DCU

DUBLIN CITY UNIVERSITY

SEMESTER ONE EXAMINATIONS 2012

MODULE:	CA427F, CA427D, CA427 Operations Research	
COURSE:	CAIS4	BSc in Computer Applications (Inf. Systems)
	CASE4	BSc in Computer Applications (SW Eng.)
	BSSAX	Study Abroad (DCU Business School)
YEAR:	4	
EXAMINERS:	Prof. F. O'Sullivan	
	Dr. Frank Bannister	
	Dr. J. Power	
	Dr W.G. Tuohey, Ext no. 8728	
TIME ALLOWED:	2 Hours	
INSTRUCTIONS:	Answer any **THREE** questions	
	All questions carry equal marks	

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The use of programmable or text storing calculators is expressly forbidden.
Please note that where a candidate answers more than the required number of questions, the examiner will mark all questions attempted and then select the highest scoring ones

Question 1 **[Total marks: 33]**

1(a) [7 marks]

A manufacturing firm makes three kinds of product (*A, B* and *C*). Each unit of *A* made requires 7 hours of labour, 4 Kg of material type I and 6 kg of material type II. For each unit of *B* made, the requirements are 3 hours of labour, 4 Kg of material type I and 3 Kg of material type II, while for *C* the unit requirements are 6 hours of labour, 5 Kg of material type I and 4 Kg of material type II. The company expects to make profits of €40, €20 and €30, respectively, for every unit of *A*, *B* and *C* sold. Each day the company has available 150 hours of labour, 200 Kg of material of type I and 220 Kg of material of type II. Formulate this as a linear programming problem to maximise the total daily profit for the manufacturer.

1(b) [16 marks]

Find the solution to the problem formulated in (a) using the **Simplex Method.**

1(c) [6 marks]

Within certain limits, the optimal profit calculated in (b) will change by a specific amount for each hour of change in labour availability. What is this amount and what are the limits?

1(d) [4 marks]

Express the linear programming problem formulated in part 1(a) in AMPL syntax.

--[End of Question 1]—

Question 2 **[Total marks: 33]**

2(a) [6 marks]

Precooked dinners of beef, chicken and fish cost €3.19, €2.59 and €2.29 per package, respectively. These dinners provide the following percentages, per packet, of the minimum daily requirements for vitamins A, C, B1 and B2:

	A	**C**	**B1**	**B2**
Beef	60	20	10	15
Chicken	8	0	20	20
Fish	8	10	15	10

It is required to find the cheapest combination of packages that will meet a ***week***'s requirements. Formulate this as a linear programming problem.

2(b) [8 marks]

Transform the linear programming problem of part 2(a) into standard form using surplus variables, and then into canonical form (that is, having an initial basis) using artificial variables. Hence, form the preliminary tableau for the Simplex Method. (***NB:*** *you are not asked to solve this problem*).

2(c) [7 marks]

Using the syntax of an AMPL model, generalise the problem of part 2(a) to where there is an arbitrary set of food items (not just 3) and an arbitrary set of nutrients or ingredients (not just 4). In addition to not-negative parameters specifying the unit cost of each food item and the amount of nutrient per unit food item, your model should include parameters to specify lower and upper bounds on both food and nutrient items.

2(d) [12 marks]

The diet problem may be viewed as a particular case of a general class of input-output problems where food items make up the set of inputs and nutrients (or ingredients) make up the set of outputs. In this regard, re-state the model for part 2(c) in terms of general sets of inputs and outputs, and present an AMPL data file for the re-stated model to specify the following particular problem.

You have been advised to burn off at least 2000 extra calories per week by some combination of walking, jogging, swimming, exercise-machine, and team sport. You have a limited tolerance for each activity in hours/week and each expends a certain number of calories per hour:

	Walk	Jog	Swim	Machine	Team
Calories	100	200	300	150	300
Tolerance	5	2	3	3.5	3

How should you divide your exercising among these activities to minimize the amount of time you spend?

--[End of Question 2]—

Question 3 **[Total marks: 33]**

3(a) [8 marks]

List the main assumptions underlying the existence of the steady state in an M/M/1 queuing system. Also, state Little's formula relating, under steady state conditions, the mean time spent in the system (or queue) to the mean number in the system (or queue).

3(b) [12 marks]

A post-office is open 10 hours per day and can serve an average of 10 persons per hour when one server is working. The average number of daily visitors is 70 and Poisson arrivals and exponential services (that is, an M/M/1 system) are assumed.

Calculate the following:

- Probability that there is no customer in the post-office
- Probability of there being 2 or 3 customers in line (including being served)
- Expected (average) number of customers waiting to be served
- Expected length of time a person spends in the post-office

3(c) [13 marks]

In December, because of the Christmas rush, there are on average 100 daily visitors. Moreover, the service rate decreases to 8 per hour on average.

Calculate

- Probability that there is no customer in the post-office
- Expected length of time a person spends waiting for service

when (i) two and (ii) three (equally efficient) servers are employed.

Note: for an M/M/s queuing system:

$$P_0 = 1 \Big/ \left(\sum_{n=0}^{s-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^s}{s!(1-\frac{\lambda}{s\mu})} \right)$$ (probability of zero items in the system)

$$L_q = \frac{P_0(\lambda/\mu)^s \rho}{s!(1-\rho)^2}$$ (average number of items in the queue), where $\rho = \frac{\lambda}{s\mu}$

--[End of Question 3]—

Question 4 **[Total marks: 33]**

4(a) [10 Marks]

Establish the classic economic lot-size formula

$$\sqrt{\frac{2KD}{h}}$$

where D is the constant demand (per time period), K is the set-up cost per order, and h is the inventory holding cost (per unit per time period). State clearly the assumptions underlying this formula.

4(b) [6 Marks]

A video manufacturer produces 8000 video recorders on a continuous production line every month. Each video requires a tape head unit and these are produced very quickly in batches. It costs €12,000 to set up the machinery to produce a batch and €0.30 a month to store and insure a tape head once made. How large should the batch size be to minimize total costs and how often will they have to set up a production run?

4(c) [6 Marks]

As for 4(a) one can establish (*but you are **not** required to establish it here*)

$$\sqrt{\frac{2KD(s+h)}{hs}}$$

for the optimum number of units per production run in the case where there is backlogged demand. In this formula, D, K and h are as in 4(a) and s is the shortage cost or the cost of being short of one item for one time unit. Assume that the company described in Part 4(b) can, if it runs out of tape head units, continue to manufacture videos and install the tape heads later but that this will incur an inconvenience cost of €1.10 per tape head per month. In this situation, what is the optimal batch size and how often should goods be ordered?

4(d) [11 Marks]

The demand for a seasonal item is subject to fluctuations over the course of a year and its manufacturer wishes to find a production schedule that minimises the costs due to output variations and inventories.

Let

x_t = number of units produced in month t (**production**)
r_t = number of finished units that must be available in month t (**requirement**)
s_t = number of finished units that are not required in month t (**storage**)
(s_0 = amount of product in storage at the beginning of the first month)
a = cost of increasing production by 1 unit from month t-1 to month t
b = cost of storing 1 unit for one month.

Outline a linear programming formulation of this problem in terms of parameter $\lambda = a/b$, that is the cost of a unit increase in output relative to that of storing a unit for a month.

--[End of Question 4] --

--[End of Exam] --