On the Intensity of 834 Å from Interplanetary Oxygen

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1 The Problem

8 Rayleigh's of intensity at 834Å were measured in a rocket experiment conducted in 2001. This flight was conducted at night and the line of sight remains in the dark for over 1,000 km above ground, and is therefore above where O+ usually dwells in the atmosphere. A possible solution to this problem is the interplanetary oxygen in the solar system could be contributing enough emission to create this intensity. This paper seeks to find an upper bound on the expected intensity from interplanetary sources to see if this solution is indeed feasible.

2 Preliminary Information

The interplanetary medium that fills the solar system contains noticeable amounts of oxygen, roughly 477 ppm. Most of this is neutral and not the ionized oxygen (O II or O III) that have 834Å lines. By assuming that all of the oxygen in the solar system is singly or double ionized we can create an upper bound to the expected intensity at this wavelength. We compute the intensity by solving the transfer equation from radiative transfer. This equation roughly states the following:

$$\text{Change in Intensity} = \text{Intensity Gained from Emission} - \text{Intensity Lost to Absorption} \tag{1}$$

This is equivalent to the differential equation:

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu \tag{2}$$

Where α_ν is the absorption coefficient, j_ν is the emission coefficient, and I_ν is the specific intensity. By dividing throughout by α_ν we get the equivalent equation:

$$\frac{dI_\nu}{dt_\nu} = -I_\nu + S_\nu \tag{3}$$

Where S_ν is the source function, and τ_ν is the optical depth. We can solve this directly from straightforward integration by introducing an appropriate multiplying factor.

$$\begin{aligned}
\frac{dI_\nu}{d\tau_\nu} &= -I_\nu + S_\nu && \text{(Transfer Equation)}\\
\Rightarrow e^{\int^\tau d\xi}[\frac{dI_\nu}{d\tau_\nu} + I_\nu] &= e^{\int^\tau d\xi} S_\nu\\
\frac{d}{d\tau}[e^\tau I_\nu] &= e^\tau S_\nu && \text{(Product Rule)}\\
I_\nu &= e^{-\tau}\int^\tau e^\xi S_\nu(\xi) d\xi && \text{(Solution)}
\end{aligned}$$

The solution can be further detail with information about the absorption and emission coefficients. The emission coefficient is the solution to the differential form $dE = j_\nu dV d\Omega dt d\nu$. This is related to the emissivity ϵ_ν and the density ρ of the material by $j_\nu = \frac{\epsilon_\nu \rho}{4\pi}$. Quantum mechanical considerations allow us to relate this to the Einstein coefficient A_{21}:

$$j_\nu = \frac{h\nu_0}{4\pi} n_2 A_{21}\phi(\nu) \tag{4}$$

Where $\phi(\nu)$ is the line profile function, and ν_0 is the point where $\frac{d\phi}{d\nu} = 0$, h is Planck's constant, and n_2 is the number density of atoms in the level 2 state. The line profile function has the closed form $\phi(\nu) = \frac{1}{\Delta\nu\sqrt{\pi}} e^{-\frac{(\nu-\nu_0)^2}{\Delta\nu}}$, where $\Delta\nu = \frac{\nu_0}{c}\sqrt{\frac{2kT}{m}}$. The absorption is related to the number density of the absorbing material and its cross-section:

$$\alpha_\nu = n_A \sigma_\nu \tag{5}$$

Piecing this together, we get a solution to the general transfer equation:

$$I_\nu = e^{-\tau}\int_0^\tau e^\xi \cdot \frac{h\nu_0 n_2(\xi) A_{21}\phi(\nu)}{4\pi n_A(\xi)\sigma_\nu} d\xi \tag{6}$$

Which simplifies to:

$$I_\nu = e^{-\tau}\frac{h\nu_0 A_{21}\phi(\nu)}{4\pi\sigma_\nu}\int_0^\tau \frac{n_2(\xi)}{n_A(\xi)} e^\xi d\xi \tag{7}$$

3 Calculating the Intensity

We can obtain an upper bound to the intensity from the interplanetary medium by replacing the exact functional solution $I_\nu = F[S_\nu]$ with an approximate solution that considers only the bounds on the source function. A reasonable approximation is that $\frac{n_2}{n_A}$ is a constant. We obtain an even greater bound on I_ν by replacing n_2 with n_E. This bound is greater as $n_E = n_1 + n_2 \geq n_2$. For our problem n_E is the number density of the emitting material, oxygen, and n_A is the number density of the absorbing material, hydrogen. This ratio is well known in the solar system and is known from the solar abundance. We compute the following inequality:

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\begin{aligned}
I_\nu &= e^{-\tau}\frac{h\nu_0 A_{21}\phi(\nu)}{4\pi\sigma_\nu}\int_0^\tau \frac{n_2(\xi)}{n(\xi)}e^\xi d\xi && \text{(Solution to Transfer Equation)} \\
I_\nu &= I_\nu(0)e^{-\tau} + S_\nu(1-e^{-\tau}) && \text{(Constancy of Oxygen to Hydrogen Ratio)} \\
I_\nu &\leq S_\nu && \text{(Boundary Conditions)} \\
I_\nu &\leq \frac{h\nu_0 A_{21}\phi(\nu)}{4\pi\sigma_\nu}\frac{n_E}{n_A} \\
I_\nu &\leq \frac{h\nu_0 A_{21}\phi(\nu_0)}{4\pi\sigma_\nu}\frac{n_E}{n_A} && \text{(Definition of Line Profile Function)} \\
&= \frac{h\nu_0 A_{21}}{4\pi\sigma_\nu}\frac{n_E}{n_A}\frac{c}{\nu_0}\sqrt{\frac{m}{2kT}} && \text{(Closed Form of Line Profile)} \\
&= \frac{hA_{21}}{4\pi\sigma_\nu}\frac{n_E}{n_A}c\sqrt{\frac{m}{2kT}}
\end{aligned}
$$

We have that $n_E = 477, n_A = 909,964, A_{21} = 8.8\times10^8 s^{-1}, \sigma_\nu = 1.12\times10^{-13}cm^2, m = 2.6\times10^{-23}g, T = 1,000K, k = 1.38\times10^{-16}ergsK^{-1}, h = 6.63\times10^{-37}erg\cdot s, c = 3\times10^{10}cm\cdot s^{-1}$. Plugging this mess in gives us $I_\nu \leq 6.33\times10^{-4}R$. Much less then the 8 Rayleigh's observed.

4 Conclusion

The upper bound for the intensity from the interplanetary medium took some massive approximations to bump the number much higher, and still falls far too short of the observed quantity. The replacement of n_2 with n_E is a massive boost as this is equivalent to saying all of the oxygen in the solar system is in the upper level, which would require the sun to have infinite brightness. The number density that was used for O II is actually the number density of all oxygen, a much larger number. It was also shown that $I_\nu \propto \phi(\nu)$, and thus by replacing the line profile function with its upper bound we are raising the bound on the intensity. That being said, the upper bound we have obtained is several many orders of magnitude above the actual intensity that one would expect to observe from the interplanetary medium, and is still much to small to be a solution to our problem.