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Math 351
HW8
Due Friday, June 9th

1.) Composite Simpson's rule

(a) Matlab code

comp_simpson.m

```
% adapted from comp_trap.m
function I = comp_simpson(x,y)
    h = x(2)-x(1);      %assume equal spacing
    I = y(1) + y(end); % beginning and end
    I = I + 4*sum(y(2:2:end-1)); %odd
    I = I + 2*sum(y(3:2:end-1)); %even
    I = (h/3)*I;
end
```

Testing:

```
test_f = @(x) x.^3 + x.^2 + x + 1;
xs = 1:0.1:2
ys = test_f(xs)
true_ans = 8.583333333333333333;
our_ans = comp_simpson(xs,ys);

true_ans - our_ans

>> ans = -7.1054e-15
```

(b) Error

```
f_b = @(x) log(x);

h = 1;
xs = 1:h:9;
ys = f_b(xs);

true_val = 9*log(9)-8;
our_val = comp_simpson(xs,ys);

diff = true_val - our_val

>> diff = 0.005782700025250
```

(c) Upper bound on the error in the approximation

4/15 ~= .266666

See last page for calculations

How much bigger is it than the true error from part b?

It is just over 46 times larger than the true error.

(d) Asymptotic error bound

364/32805 ~= 0.01109

See last page for calculations

How does it compare to the true error?

It is almost twice as large (~ 1.918)

2.) Applying the composite trapezoid rule and Romberg's method to three different integrals

(a) Compute error and display results in a table

	(i)	(ii)	(iii)
Trapezoid rule error	-0.019383341810551	3.261133585397147e-09	-0.006170939578877
Romberg's method error	-4.354525628968986e-10	5.329363361701311e-04	-0.002143763983338

(b) Comment on the performance

(i) For the composite exponential/sine integral, Romberg's method was much more accurate. This is the expected result.

(ii) For the 1/(2 - cos(x)) integral, Romberg's method was much less accurate. I believe the reason could be case 3 from the May 30th lecture notes. The stated problem is that the integrand is periodic and therefore the error in the trapezoid rule approaches zero more rapidly than assume by Romberg's method.

(iii) Lastly, for the square root of an absolute value integral, the results were fairly similar for both - they both performed rather poorly. This is because of the sharp corner of the function at x=0; here it is not differentiable and so the Euler-Maclaurin expansion isn't valid (Case 1 from May 30th lecture notes).