DCU

DUBLIN CITY UNIVERSITY

SEMESTER 2 EXAMINATIONS 2013/2014

MODULE: CA429 – Operations Research/Management Science

PROGRAMME(S):

CASE	BSc in Computer Applications (Sft.Eng.)
ECSA	Study Abroad (Engineering & Computing)
ECSAO	Study Abroad (Engineering & Computing)

YEAR OF STUDY: 4,O,X

EXAMINERS:

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TIME ALLOWED: 2 Hours

INSTRUCTIONS: Answer 3 questions. All questions carry equal marks.

PLEASE DO NOT TURN OVER THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO
The use of programmable or text storing calculators is expressly forbidden.
Please note that where a candidate answers more than the required number of questions, the examiner will mark all questions attempted and then select the highest scoring ones.

Requirements for this paper (Please mark (X) as appropriate)

- [ ] *Log Tables*
- [ ] *Graph Paper*
- [ ] *Dictionaries*
- [ ] *Statistical Tables*
- [ ] *Thermodynamic Tables*
- [ ] *Actuarial Tables*
- [ ] *MCQ Only – Do not publish*
- [ ] *Attached Answer Sheet*

QUESTION 1 *[TOTAL MARKS: 33]*

Q 1(a) **[15 Marks]**

Land cover maps derived from satellite images are used to classify a particular landscape. There are four classifications labelled SI, SE, UR and OG. The maps are generated at regular intervals and the following matrix of transition probabilities describes the observed changes:

	Destination			
Origin	**SI**	**SE**	**UR**	**OG**
SI	7/10	2/10	1/10	0
SE	0	7/10	2/10	1/10
UR	2/10	0	7/10	1/10
OG	3/10	1/10	0	6/10

Assuming stationarity, find the long term proportions of the four landscape classes.

Q 1(b) **[18 Marks]**

For a random walk with absorbing barriers, let the possible states be E_0, E_1, ..., E_n with corresponding transition matrix:

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ q & 0 & p & 0 & \dots & 0 & 0 & 0 \\ 0 & q & 0 & p & \dots & 0 & 0 & 0 \\ . & . & . & . & \dots & . & . & . \\ . & . & . & . & \dots & . & . & . \\ 0 & 0 & 0 & 0 & \dots & q & 0 & p \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 1 \end{bmatrix}$$

For each of the "interior" states E_1, E_2, ..., E_{n-1} transitions are possible to the right and left neighbours with probabilities p and $q = 1 - p$, respectively.

(i) Determine the fundamental matrix for the case n = 5. You should make use of the note below in your answer.

(ii) For the case n = 5, find the long run probabilities of each internal state transitioning to either E_0 or E_n. What are the values of these probabilities in the specific case of $p = q = 1/2$?

<u>Note</u>:

$$\begin{bmatrix} 1 & -p & 0 & 0 \\ -q & 1 & -p & 0 \\ 0 & -q & 1 & -p \\ 0 & 0 & -q & 1 \end{bmatrix}^{-1} = \frac{1}{1-3pq+p^2q^2} \begin{bmatrix} 1-2pq & p-p^2q & p^2 & p^3 \\ q-pq^2 & 1-pq & p & p^2 \\ q^2 & q & 1-pq & p-p^2q \\ q^3 & q^2 & q-pq^2 & 1-2pq \end{bmatrix}$$

[End of Question 1]

QUESTION 2 *[TOTAL MARKS: 33]*

Q 2(a) **[7 Marks]**

State the recurrence relationship of the general "value iteration algorithm" of Dynamic Programming, defining each of its elements precisely.

Q 2(b) **[26 Marks]**

The ***Knapsack Problem*** is that of a hiker who wishes to carry items of different kinds in his or her knapsack, subject to an overall limit on weight. The items of each kind have a weight and a value (or measure of utility). The objective is to maximize the overall value of the knapsack contents. Use Dynamic Programming (*marks will not be given if any other method is used*) to solve the following specific ***Knapsack Problem*** involving items of 3 kinds:

Item	Weight per unit	Value per unit
1	2	5
2	4	11
3	3	8.5

[End of Question 2]

QUESTION 3 *[TOTAL MARKS: 33]*

Q 3(a) **[7 Marks]**

Explain the roles of ***deviational variables*** and ***pre-emptive priorities*** in Goal Programming.

Q 3(b) **[26 Marks]**

A company use labour and materials to make three products for which the following unit data are specified:

	Labour (hrs/unit)	Materials (Kg/unit)	Profit (€/unit)
Product 1	5	4	3
Product 2	2	6	5
Product 3	4	3	2
Normal availability (per day)	300	500	

It is required to find the number of units to make daily of each of the three products in order to satisfy the following goals, listed in order of priority (*1 being the highest*):

1. Avoid under-utilisation of labour capacity ("no layoffs")
2. Minimise overtime
3. Achieve a "satisfactory" profit of €500 per day
4. Minimise purchase of additional materials (above 500 Kg)

(i) Formulate this as a Goal Programming problem.

(ii) Apply the Simplex Method, first ensuring that the initial tableau is in canonical form. It is not required to seek an integer solution. Perform two iterations of the Simplex method. Comment on the extent to which the feasible solution after the two iterations satisfies Goals 1 to 4.

[End of Question 3]

QUESTION 4 *[TOTAL MARKS: 33]*

Q 4(a) **[7 Marks]**

Show that a linear programming problem containing a production cost function of the following form can be expressed as a mixed integer programming formulation:

$$C_j(x_j) = K_j + c_j x_j \quad \text{for } x_j > 0$$
$$\qquad\;\; = 0 \qquad\qquad\;\, \text{for } x_j = 0$$

Q 4(b) **[19 Marks]**

Consider the following problem:

Maximize $3X_1 + 5X_2$

Subject to
$X_1 + 2X_2 \leq 5$
$6X_1 + 8X_2 \leq 21$
$X_1, X_2 \geq 0$

The optimal simplex tableau for this problem is:

	X_1	X_2	S_1	S_2	**b**
Z	0	0	3/2	1/4	51/4
X_2	0	1	3/2	-1/4	9/4
X_1	1	0	-2	1/2	1/2

where S_1 and S_2 are slack variables.

Find, using the cutting plane method, the optimal solution to the above problem if it is required that both X_1 and X_2 should have integer values.

Q 4(c) **[7 Marks]**

Show how the following quadratic programming problem may be solved by the method of Lagrange multipliers.

Minimize $(X_1 - a)^2 + (X_2 - b)^2 + (X_3 - c)^2$

Subject to $X_1 - 2X_2 - 3X_3 = 2$

where a, b and c are constant parameters. You should derive the system of four simultaneous linear equations arising from application of the method but you are ***not*** required to solve them.

[End of Question 4]

[END OF EXAM]