April 4, 2016 Lab Section: Monday 11-2pm

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ME C134 Lab 5: Magnetic Levitation

1 Purpose

The goal of this lab is to design a controller for the magnetic levitation system in Lab and implement it with analog circuitry. To achieve this, the transfer function of the analog controller is derived from circuit diagrams and component values for the analog compensator are determined from experimentally determined linearized system gains. After building the controller circuit, resistor values are tuned to achieve the desired levitation response.

2 Pre-lab 5a

2.1 System Setup

The Pre-lab begins with the following high-level block diagram of the MagLev system.

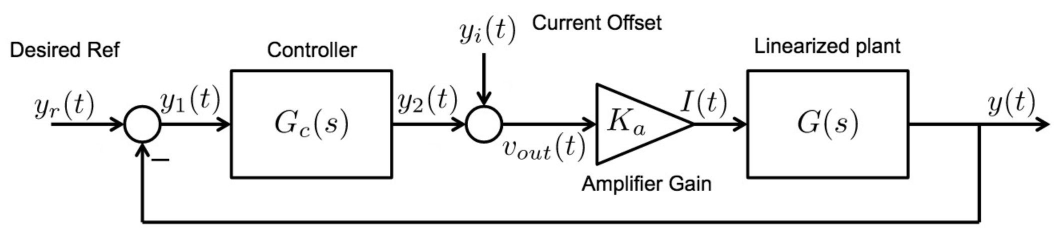

Figure 1: A block diagram of the MagLev system.

Figure 2 shows the circuitry comprising the block diagram in Figure 1. The values of resistors R, R1, R2 and capacitor C will be chosen to stabilize a levitating ball about an equilibrium point.

2.2 Desired Position/ Output Offset Circuitry

The equations of motion of the ball are

$$m\ddot{x} = f(I, x) - mg$$
$$y = h(x)$$

Letting $y_{ref} := Y_0$ in the magenta box in Figure 2, derive a relationship between the signal y_1, the voltage y from the photoresistor, and the reference voltage (tuned with a potentiometer).

$$V^+ = V^- = Y_0 = y_{ref}$$

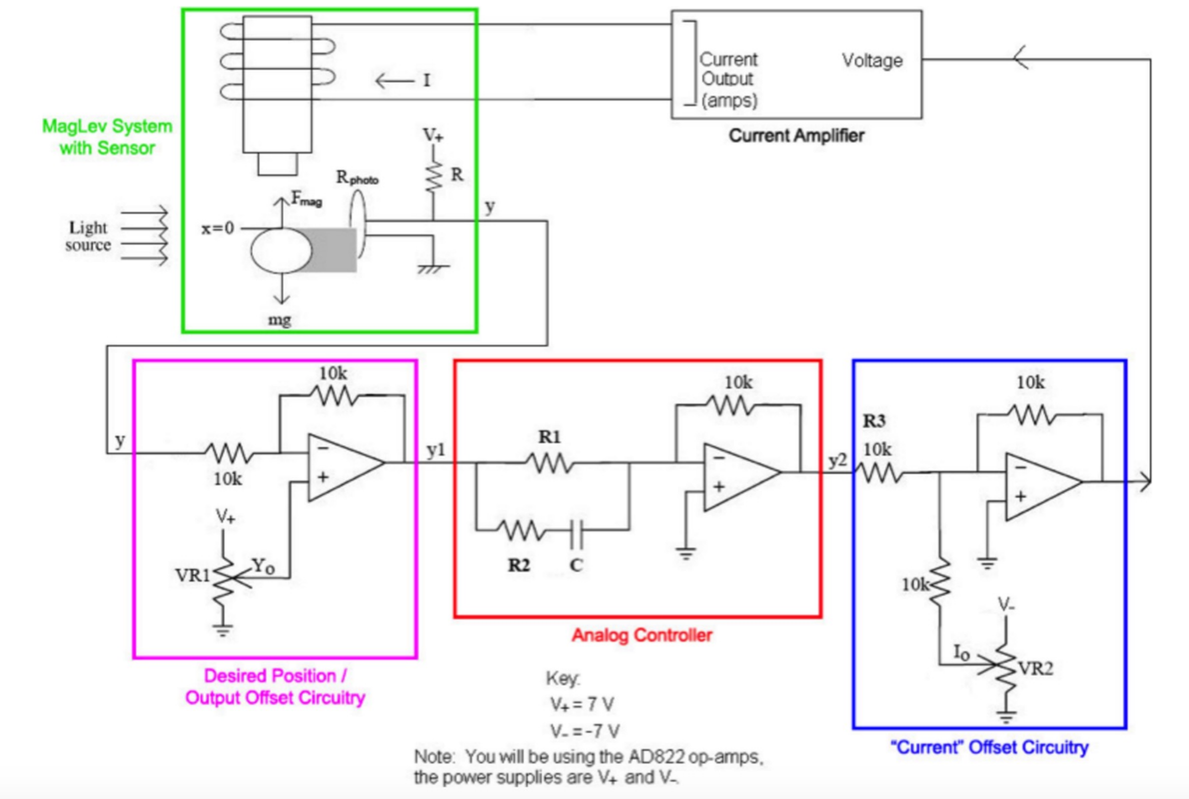

Figure 2: A diagram of the circuitry in the MagLev system.

$$i^+ = i^- = 0$$
$$y - I_1R_1 = V^- = y_{ref}$$
$$I_1 = \frac{y - y_{ref}}{R_1}$$
$$y_{ref} - I_1R_2 = y_1$$
$$y_{ref} - \frac{R_2}{R_1}y - y_{ref} = y_1$$
$$y_1 = 2y_{ref} - y$$

2.3 Derive Analog Controller Transfer Function

The transfer function of the analog controller is derived as follows.

Let I_1 be the current through resistor R_1, I_2 the current through R_2 and C, and I_3 the current through the 10k resistor from the op-amp output.

$$I_1 + I_2 = -I_3$$
$$y_1 - I_1R_1 = 0$$
$$I_1(s) = \frac{Y_1}{R_1}$$

$$y_1 = I_2 R_2 - \int \frac{I_2}{C} = 0$$
$$I_2(s) = \frac{Y_1 Cs}{R_2 Cs + 1}$$
$$y_2 - I_3 * 10k\Omega = 0$$

$$Y_2(s) = 10k\Omega(\frac{-Y_1}{R_1} - \frac{y_1 Cs}{R_2 Cs + 1}$$
$$Y_2(s) = 10k\Omega(\frac{-Y_1(s)(R_2 Cs + 1) - R_1 Cs Y_1(s)}{R_1(R_2 Cs + 1)})$$
$$\frac{Y_2(s)}{Y_1(s)} = \frac{-10^4\Omega}{R_1} * \frac{(R_1 + R_2)Cs + 1}{R_2 Cs + 1}$$

2.4 Current Offset Circuitry

In the blue box in Figure 2, let V_out be the output of the op-amp and y_i be the voltage drop across the potentiometer.

$$y_2 = I_1 * 10k\Omega = 0$$
$$I_1 = \frac{y_2}{10k\Omega}$$
$$V_{out} = -I_2 10k\Omega = 0$$
$$I_2 = \frac{V_{out}}{10k\Omega}$$
$$y_i + I_3 10k\Omega = 0$$
$$I_1 + I_2 = I + 3$$
$$y_1 = -\frac{y_2 + V_{out}}{10k\Omega} * 10k\Omega$$
$$V_{out} = -y_2 - y_i$$

2.5 Linearizing the System

Because the ball's equations of motion are non-linear, they can be linearized to obtain

$$m\ddot{x} = f(I, x) - mg \approx f(I_0, x_0) + K_i \delta I + K_x \delta x - mg$$

where $\delta I = I - I_0$ and $\delta x = x - x_0$. The output is also linearized such that the new equations become

$$m\ddot{x} = K_i \delta I + K_x \delta x$$
$$y = a\delta x$$

The linearized transfer function is derived as

$$m\delta x s^2 = K_i \delta I + K_x \delta x$$

$$y = a\delta x$$
$$Y(s) = a\delta x$$

$$\delta x = \frac{K_i \delta I}{ms^2} + \frac{K_x \delta x}{ms^2}$$
$$\frac{Y(s)}{a} = \frac{aK_i I(s)}{ms^2} + \frac{K_x Y(s)}{ams^2}$$
$$Y(s)(1 - \frac{K_x}{ms^2}) = \frac{aK_i I(s)}{ms^2}$$
$$G(s) = \frac{Y(s)}{I(s)} = \frac{aK_i}{m(s^2 - \frac{K_x}{m})}$$

3 Lab

3.1 System Identification

After setting an equilibrium height for the ball, the following range of resistance of the photo-resistor was measured at various displacements from equilibrium. Based on this data, a suitable value for resistor R is found.

Displacement (mm)	Resistance (ohms)
-2	940
-1.5	952
-1	1050
-.5	1214
0	1500
.5	2000
1	2700
1.5	3100
2	3290

No value of R will result in a power dissipation across the photoresistor greater than 250 mW, as the power dissipation is calculated as $P = \frac{V_r^2}{R_photo} = \frac{V_+^2 R_{photo}}{(R + R_{photo})^2}$ differentiated with respect to R_{photo} and set to 0. The only way this can yield a power dissipation of 250mW is if R is negative, which isn?t possible. Thus, there is no restriction on R in terms of power dissipation. It is important that R gives good resolution. We measured the range of R_{photo} as the position varied and plotted it. We set R to be the value of R_{photo} where the graph changed concavity. This made is such that the voltage divider would give symmetry to the voltage measurement and pseudolinearity near the equilibrium point.

To find a, seven voltage measurements were taken and linearized about the 5 points closest to the equilibrium point using the Matlab command polyfit, giving $y = a\delta x + b$ where $a = 0.810V/mm = 810V/m$ and $b = 3.123V$.

Displacement (mm)	Voltage (V)
-1.5	2.32
-1	2.47
-.5	2.73
0	3.1
.5	3.54
1	4.11
1.5	4.35

3.2 Finding K_i

K_i was found by increasing the voltage to the current amplifier until the ball was effectively weightless (the scale was retared twice near 0g to get accurate readings), and then slowly decreasing that voltage, recording the current/weight pairs as the voltage was decreased, recording more points near the equilibrium point. The following data was recorded:

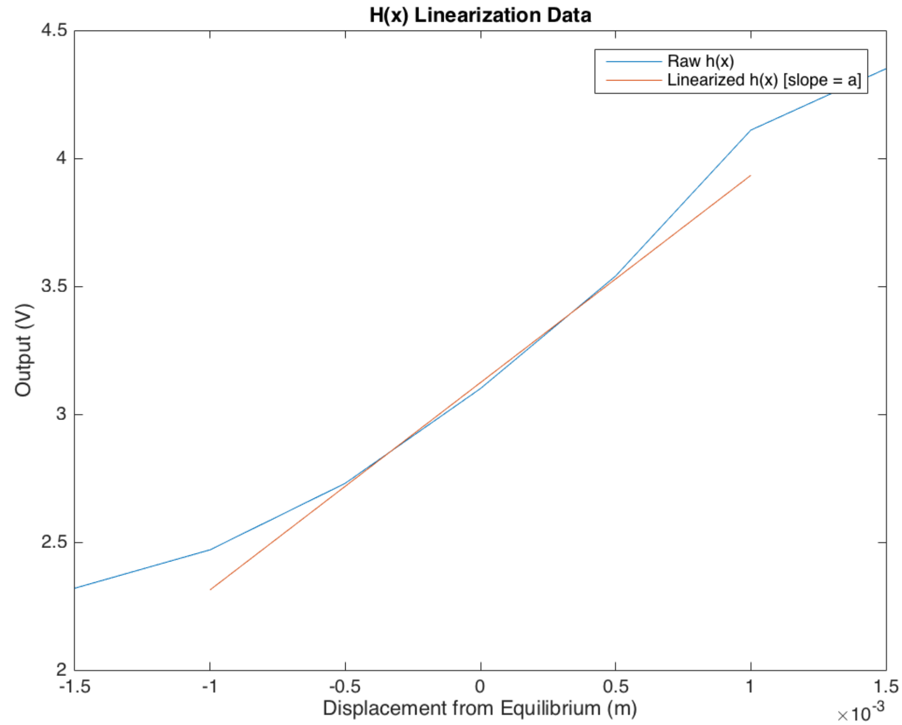

Figure 3: Raw and linearized h(x) data taken by measuring the voltage output y as a function of displacement from equilibrium. The three points closest to equilibrium were used to find the slope a of the linearization.

Current (amps)	Weight (grams)
2.2067	5.1
2.4067	3.7
2.54	2.8
2.62	2.2
2.8	1.1
2.8733	0.6
2.9267	0.2
2.94	0.1

The Matlab command polyfit was used to linearize the data (all data points were used since the data was nearly perfectly linear). This gave $F = K_i \delta I + c$ where $K_i = 0.441 N/A$ and $c = 0.196 N$.

3.3 Finding K_x

K_x was found by increasing the voltage to the current amplifier until the ball was effectively weightless (the scale was retared twice near 0g to get accurate readings), and then slowly decreasing the height of the ball, recording the height/weight pairs as the height was decreased. The following data was recorded:

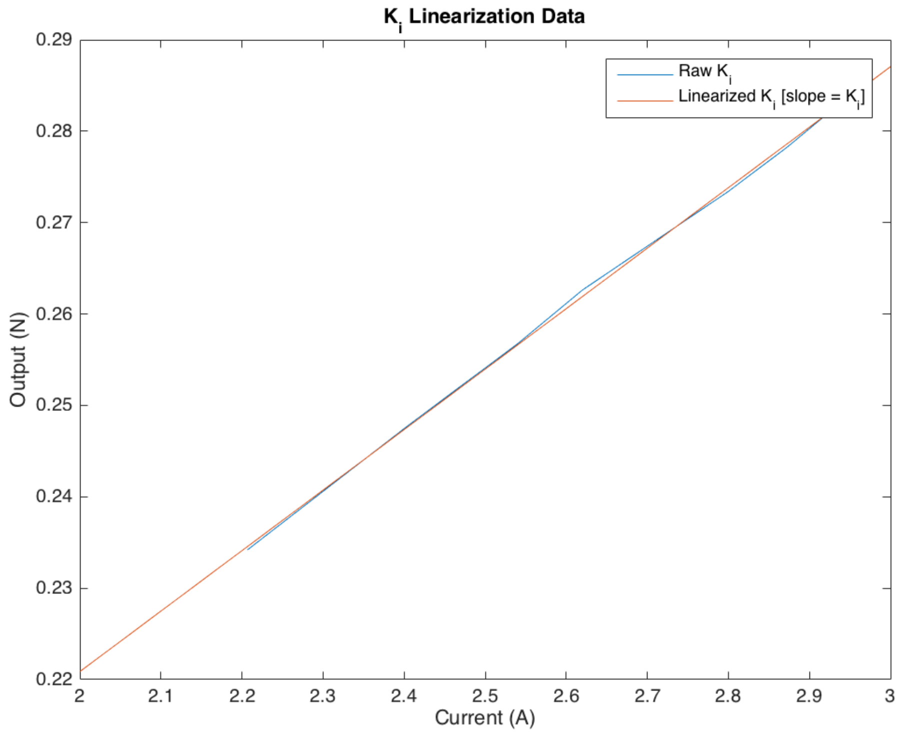

Figure 4: Raw and linearized K_i data taken by measuring Newtons of force felt by the steel ball as a function of current. All data points were used because the relationship was very linear.

Height from equilibrium (mm)	Weight (grams)
0	0.1
-0.5	2.2
-1	4.6
-1.5	6.1
-2	8

The Matlab command polyfit was used to linearize the data (all data points were used since the data was nearly perfectly linear). This gave $F = K_x \delta x + d$ where $K_x = -0.039N/md = 0.003N$. Also, $I_0 =?.441A$, the current recorded at the equilibrium position.

Given a desired DC gain of 1000 for the circuit and assuming the current amplifier $K_a = 2A/V$, the DC controller gain $K_c = \dfrac{DCGain}{aK_a} = 617$.

3.4 Prelab 5b

From the previous pre-lab and constants derived above, the system $G(s) = \dfrac{aK_i}{m(s^2 - \frac{K_x}{m})} = \dfrac{0.32}{0.029s^2 + 0.039}$. The root locus is plotted in the figure below.

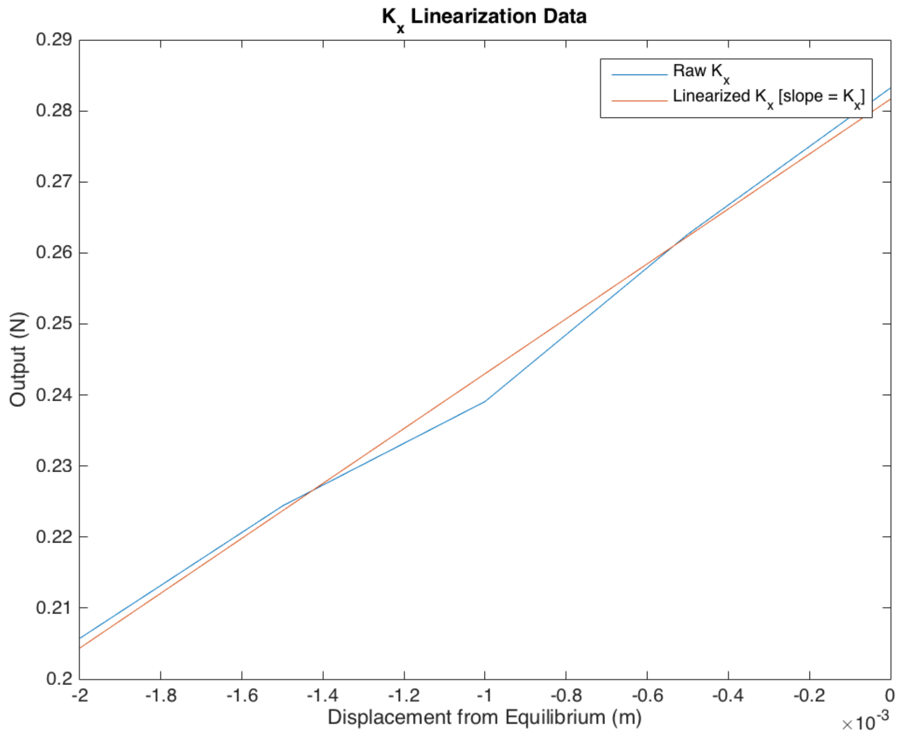

Figure 5: Raw and linearized K_x data taken by measuring Newtons of force felt by the steel ball as a function of displacement from equilibrium. All data points were used because the relationship was very linear.

The compensator $G_c(s)$ will be a lead compensator of the form $K_c\frac{\frac{s}{z_c}+1}{\frac{s}{p_c}+1}$. The combined open loop system must have a DC gain of approximately 2 and a $\frac{z_c}{p_c}$ ratio of approximately 1/20. The combined closed loop system has stable poles, overshoot of 0%, and a settling time less than 0.25 seconds. A lead compensator was chosen to increase the speed of the transient response and lower the settling time to an acceptable number less than 0.25 seconds.

To design the compensator, first the value of K_c was set using the given DC gain.

$$K_c = \frac{2K_x}{aK_iK_a} = 0.7227$$

To move the root locus left-ward (and thus speed the transient response) the compensator zero is placed at -37 with the compensator pole placed twenty times farther from the origin at -740. This location was determined by experimentation to be the farthest zero location that produced zero overshoot at the specified K_c value.

Listing 1: Matlab code to set up plant and compensator transfer functions and analyze combined system response.

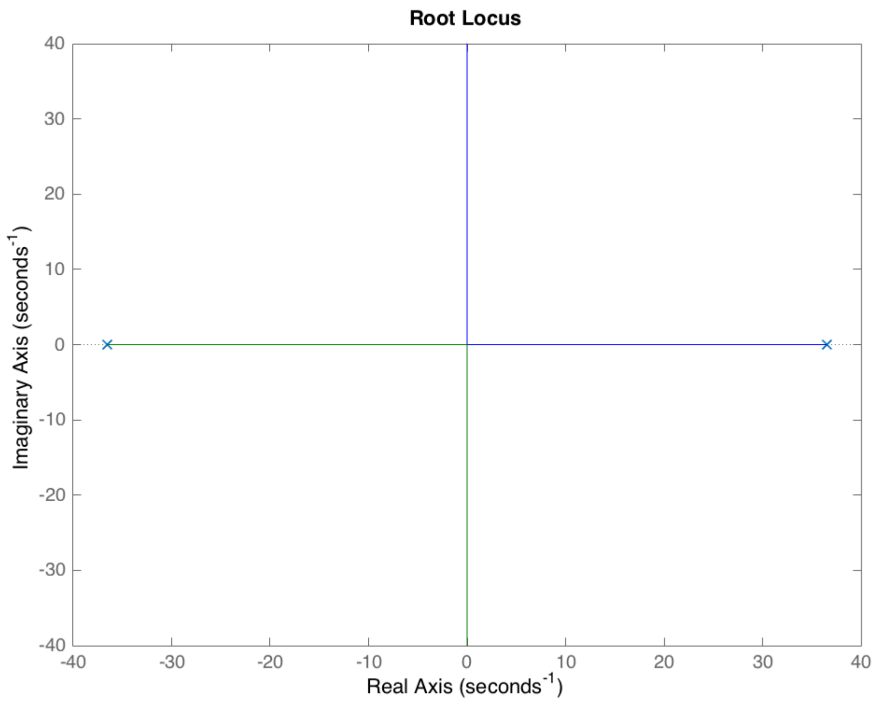

Figure 6: Root locus plotted for $G(s)$ defined above. Depending on the gain K, the uncompensated system is unstable (one root in RHP) or marginally stable (both roots on the imaginary axis).

```
% define constants
m=.029;
Ki=0.066;
Ka=2;
a=810;
Kx=38.638;
Kc=2/Ki/a/Ka*Kx;

G=tf([Ka*Ki*a],[m 0 -Kx]); % plant transfer function
zc=37; % set compensator zero location
pc=20*zc; % set compensator zero location
Gc=tf(Kc*[1/zc 1],[1/pc 1]); % compensator transfer function
sys=series(G,Gc);
T=feedback(sys,1);
step(T)
stepinfo(T) % step info to find settling time
figure(2)
rlocus(sys) % plot root locus
```

Using this controller design, the resistor and capacitor values can be determined from the relationship $G_c(s) = 617\frac{\frac{s}{37}+1}{\frac{s}{740}+1} = -\frac{10^4\Omega}{R_1} * \frac{(R_1+R_2)Cs+1}{R_2Cs+1}$. Ignoring the negative sign (as another portion of the analog circuit is inverting and cancels this out), the values

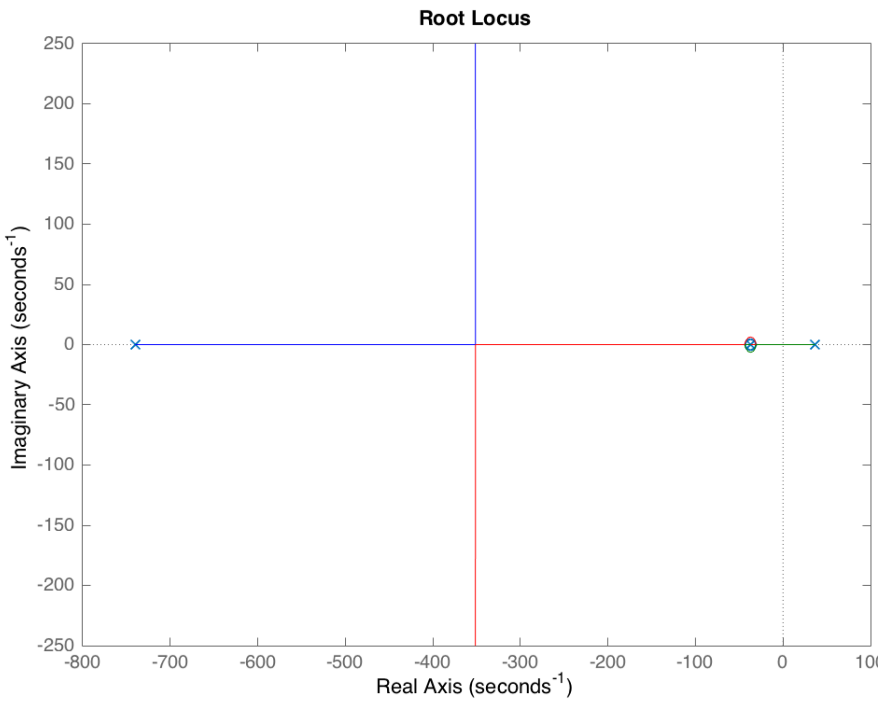

Figure 7: Root locus plotted for the combined system$G_c(s)G(s)$ defined above.

are easily computed giving $C = 1.85 * 10^{-6}F$, $R_1 = 13.8k\Omega$, and $R_2 = 0.728k\Omega$.

3.5 Lab 5b

To get the ball to levitate, the procedure outlined in section 4.2 was first used to tune the potentiometer resistance values. After completing this procedure, the ball would not levitate when placed under the magnet. When the ball was held in the equilibrium location by hand, its effective weight was noticeably reduced; however, when the ball was lowered by hand its effective weight increased quickly over several millimeters until it was too far to experience the force of the electromagnet. If the ball was released at any time it would fall as the force was not enough to levitate it. The tuning procedure was repeated but didn't significantly alter the system's behavior. To increase the force output of the electromagnet, resistance R_2 of the potentiometer was adjusted until the ball remained levitating unsupported. This worked, but required very fine adjustments of the 3/4 turn potentiometers–slight over or under-adjustments would result in the ball falling or sticking to the magnet.

Decreasing R2 causes settling time to decrease but overshoot to increase, while increasing R2 causes settling time to decrease.

After the steel ball levitated successfully, the levitation of other objects was attempted. The gear removal tool from the previous cart labs and an adjustment knob from a desk chair were successfully levitated. To do so, the resistance of R2 was adjusted to increase

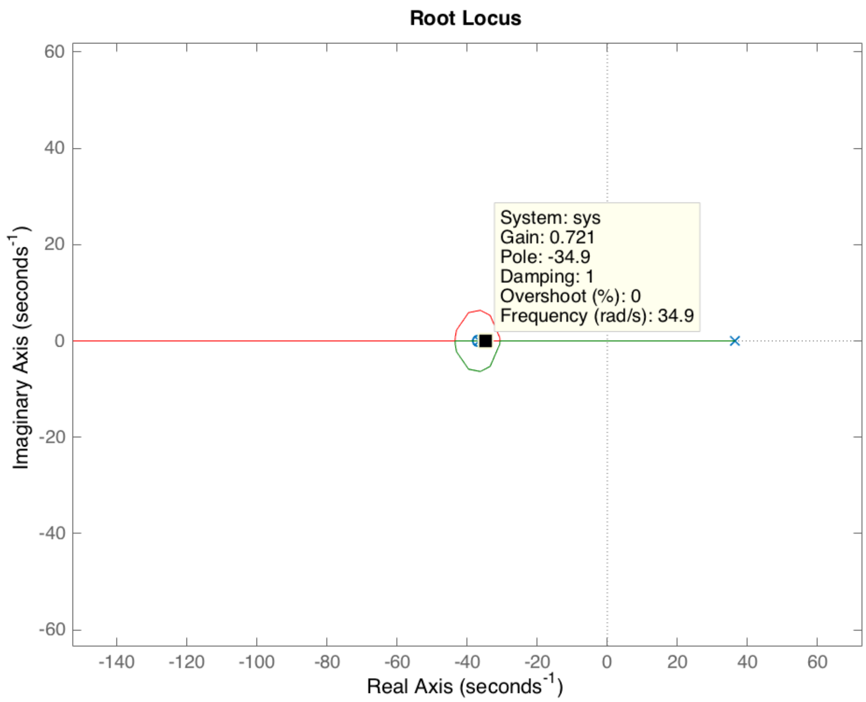

Figure 8: Zoomed in view of combined system root locus. With the compensator zero at -37 and $K_c = 0.7227$, overshoot is 0 % and settling time is computed as 0.0932 seconds with stepinfo.

or decrease the force exerted on the object below. The proper adjustment was determined experimentally–iteratively adjusting R2 and releasing the object underneath the magnet until it remained levitating.

A youtube video with the results can found at http://youtu.be/VHlP4Z12aBs.