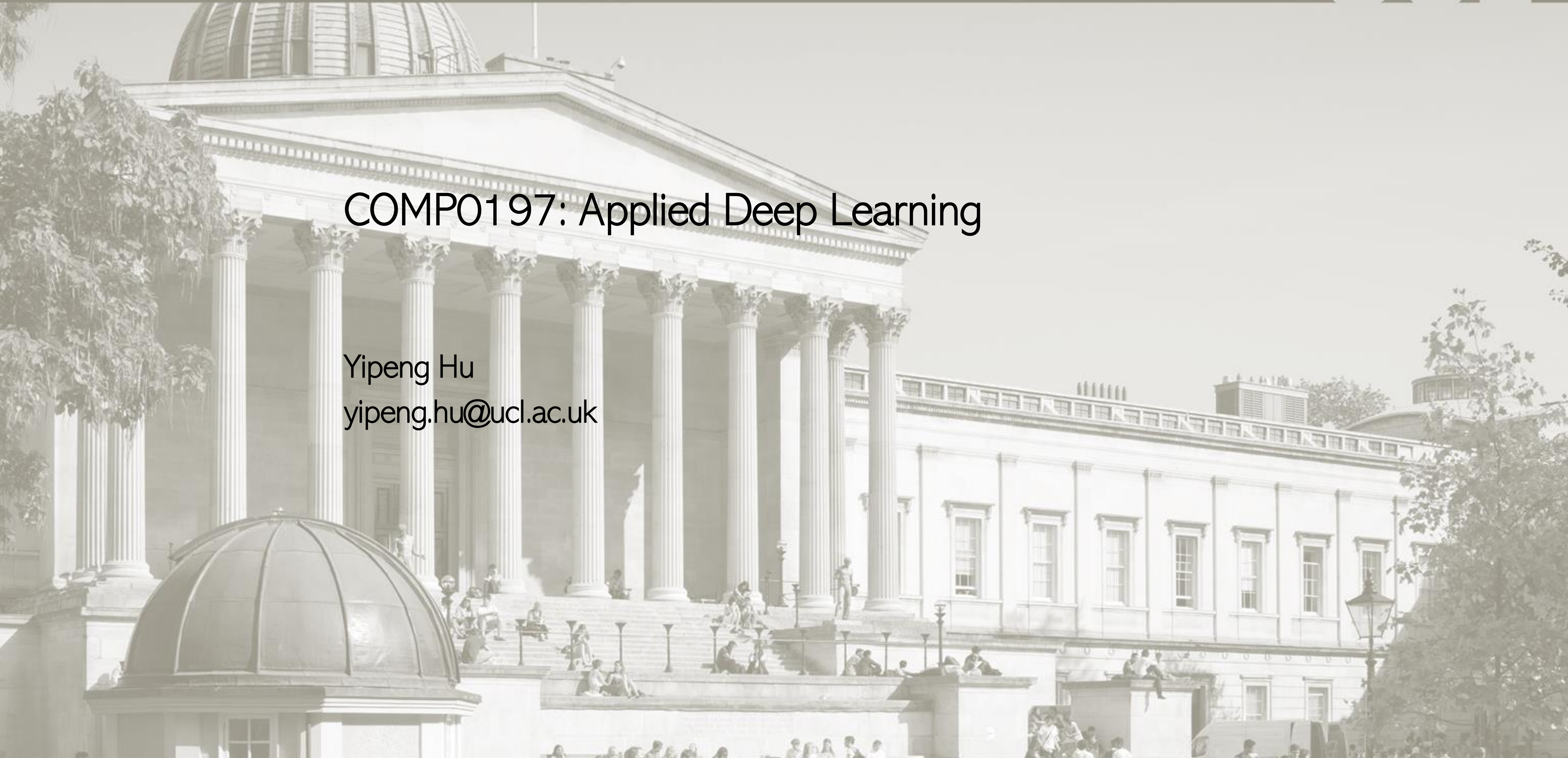
Department of Computer Science



# COMP0197: Applied Deep Learning

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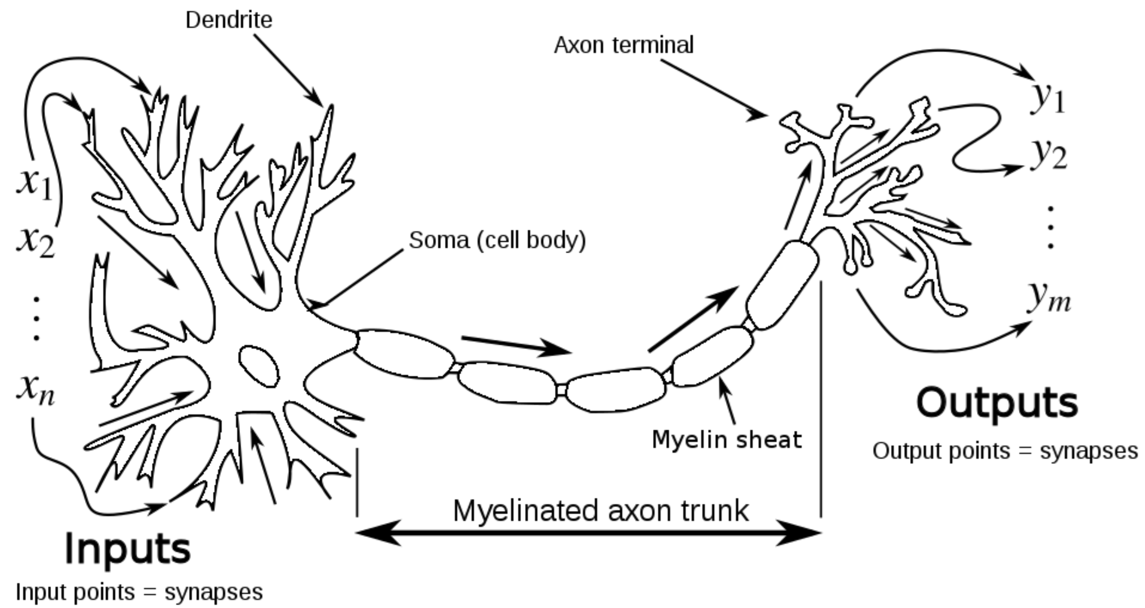
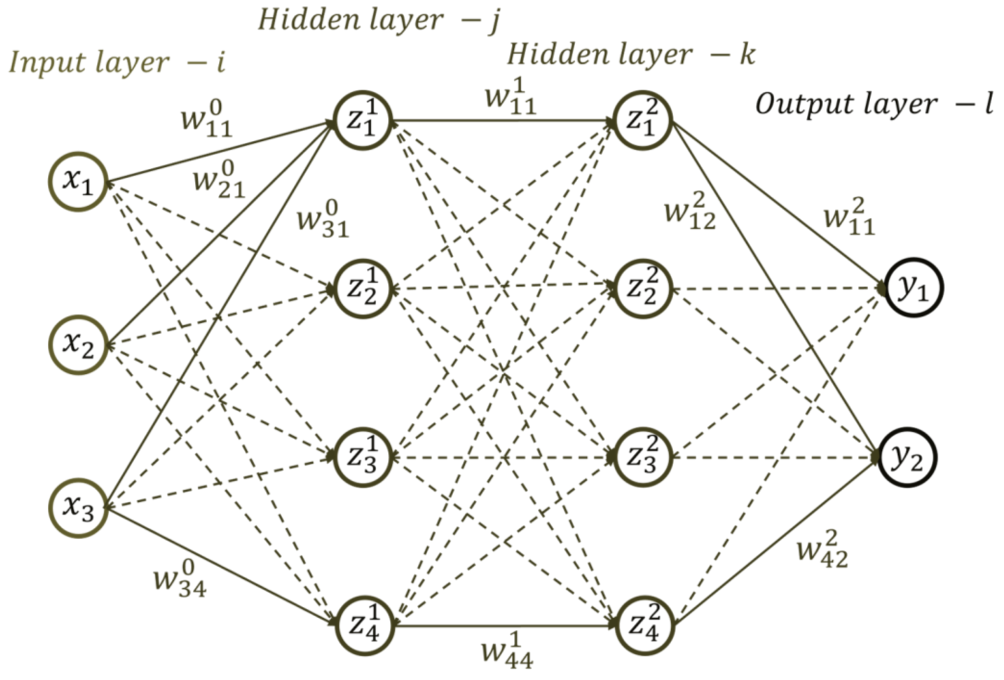
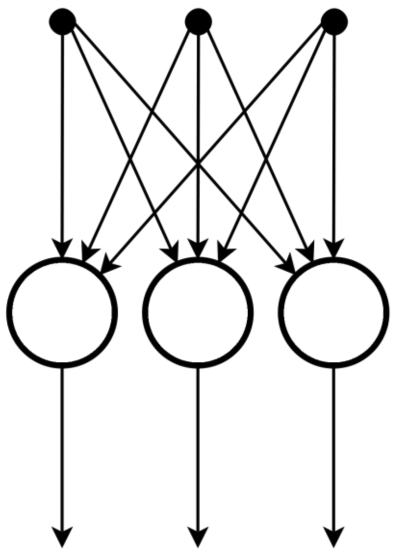
# Deep Feedforward Neural Networks

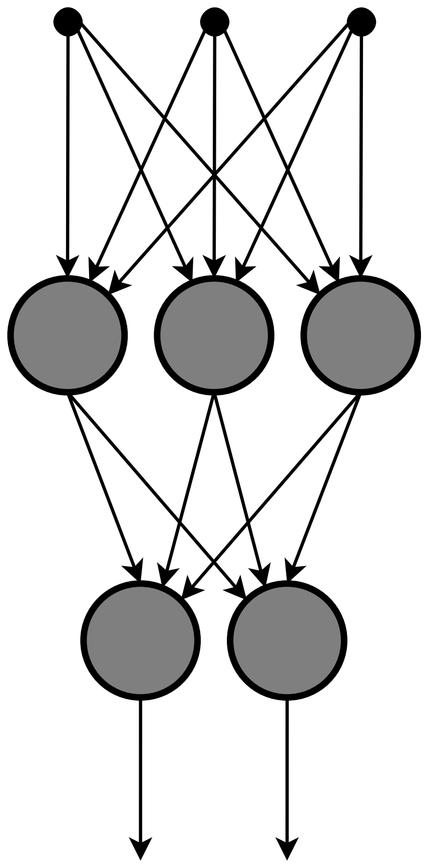
## Biological and artificial neurons



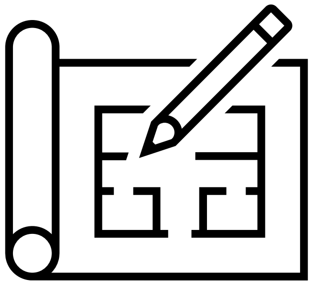



## Feedforward models



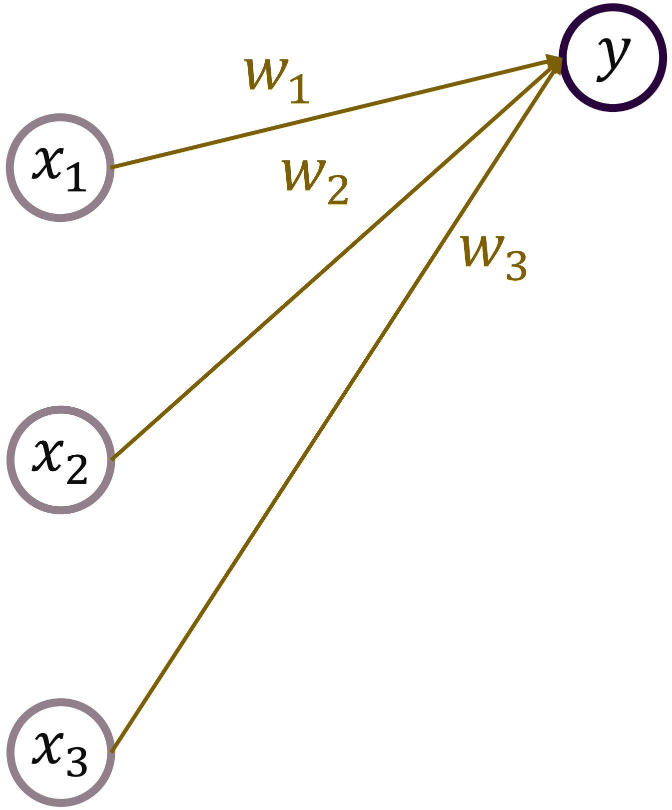


- Multilayer Perceptron (MLP)
- Anatomy of A Layer
- Activation Functions
- Architecture
    Width and Depth | Branching and Joining | Skipping | Sampling | “Ignoring”
- Architecture Search



# Deep Feedforward Neural Networks | MLP

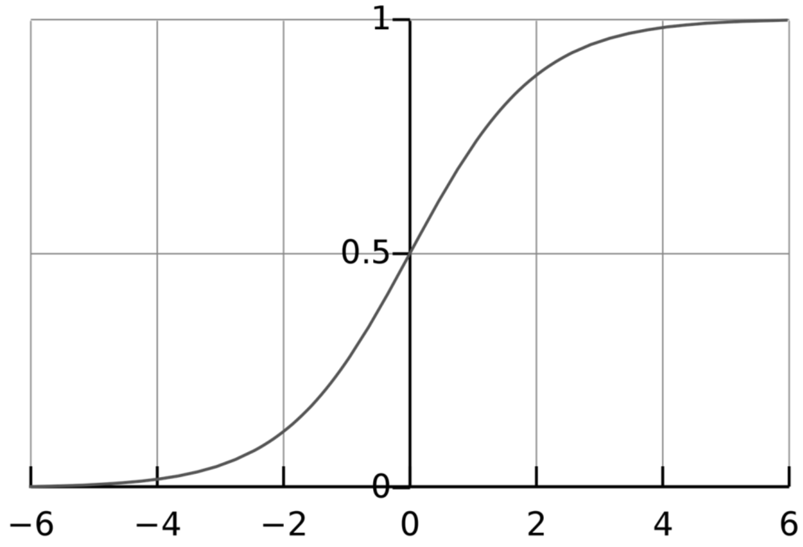
## Logistic regression

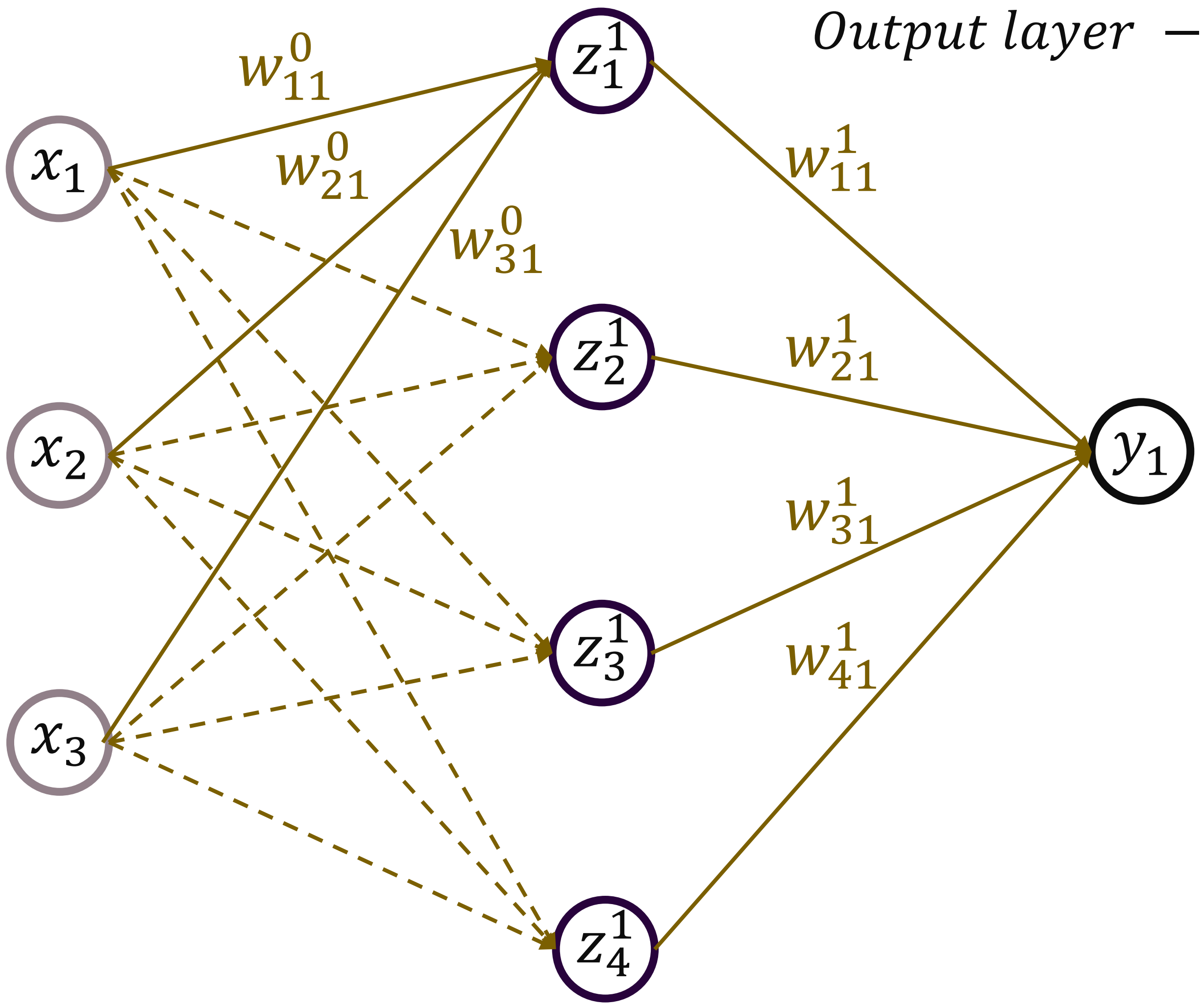



$x$: Input
$y$: Output
$\mathbf{w}$: Parameters/weights

$$y = \sigma\left[\sum_{i=1}^{i=3} x_i \mathrm{w}_i \;\; + b\right] = \sigma[\mathbf{w}^{\mathrm{T}}\mathbf{x} + b]$$

Activation function: $\sigma(z) = \frac{1}{1+\exp(-z)}$






Activation function: $\sigma \rightarrow g$

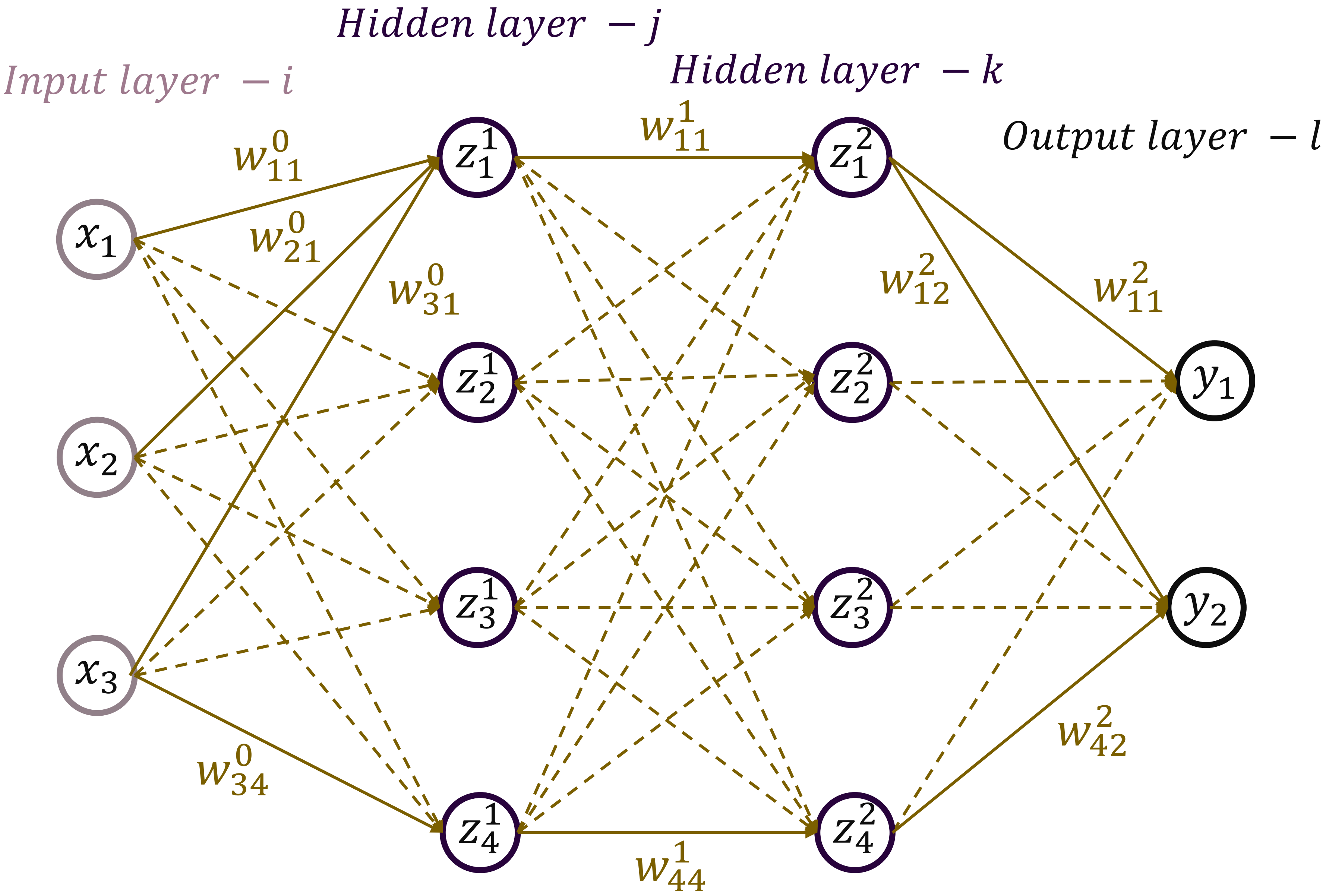
Input $\longrightarrow$ H1: i $\longrightarrow$ j

$$z_j^1 = g_j^1\left[\sum_{i=1}^{i=3} x_i w_{ij}^0 + b_j^1\right] = g_j^1\left[\left(\mathbf{w}_j^0\right)^T \boldsymbol{x} + b_j^1\right]$$

H1 $\longrightarrow$ Output: j $\longrightarrow$ k

$$f(\mathbf{x}, \mathbf{W}) = y_k = g_k\left[\left(\mathbf{w}_k^1\right)^T \mathbf{z}^1 + b_k\right]$$

*where* $\mathbf{z}^1 = [z_1^1, z_2^1, z_3^1, z_4^1]^T, k = 1$




Input → H1: i → j

$$z_j^1 = g_j^1\left[\left(\mathbf{w}_j^0\right)^{\mathrm{T}}\mathbf{x} + b_j^1\right]$$

H1 → H2: j → k

$$z_k^2 = g_k^2\left[\left(\mathbf{w}_k^1\right)^{\mathrm{T}}\mathbf{z}^1 + b_k^2\right]$$

H2 → Output: k → l

$$y_l = g_l\left[\left(\mathbf{w}_l^2\right)^{\mathrm{T}}\mathbf{z}^2 + b_l^2\right]$$

# Deep Feedforward Neural Networks | Anatomy of A Layer

$$h = g(\mathbf{w}^{\mathrm{T}}\mathbf{x} + b)$$

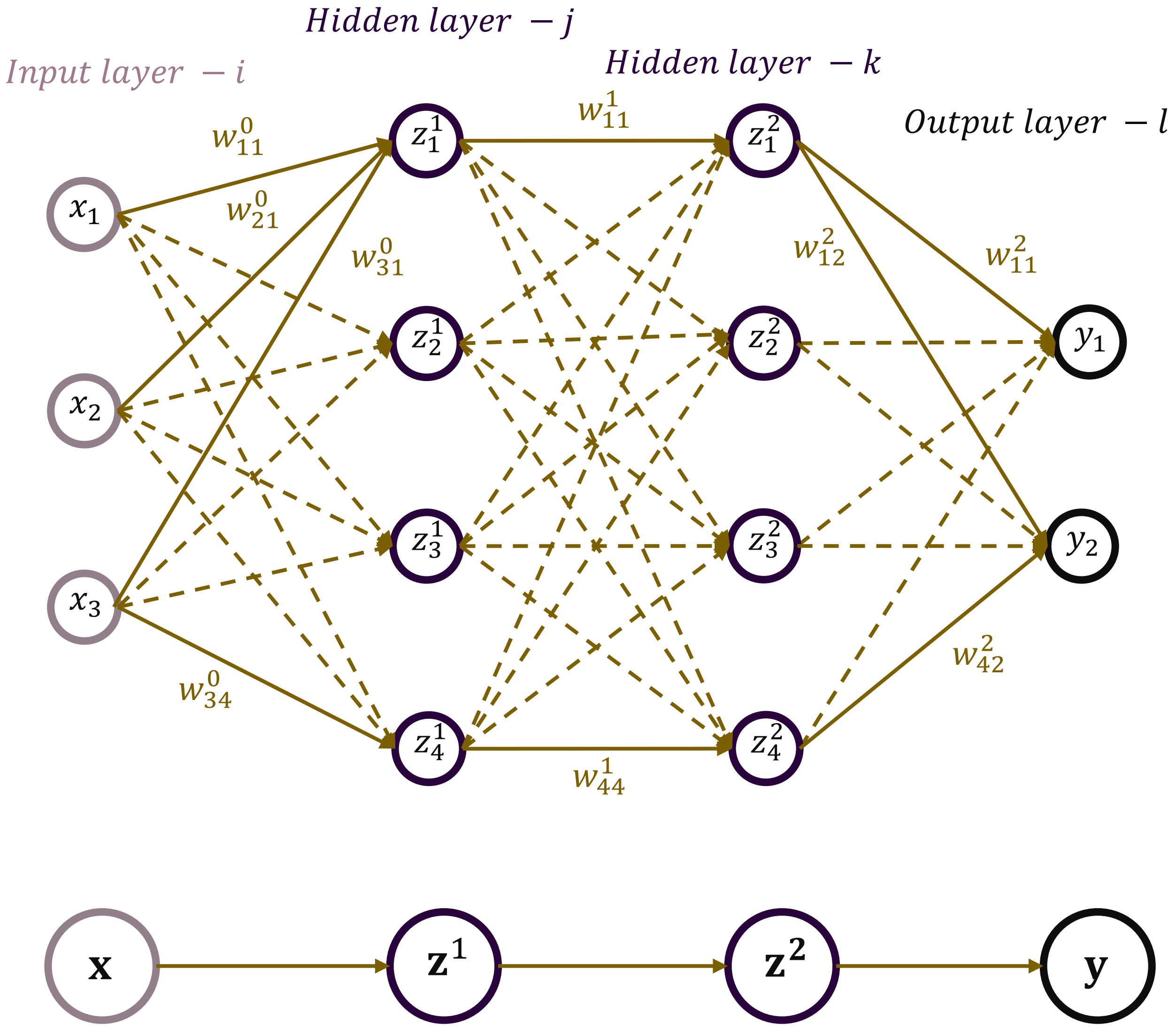
$h$: Hidden layer & output feature (vectors)

$\mathbf{x}$: Input feature vector

$\mathbf{w}$: Weights – network parameters

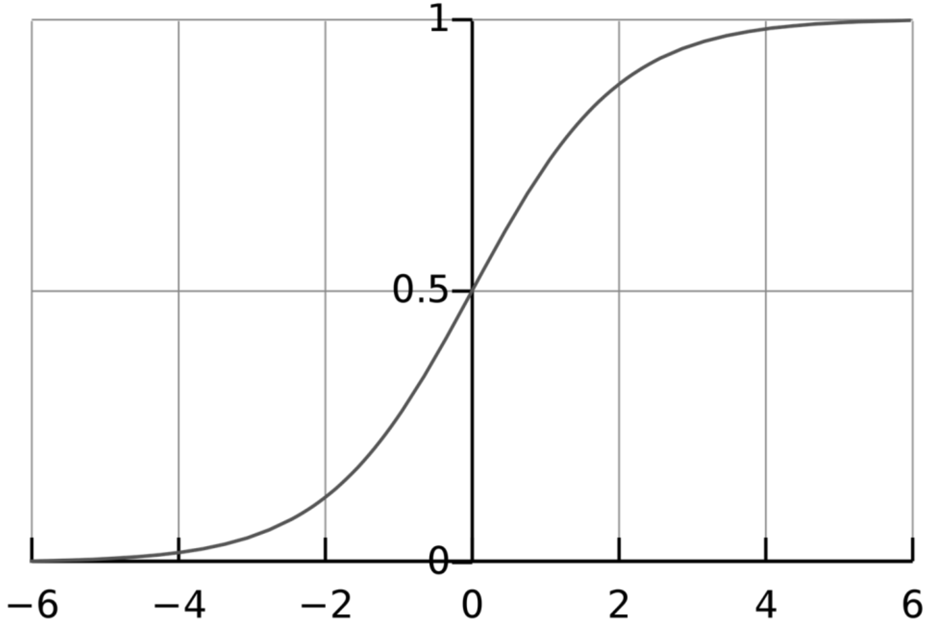
$b$: Bias

$g(\cdot)$: Activation function



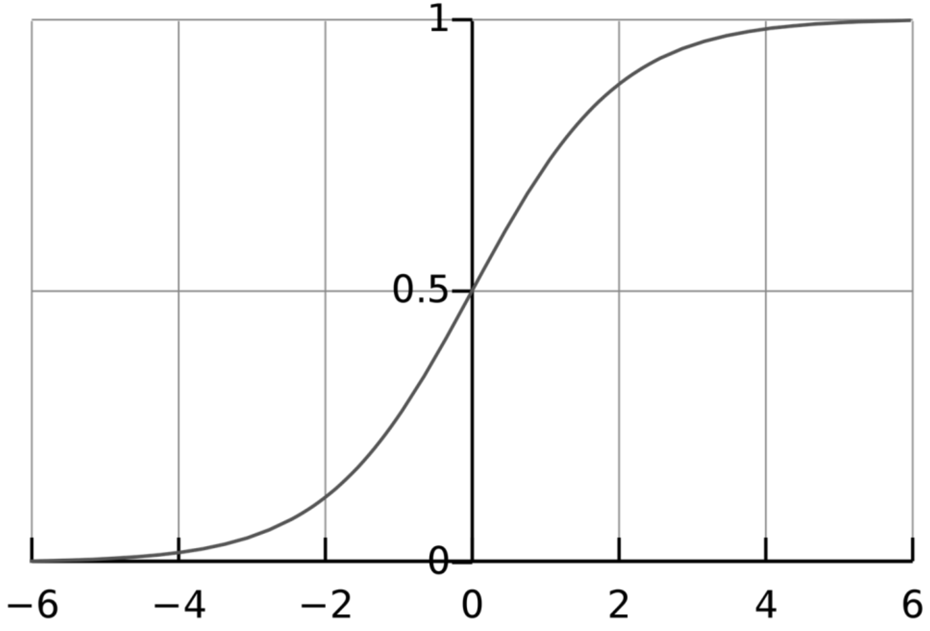
# Deep Feedforward Neural Networks | Activation Functions

# Nonlinearity?



## Nonlinearity

$$
\begin{aligned}
y &= \mathbf{w}_j^{\mathrm{T}}\left(\mathbf{w}_i^{\mathrm{T}}\mathbf{x} + \mathbf{b}_i\right) + \mathbf{b}_j \\
&= \mathbf{w}_j^{\mathrm{T}}\mathbf{w}_i^{\mathrm{T}}\mathbf{x} + \left(\mathbf{w}_j^{\mathrm{T}}\mathbf{b}_i + \mathbf{b}_j\right) \\
&= \widetilde{\mathbf{w}}^{\mathrm{T}}\mathbf{x} + \tilde{\mathbf{b}}
\end{aligned}
$$

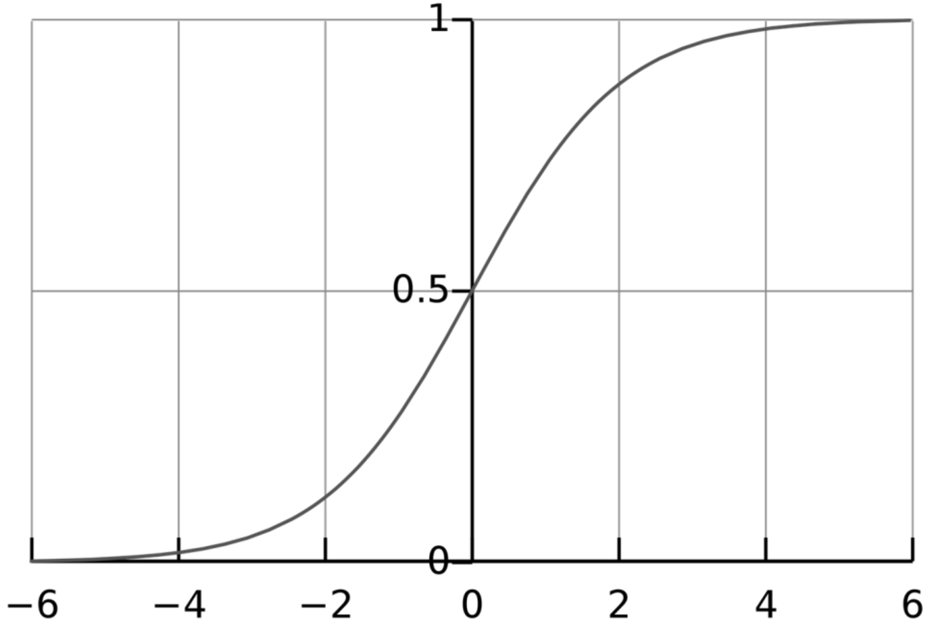


Nonlinearity

Range $[\mathbf{0}, \mathbf{1}]$

The squashing effect

A “unit” usually has one activation value

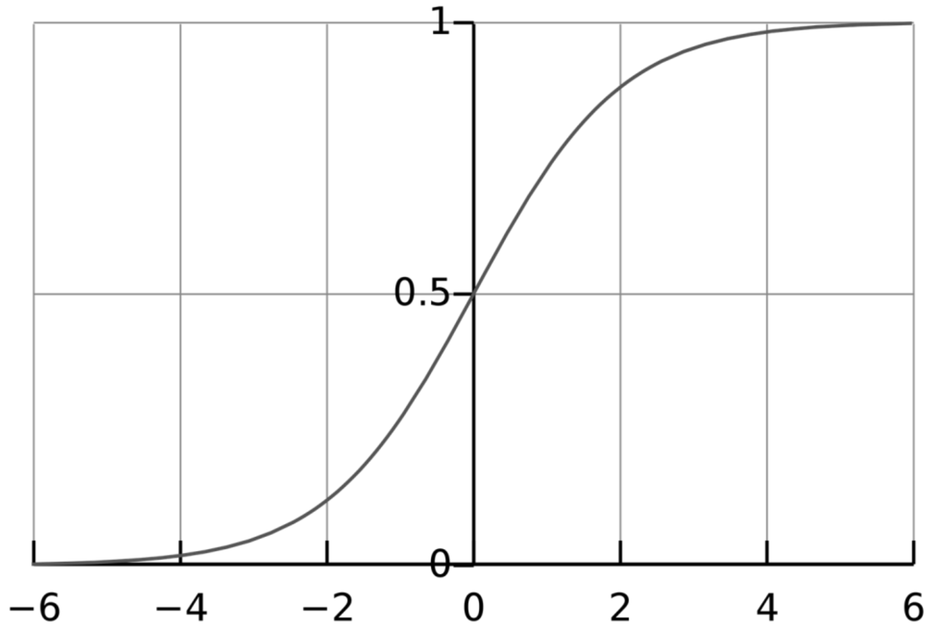


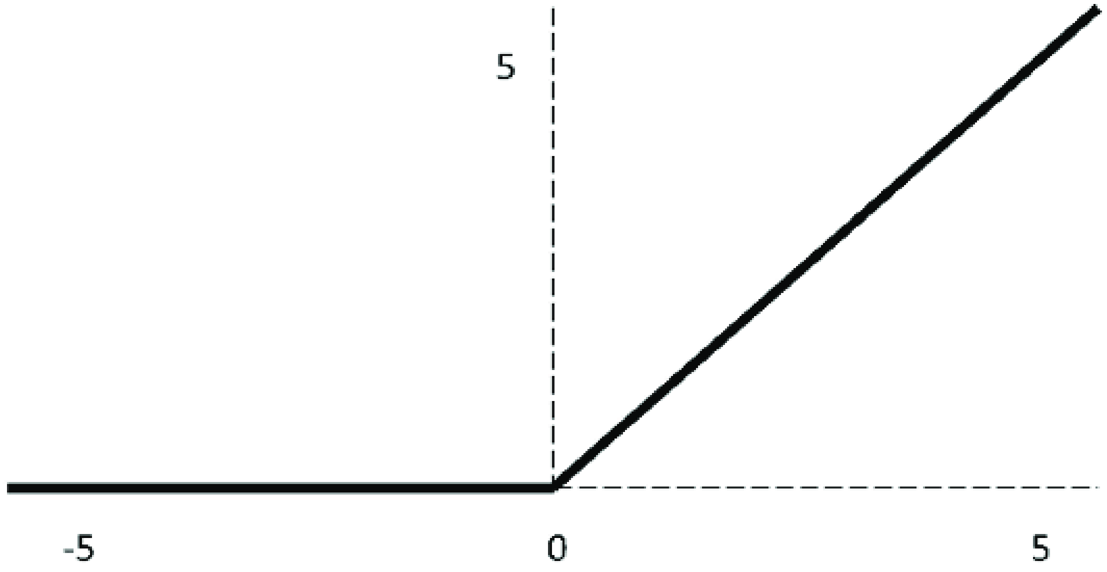

$$g(z) = \frac{1}{1 + e^{-z}}$$

The sigmoid function

## For hidden units

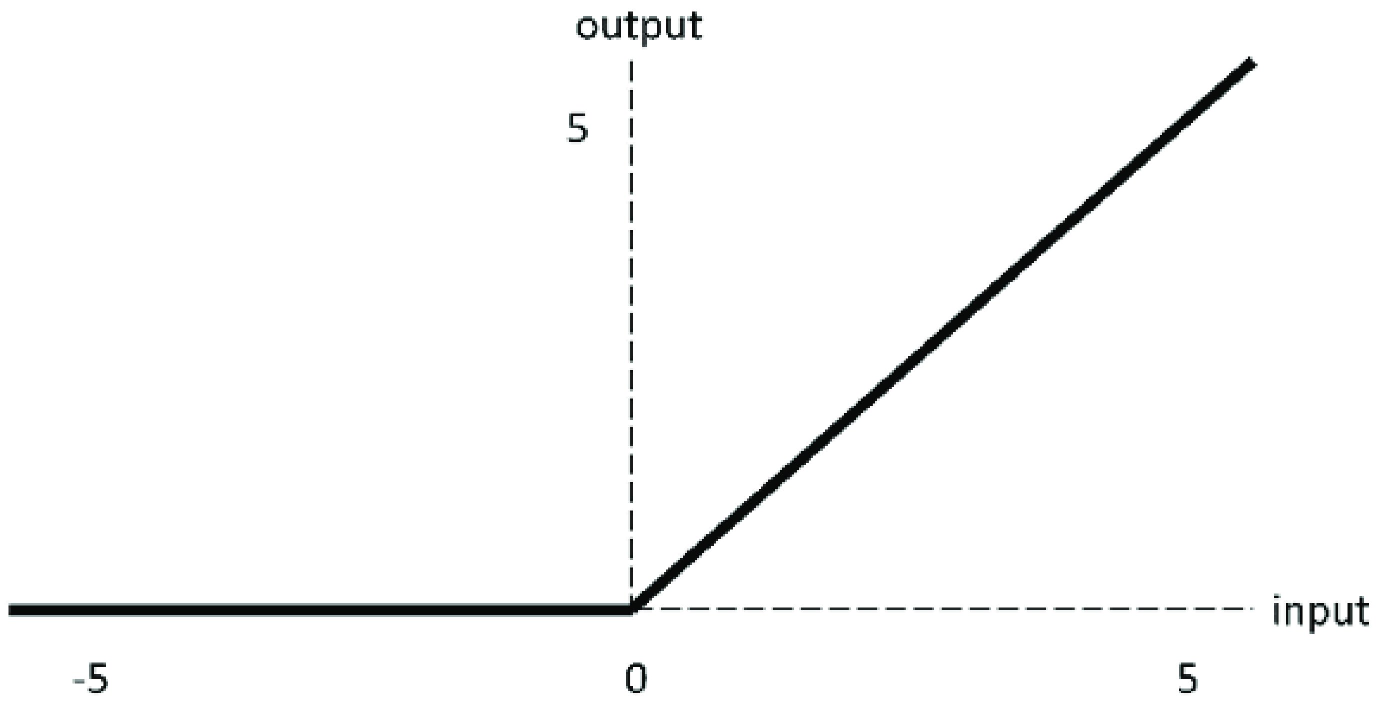
| | |
|---|---|
| Logistic sigmoid | $g(z) = \sigma(z) = \frac{1}{1+e^{-z}}$ |
| Hyperbolic tangent | $g(z) = \tanh(z) = 2\sigma(2z) - 1$ |
| Rectified linear unit (ReLU) | $g(z) = \max\{0, z\}$ |



## For hidden units

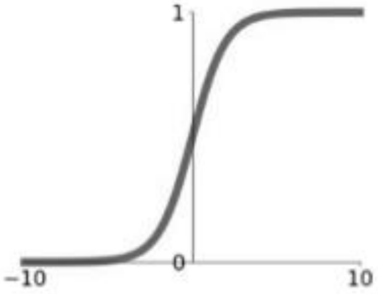
Rectified linear unit (ReLU) $g(z) = \max\{0, z\}$



For hidden units

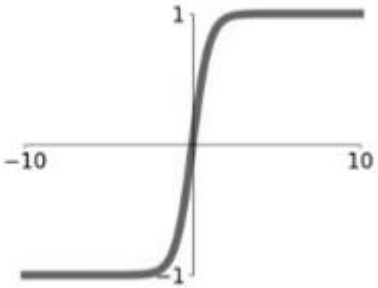
**Sigmoid**

$$\sigma(x) = \frac{1}{1+e^{-x}}$$



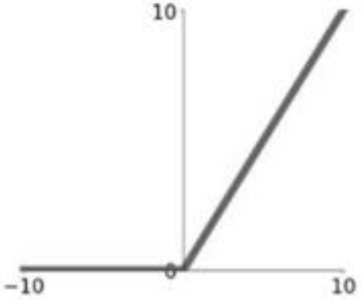

**tanh**

$$\tanh(x)$$



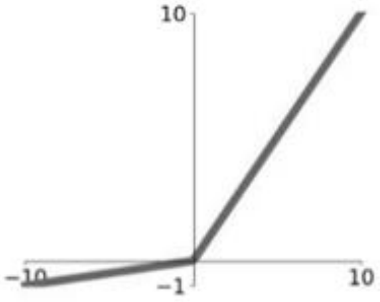

**ReLU**

$$\max(0, x)$$




**Leaky ReLU**

$$\max(0.1x, x)$$

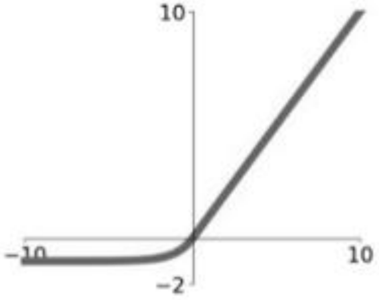



**Maxout**

$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

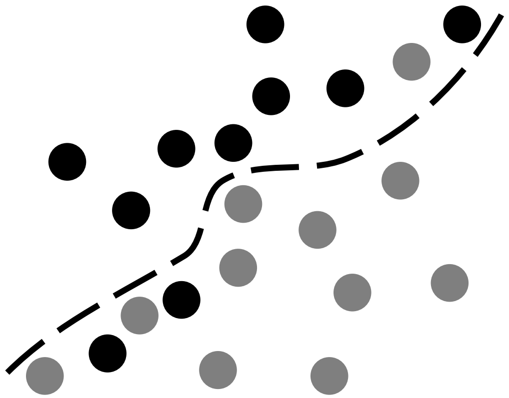
**ELU**

$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



## For output units

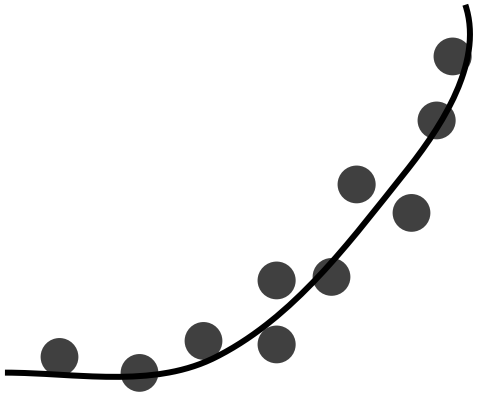
Bernoulli output (logistic sigmoid): $y = g(z) = \frac{1}{1+e^{-z}}$



Multinoulli output (softmax): $y_c = g(z_c) = \frac{\exp(z_c)}{\sum_{j=1}^{C} \exp(z_j)}$

$$\rightarrow \begin{cases} \sum_{j=1}^{C} y_c = 1 \\ 0 \leq y_c \leq 1 \end{cases}$$

Gaussian output



## Other activation functions

Permutation-invariant, e.g. max, mean, min...

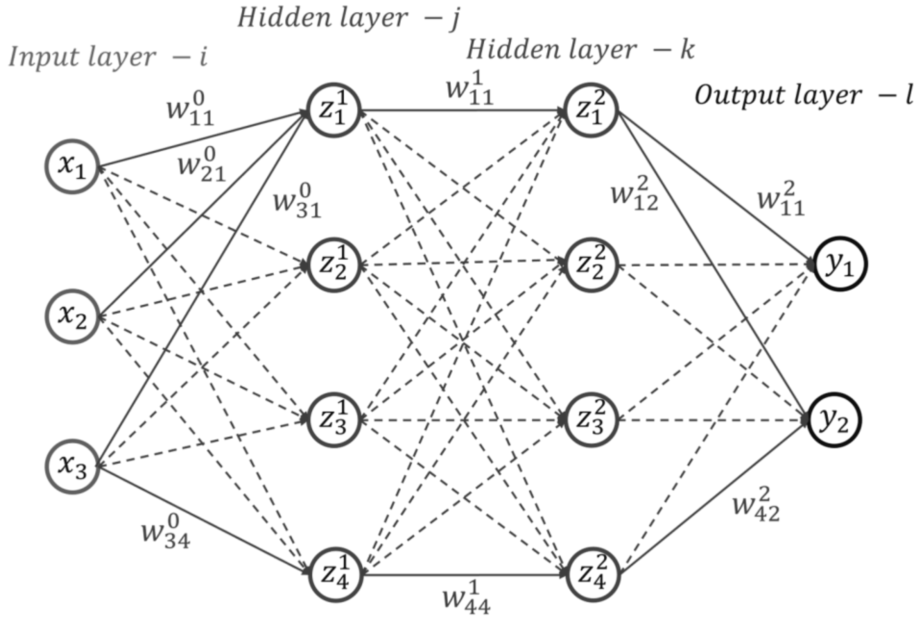
Known specific range and distribution

e.g. displacement/velocity (optical flow, registration), function parameters (camera position, pose)

# Deep Feedforward Neural Networks | Architecture

## Width and depth

How many layers?




Universal approximation theorem

A sufficiently-wide network approximates any "practically useful" functions

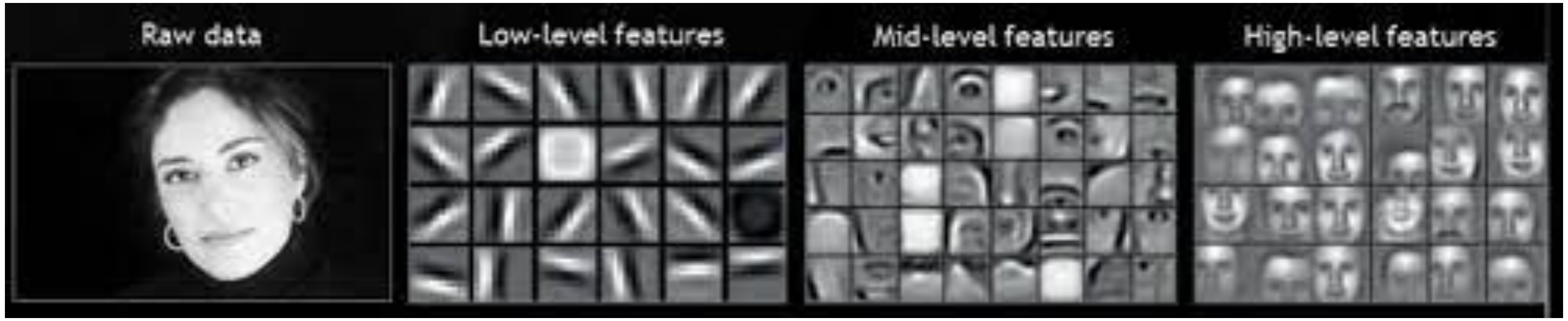
The wider the better?

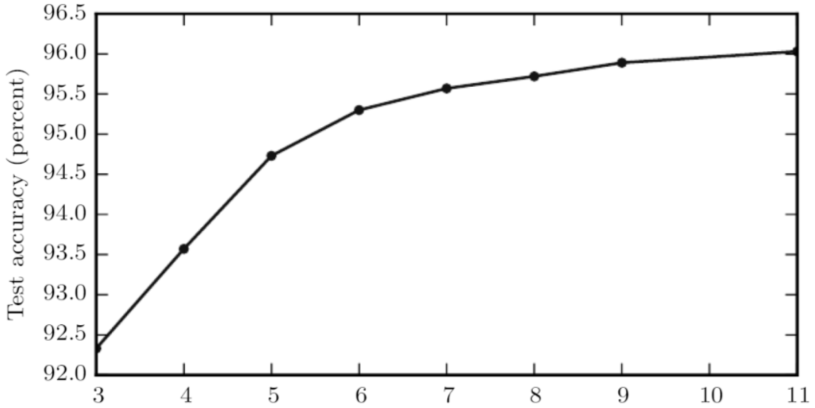
The deeper the better?

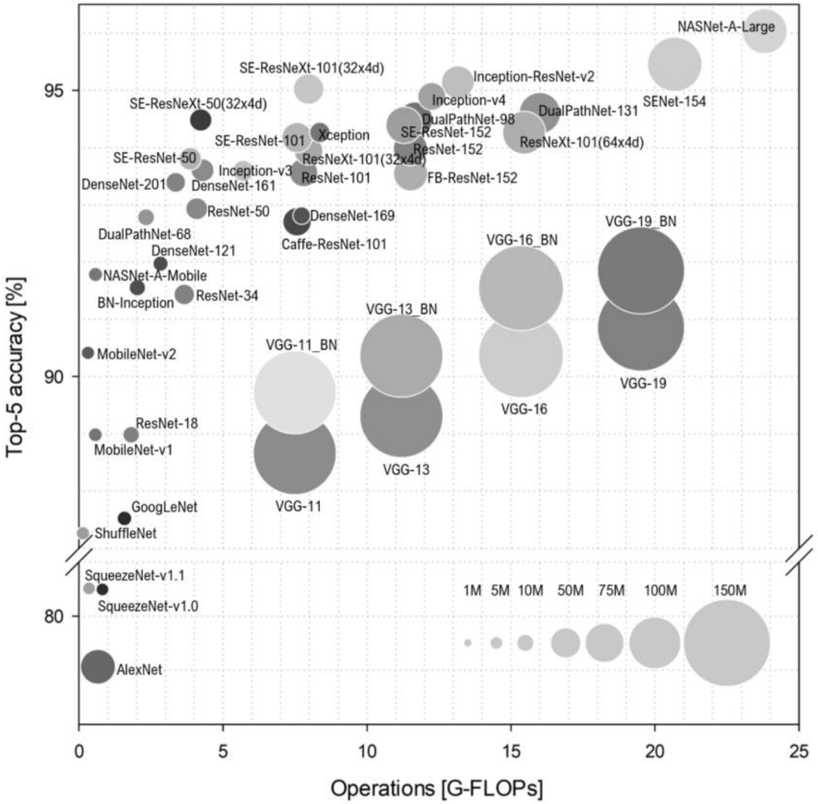
## Width and depth

Why deep?

Hierarchical representation learning, Empirical results



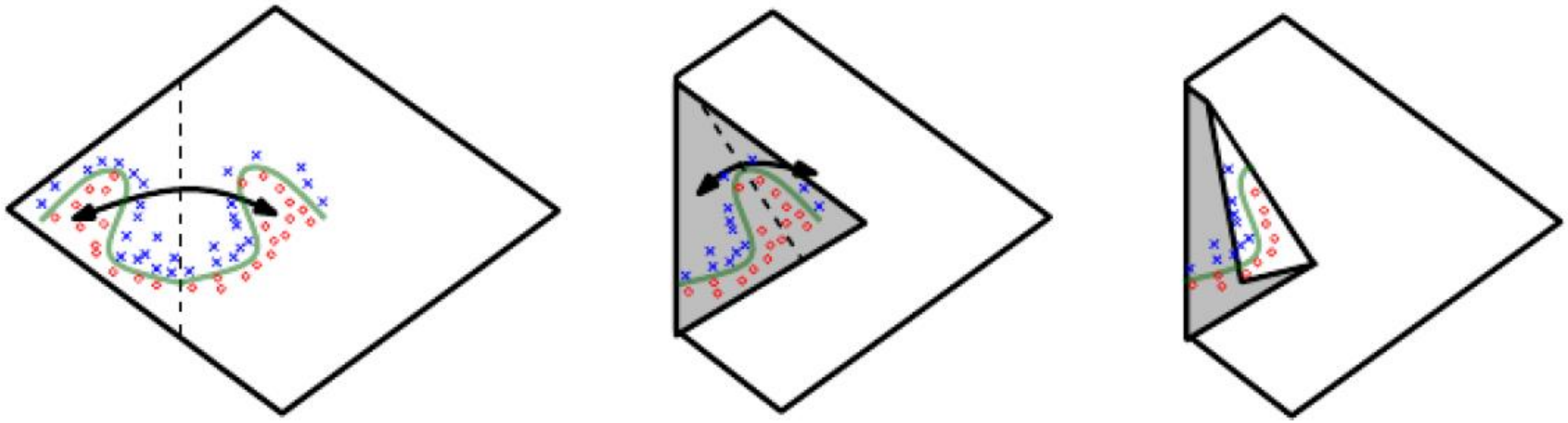



## Width and depth

Why deep?

Hierarchical representation learning, Empirical results, Efficiency



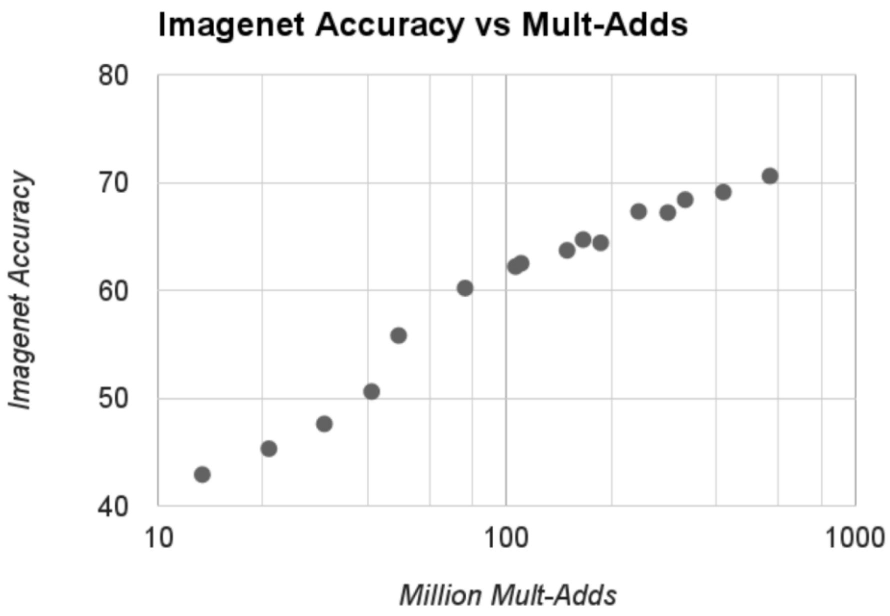
## Size

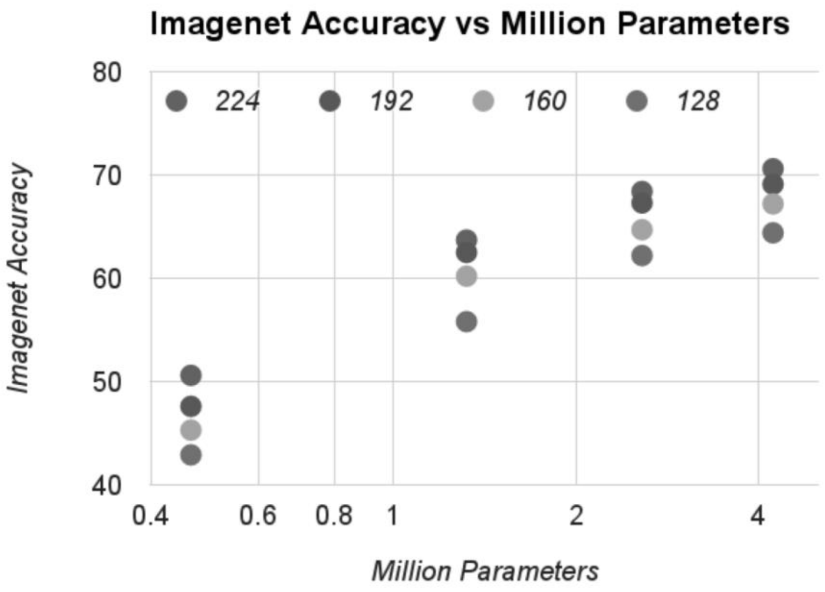
Number of parameters vs storage size ?

Memory consumption ?

Trade-off between accuracy and size

- "MobileNet" & "EfficientNet"*



## Branching and joining

- Multi-stream

- Multi-task*

- “Inception”

- Types of input

- Types of output/loss

- Types of processing

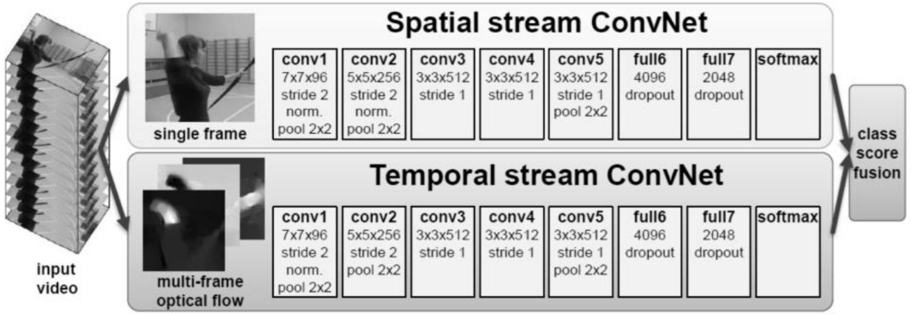
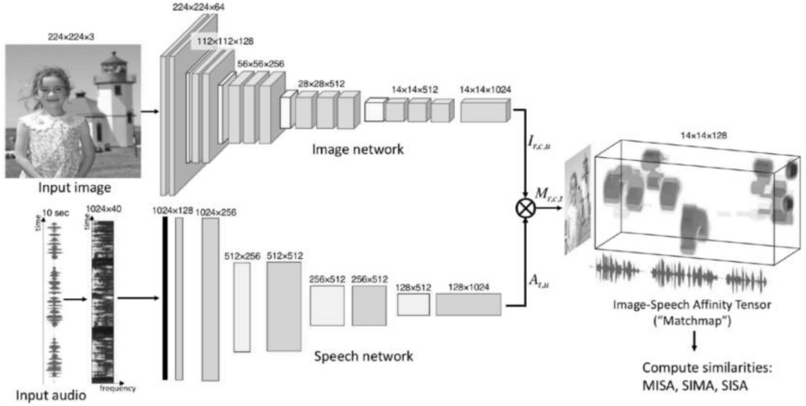
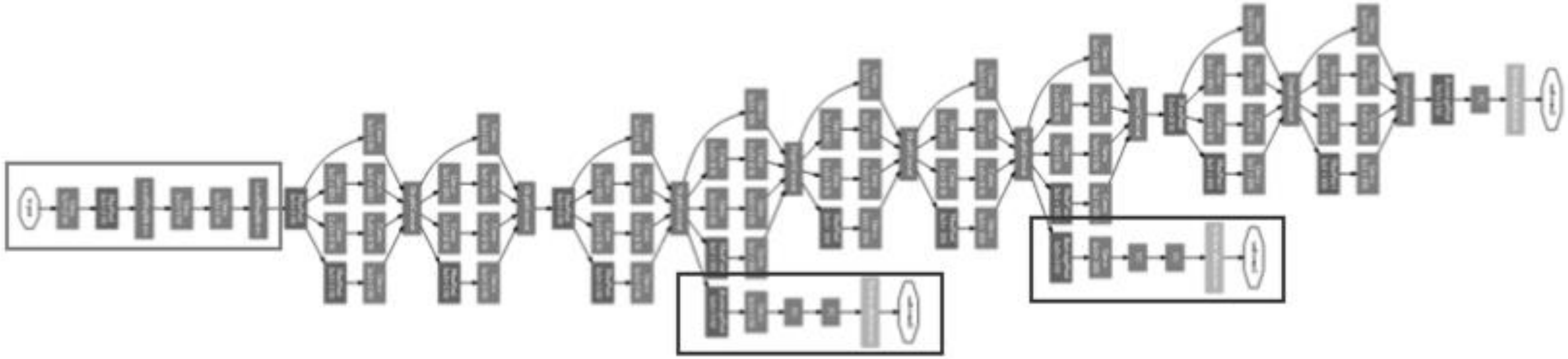



Figure 1: **Two-stream architecture for video classification.**



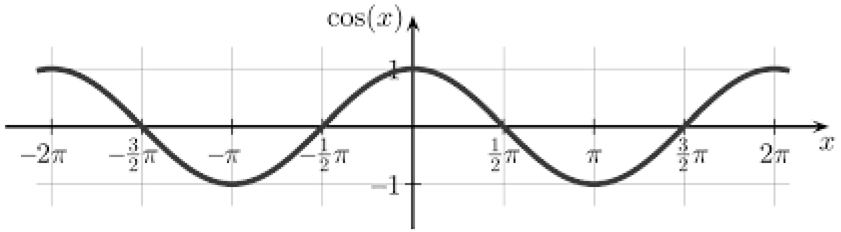
## Branching and joining

- Implementing branching with two sets of (non-)shared weights:

$$h_1 = g\left(\mathbf{w}_1^{\mathrm{T}}\mathbf{x} + b_1\right) \text{ and } h_2 = g\left(\mathbf{w}_2^{\mathrm{T}}\mathbf{x} + b_2\right)$$

- Implementing joining:

Dot product: $\mathbf{h}_1 \cdot \mathbf{h}_2 = |\mathbf{h}_1||\mathbf{h}_2| \cos\theta$ ?




Concatenation vs. summation

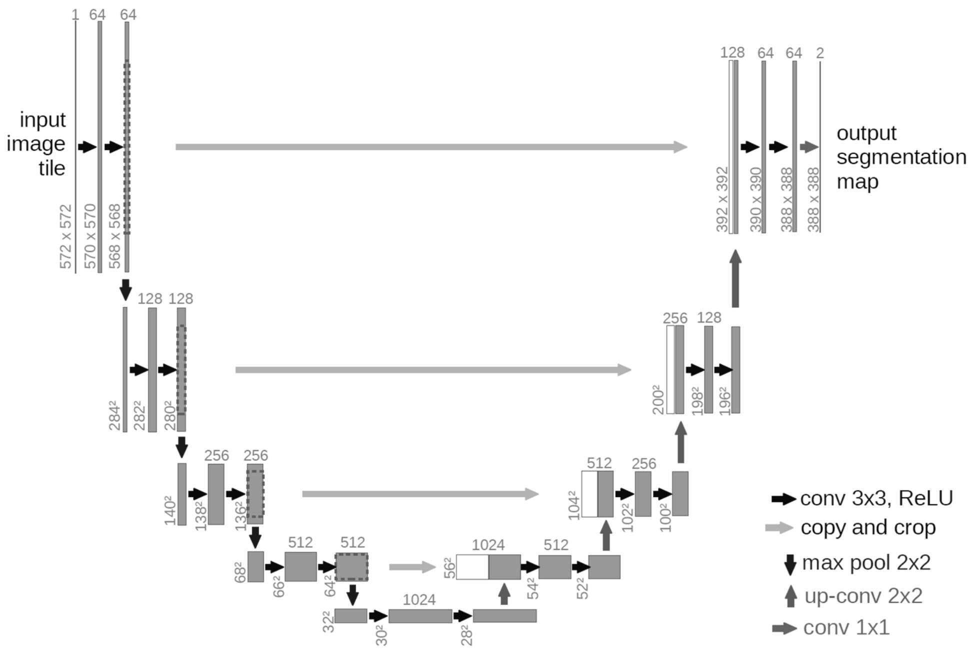
$$(\mathbf{h}_1, \mathbf{h}_2) \rightarrow [\mathbf{h}_1^T, \mathbf{h}_2^T]^T$$

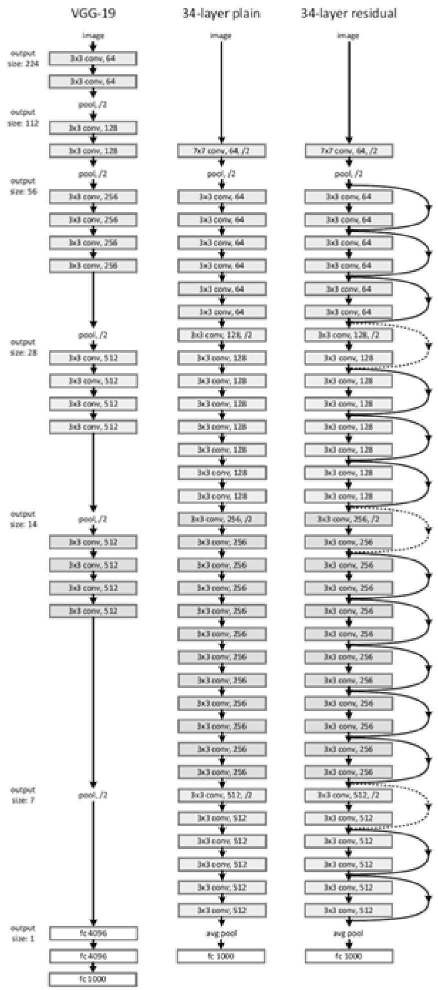
$$(\mathbf{h}_1, \mathbf{h}_2) \rightarrow \mathbf{h}_1 + \mathbf{h}_2$$

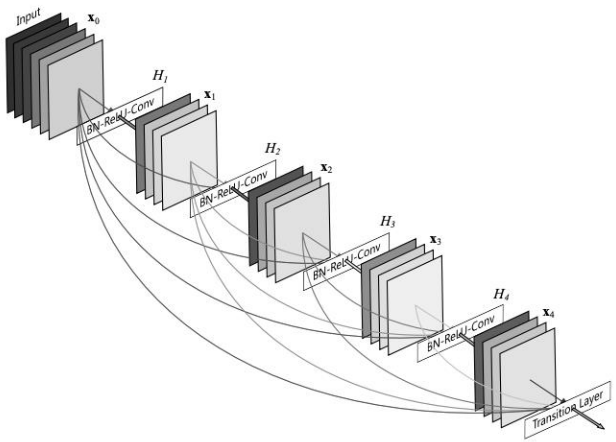
# Skipping

- Shortcuts, skip layers, residual connections

- “U-Net”*



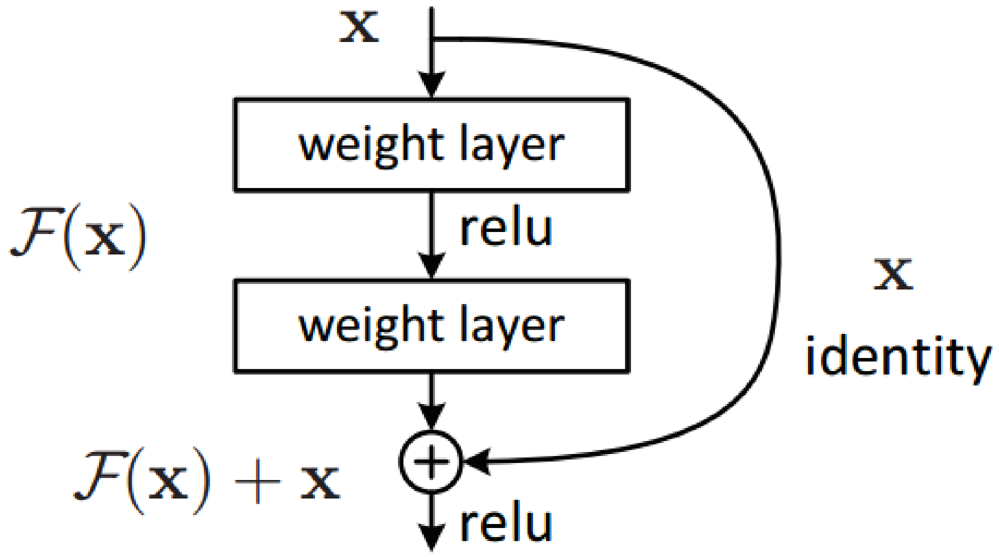



## Skipping

- Shortcuts, skip layers, residual connections

- “U-Net”*

- “ResNet”*




$$\mathbf{y} = \mathcal{F}(\mathbf{x}, \{W_i\}) + \mathbf{x}.$$

## Sampling

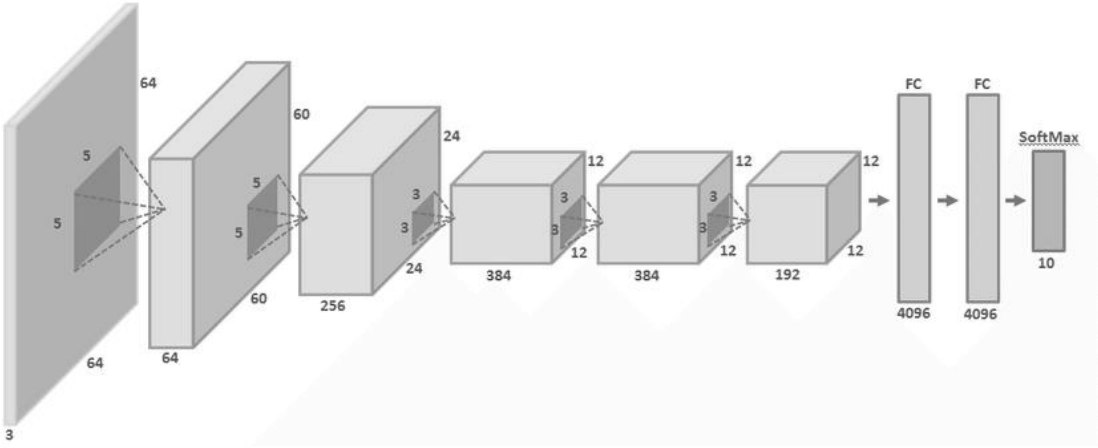
- Down-sampling for encoding ?

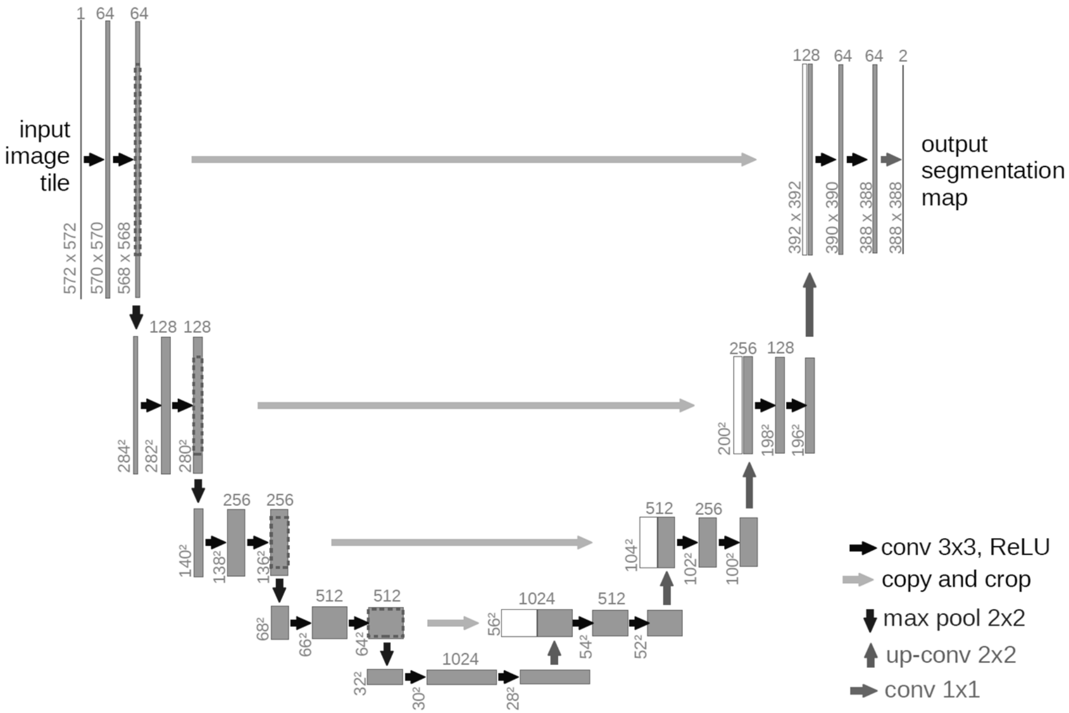
- “AlexNet” & “VGG”

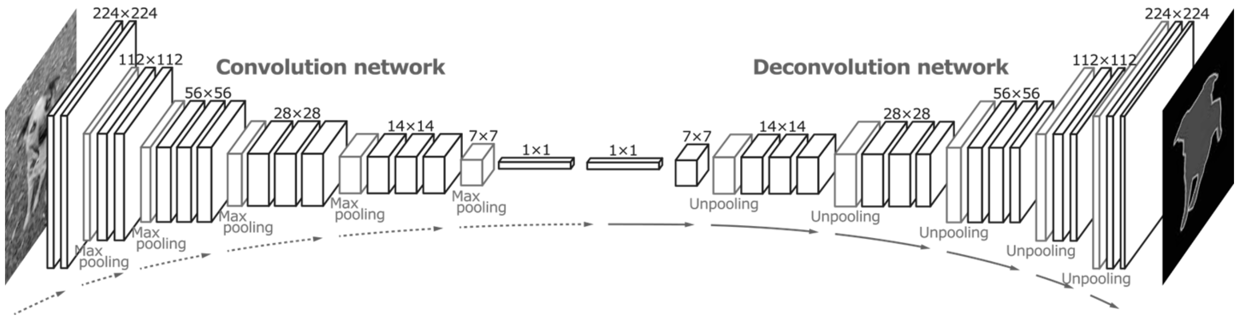
- Up-sampling for decoding ?

- “U-Net” & “Autoencoder”

- Sampling for convolution*



## Sampling

- Implement vector down-sampling: averaging, matrix multiplication

- Implement vector up-sampling: matrix multiplication, interpolation ?

$$h_n = g\left(\mathbf{w}_n^{\mathrm{T}}\mathbf{x} + b_n\right)$$

$$\begin{bmatrix} h_1 \\ \vdots \\ h_N \end{bmatrix} = g\left(\begin{bmatrix} \mathbf{w}_1^{\mathrm{T}}\mathbf{x} + b_1 \\ \vdots \\ \mathbf{w}_N^{\mathrm{T}}\mathbf{x} + b_N \end{bmatrix}\right) = g([\mathbf{w}_1 \quad \dots \quad \mathbf{w}_N]^T\mathbf{x} + [b_1 \quad \dots \quad b_N]^T)$$

$$\mathbf{h}^{(N\times 1)} = \mathbf{W}^{(N\times M)}\mathbf{x}^{(M\times 1)} + \mathbf{b}^{(N\times 1)}$$

- Implement feature map sampling*

"Ignoring"

$$f(\{x_1, x_2, x_3, \ldots x_n\}) \approx g(\{h(x_1), h(x_2), h(x_3), \ldots h(x_n)\})$$

## “Ignoring”

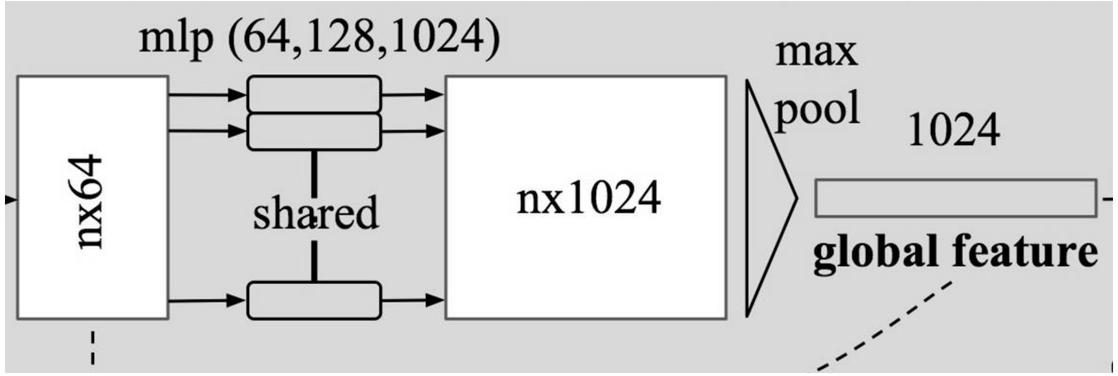
- Permutation-invariant

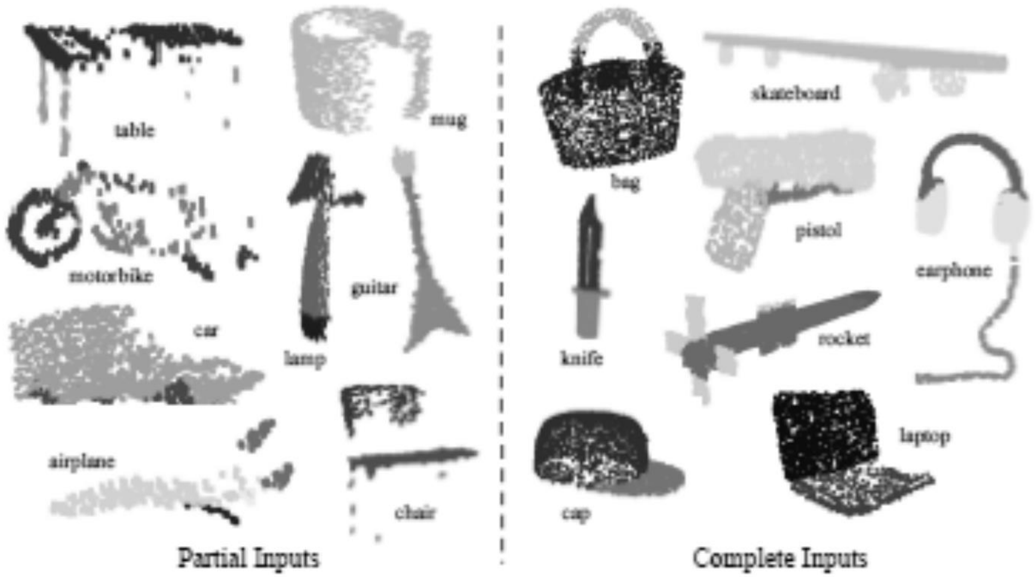
$$f(\{x_1, x_2, x_3, \dots x_n\}) \approx g(\{h(x_1), h(x_2), h(x_3), \dots h(x_n)\})$$

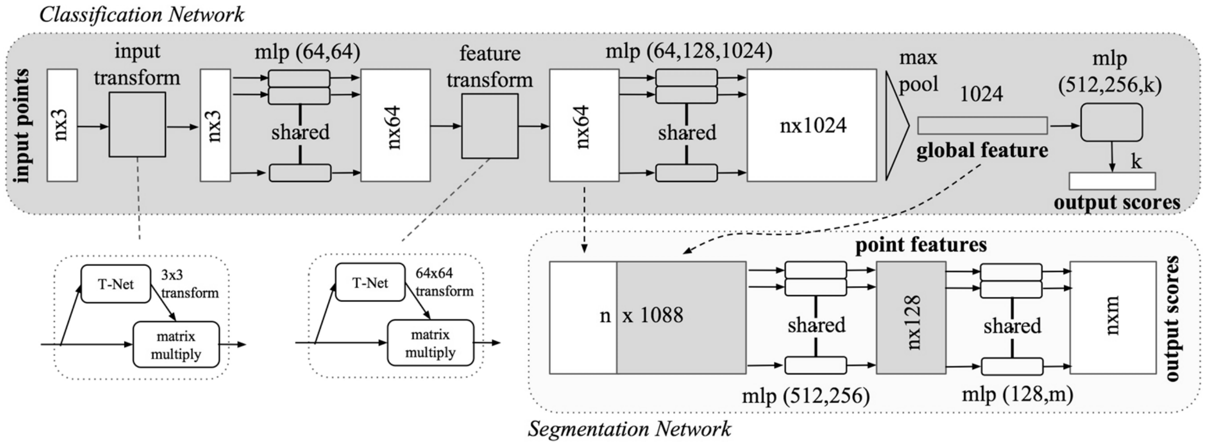
- “PointNet”

max, min, mean…

- Translation-invariant* (pooling, convolution)



# Deep Feedforward Neural Networks | Architecture Search

## Hyperparameters

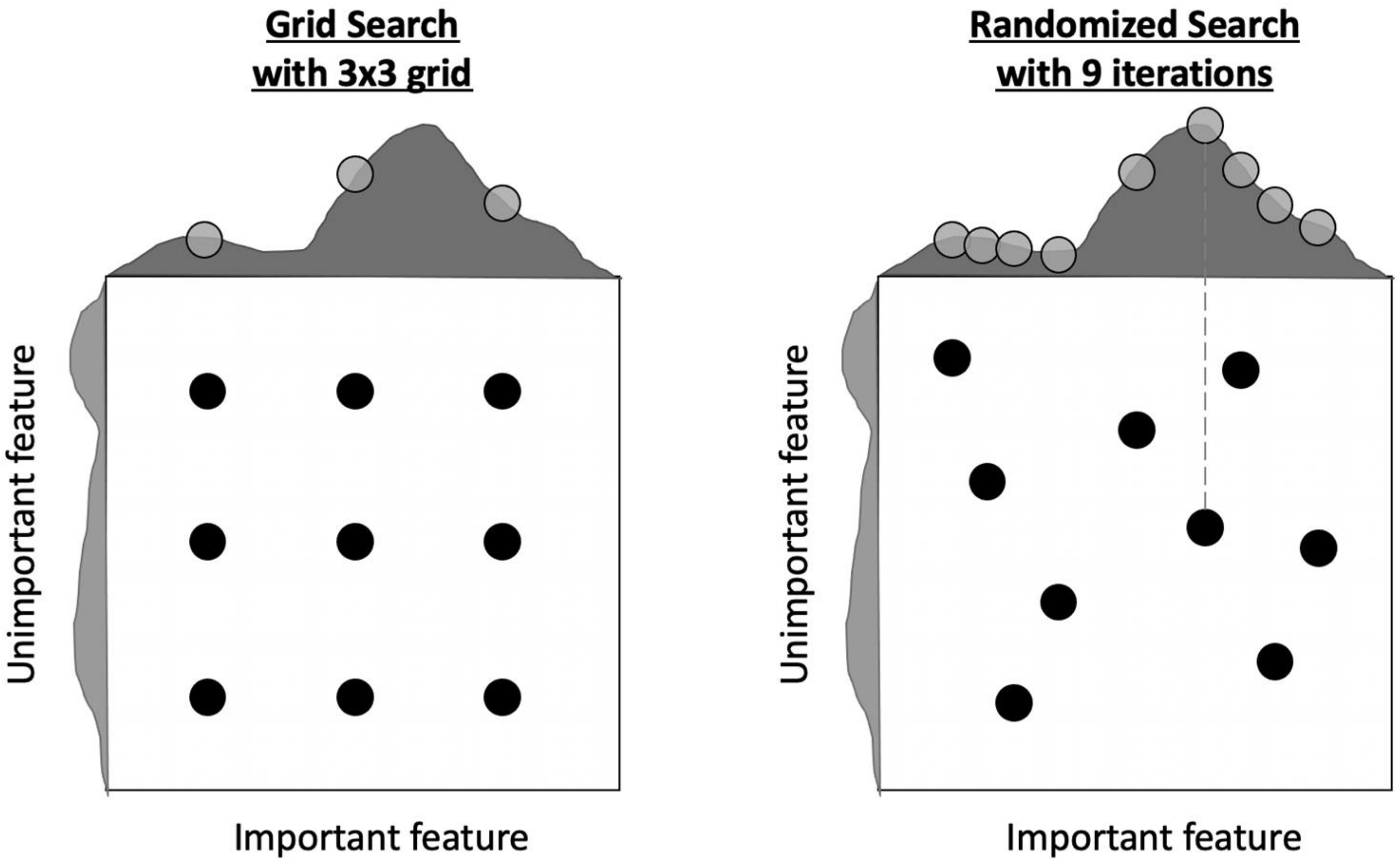
- Predefined
- Might not optimisable
- Should not be optimised
- e.g. degree of polynomial, NN architecture choice (width, depth, branches…)

## Hyperparameter tuning

## Hyperparameter search

Training-validation-test 3-way split

Grid search vs. random search

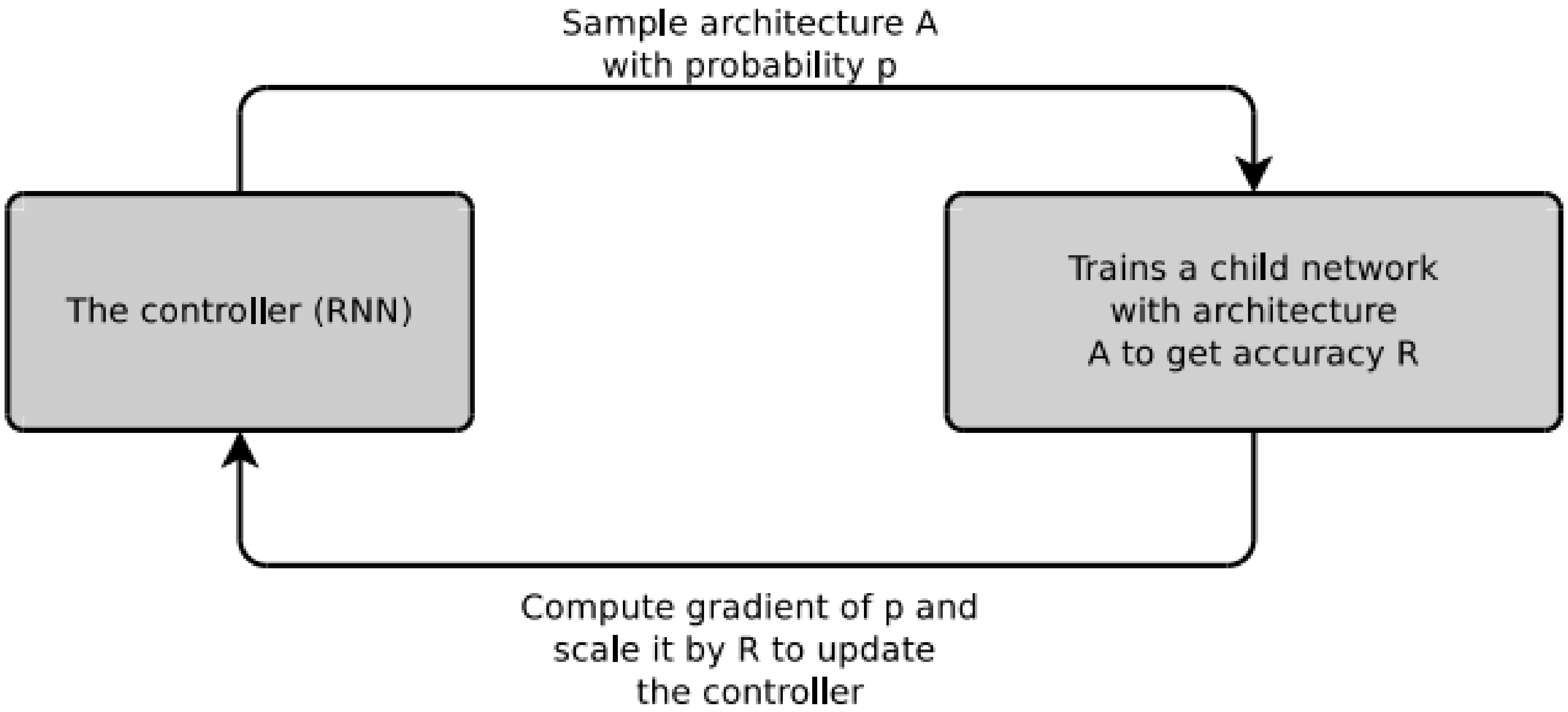



## As an optimisation,

e.g. evolution algorithms

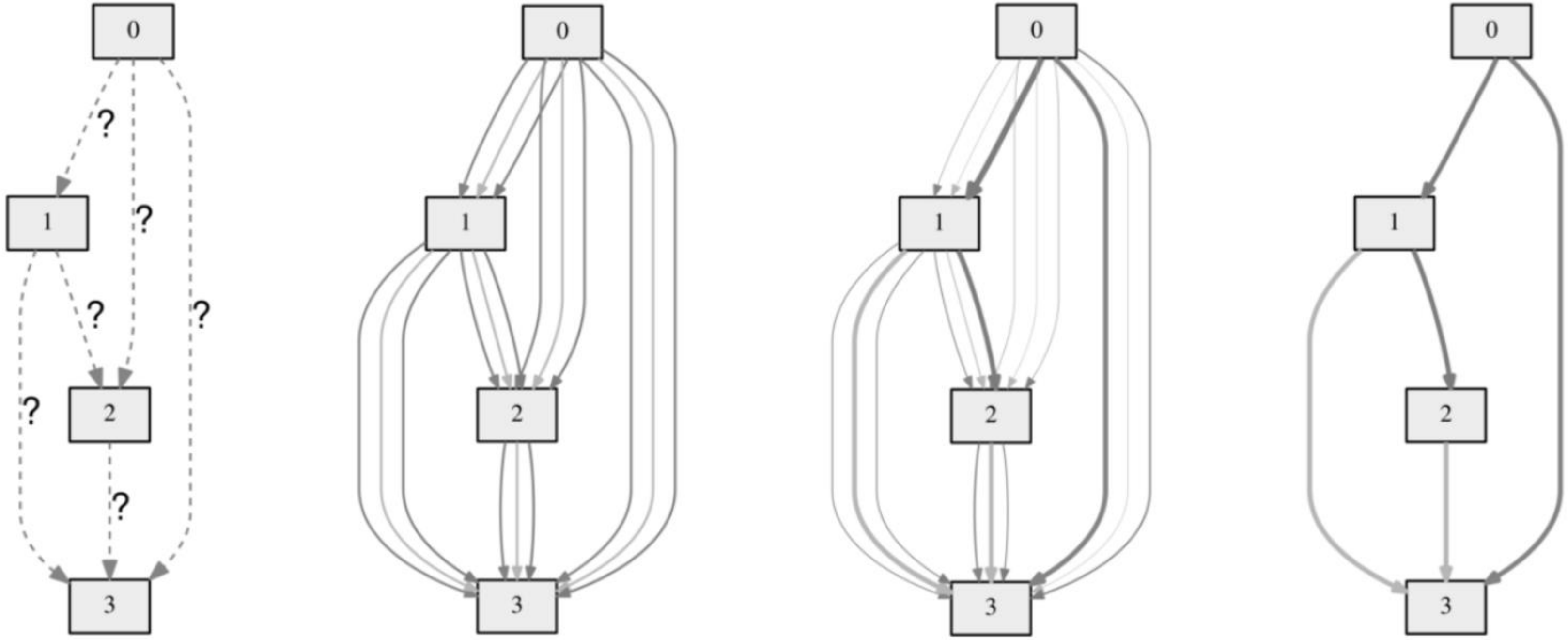
- “EfficientNet”*

# Meta-learning | reinforcement learning



# Meta-learning | differentiable architecture search

- "DARTS"




(a) (b) (c) (d)

- Multilayer Perceptron (MLP)
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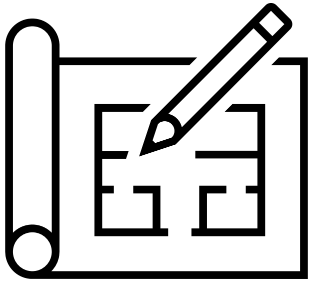

Using a fully-connected network for the “image classification” tutorial