**DUBLIN CITY UNIVERSITY**

# AUGUST/RESIT EXAMINATIONS 2014/2015

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| **MODULE:** | CA427/F – Operations Research |
| **PROGRAMME(S):** | |
| CASE | BSc in Computer Applications (Sft.Eng.) |
| CAIS | BSc in Computer Applications (Inf.Sys.) |
| **YEAR OF STUDY:** | 4 |
| **EXAMINERS:** | Dr Liam Tuohey (Ext:8728)<br>Prof. Finbarr O'Sullivan<br>Dr. Ian Pitt |
| **TIME ALLOWED:** | 2 Hours |
| **INSTRUCTIONS:** | Answer 3 questions. All questions carry equal marks. |

**PLEASE DO NOT TURN OVER THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO**
The use of programmable or text storing calculators is expressly forbidden.
Please note that where a candidate answers more than the required number of questions, the examiner will mark all questions attempted and then select the highest scoring ones.

*Requirements for this paper (Please mark (X) as appropriate)*

| | | | |
|---|---|---|---|
| ☐ | *Log Tables* | ☐ | *Thermodynamic Tables* |
| ☐ | *Graph Paper* | ☐ | *Actuarial Tables* |
| ☐ | *Dictionaries* | ☐ | *MCQ Only – Do not publish* |
| ☐ | *Statistical Tables* | ☐ | *Attached Answer Sheet* |

## *QUESTION 1* *[TOTAL MARKS: 33]*

### Q 1(a) [5 Marks]

Three products are to be manufactured. A single unit of product A requires 2.4 minutes of punch press time and 5.0 minutes of assembly time. A single unit of product B requires 3.0 minutes of punch press time and 2.5 minutes of welding time. A single unit of product C requires 2.0 minutes of punch press time, 1.5 minutes of welding time, and 2.5 minutes of assembly time. The profits per unit of products A, B and C are €0.60, €0.70 and €0.50, respectively. The available weekly capacities of the punch press, welding and assembly departments are 1200 minutes, 600 minutes and 1500 minutes, respectively. Formulate this as a linear programming problem to maximise the total weekly profit for the manufacturer.

### Q 1(b) [16 Marks]

Find the solution to the problem formulated in (a) using the **Simplex Method.**

### Q 1(c) [6 Marks]

Find the range of values for each of the product profit contributions for which the current solution will remain optimal

### Q 1(d) [6 Marks]

Find the range of values within which each of the available capacities of the punch press, welding and assembly departments must lie in order for the current solution to remain feasible.

*[End of Question 1]*

***QUESTION 2*** ***[TOTAL MARKS: 33]***

**Q 2(a)** **[9 Marks]**

Explain the use of **Slack**, **Surplus** and **Artificial Variables** in linear programming.

**Q 2(b)** **[8 Marks]**

A machine tool company conducts a job-training program for machinists. Trained machinists are used as teachers in the program at a ratio of one for every ten trainees. The training program lasts for one month. From past experience it has been found that out of ten trainees hired, only six complete the program successfully (the unsuccessful trainees are released).

Trained machinists are also needed for machining and the company's requirements for the next three months are as follows:

| | |
|---|---|
| January | 110 |
| February | 150 |
| March | 200 |

In addition, the company requires 230 trained machinists by April. There are 130 trained machinists available at the beginning of the year.

Payroll costs per month are:

| | |
|---|---|
| Each trainee | €400 |
| Each trained machinist (machining or teaching) | €700 |
| Each trained machinist idle (Union forbids firing them!) | €500 |

Set up the linear programming problem that will produce the hiring and training schedule of minimum cost, and will meet the company's requirements.

**Q 2(c)** **[6 Marks]**

Transform the linear programming problem into canonical form, ready for application of the Simplex Method. (You do ***not*** have to carry out Simplex Method).

**Q 2(d)** **[10 Marks]**

Present a general AMPL model for the class of problems of which the problem in part 2(b) is a particular case. Also, present the corresponding AMPL data file for the case of the problem of part 2(b).

***[End of Question 2]***

**QUESTION 3** **[TOTAL MARKS: 33]**

**Q 3(a)** **[6 Marks]**

(i) State Little's formula relating mean time spent in a system (queue) to mean number in the system (queue). Also, state how mean waiting times in system (W) and queue ($W_q$) are related. Use the notation that $\lambda$ = arrival rate and $\mu$ = service rate.

(ii) For a steady-state, M/M/1 queue the mean number in the system is $L = \lambda/(\mu - \lambda)$. Deduce that the mean number in the queue is $L_q = \lambda^2/(\mu(\mu - \lambda))$.

**Q 3(b)** **[21 Marks]**

A haulage company has a tyre puncture repair shop at which truck drivers arrive at a rate of four per day and for which the tyre repair potential (of one repair attendant) is five per day. The daily wage paid to the attendant at the service centre is €120 and the daily cost of a truck driver away from his or her work is €300.

Under M/M/1 assumptions

(i) Calculate the average number of truck drivers being served or waiting to be served at any given time

(ii) Calculate the average time a truck driver spends waiting for service.

(iii) Calculate the total daily cost of operating the system.

(iv) Calculate the cost of the system if there were two repair attendants (working independently), each paid €120 per day and each able to service on average five trucks per day.

(v) Would it be worth employing a third repair attendant, at the same rate of pay and tyre repair rate? Justify your answer.

Note: For an M/M/s queue

$$P_0 = 1\Bigg/\left(\sum_{n=0}^{s-1}\frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^s}{s!(1-\frac{\lambda}{s\mu})}\right) \quad \text{and} \quad L_q = \frac{P_0(\lambda/\mu)^s\rho}{s!(1-\rho)^2} \text{ where } \rho = \frac{\lambda}{s\mu}$$

**Q 3(c)** **[6 Marks]**

In queue systems where both the arrival rate ($\lambda_n$) and service rate ($\mu_n$) are dependent on the number in the system (n) it can be shown that the probability of there being n customers in the system is

$P_n = \dfrac{\lambda_0\lambda_1\ldots\lambda_n}{\mu_1\mu_2\ldots\mu_{n+1}} P_0$ where $P_0 = \{1+\dfrac{\lambda_0}{\mu_1}+\dfrac{\lambda_9\lambda_1}{\mu_1\mu_2}\ldots\}^{-1}$.

What form should $\lambda_n$ and $\mu_n$ take to represent (i) an immigration-death process and (ii) a queue with limited waiting room (R)?

***[End of Question 3]***

## *QUESTION 4* *[TOTAL MARKS: 33]*

**Q 4(a)** **[24 Marks]**

A small oil company intends to carry out a speculative drill and seismic test. The main activities have the following durations in weeks:

| Activity | Normal time | Crash time | Crash cost (per week) |
|---|---|---|---|
| **A Obtain finance** | 6 | 5 | €1000 |
| **B Obtain platform** | 5 | 3 | € 600 |
| **C Recruit workforce** | 6 | 5 | €500 |
| **D Fit and tow platform** | 8 | 5 | €600 |
| **E Train personnel** | 3 | 2 | €300 |
| **F Prepare seismic equipment** | 2 | 2 | - |
| **G Do seismic tests** | 6 | 4 | €200 |
| **H Do test drill** | 9 | 6 | €900 |

Finance for the project must be obtained before any other activity can be started. The platform must be obtained before fitting and towing commences, but recruitment may be carried out while the platform is being obtained. When recruitment is complete, training may commence. Test drilling may not commence until training is complete and the platform has been towed to its position. Preparing the seismic equipment uses a special team already assembled plus some unskilled labour, and so may commence once recruitment is complete. Seismic tests are carried out from a survey ship and do not require the platform to be in place, just the equipment prepared.

(i) Draw the activity network for this project. [7 Marks]

(ii) Find the critical path, using the given normal activity durations. [5 Marks]

(iii) [12 Marks]

For each week the project is taking place an overhead cost of €1000 is incurred. If it is possible to reduce the activity durations down to times no lower than the crash times given in the table, incurring the crash cost in each case as shown in the table, determine the duration for each activity which will result in a minimum total cost (i.e. overhead cost + crash cost) for the project.

**Q 4(b)** **[9 Marks]**

Describe generally how planning problems such as that in 4(a) may be modelled and solved using a mathematical programming approach. In particular, formulate (***not*** solve) the "crash" problem of part (iii) of 4(a) as a linear programming problem.

***[End of Question 4]***

***[END OF EXAM]***