Section 3

Formalizing Theories with Difference Equations

Theories **explain** phenomena

Phenomenon: My coffee cools faster in the winter than it does in the summer



Theories **explain** phenomena

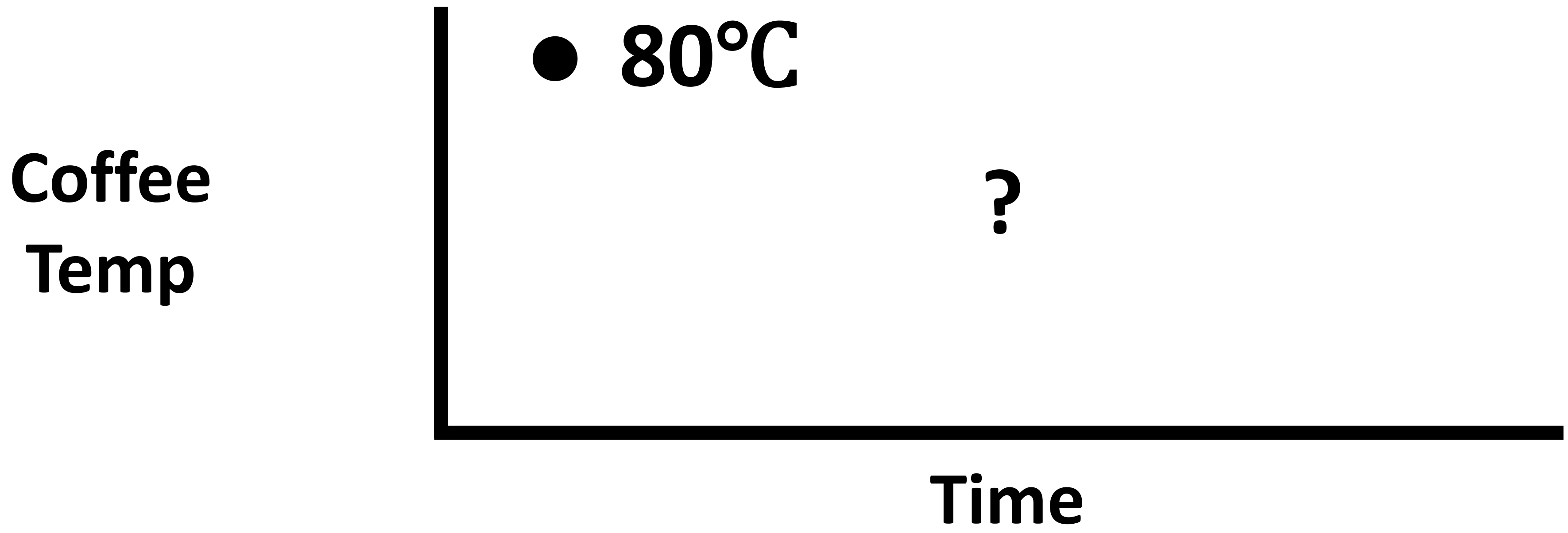
Verbal theory: My coffee's temperature will change proportional to the difference between its own temperature and the ambient temperature



Theories **explain** and **predict**

Verbal theory: My coffee’s temperature will change proportional to the difference between its own temperature and the ambient temperature

What does the theory predict?


80°C
?
Coffee
Temp
Time

Formal theory: $T_{t+1} = T_t + r(T_t - T_A)$

Difference equations tell us where a variable will go next, based on where it is now

Allows us to simulate the behavior of the variable as it evolves over time given a set of initial conditions

Formal theory: $$T_{t+1} = T_t + r(T_t - T_A)$$

$\boldsymbol{T}$ = Coffee Temperature

$\boldsymbol{t}$ = Discrete Time Step

$\boldsymbol{T_A}$= Ambient Temperature

$\boldsymbol{r}$ = Constant = -.20

Formal theory: $T_{t+1} = T_t \pm .20(T_t - 40)$

What does the theory predict?

Formal theory: $$T_{t+1} = T_t + r(T_t - T_A)$$

$$T_0 = 80.0$$

$$T_1 = 80.0 - .20(80.0 - 40) = 72.0$$

t	T_t
0	80.0
1	72.0
2	
3	

Formal theory: $$T_{t+1} = T_t + r(T_t - T_A)$$

$$T_0 = 80.0$$

$$T_1 = 80.0 - .20(80.0 - 40) = 72.0$$

$$T_2 = 72.0 - .20(72.0 - 40) = 65.6$$

t	T_t
0	80.0
1	72.0
2	65.6
3	

Formal theory: $T_{t+1} = T_t + r(T_t - T_A)$

$$T_0 = 80.0$$

$\boldsymbol{t}$	T_t
0	80.0
1	72.0
2	65.6
3	60.5

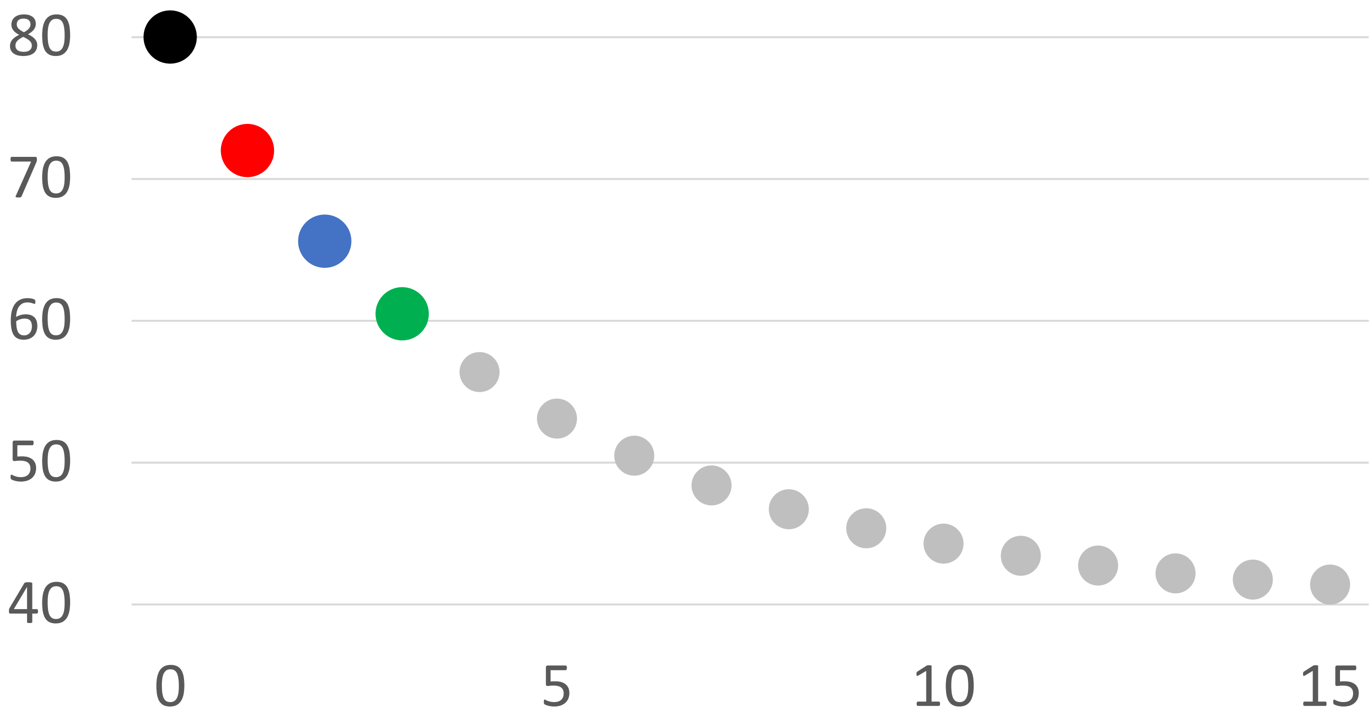
$$T_1 = 80.0 - .20(80.0 - 40) = 72.0$$

$$T_2 = 72.0 - .20(72.0 - 40) = 65.6$$

$$T_3 = 65.6 - .20(65.6 - 40) = 60.5$$

Formal theory: $T_{t+1} = T_t + r(T_t - T_A)$

t	T_t
0	80.0
1	72.0
2	65.6
3	60.5



Formal theory: $T_{t+1} = T_t + r(T_t - T_A)$

Formal theories allows us to **deduce** precisely what a theory predicts

Accurate **deduction** is a prerequisite for **explanation**

Formal theory: $$T_{t+1} = T_t + r(T_t - T_A)$$

t	T_t
0	80.0
1	72.0
2	65.6
3	60.5

t	T_t
0	80.0
1	64.0
2	51.2
3	41.1

Phenomenon: My coffee cools faster in the winter than it does in the summer

Mathematical Models and Computational Models

Formal theory: $$T_{t+1} = T_t + r(T_t - T_A)$$

Formal theory:

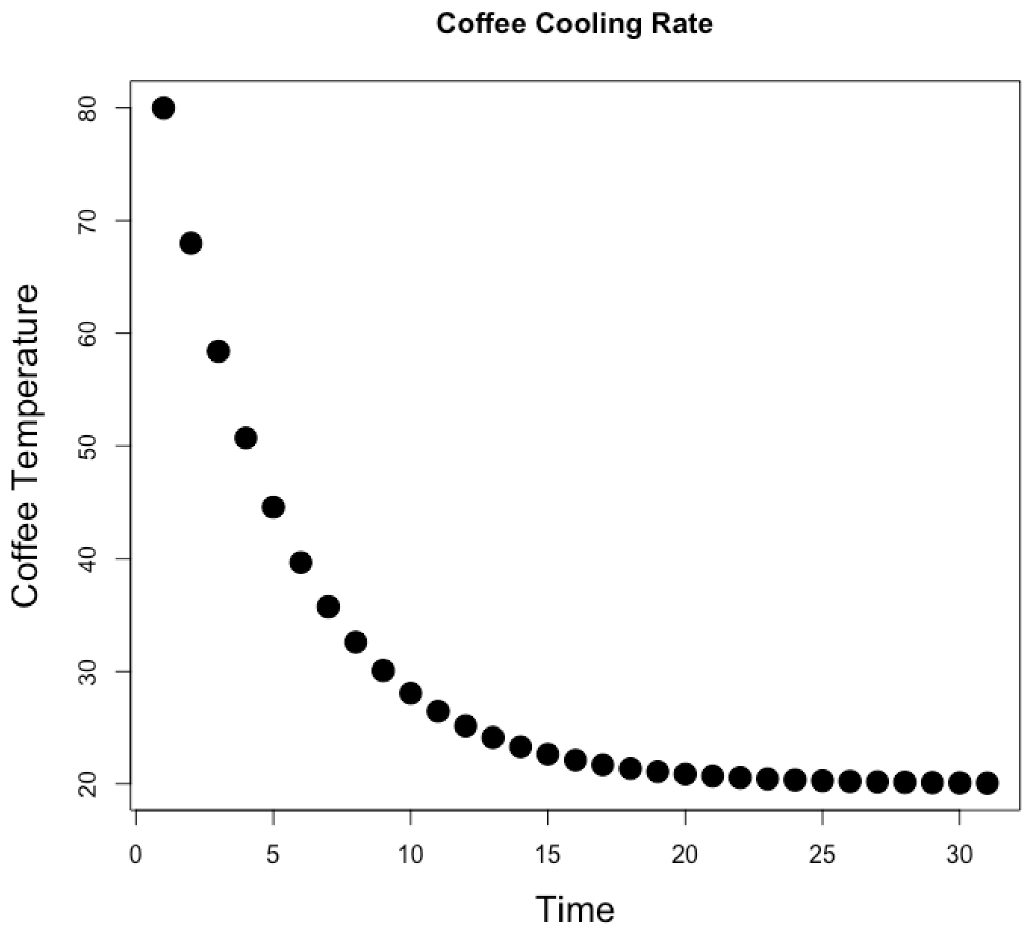
```
temp<-vector()
temp[1]<-80
time_steps<-30

for (t in 1:time_steps){
temp[t+1]<-temp[t]-.2*(temp[t]-20)}
```

A Computational Model of Coffee Temperature!

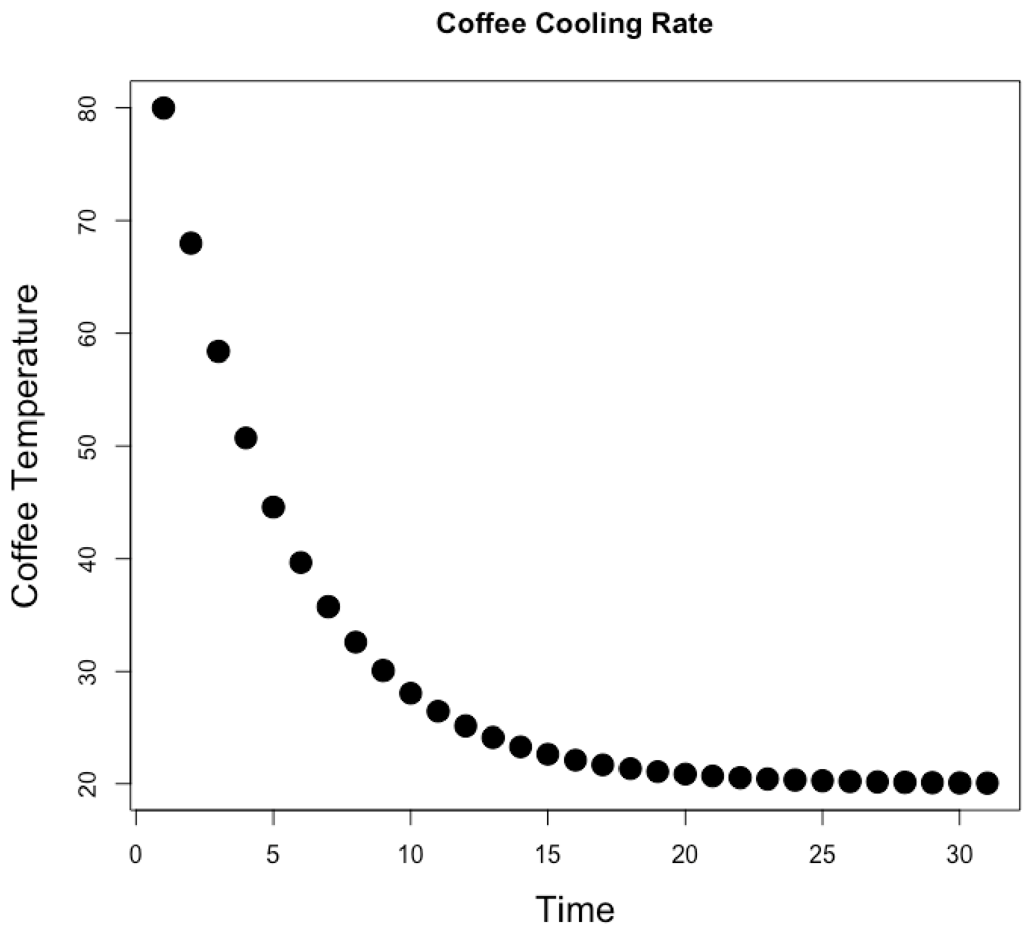
```
temp<-vector()
temp[1]<-80
time_steps<-30

for (t in 1:time_steps){
temp[t+1]<-temp[t]-.2*(temp[t]-20)}
```



A Computational Model of Coffee Temperature!

Problem: Coffee doesn't change in discrete time



Difference
<u>**Equations**</u>

Discrete Time

$$T_{t+1} = T_t - .2(T_t - 20)$$

Differential
<u>**Equations**</u>

Continuous Time

$$\frac{dT}{dt} = -.2(T - 20)$$

Differential Equations

Problem: No analytic solution for many differential equations

Continuous Time

$$\frac{dT}{dt} = -.2(T - 20)$$

Solution: Back to Difference Equations (Euler's Method)

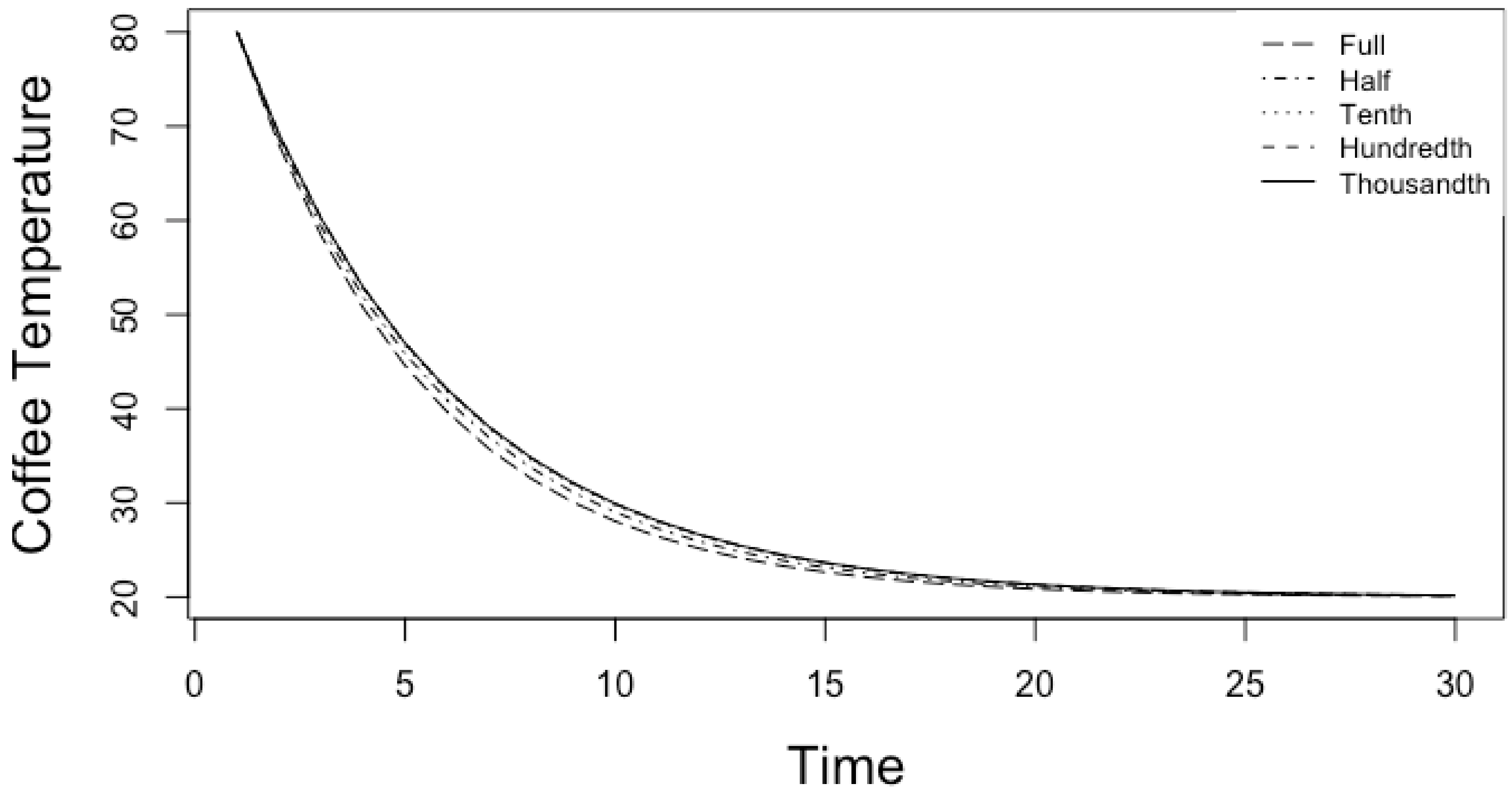
```
simTemp <- function(stepsize, subsample, temp_initial, temp_room)
{
 …
      for (t in 1:nIter){
        temp[t+1]<-temp[t]-.2*(temp[t]-temp_room)*stepsize
      }
 temp <- temp[round(seq(1, nIter, by=subsample))]
 return(temp)
}
```

Euler's Method

```
out_full<-simTemp(time_steps=30,
                  stepsize=1,
                  subsample=1/1,
                  temp_initial=80,
                  temp_room=20)

out_half<-simTemp(time_steps=30,
                  stepsize=.5,
                  subsample=1/.5,
                  temp_initial=80,
                  temp_room=20)
```

…

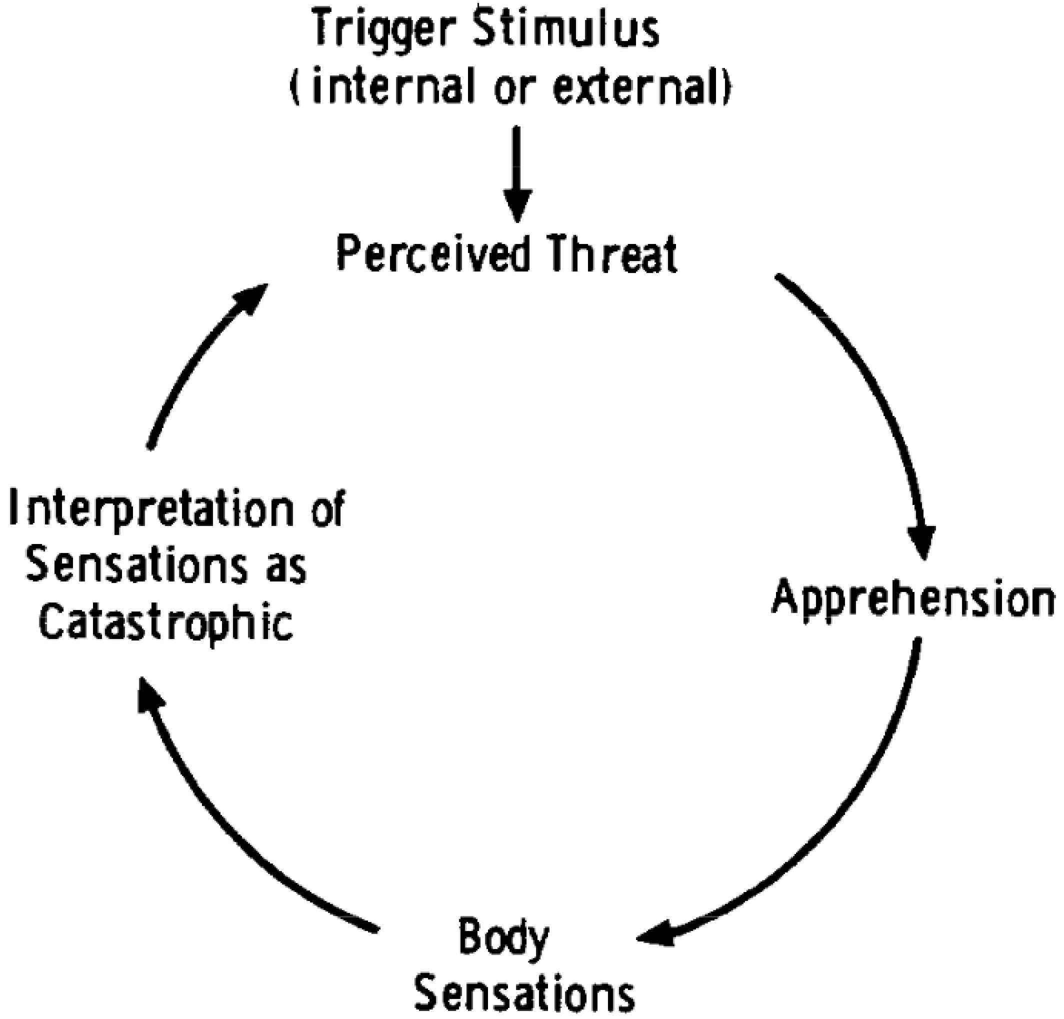


Full
Half
Tenth
Hundredth
Thousandth
Coffee Temperature
80
70
60
50
40
30
20
0
5
10
15
20
25
30
Time

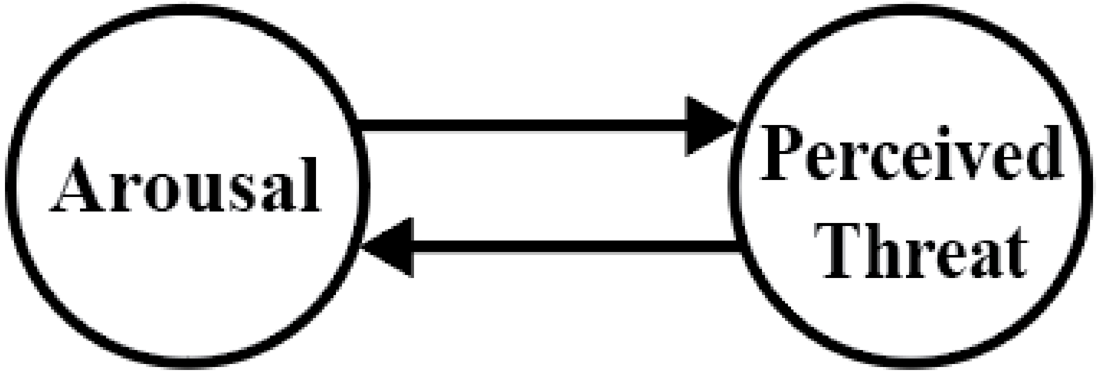
Modeling Panic Attacks with Difference Equations

Phenomenon: Panic attacks and Panic Disorder

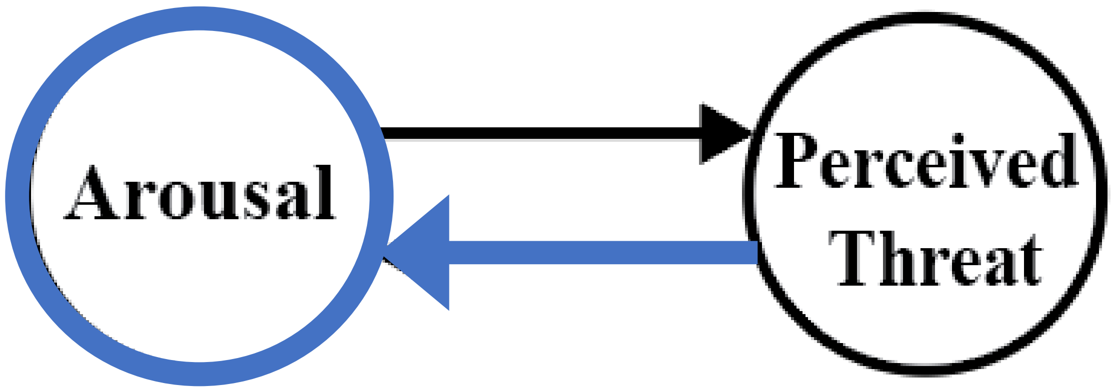
A verbal theory: If a stimulus "is perceived as a threat, a state of mild apprehension results. This state is accompanied by a wide range of bodily sensations. If these anxiety-produced sensations are interpreted in a catastrophic fashion, a further increase in apprehension occurs. This produces a further increase in body sensations and so on around in a vicious circle which culminates in a panic attack."



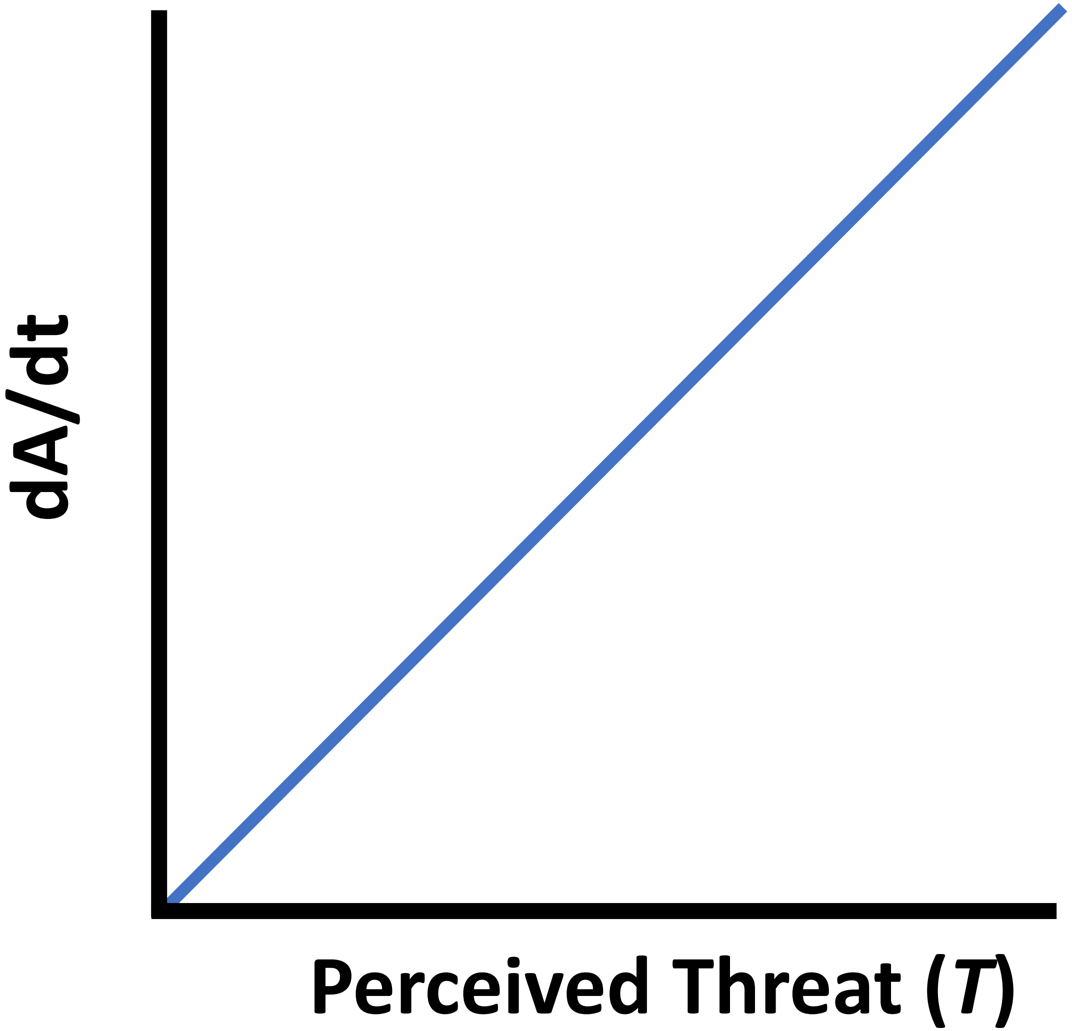
Clark, 1986, *A Cognitive Approach to Panic*

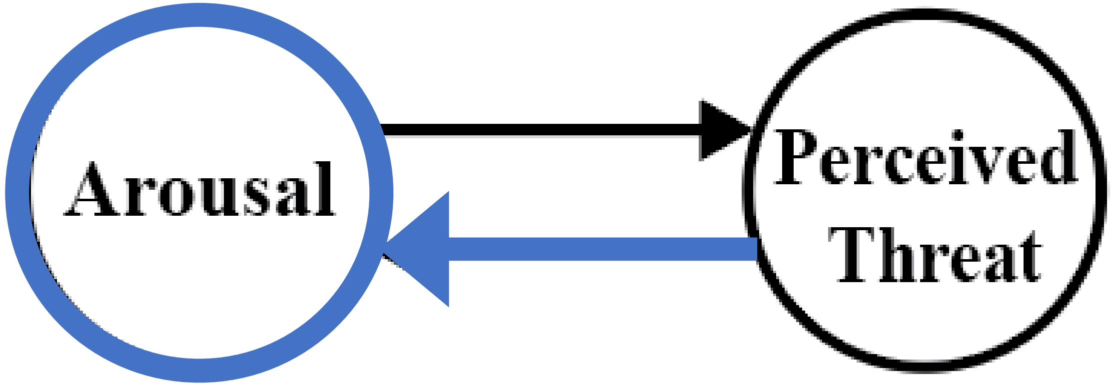

Arousal
Perceived
Threat

Formal theory: $A_{t+1} = A_t + T_t$

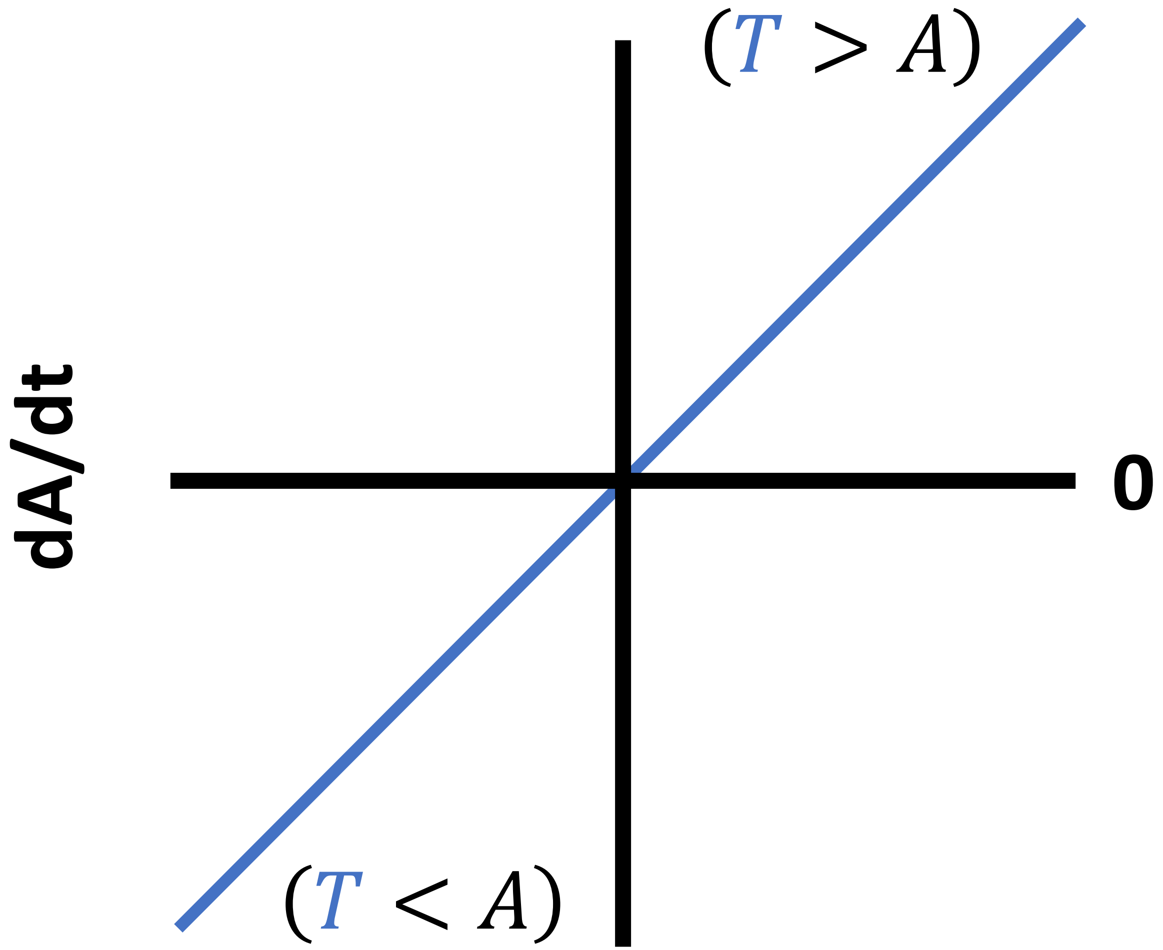


Formal theory: $\frac{dA}{dt} = (T)$

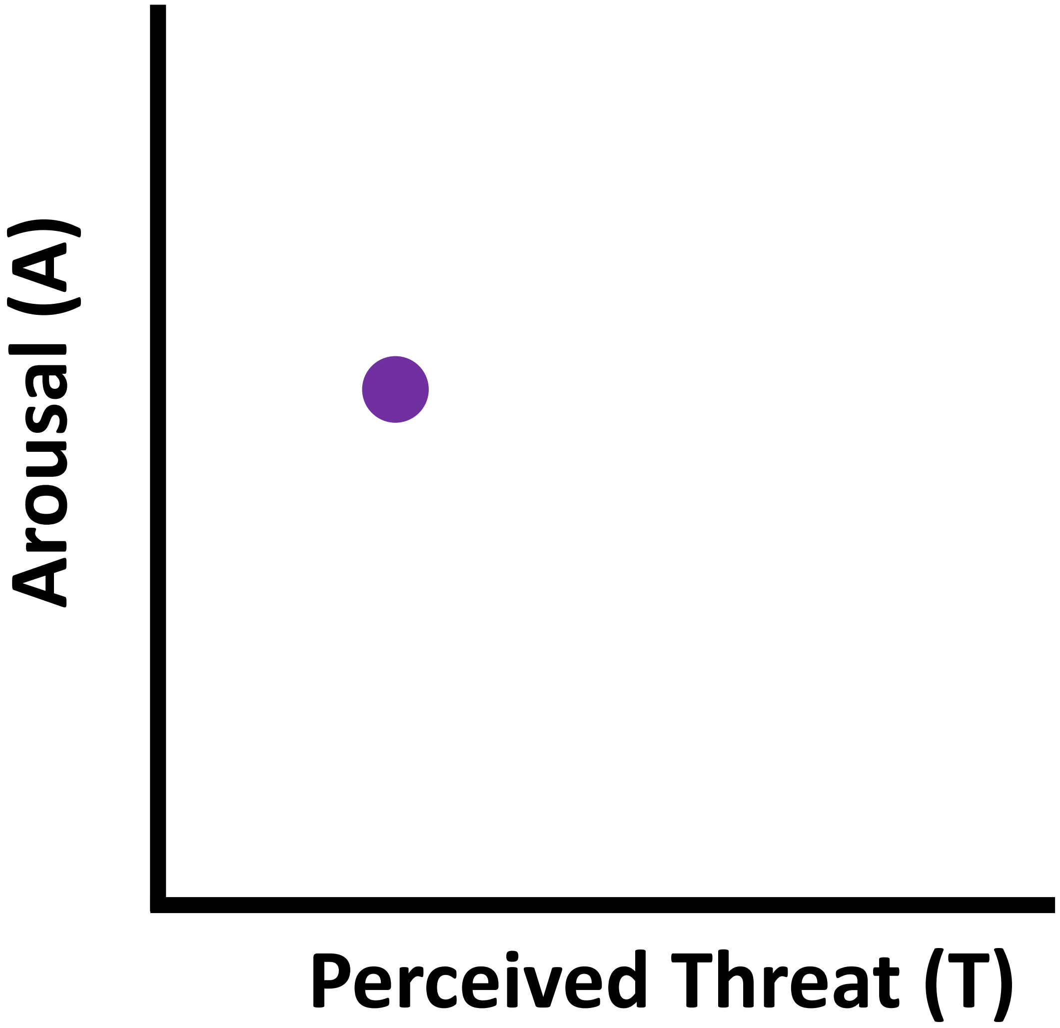




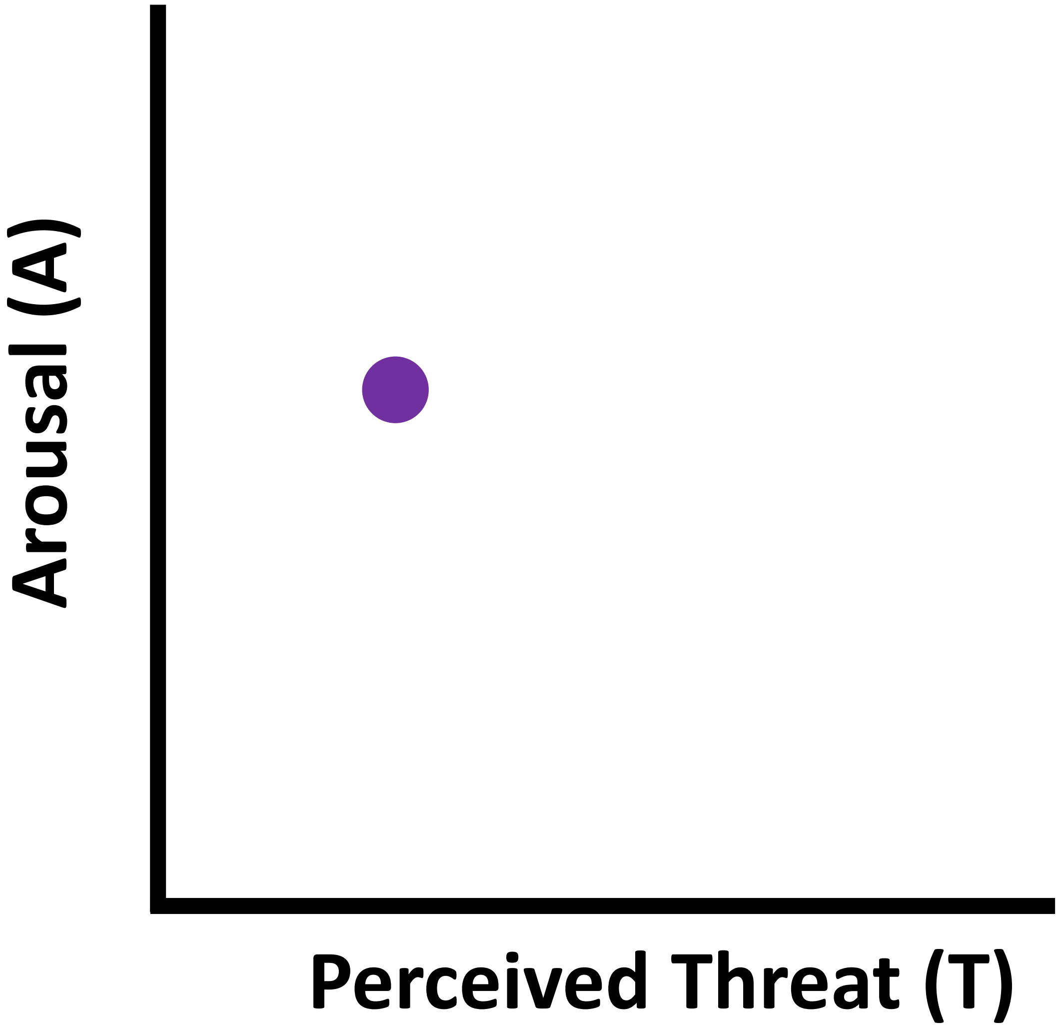
Formal theory: $$\frac{dA}{dt} = (T - A)$$



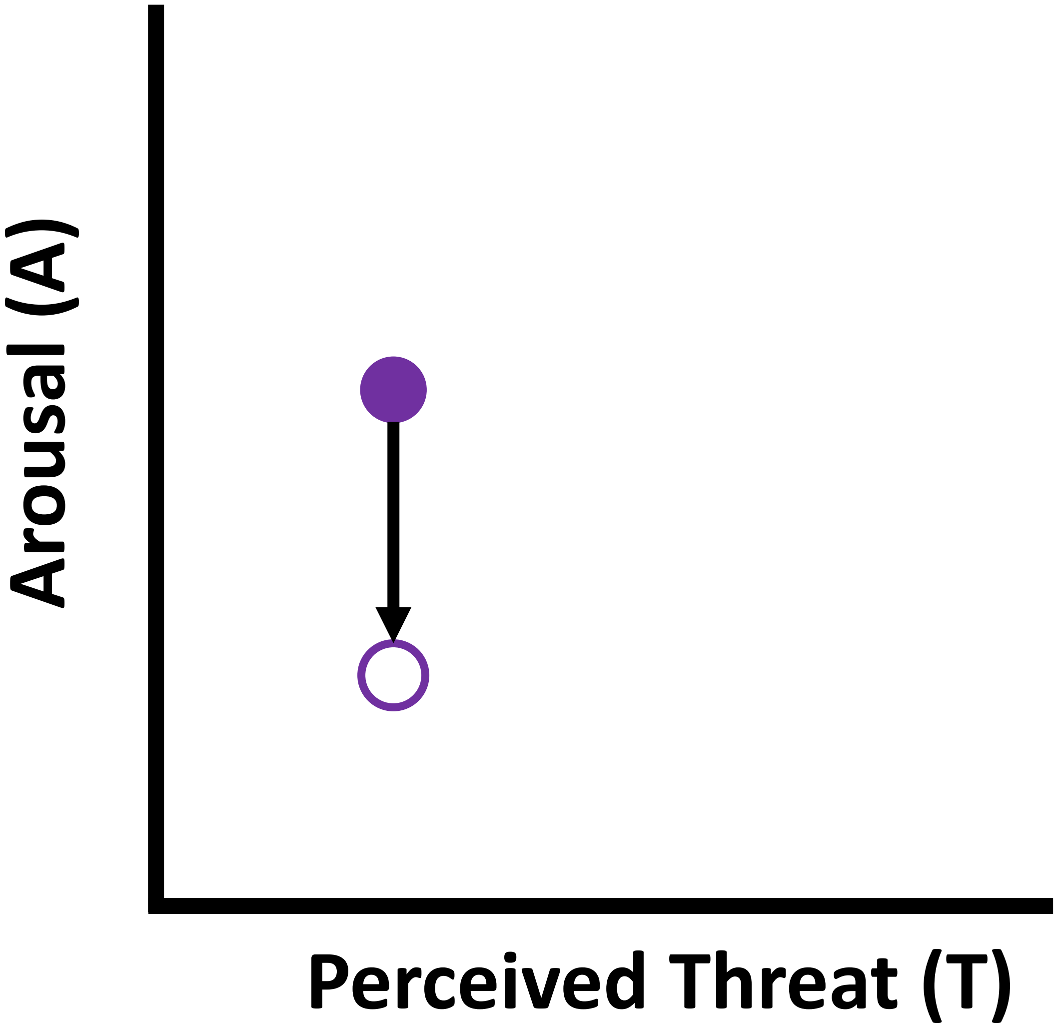
Formal theory: $$\frac{dA}{dt} = (T - A)$$



Formal theory: $$\frac{dA}{dt} = (T - A)$$

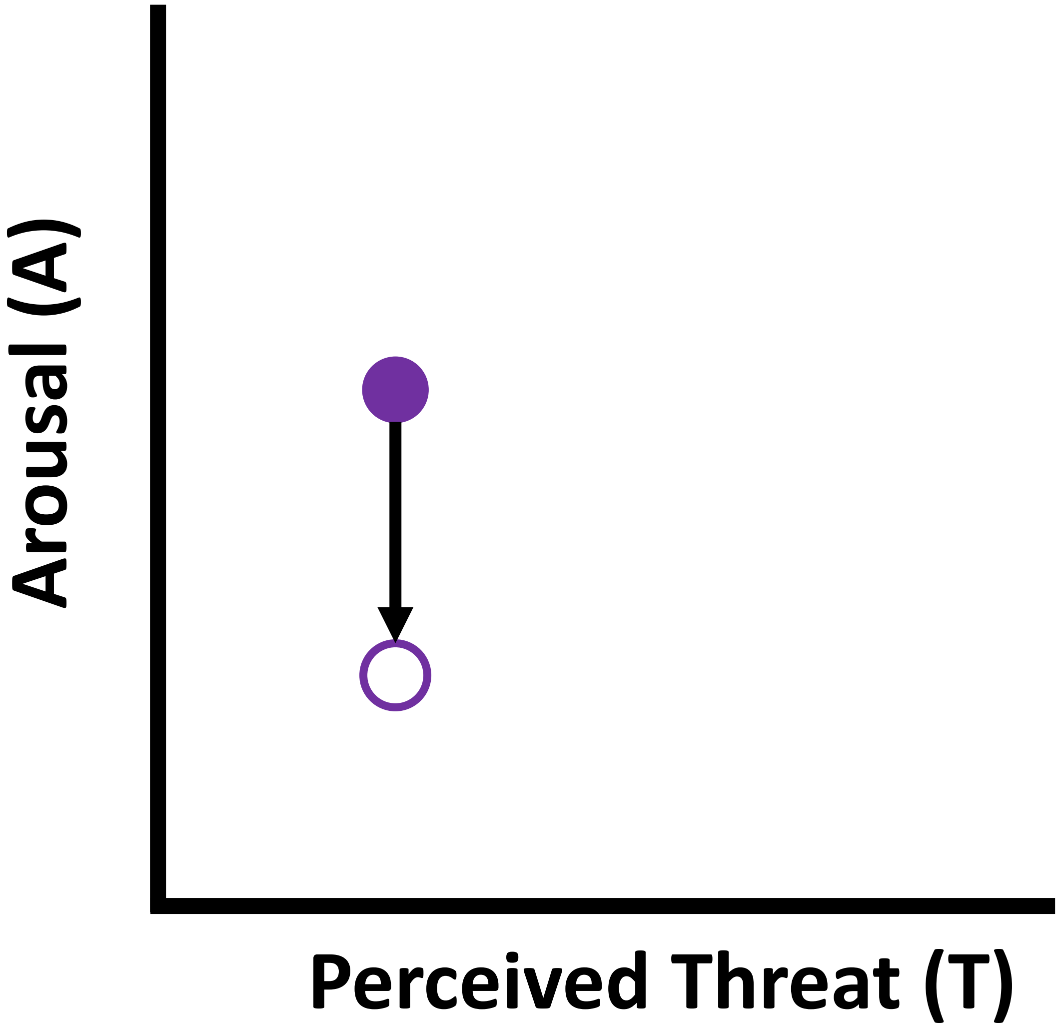


Formal theory: $$\frac{dA}{dt} = (T - A) \quad = (.25 - .50) \quad = -.25$$

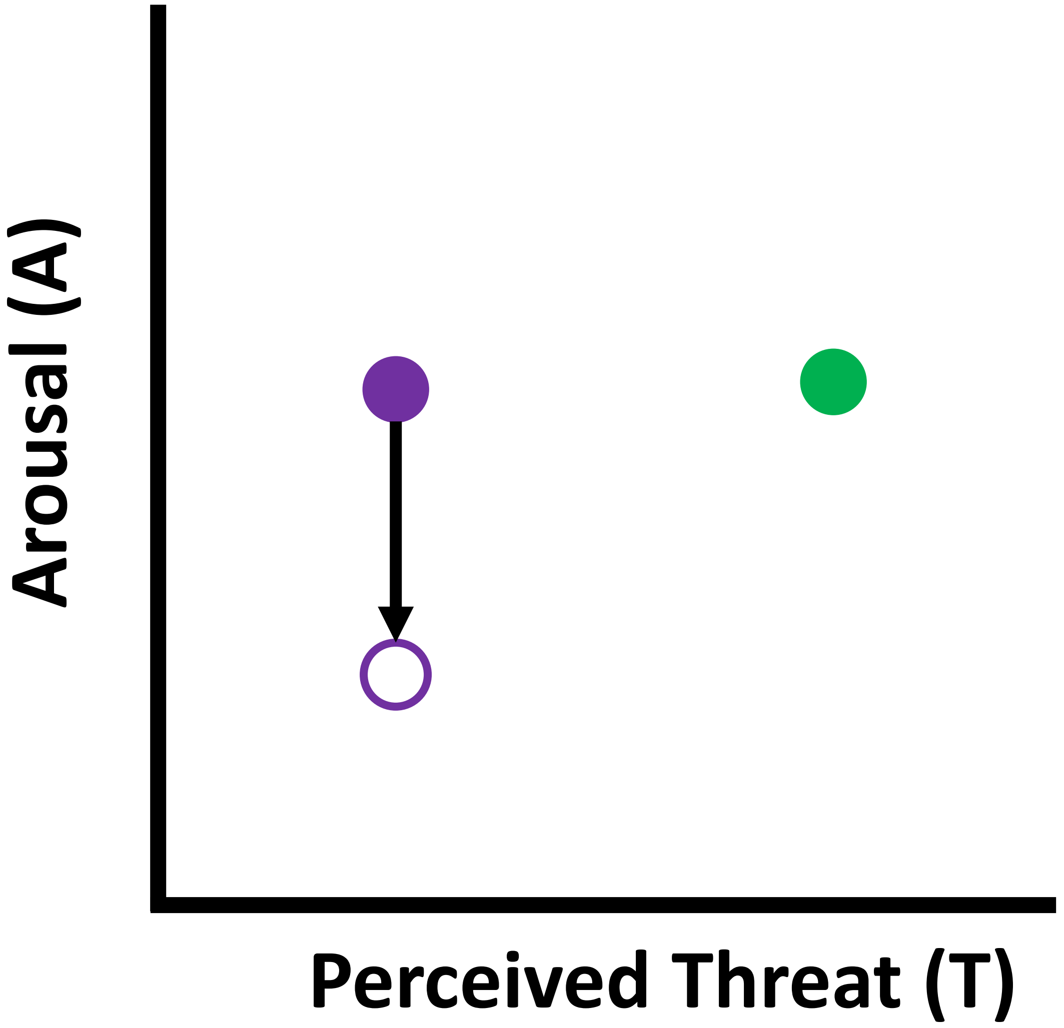


Formal theory: $\frac{dA}{dt} = (T - A)$ $= (.25 - .50)$ $= -.25$

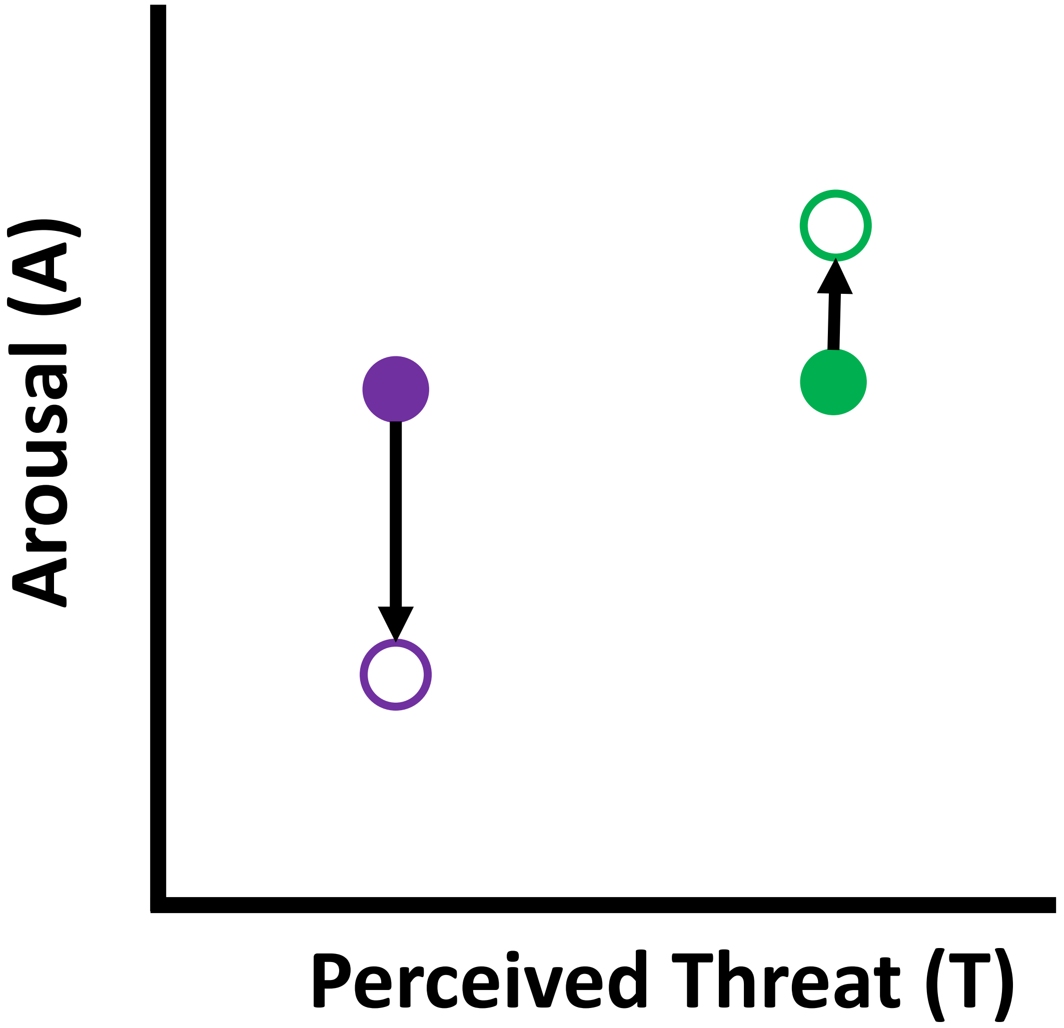
$= (.25 - .25)$ $= 0$



Formal theory: $$\frac{dA}{dt} = (T - A)$$

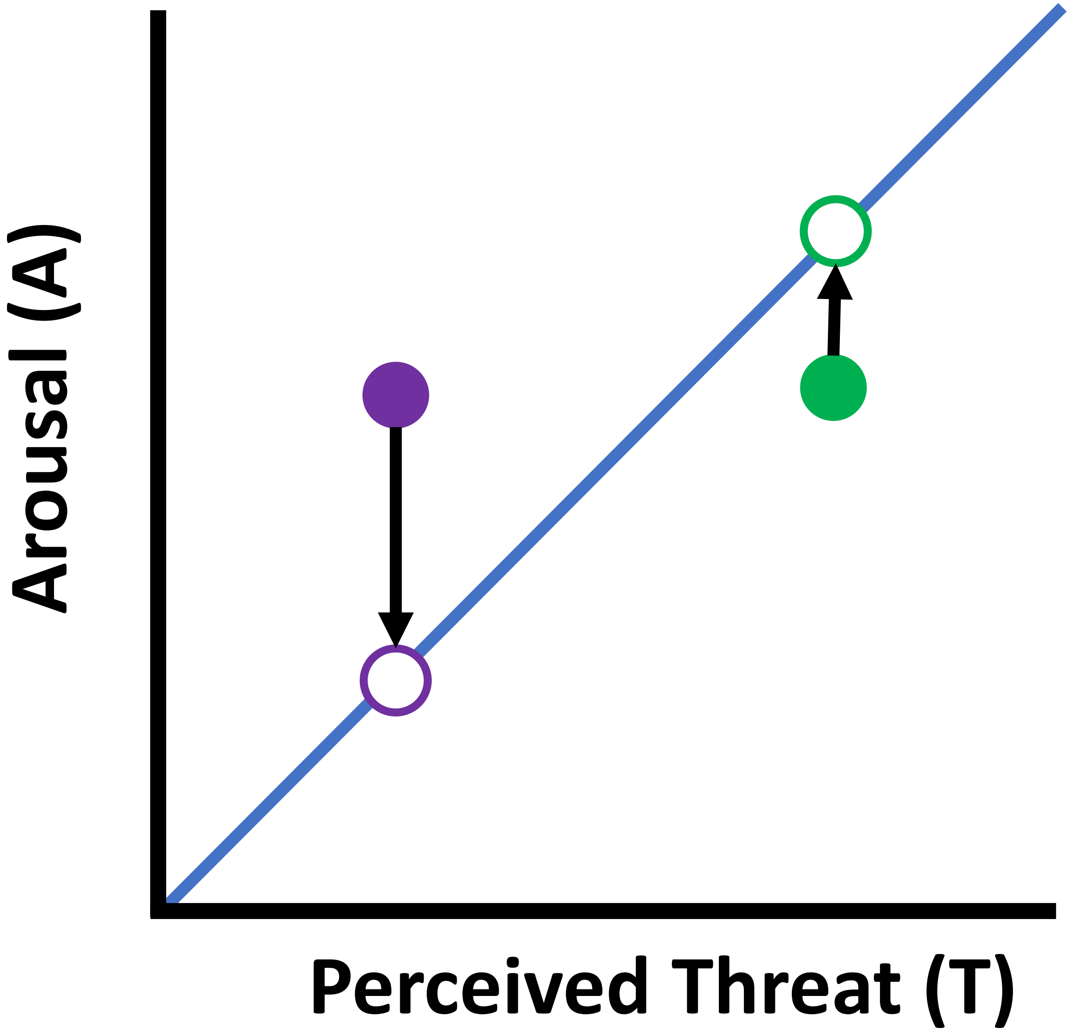


Formal theory: $\frac{dA}{dt} = (T - A) \quad = (.60 - .50) \qquad = .10$

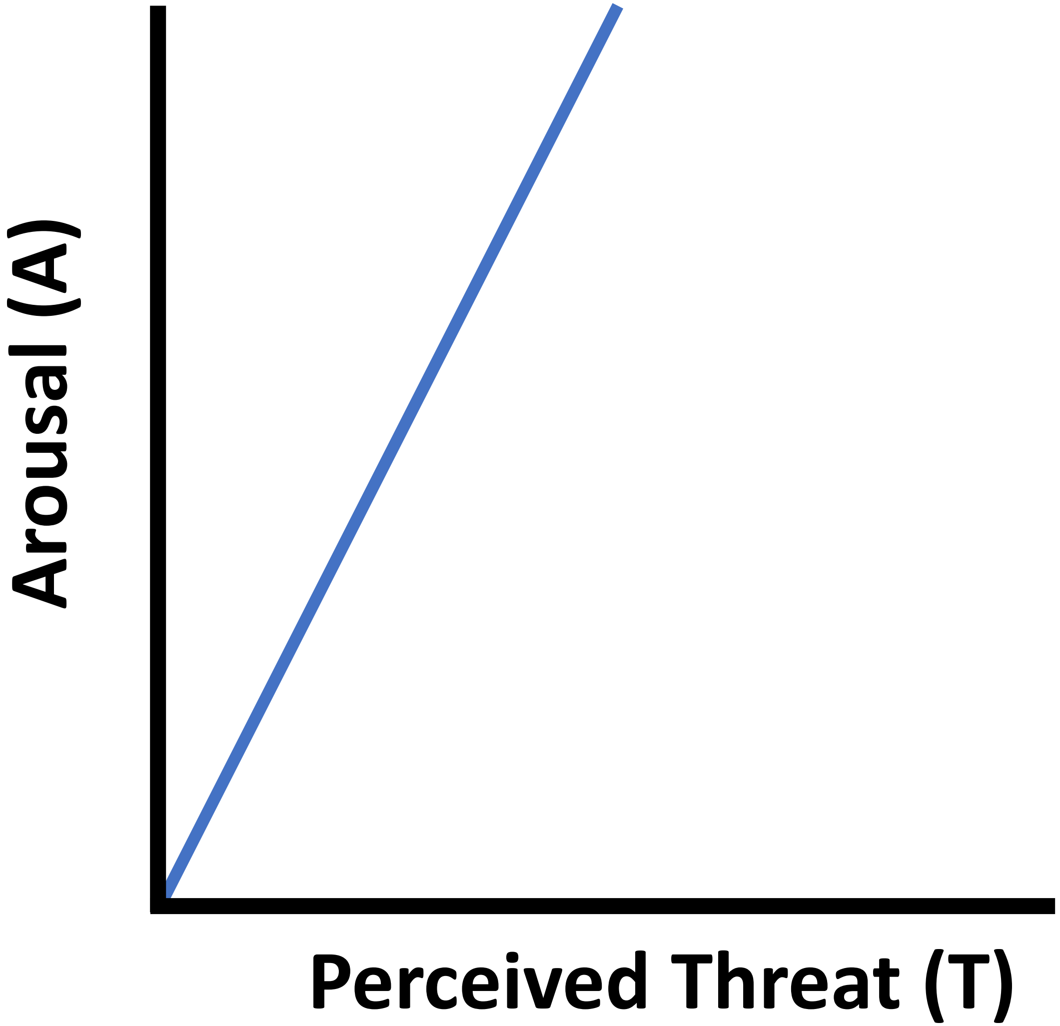


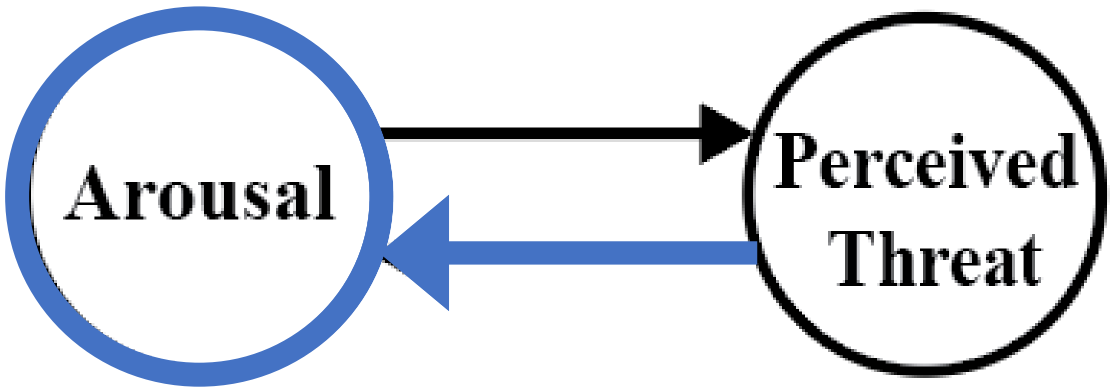
Formal theory: $\frac{dA}{dt} = (T - A)$ $= (.60 - .50)$ $= .10$

$= (.60 - .60)$ $= 0$

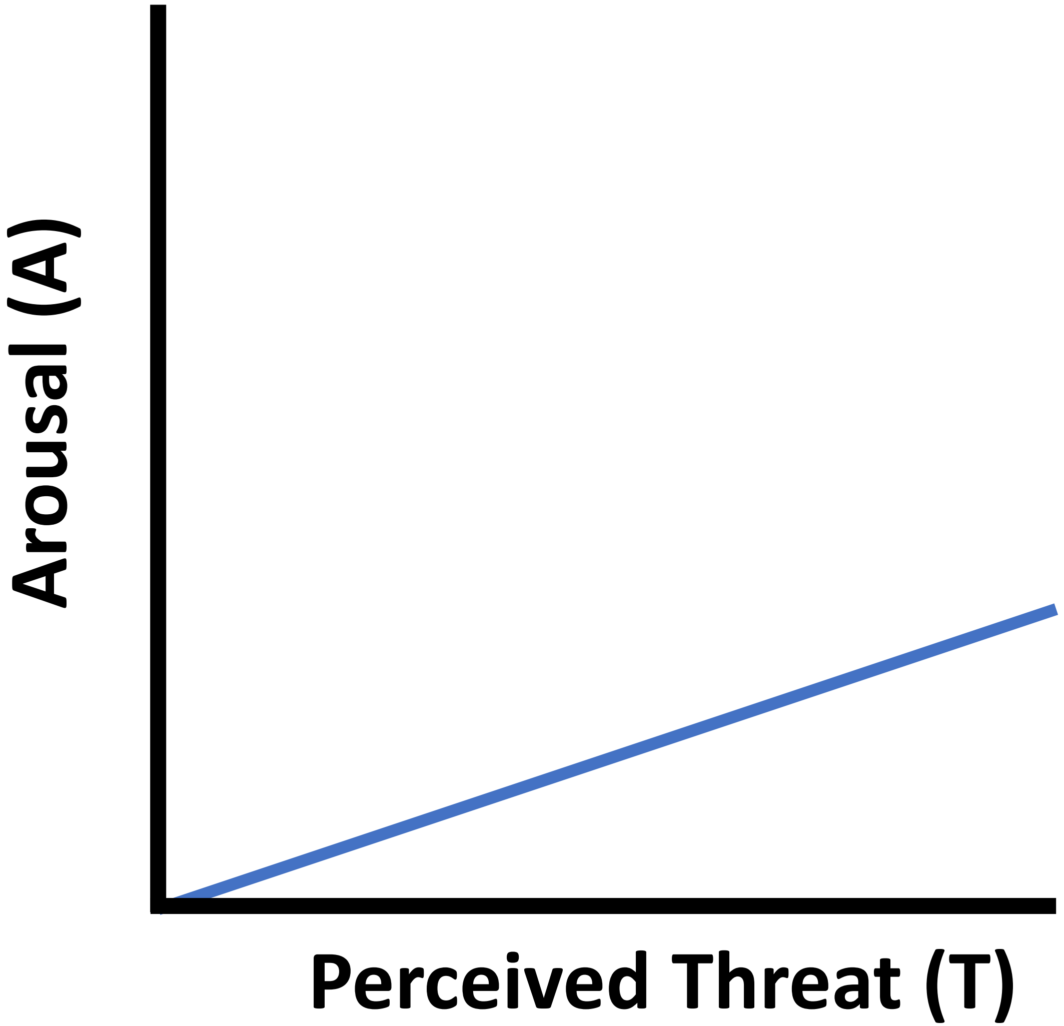


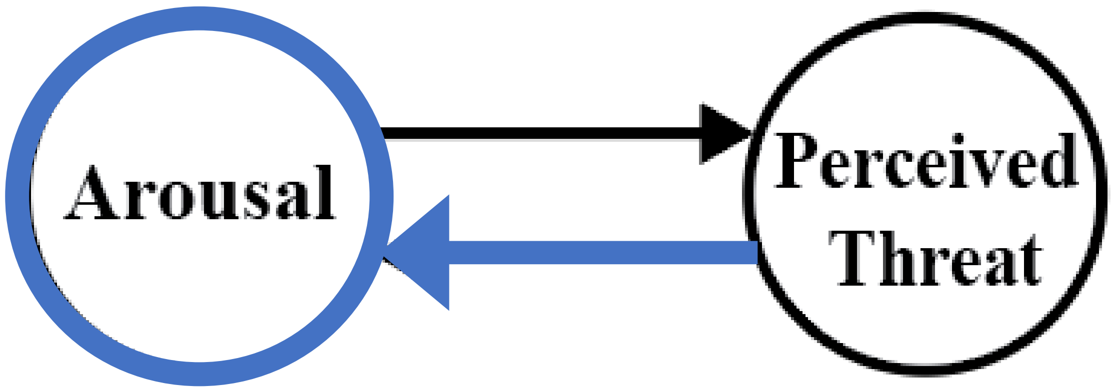
Formal theory: $\frac{dA}{dt} = (\beta T - A) \quad \beta=2$



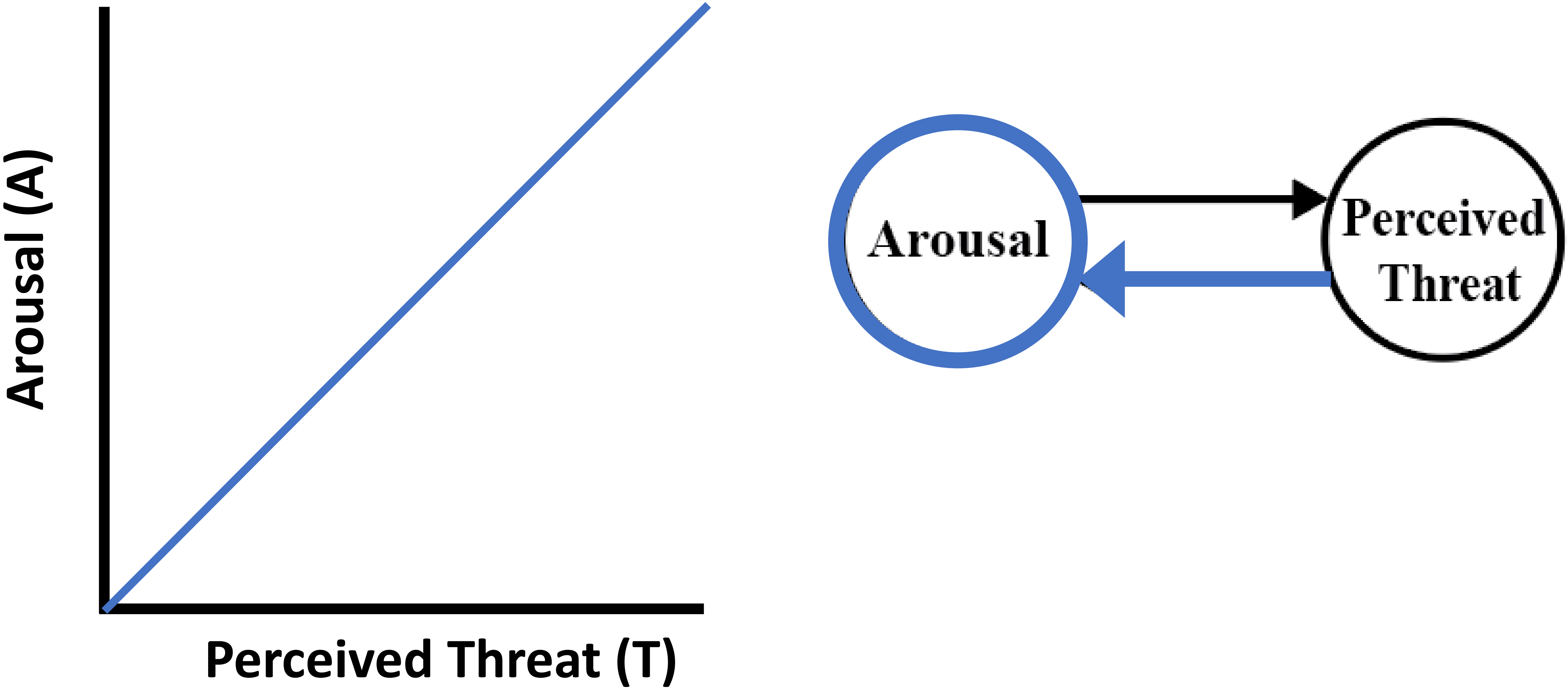


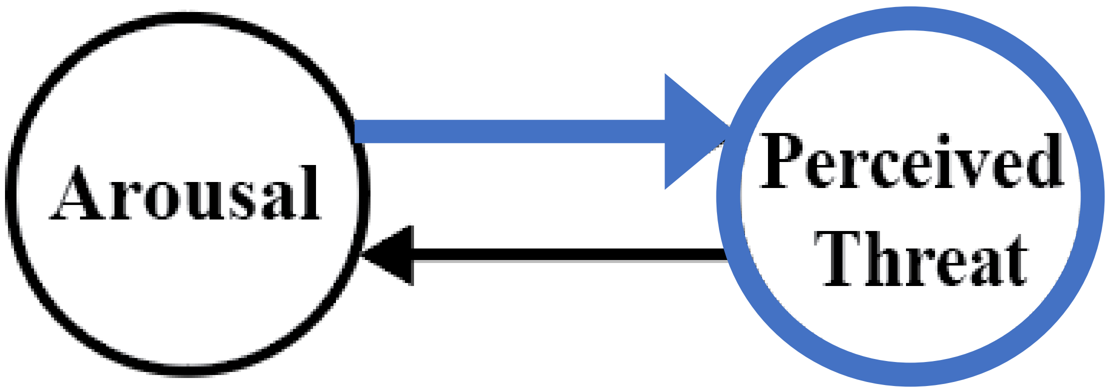
Formal theory: $\frac{dA}{dt} = (\beta T - A)$ β=.5



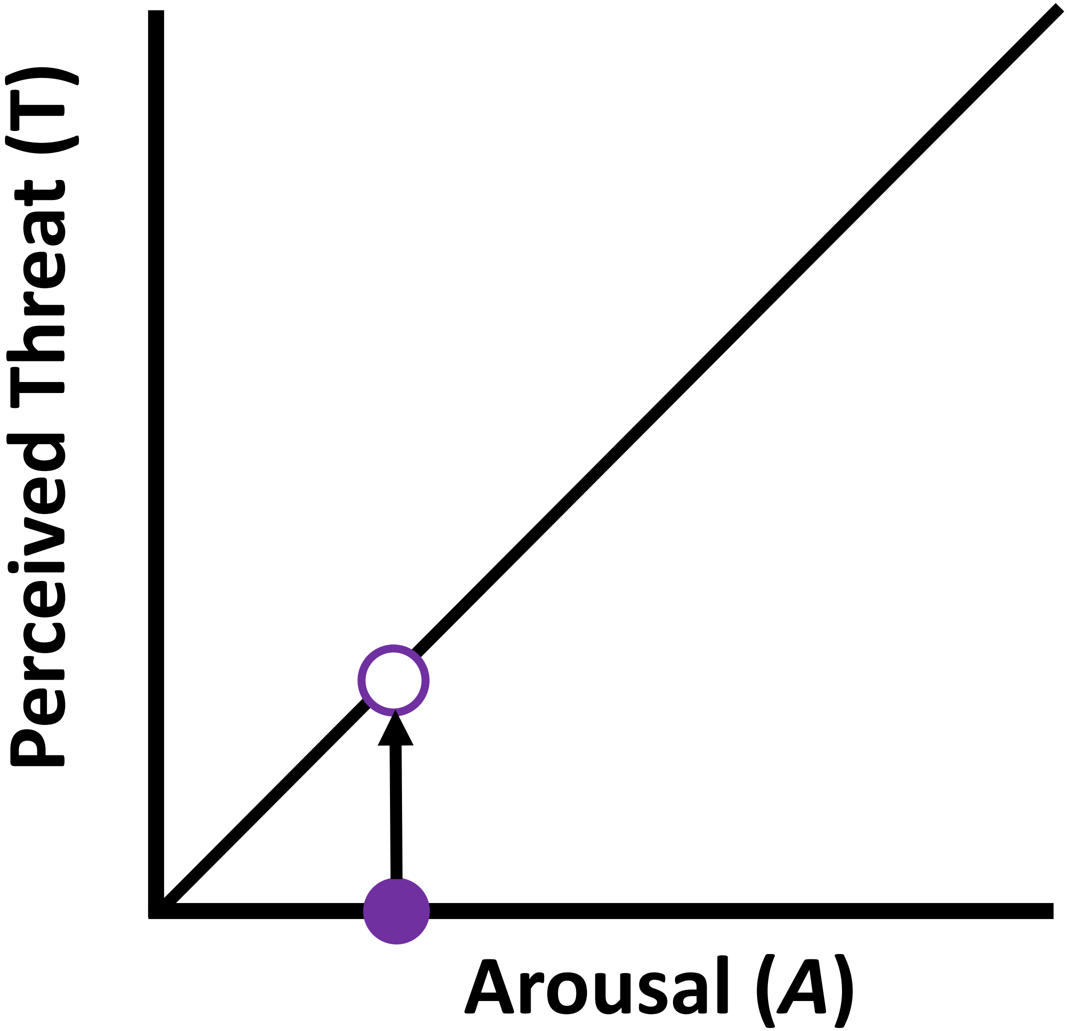


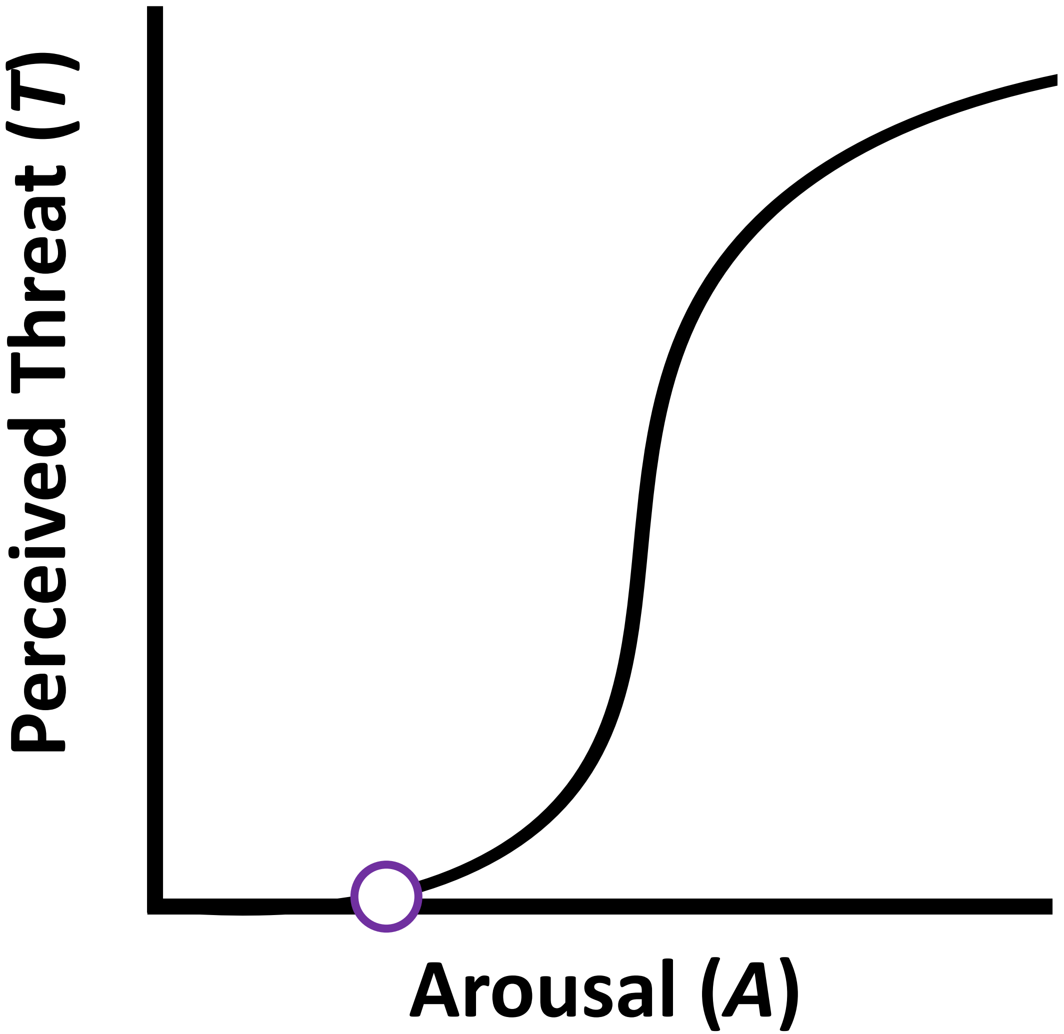
Formal theory: $\frac{dA}{dt} = (\beta T - A)$ β=1



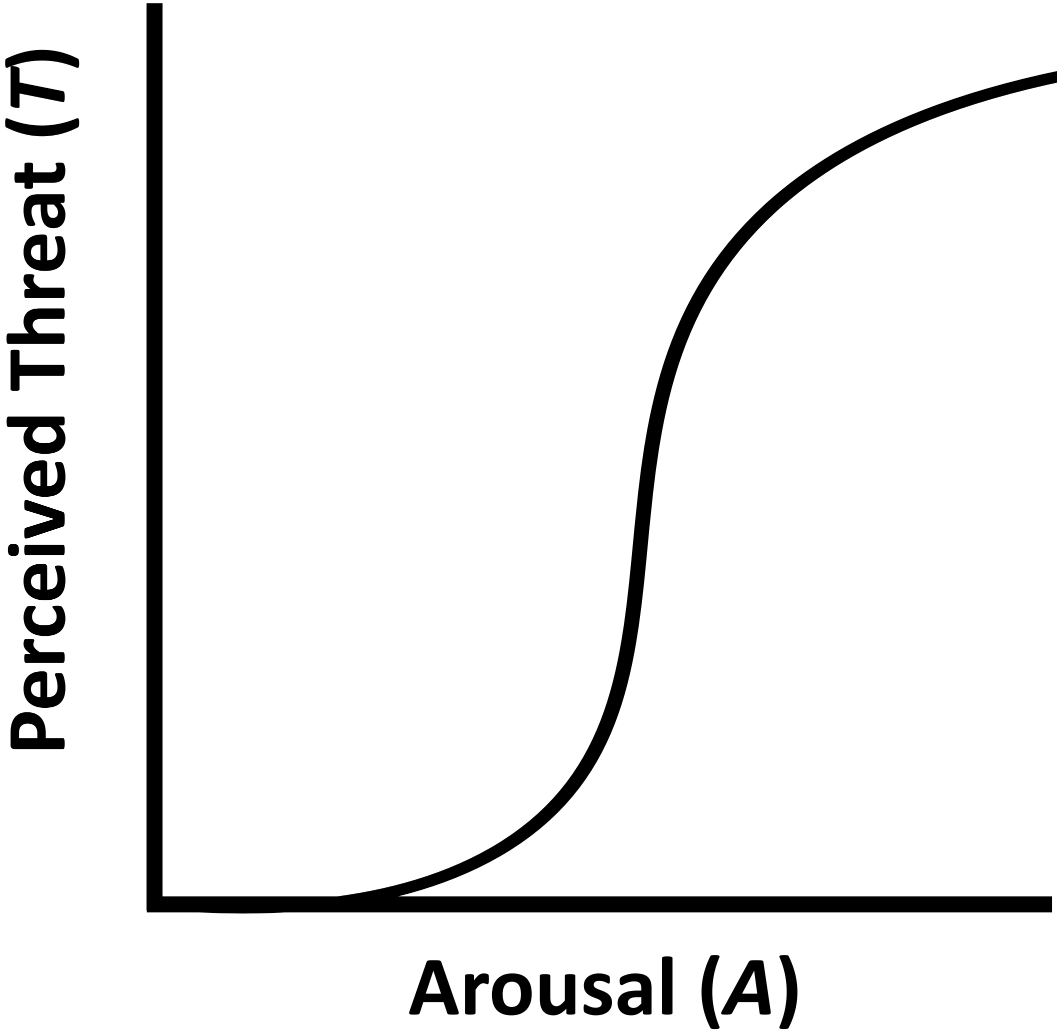

Arousal
Perceived Threat

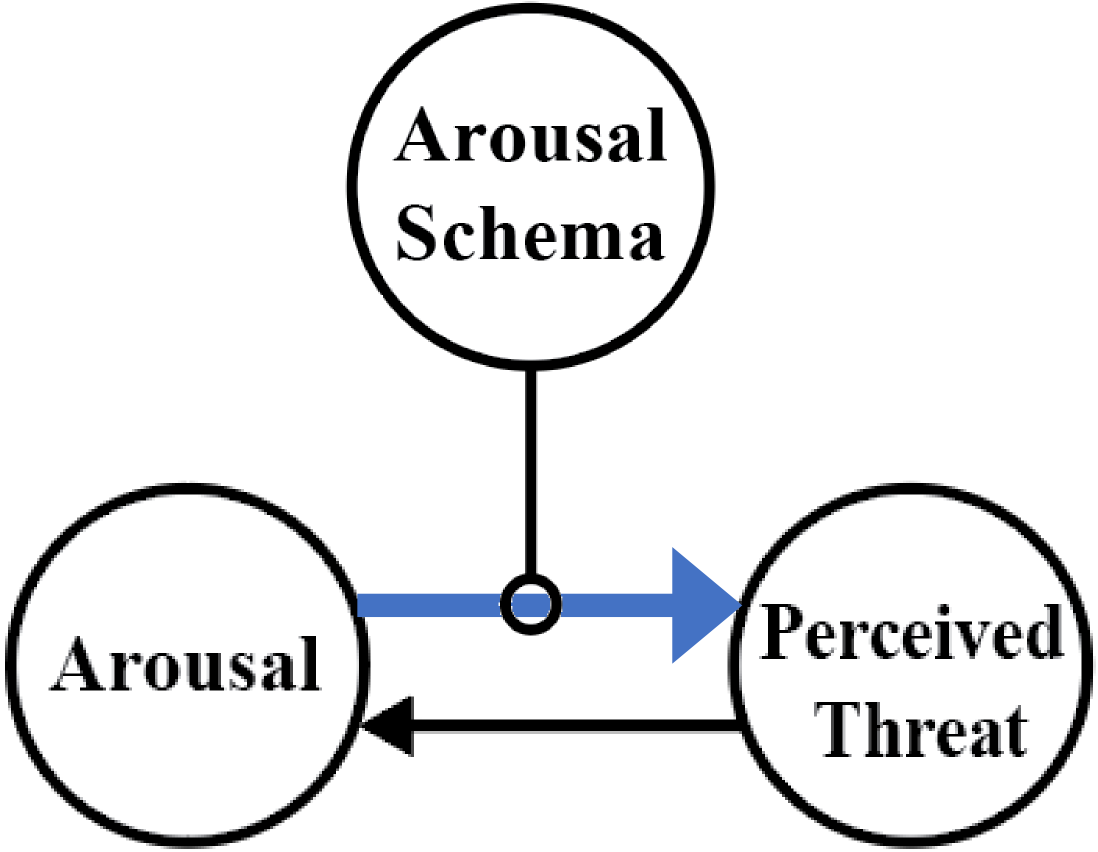
$$\frac{dT}{dt} = (\beta A - T)$$





$$\frac{dT}{dt} = \gamma\left(\frac{A^\mu}{A^\mu + \lambda^\mu} - T\right)$$

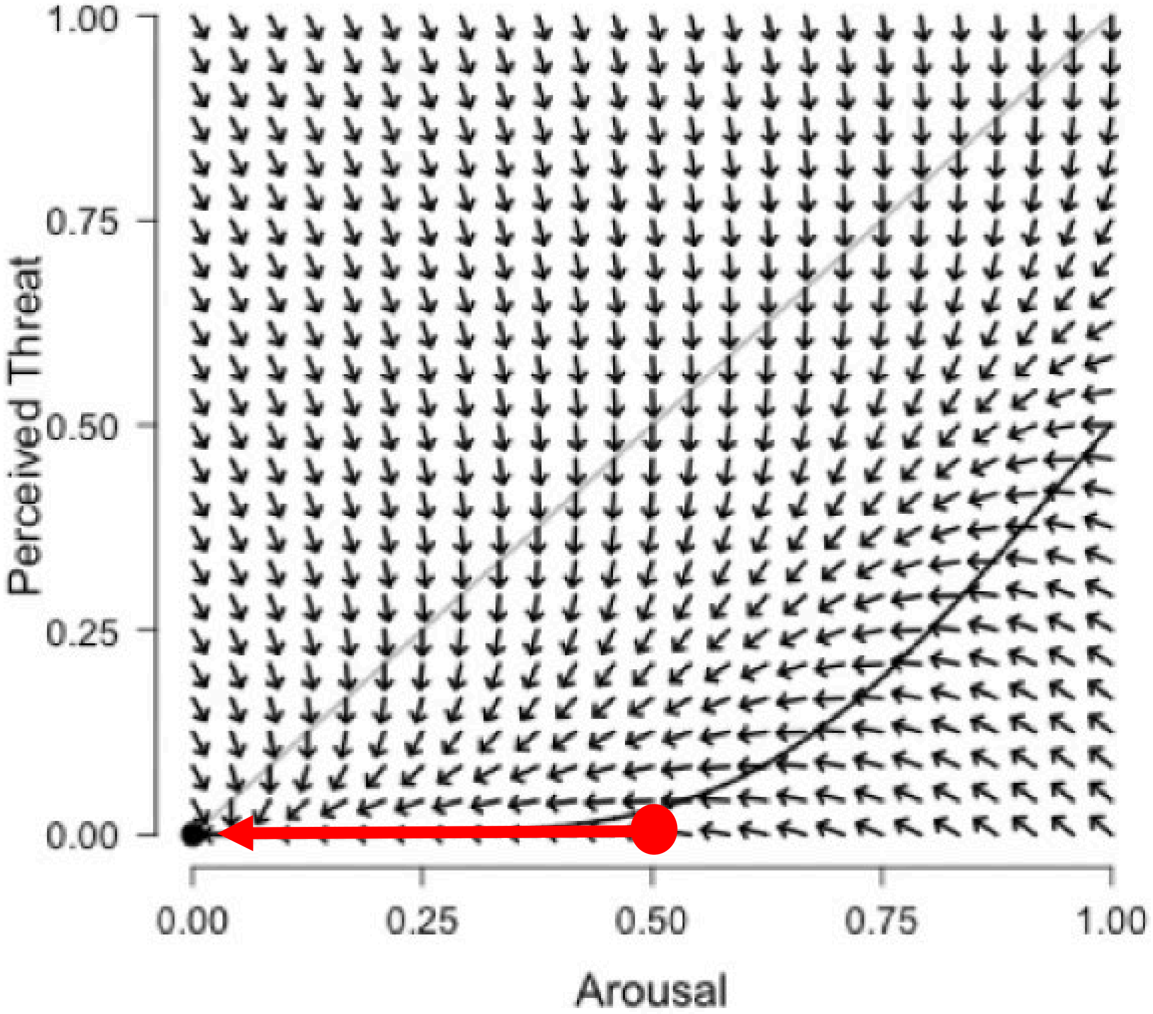


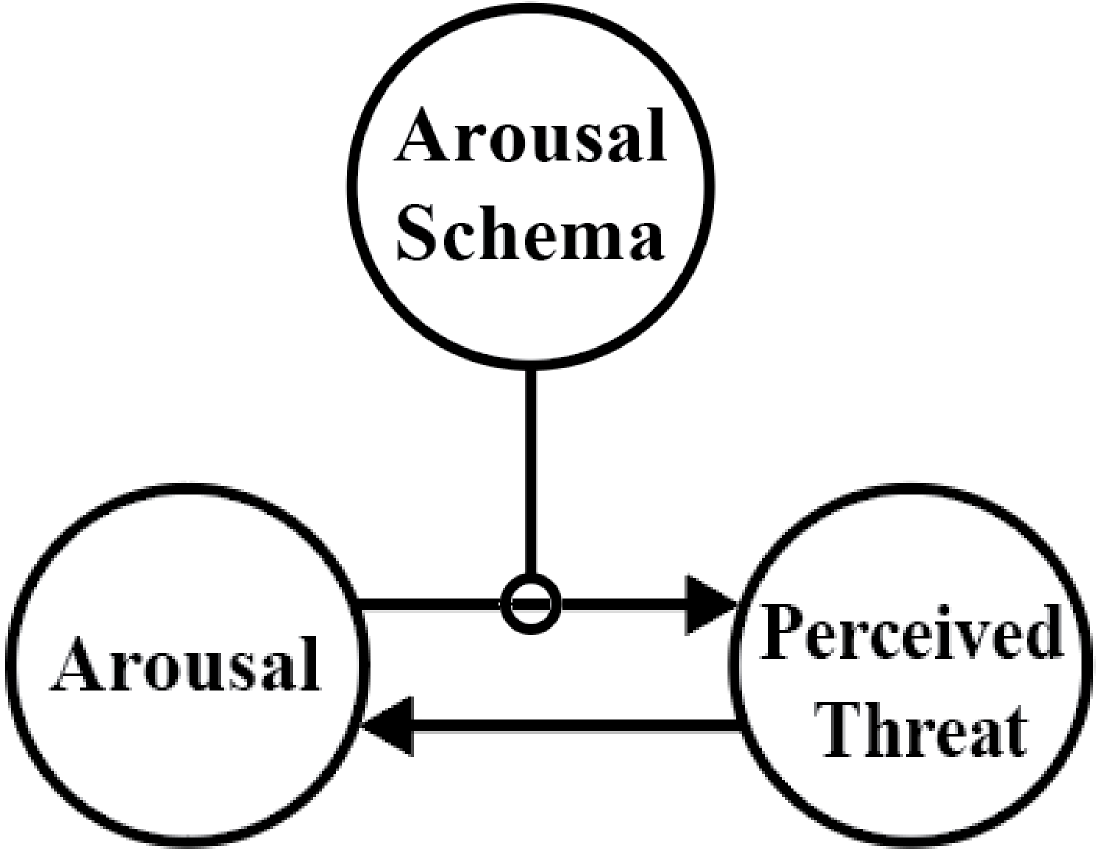


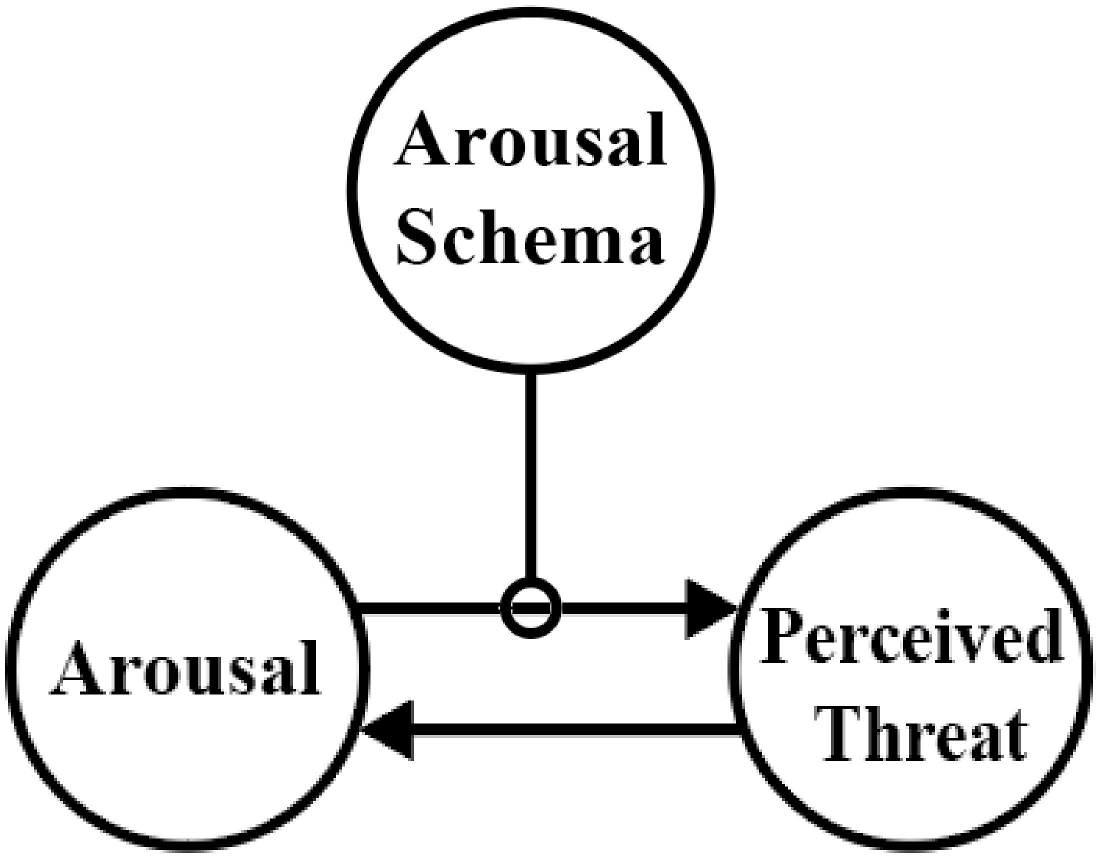
$$\frac{dT}{dt} = \gamma\left(\frac{A^{\mu}}{A^{\mu} + \lambda^{\mu}} - T\right)$$

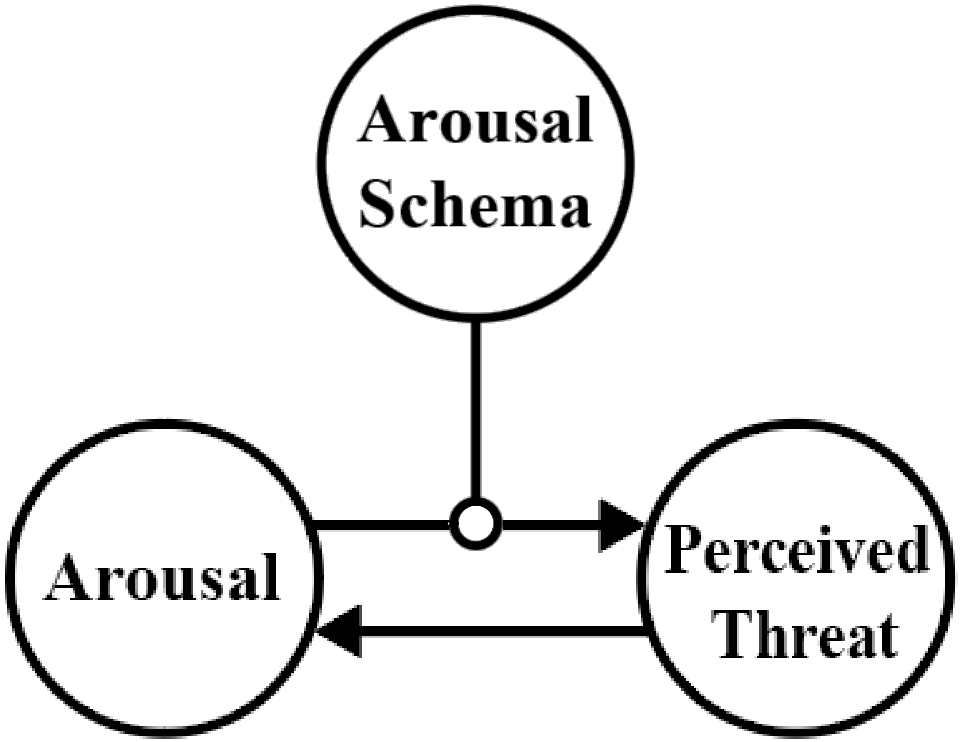
$$\lambda = 1 - \frac{S}{S + \xi}$$

Arousal Schema (S) = 0.00

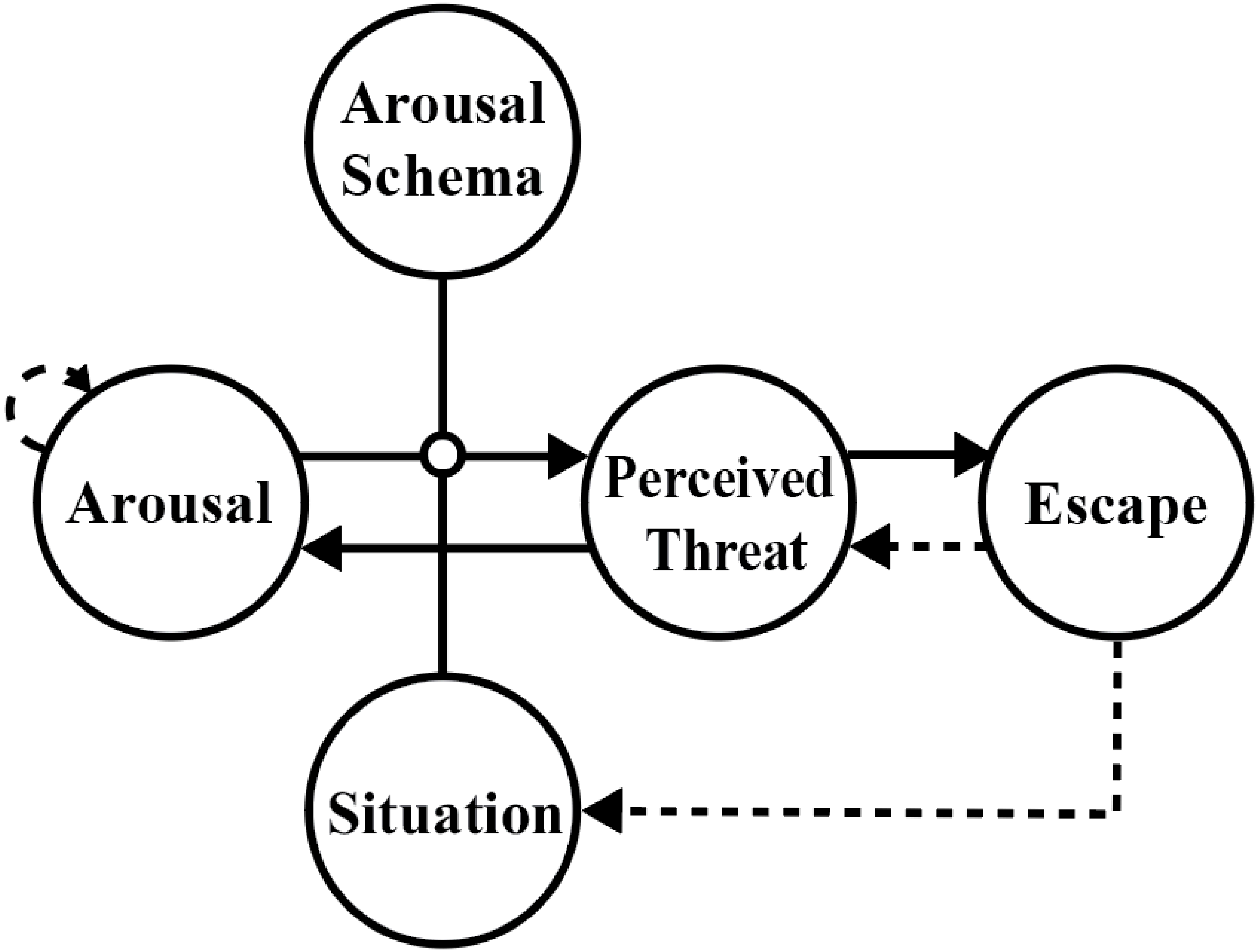





Arousal
Schema
Arousal
Perceived
Threat



Robinaugh et al., 2019, *Advancing the network theory of mental disorders: A computational model of panic disorder*



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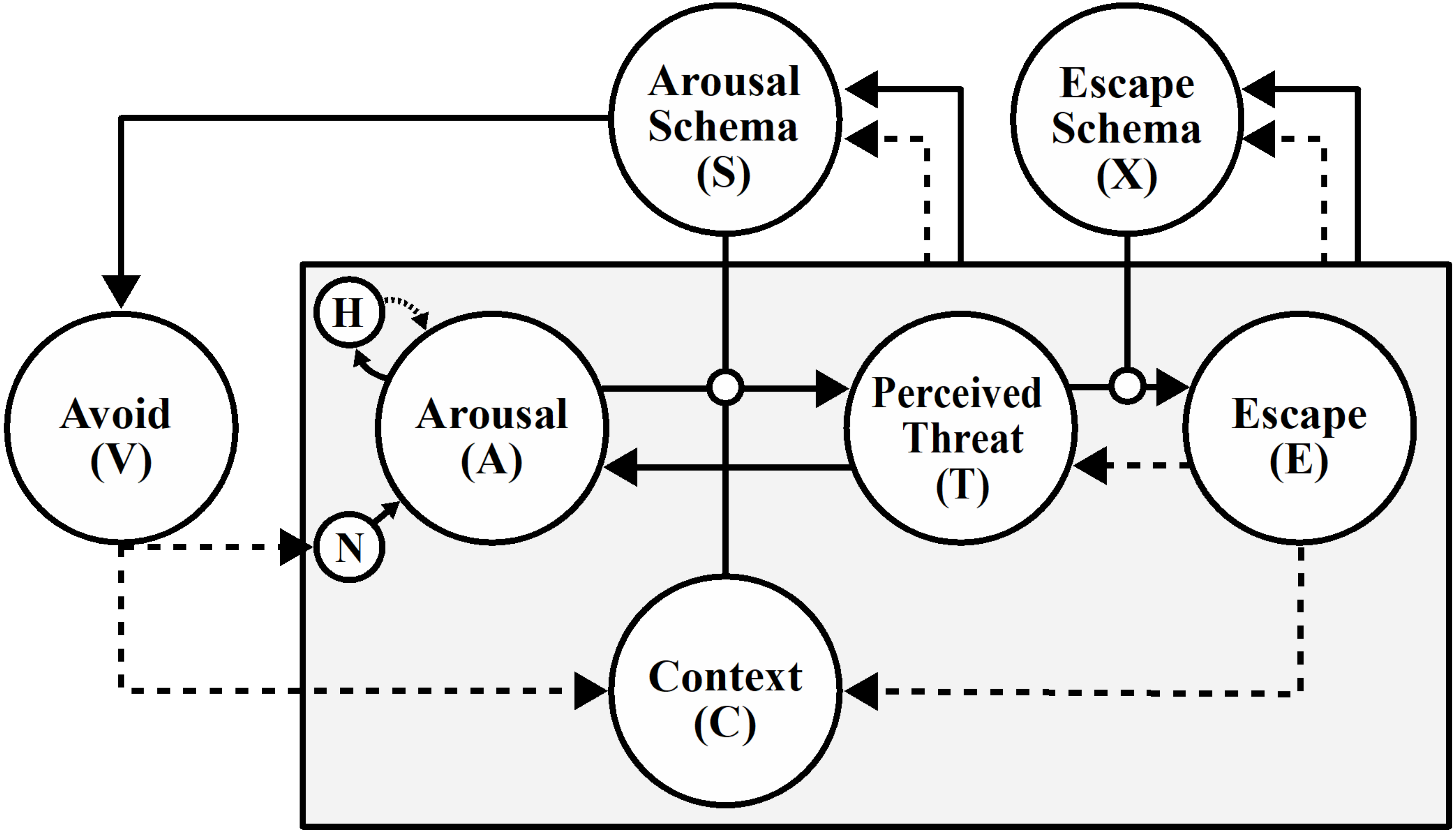
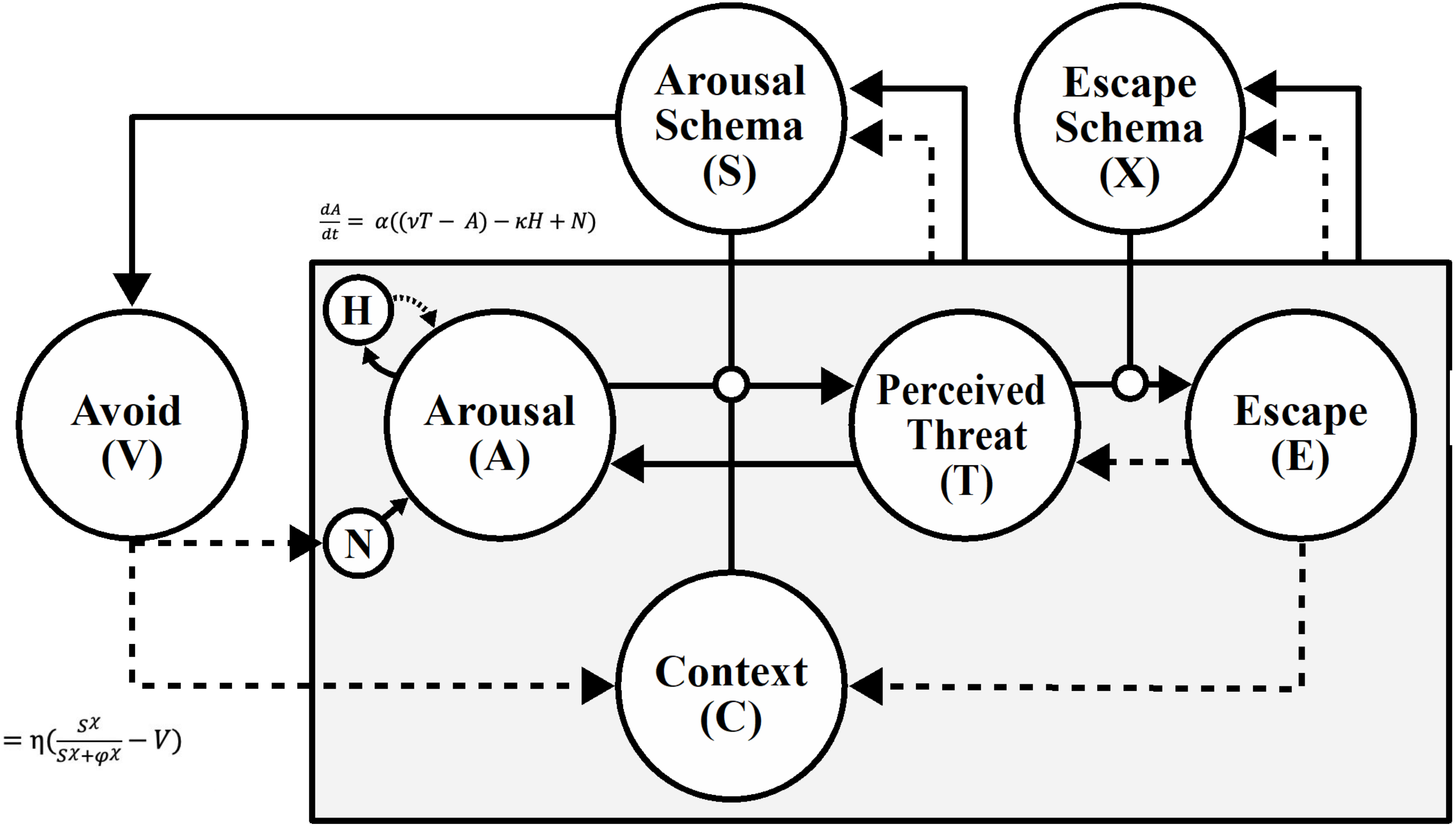


Robinaugh et al., 2019, *Advancing the network theory of mental disorders: A computational model of panic disorder*

$$\frac{dS}{dt} = \begin{cases} 0, & if\ \max(F_{t-\Omega} \ldots F_t) < \psi \\ \zeta_A(\max(T_{t-\Omega} \ldots T_t) - S), & if\ \max(F_{t-\Omega} \ldots F_t) \geq \psi, \max(E_{t-\Omega} \ldots E_t) > \omega \\ -\zeta_E\, S, & if\ \max(F_{t-\Omega} \ldots F_t) \geq \psi,\ \max(E_{t-\Omega} \ldots E_t) \leq \omega \end{cases}$$

$$\frac{dA}{dt} = \alpha((\nu T - A) - \kappa H + N)$$



$$\frac{dE}{dt} = \varepsilon\left(\frac{T^\sigma}{T^\sigma + \rho^\sigma} - E\right)$$

$$\frac{dT}{dt} = \gamma\left(\left(\frac{A^\mu}{A^\mu + \lambda^\mu} - T\right) - \tau E\right)$$

$$\frac{dV}{dt} = \eta\left(\frac{S^\chi}{S^\chi + \varphi^\chi} - V\right)$$

Robinaugh et al., 2019, *Advancing the network theory of mental disorders: A computational model of panic disorder*

A Computational Model of Panic Disorder

```
simPanic <- function(time_steps, stepsize)
{
   for(i in 1:(nIter)) {
       A_eq <- s_PT_A*PT[i]
       A_eq2 <- -s_H_A*H[i]
       A[i+1] <- A[i] + r_A*((A_eq - A[i]) + A_eq2)*stepsize
   …
   {
outlist <- list("A" = A, "PT" = PT, "H" = H, "E" = E)
return(outlist)
}
```

What does this earn us?

A tool to evaluate our theory!

Theory Evaluation

Simulation 1: Biological Challenge

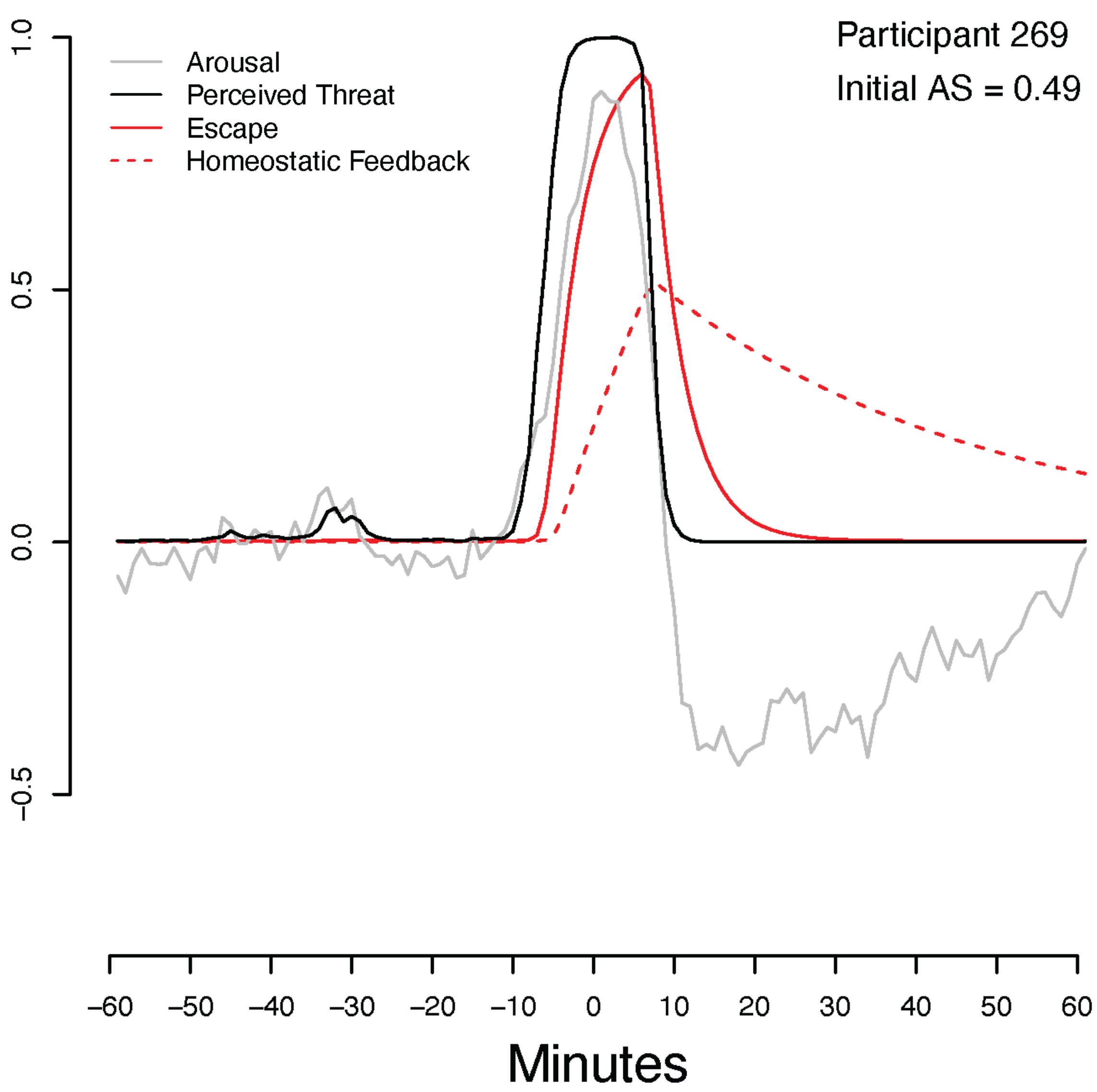
Simulation 2: 3-Month Simulation

Simulation 3: Treatment Study

Phenomenon 1

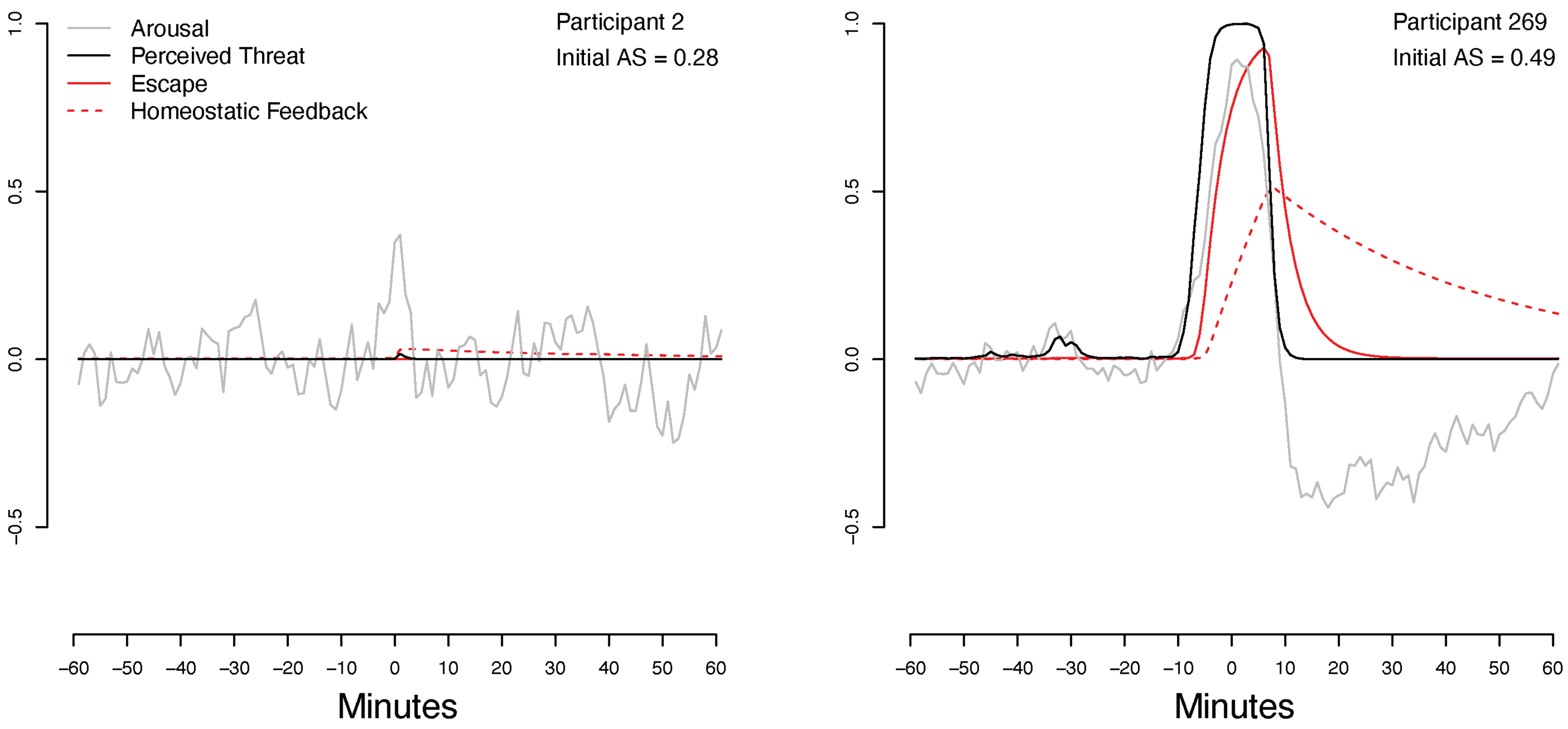
Panic Phenomenology

Some people experience surges of intense fear and somatic symptoms that come on “out of the blue.”



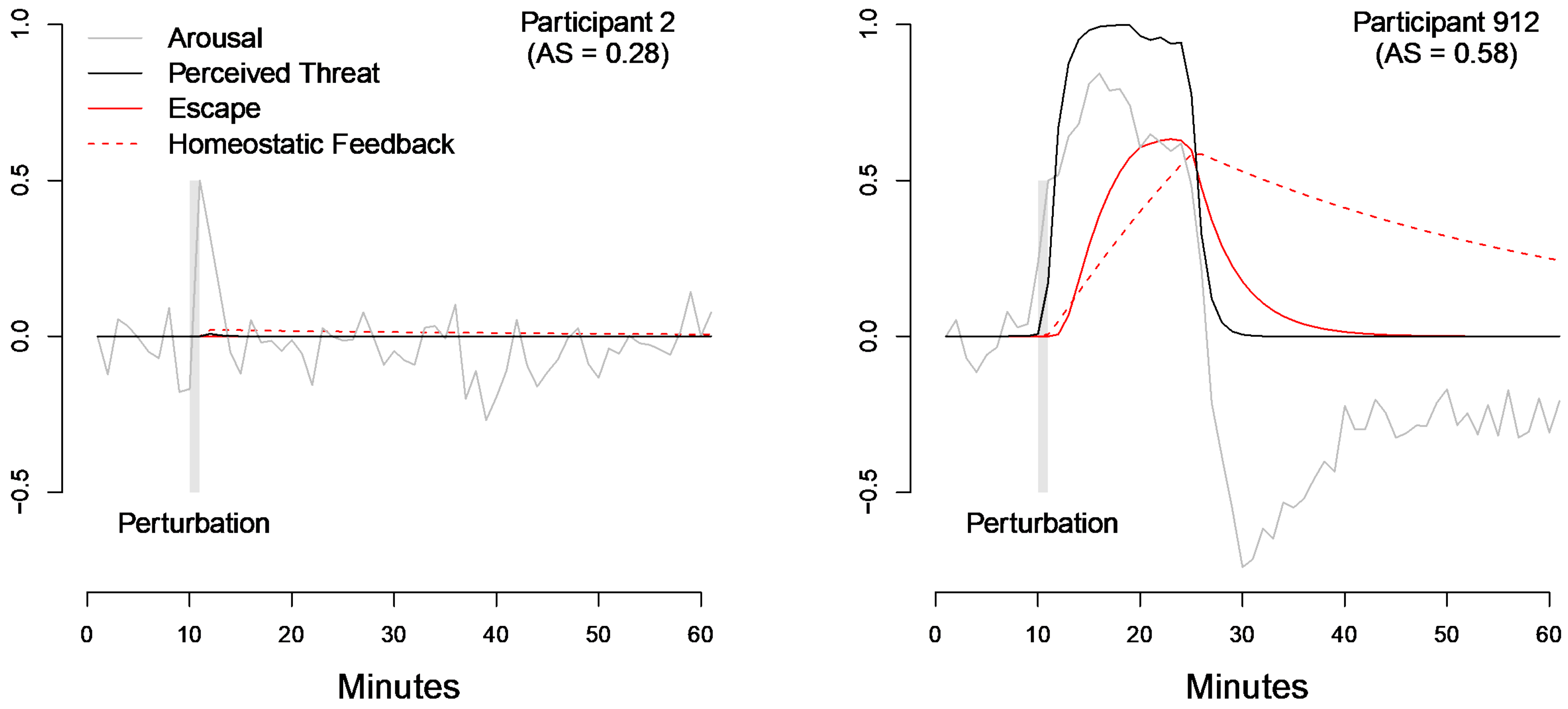
Phenomenon 2

Individual Differences



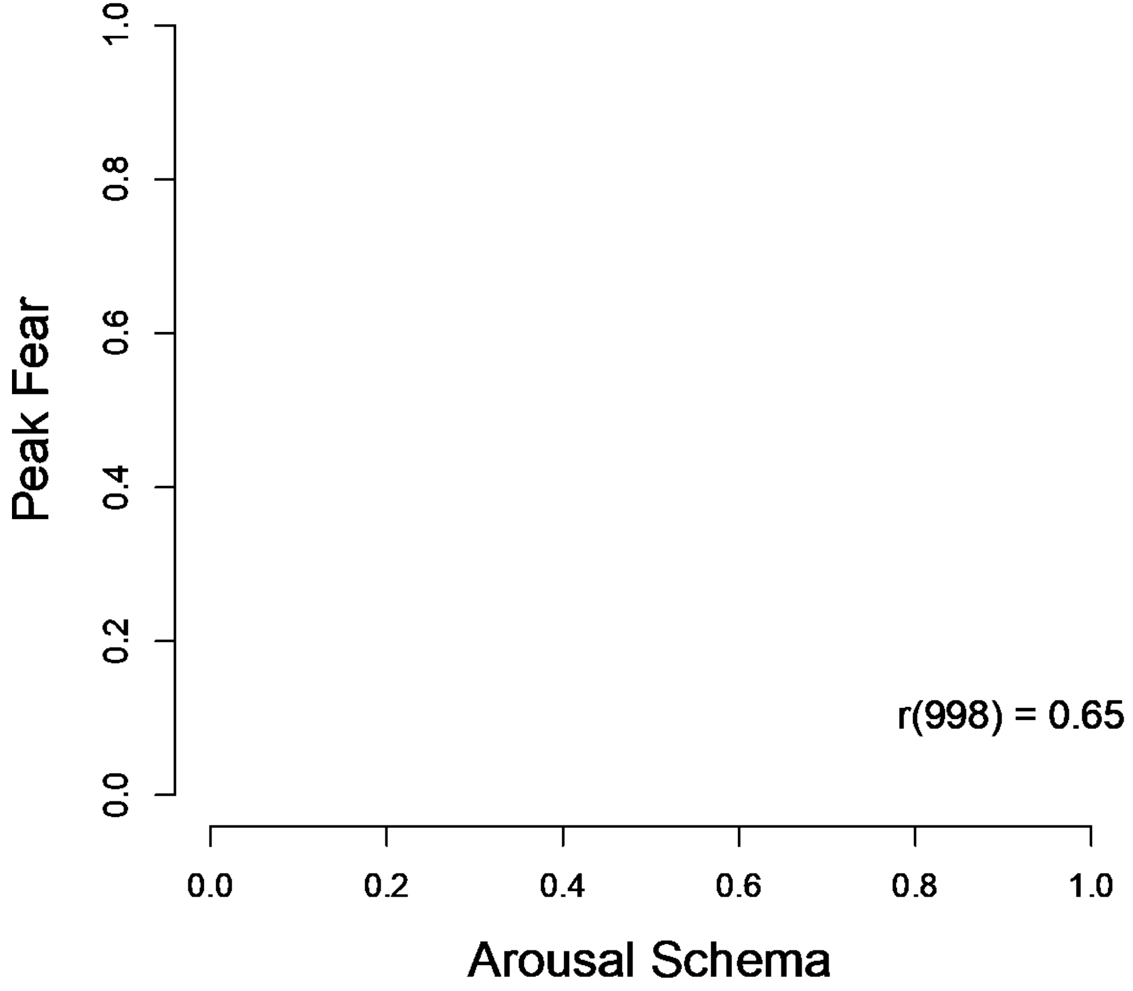
Phenomenon 2

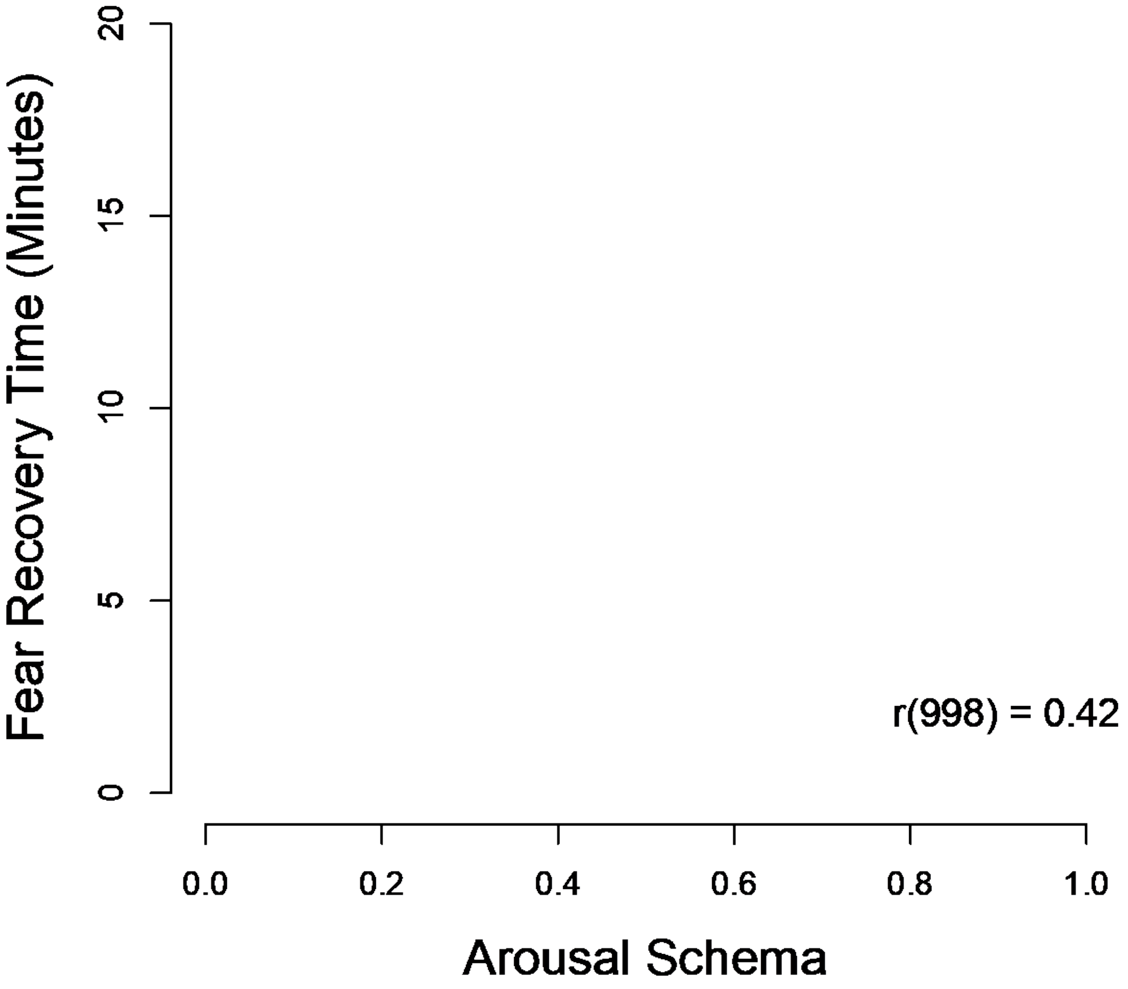
Individual Differences



Phenomenon 2

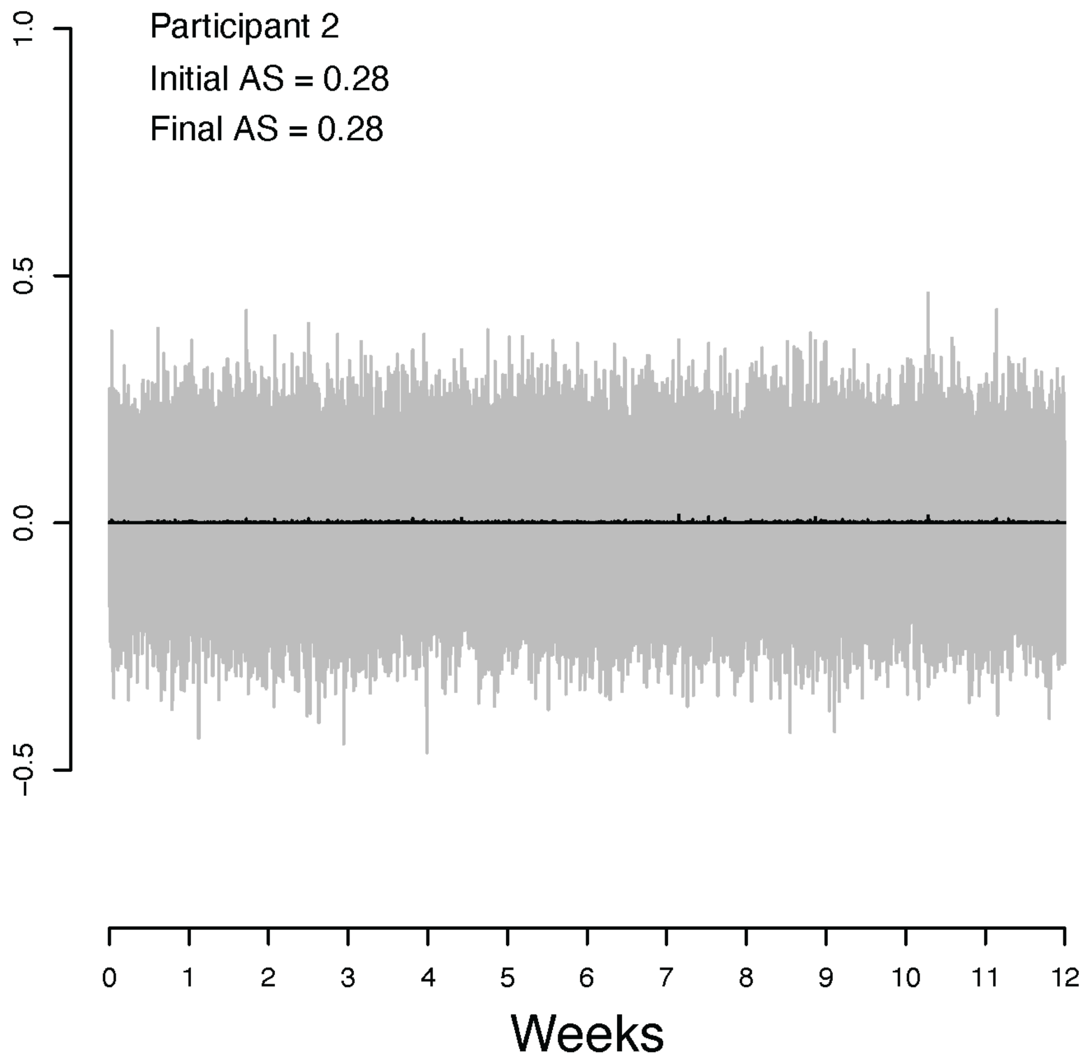
Individual Differences





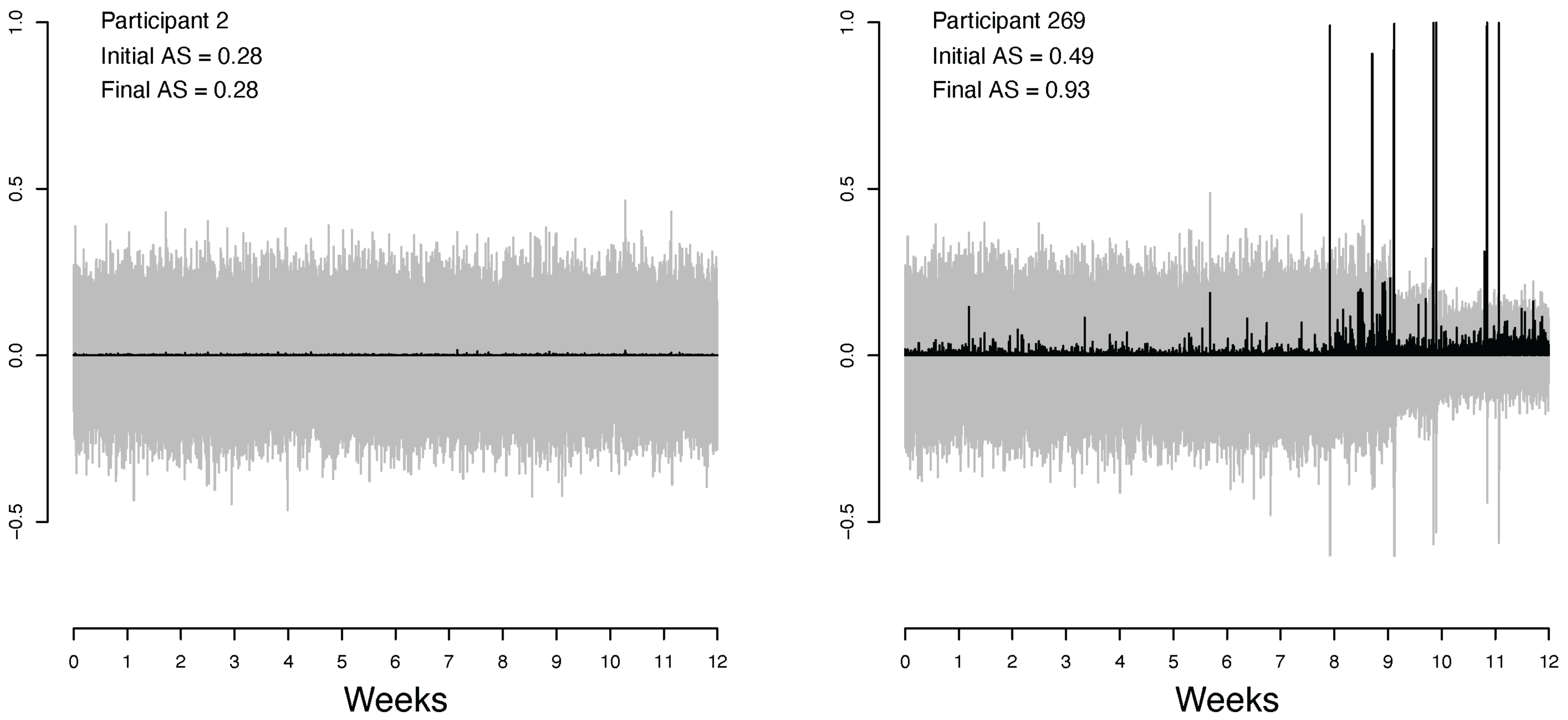
Phenomenon 3

Panic Disorder



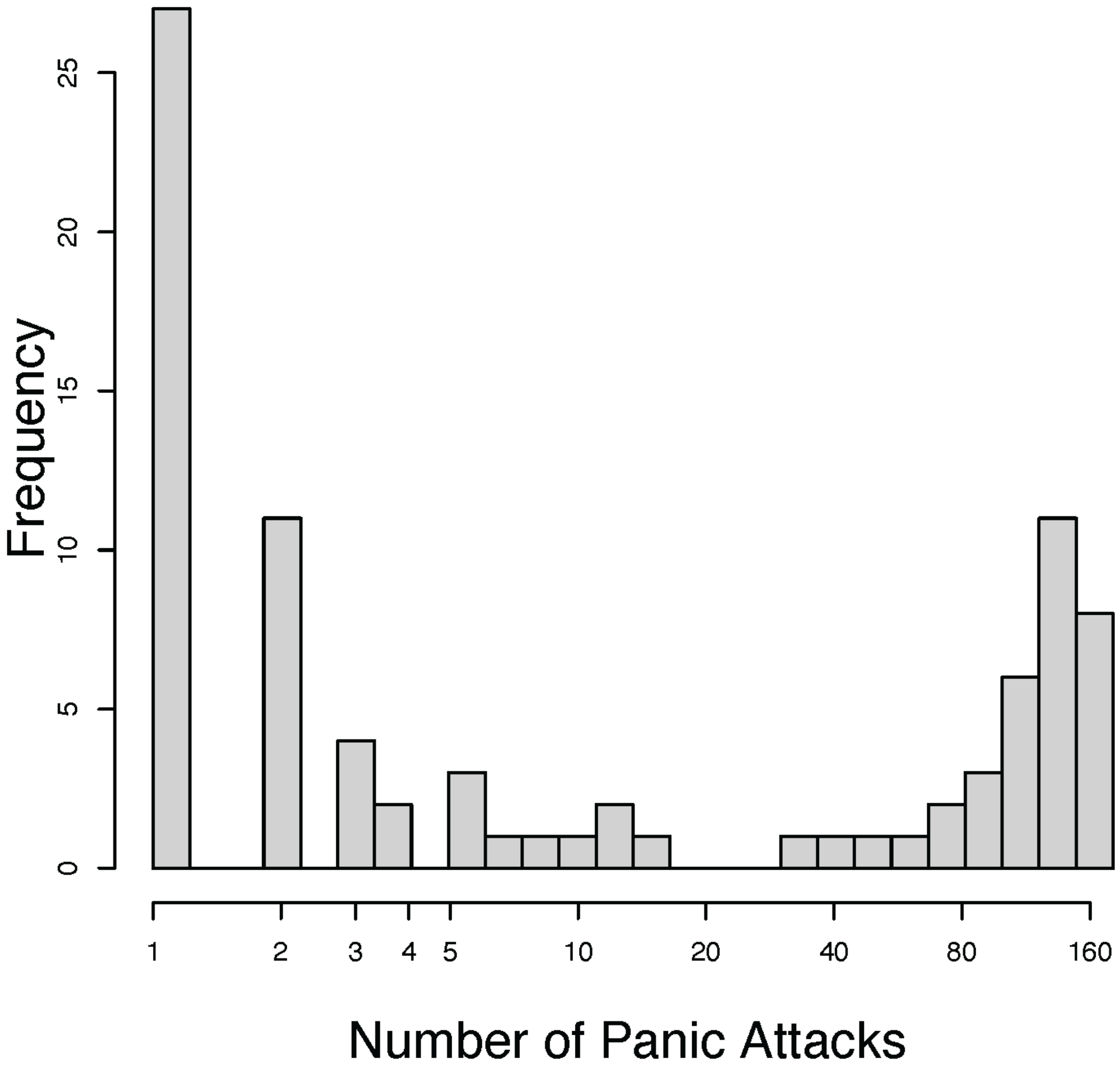
Phenomenon 3

Panic Disorder



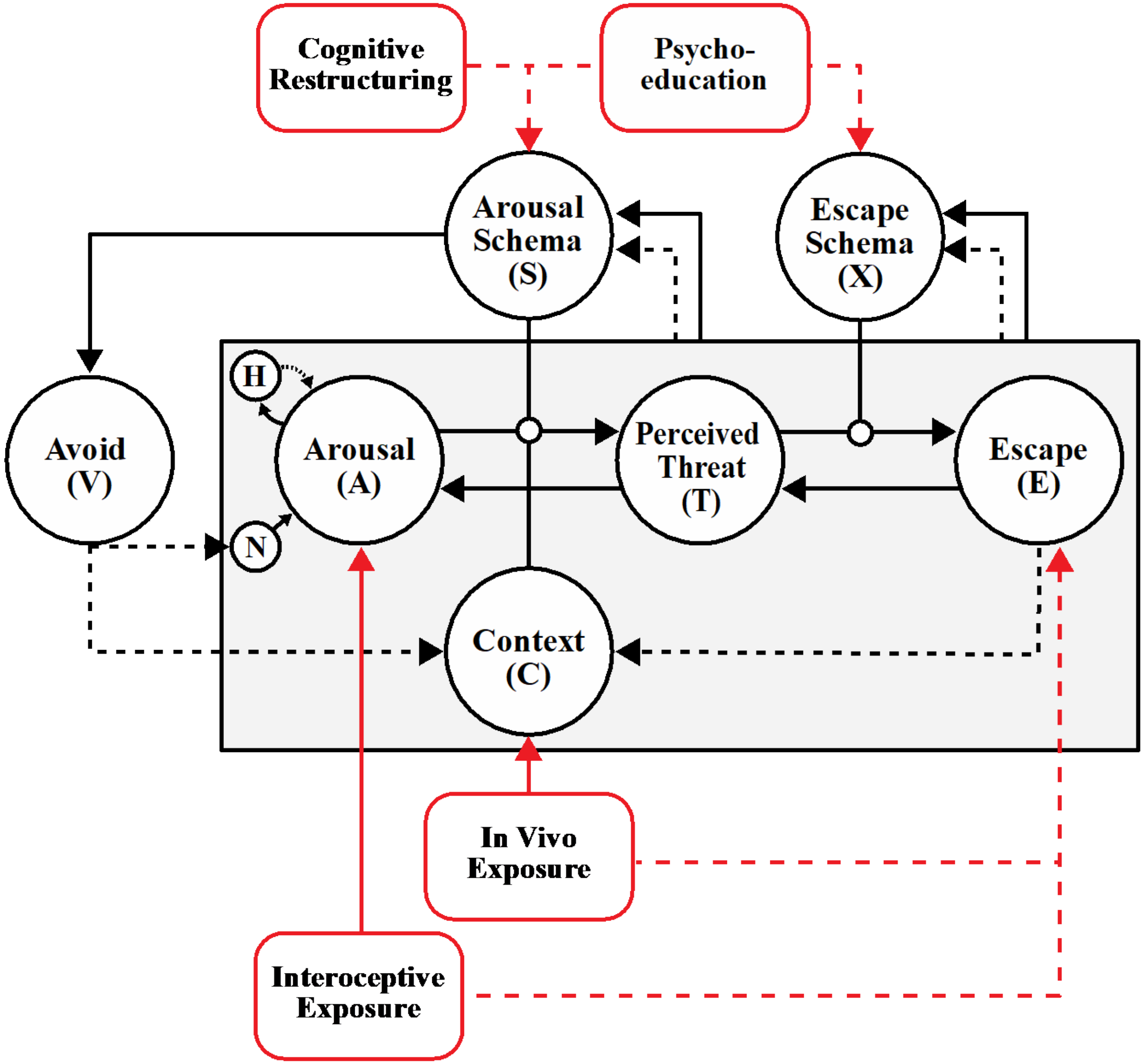
Phenomenon 4

Non-clinical Panic Attacks



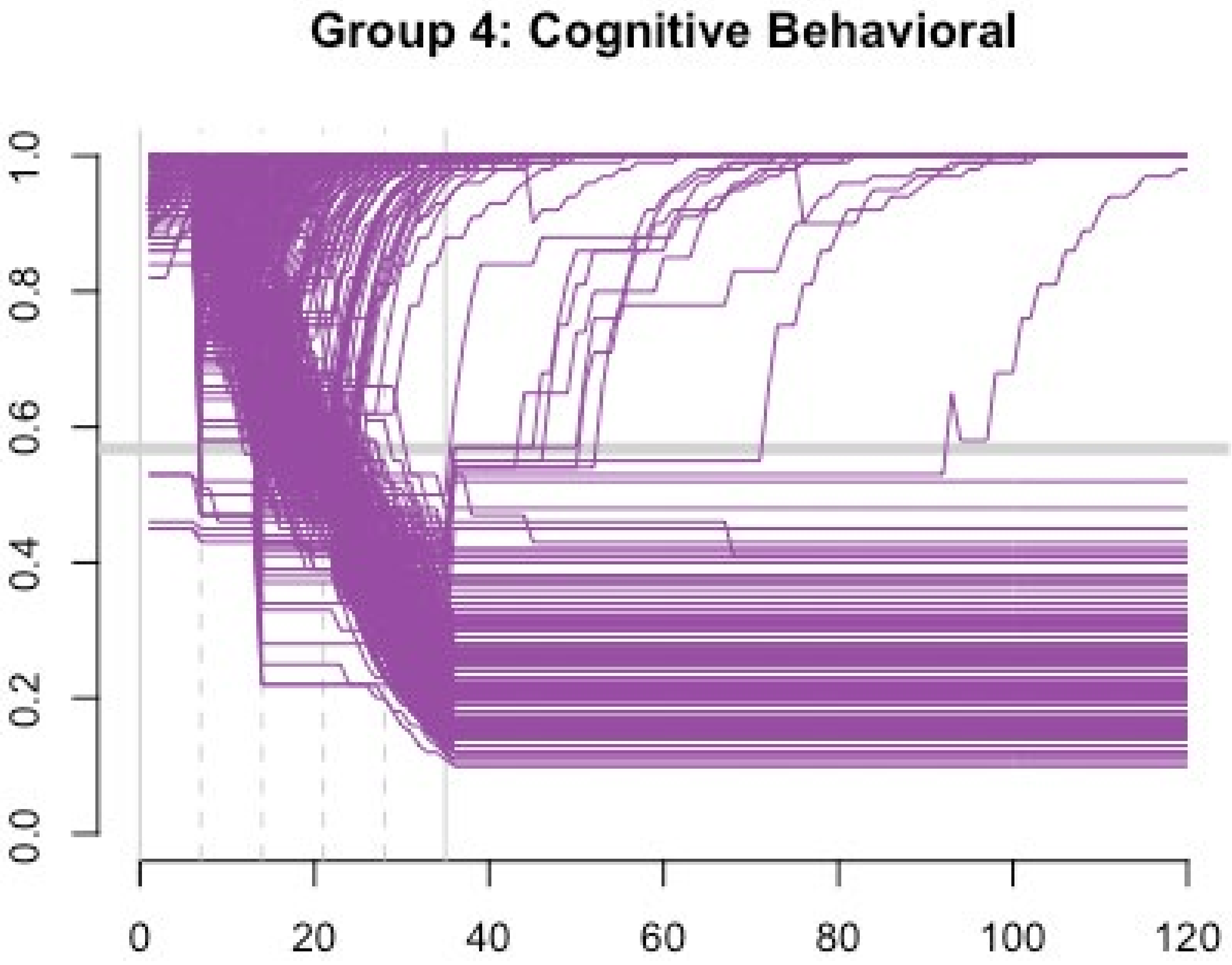
Phenomenon 5

CBT Efficacy



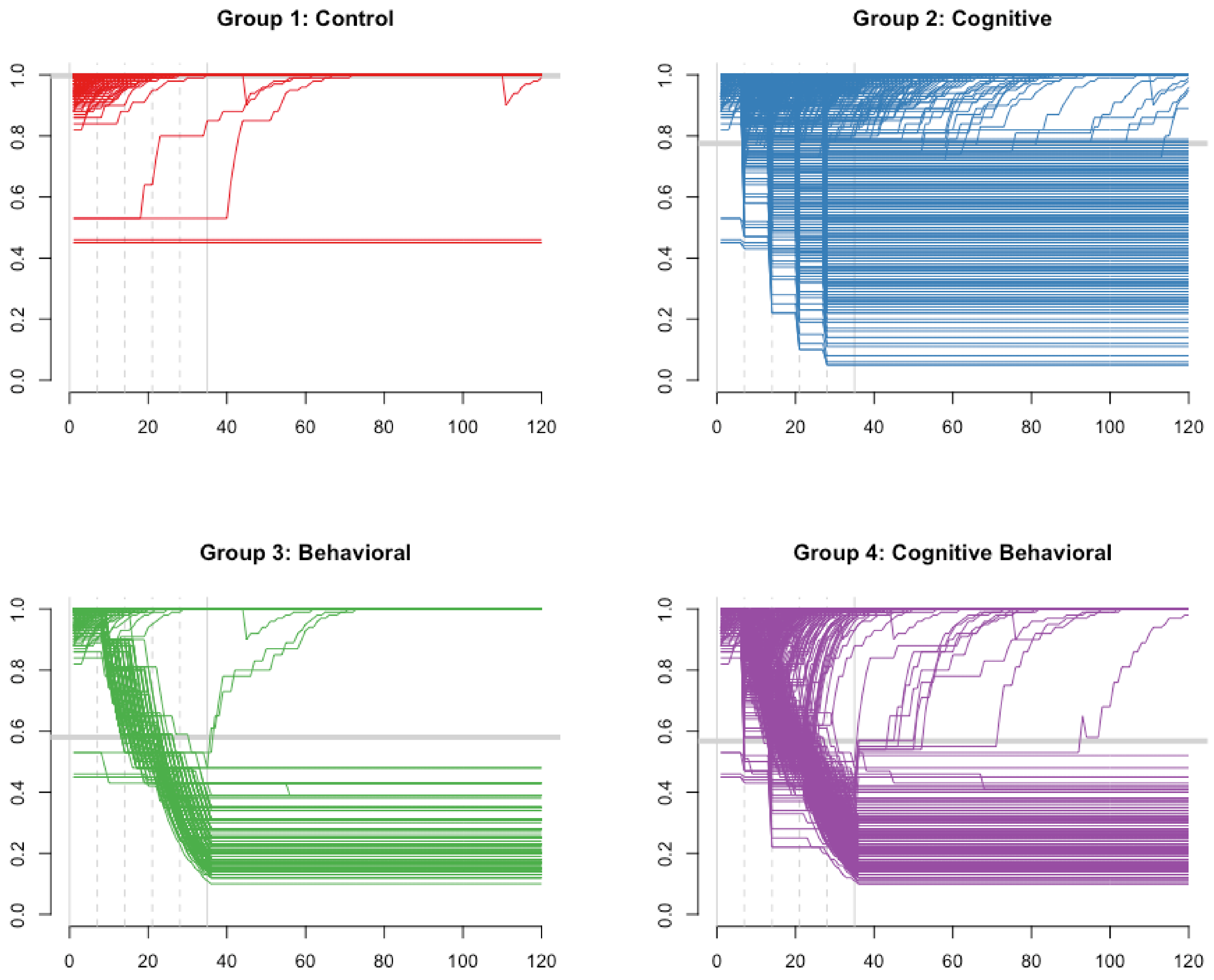
Phenomenon 5

CBT Efficacy



Phenomenon 5

CBT Efficacy



Can the theory explain core panic disorder-related phenomena?
Yes!

AND, there is still lots of room for it to explain more.