DUBLIN CITY UNIVERSITY

SEMESTER 1 EXAMINATIONS 2013/2014

MODULE: CA427 – Operations Research

PROGRAMME(S):

CASE	BSc in Computer Applications (Sft.Eng.)
BSSA	Study Abroad (DCU Business School)

YEAR OF STUDY: 4,X

EXAMINERS:

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Prof. Finbarr O'Sullivan

TIME ALLOWED: 2 Hours

INSTRUCTIONS: Answer 3 questions. All questions carry equal marks.

PLEASE DO NOT TURN OVER THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO

The use of programmable or text storing calculators is expressly forbidden.
Please note that where a candidate answers more than the required number of questions, the examiner will mark all questions attempted and then select the highest scoring ones.

Requirements for this paper (Please mark (X) as appropriate)

- [ ] *Log Tables*
- [ ] *Graph Paper*
- [ ] *Dictionaries*
- [ ] *Statistical Tables*
- [ ] *Thermodynamic Tables*
- [ ] *Actuarial Tables*
- [ ] *MCQ Only – Do not publish*
- [ ] *Attached Answer Sheet*

QUESTION 1 *[TOTAL MARKS: 33]*

Q 1(a) **[6 Marks]**

An investment broker manages share portfolios for a number of clients. A new client has asked the broker to handle an investment portfolio of €80,000. The portfolio is restricted to investments A and B only, for which the following information is available:

Investment	Price/share	Annual Return per share	Risk index per share
A	€25	€3	0.50
B	€50	€2	0.25

The risk index measures the relative risks of the two alternatives (A is therefore relatively riskier than B). By constraining the total risk for the portfolio, the broker avoids placing excessive amounts of the portfolio in potentially high return but also high risk investments. An upper limit of 700 has been set for the total risk of this particular portfolio. Formulate this as a linear programming problem to maximise the total annual return for the client.

Q 1(b) **[16 Marks]**

Find, using the **Simplex Method**, the solution to the problem formulated in (a).

Q 1(c) **[3 Marks]**

What is the optimal solution to the dual problem and how should it be interpreted?

Q 1(d) **[4 Marks]**

Within what range of values for the annual return for investment A and for investment B would the current solution remain optimal?

Q 1(e) **[4 Marks]**

Within what range of values for the total risk, and for the investment portfolio would the current solution remain optimal?

[End of Question 1]

QUESTION 2 *[TOTAL MARKS: 33]*

Q 2(a) **[9 Marks]**

Explain the use of **Slack**, **Surplus** and **Artificial Variables** in linear programming.

Q 2(b) **[8 Marks]**

A machine tool company conducts a job-training program for machinists. Trained machinists are used as teachers in the program at a ratio of one for every ten trainees. The training program lasts for one month. From past experience it has been found that out of ten trainees hired, only six complete the program successfully (the unsuccessful trainees are released).

Trained machinists are also needed for machining and the company's requirements for the next three months are as follows:

January	110
February	150
March	200

In addition, the company requires 230 trained machinists by April. There are 130 trained machinists available at the beginning of the year.

Payroll costs per month are:

Each trainee	€400
Each trained machinist (machining or teaching)	€700
Each trained machinist idle (Union forbids firing them!)	€500

Set up the linear programming problem that will produce the hiring and training schedule of minimum cost, and will meet the company's requirements.

Q 2(c) **[6 Marks]**

Transform the linear programming problem into canonical form, ready for application of the Simplex Method. (You do ***not*** have to carry out Simplex Method).

Q 2(d) **[10 Marks]**

Present a general AMPL model for the class of problems of which the problem in part 2(b) is a particular case. Also, present the corresponding AMPL data file for the case of the problem of part 2(b).

[End of Question 2]

QUESTION 3 ***[TOTAL MARKS: 33]***

Q 3(a) **[10 Marks]**

(i) [3 marks]

State Little's formula relating, under steady state conditions, the mean time spent in a system (or queue) to the mean number in the system (or queue). Also, state how the mean waiting time in system (W) is related to the mean waiting time in queue (W_q).

(ii) [7 marks]

Show, for a steady-state, M/M/1 queue, that the probability (P_n) that there are n customers in the system, satisfies the equations

$P_1 = (\lambda/\mu)P_0$ (for n = 0)

$P_{n+1} = ((\lambda + \mu)/\mu)P_n - (\lambda/\mu)P_{n-1}$ (for n > 0)

where λ and μ are the arrival and service rates, respectively.

Note: It follows (*but you are **not** required to prove*) that $P_n = (\lambda/\mu)^n P_0$, and, using $P_0+P_1+P_2+\ldots = 1$, that $P_0 = 1 - \lambda/\mu$. Also, that mean number in the system is $L = \lambda/(\mu - \lambda)$.

Q 3(b) **[20 Marks]**

A factory has a number of machines which break down at an average rate of one every 10 minutes. Fixing the machines is relatively simple and takes on average 15 minutes. Two mechanics are employed to do this work and are paid €20 per hour. Every hour a machine is not available for production costs the company €15. If you assume that the rate at which machines break down and the rate at which they are repaired are described by Poisson distributions, would the company save money by employing an additional mechanic?

Note: For an M/M/s queue

$$P_0 = 1 \Big/ \left(\sum_{n=0}^{s-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^s}{s!(1-\frac{\lambda}{s\mu})} \right)$$

and

$$L_q = \frac{P_0(\lambda/\mu)^s \rho}{s!(1-\rho)^2} \quad \text{where } \rho = \frac{\lambda}{s\mu}$$

Q 3(c) **[3 Marks]**

Explain why it would not make sense to employ just one mechanic for the machine repair system of Part 3(b).

[End of Question 3]

QUESTION 4 *[TOTAL MARKS: 33]*

An auctioneering company helps individuals to sell their homes. In order to complete a sale the following tasks must be completed:

Task	Description	Nominal task time (days)	Possible reduction (days)	Reduction cost per day
A	Inspect the house	4	2	€80
B	Assess the house	3	1	€60
C	Do the title search	4	2	€50
D	Get tax clearance	5	2	€40
E	Get a sales permit	2	0	-
F	Find a buyer	21	5	€80
G	Get a mortgage	14	4	€70
H	Get the legal documents	10	3	€40
I	File the legal documents	1	0	-
J	Final closing	1	0	-

The assessment is done after the inspection. To get the sales permit, you must first obtain the tax clearance. A buyer cannot be found until the house is assessed, the title search is complete, and the sales permit is obtained. After a buyer is found, the legal documents can be prepared and the buyer can obtain a mortgage. Once the legal documents are obtained, they can be filed with the county. The final closing takes place once the mortgage is obtained and the legal documents are filed.

Q 4(a) [16 Marks]

Using the given **nominal** data, draw the activity network and determine the amount of time a sale will take, and the critical path.

Q 4(b) [9 Marks]

As indicated by the fourth and fifth columns of the above table, it is possible to reduce some of the task times by allocating additional staff to the sale of a particular house. For every day the auctioneering company has the house on its books there is an overhead cost of €100. Determine how long each task will take if total cost is to be minimised (i.e. overhead cost + cost of reducing task times).

Q 4(c) [8 Marks]

Describe generally how planning problems such as the above may be modelled and solved using a mathematical programming approach. In particular, formulate (***not*** solve) the "crash" problem of part 4(b) as a linear programming problem.

[End of Question 4]

[END OF EXAM]