

		1	2				
		3	4				
5	6	9	10	13	14	17	18
7	8	11	12	15	16	19	20
		21	22				
		23	24				

Cyclic Definition

$R := (13,14,16,15)(10,2,19,22)(12,4,17,24)$

$D := (21,22,24,23)(11,15,19,7)(12,16,20,8)$

$B := (17,18,20,19)(2,5,23,16)(1,7,24,14)$

$G := \langle R,D,B \rangle$

Cardinality of the Group

Base: $8! \times \frac{3^8}{3} = 88,179,840$

With Isometries: $8! \times \frac{3^8}{3} \times \frac{1}{24} = 3,674,160$
 $= 7! \times \frac{3^7}{3}$

Symmetries

Isometries: Physical rotations of a cube doesn't affect cube state. 24-Fold Symmetry.

Color Permutations: Colors can be arranged differently and be the same cube. A color-blind man solves the cube in the same way. 24-Fold Symmetry (Some overlap with Rotational)

Reflectional Symmetry: The difference in solving with any move or its inverse is just a clockwise vs counterclockwise turn. 2-Fold Symmetry (Some overlap with previous two)

God's Number

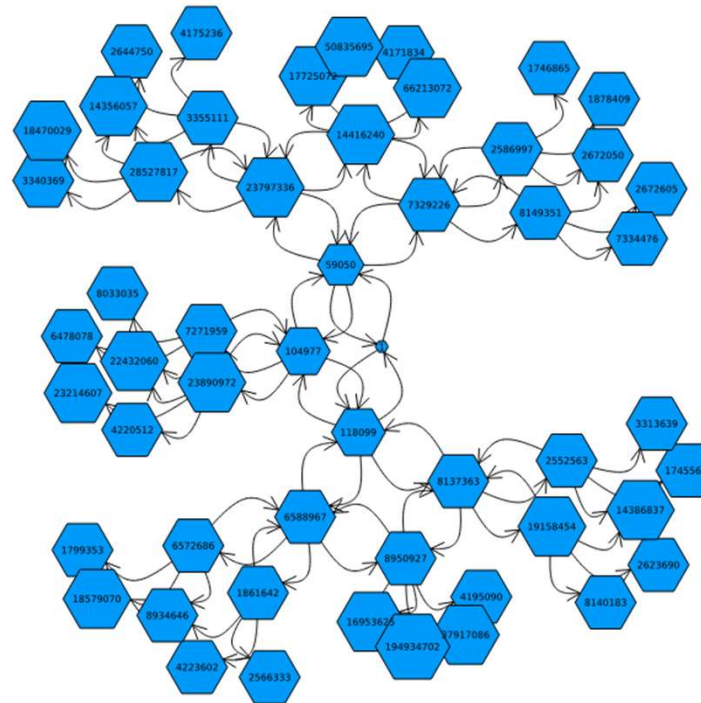
If one were to hand God a Rubik's cube, They would solve it in the least number of moves. God's number is the maximum number of moves god can perform to solve some state of the Rubik's cube. For the pocket cube, this is 14. The standard cube? 26.

Exploration of the Pocket Rubik's Cube

Ryan Brown

University of Massachusetts Lowell

Directed Graph allowing Rotational and Reflectional Symmetries



Further Questions

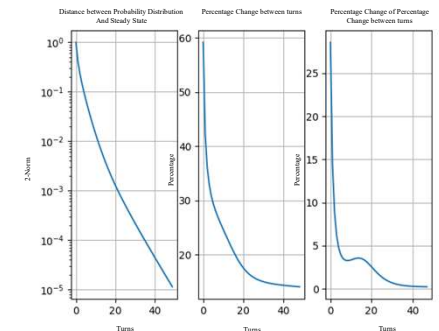
- How many elements are in the Pocket Cube group when accounting for all symmetries?
- How do different mixing algorithms affect the mixing efficiency?
- How do different solving algorithms affect the ideal mixing length?
- Can you solve the Pocket Cube?

How many turns does it take to properly mix a Pocket Cube?

34, after that mixing doesn't significantly change the distribution

Mixing Efficiency Using Soft Markov Chain Analysis

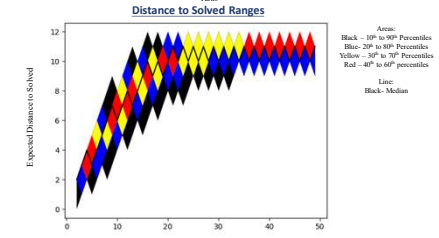
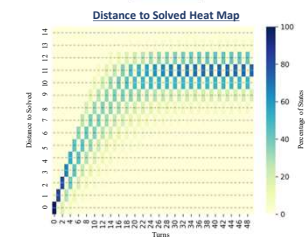
Distance from Steady State



While the mixing does reduce efficiency around 17, the (pseudo) second derivative doesn't bottom out until around mid 30s.

Expected Number of Moves for God to Solve

Solve



Conclusion

When any area disappears, it is actually hidden behind another. Meaning 34 turns away guarantees, 80% of the states to take God either 10 or 12 moves. The heat map further says over 90% of the states are