Appendix C Notes

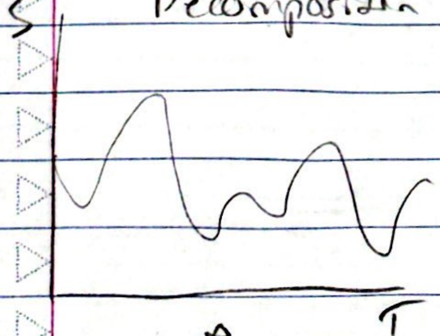
$$y(t) = \sum_{j=1}^{S} Y_j \sin(2\pi f_j t + \phi_j)$$

Any signal can be written this way in its "decomposed" parts

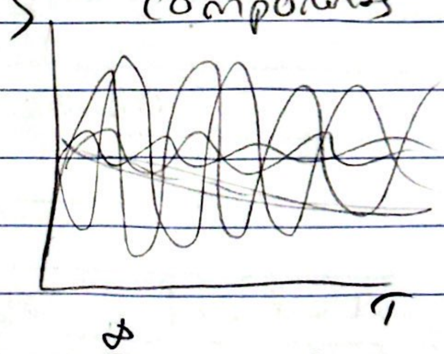
$$y(t) = \int_{-\infty}^{\infty} Y(f)\, e^{-2\pi i f t}\, df = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(\omega/2\pi)\, e^{i\omega t}\, d\omega$$

$$e^{-2\pi i f t} = \cos(2\pi f t) - i\sin(2\pi f t)$$

Decomposition

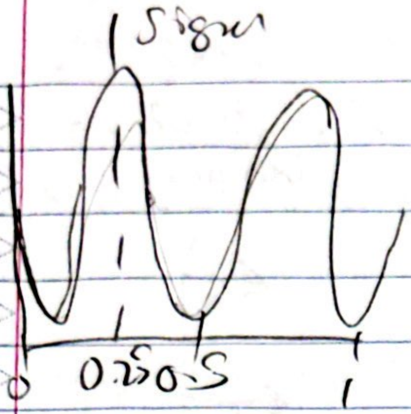


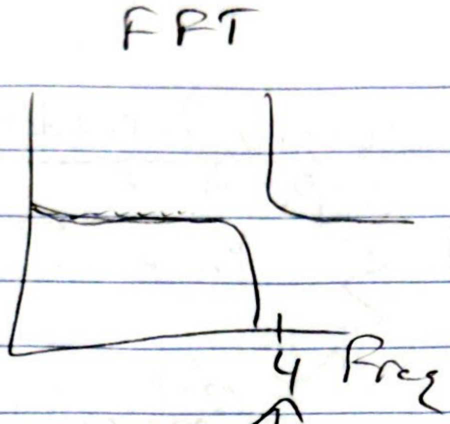
components



$$Y(f) = \int_{-\infty}^{\infty} y(t)\, e^{2\pi i f t}\, dt = \int_{-\infty}^{\infty} y(t)\, e^{i\omega t}\, dt$$

Forward and Inverse Fourier transforms





$T = 0.25$ $F = \frac{1}{T} = 4$

$$P \propto \int_{-\infty}^{\infty} |Y(f)|^2 \, df$$

$$P_j = Y_j(\text{real})^2 + Y_j(\text{imag})^2$$

Discrete Fourier Transform - Amplitude is known

$$y_m = \frac{1}{N}\sum_{n=0}^{N-1} Y_n e^{-2\pi i m n/N}$$

$$Y_N = \sum_{m=0}^{N-1} y_m e^{2\pi i m n/N}$$

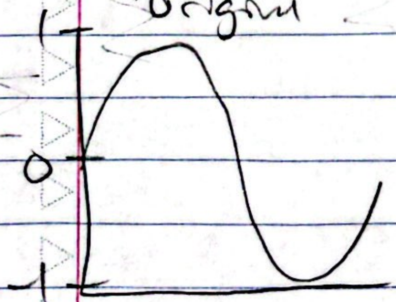
$$f_n = n(N\Delta T)$$

Fast Fourier Transform

$$Y_n = Y_n^e + \omega^n Y_n^o \qquad \omega = e^{2\pi i/N}$$

$$= \sum_{m'=0}^{3} y_{2m'}\, \omega^{2m'n} + \omega^n \sum_{m'=0}^{3} y_{2m'+1}\, \omega^{2m'n}$$

Origin



FFT