DUBLIN CITY UNIVERSITY

SEMESTER 1 EXAMINATIONS 2014/2015

MODULE:	CA427/F – Operations Research
PROGRAMME(S):	
CASE	BSc in Computer Applications (Sft.Eng.)
CAIS	BSc in Computer Applications (Inf.Sys.)
YEAR OF STUDY:	4
EXAMINERS:	Dr Liam Tuohey (Ext:8728) Prof. Finbarr O'Sullivan Dr. Ian Pitt
TIME ALLOWED:	2 Hours
INSTRUCTIONS:	Answer 3 questions. All questions carry equal marks.

PLEASE DO NOT TURN OVER THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO
The use of programmable or text storing calculators is expressly forbidden.
Please note that where a candidate answers more than the required number of questions, the examiner will mark all questions attempted and then select the highest scoring ones.

Requirements for this paper (Please mark (X) as appropriate)

☐	*Log Tables*	☐	*Thermodynamic Tables*
☐	*Graph Paper*	☐	*Actuarial Tables*
☐	*Dictionaries*	☐	*MCQ Only – Do not publish*
☐	*Statistical Tables*	☐	*Attached Answer Sheet*

QUESTION 1 ***[TOTAL MARKS: 33]***

Q 1(a) **[6 Marks]**

United Metal produces alloys B_1 (special brass) and B_2 (yellow tombac). B_1 contains 50% copper and 50% zinc. B_2 contains 75% copper and 25% zinc. Net profits are €120 per ton of B_1 and €100 per ton of B_2. The daily copper and zinc supplies are 15 and 10 tons, respectively. Formulate this as a linear programming problem to maximize the net profit of the daily production.

Q 1(b) **[13 Marks]**

Find, using the **Simplex Method**, the solution to the problem formulated in Q1(a).

Q 1(c) **[3 Marks]**

What is the optimal solution to the dual problem and how should it be interpreted?

Q 1(d) **[4 Marks]**

Within what range of values for the copper and zinc supplies (considered individually) would the current solution remain optimal?

Q 1(e) **[7 Marks]**

Consider a generalisation of the problem stated in Q1(a) in which there is a set of several alloys produced, not just B_1 and B_2, and in which there are several raw materials, not just copper and zinc. Write an ***ampl*** model to represent this more general problem. Also, present the corresponding ***ampl*** data file for the case of the problem of Q1(a).

[End of Question 1]

QUESTION 2 *[TOTAL MARKS: 33]*

Q 2(a) **[12 Marks]**

A local manufacturing plant runs 24 hours a day, 7 days a week. During various times during the day, different numbers of workers are needed to run the various machines. Below is the minimum number of people needed to safely run the plant during the various times.

Period Number	Period Times	Minimum no. of employees needed
1	00.00-04.00	8
2	04.00-08.00	9
3	08.00-12.00	15
4	12.00-16.00	14
5	16.00-20.00	13
6	20.00-00.00	11

Each worker works two consecutive 4-hour periods. Formulate a linear programming problem to determine the minimum number of workers to safely run the plant.

Note: Someone who starts work at 20.00 on day K will work through to 04.00 on day K+1.

Q 2(b) **[6 Marks]**

Transform the linear programming problem you obtained in Q2(a) into canonical form, ready for application of Simplex Method. (Do ***not*** carry out Simplex Method.)

Q 2(c) **[7 Marks]**

A baking company has two bakeries where they bake their goods, which they then ship to four different stores to sell. Each bakery produces 50 truck-loads of baked goods per week while the demands, in truck-loads, are anticipated to be 30, 25, 40 and 5 from stores 1, 2, 3 and 4, respectively. Each bakery can supply any of the stores, and the unit cost per truck-load of shipping from each bakery to each store is given in the following table:

	Store 1	Store 2	Store 3	Store 4
Bakery 1	€20	€45	€35	€30
Bakery 2	€35	€35	€50	€40

Set up (but do **not** solve) the transportation problem to minimize the total shipping cost, using all goods produced and exactly satisfying demand.

Q 2(d) **[8 Marks]**

Present a general ***ampl*** model for the class of problems of which the problem in Q2(c) is a particular case. Also, present the corresponding ***ampl*** data file for the case of the problem of Q2(c).

[End of Question 2]

QUESTION 3 **[TOTAL MARKS: 33]**

Q 3(a) **[6 Marks]**

(i) State Little's formula relating, under steady state conditions, the mean time spent in a system (or queue) to the mean number in the system (or queue). Also, state how the mean waiting time in system (W) is related to the mean waiting time in queue (W_q). Use the notation that λ = arrival rate and μ = service rate.

(ii) For a steady-state, M/M/1 queue the mean number in the system is $L = \lambda/(\mu - \lambda)$. Deduce, using your answer to Q3(a), that the mean number in the queue is $L_q = \lambda^2/(\mu(\mu - \lambda))$.

Q 3(b) **[21 Marks]**

A haulage company has a tyre puncture repair shop at which truck drivers arrive at a rate of four per day and for which the tyre repair potential (of one repair attendant) is five per day. The daily wage paid to the attendant at the service centre is €120 and the daily cost of a truck driver away from his or her work is €300.

Under M/M/1 assumptions

(i) Calculate the average number of truck drivers being served or waiting to be served at any given time

(ii) Calculate the average time a truck driver spends waiting for service.

(iii) Calculate the total daily cost of operating the system.

(iv) Calculate the cost of the system if there were two repair attendants (working independently), each paid €120 per day and each able to service on average five trucks per day.

(v) Would it be worth employing a third repair attendant, at the same rate of pay and tyre repair rate? Justify your answer.

Note: For an M/M/s queue

$$P_0 = 1 \Bigg/ \left(\sum_{n=0}^{s-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^s}{s!(1-\frac{\lambda}{s\mu})} \right)$$

and

$$L_q = \frac{P_0(\lambda/\mu)^s \rho}{s!(1-\rho)^2} \quad \text{where } \rho = \frac{\lambda}{s\mu}$$

Q 3(c) **[6 Marks]**

In queue systems where both the arrival rate (λ_n) and service rate (μ_n) are dependent on the number in the system (n) it can be shown that the probability of there being n customers in the system is

$P_n = \dfrac{\lambda_0\lambda_1\ldots\lambda_n}{\mu_1\mu_2\cdots\mu_{n+1}} P_0$ where $P_0 = \{1+\dfrac{\lambda_0}{\mu_1}+\dfrac{\lambda_9\lambda_1}{\mu_1\mu_2}\ldots\}^{-1}$.

What form should λ_n and μ_n take to represent (i) an immigration-death process and (ii) a queue with limited waiting room (R)?

[End of Question 3]

QUESTION 4 **_[TOTAL MARKS: 33]_**

Q 4(a) **[8 Marks]**

Establish the classic economic lot-size formula $\sqrt{\frac{2KD}{h}}$ where D is the constant demand (per time period), K is the set-up cost per order, and h is the inventory holding cost (per unit per time period). State clearly the assumptions underlying this formula.

Q 4(b) **[7 Marks]**

A manufacturer produces 100,000 memory sticks on a continuous production line every month. Each memory stick requires a casing and these are produced very quickly in batches. It costs €12,000 to set up machinery to produce a batch and €0.01 a month to store and insure a casing once made. How large should the batch size be to minimize total costs, and how often must a production run be set up?

Q 4(c) **[10 Marks]**

A company manufacturing computer tablets has to purchase components regularly from a supplier who offers the following pricing regime:

Quantity Purchased	Price per Component
< 400	€10.00
400 – 800	€ 9.80
800 – 1600	€ 9.60
≥ 1600	€ 9.50

The company has an annual demand for 5000 of these components. Furthermore, it costs the company €50 to place up an order (regardless of order size), and there is an inventory holding charge of 30% of the value of the stock per annum. What order quantity will minimise the total annual cost (i.e. purchasing cost + ordering cost + holding cost)?

Q 4(d) **[8 Marks]**

The demand for a seasonal item is subject to fluctuations over the course of a year and its manufacturer wishes to find a production schedule that minimises the costs due to output variations and inventories.

Let x_t = number of units produced in week t
r_t = number of finished units that must be available in week t
s_t = number of finished units that are not required in week t
(s_0 = amount of product in storage at the beginning of the first week)
a = cost of increasing production by 1 unit from week t-1 to week t
b = cost of storing 1 unit for one week.

Present a linear programming formulation of this problem in terms of parameter λ = a/b, the cost of a unit increase in output relative to that of storing a unit for a week.

[End of Question 4]

[END OF EXAM]