Math 228B: Homework 4

1a: Derive the Galerkin formulation of

$$\partial_x^4 u = 480x - 120$$

$$u(0) = u'(0) = u(1) = u'(1) = 0$$

If there was a function u defined on $[0, 1]$ which satisfies the above PDE then it must be the case that for any function ϕ with $\phi(0) = \phi'(0) = \phi(1) = \phi'(1) = 0$ that

$$\int_0^1 \partial_x^4 u \phi dx = \int_0^1 f \phi dx$$

Integrating by parts twice we must also have

$$\int_0^1 \partial_x^2 u \partial_x^2 \phi dx + \partial^3 u \phi|_0^1 - \partial_x^2 u \partial_x \phi|_0^1 = \int_0^1 f \phi dx$$

but since we chose ϕ to be 0 to first order on the boundaries this is

$$\int_0^1 \partial_x^2 u \partial_x^2 \phi dx = \int_0^1 f \phi dx.$$

Now, this will determine u uniquely if we check this against enough ϕ so that $\partial^2 \phi$ span a dense subspace of the image of $\partial_x^2 u$ on the space we look for a solution u, *as long as* ∂_x^2 *is*