DCU

DUBLIN CITY UNIVERSITY

SEMESTER 2 EXAMINATIONS 2014/2015

MODULE: CA429/F – Operations Research/Management Science

PROGRAMME(S):

CASE	BSc in Computer Applications (Sft.Eng.)
ECSA	Study Abroad (Engineering & Computing)
ECSAO	Study Abroad (Engineering & Computing)
CAIS	BSc in Computer Applications (Inf.Sys.)

YEAR OF STUDY: 4,O,X

EXAMINERS:

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Prof. Finbarr O'Sullivan

TIME ALLOWED: 2 Hours

INSTRUCTIONS: Answer 3 questions. All questions carry equal marks.

PLEASE DO NOT TURN OVER THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO
The use of programmable or text storing calculators is expressly forbidden.
Please note that where a candidate answers more than the required number of questions, the examiner will mark all questions attempted and then select the highest scoring ones.

Requirements for this paper (Please mark (X) as appropriate)

	Log Tables		*Thermodynamic Tables*
	Graph Paper		*Actuarial Tables*
	Dictionaries		*MCQ Only – Do not publish*
	Statistical Tables		*Attached Answer Sheet*

QUESTION 1 ***[TOTAL MARKS: 33]***

Q 1(a) **[18 Marks]**

(i) [10 marks]

A lathe in a factory is examined at the end of each day to determine its status with respect to accuracy. The inspection results are classified as follows:

State	Lathe Condition
1	Excellent
2	Acceptable – some deterioration in accuracy
3	Marginal – significant deterioration in accuracy
4	Unacceptable accuracy –repairs required

Based on past data collected concerning the operation of the lathe, its condition over a period of time has been described by the following transition matrix:

		State on the following day			
		1	***2***	***3***	***4***
State on one day	***1***	0	0.9	0.1	0
	2	0	0.6	0.3	0.1
	3	0	0	0.5	0.5
	4	1	0	0	0

The expected costs associated with the various states of the machine are as follows:

State	Expected cost per day
1	€0
2	€100 (some re-work)
3	€900 (significant re-work)
4	€2000 (repair cost and downtime)

What is the expected cost associated with the above policy of repairing the machine to a like-new condition each time it reaches state 4?

(ii) [8 marks]

The company is considering implementing a new policy whereby when the accuracy is found to be marginal, a minor repair is carried out, at a cost of €500, whereby the accuracy is returned to acceptable (state 2). Would it be worthwhile implementing this policy?

Q 1(b) [15 Marks]

For a random walk with absorbing barriers, let the possible states be E_0, E_1, ..., E_n with corresponding transition matrix:

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ q & 0 & p & 0 & \dots & 0 & 0 & 0 \\ 0 & q & 0 & p & \dots & 0 & 0 & 0 \\ . & . & . & . & \dots & . & . & . \\ . & . & . & . & \dots & . & . & . \\ 0 & 0 & 0 & 0 & \dots & q & 0 & p \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 1 \end{bmatrix}$$

For each of the "interior" states E_1, E_2, ..., E_{n-1} transitions are possible to the right and left neighbours with probabilities p and $q = 1 - p$, respectively.

Determine the fundamental matrix for the case n = 5 when p=1/3 and q = 2/3, and hence find the corresponding long run probabilities of each internal state transitioning to either E_0 or E_5.

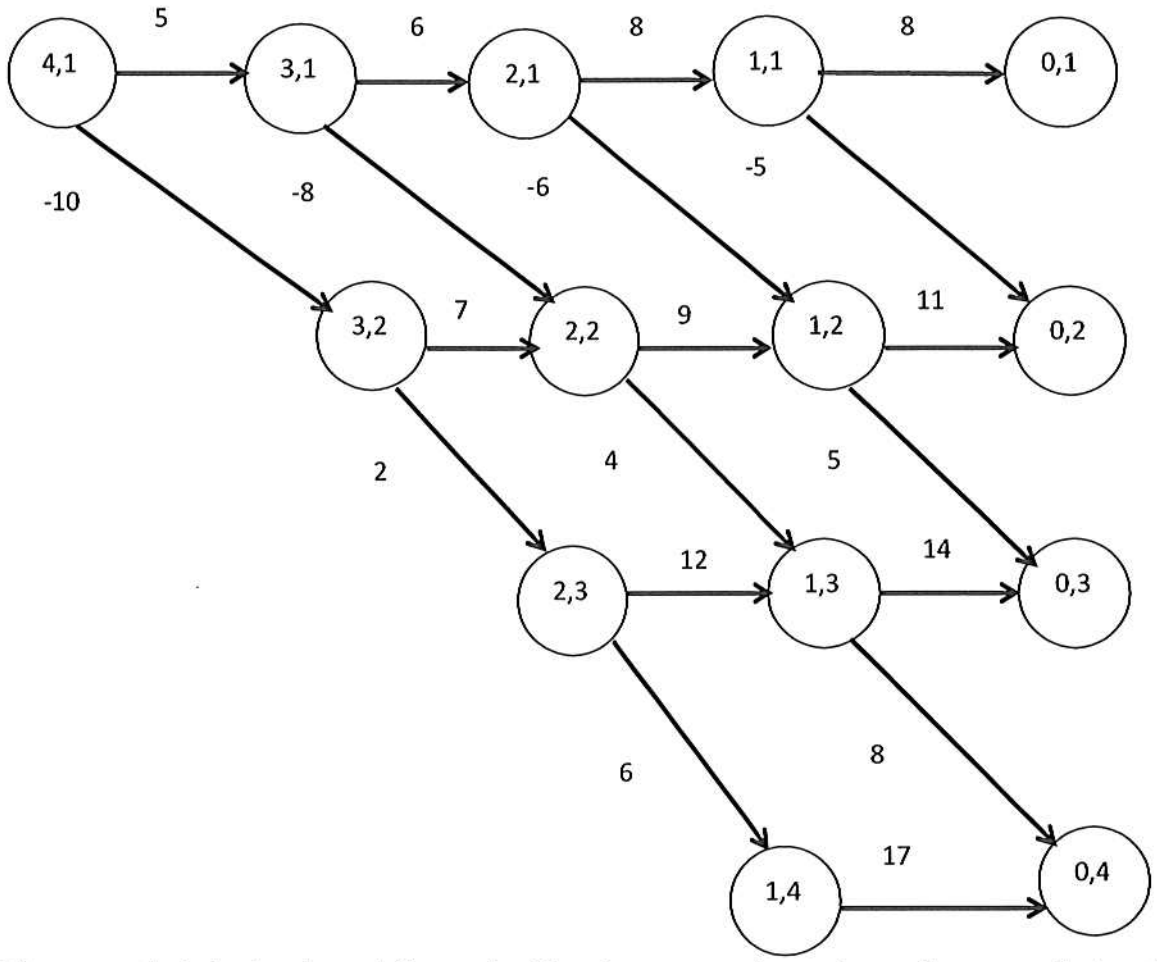
Note:

$$\begin{bmatrix} 1 & -1/3 & 0 & 0 \\ -2/3 & 1 & -1/3 & 0 \\ 0 & -2/3 & 1 & -1/3 \\ 0 & 0 & -2/3 & 1 \end{bmatrix}^{-1} = \frac{1}{31}\begin{bmatrix} 45 & 21 & 9 & 3 \\ 42 & 63 & 27 & 9 \\ 36 & 54 & 63 & 21 \\ 24 & 36 & 42 & 45 \end{bmatrix}$$

[End of Question 1]

QUESTION 2 ***[TOTAL MARKS: 33]***

A company has a manufacturing plant which, at present, is operating at capacity level 1. At the beginning of each year, the company decides whether to maintain the plant at its current capacity level or to increase it to the next level. The following figure depicts the expected net returns associated with the possible decisions, over a four year period. For example, at the start of Year 1, the possible actions are to maintain level 1 capacity (net return 5) or to expand to level 2 capacity (net return -10).



The expected plant value at the end of the four years depends on the capacity level as follows:

Capacity level	1	2	3	4
Expected plant value after 4 years	0	4	9	14

Find, ***using Dynamic Programming***, the maximum present value of the plant, and the associated set of decisions that the company should take over the four years to achieve this. Assume an annual interest rate of 20%.

[End of Question 2]

QUESTION 3 ***[TOTAL MARKS: 33]***

Consider the following problem:

Maximise $2X_1 + 3X_2$

Subject to $X_1 + 2X_2 \leq 3$
$6X_1 + 8X_2 \leq 15$

$X_1, X_2 \geq 0$

for which the optimal simplex tableau is

	X_1	X_2	S_1	S_2	b
Z	0	0	½	¼	5¼
X_2	0	1	1½	-¼	¾
X_1	1	0	-2	½	1½

where S_1 and S_2 are slack variables.

Q 3(a) **[16 Marks]**
Using the ***cutting plane*** method find the optimal solution to the above problem if it is required that both X_1 and X_2 should have integer values.

Q 3(b) **[17 Marks]**
Solve the problem of **Q 3(a)** using the ***branch and bound*** method.

[End of Question 3]

QUESTION 4 ***[TOTAL MARKS: 33]***

Q 4(a) **[7 Marks]**

Explain the roles of ***deviational variables*** and ***pre-emptive priorities*** in Goal Programming.

Q 4(b) **[16 Marks]**

A city parks department has been given a grant of €600million to expand its public recreational facilities. Two different types of facilities have been requested by the city council members, namely gymnasiums and athletic fields. The total demand by various neighbourhoods has been estimated to be seven gyms and ten athletic fields. Each facility costs a certain amount, requires a certain number of hectares and has an expected usage as shown in the following table:

Facility	Cost (€million)	Required hectares	Expected usage (people/week)
Gymnasium	80	4	1500
Athletic field	24	8	3000

The parks department has located a total of 50 hectares of land for construction.

The city council has established the following list of prioritised goals:

1. The parks department must spend the total grant.
2. The facilities should be used by 20,000 or more people weekly.
3. Avoid having to acquire more land than the 50 hectares available at present.
4. The parks department would like to meet the demands of the various neighbourhoods. However, this priority should be weighted according to the number of people expected to use each facility.

Formulate the above as a goal programming problem.

Q 4(c) **[10 Marks]**

Set up the initial Simplex tableau for the above problem, including conversion to canonical form.

[End of Question 4]

[END OF EXAM]