DCU

DUBLIN CITY UNIVERSITY

SEMESTER 2 RESIT EXAMINATIONS 2012/2013

MODULE: CA429D/F – Operations Research/Management Science

PROGRAMME(S):

CASE BSc in Computer Applications (Sft.Eng.)

CAIS BSc in Computer Applications (Inf.Sys.)

YEAR OF STUDY: 4

EXAMINERS:

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Prof. Finbarr O'Sullivan

TIME ALLOWED: 2 Hours

INSTRUCTIONS: Answer 3 questions. All questions carry equal marks.

PLEASE DO NOT TURN OVER THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO

The use of programmable or text storing calculators is expressly forbidden.

Please note that where a candidate answers more than the required number of questions, the examiner will mark all questions attempted and then select the highest scoring ones.

Requirements for this paper (Please mark (X) as appropriate)

- [ ] *Log Tables*
- [ ] *Graph Paper*
- [ ] *Dictionaries*
- [ ] *Statistical Tables*
- [ ] *Thermodynamic Tables*
- [ ] *Actuarial Tables*
- [ ] *MCQ Only – Do not publish*

QUESTION 1 **[TOTAL MARKS: 33]**

1(a) **[6 Marks]**

In the context of Markov chains:

(i) Define an ***irreducible*** transition matrix and hence state a sufficient condition which, when true, will guarantee that a steady-state exists. **[3 marks]**

(ii) Define an ***absorbing*** state. If there are multiple absorbing states can a steady-state exist? **[3 marks]**

1(b) **[12 Marks]**

Consider a society with three social classes. Each individual may belong to the lower class (state 1), the middle class (state 2), or the upper class (state 3). Thus, the social class occupied by an individual in generation t may be denoted by $s_t \in \{1, 2, 3\}$. Further suppose that each individual in generation t has exactly one child in generation t+1, who has exactly one child in generation t+2, and so on. Finally, suppose that intergenerational mobility is characterized by a (3×3) transition matrix which does not change over time. Under these conditions, a single "family history" – the sequence of social classes (s_0, s_1, s_2, . . .) – is a Markov chain. The following transition matrix is a specific example:

	State in "t+1"		
State in generation t	0.6	0.4	0.0
	0.3	0.4	0.3
	0.0	0.7	0.3

For example, a child with a lower class parent has a 40% chance of becoming middle class, and a child of an upper class parent has a 70% chance of becoming middle class.

(i) Explain why a steady-state exists for this example. **[2 marks]**

(ii) Calculate the steady-state probabilities, and interpret them. **[7 marks]**

(iii) What is the probability that a particular "family history" over eight generations is "Upper-Upper-Upper-Middle-Middle-Lower-Middle-Lower"? **[3 marks]**

1(c) **[15 Marks]**

States {A, B, C, D, E} have been identified for final testing of a software product, where A = "Ready for entry to final testing", B = "Alpha test set completed", C = "Beta test set completed", D = "Released" and E = "Discarded".

The probability of going from A to B is 1. From B, there is 0.4 probability of going to A, and 0.2 of going to each of C, D and E. From C, there is 0.3 probability of going to A, 0.6 of going to D and 0.1 of going to E. If state D is entered then the probability is 1 of remaining in that state, and similarly for state E.

(i) Write out the matrix of transition probabilities and explain why a steady-state does not exist in this case. **[2 marks]**

(ii) Calculate the ***fundamental matrix*** for this system. **[9 marks]**

(iii) What are the probabilities that the software product will **[4 marks]**

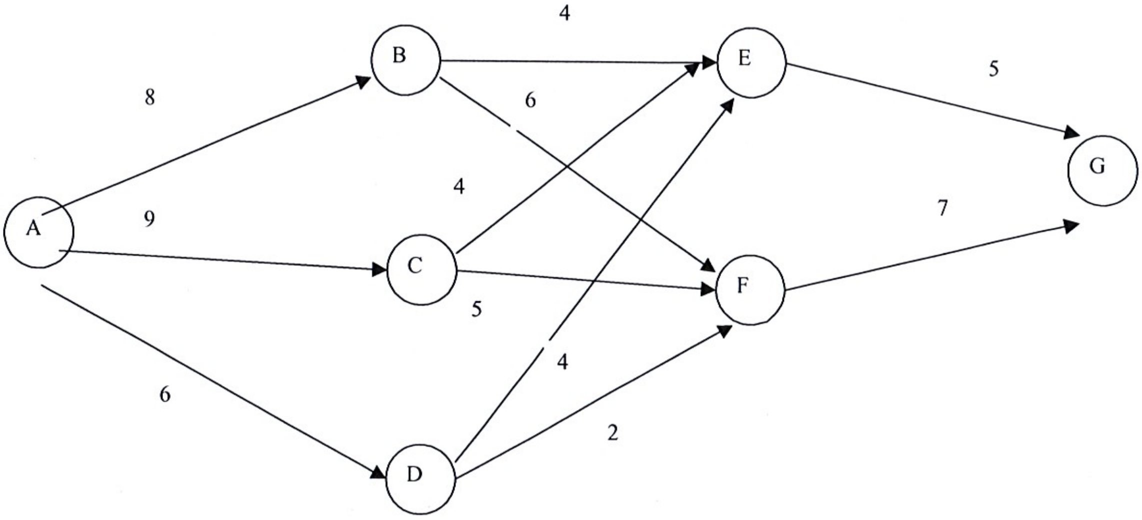
(a) be released eventually in the case that the Alpha test has been completed,

(b) be discarded eventually in the case that the Beta test has been completed?

[End of Question 1]

QUESTION 2 **[TOTAL MARKS: 33]**

The following diagram depicts distances and pathways between the set of nodes {A, B, C, D, E, F, G}. The arrows indicate the allowed direction of travel; for example, it is possible to go from C to E but not from E to C. It is impossible to travel directly between nodes that are not joined by an arrow.



2(a) **[14 Marks]**

Making use of the stage structure of the above system, ***use dynamic programming*** to calculate the shortest distance and corresponding path from A to G.

2(b) **[19 Marks]**

Suppose that additional pathways are added to the above diagram as follows:

Pathway	A to E	A to F	B to G	C to G	G to D
Distance	10	9	11	13	4

For this modified system, where there is no longer a stage structure, ***use dynamic programming*** to find the shortest distance and corresponding path to G from any of nodes A to F.

Note: Answers to this question should demonstrate ***knowledge of the appropriate dynamic programming methods***. It is clear that the solution may be found by other approaches but that is not the purpose of the question.

[End of Question 2]

QUESTION 3 **[TOTAL MARKS: 33]**

3(a) **[10 Marks]**

Consider the following two constraints in a linear programming problem:

$$2X_1 + X_2 \leq 20 \qquad (1)$$
$$X_1 + 4X_2 \leq 16 \qquad (2)$$

If it is required that **<u>either</u>** constraint (1) **<u>or</u>** constraint (2) must hold at the solution, how would you formulate this situation?

3(b) **[23 Marks]**

Consider the following problem:

Max $\quad Z = 5X_1 + 3X_2$

Subject to

$$3X_1 + 5X_2 \leq 15$$
$$5X_1 + 2X_2 \leq 10$$
$$X_1 + X_2 + X_3 \leq 4$$

$$X_1, X_2, X_3 \geq 0$$

The optimal simplex tableau for this problem can be shown to be:

	X_1	X_2	X_3	S_1	S_2	S_3	**b**
Z	0	0	0	5/19	16/19	0	235/19
X_2	0	1	0	5/19	-3/19	0	45/19
X_1	1	0	0	-2/19	5/19	0	20/19
S_3	0	0	1	-3/19	-2/19	1	11/19

where S_1, S_2 and S_3 are slack variables corresponding to the first, second and third constraints, respectively. Hence, the optimal solution is $X_1 = 20/19$, $X_2 = 45/19$ and $X_3 = 0$, with the minimum of Z equal to 235/19.

Using the cutting plane method, find the optimal solution to the above problem if it is required that each of X_2 and X_3 should have integer values.

<u>Note</u>: In order to limit the amount of calculations, it is not required to find the solution where X_1 is also an integer.

[End of Question 3]

QUESTION 4 **[TOTAL MARKS: 33]**

4(a) **[17 Marks]**

Show **how** the following problem may be solved approximately using Separable Programming:

$$\begin{aligned} &\text{Maximise} \quad 4X_1^2 - 3X_1 + 3X_2 \\ &\text{Subject to} \quad 3X_1^2 + 4X_2^2 \leq 7 \\ &\qquad\qquad 0 \leq X_1 \leq 2 \\ &\qquad\qquad 0 \leq X_2 \leq 2 \end{aligned}$$

You should clearly indicate the form of the objective function and of all constraints in the approximate formulation. However, you are **not** required to actually calculate the corresponding approximate solution.

4(b) **[16 Marks]**

Solve the following problem using the method of Lagrange Multipliers:

$$\text{Minimise} \quad Z = (X_1 - 1)^2 + (X_2 - 2)^2 + (X_3)^2$$

$$\text{subject to} \quad X_1 - 2X_2 + 3X_3 = -1$$

[End of Question 4]

[END OF EXAM]