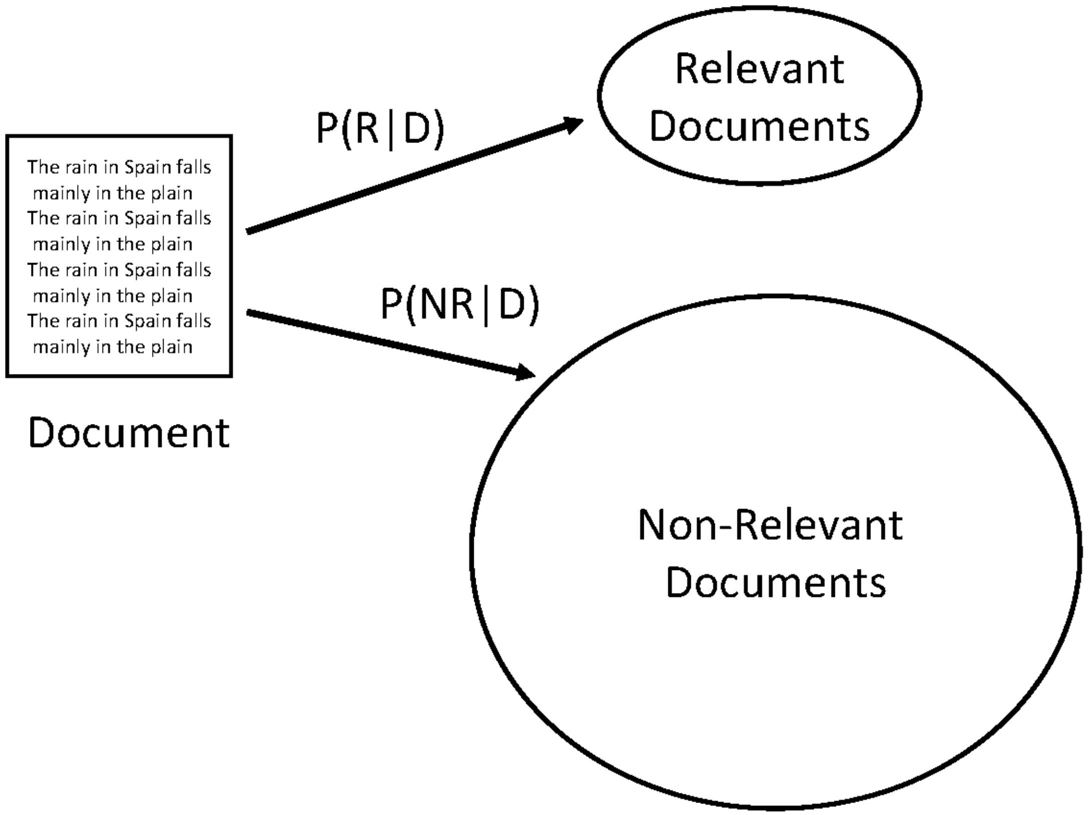
Retrieval Models Overview

- Older models
  - Boolean retrieval
  - Vector Space model
- Probabilistic Models
  - Language models
  - BM25
- Combining evidence
  - Inference networks
  - Learning to Rank

Probability Ranking Principle

- Robertson (1977)
  - “If a reference retrieval system’s response to each request is a ranking of the documents in the collection in order of decreasing probability of relevance to the user who submitted the request,
  - where the probabilities are estimated as accurately as possible on the basis of whatever data have been made available to the system for this purpose,
  - the overall effectiveness of the system to its user will be the best that is obtainable on the basis of those data.”

IR as Classification



Bayes Classifier

- Bayes Decision Rule
  - A document D is relevant if $P(R|D) > P(NR|D)$
- Estimating probabilities
  - Use Bayes Rule

$$P(R|D) = \frac{P(D|R)P(R)}{P(D)} \qquad P(NR\,|\,D) = \frac{P(D\,|\,NR)P(NR)}{P(D)}$$

  - Classify a document as relevant if

$$\frac{P(D\,|\,R)P(R)}{P(D)} > \frac{P(D\,|\,NR)P(NR)}{P(D)}$$

    - lhs is *likelihood ratio*

$$\frac{P(D|R)}{P(D|NR)} > \frac{P(NR)}{P(R)}$$

Estimating P(D|R)

- Assume independence

$$P(D|R) = \prod_{i=1}^{t} P(d_i|R)$$

- *Binary independence model*
  - document represented by a vector of binary features indicating term occurrence (or non-occurrence)
  - Assume:
    - p_i is probability that term i occurs (i.e., has value 1) in relevant document
    - s_i is probability of occurrence in non-relevant document

Binary Independence Model

$$\frac{P(D|R)}{P(D|NR)} = \prod_{i:d_i=1} \frac{p_i}{s_i} \cdot \prod_{i:d_i=0} \frac{1-p_i}{1-s_i}$$

$$= \prod_{i:d_i=1} \frac{p_i}{s_i} \cdot \left(\prod_{i:d_i=1} \frac{1-s_i}{1-p_i} \cdot \prod_{i:d_i=1} \frac{1-p_i}{1-s_i}\right) \cdot \prod_{i:d_i=0} \frac{1-p_i}{1-s_i}$$

$$= \prod_{i:d_i=1} \frac{p_i(1-s_i)}{s_i(1-p_i)} \cdot \prod_i \frac{1-p_i}{1-s_i}$$

Binary Independence Model

- Scoring function is

$$\sum_{i:d_i=1} \log \frac{p_i(1-s_i)}{s_i(1-p_i)}$$

- Query provides information about relevant documents
- If we assume p_i constant, s_i approximated by entire collection, get *idf*-like weight

$$\log \frac{0.5(1-\frac{n_i}{N})}{\frac{n_i}{N}(1-0.5)} = \log \frac{N-n_i}{n_i}$$

Contingency Table

	Relevant	Non-relevant	Total
$d_i = 1$	r_i	$n_i - r_i$	n_i
$d_i = 0$	$R - r_i$	$N - n_i - R + r_i$	$N - n_i$
Total	R	$N - R$	N

$$p_i = (r_i + 0.5)/(R + 1)$$

$$s_i = (n_i - r_i + 0.5)/(N - R + 1)$$

Gives scoring function:

$$\sum_{i:d_i=q_i=1} \log \frac{(r_i+0.5)/(R-r_i+0.5)}{(n_i-r_i+0.5)/(N-n_i-R+r_i+0.5)}$$

BM25

- Popular and effective ranking algorithm based on binary independence model
  - adds document and query term weights

$$\sum_{i \in Q} \log \frac{(r_i+0.5)/(R-r_i+0.5)}{(n_i-r_i+0.5)/(N-n_i-R+r_i+0.5)} \cdot \frac{(k_1+1)f_i}{K+f_i} \cdot \frac{(k_2+1)qf_i}{k_2+qf_i}$$

  - k_1, k_2 and K are parameters whose values are set empirically

$$K = k_1((1-b) + b \cdot \frac{dl}{avdl})$$

  - *dl* is document length
  - *avdl is* average document length
  - Typical value for k_1 is 1.2, k_2 varies from 0 to 1000, b = 0.75

BM25 Example

- Query with two terms, “president lincoln”, (*qf = 1)*
- No relevance information (*r and R are* zero)
- N = 500,000 documents
- *“president”* occurs in 40,000 documents (n_1 = 40,000)
- *“lincoln”* occurs in 300 documents (n_2 = 300)
- “president” occurs 15 times in doc (f_1 = 15)
- *“lincoln”* occurs 25 times (f_2 = 25)
- document length is 90% of the average length ($dl/avdl$ = .9)
- k_1 = 1.2, b = 0.75, and k_2 = 100
- $K = 1.2 \cdot (0.25 + 0.75 \cdot 0.9) = 1.11$

BM25 Example

BM25 (Q,D) = $\sum_{i \in Q} \log \frac{(r_i+0.5)/(R-r_i+0.5)}{(n_i-r_i+0.5)/(N-n_i-R+r_i+0.5)} \cdot \frac{(k_1+1)f_i}{K+f_i} \cdot \frac{(k_2+1)qf_i}{k_2+qf_i}$

$$
\begin{aligned}
BM25(Q,D) \quad = & \\
& \log \frac{(0+0.5)/(0-0+0.5)}{(40000-0+0.5)/(500000-40000-0+0+0.5)} \\
& \times \frac{(1.2+1)15}{1.11+15} \times \frac{(100+1)1}{100+1} \\
& + \log \frac{(0+0.5)/(0-0+0.5)}{(300-0+0.5)/(500000-300-0+0+0.5)} \\
& \times \frac{(1.2+1)25}{1.11+25} \times \frac{(100+1)1}{100+1}
\end{aligned}
$$

$$
\begin{aligned}
= \quad & \log 460000.5/40000.5 \cdot 33/16.11 \cdot 101/101 \\
& + \log 499700.5/300.5 \cdot 55/26.11 \cdot 101/101 \\
= \quad & 2.44 \cdot 2.05 \cdot 1 + 7.42 \cdot 2.11 \cdot 1 \\
= \quad & 5.00 + 15.66 = 20.66
\end{aligned}
$$

BM25 Example

- Effect of term frequencies

Frequency of "president"	Frequency of "lincoln"	BM25 score
15	25	20.66
15	1	12.74
15	0	5.00
1	25	18.2
0	25	15.66

Summary

- Probabilistic models for information retrieval
  - Language models
  - BM25

- More advanced models available
  - Learning to rank
  - Neural/dense retrieval models