose that General Electric has a **comparative advantage** to borrowing in USD and Qantas Airways has a **comparative advantage** to borrowing in AUD. A financial institution could reduce both of their costs by taking on FX risk:



It might be more cost-effective for Qantas Airways to bear some FX risk:



Or it might be more cost-effective for General Electric to bear some FX risk:



Other swaps

equity swap: agreement to exchange the total return (dividends plus gains) of an equity index for a fixed or floating rate of interest.

credit default swap: agreement that generates a payment if a particular company (the **reference entity** defaults

- The **protection buyer** pays the **CDS spread** (an insurance premium) over the life of the contract.
- In the event of default, the **protection seller** pays an amount that would restore the value of a hypothetical portfolio of the bonds of the reference entity to the value of its principal.

Options:

- **extendable swap**: one party can extend the swap arrangement
- **puttable swap**: one party can terminate the swap arrangement early
- **swaption**: option on a swap

Thank You

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