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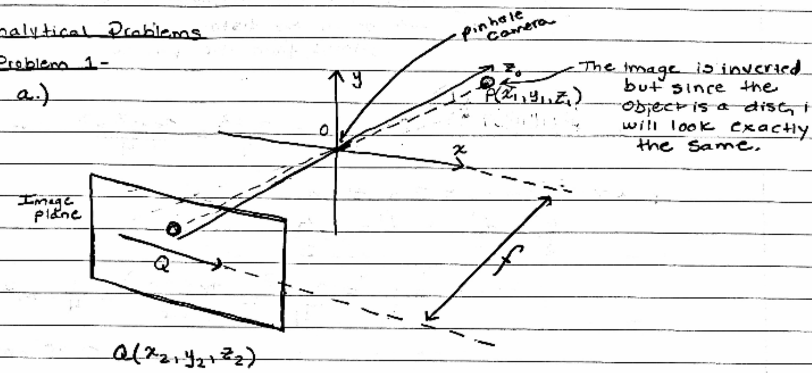
Computer Vision, Spring 2021
Professor Nayar
Homework 1

Analytical Problems

Problem 1 -

a.)



- x_1, y_1, z_1 are object coordinates in 3D and x_2, y_2, z_2 are image coordinates in 3D
- f = focal length

- Therefore,

$$x_2 = \frac{-f}{z_1}(x_1) \quad \text{and} \quad y_2 = \frac{-f}{z_1}(y_1)$$

- There are no $(z_2 = 0)$

- So, the image will be inverted to the object. In this case, the object is a disc so the image of the disk is also a circular disk!

b.) Let the diameter of the circular disc $= D$ so the angle made by the circular disc and the image will be the same. Meaning,

$$\frac{D}{1} = \frac{d}{x} \quad \begin{array}{l} \longrightarrow d \text{ is the diameter of the image} \\ \longrightarrow x \text{ is the distance of the image plane from the pin hole.} \end{array}$$

$$\Rightarrow d(1) = D(x)$$

and the area of the image $= \dfrac{\pi d^2}{4} = \dfrac{\pi D^2 x^2}{4} = 1\,mm^2$

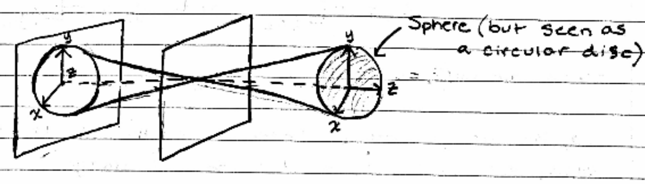
when distance $= 2m$ then,

$$\frac{D}{2} = \frac{d'}{x} \Rightarrow d' = \frac{D(x)}{2}$$

$$\text{new area} = \frac{\pi d'^2}{4} = \frac{\pi D^2 x^2}{16} = \frac{1}{4}\,mm^2$$

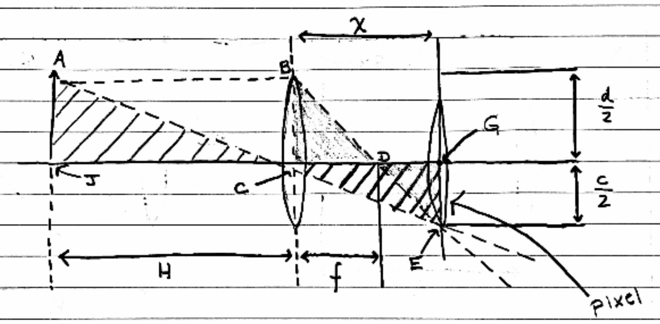
$$= 0.25\,mm^2$$

c.) When replacing the disc with a sphere, the shape of the image of the sphere will still be a circular disc because the x and y axis is seen in the image; However, the z-axis cannot be seen because the optical axis is on the z-axis.



2.) Formula for hyperfocal distance = $H = \frac{f^2}{Nc} + f$

$H \rightarrow$ hyperfocal distance
$N \rightarrow$ f number ($\frac{f}{d}$ for aperture diameter d)
$c \rightarrow$ size of pixel
$f \rightarrow$ focal length



- Above is my attempt at a geometric drawing of this problem to show how I will be deriving $H = \frac{f^2}{Nc} + f$

- By looking at this, we can see the two of the triangles are almost identical

$\therefore \Delta BCD \equiv \Delta DEG$ Taking into account ΔBCD and
$\therefore \Delta AJC \equiv \Delta CDE$ ΔDEG we get,

$$\frac{x-f}{\frac{c}{2}} = \frac{f}{\frac{d}{2}}$$

$$\therefore x - f = \frac{cf}{d}$$

$$\therefore x = f + \frac{cf}{d}$$

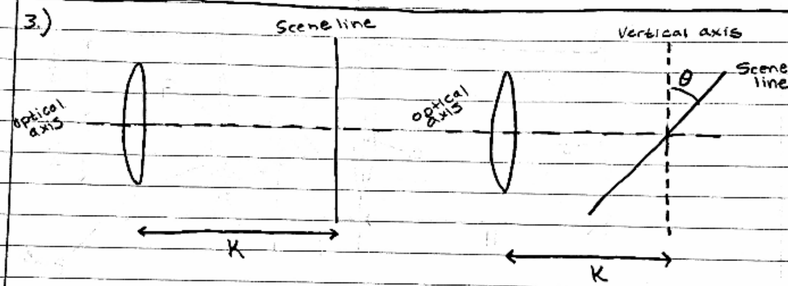
Taking into account ΔAJC and ΔCDE we get,

$$\therefore \frac{H}{\frac{d}{2}} = \frac{x}{\frac{c}{2}} \Rightarrow H = \frac{dx}{c}$$

$$\Rightarrow \frac{d}{c}\left(f + \frac{cf}{d}\right)$$

$$\Rightarrow \frac{df}{c} + f = \frac{f^2}{Nc} + f$$

3.)



a.) focal length $= f$

object distance $(d_o) = K$
(scene distance)

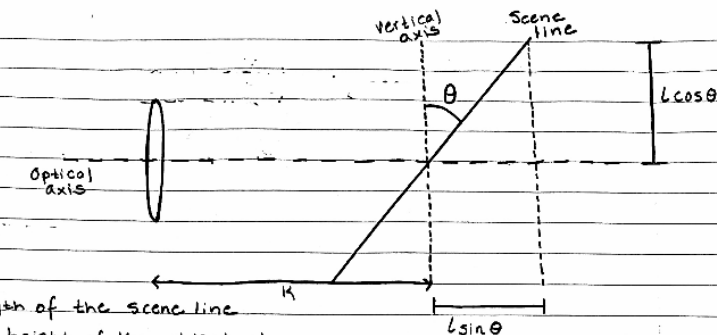
$$\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f}$$

let $K = d_o$

$$\frac{1}{d_i} + \frac{1}{K} = \frac{1}{f}$$

$$d_i = \frac{Kf}{K-f}$$

Hence, a focused image of the scene line is formed at distance of $\frac{Kf}{K-f}$ from lens c.)



L = length of the scene line

$L\cos\theta$ = height of the object above the principal axis

$\frac{Kf}{K-f}$ = forms an image when object is at a distance of K from the lens

m = Lateral magnification

$$m = -\frac{D_i}{D_o}$$

$$= -\frac{h_i}{h_o}$$

$$= -\frac{\left(\frac{Kf}{K-f}\right)}{K}$$

$$= \frac{f}{K+f}$$

The Longitudinal magnification $\frac{Ad_i}{Ad_o} = \frac{d_i^2}{d_o^2}$

L' = The length of the image tilted at ϕ

$$\frac{L'\cos\phi}{L\cos\theta} = \frac{f}{K+f} \Rightarrow L'\cos\phi = \left(\frac{f}{K+f}\right)L\cos\theta$$

$$\frac{L'\sin\phi}{L\sin\theta} = \left(\frac{f}{K+f}\right)^2 \Rightarrow L'\sin\phi = \left(\frac{f}{K+f}\right)^2 L\sin\theta$$

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$$\tan\phi = \frac{L'\sin\phi}{L'\cos\phi} = \frac{\left(\frac{f}{K+f}\right)^2 L\sin\theta}{\left(\frac{f}{K+f}\right) L\cos\theta} = \left(\frac{f}{K+f}\right)\frac{\sin\theta}{\cos\theta}$$

$$= \boxed{\tan\theta\left(\frac{f}{K+f}\right)}$$