Due Wednesday, April 30 by the end of class.

1. **[7 points]** The file `a4q1.mat` contains a data set representing the fuel economy in MPG of a number of cars (denoted y) as a function of engine size in HP (denoted x). The file `a4q1.m` loads this data set and then fits a quadratic function to it using least squares. (This is the same demo that was shown in class).

   (a) Fit the following functions to the data using least squares fitting:

      i. An exponential function of the form $y = ae^{bx}$.

      ii. A power function of the form $y = ax^b$.

   Give the values of a and b that you get for each case.

   (b) Plot the data $[x_i, y_i]$ as discrete points along with the quadratic, exponential and power fit functions, labelled clearly. Comment on any noticeable differences between the three least squares fit functions.

   Please include: A printout of your code and of the plot, as well as written responses to parts (a) and (b).

2. **[7 points]** Suppose we want to approximate the value of $\sqrt[3]{20}$ using a root-finding method.

   (a) For which function $f(x)$ do you need to find a root?

   (b) Suppose one uses bisection to find the root, beginning with the interval [2,3]. Based on the formula seen in class, how many iterations of bisection does one need to run to be guaranteed an absolute error of less than 10^{-6}?

   (c) Modify `bisect.m` to find the desired root, and run it for the number of iterations found in part (b). Verify that the error is less than 10^{-6} by comparing against the value computed by Matlab, `12^(1/3)`.

   (d) Modify `newtons.m` to find the desired root, and run it with the same error tolerance as above, starting from $x_0 = 1$. Is the convergence quadratic? Explain.

   (e) Show that the limit of the ratio of quadratic errors in part (d) is roughly equal to the value predicted by the error formula for Newton's method.

3. **[6 points]** Recall that the minimum of a function occurs when the **derivative** of that function is equal to zero (provided that the second derivative of the function is positive at that point).

   (a) Explain how you can use Newton's method to find the point (x, y) on the graph of the function $y = x^2$ that is closest to the point $(1, 0)$. Write out the Newton's method iteration in the form

   $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

   for the appropriate function $f(x)$.

   Hint: The distance of a point (a, b) from a point on the function $y(x)$ is given by

   $$D(x) = \sqrt{[x-a]^2 + [y(x) - b]^2}.$$

   Instead of minimizing $D(x)$, it is easier to minimize the square of the distance instead, however.

   (b) Use `newtons.m` to find the value of (x, y).