UCL

Advanced Machine Learning in Finance

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LECTURE 3

- K-Nearest Neighbors (KNN)
- Random Forest
- Support Vector Machines (SVM)
- Perceptron
- Multilayer Perceptron
- Case study: Credit Crisis Prediction

Supervised Learning

Let us consider a set of samples, a paired list of features x and labels y

x_{11}					x_{1N}
x_{P1}					x_{PN}

y_1
y_P

K-Nearest Neighbors

K-Nearest Neighbors is a supervised algorithm which - quite literally - takes a test sample and looks at its **k nearest neighbors** (e.g. defined with a Euclidean norm in the space of features) in the training set.

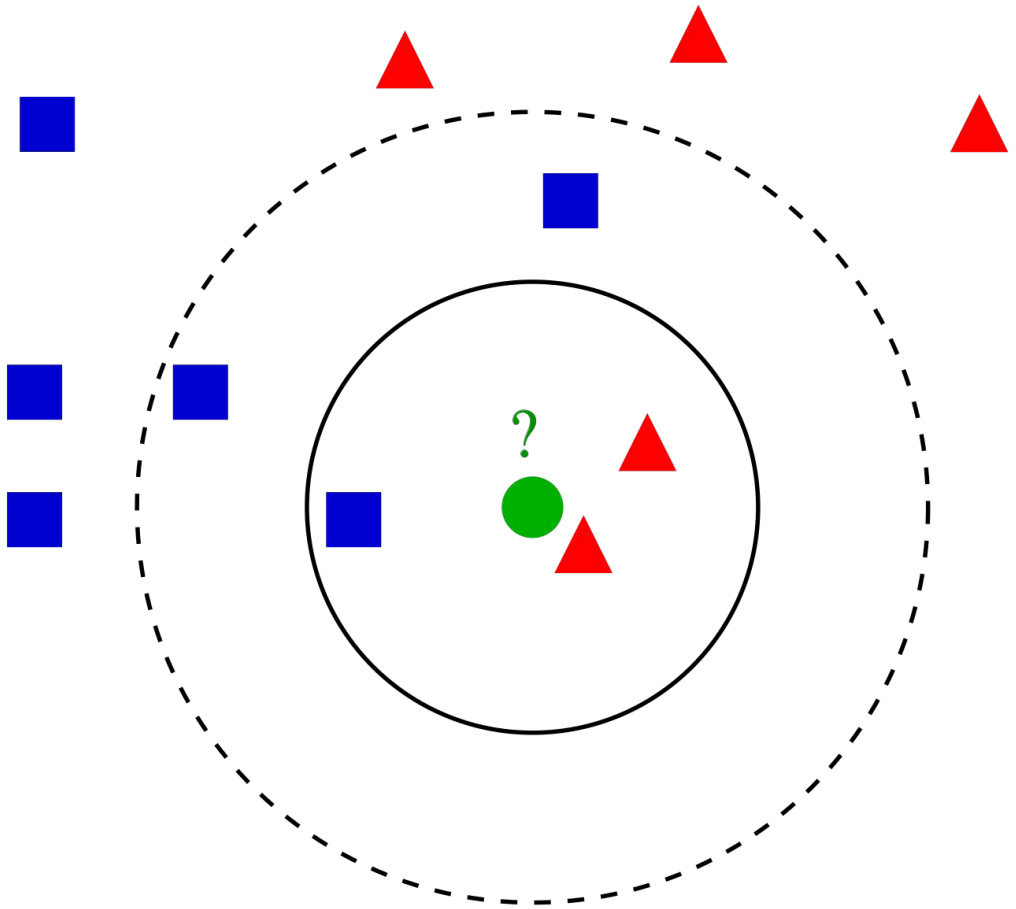
K-Nearest Neighbors

K-NN

- for classification, it assigns the test sample to the class (y label) of the majority of its neighbors
- for regression, it takes the average value of the y values of its neighbors

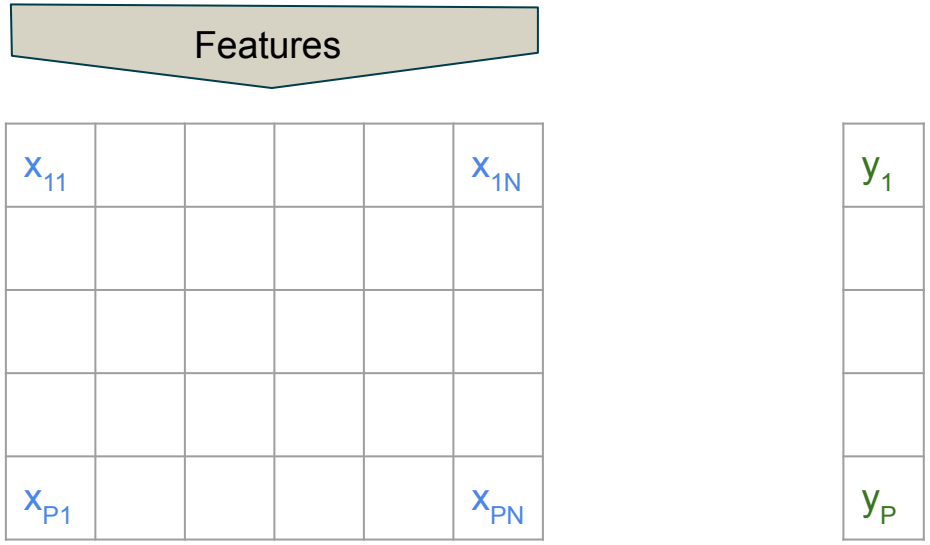
k is an hyper-parameters which can be optimised, e.g. using a validation set.

K-Nearest Neighbors



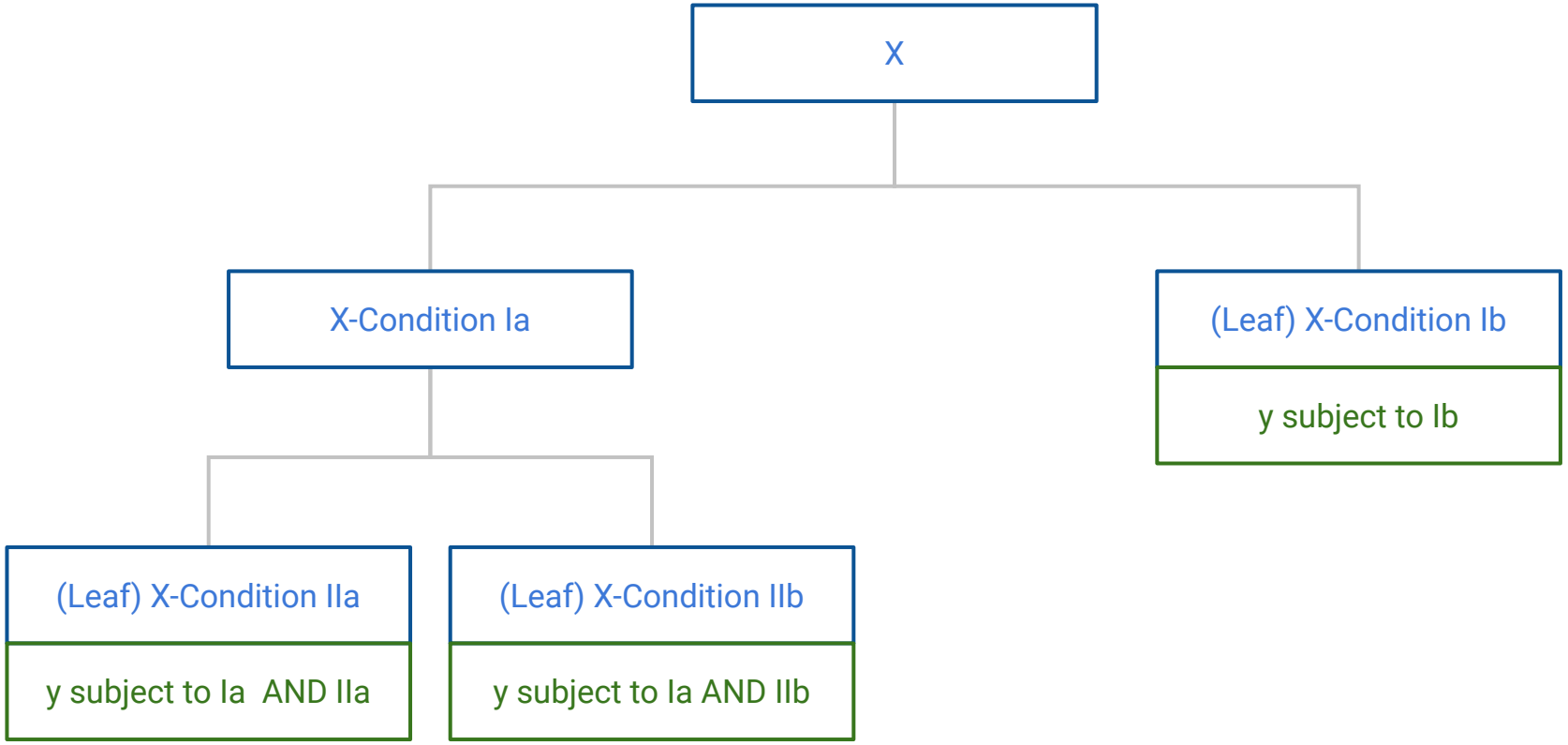
Random (Decision) Forest

A forest of random decision trees, i.e. ensemble learning with decision trees.

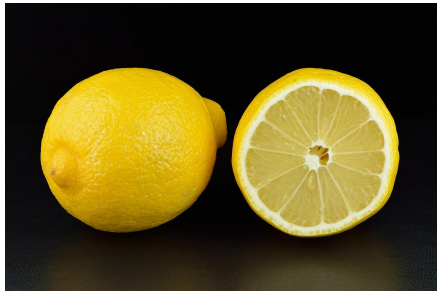


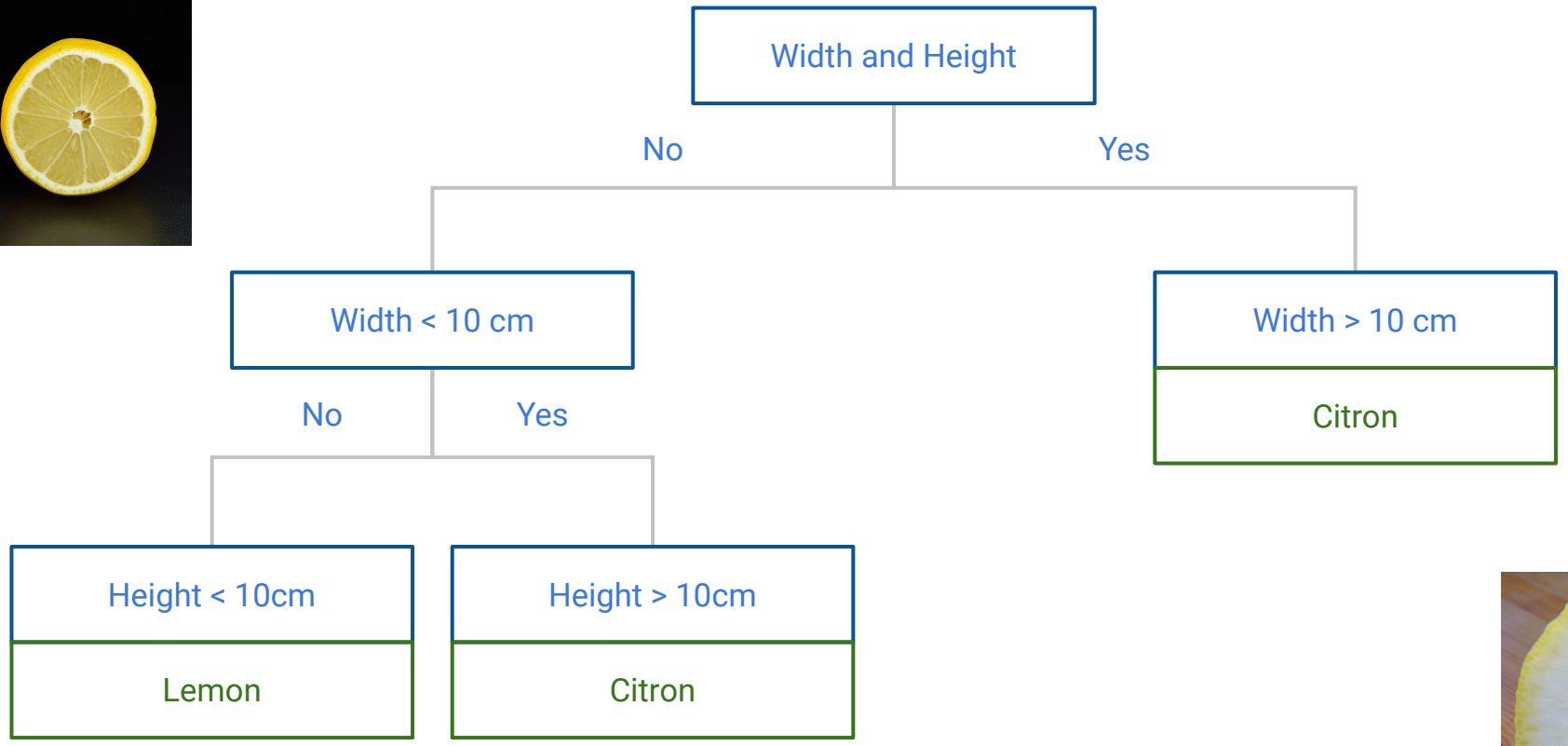
NOT a forest of random decisions!

A decision tree

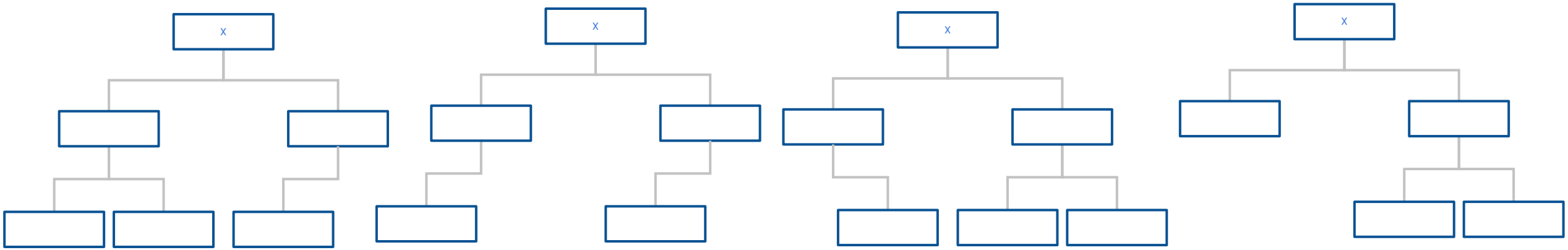


A decision tree (example)





A forest of decision trees



A forest of decision trees

Each tree is NOT completely random,

so exactly what is random about the tree?

A forest of decision trees

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The subset of features that is used for learning each decision tree is random, i.e. each tree considers a random subset of features (a random subset of columns in the feature matrix) so that the decision problem for each tree takes little time.

A forest of decision trees

This is because one needs to compare a few features at a time and each decision (branch) is made based on computing the information gain given by each feature in the dataset, e.g. the conditional entropies.

$$
\begin{aligned}
H(Y|X) &= \sum_{x \in X} p(x) H(Y|X = x) \\
&= -\sum_{x \in X} \sum_{y \in Y} p(x, y) \log_2 p(y|x)
\end{aligned}
$$

A random subset of features

A random forest algorithm

Algorithm 1 Random Forest

Precondition: A training set $S := (x_1, y_1), \ldots, (x_n, y_n)$, features F, and number of trees in forest B.

```
function RandomForest(S, F)
    H ← ∅
    for i ∈ 1, ..., B do
        S^(i) ← A bootstrap sample from S
        h_i ← RandomizedTreeLearn(S^(i), F)
        H ← H ∪ {h_i}
    end for
    return H
end function
function RandomizedTreeLearn(S, F)
    At each node:
        f ← very small subset of F
        Split on best feature in f
    return The learned tree
end function
```

http://pages.cs.wisc.edu/~matthewb/pages/notes/pdf/ensembles/RandomForests.pdf

Ensemble learning (a committee of trees)

Each random tree provides a prediction for the label given the (subset of) features and we can take the ensemble prediction, that is:

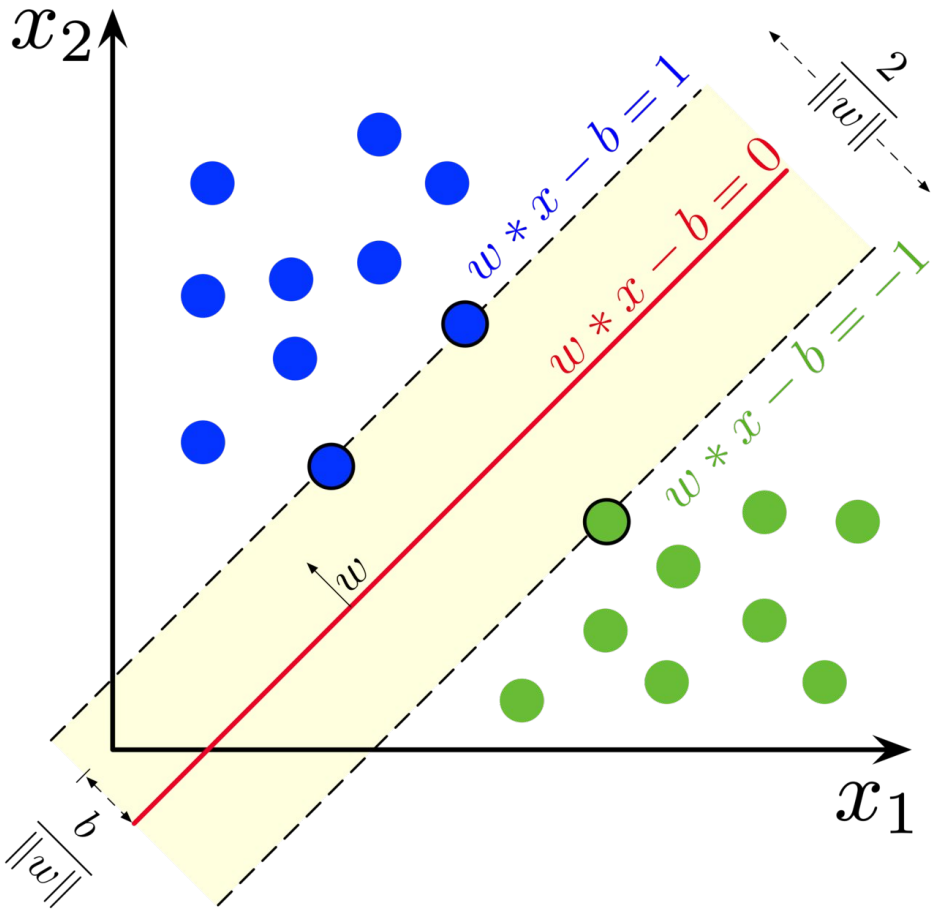
- the value of y which is predicted by the majority of trees for discrete labels (classification)
- y averaged over the trees for continuous labels (regression)

Support Vector Machines

Support vector machines identify a decision boundary,
by introducing a **support vector**.

The support vector defines an hyperplane in the space of features or a
in a transformed space of features.

Support Vector Machines



Support Vector Machines

The original linear version is the support vector classifier, which solves the following problem for the support vector w,

- Hard-margin. Minimise norm(w) subject to $y_i(\mathbf{w}^T\mathbf{x}_i - b) \geq 1$ for $i = 1, \ldots, n$.

- Soft-margin. Minimise $$\lambda\|\mathbf{w}\|^2 + \left[\frac{1}{n}\sum_{i=1}^{n}\max\left(0, 1 - y_i(\mathbf{w}^T\mathbf{x}_i - b)\right)\right]$$

Support Vector Machines

Support vector machines also consider a kernel function which allows a non-linear transformation of the original space of features.

In order to introduce the non-linear transformation the optimisation problem is

(1) mapped into its Lagrangian dual and then

(2) the feature space vector is transformed through a function $\varphi(\mathbf{x}_i)$ the kernel

Lagrangian Dual

$$\text{maximize } f(c_1 \dots c_n) = \sum_{i=1}^{n} c_i - \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} y_i c_i (\mathbf{x}_i^T \mathbf{x}_j) y_j c_j,$$

$$\text{subject to } \sum_{i=1}^{n} c_i y_i = 0, \text{ and } 0 \leq c_i \leq \frac{1}{2n\lambda} \text{ for all } i.$$

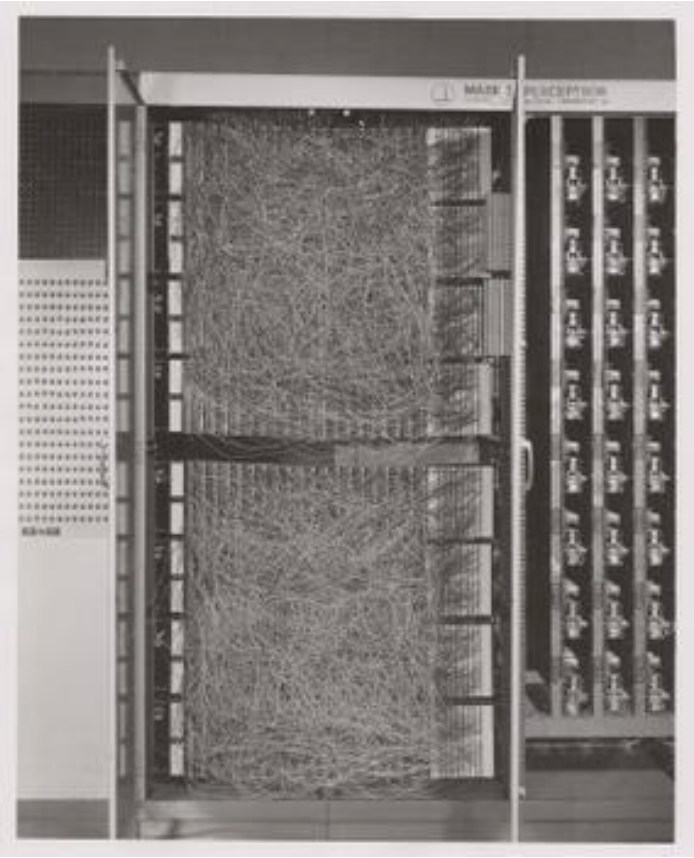
Lagrangian Dual with Kernel

$$\text{maximize } f(c_1 \dots c_n) = \sum_{i=1}^{n} c_i - \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} y_i c_i (\varphi(\mathbf{x}_i) \cdot \varphi(\mathbf{x}_j)) y_j c_j$$

$$= \sum_{i=1}^{n} c_i - \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} y_i c_i k(\mathbf{x}_i, \mathbf{x}_j) y_j c_j$$

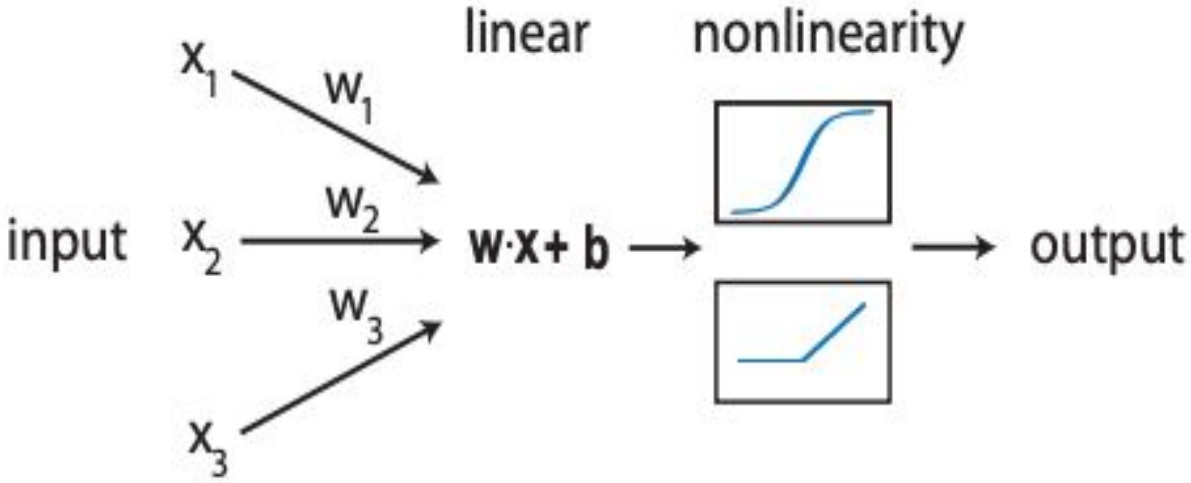
$$\text{subject to } \sum_{i=1}^{n} c_i y_i = 0, \text{ and } 0 \leq c_i \leq \frac{1}{2n\lambda} \text{ for all } i.$$

Perceptron



First perceptron Mark I machine for image recognition, it was connected to a 20x20 pixel camera, based on the adaptive algorithm by Frank Rosenblatt 1958.

Perceptron



$$\hat{y} = \Theta(\sum_{j=1}^{N} w_{1j} x_j + b_1)$$

Figures from A high-bias, low-variance introduction to Machine Learning for physicists. arXiv:1803.08823. Pankaj Mehta, Marin Bukov, Ching-Hao Wang, Alexandre Day, Clint Richardson, Charles Fisher, David Schwab.

Perceptron

The perceptron problem

- I have a training set, $training = \{\vec{x}_k, y_k\}_{k=1}^{P}$
- Can I find a set of weights, w, and biases, b, such that when I feed the training input to the perceptron its output coincides with the training output?
- Or equivalently, can we send the cost function, E, to zero?

$$E(w,b) = \sum_{k=1}^{P} (\hat{y}_k(w,b) - y_k)^2$$

Perceptron

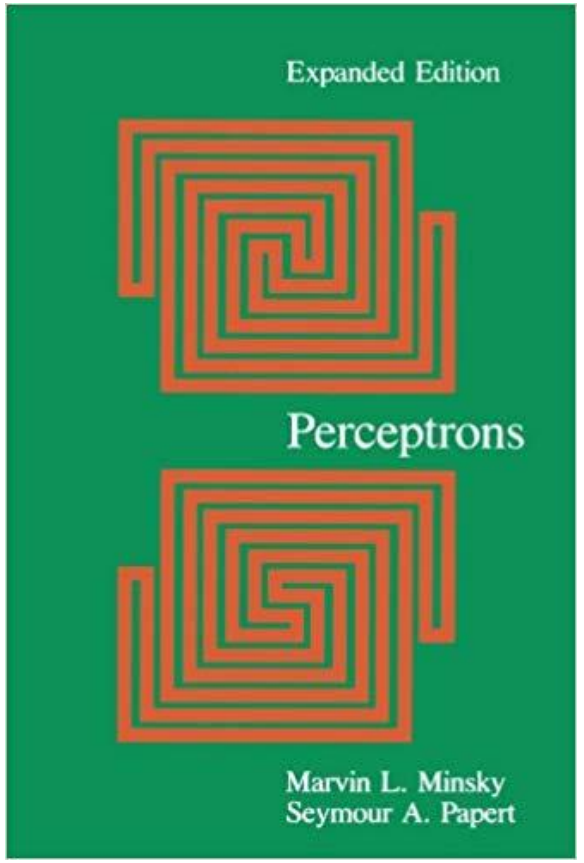
The perceptron problem

- I have a training set, $training = \{\vec{x}_k, y_k\}_{k=1}^{P}$
- Can I find a set of weights, w, and biases, b, such that when I feed the training input to the perceptron its output coincides with the training output?
- Yes, if P not too large with respect to N there exist values w* and b* such E(w*,b*)=0

$$E(w,b) = \sum_{k=1}^{P} (\hat{y}_k(w,b) - y_k)^2$$

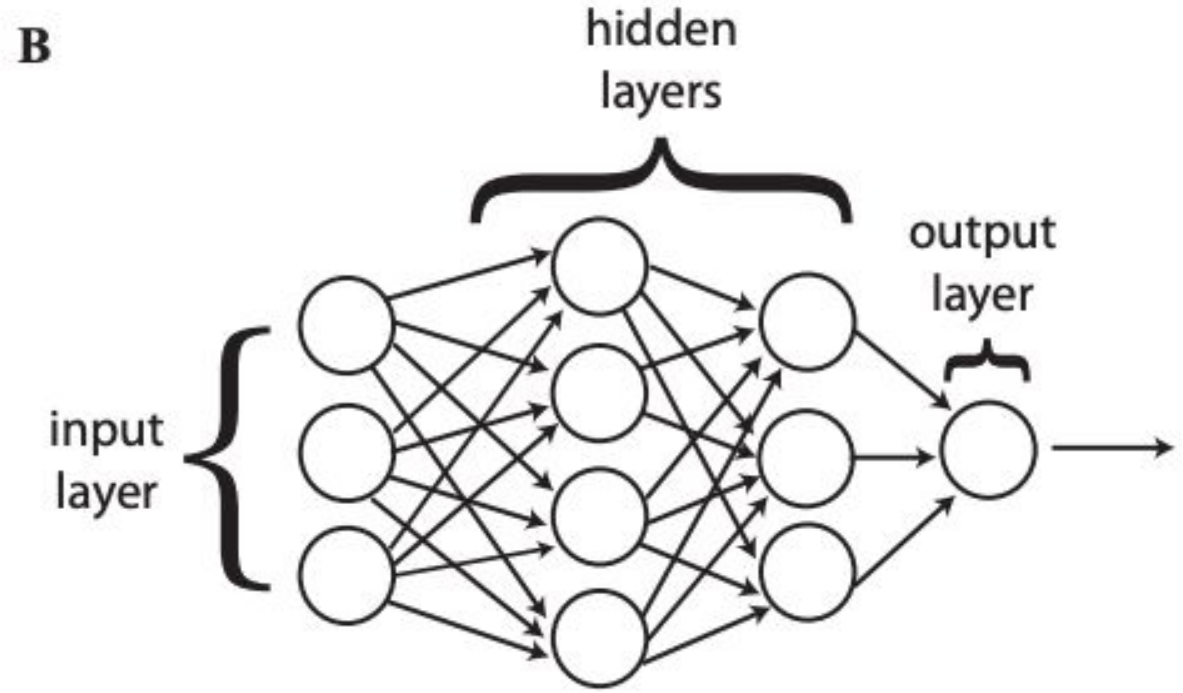
Perceptron

- Long known and studied in computer science and statistical physics literature
- Thought incapable to solve specific tasks in feasible simulations
- Minsky and Papert proved that the perceptron '*cannot compute parity under the condition of conjunctive localness and showed that the size required for a perceptron to compute connectivity grew impractically large'*



Minsky, Marvin; Papert, Seymour (1988). *Perceptrons: An Introduction to Computational Geometry*. MIT Press.

Multilayer Perceptron



$$\hat{y} = f_L\left(\sum_{j=1}^{N_{L-1}} w_{1j}^{(L)} a_j^{(L-1)}(x) + b_1^{(L)}\right)$$

$$z_i^{(l)} = (w^{(l)T} a^{(l-1)} + b^{(l)})_i \qquad a_i^{(l)}(x) = f_l\left(\sum_{j=1}^{N_{l-1}} w_{ij}^{(l)} a_j^{(l-1)} + b_i^{(l)}\right) = f_l(z_i^{(l)})$$

Figures from A high-bias, low-variance introduction to Machine Learning for physicists. arXiv:1803.08823. Pankaj Mehta, Marin Bukov, Ching-Hao Wang, Alexandre Day, Clint Richardson, Charles Fisher, David Schwab.

Credit Crisis Prediction

Let us consider the paper by Daniel Fricke

Financial Crisis Prediction: A Model Comparison,

Daniel Fricke
Deutsche Bundesbank; University College London; London School of Economics & Political Science (LSE) - Systemic Risk Centre