

Two more Classification Techniques

Classification

- Regression: Target variable (y) is numerical
- Classification: Target variable (y) is binary or categorical
- Examples:
 - Is an email legitimate or spam?
 - Will a flight be on time or late?
 - Will a student receive an High Pass, Pass, Low Pass or Fail?
 - Is it an appropriate time to buy or sell a stock?
 - Will a customer choose to buy or not buy?
- The prediction in many Classification techniques is the probability of categories (or labels) for given set of input variables (X)

Default data

- Default of individual credit card payment; 10,000 records
- Target: default (y)
 - Yes, No
- Features (Predictors) (X)
 - student (Yes(1), No(0)), balance, income

	default	student	balance	income
0	No	0	729.526495	44361.625070
1	No	1	817.180407	12106.134700
2	No	0	1073.549164	31767.138950
3	No	0	529.250605	35704.493940
4	No	0	785.655883	38463.495880
5	No	1	919.588530	7491.558572
6	No	0	825.513330	24905.226580
7	No	1	808.667504	17600.451340
8	No	0	1161.057854	37468.529290
9	No	0	0.000000	29275.268290

Descriptive Analysis

```
no=df_default['default']=='No'
yes=df_default['default']=='Yes'
```

- Individuals who default tend to have higher balance
- Individuals who default tend to have lower income

df_default[no].describe()

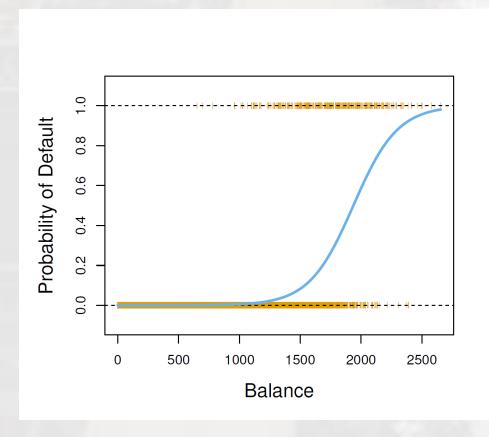
	student	balance	income
count	9667.000000	9667.000000	9667.000000
mean	0.291404	803.943750	33566.166625
std	0.454433	456.476236	13318.251249
min	0.000000	0.000000	771.967729
25%	0.000000	465.714646	21405.060665
50%	0.000000	802.857102	34589.488060

df_default[yes].describe()

	student	balance	income
count	333.000000	333.000000	333.000000
mean	0.381381	1747.821690	32089.147124
std	0.486457	341.266808	13804.221110
min	0.000000	652.397134	9663.788159
25%	0.000000	1511.610952	19027.508630
50%	0.000000	1789.093391	31515.344490

Simple Logistic Regression

• Let's consider probability that an individual will default depends only on "balance"



$$Pr(default = Yes) = \frac{e^{\beta_0 + \beta_1 \text{ balance}}}{1 + e^{\beta_0 + \beta_1 \text{ balance}}}$$

(Logistic function)

$$Pr(default = No) = \frac{1}{1 + e^{\beta_0 + \beta_1 \text{ balance}}}$$

$$Pr(default = Yes) + Pr(default = No) = 1$$

Manipulation

$$\frac{\Pr(\text{default} = \text{Yes})}{1 - \Pr(\text{default} = \text{Yes})} = e^{\beta_0 + \beta_1 \text{ balance}}$$

- Left hand side of the above equation is call "odds"
- Odds close to 0 indicate low probability of default
- Odds close to 1 indicate <u>high</u> probability of default

$$\ln\left(\frac{\Pr(\text{default} = \text{Yes})}{1 - \Pr(\text{default} = \text{Yes})}\right) = \beta_0 + \beta_1 \text{ balance}$$

Left hand side of the above example is call "log-odds"

Logistic Regression in scikit-learn

```
X,y=df_default[['balance']],df_default['default']
from sklearn.model_selection import train_test_split
X_train, X_test, y_train, y_test=train_test_split(X,y,random_state=0)

from sklearn.linear_model import LogisticRegression

log_reg=LogisticRegression()
log_reg.fit(X_train,y_train)
```

```
print('Log reg acc on train: {:.3f}'.format(log_reg.score(X_train,y_train)))
print('Log reg acc on test: {:.3f}'.format(log_reg.score(X_test,y_test)))

Log reg acc on train: 0.973
Log reg acc on test: 0.968
```

```
p1=[1000]
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   p2=[2000]
   p3=[3000]
   print('Predcited traget for p1, p2, and p3:',log_reg.predict([p1,p2,p3]))
  Predcited traget for p1, p2, and p3: ['No' 'No' 'Yes']
log_reg.predict_proba([p1,p2,p3])
array([[0.98723001, 0.01276999],
       [0.54594197, 0.45405803],
       [0.01835677, 0.98164323]])
log_reg.classes_
array(['No', 'Yes'], dtype=object)
print('Probablities for p1, p2, and p3:',log_reg.predict_proba([p1,p2,p3])[:,1])
Probablities for p1, p2, and p3: [0.01276999 0.45405803 0.98164323]
```

Logistic Regression

Pr(target vriable) = logitic(
$$\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_p x_p$$
)

Pr(target vriable) =
$$\frac{e^{(\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_p x_p)}}{1 + e^{(\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_p x_p)}}$$

$$\ln\left(\frac{\text{Pr(target vriable)}}{1 - \text{Pr(target vriable)}}\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_p x_p$$

• Coefficients (β_0 , β_1 ,...) of the model are estimated using "Maximum Likelihood"

```
X,y=diabetes.iloc[:,:-1],diabetes['Outcome']
            X_train, X_test, y_train, y_test=train_test_split(X,y,random_state=0)
Logis
            log_reg2=LogisticRegression()
log_reg2.fit(X_train,y_train)
            LogisticRegression(C=1.0, class_weight=None, dual=False, fit_intercept=True,
                       intercept_scaling=1, max_iter=100, multi_class='ovr', n_jobs=1,
                       penalty='12', random_state=None, solver='liblinear', tol=0.0001,
                       verbose=0, warm start=False)
             print('Log reg2 acc on train: {:.3f}'.format(log_reg2.score(X_train,y_train)))
             print('Log reg2 acc on test: {:.3f}'.format(log_reg2.score(X_test,y_test)))
            Log reg2 acc on train: 0.757
            Log reg2 acc on test: 0.807
             p1=[3,150,80,22,10,40,2.3,66]
            log reg2.predict([p1])
            array([1], dtype=int64)
            log_reg2.predict_proba([p1])
            array([[0.14143716, 0.85856284]])
            log reg2.classes
            array([0, 1], dtype=int64)
```

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Logistic Regression

- By default, logistic regression applies an L2 regularization (in the same way that Ridge does for regression)
- L2 regularization, forces coefficients' magnitude to be closer to zero
- For *LogisticRegression*, the trade-off parameter that determines the strength of the regularization is called *C*.
- Higher values of C correspond to <u>less</u> regularization
 - Higher C, LogisticRegression tries to fit the training set as best as possible
 - Lower C, more regularization, coefficients (β_i s) closer to zero

```
cancer=pd.read_csv('breast_cancer_data.csv',index_col=0)
cancer.head()
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```

diagnosis radius_mean texture_mean perimeter_mean area_mean sn

id					
842302	M	17.99	10.38	122.80	1001.0
842517	M	20.57	17 77	132 90	1326 በ

```
X,y=cancer.iloc[:,1:],cancer['diagnosis']
X_train,X_test,y_train,y_test=train_test_split(X,y,random_state=0)
```

```
log_reg3=LogisticRegression()
log_reg3.fit(X_train,y_train)
print('Log reg3 acc on train: {:.3f}'.format(log_reg3.score(X_train,y_train)))
print('Log reg3 acc on test: {:.3f}'.format(log_reg3.score(X_test,y_test)))
```

Log reg3 acc on train: 0.960 Log reg3 acc on test: 0.958

Logistic Regression in scikit-learn

```
log_reg4=LogisticRegression(C=100)
log_reg4.fit(X_train,y_train)
print('Log reg4 acc on train: {:.3f}'.format(log_reg4.score(X_train,y_train)))
print('Log reg4 acc on test: {:.3f}'.format(log_reg4.score(X_test,y_test)))
Log reg4 acc on train: 0.967
Log reg4 acc on test: 0.965
log_reg5=LogisticRegression(C=.1)
log_reg5.fit(X_train,y_train)
print('Log reg5 acc on train: {:.3f}'.format(log_reg5.score(X_train,y_train)))
print('Log reg5 acc on test: {:.3f}'.format(log_reg5.score(X_test,y_test)))
Log reg5 acc on train: 0.951
Log reg5 acc on test: 0.944
```

Transforming the data (scaling)

- Some techniques are sensitive to the scale of variables in data
 - Different scales in various variables
 - High variability in a single variable (default example with balance as the only feature)
- Therefore, before developing these models, it is a good idea to transform all the variables to the same scale
- One way to do this is using the MinMaxScaler function
- This function transforms all variables to a between 0 and 1 scale
- It uses this formula:

$$x_{new} = \frac{x - x_{min}}{x_{max} - x_{min}}$$



Transforming the data (scaling)

x1	x2
100	1.1
110	1
130	1.2
115	3
125	2.2

		x1	x1_new
max	130	100	0
min	100	110	0.3
		130	1
		115	0.5
		125	0.8

		x2	x2_new
max	3	1.1	0.05
min	1	1	0
		1.2	0.1
		3	1
		2.2	0.6

x1_new	x2_new
0.0	0.05
0.3	0
1.0	0.1
0.5	1
0.8	0.6

Evaluating the Effect of Parameters

- validation_curve function
 - Determines training and test scores for varying parameter values.
- Function inputs:
 - estimator: object type that implements the "fit" and "predict" methods
 - X: Training features
 - y: Target
 - param_name: Name of the parameter that will be varied.
 - param_range: The values of the parameter that will be evaluated.
 - cv : Determines the cross-validation splitting strategy

3 breast cancer data

```
1 # reading the data
 cancer=pd.read_csv('breast_cancer_data.csv',index_col=0)
 3 cancer.head(1)
       diagnosis radius_mean texture_mean perimeter_mean area_mean smoothness_mean compa
    id
              M
                       17.99
                                   10.38
                                                  122.8
                                                           1001.0
                                                                           0.1184
842302
1 rows × 31 columns
 1 X_cancer, y_cancer=cancer.iloc[:,1:], cancer['diagnosis']
 2 display(X_cancer.head(1))
 3 display(y cancer.head(2))
       radius_mean texture_mean perimeter_mean area_mean smoothness_mean compactness_mea
    id
842302
             17.99
                          10.38
                                        122.8
                                                  1001.0
                                                                  0.1184
                                                                                   0.277
1 rows × 30 columns
id
842302
          Μ
842517
Name: diagnosis, dtype: object
```

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```
1 from sklearn.preprocessing import MinMaxScaler
)]:
    1 | scaler1=MinMaxScaler()
    2 X_cancer_trns=scaler1.fit_transform(X_cancer)
    3 X_cancer_trns
0.41886396],
        [0.64314449, 0.27257355, 0.61578329, ..., 0.63917526, 0.23358959,
         0.22287813],
        [0.60149557, 0.3902604, 0.59574321, ..., 0.83505155, 0.40370589,
```

```
2 from sklearn.model selection import validation curve
                                        1 | C range=[.1,.5,1,10,100,200]
                                        1 # validation_curve has two outputs: 1- score on traning sets 2- scores on test sets
                                        2 # order is important
                                           train scores, test scores=validation curve(LogisticRegression(solver='liblinear'),
                                                                                         X cancer, y cancer,
                                                                                        param name='C',param range=C range, cv=4)
 1 train scores.round(4)
array([[0.9531, 0.9532, 0.9438, 0.9578],
       [0.9577, 0.9555, 0.9508, 0.9672],
      [0.9554, 0.9578, 0.9555, 0.9649],
      [0.9695, 0.9696, 0.9649, 0.9696],
       [0.9765, 0.9696, 0.9672, 0.9789],
       [0.9742, 0.9742, 0.9789, 0.9742]])
 1 print('ave cross val scores on train:',train scores.mean(axis=1).round(4))
ave cross val scores on train: [0.952 0.9578 0.9584 0.9684 0.9731 0.9754]
 1 test scores.round(4)
array([[0.9021, 0.9648, 0.9577, 0.9085],
      [0.9301, 0.9366, 0.9718, 0.9296],
      [0.9301, 0.9366, 0.9718, 0.9366],
       [0.9371, 0.9507, 0.9718, 0.9437],
       [0.9301, 0.9507, 0.9718, 0.9507],
      [0.9301, 0.9437, 0.9718, 0.9437]])
 print('ave cross val scores on test:',test_scores.mean(axis=1).round(4))
ave cross val scores on test: [0.9333 0.942 0.9438 0.9508 0.9508 0.9473]
```

Support Vector Machines

Support Vector Machine

- Approach for Classification (also for regression, but in this class we only use it for classification)
- Developed by Computer Scientists in 1990's
- In short stated as "SVM"
- The are two type of SVMs:
 - Linear SVM
 - Non-linear SVM

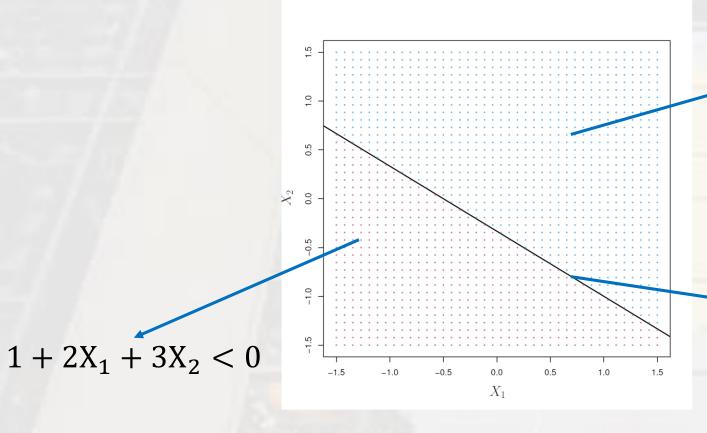
Terminology

- Hyperplane
 - In two dimension space: a line
 - In three dimension space: a plane
- Equation of the hyperplane in a p-dimensional space:

•
$$\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_p X_p = 0$$

• Example : A hyperplane in two-dimensional space $1 + 2X_1 + 3X_2 = 0$

Terminology



$$1 + 2X_1 + 3X_2 > 0$$

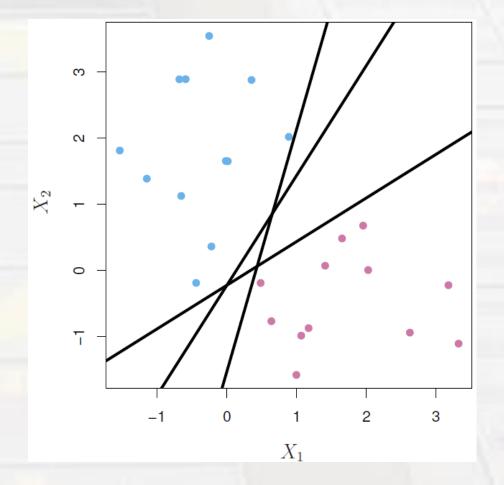
$$1 + 2X_1 + 3X_2 = 0$$

Separating Hyperplane (decision boundary)

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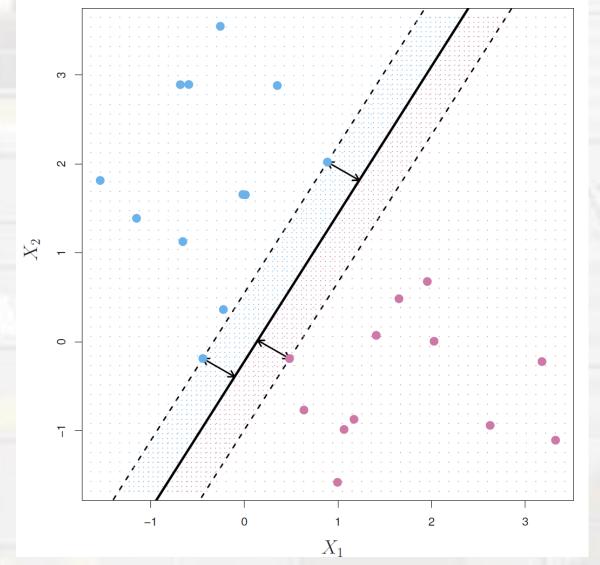
Classification using Separating Hyperplane

- Suppose we have training data of "n" observations and "p" variables
- Assume each of the observations fall into one of the two classes {-1, 1}
- Our goal is to classify a new test data,
 x* with "p" variables into one of these classes
- All are hyperplanes shown in black are equally good



Maximum Margin Classifier

- Among all the hyperplanes, maximum margin classifier is the one that makes the biggest gap between the two classes
- The margin is the width that the decision boundary can be increased before hitting a data point.
- The objective in SVM
 - Minimizing the classification error and maximizing the margin between two classes



Support Vector Machine

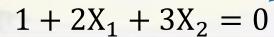
Feature vector

Class value (target prediction)

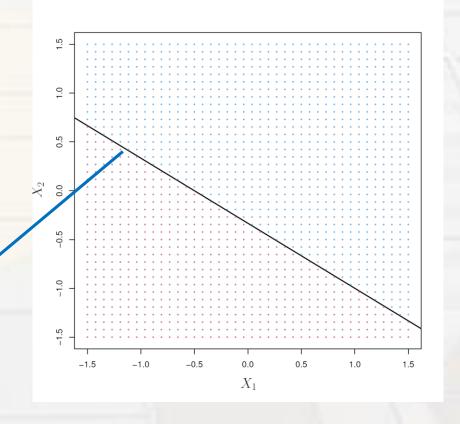


•
$$f(X, \beta) = sign(\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_p X_p)$$

- X=(-1, 0) what is the class?
- X=(1, 1)



Separating Hyperplane (decision boundary)



Transforming the data (scaling) before using business SVM

- SVM is very sensitive to the scale of variables in data
 - Different scales in various variables
 - High variability in a single variable (default example with balance as the only feature)
- Therefore, before developing SVM models, it is a good idea to transform all the variables to the same scale
- One way to do this is using the MinMaxScaler function
- This function transforms all variables to a between 0 and 1 scale
- It uses this formula:

$$x_{new} = \frac{x - x_{min}}{x_{max} - x_{min}}$$

Transforming the data (scaling) before using business SVM

x1	x2
100	1.1
110	1
130	1.2
115	3
125	2.2

		x1	x1_new
max	130	100	0
min	100	110	0.3
		130	1
		115	0.5
		125	0.8

		x2	x2_new
max	3	1.1	0.05
min	1	1	0
		1.2	0.1
		3	1
		2.2	0.6

x1_new	x2_new
0.0	0.05
0.3	0
1.0	0.1
0.5	1
0.8	0.6

```
: from sklearn.svm import LinearSVC
```

: df_default.head(n=2)

•

		default	student	balance	income
Ī	0	No	0	729.526495	44361.62507
	1	No	1	817.180407	12106.13470

X_def, y_def=df_default[['balance']],df_default['default']

91]: X_def.describe()

91]:

	balance
count	10000.000000
mean	835.374886
std	483.714985
min	0.000000
25%	481.731105
50%	823.636973
75 %	1166.308387
max	2654.322576

- LinearSVC, has its own random number generator
- If we do not specify the random_state option, every time we fit our model, we will have different scores

```
from sklearn.preprocessing import MinMaxScaler
     scaler1=MinMaxScaler()
73]: X_def_trs=scaler1.fit_transform(X_def)
     X def trs
73]: array([[0.2748447],
            [0.30786778],
            [0.40445316],
            [0.31850386],
            [0.59111468],
            [0.07569622]])
74]: X_train,X_test, y_train, y_test=train_test_split(X_def_trs,y_def,random_state=0)
     svm=LinearSVC(random_state=0)
     svm.fit(X train,y train)
```

Test data points have to be transformed before making predictions

```
i]: svm.coef_
i]: array([[3.01440675]])

i]: svm.intercept_
i]: array([-2.41896078])

i]: # -2.418+3.014x
```

```
print('svm on train: {:.2%}'.format(svm.score(X train,y train)))
93]:
     print('svm on test: {:.2%}'.format(svm.score(X_test,y_test)))
     svm on train: 97.11%
     svm on test: 96.60%
     p=[[800],[5000]]
     p trs=scaler1.transform(p)
     p trs
94]: array([[0.30139517],
            [1.8837198]])
95]:
     svm.predict(p_trs)
95]: array(['No', 'Yes'], dtype=object)
```

Regularization Parameter

- Similar to LogisticRegression,
- By default, *LinearSVC* applies an L2 regularization (in the same way that Ridge does for regression)
- L2 regularization, forces coefficients' magnitude to be closer to zero
- For *LinearSVC*, the trade-off parameter that determines the strength of the regularization is called *C*.
- Higher values of C correspond to <u>less</u> regularization
 - Higher C, LogisticRegression tries to fit the training set as best as possible
 - Lower C, more regularization, coefficients (β_is) closer to zero

1.2 breast cancer data

```
1 cancer=pd.read csv('breast cancer data.csv', index col=0)
 2 cancer.head(1)
       diagnosis radius_mean texture_mean perimeter_mean area_mean smoothness_mean compactness_mean conca
    id
842302
                                                  122.8
                       17.99
                                    10.38
                                                           1001.0
                                                                            0.1184
                                                                                              0.2776
1 rows × 31 columns
 1 # create features and traget sets
 2 X_cancer, y_cancer=cancer.iloc[:,1:], cancer['diagnosis']
 3 display(X_cancer.head(1))
 4 display(y cancer.head(2))
       radius_mean texture_mean perimeter_mean area_mean smoothness_mean compactness_mean concavity_mean
    id
                                        122.8
                                                                                    0.2776
                                                                                                   0.3001
842302
              17.99
                          10.38
                                                  1001.0
                                                                   0.1184
1 rows × 30 columns
id
842302
842517
Name: diagnosis, dtype: object
```

```
1 # transforming the features
 from sklearn.preprocessing import MinMaxScaler
 3 from sklearn.svm import LinearSVC
 4 from sklearn.model selection import validation curve
 1 scaler2=MinMaxScaler()
 2 X cancer trns=scaler2.fit transform(X cancer)
 3 X cancer trns[:1,:]
array([[0.52103744, 0.0226581, 0.54598853, 0.36373277, 0.59375282,
       0.7920373 , 0.70313964, 0.73111332, 0.68636364, 0.60551811,
       0.35614702, 0.12046941, 0.3690336, 0.27381126, 0.15929565,
       0.35139844, 0.13568182, 0.30062512, 0.31164518, 0.18304244,
       0.62077552, 0.14152452, 0.66831017, 0.45069799, 0.60113584,
        0.61929156, 0.56861022, 0.91202749, 0.59846245, 0.41886396])
    C_range=[.1,1,5,10,50,100]
    train scores, test scores=validation curve(LinearSVC(random state=0, max iter=100000)
                                               ,X cancer trns,y cancer,
                                               param_name='C', param_range=C_range, cv=4)
```

```
train scores.round(4)
42]:
42]: array([[0.9624, 0.9578, 0.9672, 0.9696],
            [0.9812, 0.9789, 0.9859, 0.9766],
            [0.9883, 0.9859, 0.9883, 0.9813],
            [0.9883, 0.9883, 0.9906, 0.9813],
            [0.993, 0.9906, 0.9906, 0.9836],
            [0.9953, 0.9906, 0.9906, 0.9836]])
         test scores.round(4)
43]:
43]: array([[0.951, 0.9437, 0.9577, 0.9718],
            [0.972, 0.9859, 0.9648, 0.9859],
            [0.965, 0.9789, 0.9648, 0.9859],
            [0.972, 0.9789, 0.9648, 0.9789],
            [0.958, 0.9789, 0.9859, 0.9648],
            [0.951, 0.9577, 0.9859, 0.9577]])
          print('ave cross val scores on train:',train_scores.mean(axis=1).round(3))
     ave cross val scores on train: [0.964 0.981 0.986 0.987 0.989 0.99 ]
       print('ave cross val scores on test:',test scores.mean(axis=1).round(3))
     ave cross val scores on test: [0.956 0.977 0.974 0.974 0.972 0.963]
```

Linear Models parameters, strengths, weaknesses

Linear Models parameters, strengths, weaknesses

- Linear regression, Lasso, Ridge
- Logistic regression, Linear support vector machine
- The main parameter is the regularization parameter
 - alpha in regression models and C in LinearSVC and LogisticRegression
 - Large values for alpha or small values for C mean simpler models
 - In particular for regression models, tuning this parameter is very important



Linear Models

parameters, strengths, weaknesses

- The other parameter in linear models is the regularization type
 - L1 versus L2
 - Default for LinearSVC, LogisticRegression and Ridge is L2
 - Lasso uses L1
 - If you have many features and think only a few of them are important, you should use L1, otherwise use the default (L2)
 - As L1 only uses a few features, it is easier to explain

Linear Models

parameters, strengths, weaknesses

- Linear models are very fast to train, also fast to predict
- They are suitable for very large and sparse datasets
- They perform well when the number of features is large compared to the number of samples (records)
- Linear models are usually easy to understand because of their linear nature!
- Linear models might not perform well in lower-dimensional spaces (fewer features in data)