Homework 4

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1 Part 1

1. Check code

2. For the given Metropolis updating method, the acceptance ratio for proposed move θ^* from a current value θ with a jumping probability π is given as

$$\frac{Ac(\theta^*|\theta)}{Ac(\theta|\theta^*)} = \frac{P(\theta^*)\pi(\theta|\theta^*)}{P(\theta)\pi(\theta^*|\theta)} \tag{1}$$

   where the acceptance probability for such a move is defined as

$$Ac(\theta^*|\theta) = min(1, \frac{P(\theta^*)\pi(\theta|\theta^*)}{P(\theta)\pi(\theta^*|\theta)}) \tag{2}$$

   Rearranging (1), we get

$$Ac(\theta^*|\theta)P(\theta)\pi(\theta^*|\theta) = Ac(\theta|\theta^*)P(\theta^*)\pi(\theta|\theta^*) \tag{3}$$

   Plugging in (2) into (3), we finally get

$$min(P(\theta)\pi(\theta^*|\theta), P(\theta^*)\pi(\theta|\theta^*)) = min(P(\theta^*)\pi(\theta|\theta^*), P(\theta)\pi(\theta^*|\theta)) \tag{4}$$

   The ratio of fluxes for this method is simply the acceptance ratio. The probabilities on either side of (4) are exactly the same proving that the flux of going from flux($\theta^* \rightarrow \theta$) = flux($\theta \rightarrow \theta^*$)

3. For our system, there are two spin states which can be adopted. The chain is aperiodic, meaning that all nodes in the underlying directed graph are reachable with some probability. Any given coordinate in our system can adopt either of the spins given more iteration steps since random coordinates are selected to change spin states in each iteration; this proves that the system is irreducible. A stationary distribution exists for our system in which the most likely energy state of the system is reached. Thus, the system is ergodic.

2 Part 2

1. Onsager's exact T_c is the temperature at which an order-disorder transition occurs. By scaling our input temperature to T_c, we are able to calculate the free energy associated with different spin state changes imposed under the Ising Model. With higher values of T_c, the system is more robust to thermal fluctuations and is less likely to change.

2. The results of this experiment can be found as h2_normal.gif, h2_all1.gif, h2_allneg1.gif for a random, all +1, and all -1 initial lattice, respectively. For all of these, the systems are all highly disordered and are basically indistinguishable from one another. Throughout the course of the experiment, this disorder does not change significantly. This is true because the T_c is quite high, meaning that the system is more robust to fluctuations and is less likely to form aggregates.

3. The results of this experiment can be found as h05_normal.gif, h05_all1.gif, h05_allneg1.gif for a random, all +1, and all -1 initial lattice, respectively. In the random lattice, you can see even phase separation occurring as the system approaches a half spin up - half spin down environment, with aggregates forming. This implies that lower values of T_c are more likely to form aggregates. However, with the +1 and -1 lattices, even distribution of lattices were not observed. If the lattice started at all +1, most of the spins stayed +1; the same is true conversely for the all -1 lattice. This can be explained by one of two reasons: (1) maybe the equilibrium distribution has not been reached yet and I simply need to run more samples or (2) the initial conditions impart some magnetization on the system and makes the system invariant to proposed moves. To elaborate on (2), since the system is not changing its spins as much as the random lattice, it could indicate that the initial conditions act in the same way as increasing the h values, causing less aggregation to occur.

4. The results of this experiment for $t = 1.05$ can be found as h105_normal.gif, h105_all1.gif, h105_allneg1.gif for a random, all +1, and all -1 initial lattice, respectively. The results of this experiment for $t = 1.10$ can be found as h110_normal.gif, h110_all1.gif, h110_allneg1.gif for a random, all +1, and all -1 initial lattice, respectively. I'm sure you want us to describe the results, though it's not explicitly stated. With temperatures near the critical point, we can actually see phase separation within each of the simulations; this includes the ones with homogeneous lattice spin conditions! As we stray from 1.05 to 1.1, we see slightly less phase separation happening.

5. Results for this can be found as h002_t05_normal.gif, h002_t05_all1.gif, h002_t05_allneg1.gif, h01_t05_normal.gif, h01_t05_all1.gif, h01_t2_allneg1.gif,h002_t2_normal.gif, h002_t2_all1.gif, h002_t2_allneg1.gif, h01_t2_normal.gif, h2_t05_all1.gif, and h01_t2_allneg1.gif. Distinct phase separated domains that used to be present in the random lattice were dispersed with increasing h. Disorder of the system also increased.

3 Part 3

1. You can tell your system is at equilibrium when the mean value of the lattice converges on a particular value. As T is decreased, the number of sweeps it takes for the system to get into equilibrium increases. By $t = 0.5$, the time to equilibrium is too long to observe with 100 sweeps.

2. Refer to SponMag.fig. This is for h = 0.

3. Refer to Susceptibility.fig