## Question 1

1) (a) given risk-free zero interest rates,

calculate forward interest rates

$$R_{1-2} = R_2 + \frac{(R_2 - R_1)T_1}{T_2 - T_1}$$

$$R_{3-6} = 3.2 + \frac{(3.2-3) \times 3}{6-3} = 3.4$$

$$R_{6-9} = 3.4 + \frac{(3.4-3.2) \times 6}{9-6} = 3.8$$

$$R_{9-12} = 3.5 + \frac{(3.5-3.4) \times 9}{12-9} = 3.8$$

$$R_{12-15} = 3.6 + \frac{(3.6-3.5) \times 12}{15-12} = 4.0$$

$$R_{15-18} = 3.7 + \frac{(3.7-3.6) \times 15}{18-15} = 4.2$$

II) given bond zero rates, continuously compounded, cash flow, principal

(a) bond's theoretical price:

$$\text{price} = Pe^{-r(n)n} + \sum_{t \in \{0.5, 1, 1.5, 2\}} c \cdot e^{-r(t) \cdot t}$$

$$= 1000 \times e^{-0.032 \times 2}$$

$$20 \times (e^{-0.02 \times 0.5} + e^{-0.023 \times 1} + e^{-0.027 \times 1.5} + e^{-0.032 \times 2})$$

$$= 1015.32$$

(b) sell's at theoretical price,

calculate bond's yield

$$1015.32 = Pe^{-yn} + \sum_{t\in\{0.5,1,1.5,2\}} ce^{-yt}$$

$$1015.32 = 20e^{-0.5y} + 20e^{-y} + 20e^{-1.5y} + 1020e^{-2y}$$

$\therefore\ y = 0.0318 = 3.18\%$

III) convert a 5% per annum interest rate with semi annual compounding to:

$$\left(1+\frac{r_1}{m_1}\right)^{m_1} = \left(1+\frac{r_2}{m_2}\right)^{m_2}$$

(a) annual compounding:

$$\left(1+\frac{5\%}{2}\right)^2 = \left(1+\frac{r_a}{1}\right)^1$$

$$r_a = \left(1+\frac{5\%}{2}\right)^2 - 1 = 5.06\%$$

(b) Monthly Compounding:

$$\left(1+\frac{5\%}{2}\right)^2 = \left(1+\frac{r_m}{12}\right)^{12}$$

$$r_m = 12 \times \left[\sqrt[12]{\left(1+\frac{5\%}{2}\right)^2} - 1\right] = 4.95\%$$

(c) continuous compounding:

$$\left(1+\frac{5\%}{2}\right)^2 = e^{r_c}$$

$$e = 2 \times \ln\left(1+\frac{5\%}{2}\right) = 4.94\%$$

## Question 2

1)

| | Bill | Comm |
|---|---|---|
| expected earnings | 5 per share | 5 per share |

pay out all earnings as dividends, maintaining a per perpetuel dividen flow

(a) market capitalization rate $k = 12.5\%$

calculate value per share :

dividend discount model

$$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+k)^t} = D_t \cdot \sum_{t=1}^{\infty} \frac{1}{(1+k)^t}$$

$$= 5 \times \frac{1}{1+k} \times \frac{1}{1-\frac{1}{1+k}}$$

$$= 5 \times \frac{1}{1.125} \times \frac{1}{1-\frac{1}{1.125}} = 40$$

(b) after payout the dividend, will the firms grow in value? why?

since $P_0 = \frac{D_1 + P_1}{1+r}$

we have $P_1 = P_0(1+r) - D_1$

after paying the dividend $D_1$ :

$$P_1 = 40 \times (1 + 12.5\%) - 5$$

$$= 40 = P_0$$

$\therefore$ neither firm will grow in value because their sustainable growth rate is zero.

$$g = r - \frac{D_1}{P_0} = 12.5\% - \frac{5}{40} = 0$$

(c) Comm engages in growth projects with

?

a return on investment = 15%

how to manage the plowback ratio to grow the stock value? why?

retain more earnings (higher plowback)

$g = b \times ROE$

11) one-year risk-free bond with $r = 5\%$

corporate bond $F = 1000$, $T = 1$

payoff = 1050, $P_{default} = 20\%$

3% risk premium

(a) calculate the price of bond and promised yield:

$r_p = y_e - r$

$\therefore y_e = r_p + r = 3\% + 5\% = 8\%$

$y_e = y_p \times 80\% + y_d \times 20\%$

$y_p = \frac{1050}{price} - 1$ and $y_d = \frac{0}{price} - 1 = -1$

$\therefore 8\% = \left(\frac{1050}{price} - 1\right) \times 0.8 - 1 \times 0.2$

price = 777.78

$y_p = \frac{1050}{777.78} - 1 = 35\%$

(b) what is a recovery rate?

how will price of the bond change if recovery rate = 4%

recovery rate of a bond is the bond's market value shortly after default, as a fraction of its face value.

$$\lambda = \frac{s}{1-R}$$

$$20\% = \frac{s}{1-40\%} \quad \therefore s = 0.12$$

$$s = y_P - r = \frac{1050}{P} - 1 - 5\% = 0.12$$

$$P = 897.44 \quad \therefore \text{ price increases if } R = 40\%$$

(C) you are worried about the default risk, sell or buy credit default swaps?

Buy. I will want to purchase for protection against losses from the default.

## Question 3

I) $T=3$ , corporate bond , $S=60$ basis points

$R=40\%$ , estimate the averaged hazard rate per year

$$\bar{\lambda}(3) = \frac{S}{1-R} = \frac{0.0060}{1-0.4} = 1\%$$

II) (a) spot price = 20 / barrel , storing = 2 / barrel

risk-free interest rate = 7% , continuous compounding

calculate the upper bound for the 1-year futures price

$$F = \left(20 + 2e^{-7\%}\right) \cdot e^{7\%} = 23.45$$

(b) currently $\frac{CHF}{EUR} = 1.1000$ francs per euro ,

6-month forward exchange rate = 1.0950 ,

6-month CHF interest rate = 3% per annum

(continuously compounded)

estimate the 6-month EUR interest rate

assume we have $X$ amount of CHF at $t=0$

then we can exchange for $1.1X$ EUR at $t=0$

At $t=\frac{1}{2}$ , to have no arbitrage , we should have:

$$1.1Xe^{r\cdot\frac{1}{2}} = 1.0950 \cdot Xe^{3\%\cdot\frac{1}{2}}$$

$$\therefore\ r = 2 \times \ln\left(1.095 \times e^{1.5\%}\right) = 2.09\%$$

## Question 4

1) Stock, no dividend, spot price = 40, entered a 1-year forward contract, $r = 5\%$ (continuous compounding)

(a) initial value: $f_0 = S_0 - Ke^{-rT} = 0$

(b) forward price at $t=0$:

$$F_0 = S_0 e^{rT} = 40 \times e^{5\% \times 1} = 42.05$$

(c) given $S_{\frac{1}{2}} = 45$, $r = 5\%$

$$K = S_0 e^{rT} = 42.05$$

forward price:

$$F_{\frac{1}{2}} = S_{\frac{1}{2}} e^{r(1-\frac{1}{2})}$$

$$= 45 \times e^{5\% \times \frac{1}{2}} = 46.14$$

forward value:

$$f_{\frac{1}{2}} = S_{\frac{1}{2}} - Ke^{-r(1-\frac{1}{2})}$$

$$= 45 - 42.05 \times e^{-5\% \times \frac{1}{2}}$$

$$= 3.99$$

(d) currently $f = 3.33$, how to earn a profit?

the value of the forward contract increased before maturity. (from 0 to 3.33)

this reflects a shift in the underlying spot exchange rate.

thus, Pepaw can close out the forward contract by entering into an opposite forward contract with the same expiration data at the current market rate to take the opportunity profit.

11) call option, $k = 105$, $T = 2$, $S_0 = 100$,

$P_u = 0.65$, $= 1.3$

$P_d = 0.35$, $= 1.05$

$r = 1\%$ : $P_r = 0.7$, $u_r = 0.04$

$$f_{uu} = \max(Su^2 - k, 0) = \max(169 - 105, 0) = 64$$

$$f_{ud} = \max(Sud - k, 0) = \max(136.5 - 105, 0) = 31.5$$

$$f_{dd} = \max(Sd^2 - k, 0) = \max(110.25 - 105, 0) = 5.25$$

$$f_{u_1} = e^{-r_{uu}\Delta t}(P_u \cdot f_{uu} + P_d \cdot f_{ud})$$

52.625

$$= e^{-0.09\times 1}(0.65\times 64 + 0.35\times 31.5)$$

$$= 48.10$$

$$f_{u_2} = e^{-r_{ud}\Delta t}(P_u \cdot f_{uu} + P_d \cdot f_{ud})$$

$$= e^{-0.05\times 1}\left(0.65\times 64 + 0.35\times 31.5\right)$$

$$= 50.06$$

$$f_{u_3} = e^{-r_{dd}\Delta t}\left(P_u \cdot f_{uu} + P_d \cdot f_{ud}\right)$$

$$= e^{-0.01\times 1}\left(0.65\times 64 + 0.35\times 31.5\right)$$

$$= 52.10$$

$$f_u = 0.7\times 0.7\times f_{u_1} + 0.7\times 0.3\times f_{u_2} + 0.3\times 0.3\times f_{u_3}$$

$$=$$