DUBLIN CITY UNIVERSITY

AUGUST/RESIT EXAMINATIONS 2014/2015

MODULE:	CA429/F – Operations Research/Management Science
PROGRAMME(S):	
CASE	BSc in Computer Applications (Sft.Eng.)
ECSA	Study Abroad (Engineering & Computing)
ECSAO	Study Abroad (Engineering & Computing)
CAIS	BSc in Computer Applications (Inf.Sys.)
YEAR OF STUDY:	4,O,X
EXAMINERS:	
	Dr Liam Tuohey (Ext:8728)
	Prof. Finbarr O'Sullivan
TIME ALLOWED:	2 Hours
INSTRUCTIONS:	Answer 3 questions. All questions carry equal marks.

PLEASE DO NOT TURN OVER THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO
The use of programmable or text storing calculators is expressly forbidden.
Please note that where a candidate answers more than the required number of questions, the examiner will mark all questions attempted and then select the highest scoring ones.

Requirements for this paper (Please mark (X) as appropriate)

☐	*Log Tables*	☐	*Thermodynamic Tables*
☐	*Graph Paper*	☐	*Actuarial Tables*
☐	*Dictionaries*	☐	*MCQ Only – Do not publish*
☐	*Statistical Tables*	☐	*Attached Answer Sheet*

QUESTION 1 ***[TOTAL MARKS: 33]***

Q 1(a) **[6 Marks]**

In the context of Markov chains:

(i) Define an ***irreducible*** transition matrix and hence state a sufficient condition which, when true, will guarantee that a steady-state exists. **[3 marks]**

(ii) Define an ***absorbing*** state. If there are multiple absorbing states can a steady-state exist? **[3 marks]**

Q 1(b) **[12 Marks]**

Consider a society with three social classes. Each individual may belong to the lower class (state 1), the middle class (state 2), or the upper class (state 3). Thus, the social class occupied by an individual in generation t may be denoted by $s_t \in \{1, 2, 3\}$. Further suppose that each individual in generation t has exactly one child in generation t+1, who has exactly one child in generation t+2, and so on. Finally, suppose that intergenerational mobility is characterized by a (3×3) transition matrix which does not change over time. Under these conditions, a single "family history" – the sequence of social classes (s_0, s_1, s_2, . . .) – is a Markov chain. The following transition matrix is a specific example:

	State in "t+1"		
State in generation t	0.7	0.3	0.0
	0.3	0.5	0.2
	0.0	0.8	0.2

For example, a child with a lower class parent has a 30% chance of becoming middle class, and a child of an upper class parent has an 80% chance of becoming middle class.

(i) Explain why a steady-state exists for this example. **[3 marks]**

(ii) Calculate the steady-state probabilities, and interpret them. **[9 marks]**

Q 1(c) **[15 Marks]**

States {A, B, C, D, E} have been identified for final testing of a software product, where A = "Ready for entry to final testing", B = "Alpha test set completed", C = "Beta test set completed", D = "Released" and E = "Discarded".

The probability of going from A to B is 1. From B, there is 0.4 probability of going to A, and 0.2 of going to each of C, D and E. From C, there is 0.3 probability of going to A, 0.6 of going to D and 0.1 of going to E. If state D is entered then the probability is 1 of remaining in that state, and similarly for state E.

(i) Calculate the ***fundamental matrix*** for this system. **[9 marks]**

(ii) What are the probabilities that the software product will (a) be released eventually in the case that the Alpha test has been completed, and (b) be discarded eventually in the case that the Beta test has been completed? **[6 marks]**

[End of Question 1]

QUESTION 2 ***[TOTAL MARKS: 33]***

Q 2(a) **[7 Marks]**

Write down the formulation of the 'loading problem' to determine how many units of each of several item types should be loaded into a limited capacity so as to maximise the total value of the load.

Q 2(b) **[19 Marks]**

A sawmill receives logs in 9-metre lengths, and cuts them into smaller lengths to sell them on to manufacturing companies. The lengths favoured by the manufacturing companies are as follows:

L_1 = 1 metre

L_2 = 2 metres

L_3 = 3 metres

L_4 = 4 metres

L_5 = 5 metres

and the profit the sawmill makes on each length is as follows:

L_1 €11 per unit

L_2 €21 per unit

L_3 €34 per unit

L_4 €41 per unit

L_5 €58 per unit

Determine, using **dynamic programming**, how many units of each length the sawmill should cut from each 9-metre log in order to maximise the total profit per log.

Q 2(c) **[7 Marks]**

State Bellman's principle of Optimality and describe its relevance to finding the solution to dynamic programming problems.

[End of Question 2]

QUESTION 3 [TOTAL MARKS: 33]

Q 3(a) **[10 Marks]**

Consider the following two constraints in a linear programming problem:

$$3X_1 + X_2 \leq 20 \qquad (1)$$
$$X_1 + 5X_2 \leq 19 \qquad (2)$$

If it is required that **either** constraint (1) **or** constraint (2) must hold at the solution, how would you formulate this situation?

Q 3(b) **[18 Marks]**

Consider the following problem:

Maximise $Z = 5X_1 + 3X_2$

Subject to
$$3X_1 + 5X_2 \leq 15$$
$$5X_1 + 2X_2 \leq 10$$
$$X_1 + X_2 + X_3 \leq 4$$
$$X_1, X_2, X_3 \geq 0$$

The optimal simplex tableau for this problem can be shown to be:

	X_1	X_2	X_3	S_1	S_2	S_3	b
Z	0	0	0	5/19	16/19	0	235/19
X_2	0	1	0	5/19	-3/19	0	45/19
X_1	1	0	0	-2/19	5/19	0	20/19
S_3	0	0	1	-3/19	-2/19	1	11/19

where S_1, S_2 and S_3 are slack variables corresponding to the first, second and third constraints, respectively. Hence, the optimal solution is $X_1 = 20/19$, $X_2 = 45/19$ and $X_3 = 0$, with the minimum of Z equal to 235/19.

Using the cutting plane method, find the optimal solution to the above problem if it is required that each of X_2 and X_3 should have integer values.

Q 3(c) **[5 Marks]**

Set up the additional constraint that should be introduced if it required that X_1 also be an integer. However, it is ***not*** required to calculate the new, all integer, solution.

[End of Question 3]

QUESTION 4 *[TOTAL MARKS: 33]*

Q 4(a) **[21 Marks]**

Show **how** the following problem can be solved using Separable Programming:

Maximise $2X_1^2 - 3X_1 + 2X_2$

Subject to $3X_1^2 + 4X_2^2 \leq 8$

$0 \leq X_1 \leq 2$ and $0 \leq X_2 \leq 2$

Q 4(b) **[12 Marks]**

Explain what is meant by each of the following:

(i) Linear/Non-Linear functions

(ii) Convex/non-convex functions and sets

(iii) Lagrange multiplier

[End of Question 4]

[END OF EXAM]