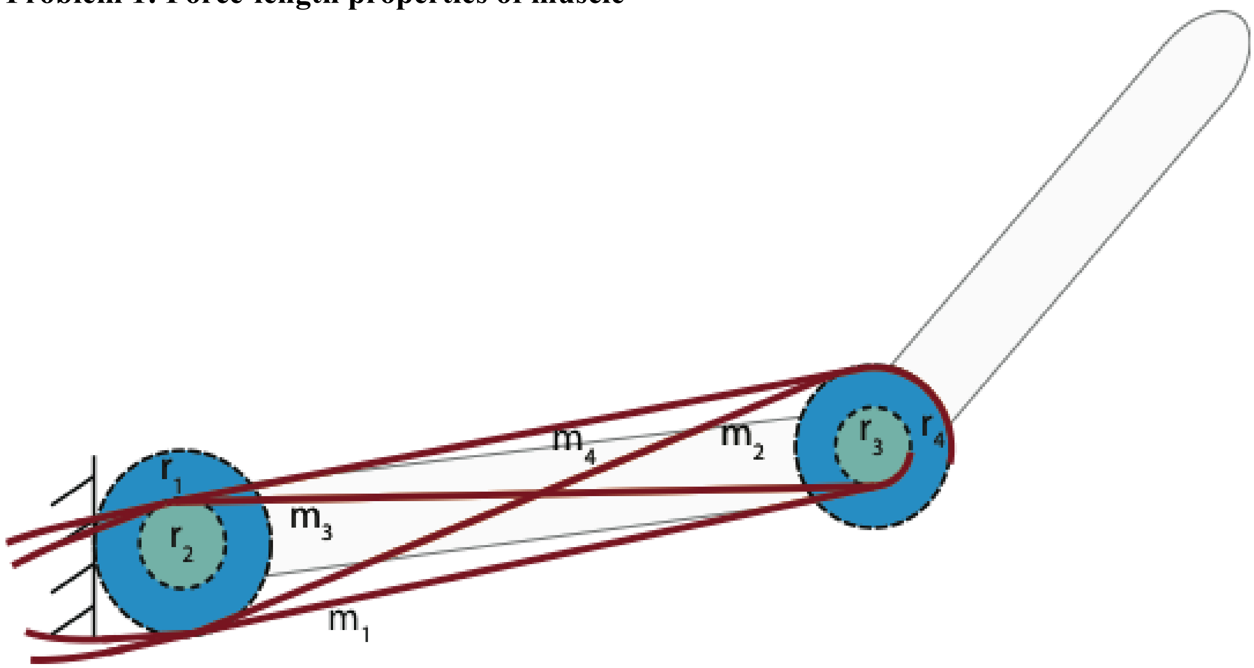
# Homework 5 Muscle Redundancy: Linear programming. Due before class on 10/21/2021

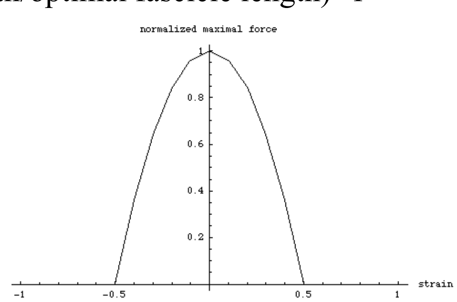
**Problem 1: Force-length properties of muscle**




**System description: A 2-link planar limb on a frictionless surface.**
l1 = 0.8; % length of first segment in meters
l2 = 0.5; % length of second segment in meters
Assume the following constant values for moment arms (i.e., the pulleys are full circles, but shown as semicircles for clarity):
r1 = 10 cm; r2 = 7 cm; r3 = 8 cm; r4 = 12 cm (note: The muscle has the same moment arm for the entire range of motion).
Range of motion: $0 \le q1 \le 90$ deg; $0 \le q2 \le 180$ deg

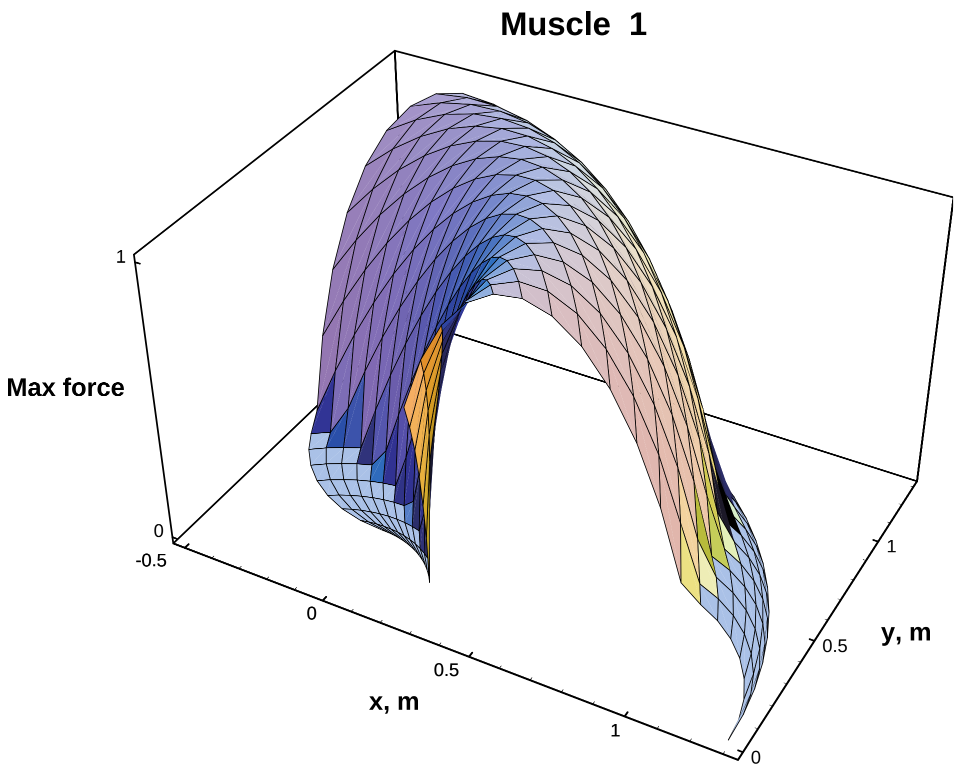
The slack length of the tendon is that which is necessary to span the distance between musculotendon origin and insertion when q1 = 45 deg; q2 = 90 deg. Assume all muscles are at optimal fiber length in this configuration. Assume musculotendons remain isometric during force production. Assume tendons are inextensible; use the following simple normalized force-length curve:

x= normalized fascicle length –1 = (muscle length/optimal fascicle length) -1
w = 0.5; shape parameter
If x <= -.5 && x >= .5, Fmax = 0
Else, Fmax = 1 - (x/w)^2

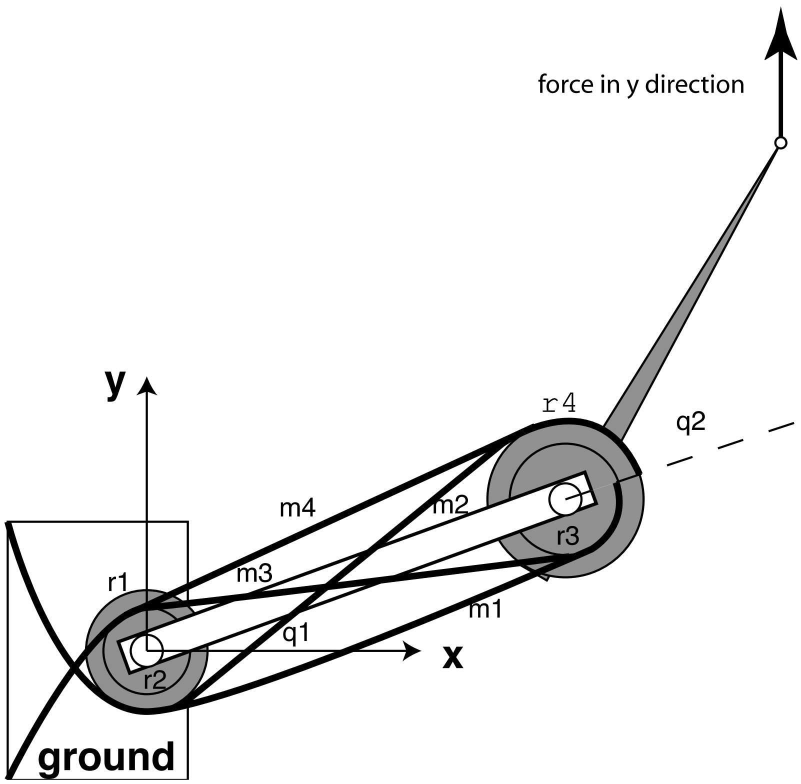


**Part 1.1: Describe the necessary steps, and write down the analytical expressions, you need to calculate the normalized maximal isometric static force of each muscle throughout the workspace of the limb.** The workspace of a limb is the range of X, Y locations the endpoint can reach. For a single joint, it is a section of a circle, for a two-joint system it will look like an incomplete crescent shape subset of a circle (see X, Y footprint of Figure below).

**Part 1.2: Write the necessary code to plot the normalized maximal isometric static force of each muscle as shown in the example below for muscle 1.**



**Problem 2: Taping the ceiling**




**System description: Same as in problem 1.**

**Muscle architecture:**

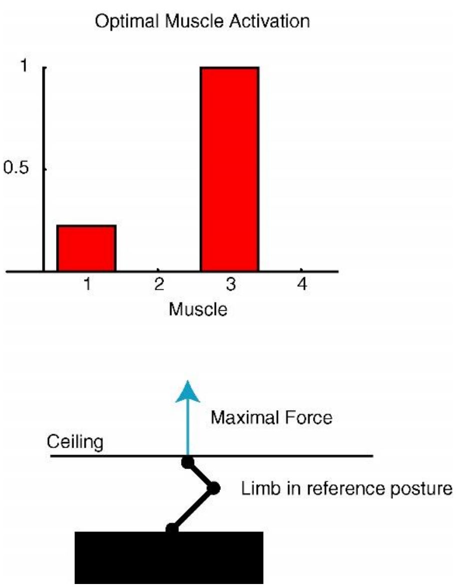
| muscle | PCSA cm^2 | lo cm |
|---|---|---|
| 1 | 10 | 20 |
| 2 | 20 | 10 |
| 3 | 15 | 20 |
| 4 | 25 | 15 |

Pennation angle = 0 for all muscles. sigmamax = 35 N/cm^2. As in the prior problem, the slack length of the tendon is that which is necessary to span the distance between musculotendon origin and insertion when q1 = 45 deg; q2 = 90 deg. Assume all muscles are at optimal fiber length in this configuration. Assume musculotendons remain isometric during force production. Assume tendons are inextensible; use the force-length curve equation used in the previous problem.

**Part 2.1:** Describe the necessary steps, and write down the analytical expressions, you need to calculate the optimal coordination pattern (distribution of muscle activations, which for each muscle can lie between 0 and 1) to maximize force along the positive y-axis (as shown in the figure) at a given limb posture. Hints: State the problem as a linear programming problem. The key to the problem is to find the necessary constraints and the cost function. Note that the mapping from activation levels to output forces is done by the matrices $J^{-T}RFo$. You do not need to solve for the constraints analytically, but you have to show them in matrix/vector form (i.e., specify the A, b and c elements of the LP problem).

**Part 2.2:** Write the necessary code to predict the optimal coordination pattern for 10 limb postures (described below) where the endpoint of the limb lies along the horizontal path y = .8 (as if the limb were an arm applying normal force to the ceiling to apply pressure sensitive tape to it) in a quasi-static way. Assume the limb is static at each of the 10 postures, and that the ceiling is resisting the maximal y-force. Plot limb configuration, muscle activation and force magnitude as a function of posture for all postures.

For example, this is the optimal solution for the reference posture q1 = 45°, q2 = 90°. The maximal force is 136.45 N:




The q1, q2 values, in radians for each of the ten postures are:

[1.1908 1.83641;
1.0621 1.87549;
0.813389 1.87549;
0.700844 1.83641;
0.601398 1.77215;
0.518223 1.68353;
0.453598 1.5708;
0.409172 1.43286;
0.386661 1.2661;
0.389248 1.06157]