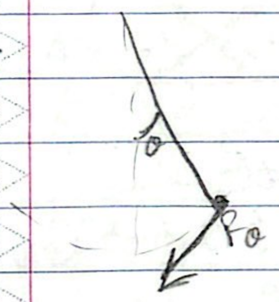
Ch 3 Notes

Elementary examples of oscilatory motion do not consider driving force

$F_\theta = -mg\sin\theta$ force opposite to displacement from vertical



$F_\theta = m\frac{d^2 s}{dt^2}$ $\quad \frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta$

$\theta = \theta_0 \sin(\Omega t + \phi)$ $\quad \Omega = \sqrt{\frac{g}{l}}$

angular frequency

$\frac{d\omega}{dt} = -\frac{g}{l}\theta$ $\quad \frac{d\theta}{dt} = \omega$

$\omega_{i+1} = \omega_i - \frac{g}{l}\theta_i \Delta t$ $\quad \theta_{i+1} = \theta_i + \omega_i \Delta t$

Using Euler method, model is not accurate, amplitude grows with time, energy not conserved

$E = \frac{1}{2} m l^2 \omega^2 + mgl(1-\cos\theta)$

$E = \frac{1}{2} m l^2 (\omega^2 + \frac{g}{l}\theta^2)$

$E_{i+1} = E_i + \frac{1}{2} mgl(\omega_i^2 + \frac{g}{l}\theta_i^2)(\Delta t)^2$

Euler solution Energy (E) will always increase with time

Euler method can model systems that don't conserve energy

Euler - Cromer

$\omega_{i+1} = \omega_i - \left(\frac{g}{\ell}\right)\theta_i \Delta t$

$\theta_{i+1} = \theta_i + \omega_{i+1}\Delta t$ (use ω_{i+1} to calculate θ_{i+1})

$t_{i+1} = t_i + \Delta t$

This method conserves energy

Damping $-q(d\theta/dt)$ $v = \ell(d\theta/dt)$

$$\frac{d^2\theta}{dt} = -\frac{g}{\ell}\theta \boxed{- q\frac{d\theta}{dt}}$$ friction

$$\theta = \theta_0 e^{-qt/2} \sin\left(\sqrt{\Omega^2 - q^2/4}\, t + \phi\right)$$

q changes graph drastically

$$\frac{d^2\theta}{dt^2} - \frac{g}{\ell}\theta - q\frac{d\theta}{dt} + F_D \sin(\Omega_D t)$$ driving force

- Driving puts more Energy into system

- $$\frac{d^2\theta}{dt^2} = -\frac{g}{\ell}\sin\theta$$ ← equation of motion

- $$\frac{d\omega}{dt} = -\frac{g}{\ell}\sin\theta - q\frac{d\theta}{dt} + F_D\sin(\Omega_D t)$$

$$\frac{d\theta}{dt} = \omega$$

Euler-Cromer for physical pendulum

$$\omega_{i+1} = \omega_i - \left[\left(\frac{g}{\ell}\right)\sin\theta_i - q\omega_i + F_D\sin(\Omega_D t_i)\right]\Delta t$$
$$\theta_{i+1} = \theta_i + \omega_i\Delta t$$
$$t_{i+1} = t_i + \Delta t$$

- Incresing F_D cause motion to become more chaotic

- Can use phase-space plot to make predictions of chaotic motion