Due Wednesday, May 21 by the end of class.

1. **[12 points]** Consider using the Lagrange polynomial to interpolate the function

$$f(x) = \cos\left(\frac{x}{2}\right),$$

using the node points $x_0 = -\pi$, $x_1 = 0$, $x_2 = \pi$.

(a) Compute by hand the second-order interpolating polynomial, $P_2(x)$, for this data using the Lagrange formulation $P_2(x) = \sum_{i=0}^{2} f(x_i)L_i(x)$.

(b) Compute $P_2(x)$ by hand using the Newton's divided difference method, and verify that it is the same polynomial that you get in part (a).

(c) Plot $f(x)$ and $P_2(x)$ on the interval $[-\pi, \pi]$, on the same set of axes. Based on the plot, will $P_2(x)$ overestimate or underestimate $f(x)$ for values of x on this interval (that are not node points)?

Note: you can use something like `x = pi*(-1:0.01:1)` in Matlab to get evenly-spaced points on the interval for plotting.

(d) Use $P_2(x)$ to give an approximation to $f\left(\frac{\pi}{2}\right)$. Give an upper bound on the error based on the formula seen in class, and verify that the true error $f\left(\frac{\pi}{2}\right) - P_2\left(\frac{\pi}{2}\right)$ is within the computed bound.

Please include a copy of your plot for part (c).

2. **[8 points]** Suppose we have $n+1$ data points (x_i, y_i), $i = 0 \ldots n$. Let $P_{n-1}(x)$ be the Lagrange interpolant of the data from $i = 0 \ldots n-1$, and let $Q_{n-1}(x)$ be the Lagrange interpolant of the data for $i = 1 \ldots n$. Show that the Lagrange polynomial that interpolates all of the points can be written as

$$P_n(x) = \frac{(x - x_0)Q_{n-1}(x) + (x_n - x)P_{n-1}(x)}{x_n - x_0}.$$

Hint: You must show that it interpolates all of the points and that it has the appropriate degree.

This gives a way of constructing a Lagrange interpolant from two lower-order interpolants, which is sometimes useful.