Ryan Napolitano (rn2473)
Professor Dear
Computational Aspects of Robotics, Spring 2021
Homework 5 - Written

Solutions

Problem 1: Spherical Arm Jacobian

a) We have the intermediate transforms as follows.

$$T_0^1 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},\ T_1^2 = \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 & 0 \\ \sin\theta_2 & 0 & -\cos\theta_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, T_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

$$\Rightarrow T_0^3 = T_0^1 \cdot T_1^2 \cdot T_2^3 = \begin{bmatrix} \cos\theta_1\cos\theta_2 & -\sin\theta_1 & \cos\theta_1\sin\theta_2 & -d_2\sin\theta_1 + d_3\cos\theta_1\sin\theta_2 \\ \sin\theta_1\cos\theta_2 & \cos\theta_1 & \sin\theta_1\sin\theta_2 & d_2\cos\theta_1 + d_3\sin\theta_1\sin\theta_2 \\ -\sin\theta_2 & 0 & \cos\theta_2 & d_3\cos\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

b) For Jacobian, we need the position of end effector, which is the last column of the transform of end effector. From this, we get the angular velocity Jacobian as

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -\sin\theta_1^* & 0 \\ 0 & \cos\theta_1^* & 0 \\ 1 & 0 & 0 \end{bmatrix}}_{J_\omega(q)} \cdot \underbrace{\begin{bmatrix} \dot\theta_1 \\ \dot\theta_2 \\ \dot d_3^* \end{bmatrix}}_{q} \tag{3}$$

The way we got this is that for ω_x, we see that it is from the cos component of $\dot\theta_2^*$ and ω_y is from the sin component of the same. And ω_z is directly from $\dot\theta_1^*$. Hence, the Jacobian is intuitively written as above. Same can be derived by using $\omega_n^0 = \sum_{i=1}^n \dot q_i z_{i-1}^0$. Note that for prismatic joing it is zero and hence third column, it is zero.

c) We have as follows.

$$v_1^2 = \begin{bmatrix} \dot d_3 \sin\theta_2 \\ -\dot d_3\cos\theta_2 \\ 0 \end{bmatrix} + \dot\theta_2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} d_3\sin\theta_2 \\ -d_3\cos\theta_2 \\ d_2 \end{bmatrix} = \begin{bmatrix} d_3\dot\theta_2\cos\theta_2 + \dot\theta_3\sin\theta_2 \\ d_3\dot\theta_2\sin\theta_2 - \dot d_3\cos\theta_2 \\ 0 \end{bmatrix} \tag{4}$$

Next with respect to frame 0, using the T_0^1,

$$v_0^1 = R_0^1 v_1^2 + \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1 \end{bmatrix} \times \left(R_0^1 \begin{bmatrix} d_3\sin\theta_2 \\ -d_3\cos\theta_2 \\ d_2 \end{bmatrix} \right) = \underbrace{\begin{bmatrix} -d_3\sin\theta_1\sin\theta_2 - d_2\cos\theta_1 & d_3\cos\theta_1\cos\theta_2 & \cos\theta_1\sin\theta_2 \\ d_3\cos\theta_1\sin\theta_2 - d_2\sin\theta_1 & d_3\sin\theta_1\cos\theta_2 & \sin\theta_1\sin\theta_2 \\ 0 & -d_3\sin\theta_2 & \cos\theta_2 \end{bmatrix}}_{J_v(q)} \cdot \underbrace{\begin{bmatrix} \dot\theta_1 \\ \dot\theta_2 \\ \dot d_3 \end{bmatrix}}_{\dot q} \tag{5}$$

This is the velocity Jacobian.

d) Clearly, $J_\omega(q)$ has maximum rank of 2 and $J_v(q)$ has maximum rank of 2. It is obvious as we see that any velocity along z_1 is never possible. One singular configuration is when $\theta_2 = 0$ in which case velocities at the tip can only be in tangential direction to θ_1 and hence is of rank 1.

Problem 2: Inverse Velocity Kinematics

a) When all three end effector velocities are specified, in general, no solution will exist as the rank of corresponding Jacobian is only 2. However, for specific combination of end effector velocities, inverse solutions will exist.

On the other hand, if only $\dot x, \dot z$, then a unique solution will exist as this will be in reacheable span of the Jacobian column vectors.

b) For the given end effector velocities, we use pseudo inverse to minimize least squares error. So, we find the pseudo inverse of the Jacobian as

$$J_{pi}(q) = (J^T J)^{-1} J^T = \begin{bmatrix} -0.1237 & -0.0714 & 0 & -0.2143 & 0.3711 & 0.7143 \\ 0.1856 & 0.1071 & 0 & -0.1786 & 0.3093 & 0.4286 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} \theta_1 \\ \theta_2 \\ d_3 \end{bmatrix} = J_{pi}(q) \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.6760 \\ 0.8520 \\ 1.0000 \end{bmatrix} \quad (6)$$

c) The code for the same is done in MATLAB in file `P2c.m` and the plot is as follows.

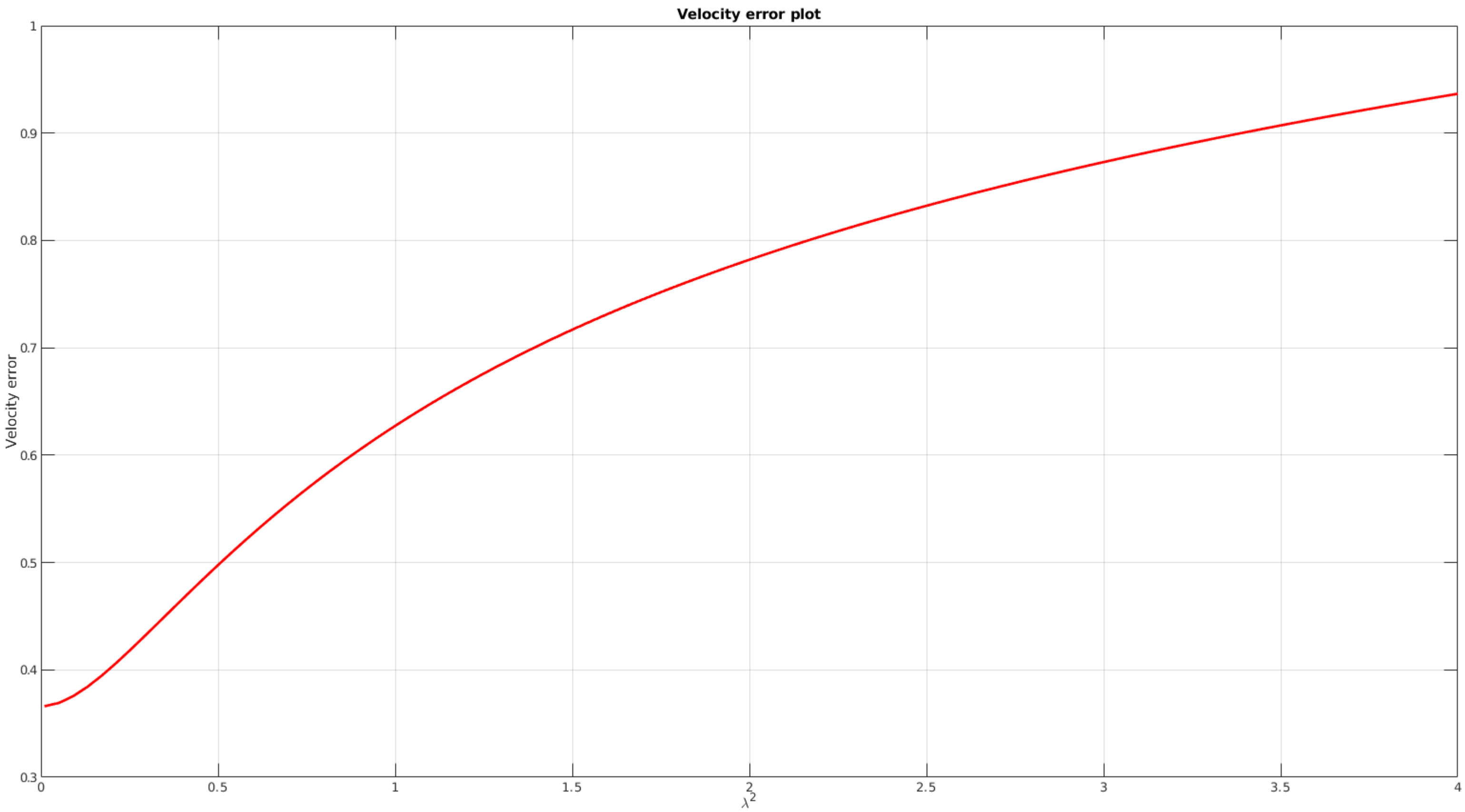

Figure 1: Velocity error plot as λ^2 is varied

Problem 3: Numerical Inverse Kinematics

The coding has been done in MATLAB in `mainfile.m` along with associated functions. The learning rate chosen after tuning for Gradient Descent was $\alpha = 0.58$ which gave iteration count to convergence of 24. The damping for DLS tuned was $\lambda^2 = 0.02$ which gave the iteration count to convergence for Newton's algorithm to 12. For both methods, the error norm at convegence under a tolerance of 10^{-5} was 0.4690.

Problem 4: Learning Inverse Kinematics

The code is done in Jupyter notebook `Problem 4.ipynb`.

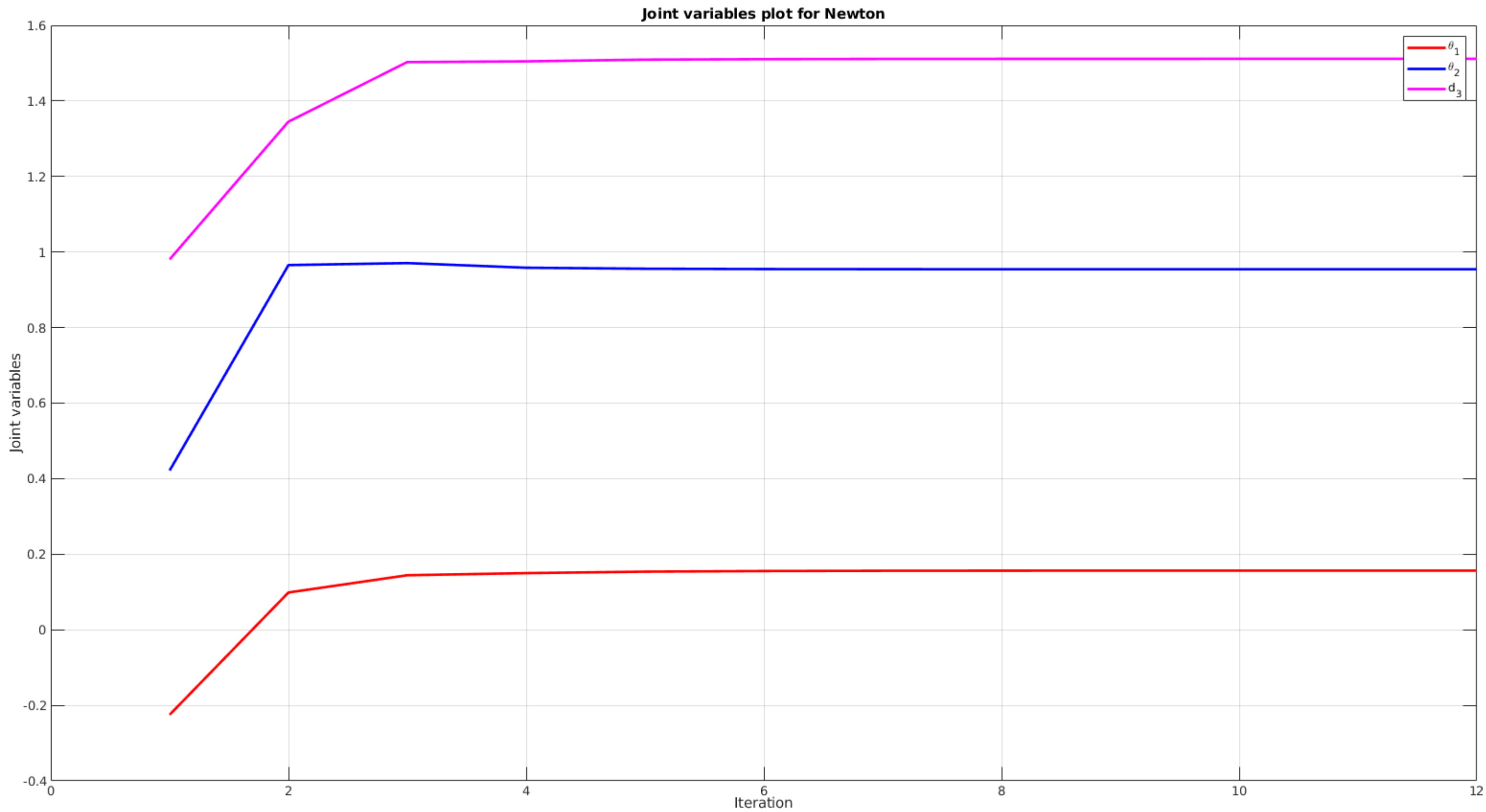

Figure 2: Newton method joint angle plot

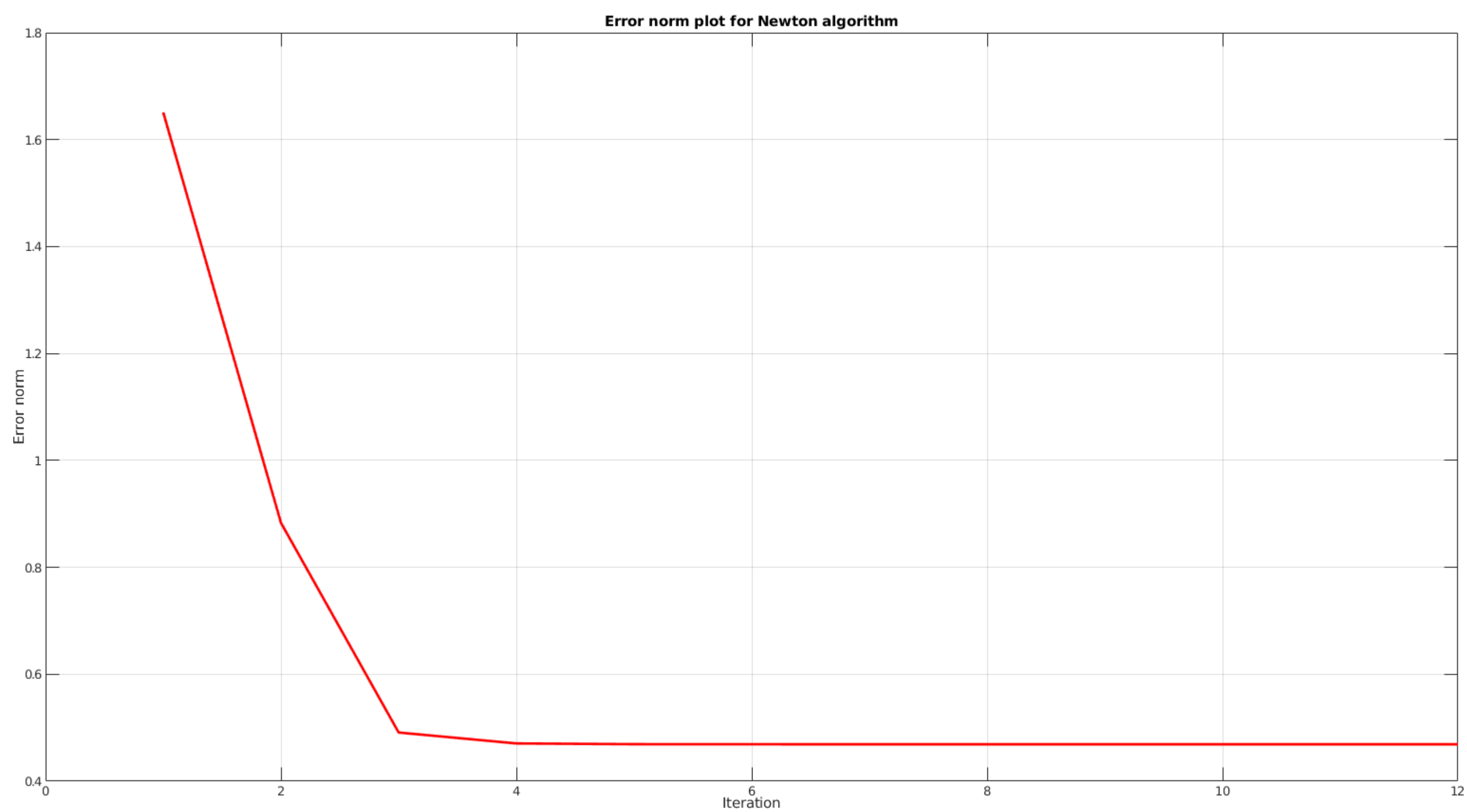

Figure 3: Newton method error norm plot

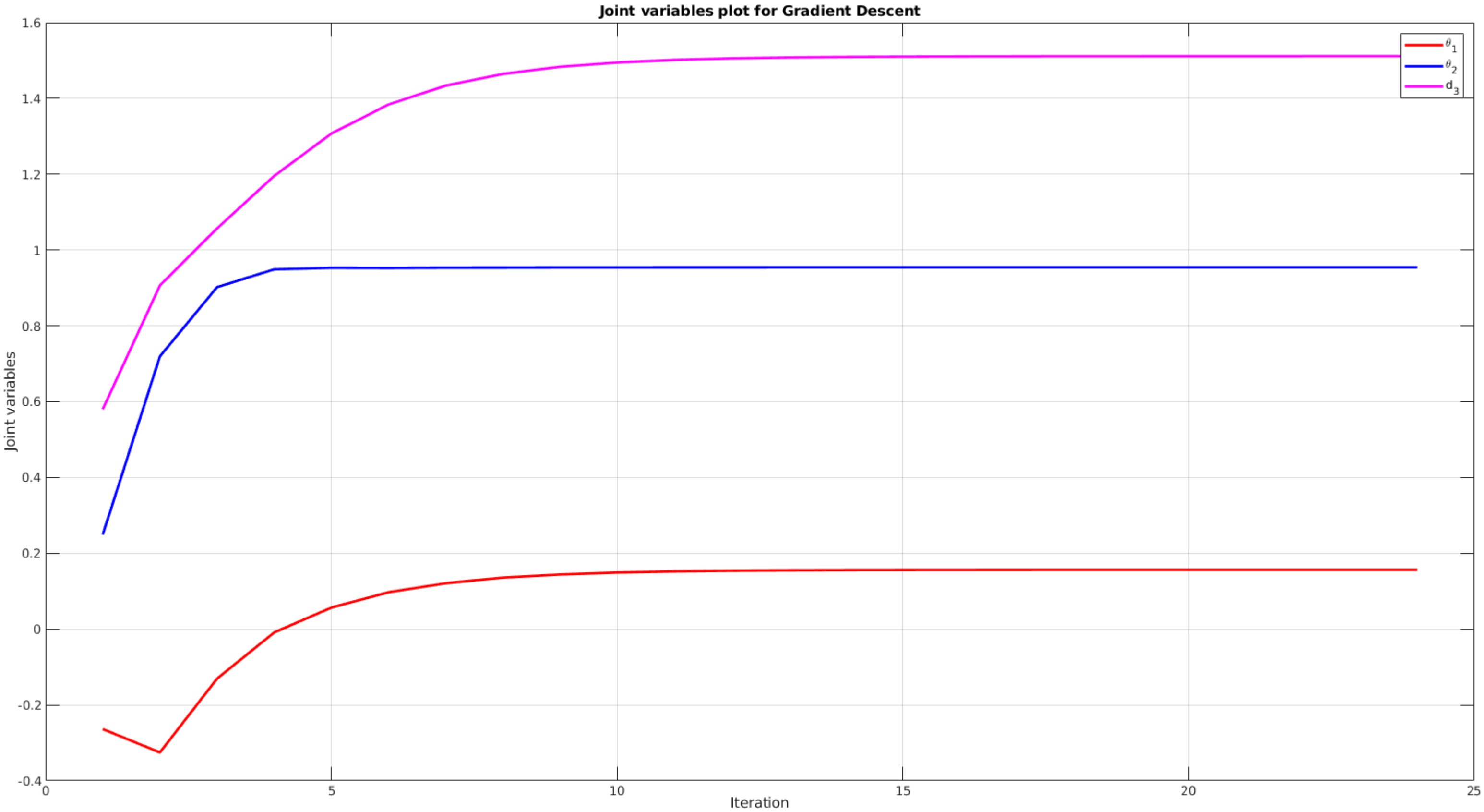

Figure 4: Gradient Descent joint angle plot

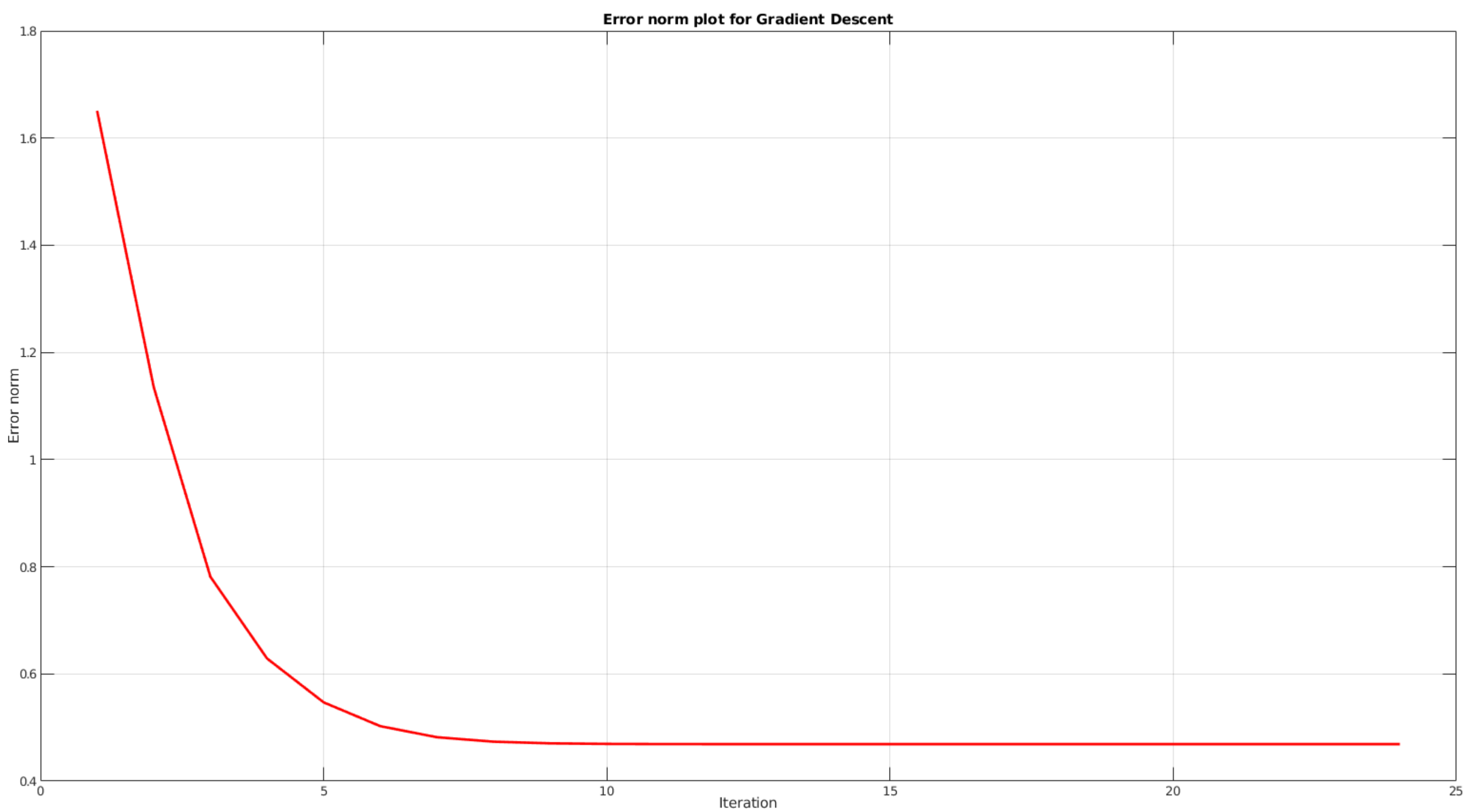

Figure 5: Gradient Descent error norm plot