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Professor Dear
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Homework 4 - Written

Problem 1: Homogeneous Transformations

a) We have for frame 1 and 0,

$$A_1^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

$$A_2^1 = \begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 0.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$A_3^2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

b) When robot moves the cube, the values of transform A_2^1, A_3^2 changes to as follows. Note, that these transforms can be just written down by inspection.

$$^nA_2^1 = \begin{bmatrix} 0 & -1 & 0 & -0.2 \\ 1 & 0 & 0 & 0.2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

$$^nA_3^2 = \begin{bmatrix} 1 & 0 & 0 & 0.3 \\ 0 & -1 & 0 & 0.3 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

c) We have by multiplication, the composite transform as follows.

$$A_3^0 = A_1^0 \cdot A_2^1 \cdot A_3^2 = A_1^0 \cdot^n A_2^1 \cdot^n A_3^2 = \begin{bmatrix} 0 & 1 & 0 & -0.5 \\ 1 & 0 & 0 & 1.5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

Clearly it does not matter when the intermediate frames change as long as the relative orientations and positions of initial(1) and final frames(3) dont change.

Problem 2: Forward Kinematics

a) We have the DH parameter table as follows. Note that in comparison to given anthropomorphic arm, the arm here has rotation axis intersecting at 90○. So, the intermediate coordinate $3'$ is introduced to correct orientation as seen in the DH parameter table.

b) We find

$$T_0^6 = T_0^1 \cdot T_1^2 \cdot T_2^3 \cdot T_3^{3'} \cdot T_{3'}^4 \cdot T_4^5 \cdot T_5^6 \tag{7}$$

#	d_i	θ_i	a_i	α_i
1	0	θ_1	0	$\frac{\pi}{2}$
2	0	θ_2	a_2	0
3	0	θ_3	0	0
3'	0	$\frac{\pi}{2}$	0	$\frac{\pi}{2}$
4	d_4	θ_4	0	$-\frac{\pi}{2}$
5	0	θ_5	0	$\frac{\pi}{2}$
6	d_6	θ_6	0	0

Table 1: DH parameter table

Now, on substituting all angles to zeros, we get the transform as

$$T_0^6(0,0,0,0,0,0) = \begin{bmatrix} 0 & 0 & 1 & a_2+d_4+d_6 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

This matrix is self explanatory as under zero angle configuration(home configuration), the end effector is stretched out and hence the link lengths are summed.

Problem 3: Bayes Filter

a) Given belief at time

$$B(x_0) \sim U(0,1) \tag{9}$$

, the unform continuous distribution. The motion model is

$$p(x_1|x_0,u_0) \sim \mathcal{N}(x_0+u_0,1) \tag{10}$$

Hence, the updated belief prediction is

$$B'(x_1) \sim \int_0^1 p(x_1|x_0,u_0=1)\cdot B(x_0)dx_0 = \int_0^1 \mathcal{N}(x_0+1,1)\cdot U(0,1)dx_0 = \int_0^1 \underbrace{\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-x_0-1}{1}\right)^2}}_{p(x_t|x_0,u_0=1)} \cdot \underbrace{\frac{1}{1}}_{B(x_0)} \cdot dx_0 \tag{11}$$

$$= \frac{1}{2}\left(\operatorname{erf}\left(\frac{x-1}{\sqrt{2}}\right) - \operatorname{erf}\left(\frac{x-2}{\sqrt{2}}\right)\right) \tag{12}$$

The plot for this looks like as follows.

b) Now, for the update with sensor model we have

$$B(x_t) = \frac{U(0,1)\cdot B'(x_1)}{\int_{-\infty}^{\infty} B(x,1)\cdot U(0,1)dx} = B'(x_t) \tag{13}$$

The plot is same as in fig 1 above. When compared, we clearly see that from a uniform distribution which is flat over [0,1], we see the belief has updated to a Gaussian with peak at ≈ 1.55.

Problem 4: Particle Filter Localization

All codes are added in the Python script `hw4.py`.

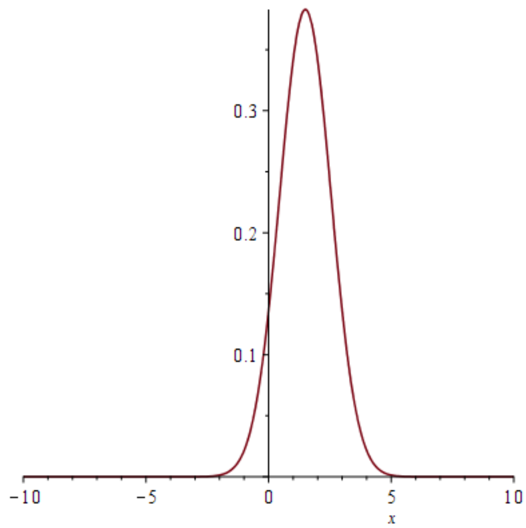

Figure 1: Q3:Belief prediction $B'(x_t)$

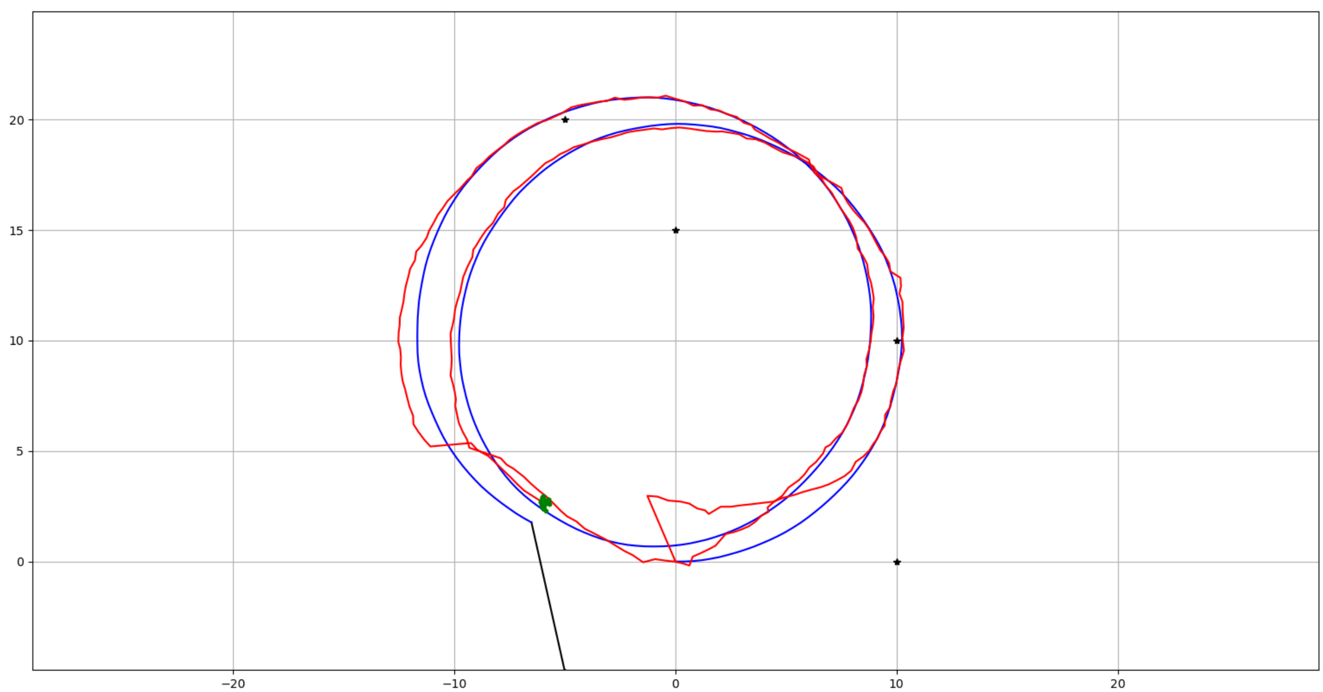

Figure 2: 4.4-Q1:Final frame of animation of trajectory - Truth and Estimate

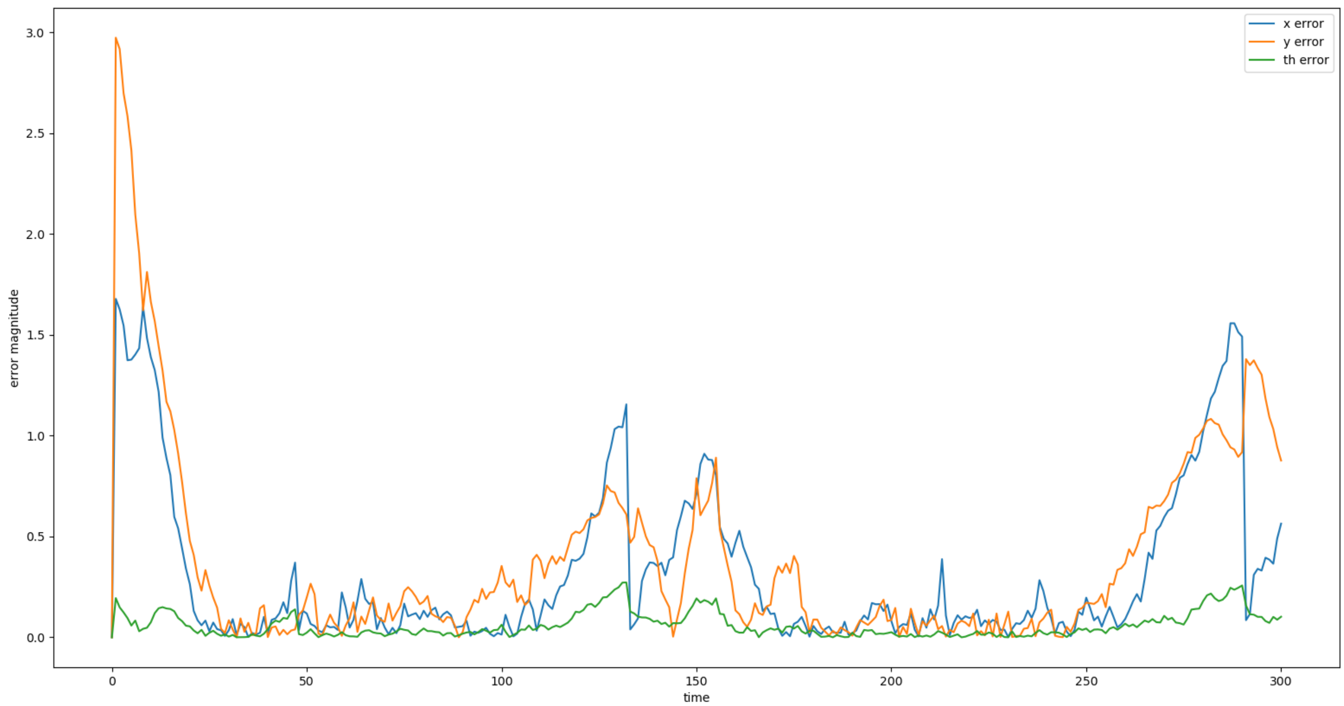

Figure 3: 4.4-Q1:Error plot of simulation run

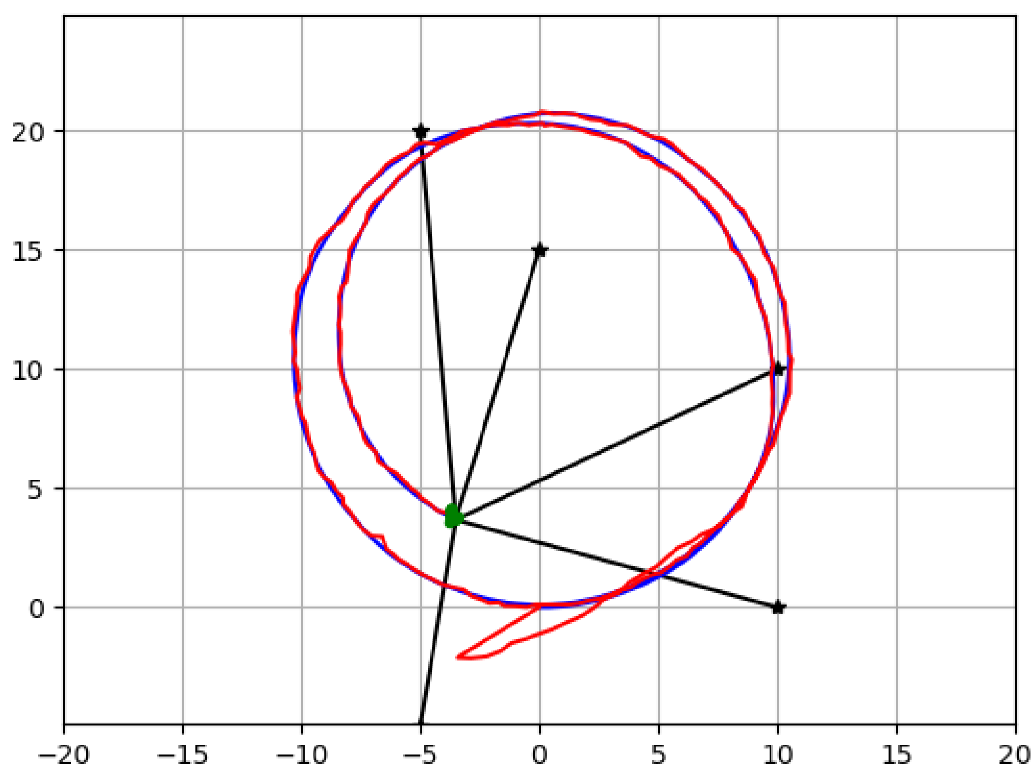

Figure 4: 4.4-Q2:Final frame of animation of trajectory - Truth and Estimate when `MAX_RANGE`=20

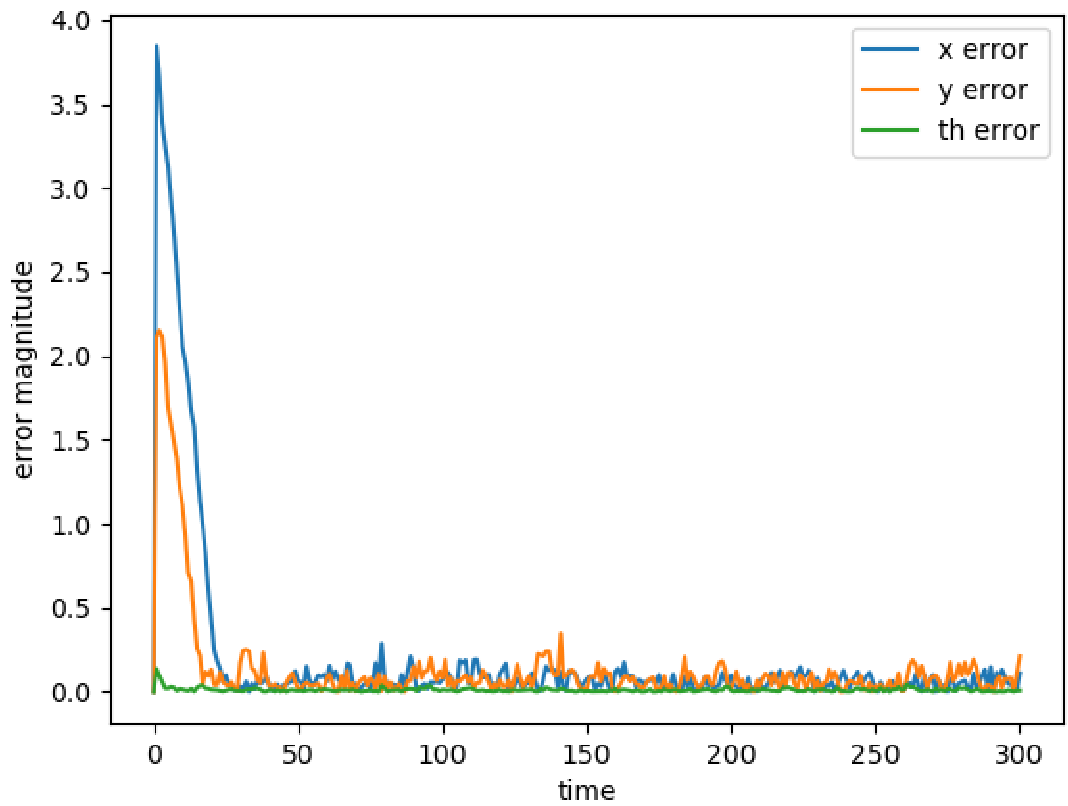

Figure 5: 4.4-Q2:Error plot of simulation run when `MAX_RANGE`=20

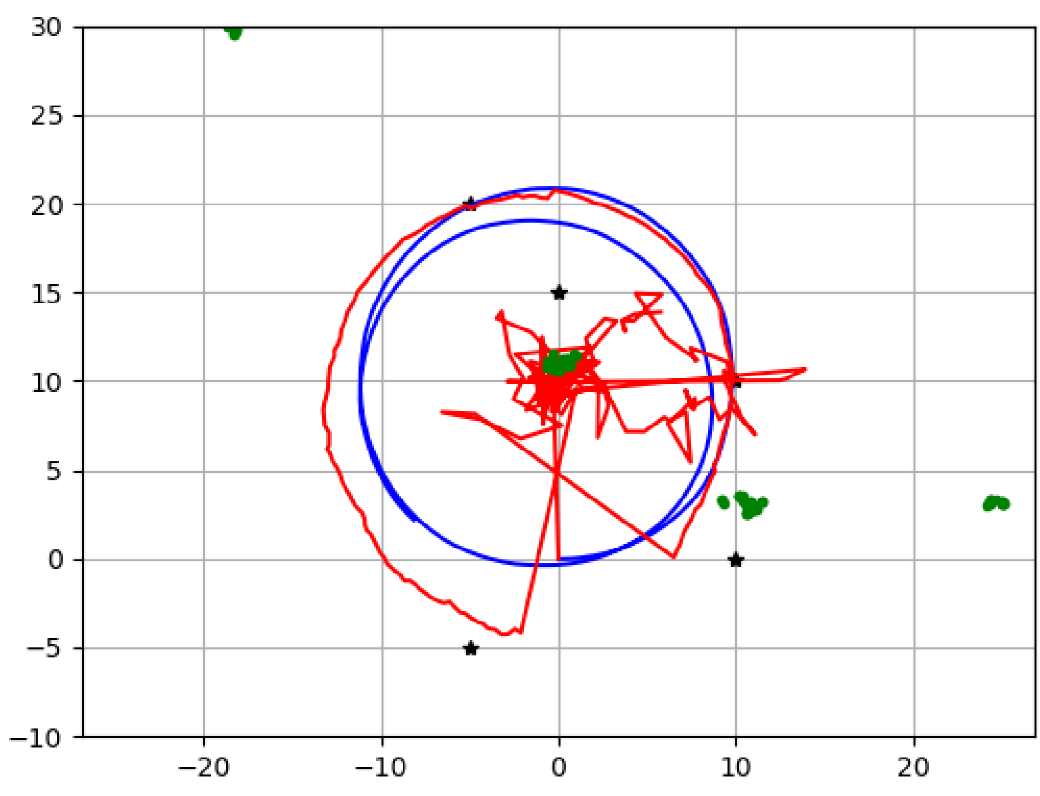

Figure 6: 4.4-Q2:Final frame of animation of trajectory - Truth and Estimate when `MAX_RANGE`=5

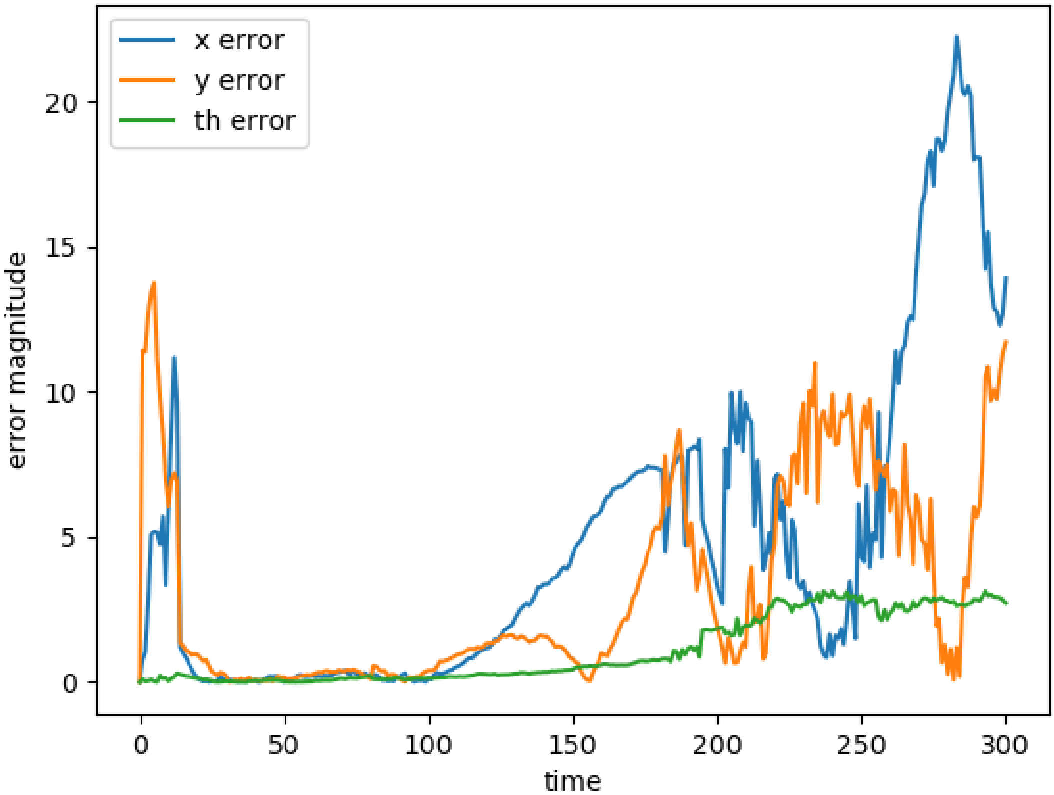

Figure 7: 4.4-Q2:Error plot of simulation run when `MAX_RANGE`=5

4.4: Analysis and Discussion

1. The plots of simulation run are as follows. So the particles in general slowly deviate from providing a good estimate when sufficient landmarks are not visible over time. Once a landmark is observed, the particle regroup and improve the estimate by removing the accumulated drift. This is because the correction step happens only when landmarks comes up, otherwise particles evolve as per prediction equations only which have their noise.

2. The plots are as follows. As can be seen, when range is high, sensor always sees more landmarks or sees them frequently thereby reducing drift and getting corrections, resulting in accurate trajecotry. Particles are always sampled close to truth. On contrary, when range reduces, the landmarks get visible very close to them and less frequently, hence, particles drift out heavily and estimate becomes much worse over time.

3. Plots are shown. As can be seen, when NP=50 also, the particles samples enough to quickly sample close to truth and hence converge to truth faster which doesnt happen so when NP=20, where somtimes, the filter loses track of the trajectory also at times. Because, when NP is too less, sampling/resampling may never give enough particles with sufficient weights(importance will be low) due to most will be discarded.

4. The scenarios are as follows.

   - When high Q and low R, the likelihoods reduce for the weights due to which particles are continuously discarded.
   - When measurement noise is low(low R) and low Q, then the likelihood of most particles even when close to truth are seen low and hence discarded due to which solutions never converge.
   - When process noise is high (high Q), the likelihoods reduce for the weights due to which particles are continuously discarded.
   - When process noise is low (low Q) and high R, then likelihoods are high due to which particles are never rejected. S drift in trajectory is very high.

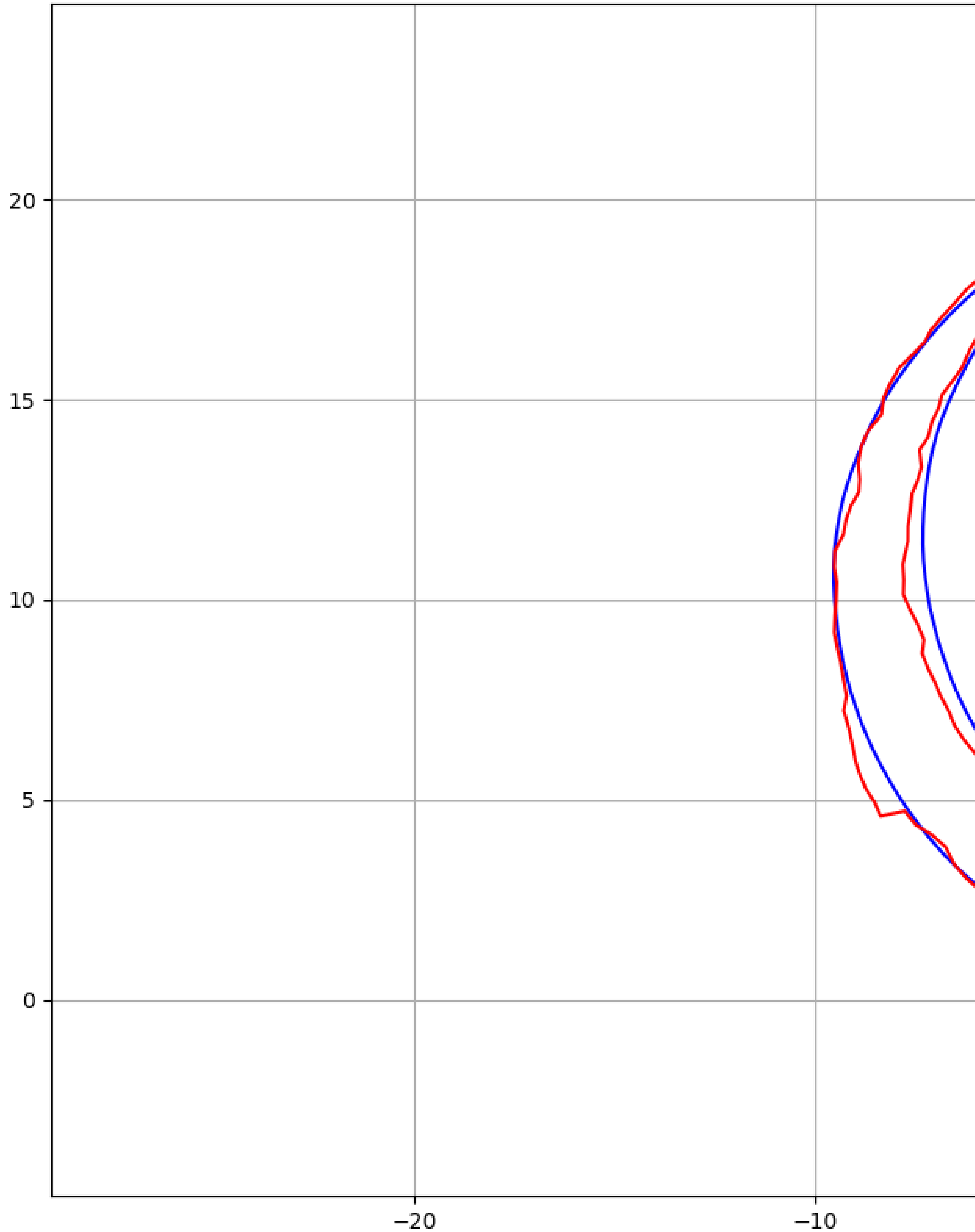

Figure 8: 4.4-Q3:Final frame of animation of trajectory - Truth and Estimate when NP=50

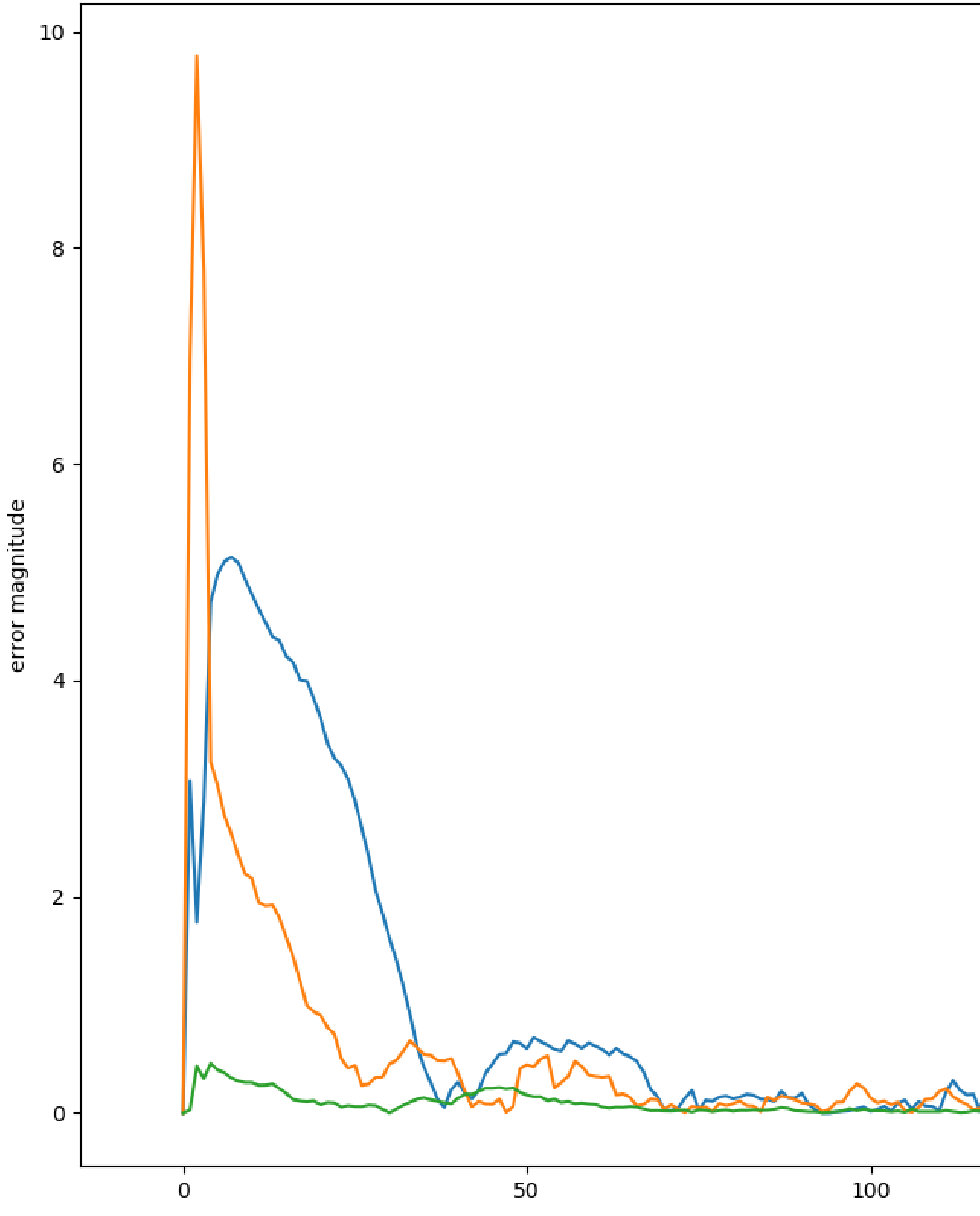

Figure 9: 4.4-Q3:Error plot of simulation run when NP=50

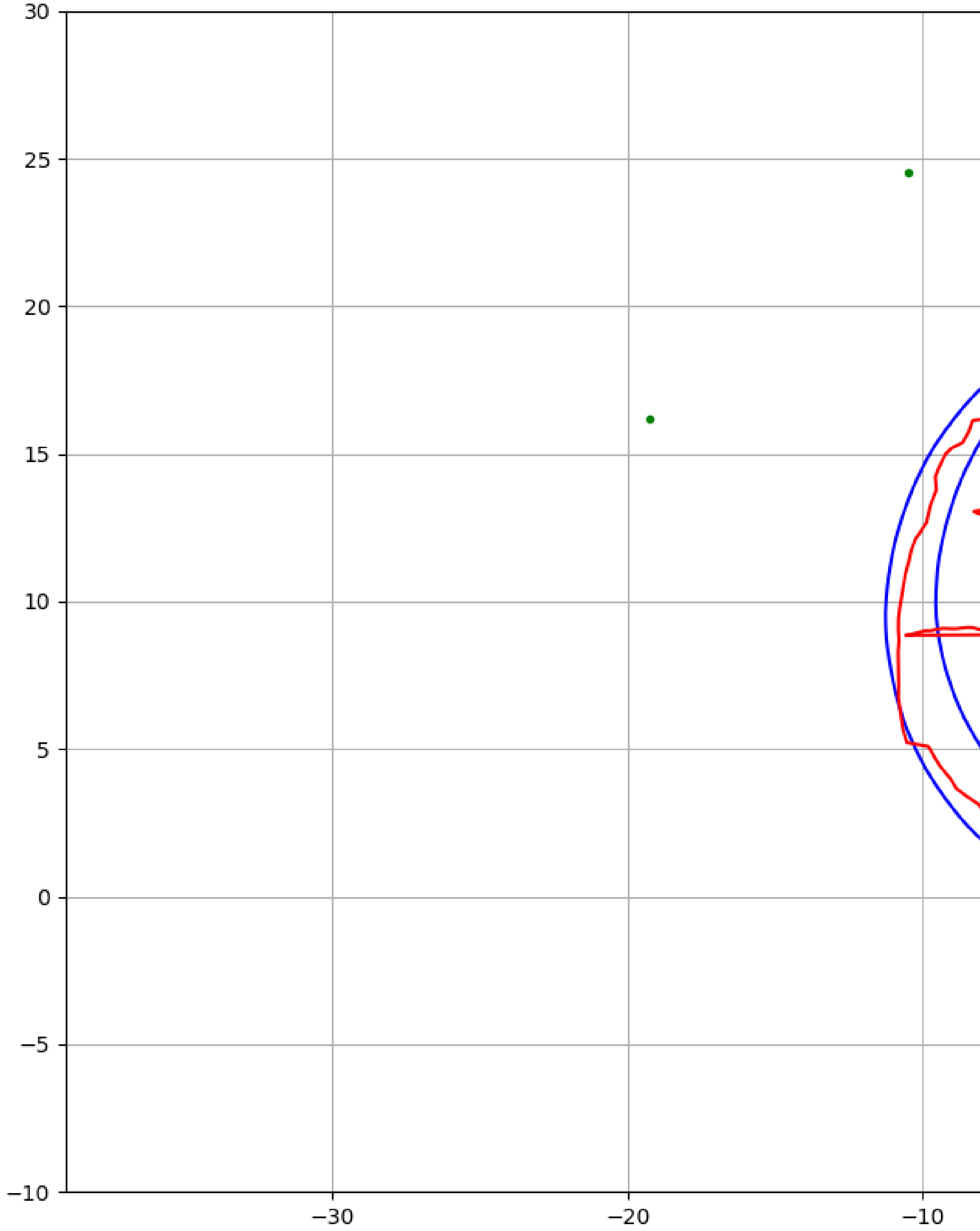

Figure 10: 4.4-Q3:Final frame of animation of trajectory - Truth and Estimate when NP=20

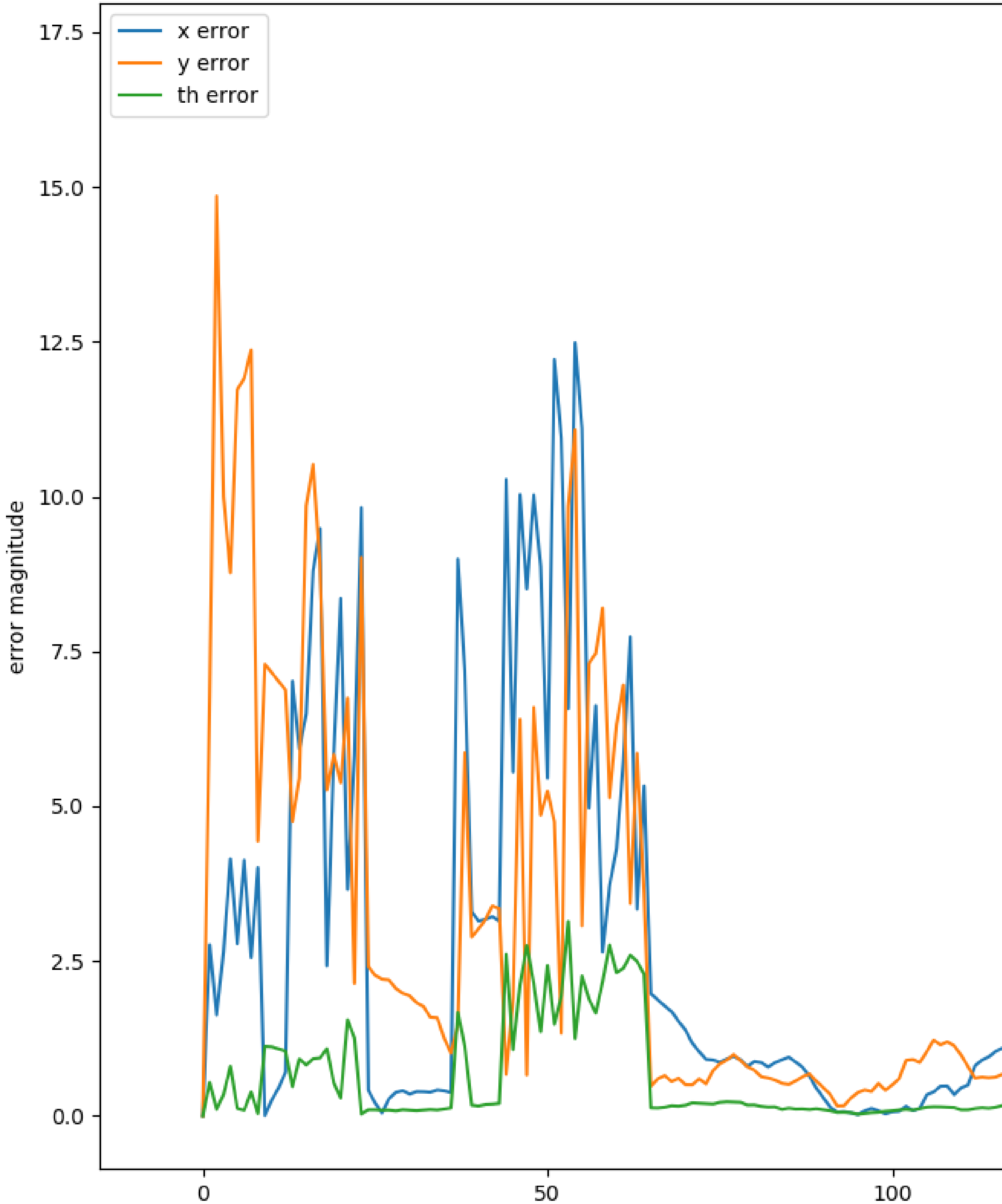

Figure 11: 4.4-Q3:Error plot of simulation run when `NP`=20