

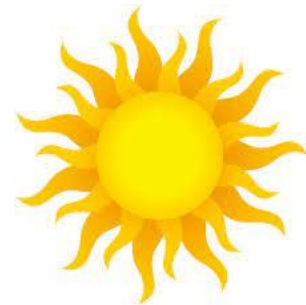


Section 3: Target Systems and Phenomena

Formalizing Theories with Difference Equations

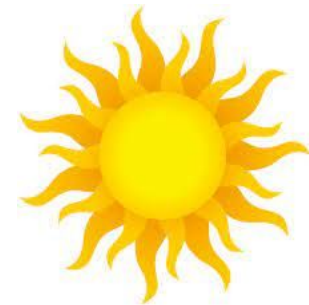
Theories **explain** phenomena

Phenomenon: My coffee cools faster in the winter than it does in the summer



Theories **explain** phenomena

Verbal theory: My coffee's temperature will change proportional to the difference between its own temperature and the ambient temperature



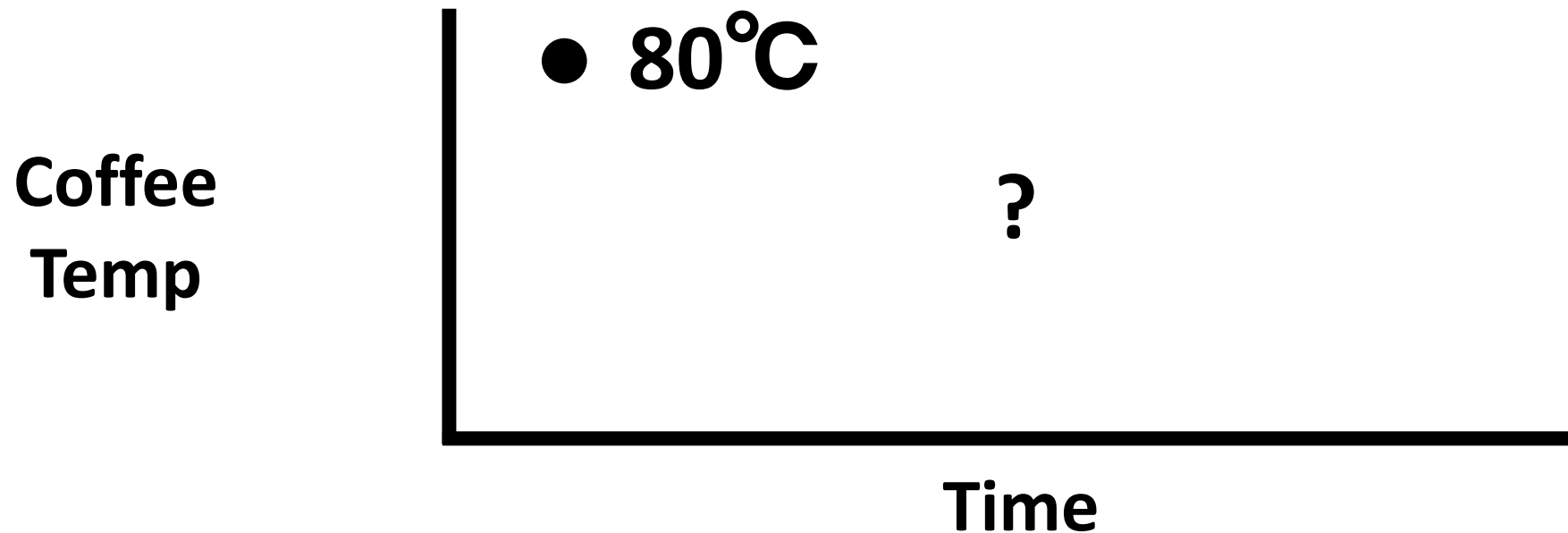
Theories **explain** and **predict**

Verbal theory: My coffee's temperature will change proportional to the difference between its own temperature and the ambient temperature

What does the theory predict?

Verbal theory: My coffee's temperature will change proportional to the difference between its own temperature and the ambient temperature

What does the theory predict?



Theories **explain** and **predict**

Formal theory: $T_{t+1} = T_t + r(T_t - T_A)$

Difference equations tell us where a variable will go next, based on where it is now

Allows us to simulate the behavior of the variable as it evolves over time given a set of initial conditions

Theories **explain** and **predict**

Formal theory: $T_{t+1} = T_t + r(T_t - T_A)$

T = Coffee Temperature

t = Discrete Time Step

T_A = Ambient Temperature

r = Constant = -.20

Theories **explain** and **predict**

Formal theory: $T_{t+1} = T_t + r(T_t - T_A)$

$$T_{t+1} = T_t \pm .20(T_t - 40)$$

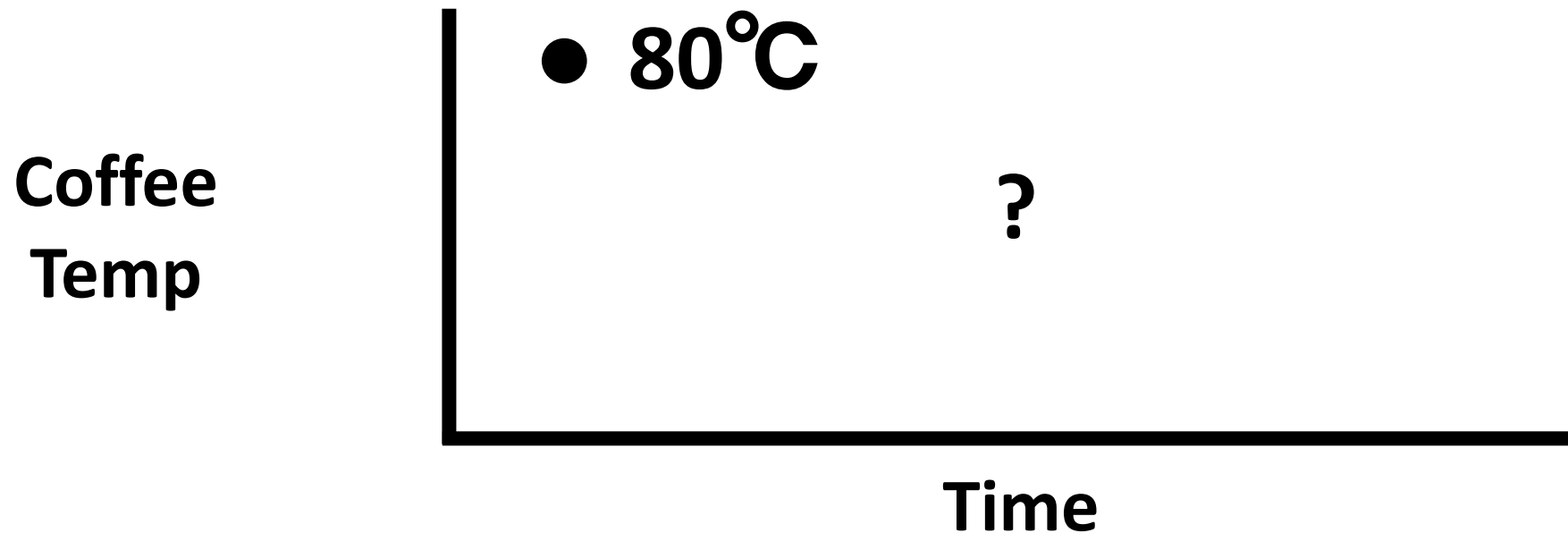
$$T_0 = 80$$

What does the theory predict?

What does the theory predict?

Formal theory: $T_{t+1} = T_t + r(T_t - T_A)$

$$T_{t+1} = T_t \pm .20(T_t - 40)$$



Formal theory: $T_{t+1} = T_t + r(T_t - T_A)$

$$T_0 = 80.0$$

$$T_1 = 80.0 - .20(80.0 - 40) = 72.0$$

t	
0	80.0
1	72.0
2	
3	

Formal theory: $T_{t+1} = T_t + r(T_t - T_A)$

$$T_0 = 80.0$$

t	
0	80.0
1	72.0
2	65.6
3	

$$T_1 = 80.0 - .20(80.0 - 40) = 72.0$$

$$T_2 = 72.0 - .20(72.0 - 40) = 65.6$$

Formal theory: $T_{t+1} = T_t + r(T_t - T_A)$

$$T_0 = 80.0$$

t	
0	80.0
1	72.0
2	65.6
3	60.5

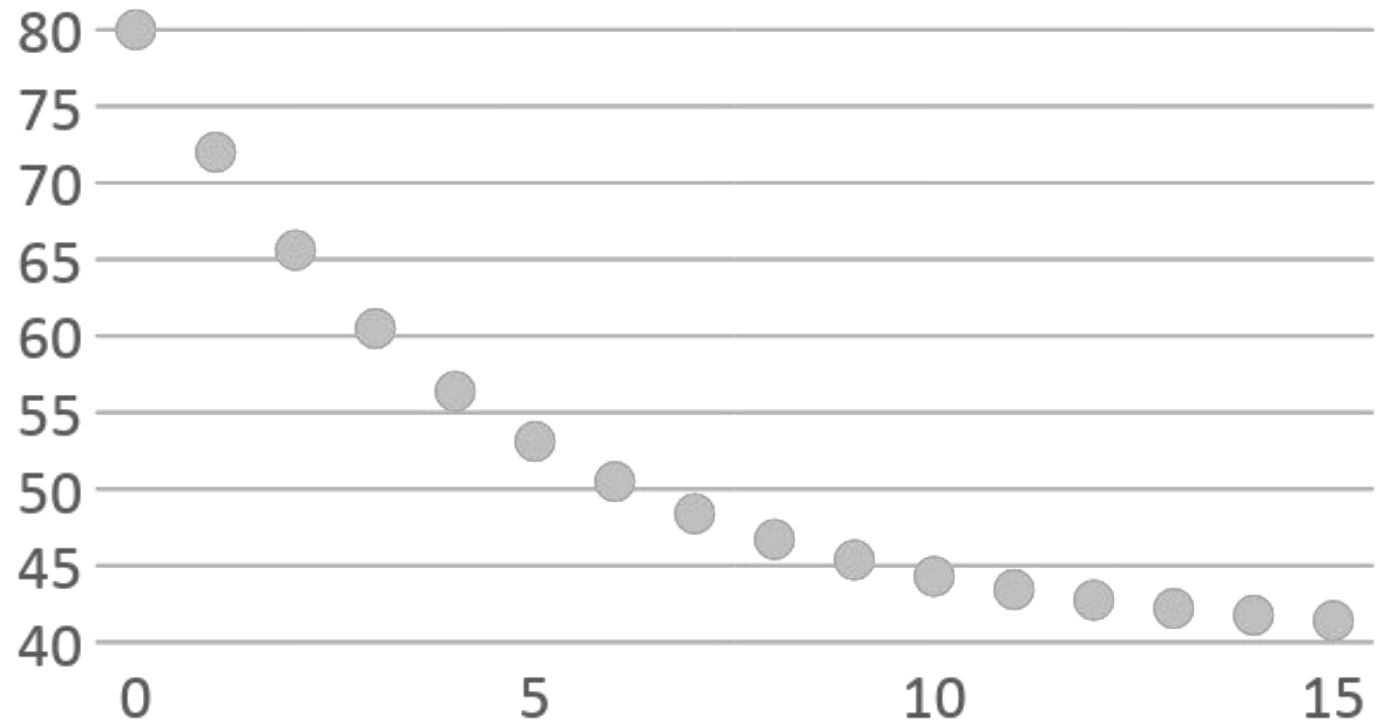
$$T_1 = 80.0 - .20(80.0 - 40) = 72.0$$

$$T_2 = 72.0 - .20(72.0 - 40) = 65.6$$

$$T_3 = 65.6 - .20(65.6 - 40) = 60.5$$

Formal theory: $T_{t+1} = T_t + r(T_t - T_A)$

t	
0	80.0
1	72.0
2	65.6
3	60.5



Theories **explain** and **predict**

Formal theory: $T_{t+1} = T_t + r(T_t - T_A)$

Formal theories allows us to **deduce** precisely what a theory predicts

Accurate **deduction** is a prerequisite for **explanation**

Formal theory: $T_{t+1} = T_t + r(T_t - T_A)$



t	
0	80.0
1	72.0
2	65.6
3	60.5



t	
0	80.0
1	64.0
2	51.2
3	41.1

Phenomenon: My coffee cools faster in the winter than it does in the summer

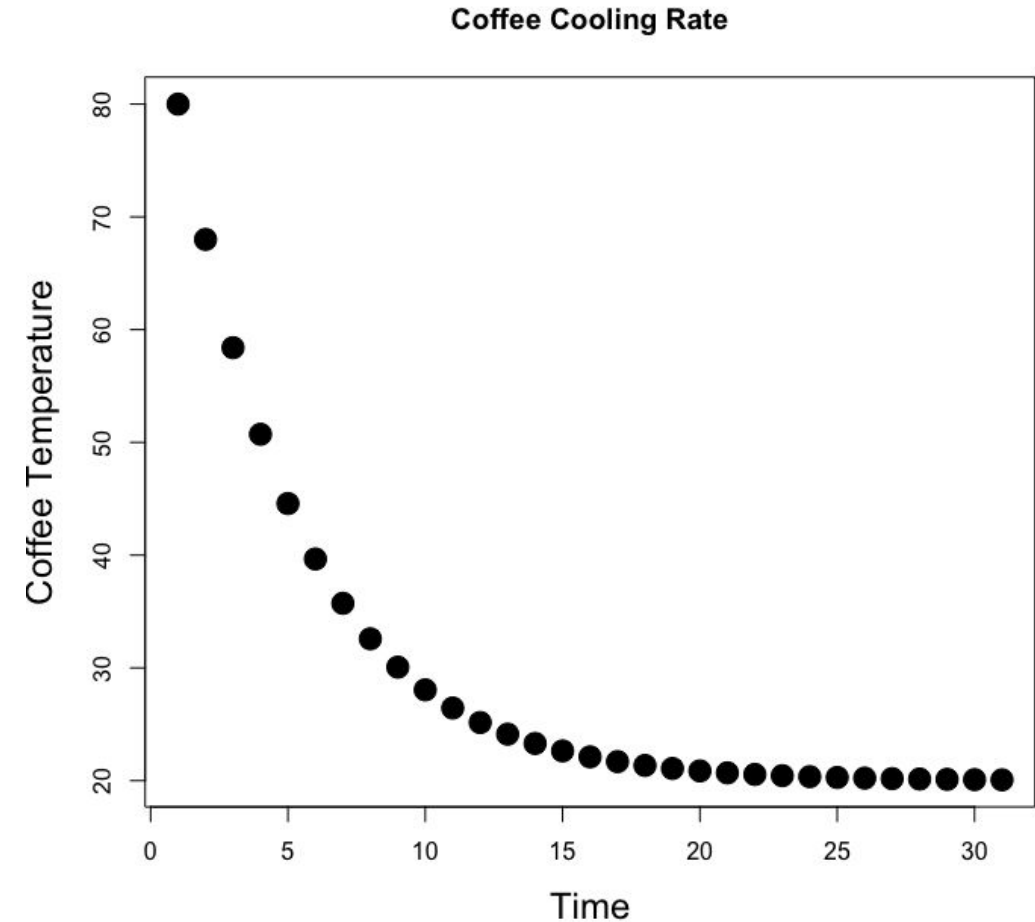
Formal theory: $T_{t+1} = T_t + r(T_t - T_A)$

Formal theory:

```
temp<-vector()  
temp[1]<-80  
time_steps<-30  
  
for (t in 1:time_steps){  
  temp[t+1]<-temp[t] -.2*(temp[t]-20) }
```

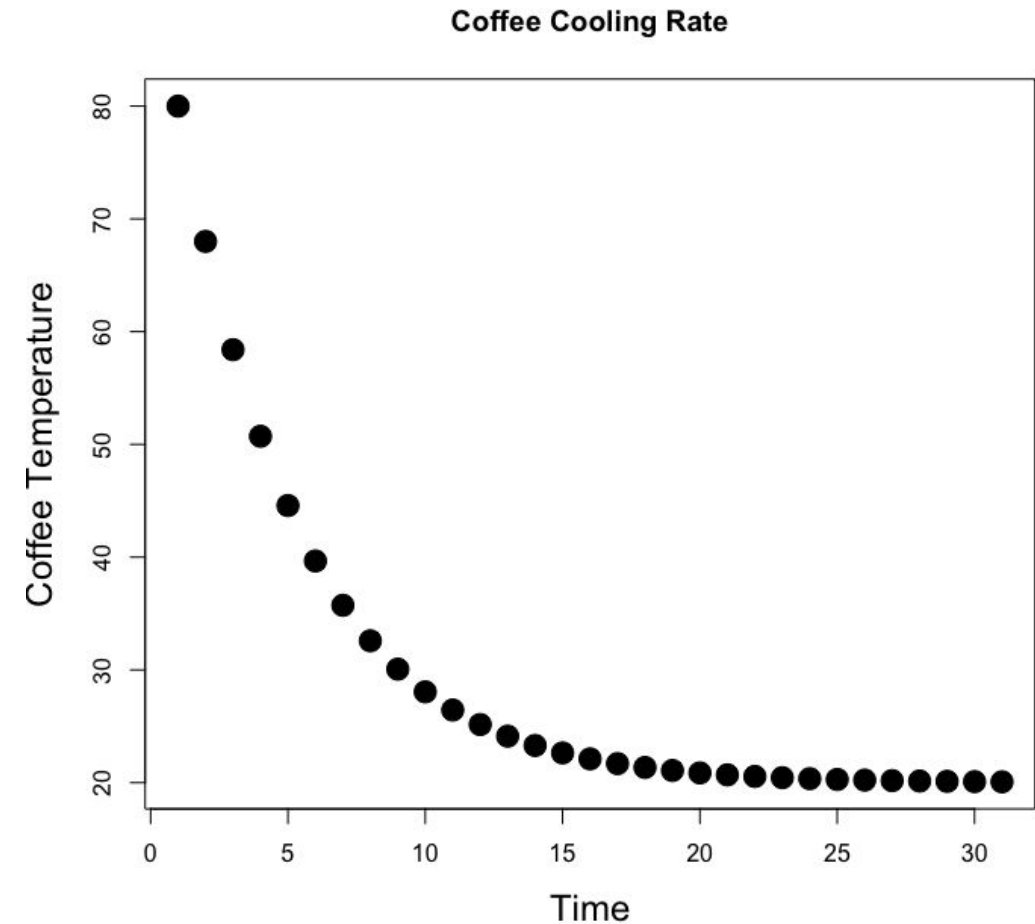

A Computational Model of Coffee Temperature!

```
temp<-vector()  
temp[1]<-80  
time_steps<-30  
  
for (t in 1:time_steps){  
  temp[t+1]<-temp[t] - .2*(temp[t] -20) }
```



A Computational Model of Coffee Temperature!

Problem: Coffee doesn't change in discrete time



Difference Equations

Discrete Time

$$T_{t+1} = T_t - .2(T_t - 20)$$

Differential Equations

Continuous Time

$$\frac{dT}{dt} = -.2(T - 20)$$

Differential Equations

Problem: No analytic
solution for many
differential equations

Continuous Time

$$\frac{dT}{dt} = -.2(T - 20)$$

Solution: Back to Difference Equations (Euler's Method)

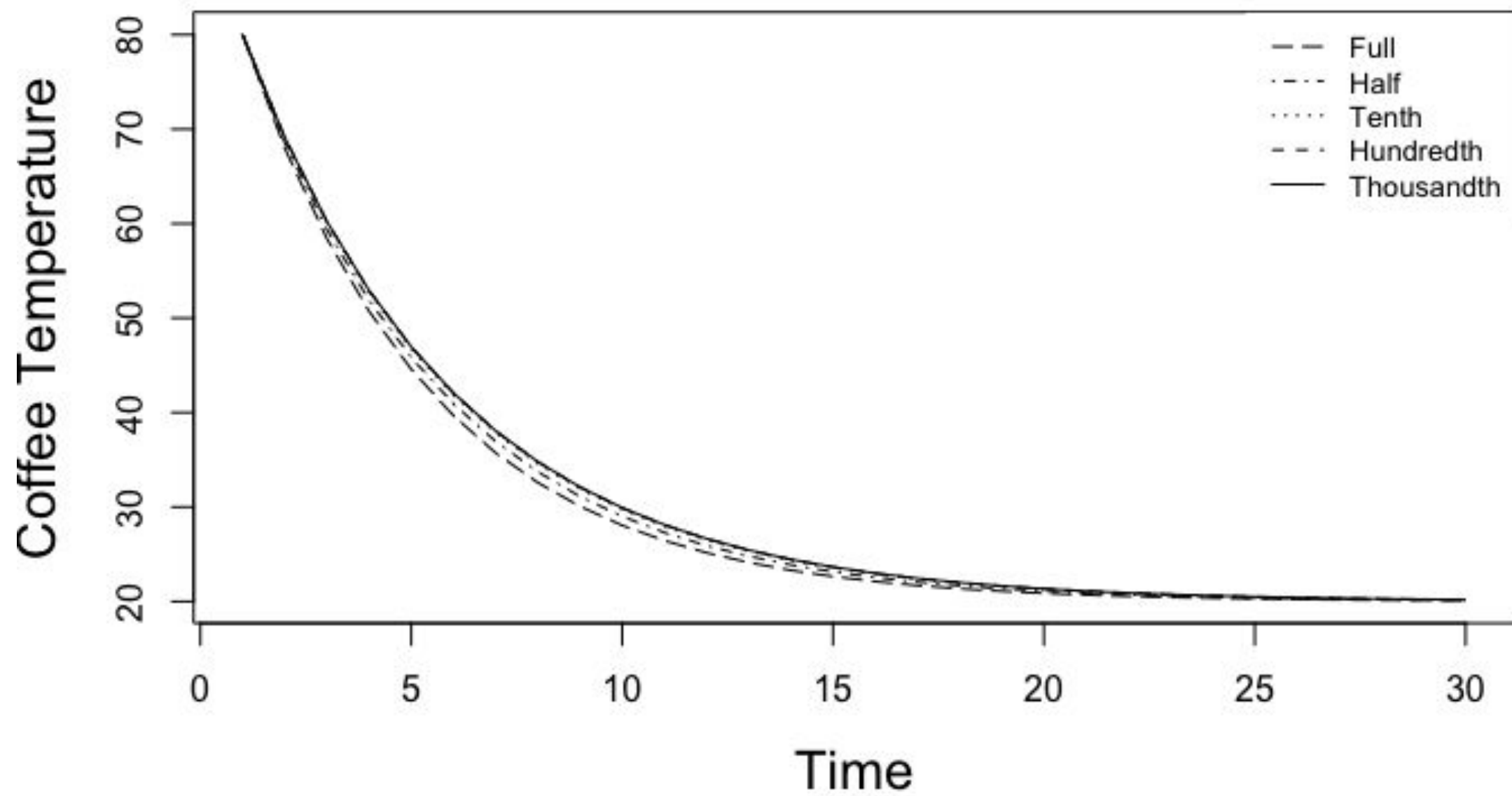
```
simTemp <- function(stepsize, subsample, temp_initial, temp_room)
{
  ...
  for (t in 1:nlter){
    temp[t+1]<-temp[t]-.2*(temp[t]-temp_room)*stepsize
  }
  temp <- temp[round(seq(1, nlter, by=subsample))]
  return(temp)
}
```

Euler's Method

```
out_full<-simTemp(time_steps=30,  
                  stepsize=1,  
                  subsample=1/1,  
                  temp_initial=80,  
                  temp_room=20)
```

```
out_half<-simTemp(time_steps=30,  
                  stepsize=.5,  
                  subsample=1/.5,  
                  temp_initial=80,  
                  temp_room=20)
```

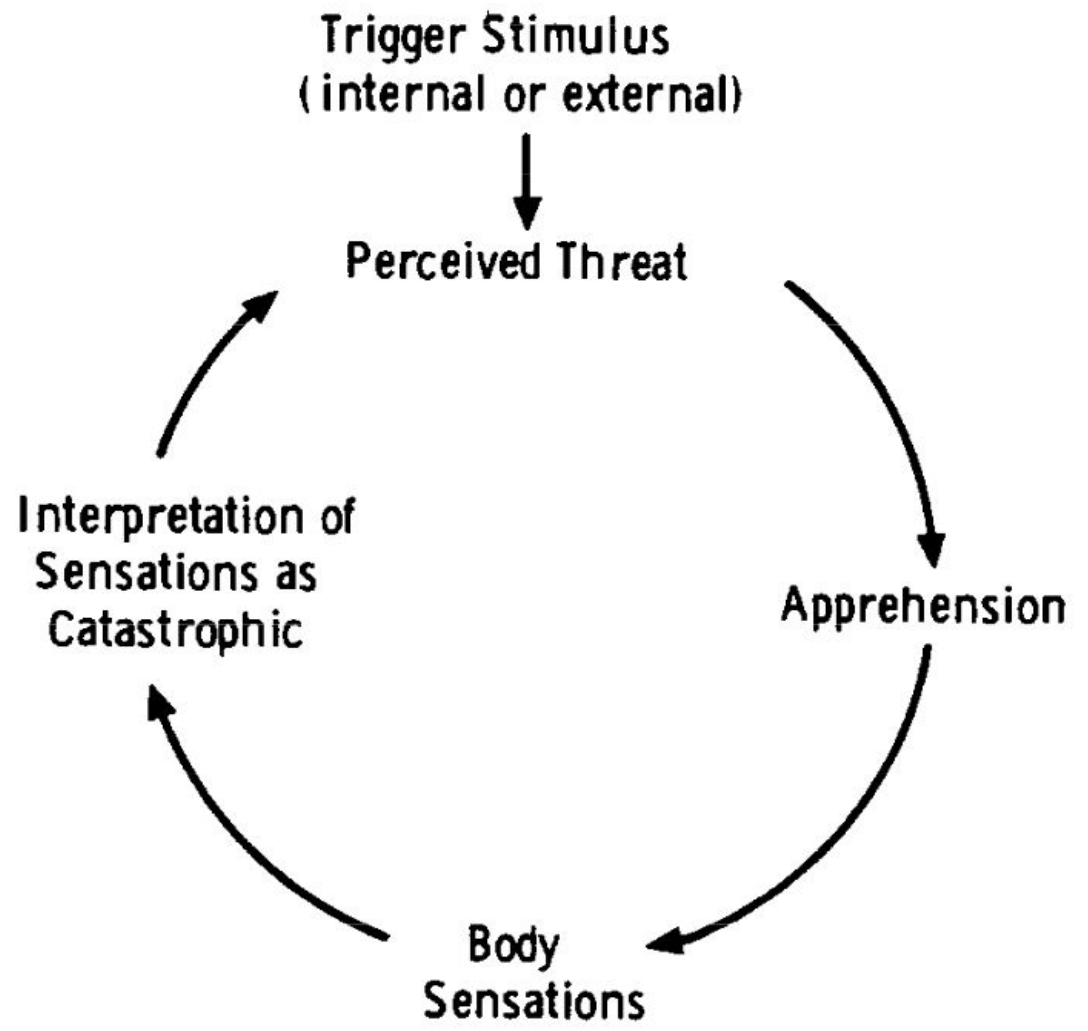
...

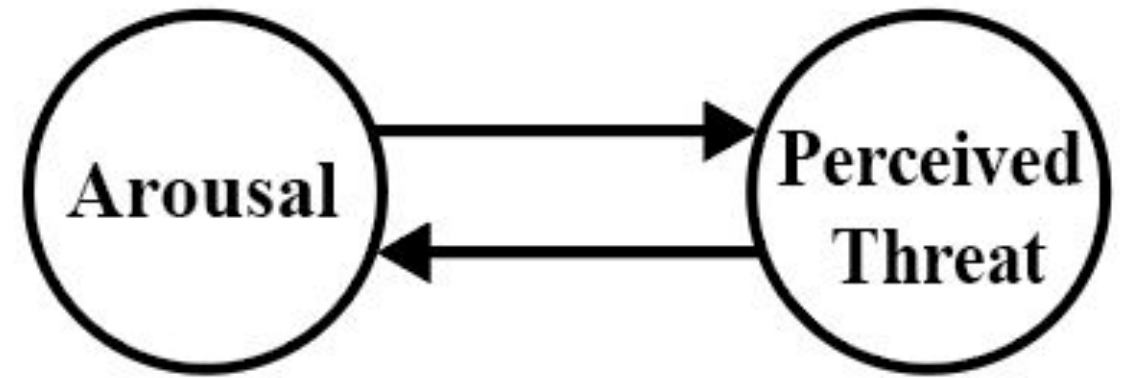


Modeling Panic Attacks with Difference Equations

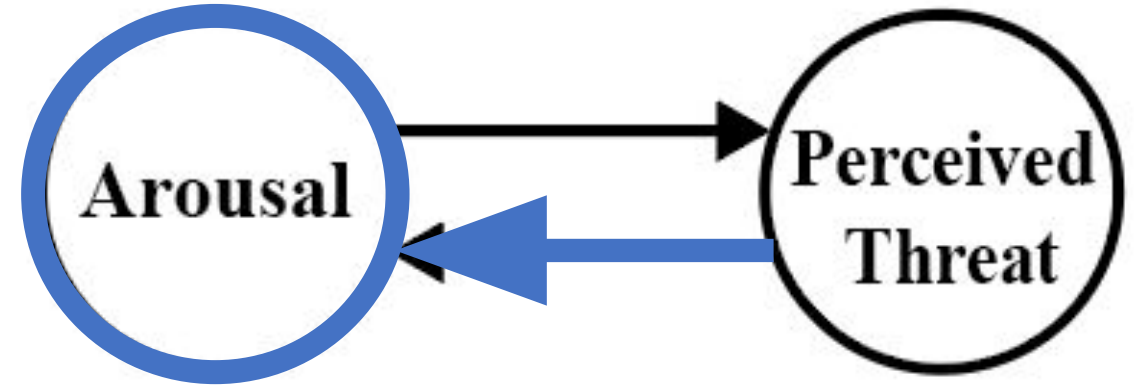
Phenomenon: Panic attacks and Panic Disorder

A verbal theory: If a stimulus “is perceived as a threat, a state of mild apprehension results. This state is accompanied by a wide range of bodily sensations. If these anxiety-produced sensations are interpreted in a catastrophic fashion, a further increase in apprehension occurs. This produces a further increase in body sensations and so on around in a vicious circle which culminates in a panic attack.”

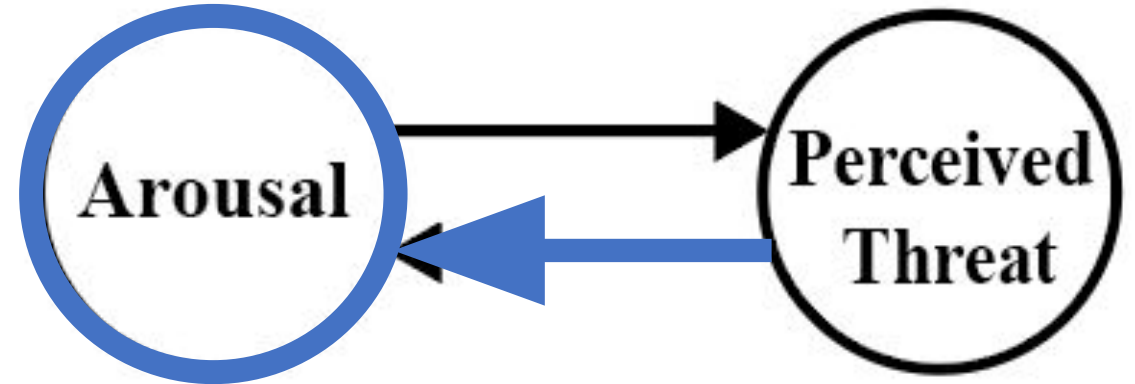
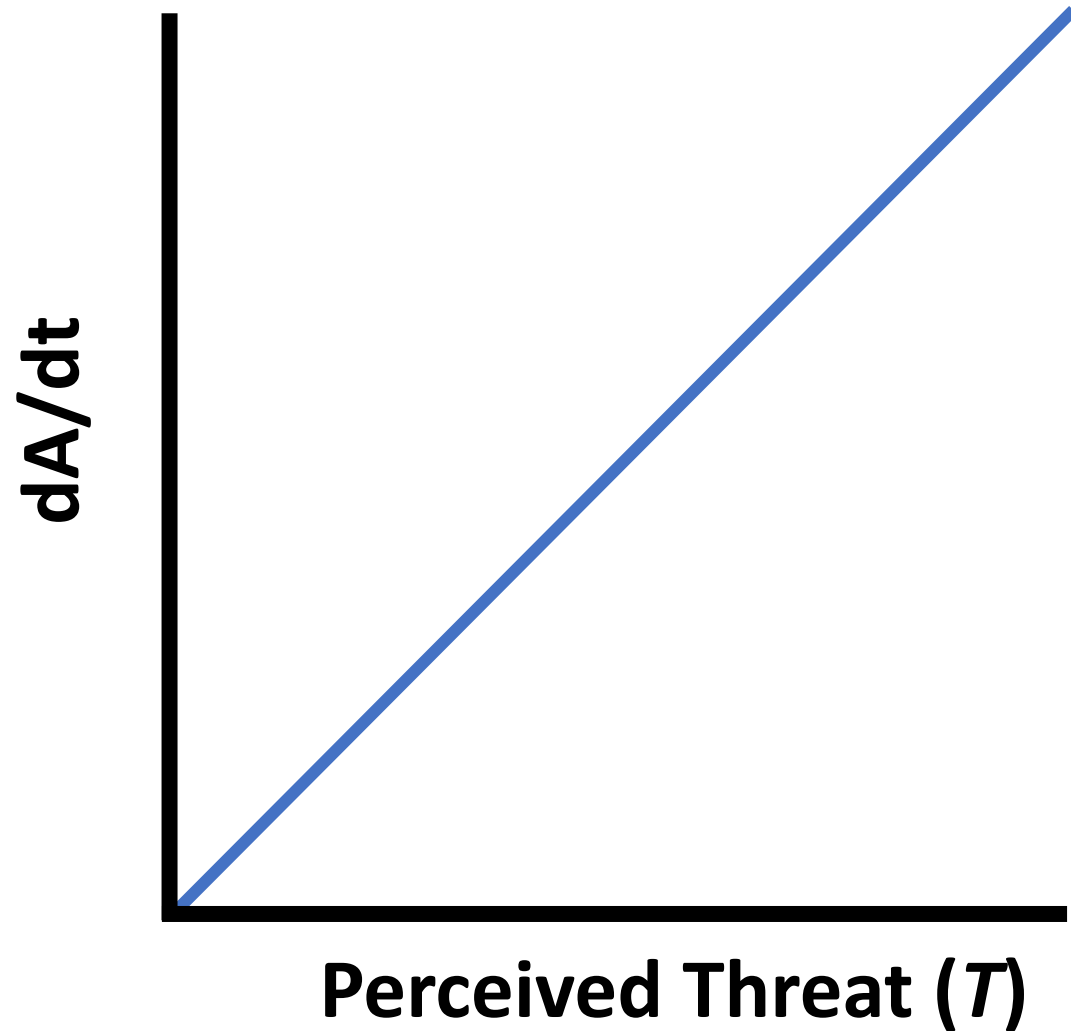




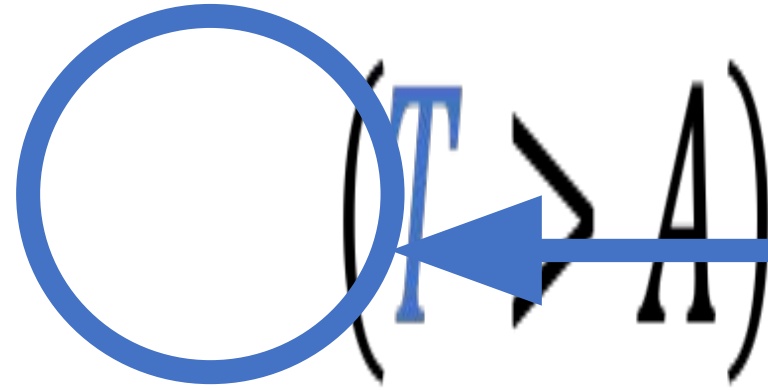
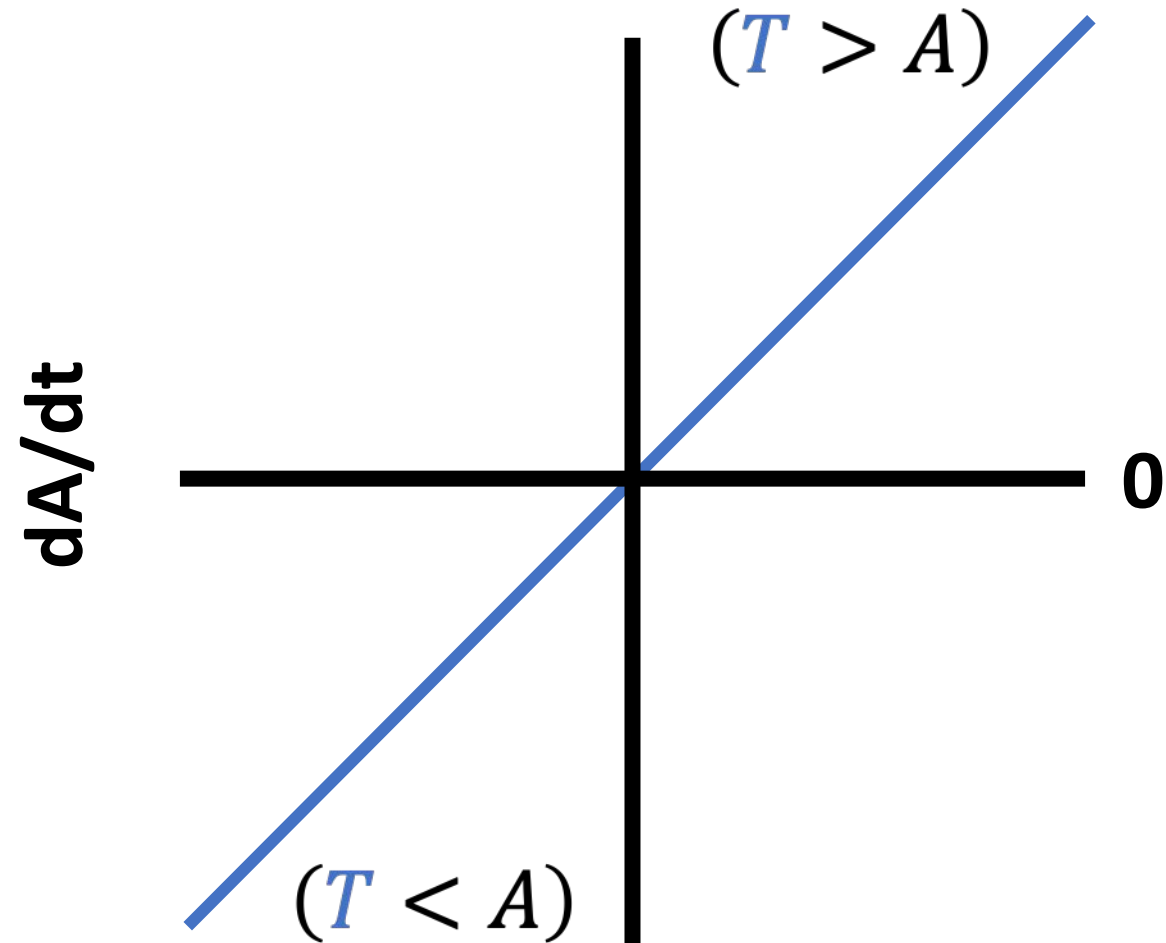
Formal theory: $A_{t+1} = A_t + T_t$



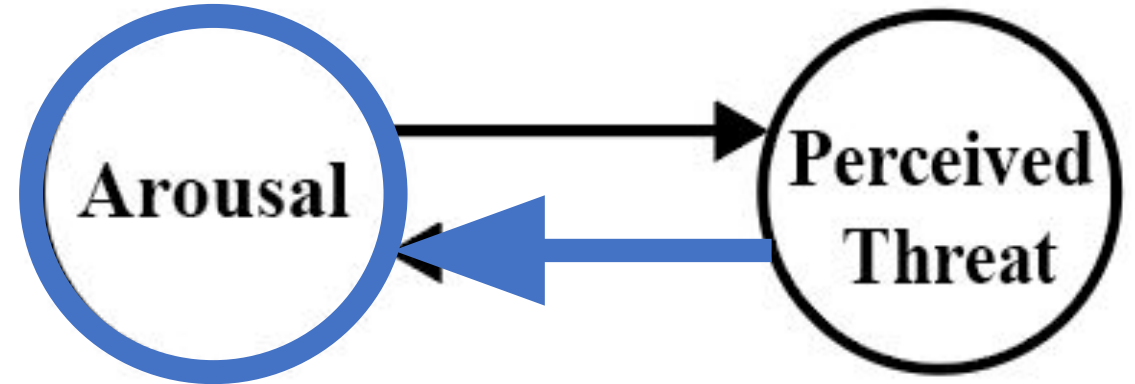
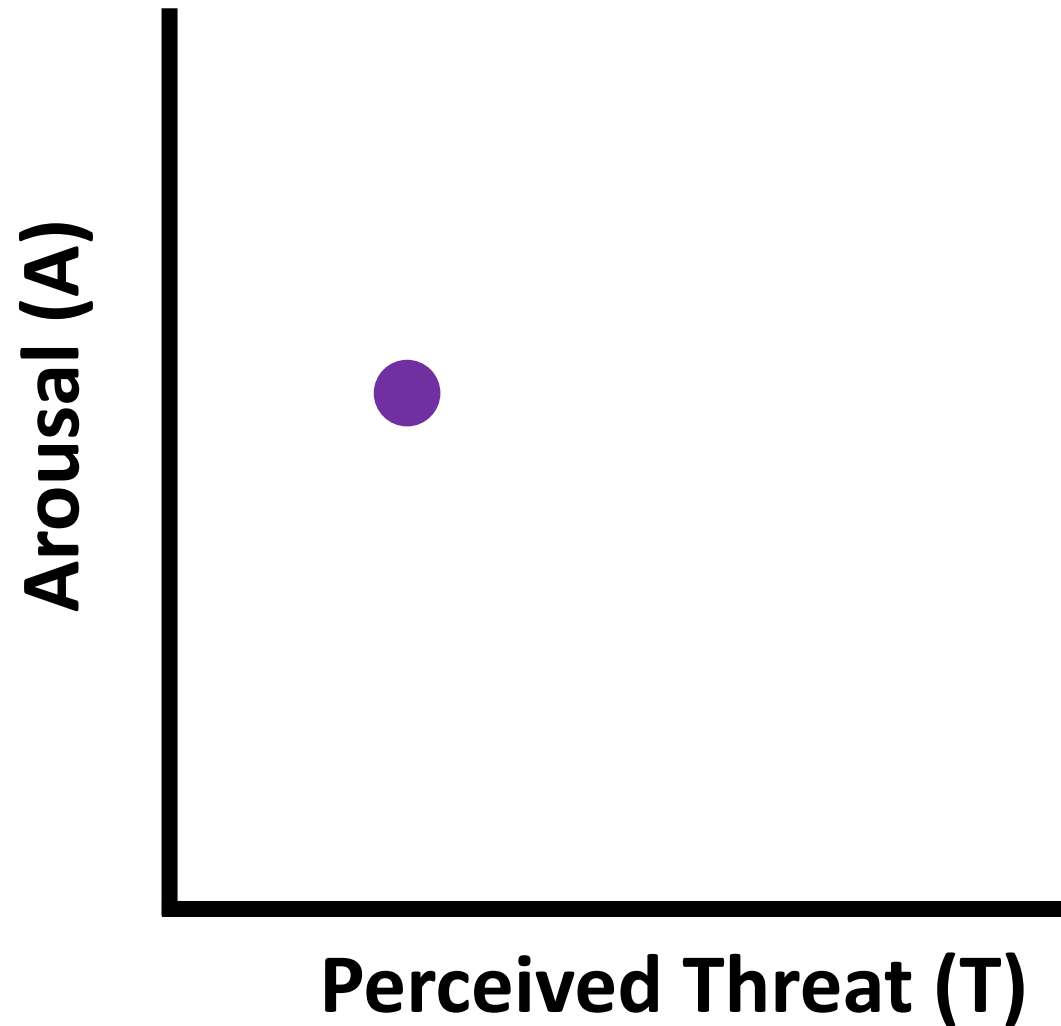
Formal theory: $\frac{dA}{dt} = (T)$



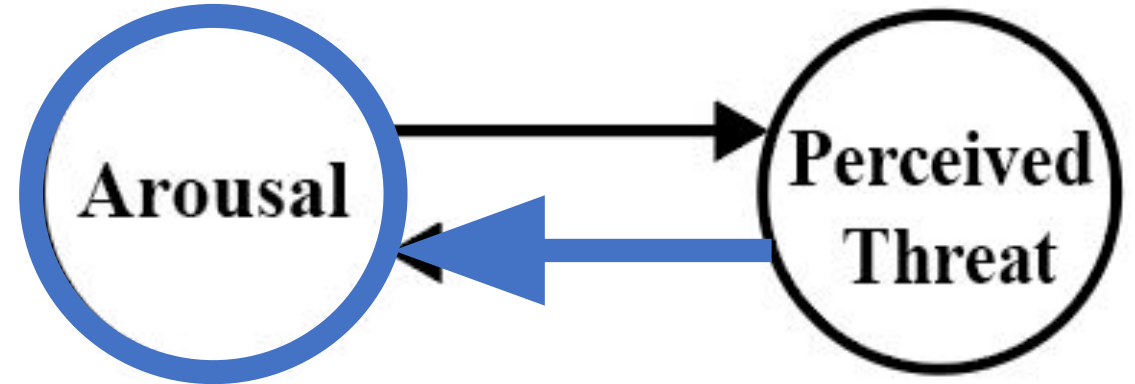
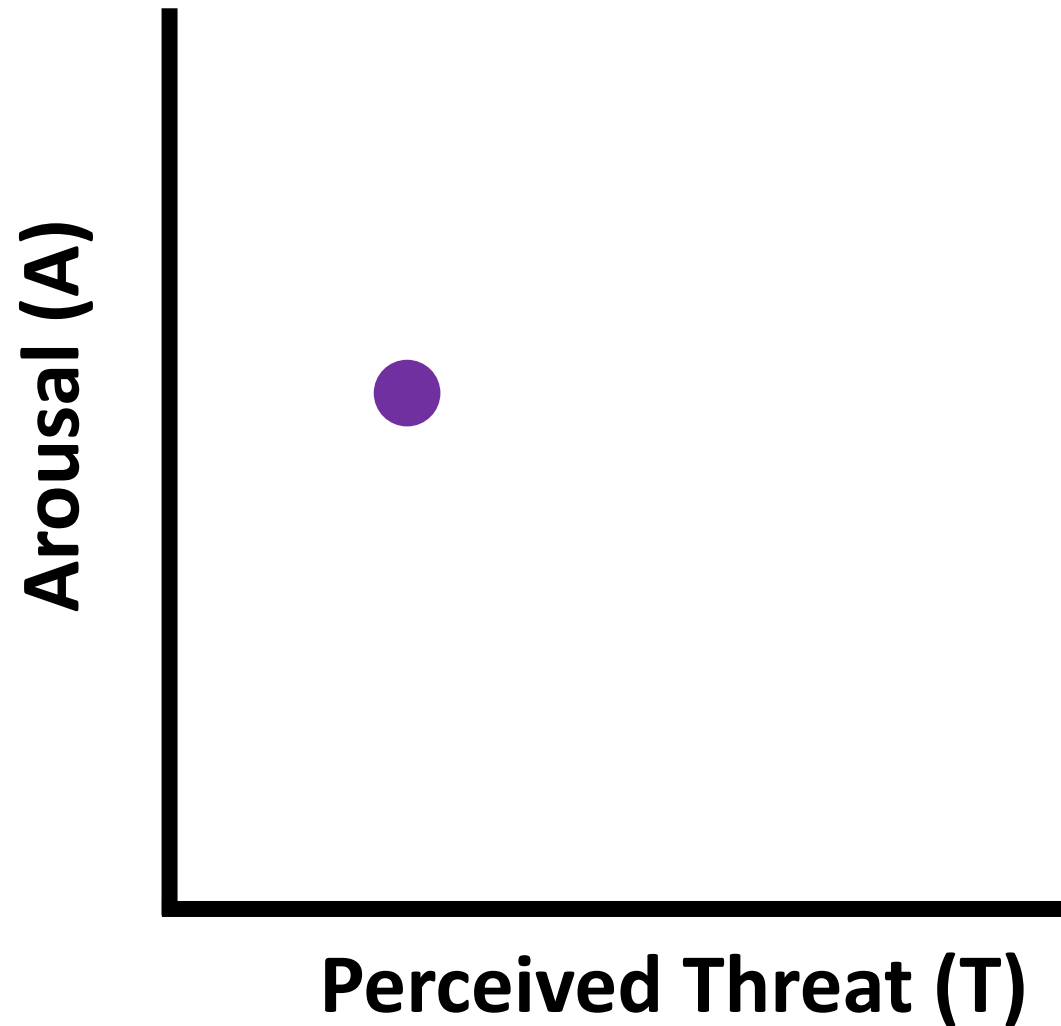
Formal theory: $\frac{dA}{dt} = (T - A)$



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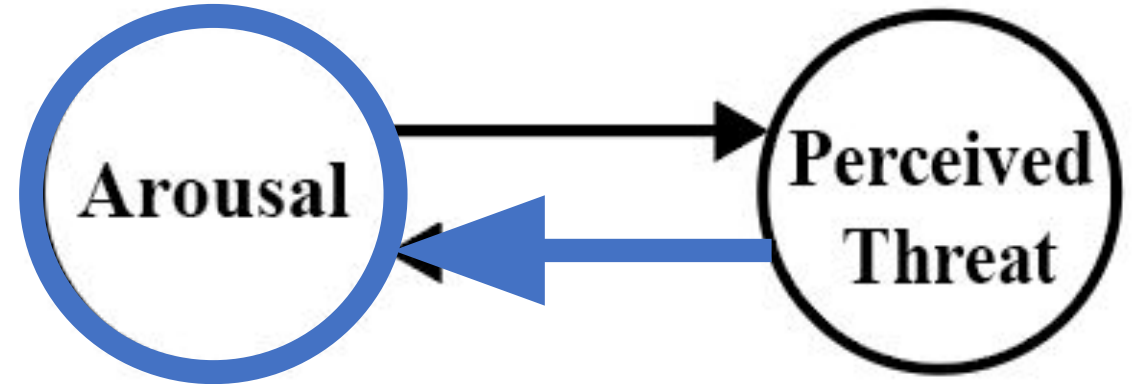
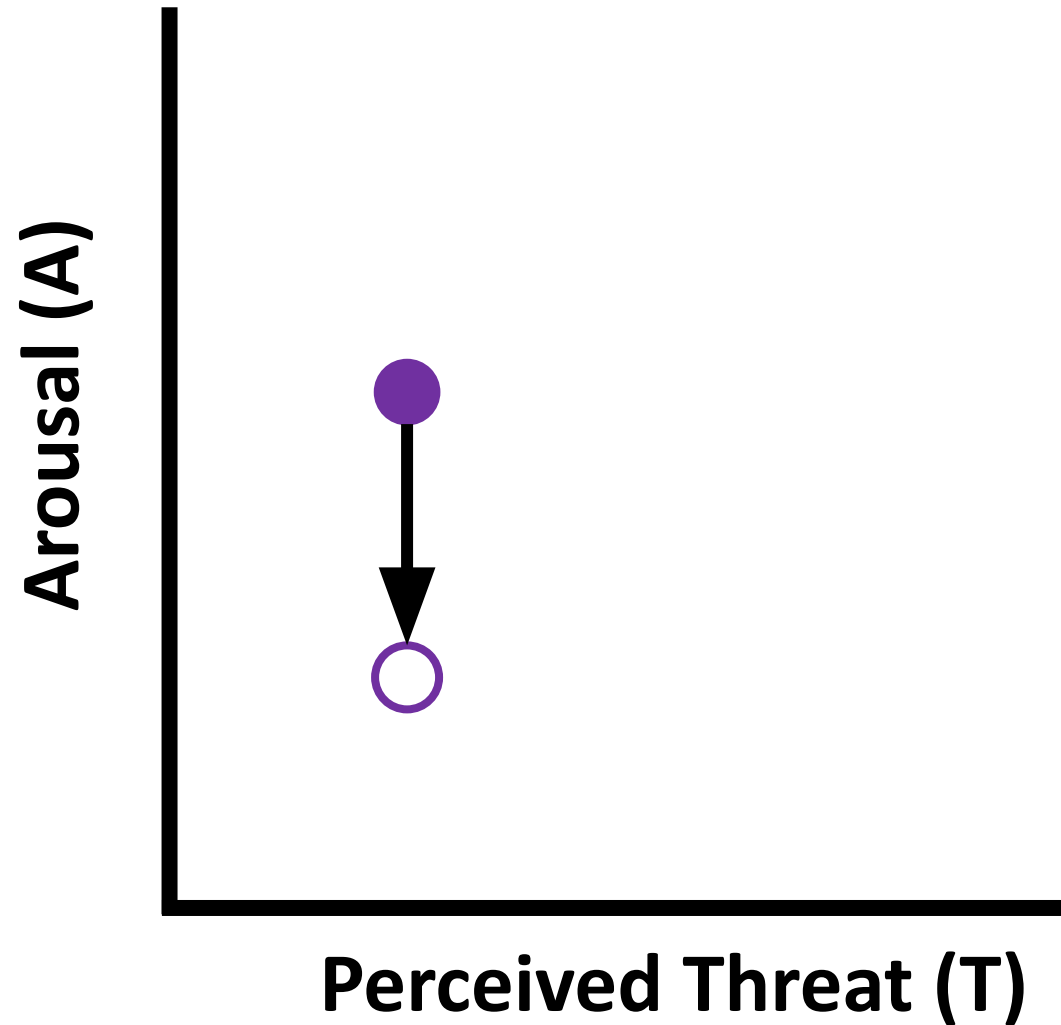


Formal theory: $\frac{dA}{dt} = (T - A)$



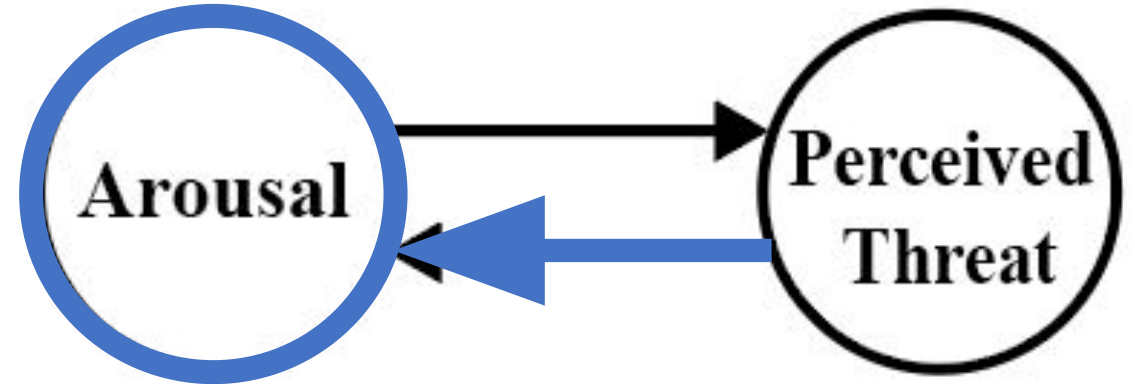
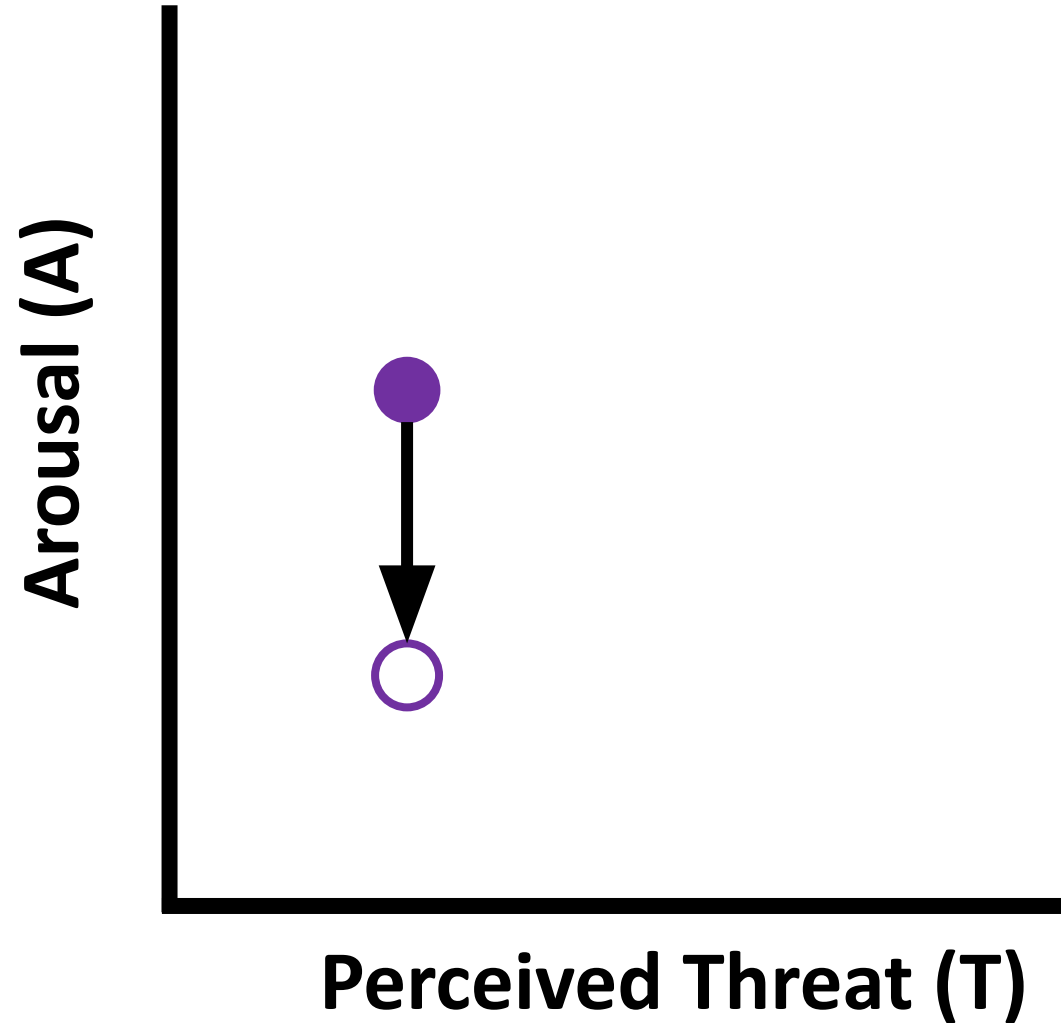
**Formal
theory:**

$$\frac{dA}{dt} = (T - A) = (.25 - .50) = -.25$$



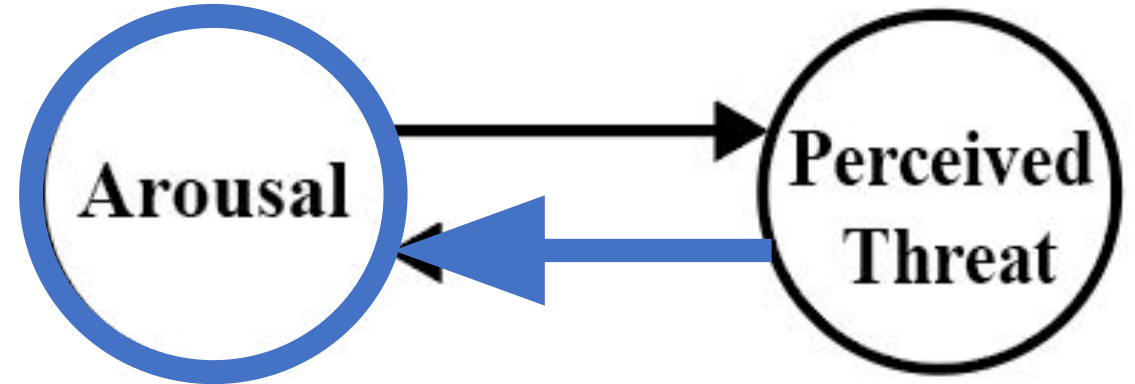
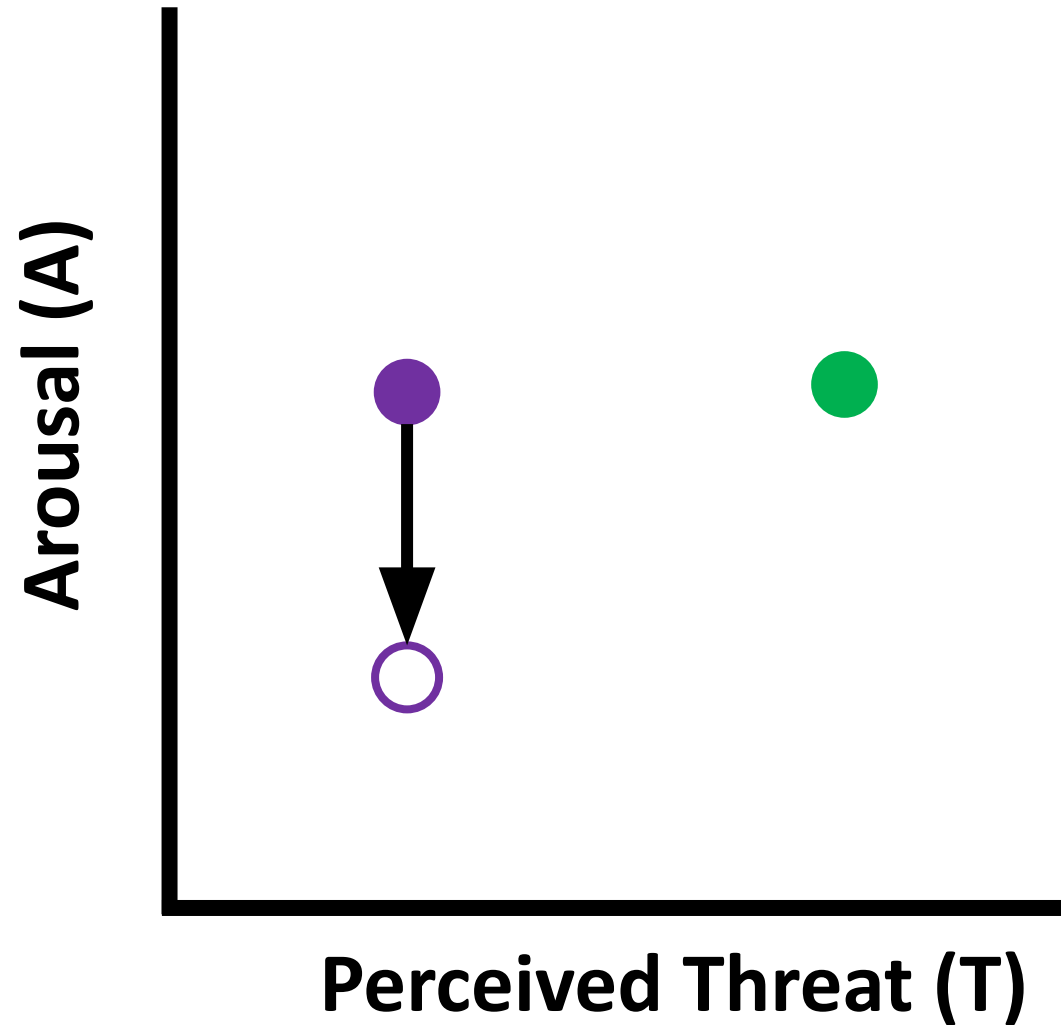
**Formal
theory:**

$$\begin{aligned}\frac{dA}{dt} &= (T - A) &= (.25 - .50) &= -.25 \\ & &= (.25 - .25) &= 0\end{aligned}$$



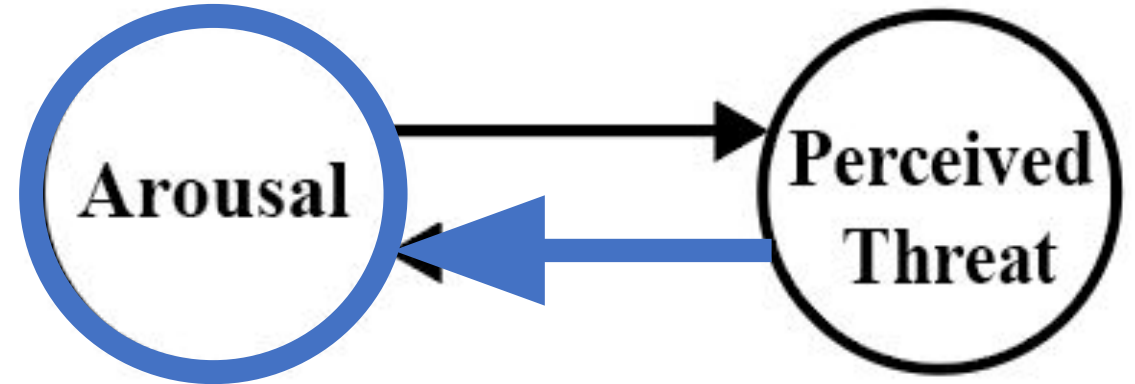
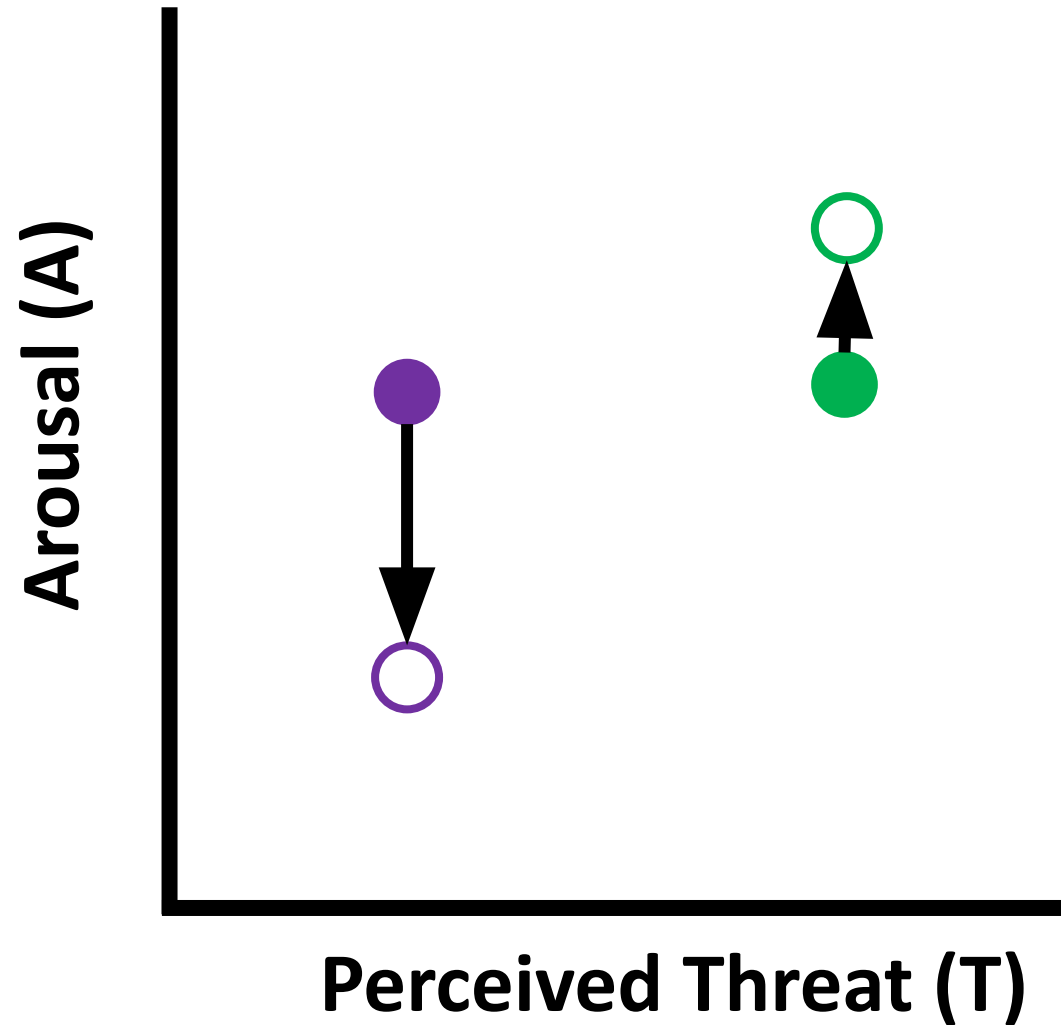
**Formal
theory:**

$$\frac{dA}{dt} = (T - A)$$



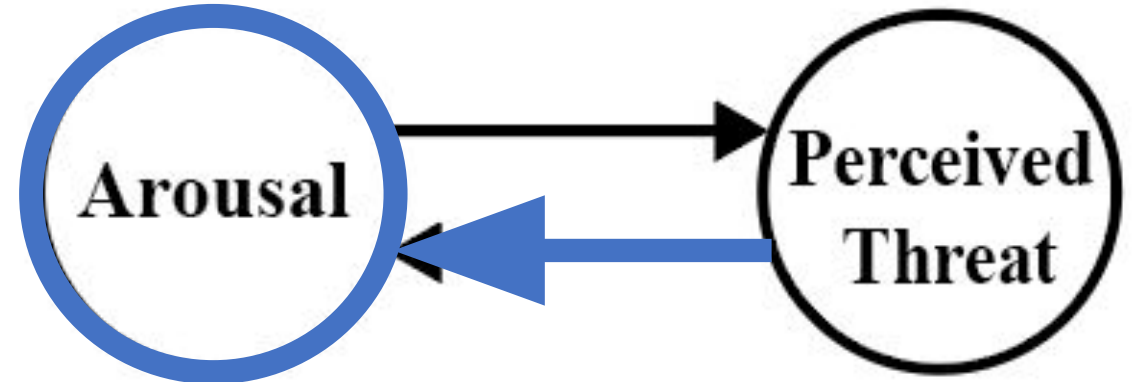
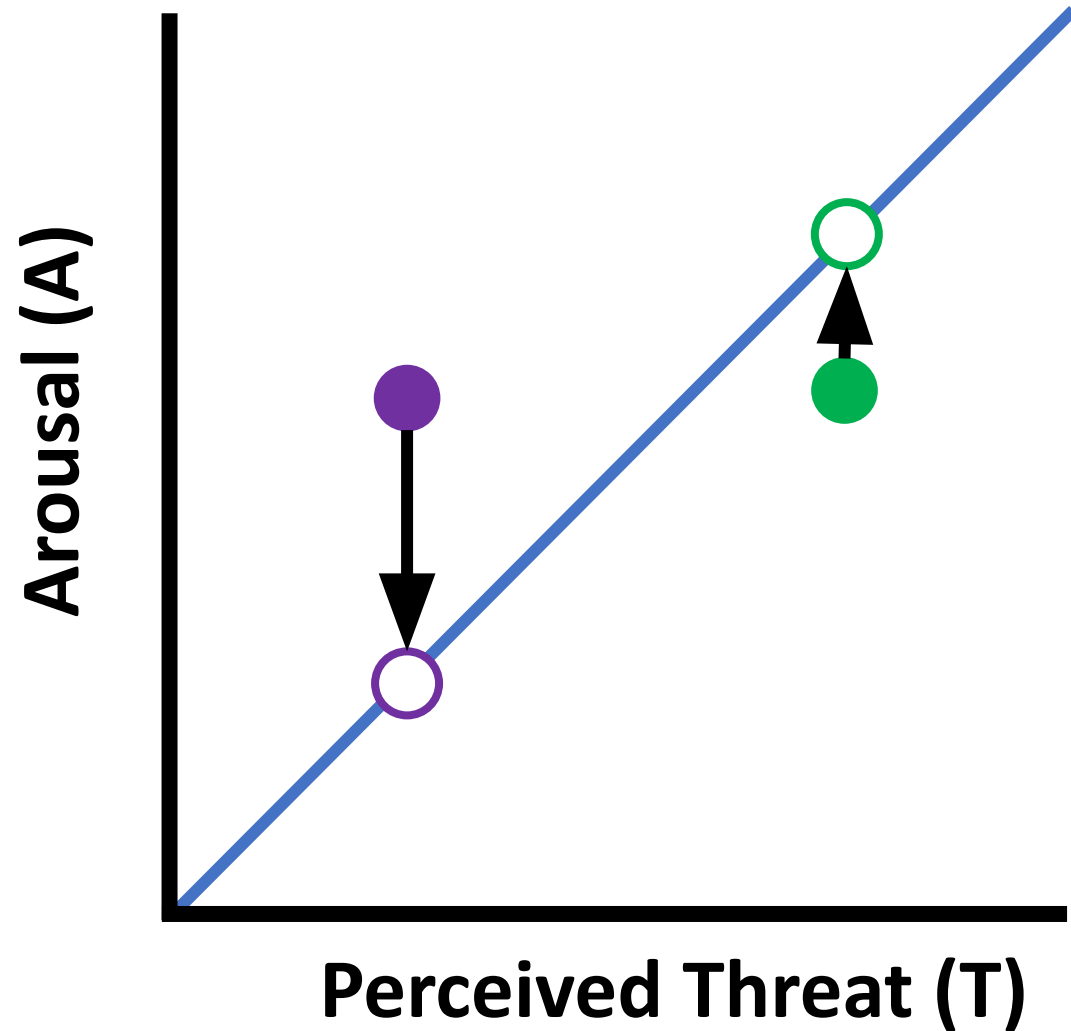
**Formal
theory:**

$$\frac{dA}{dt} = (T - A) = (.60 - .50) = .10$$



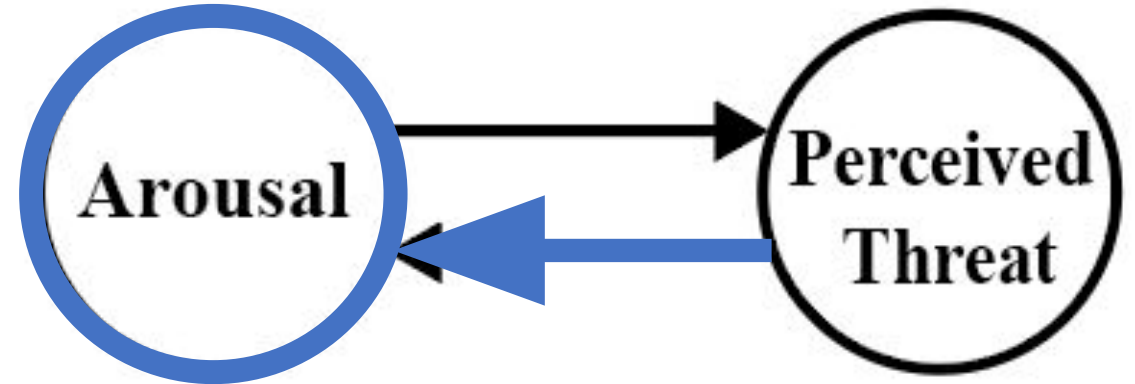
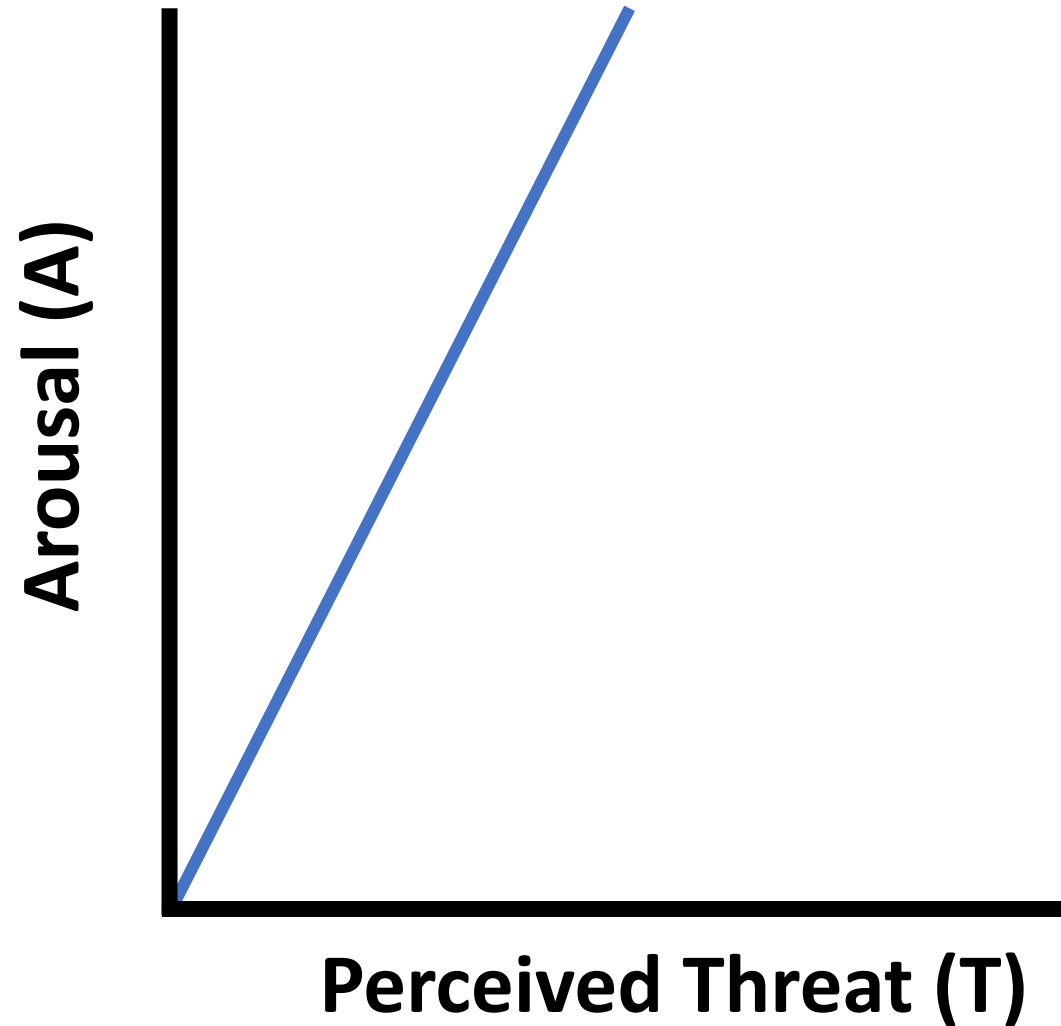
Formal theory:

$$\begin{aligned}\frac{dA}{dt} &= (T - A) &= (.60 - .50) &= .10 \\ & &= (.60 - .60) &= 0\end{aligned}$$



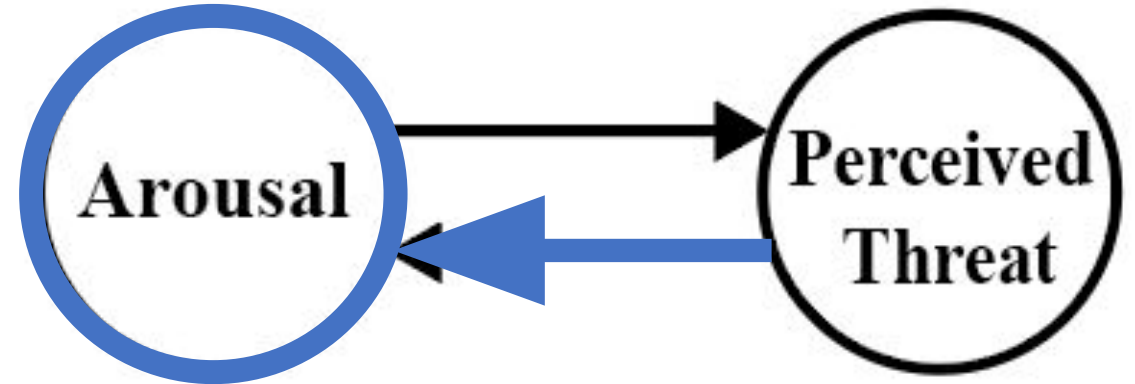
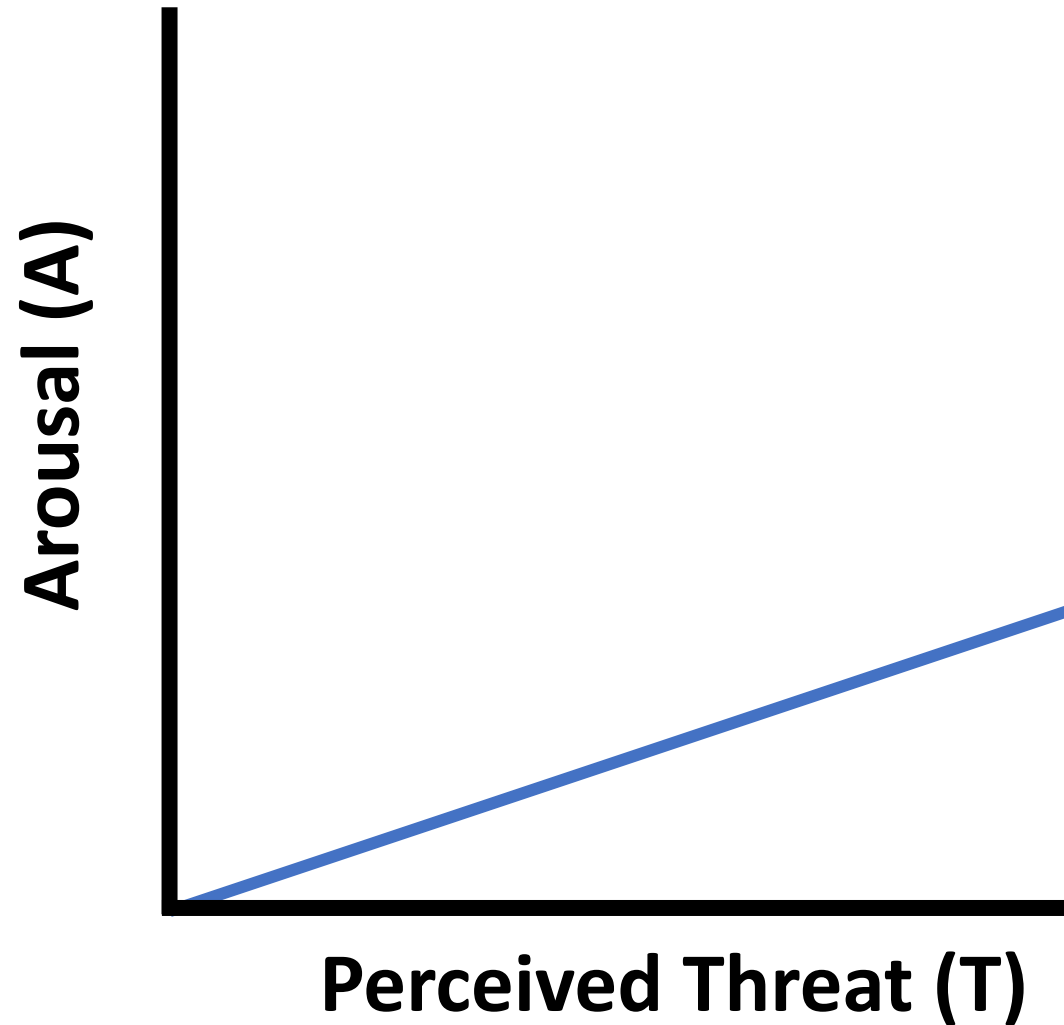
**Formal
theory:**

$$\frac{dA}{dt} = (\beta T - A) \quad \beta=2$$



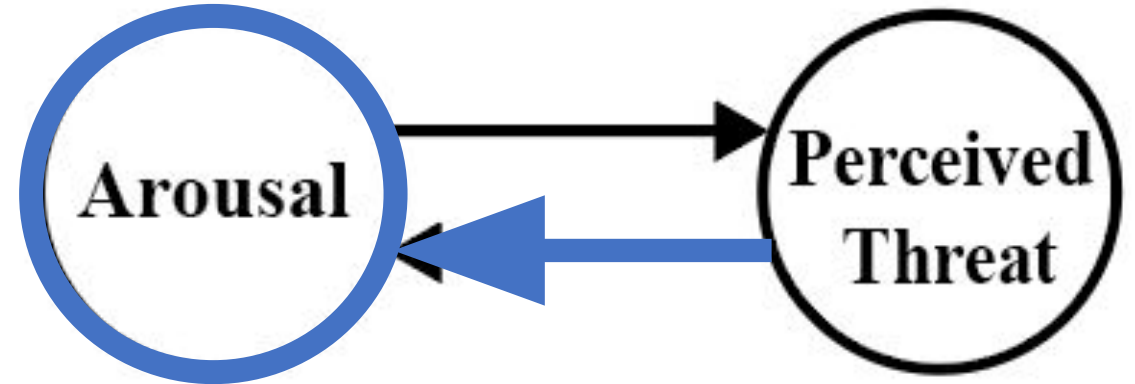
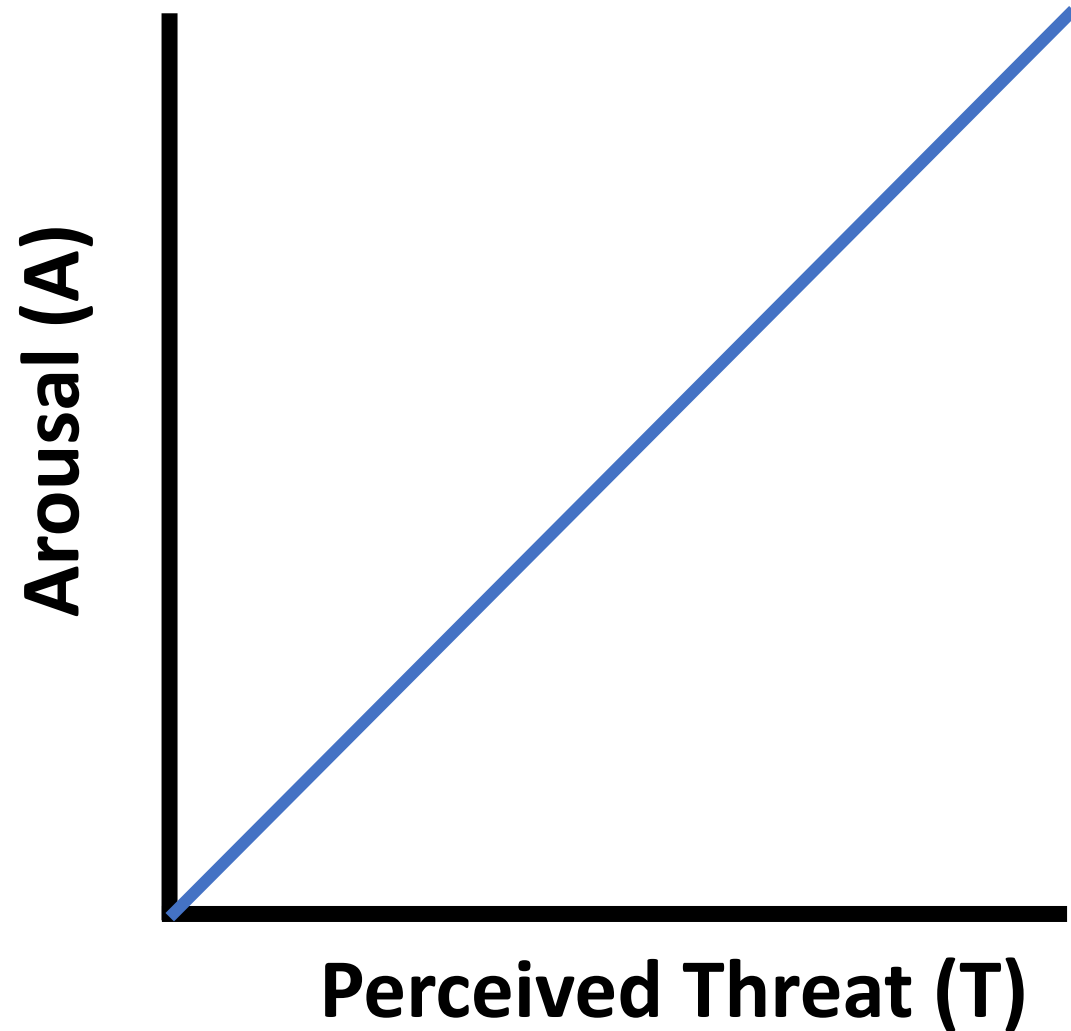
**Formal
theory:**

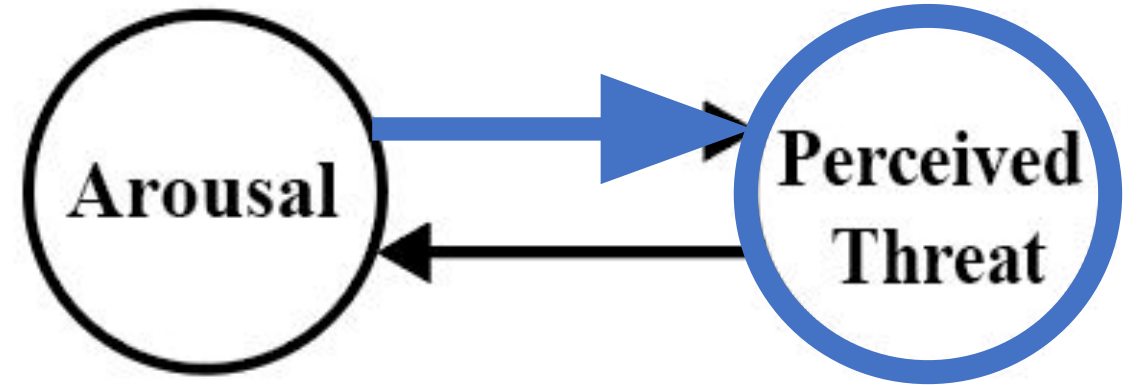
$$\frac{dA}{dt} = (\beta T - A) \quad \beta = .5$$



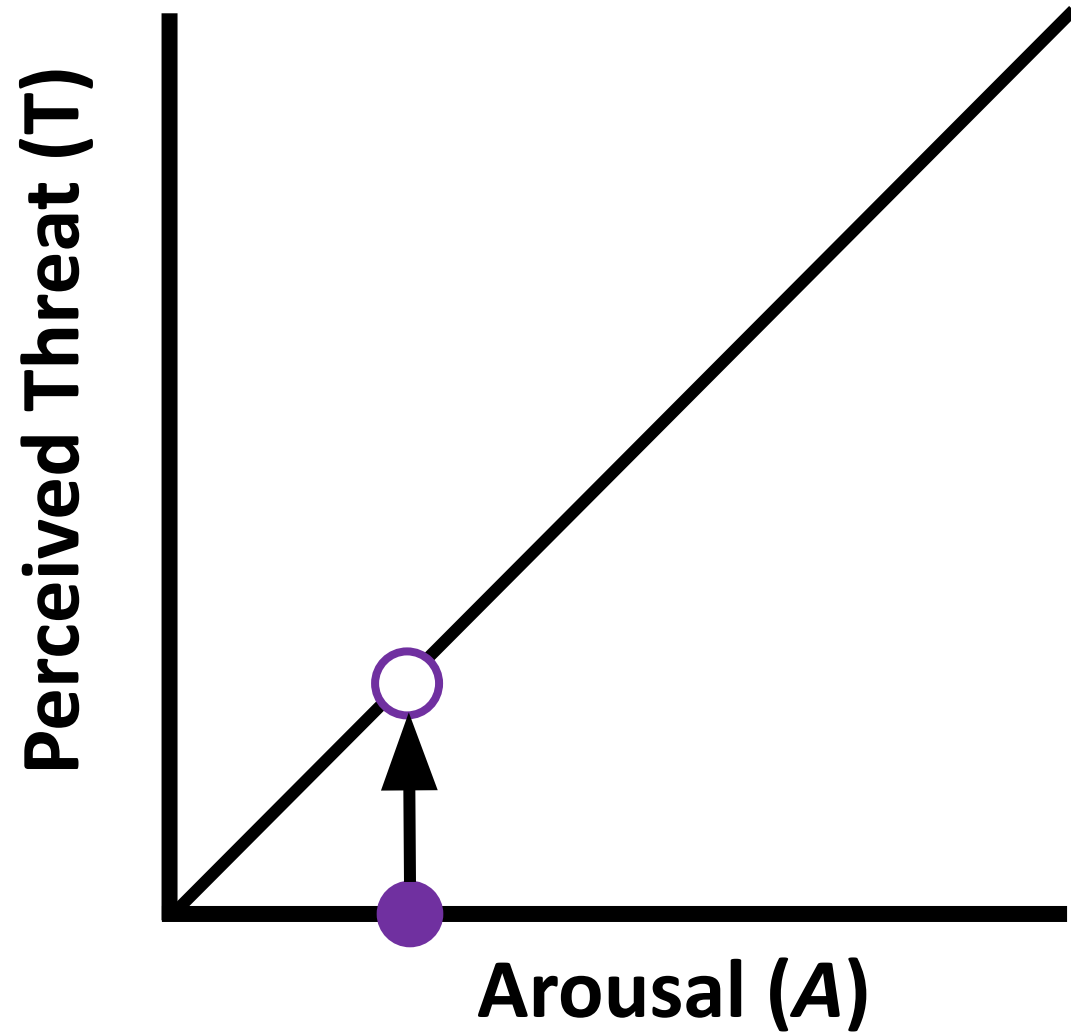
**Formal
theory:**

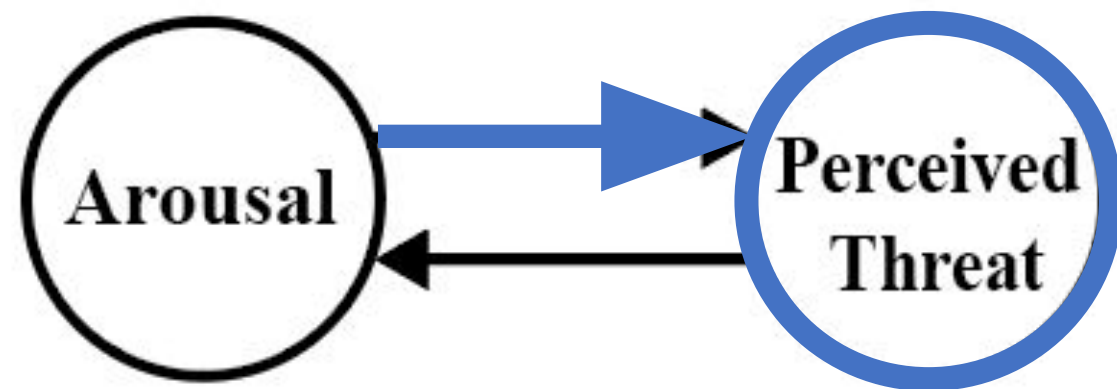
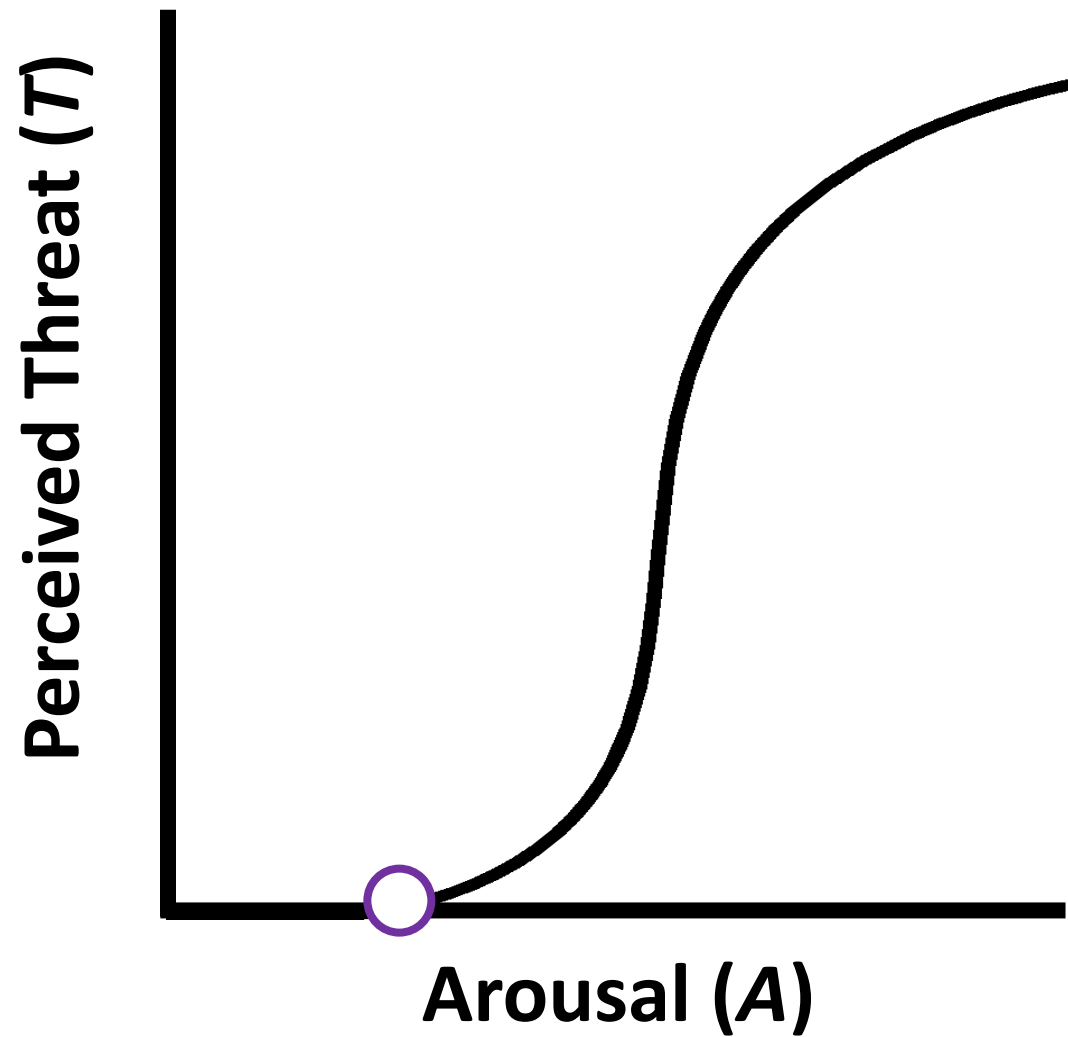
$$\frac{dA}{dt} = (\beta T - A) \quad \beta=1$$



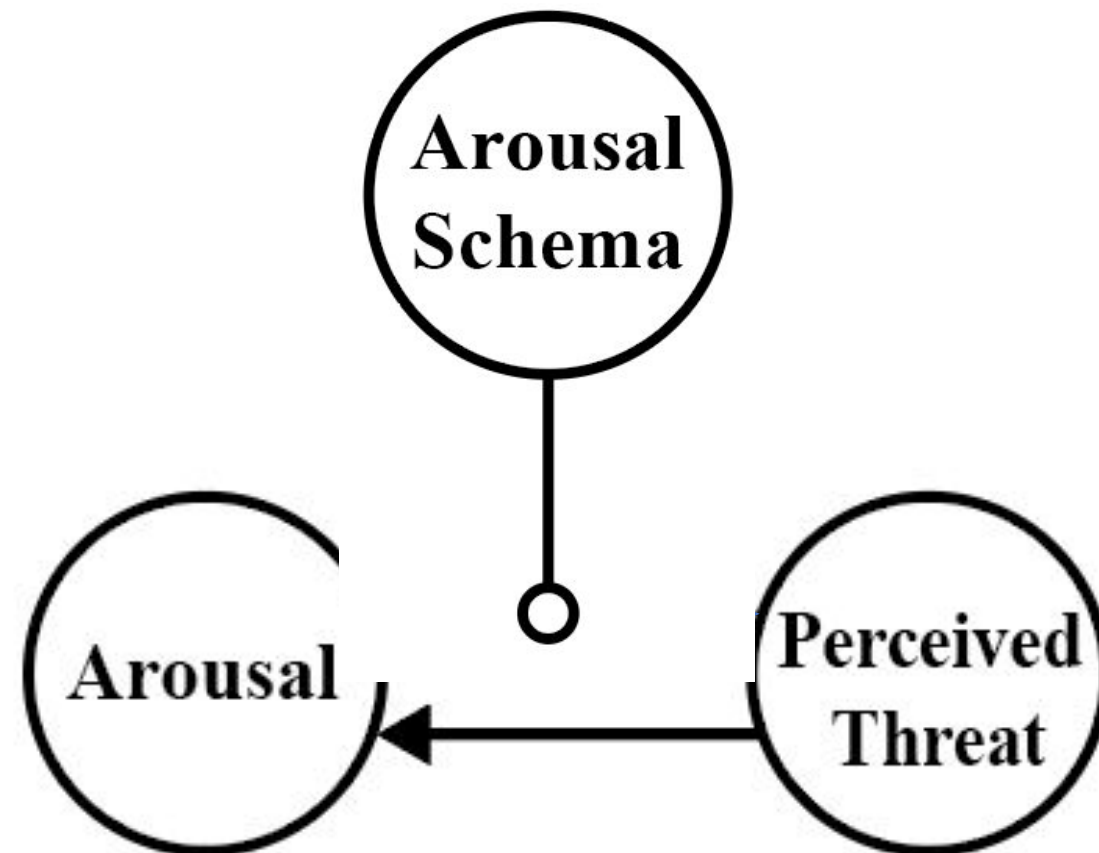
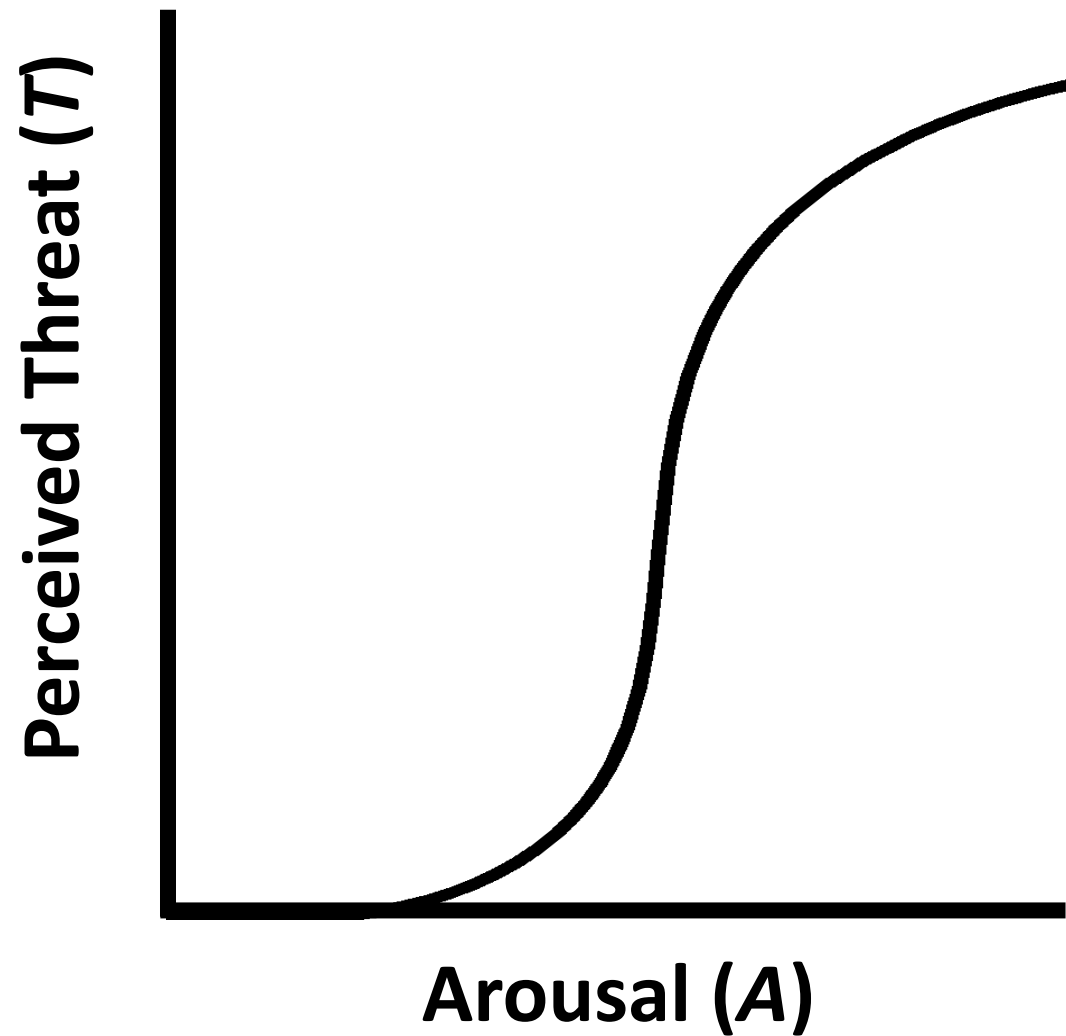


$$\frac{dT}{dt} = (\beta A - T)$$





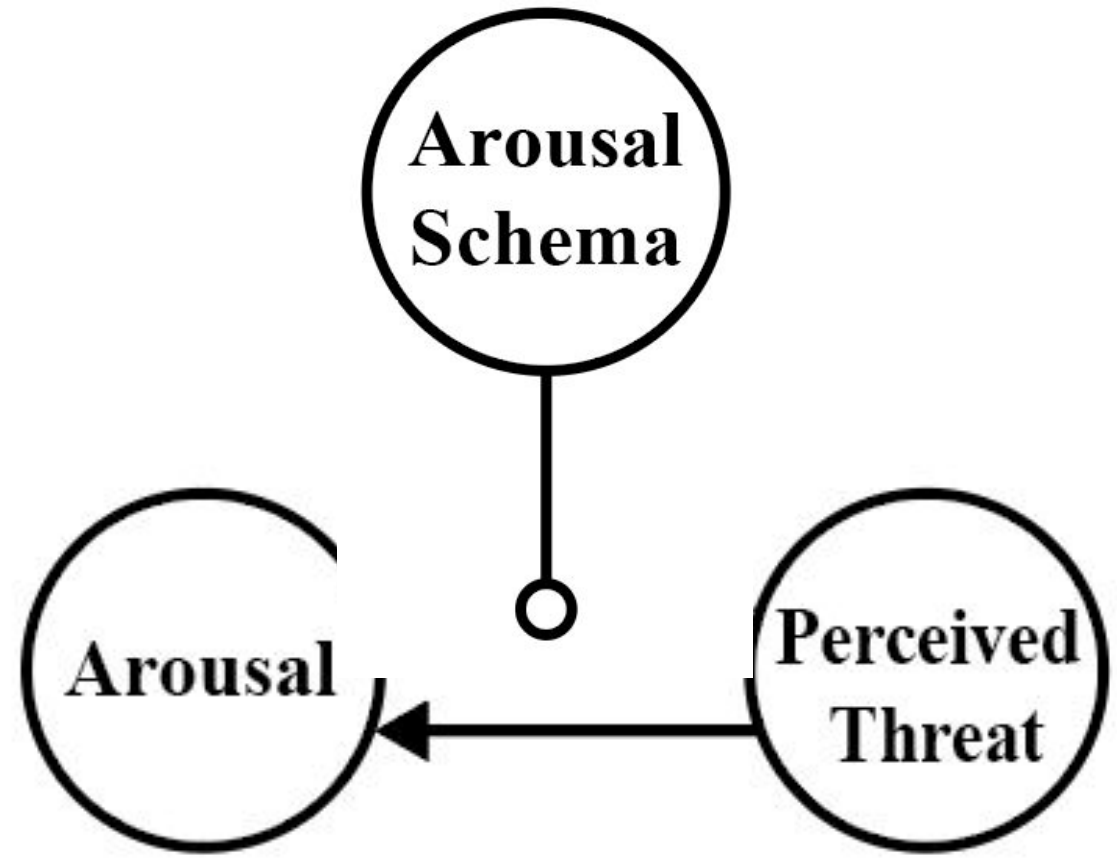
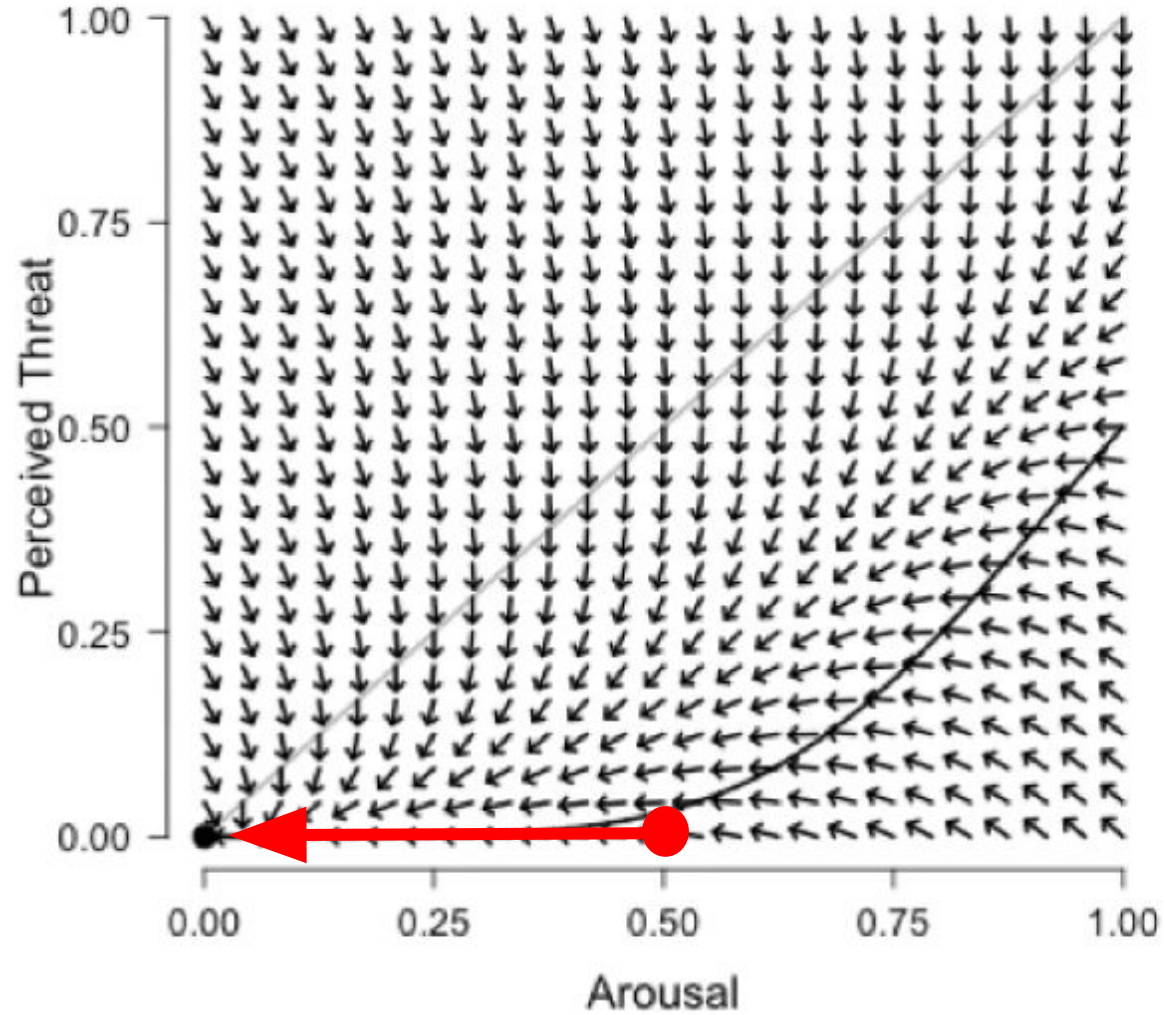
$$\frac{dT}{dt} = \gamma \left(\frac{A^\mu}{A^\mu + \lambda^\mu} - T \right)$$



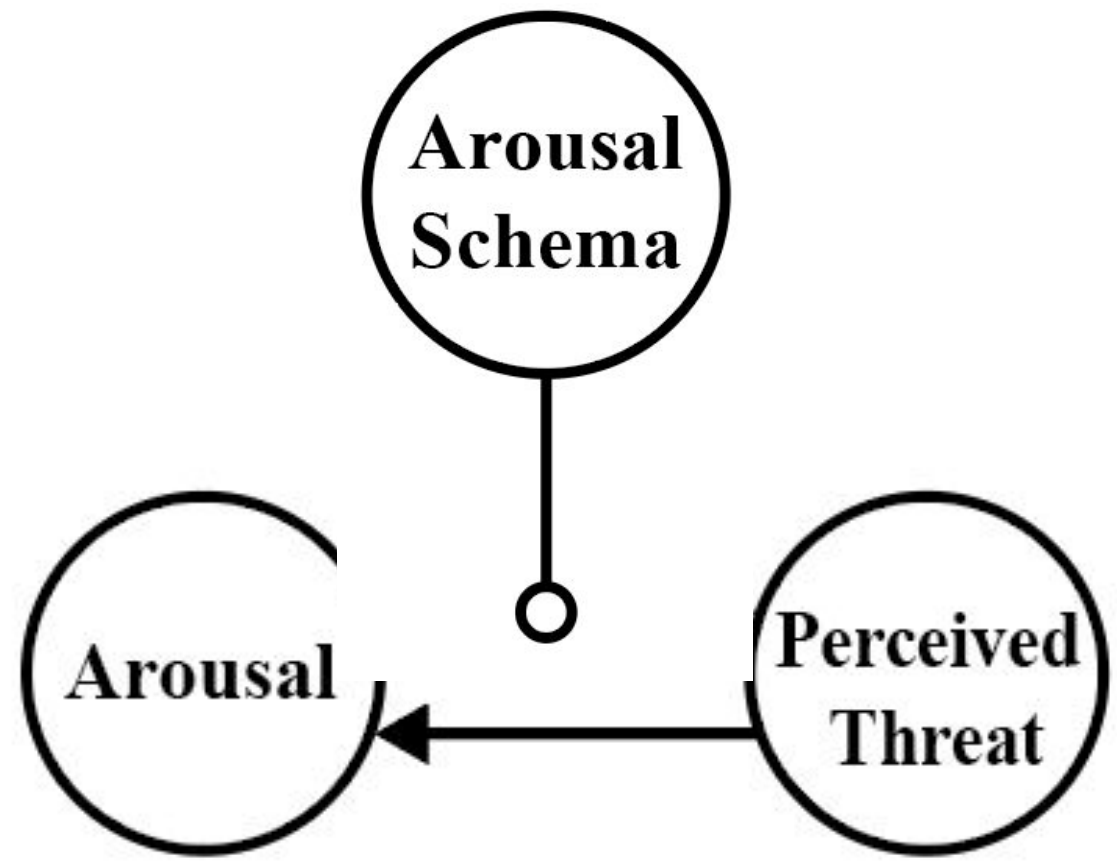
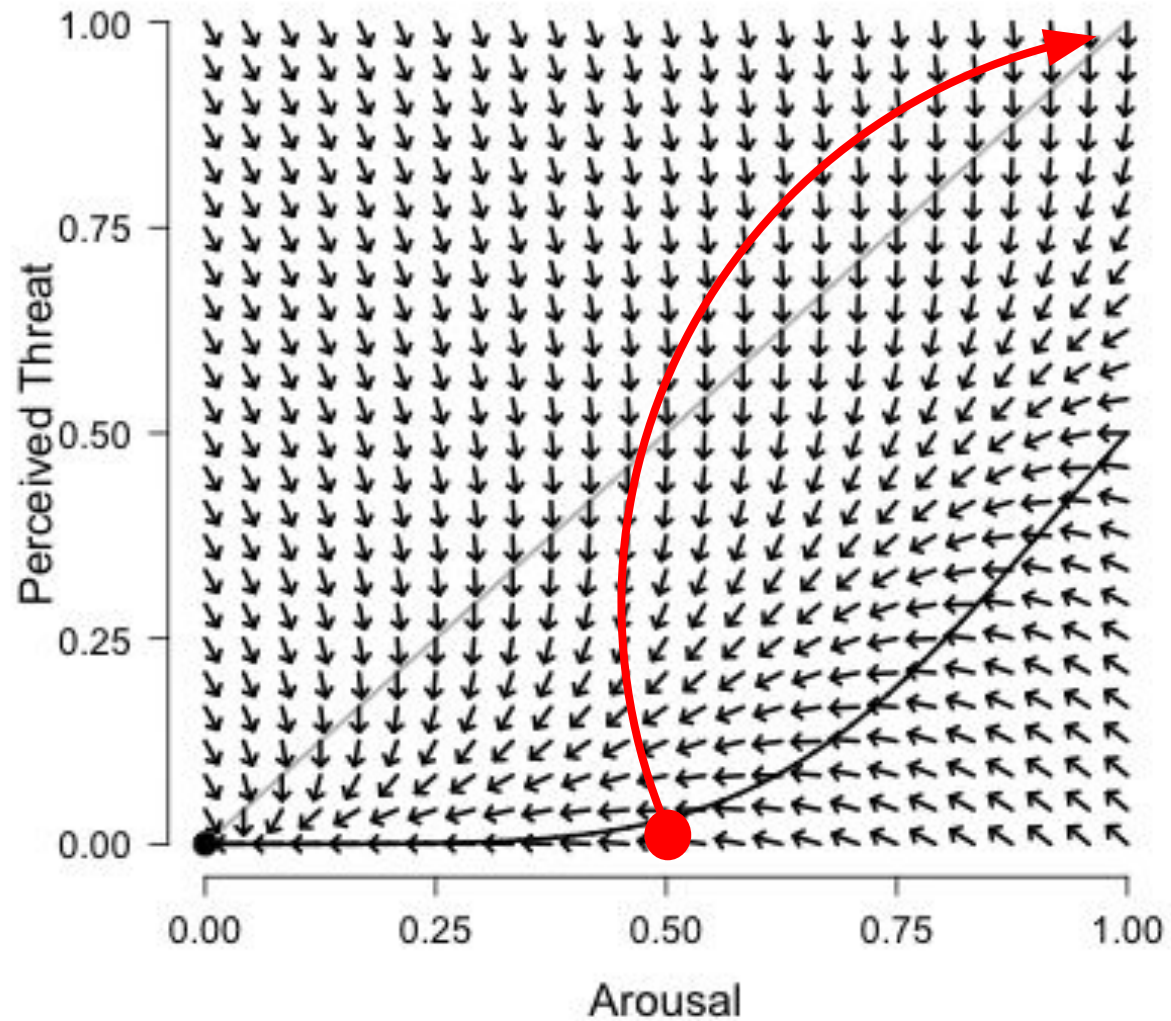
$$\frac{dT}{dt} = \gamma \left(\frac{A^\mu}{A^\mu + \lambda^\mu} - T \right)$$

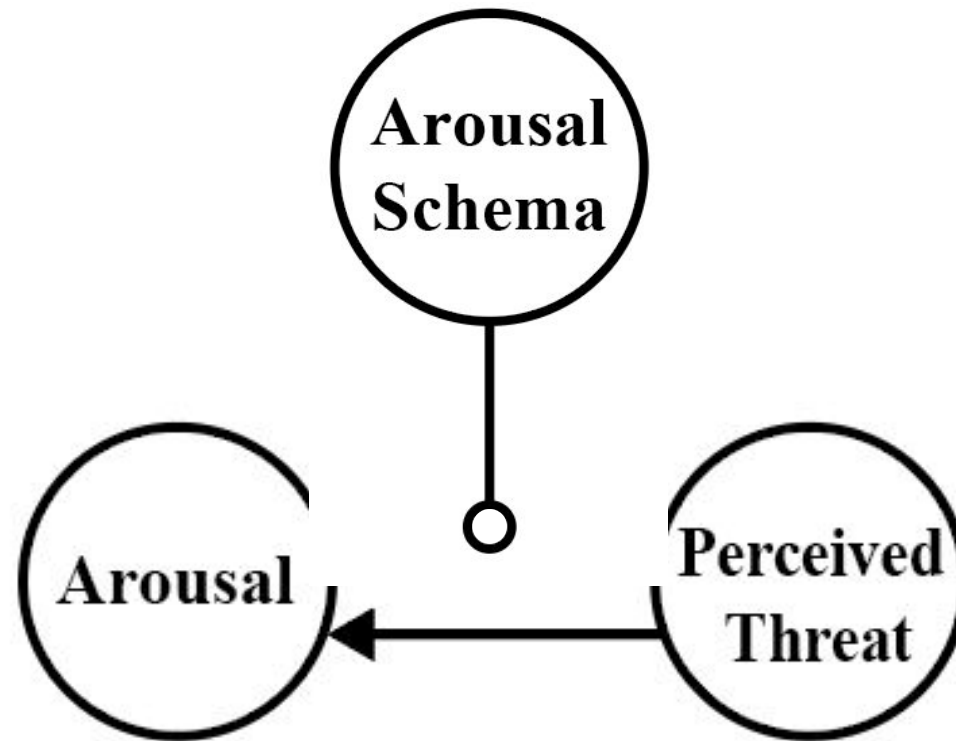
$$\lambda = 1 - \frac{S}{S + \xi}$$

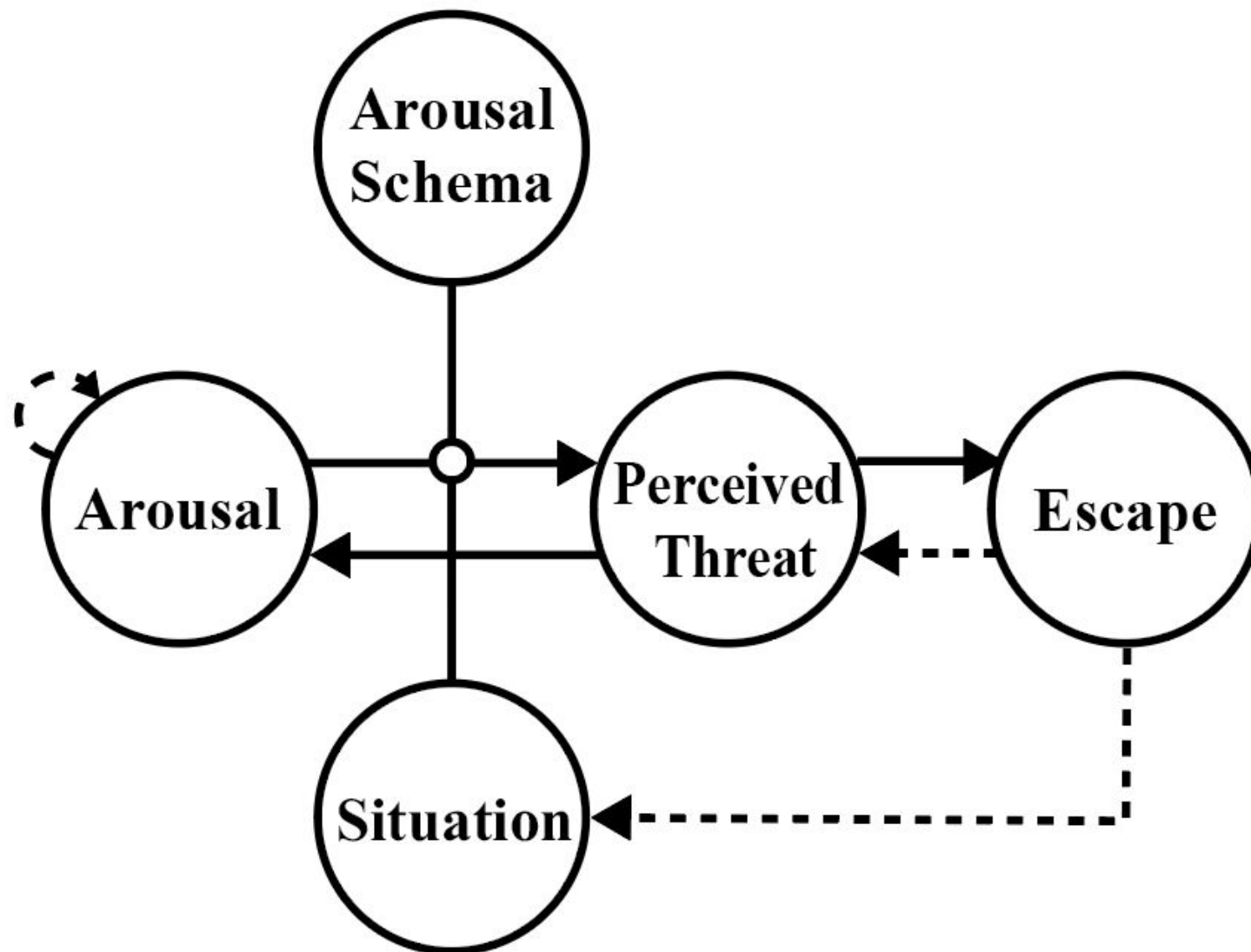
Arousal Schema (S) = 0.00

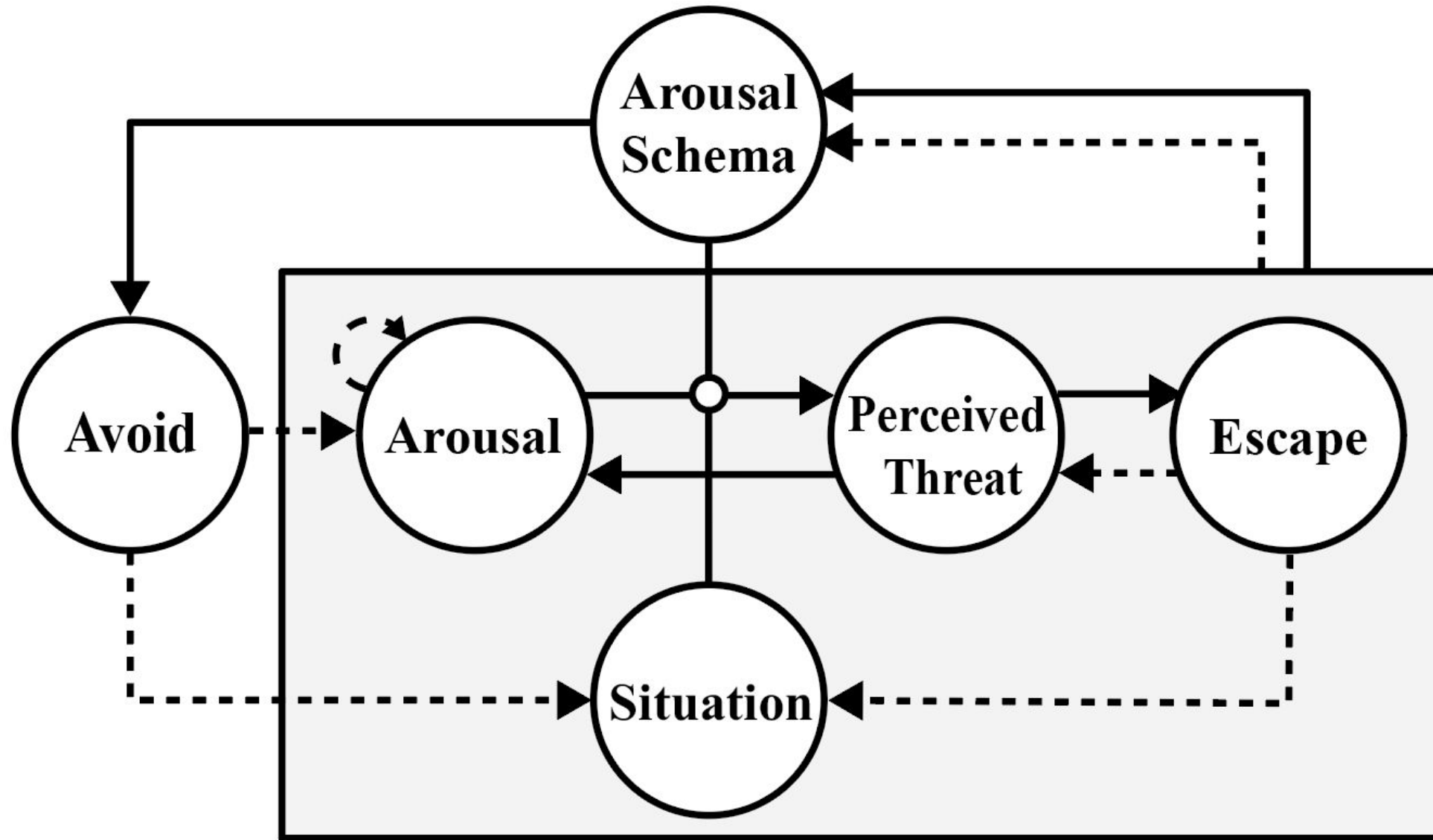


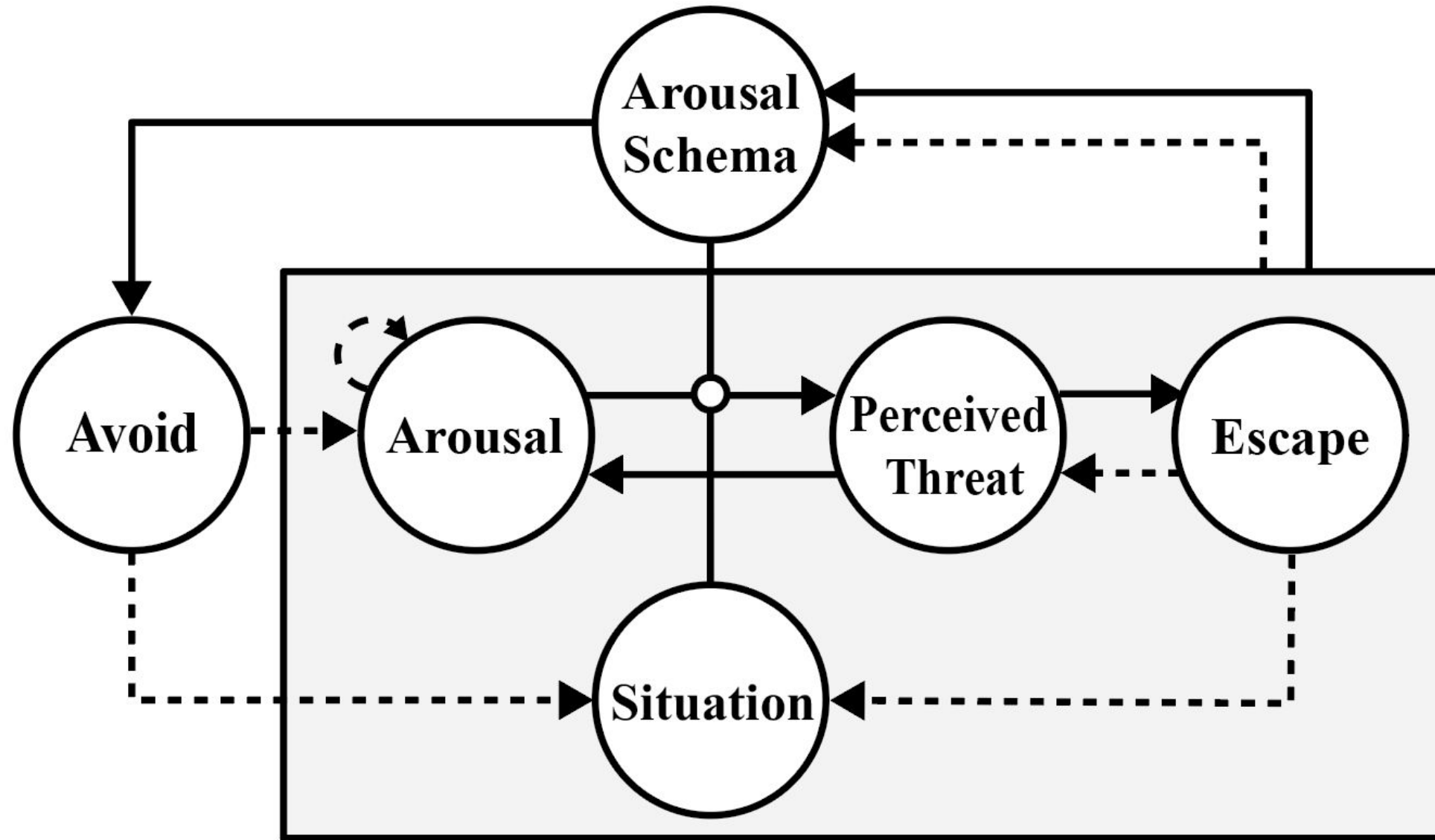
Arousal Schema (S) = 0.00

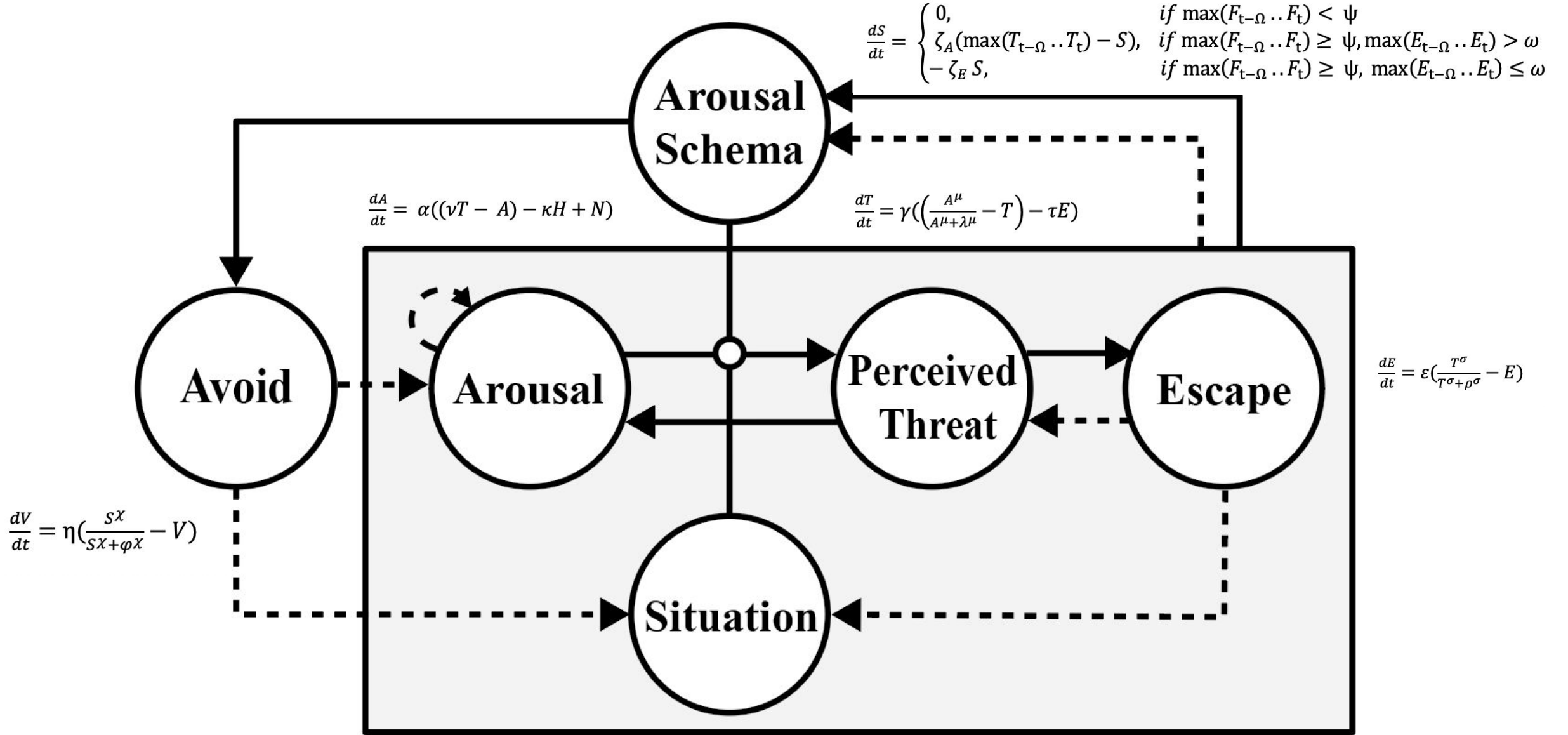












A Computational Model of Panic Disorder

```
simPanic <- function(time_steps, stepsize)
{
  for(i in 1:(nIter)) {
    A_eq <- s_PT_A*PT[i]
    A_eq2 <- -s_H_A*H[i]
    A[i+1] <- A[i] + r_A*((A_eq - A[i]) + A_eq2)*stepsize
    ...
  }
  outlist <- list("A" = A, "PT" = PT, "H" = H, "E" = E)
  return(outlist)
}
```

What does this earn us?

A tool to evaluate our theory!

Theory Evaluation

Simulation 1: Biological Challenge

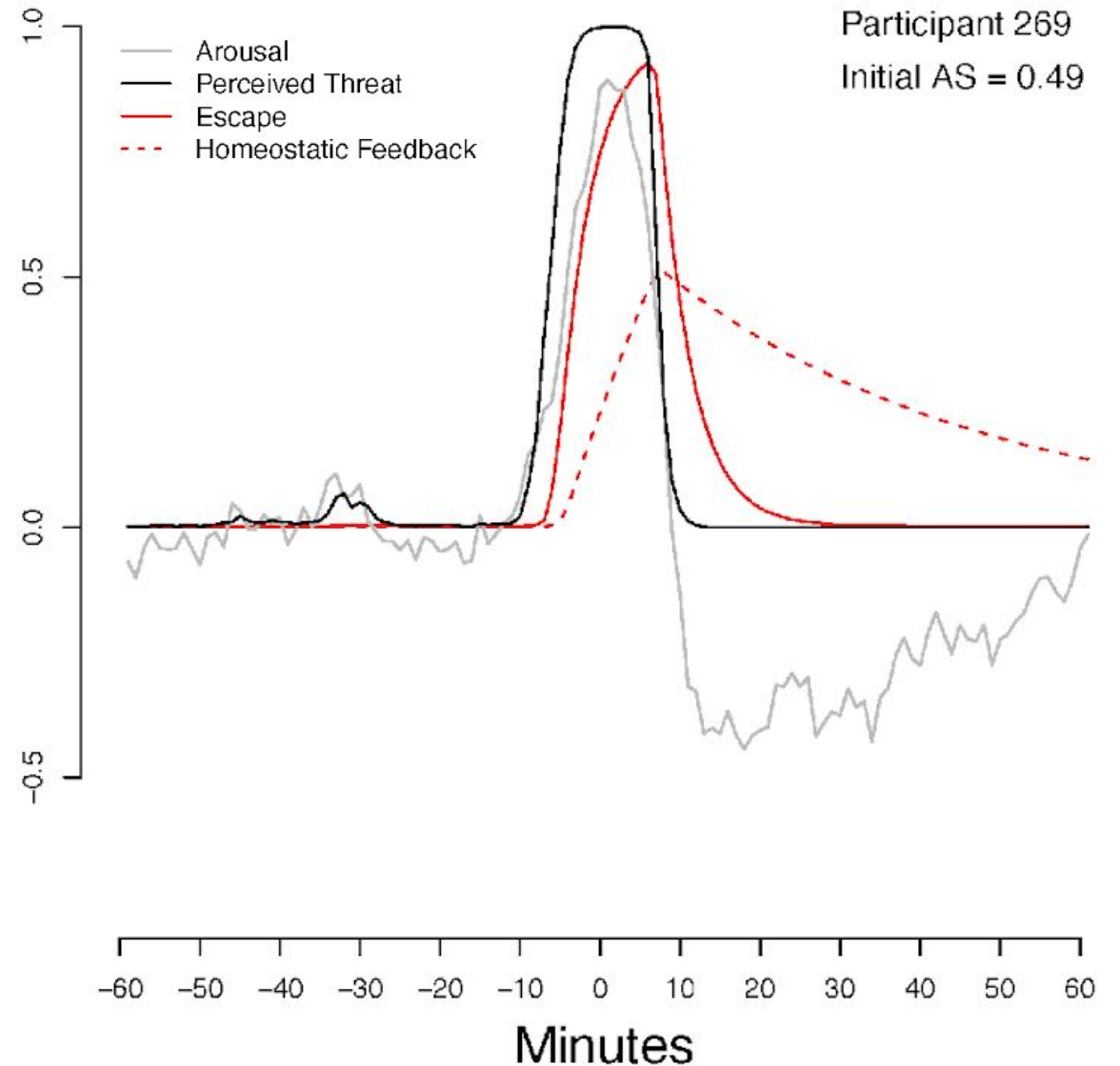
Simulation 2: 3-Month Simulation

Simulation 3: Treatment Study

Phenomenon 1

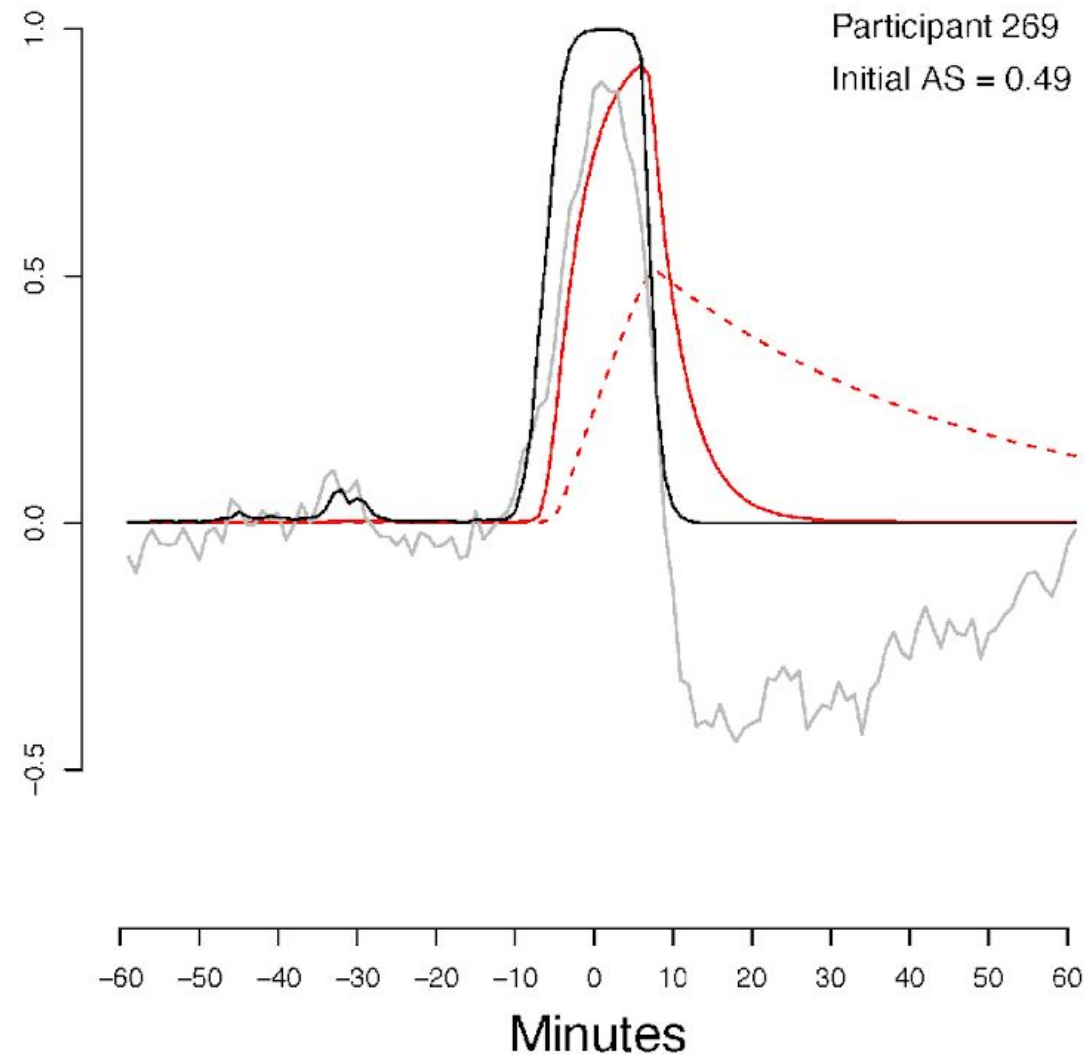
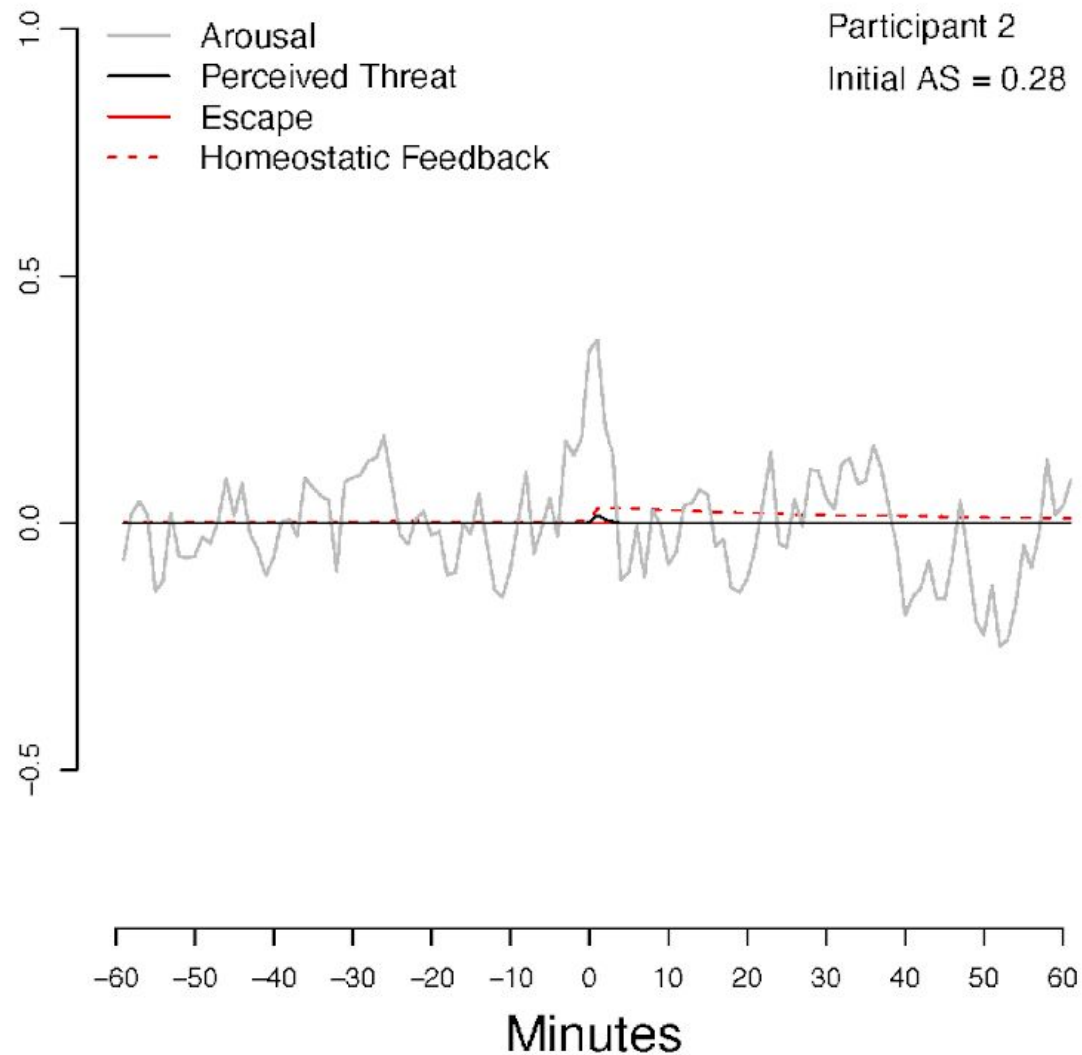
Panic Phenomenology

Some people experience surges of intense fear and somatic symptoms that come on “out of the blue.”



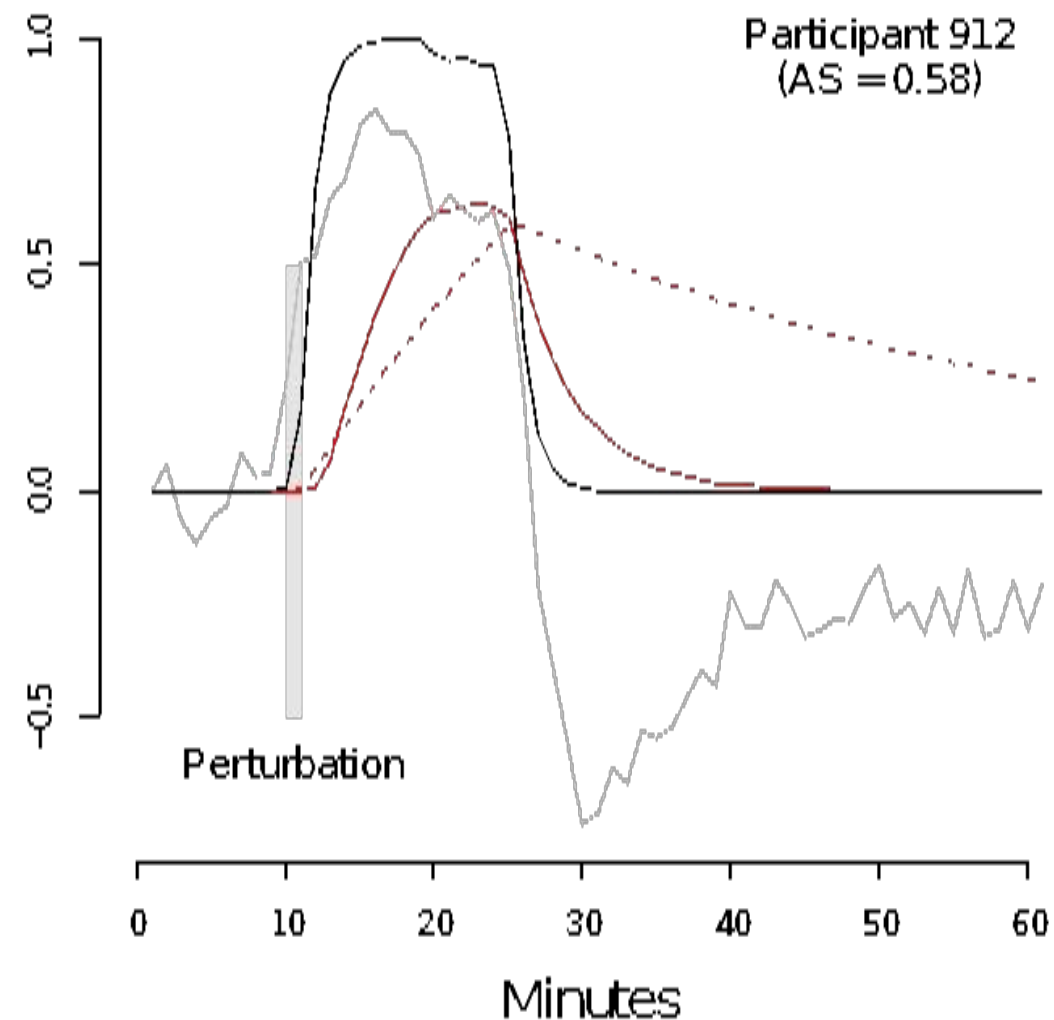
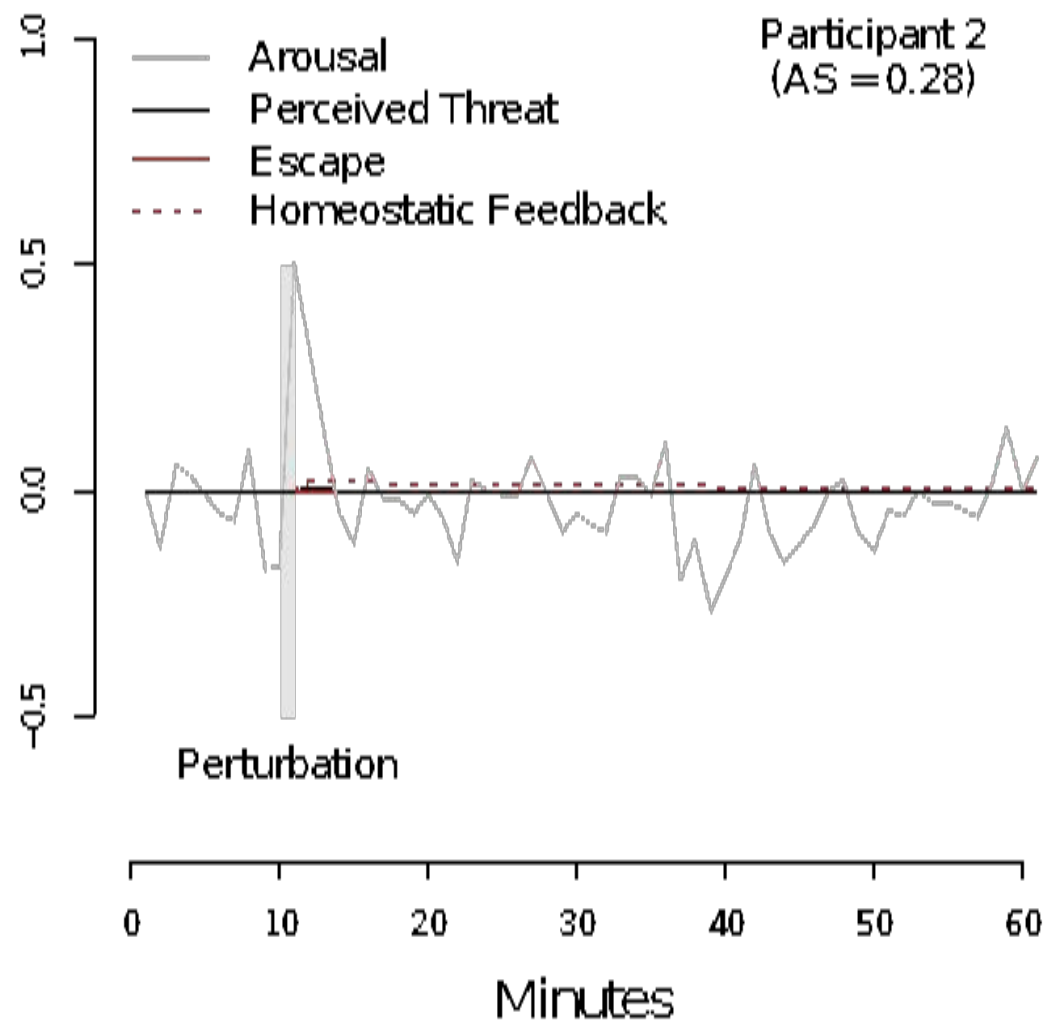
Phenomenon 2

Individual Differences



Phenomenon 2

Individual Differences



Phenomenon 2

Individual Differences

