Continuous-Time Modeling

Modeling Intensive Longitudinal Data: The basics
EAM 2021

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Intro

Previous Lecture: *Discrete-Time* (DT) models

- Current observation regressed on previous observation(s)
- *Time series analysis* tradition of economics

Continuous-Time models

- Based on *differential equations*
- Attempt to model dynamics using relationships at the *moment-to-moment* level
- *Dynamical Systems Theory* - Physics, Ecology, Biology
- Different perspective, but many shared concepts and connections

Why care about CT models?

- Flexible modeling approach with a large literature
  - Focus on *qualitative behaviour* of a system
  - Increasing attention on generative/formal models (e.g. Robinaugh et al. 2020, Haslbeck, Ryan, Robinaugh et al, 2021, Borsboom et al 2021)

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  - Increasing attention on generative/formal models (e.g. Robinaugh et al. 2020, Haslbeck, Ryan, Robinaugh et al, 2021, Borsboom et al 2021)
- Practical and Conceptual Advantages
  - *The Time-Interval Problem*
  - Unequally spaced measurements
  - New perspective on network structure, direct effects, interventions
  - Can be estimated from the same type of data as DT models

Overview

1. **Basic concepts of CT models**
2. Why should you care?
3. What can you do in practice?

CT and DT models

In many cases, CT models describe the same qualitative behaviour as DT models

CT and DT models

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But CT models use a different perspective to do so, and so, different language

- Differential Equations and Integral Solutions

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But CT models use a different perspective to do so, and so, different language

- Differential Equations and Integral Solutions

To build intuition for this, let's use the simplest possible example

A simple discrete-time model

Impulse Response Functions help us understand qualitative behaviour

1. Pick an interesting value for Y_0
2. Take your model + parameters
3. Calculate expected value of Y_1
4. Repeat and plot to visualize time-evolution of the system

A simple discrete-time model

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AR(1) Model

$$E[Y_1] = \phi Y_0$$

A simple discrete-time model

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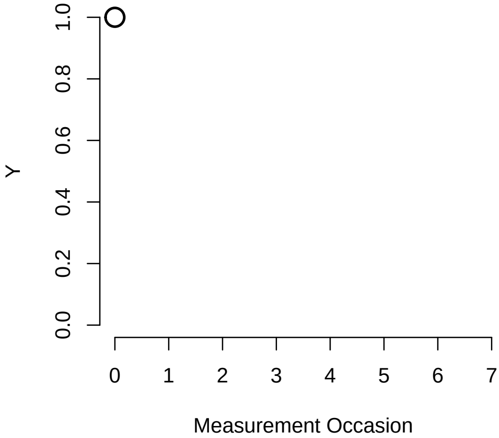
AR(1) Model

$$E[Y_1] = \phi Y_0$$

with

- $\phi = .5$
- $Y_0 = 1$

A simple discrete-time model



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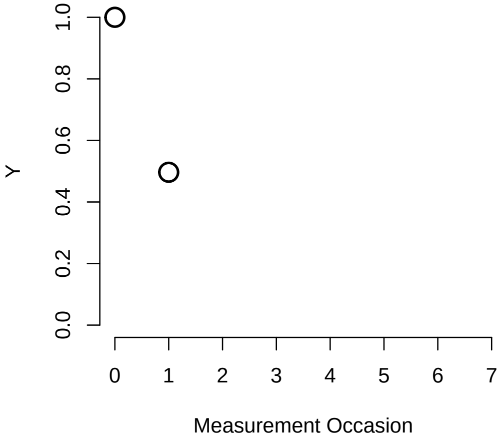

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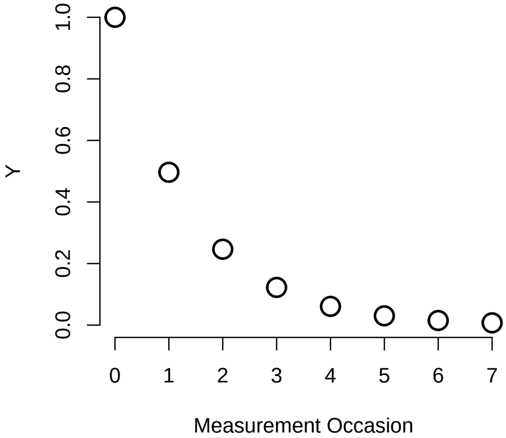

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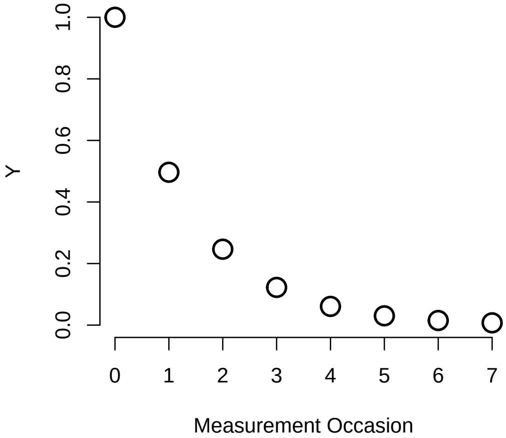


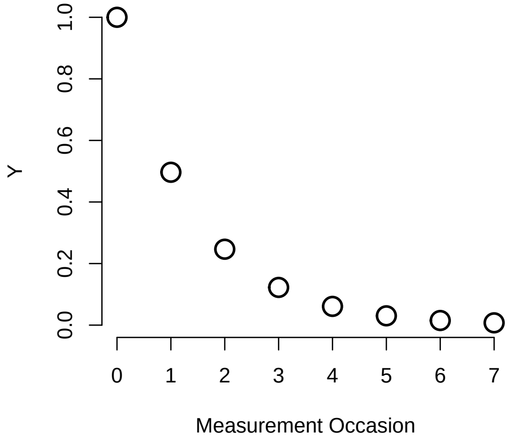
AR(1) Model

$$E[Y_2] = \phi \times \phi Y_0$$

with

- $\phi = .5$
- $Y_0 = 1$

A simple discrete-time model



AR(1) Model

$$E[Y_\tau] = \phi^\tau Y_0$$

with

- $\phi = .5$
- $Y_0 = 1$

A simple discrete-time model

The Discrete-Time AR(1) model:

- when $0 < \phi < 1$ describes an *equilibrium reverting* system
- after a *shock* the system gets pulled back towards resting state
- moves there in an *exponential decay*

A simple discrete-time model

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Dynamics understood in terms of discrete "jumps" from one *occassion* to the next

A simple discrete-time model

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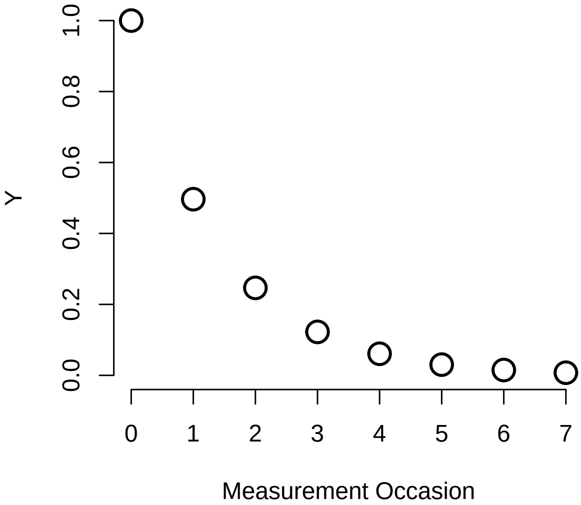
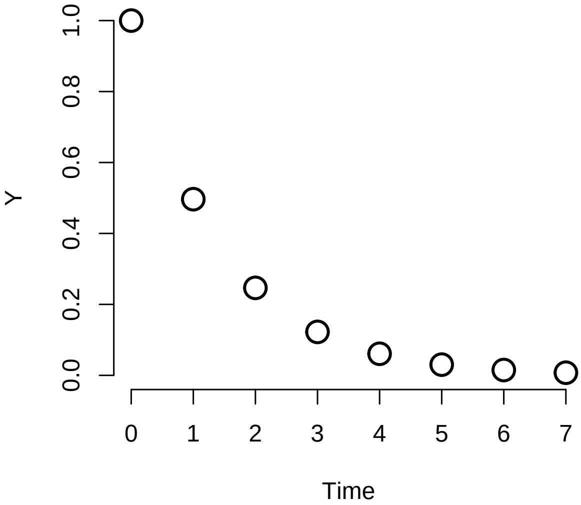
Dynamics understood in terms of discrete "jumps" from one *occassion* to the next

A **continuous-time** model can describe the same *qualitative behaviour*, but treating *time* itself as a continuous dimension



Y
0.0
0.2
0.4
0.6
0.8
1.0
0
1
2
3
4
5
6
7
Measurement Occasion



Y
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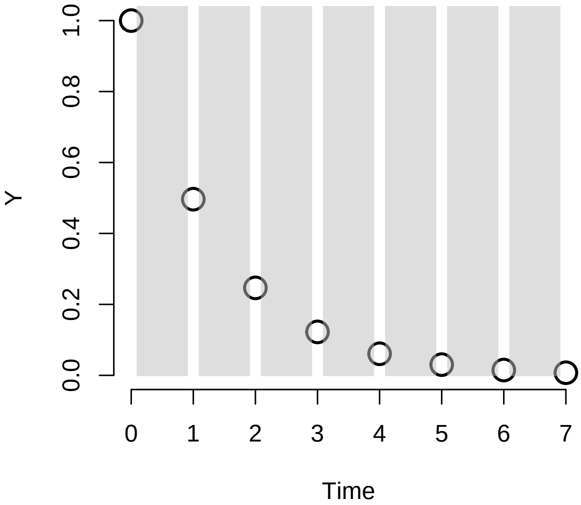
Time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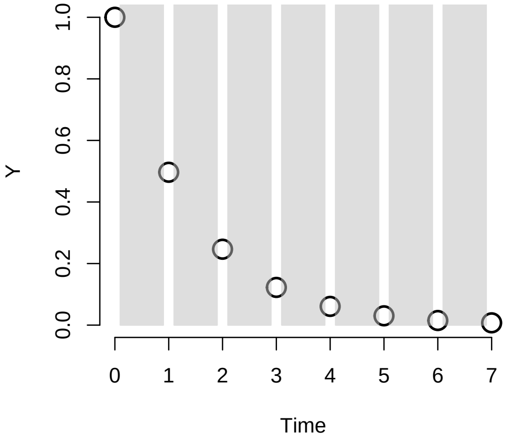
Time

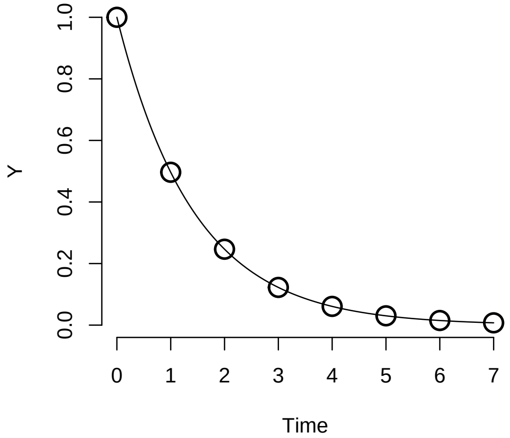
A simple continuous-time model



Continuous-Time Models

- Process takes on *some value* at every moment in time

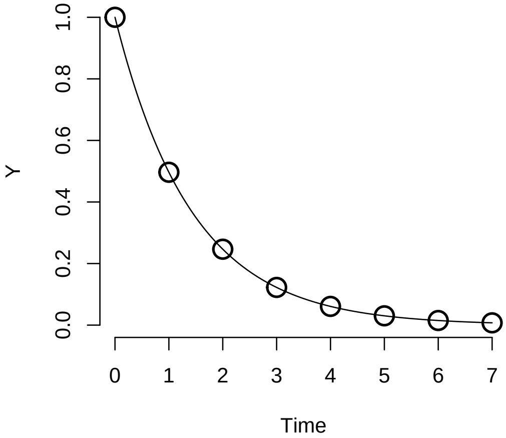
A simple continuous-time model



Continuous-Time Models

- Process takes on *some value* at every moment in time
- System evolves in a smooth and continuous manner

A simple continuous-time model

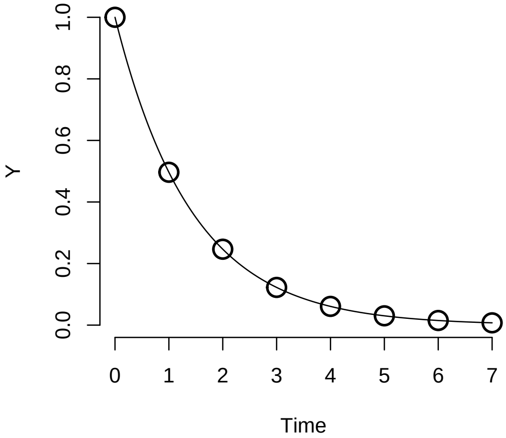


Continuous-Time Models

- Process takes on *some value* at every moment in time
- System evolves in a smooth and continuous manner

We can describe the evolution of the system using *Differential Equations*

A simple continuous-time model



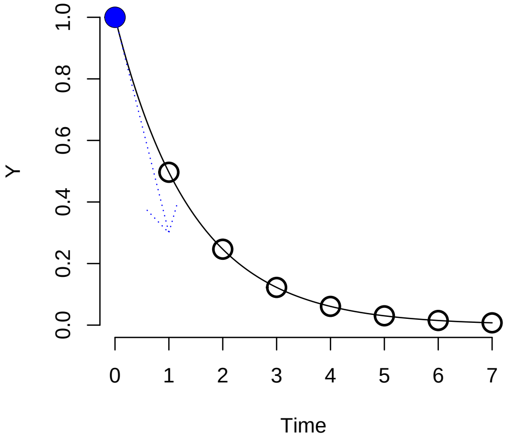
First-order DE

$$\frac{dY(t)}{dt} = A \times Y(t)$$

with

- $A = -.69$

A simple continuous-time model



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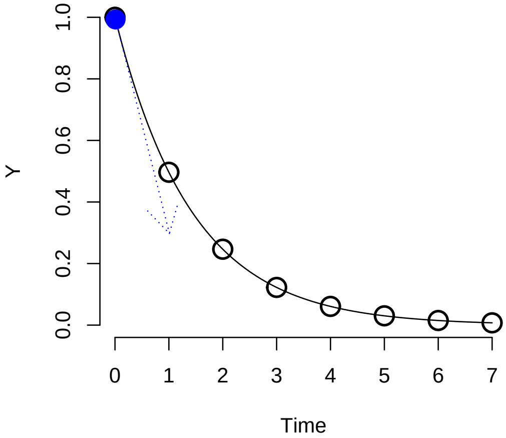

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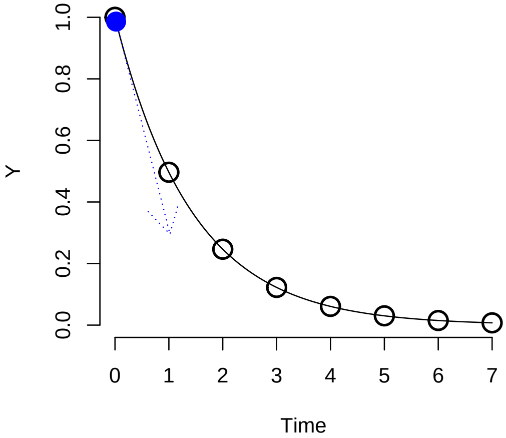

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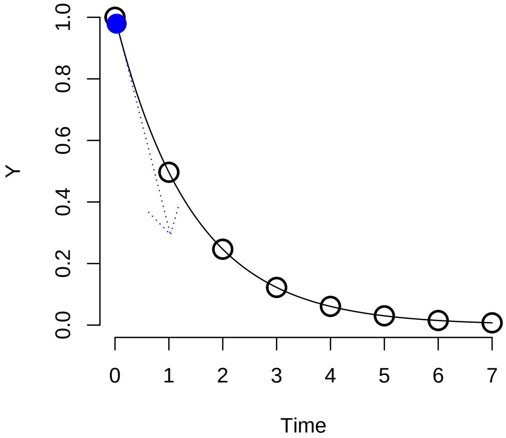

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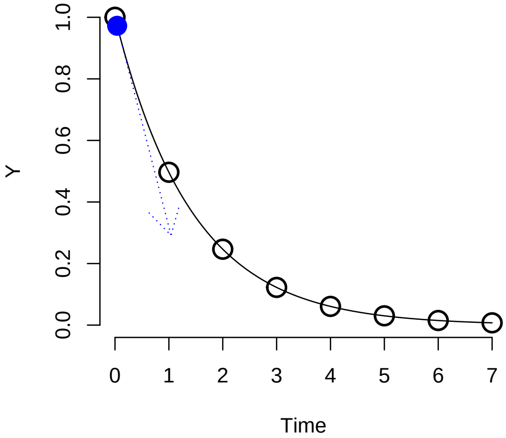

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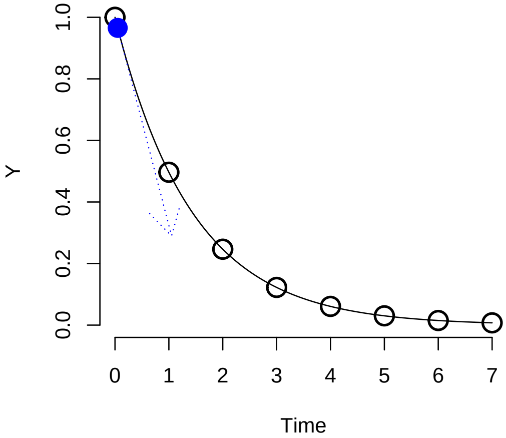

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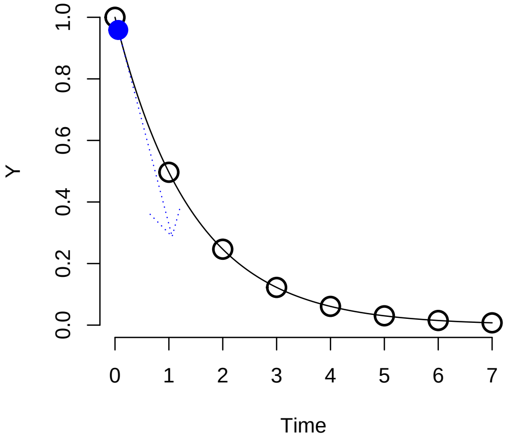

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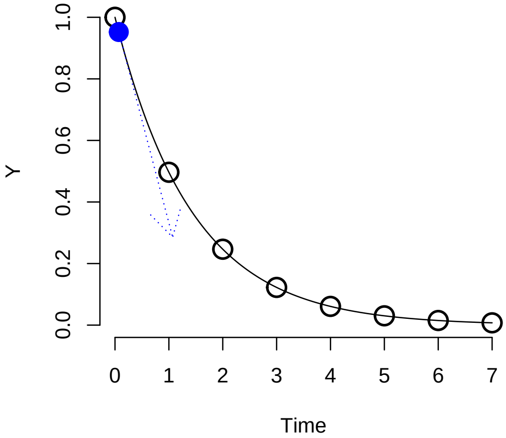

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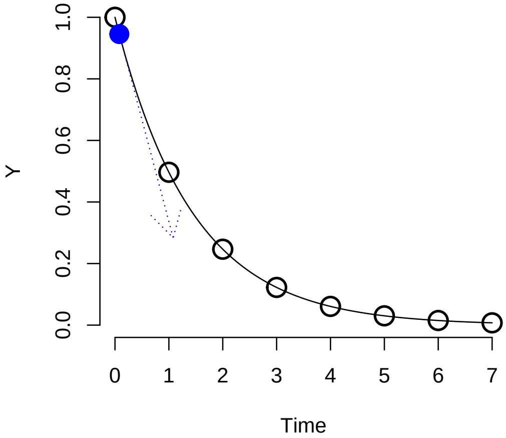

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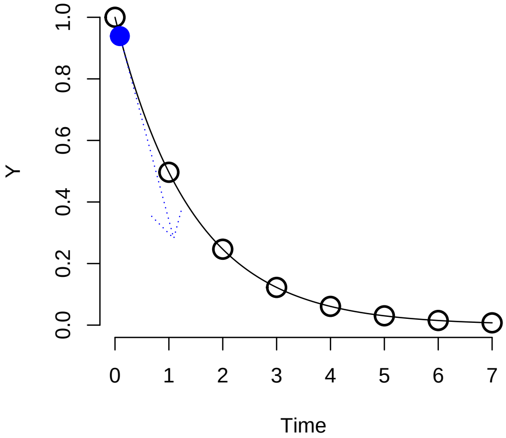

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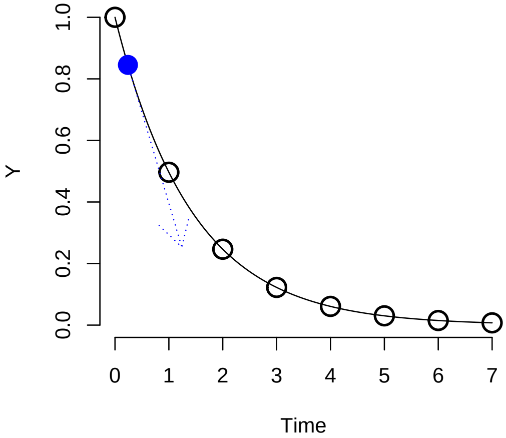

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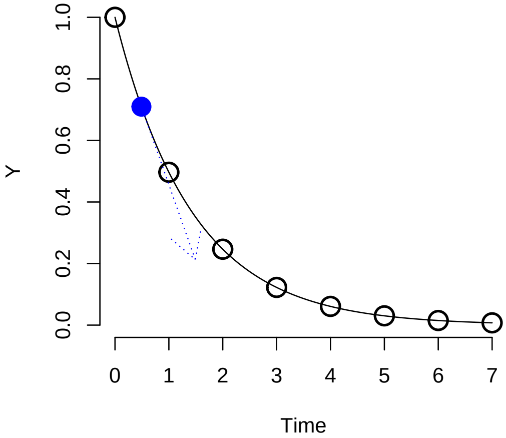

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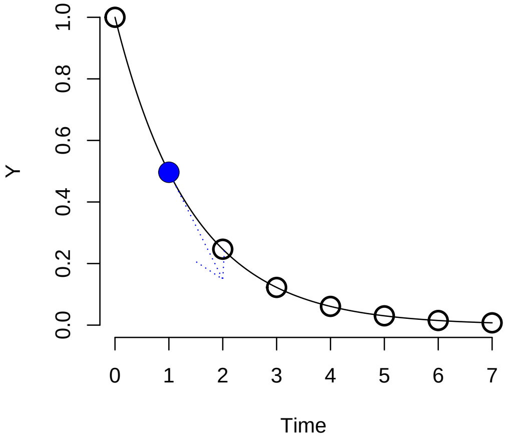

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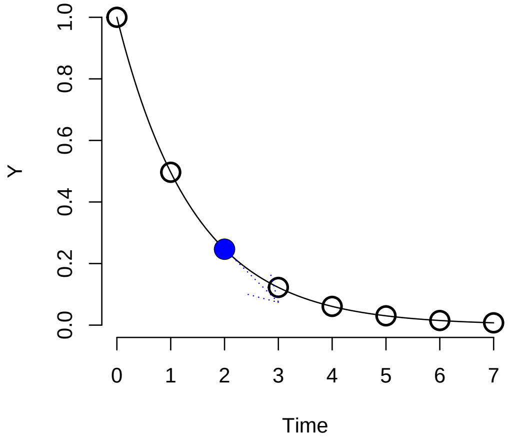

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The same, but different

AR(1) Model

$$E[Y_{\tau+1}] = \phi \times Y_\tau$$

with

- $0 < \phi < 1$
- Equally spaced measurements
- Discrete-time model

First-order DE

$$\frac{dY(t)}{dt} = A \times Y(t)$$

with

- $A < 0$
- Continuous-time model

Both models are equivalent, in the sense they describe the same qualitative behaviour

Model Equivalence

Re-write the DE so that the rhs is “Y now” and the lhs is “Y later”

$$\frac{dY(t)}{dt} = A \times Y(t)$$
$$\lim_{s \to 0} \frac{Y(t+s) - Y(t)}{s} = A \times Y(t)$$

Model Equivalence

Re-write the DE so that the rhs is "Y now" and the lhs is "Y later"

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$$\lim_{s \to 0} \frac{Y(t+s) - Y(t)}{s} = A \times Y(t)$$
$$\lim_{s \to 0} Y(t+s) = (1 + A \lim_{s \to 0} s) Y(t)$$

Model Equivalence

Re-write the DE so that the rhs is "Y now" and the lhs is "Y later"

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\begin{aligned}
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\lim_{s \to 0} Y(t+s) &= (1 + A \lim_{s \to 0} s) Y(t) \\
Y(t + \Delta t) &= \lim_{n \to \infty} (1 + A \frac{\Delta t}{n})^n Y(t)
\end{aligned}
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Model Equivalence

Re-write the DE so that the rhs is "Y now" and the lhs is "Y later"

$$\frac{dY(t)}{dt} = A \times Y(t)$$

$$\lim_{s\to 0} \frac{Y(t+s) - Y(t)}{s} = A \times Y(t)$$

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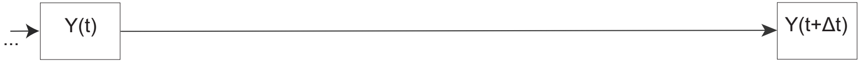
$$Y(t+\Delta t) = \lim_{n\to\infty} (1 + A\frac{\Delta t}{n})^n Y(t)$$

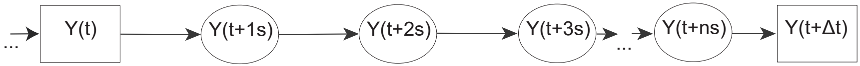
$$Y(t+\Delta t) = e^{A\Delta t} Y(t)$$

Model Equivalence

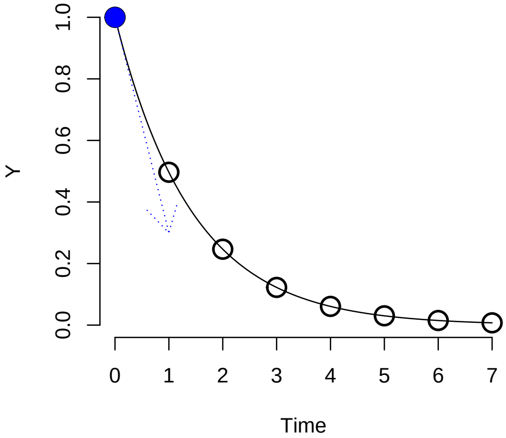
Discrete-Time AR(1)



Continuous-Time AR(1)



A simple continuous-time model



First-order DE

$$\frac{dY(t)}{dt} = A \times Y(t)$$

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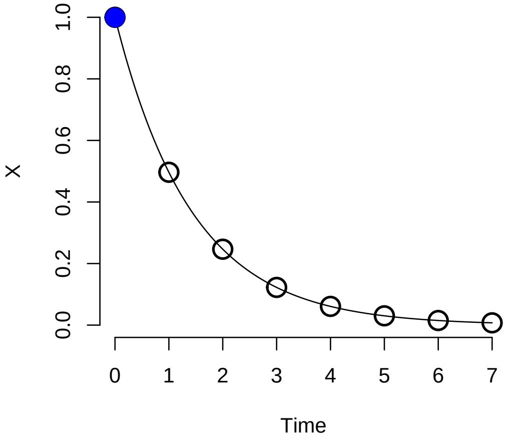

Integral Solution

$$Y(t + \Delta t) = e^{A \Delta t} Y(t)$$

with

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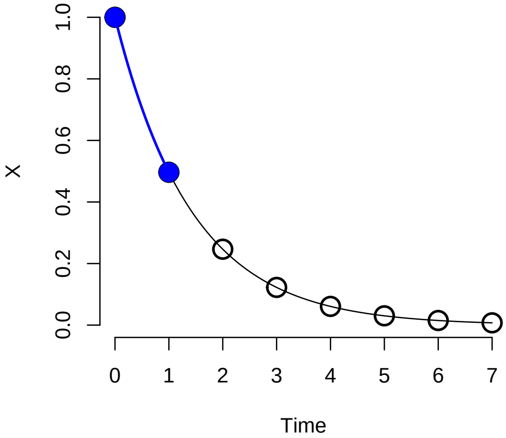
A simple continuous-time model



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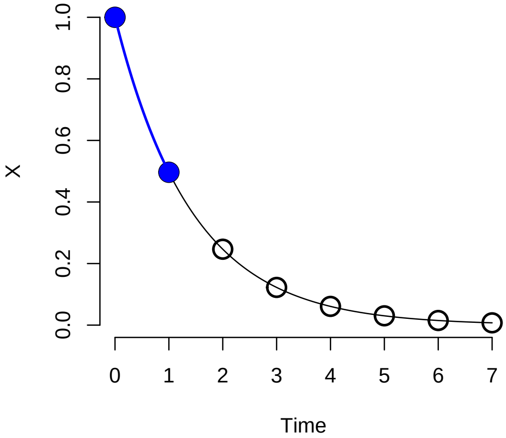


Integral Solution

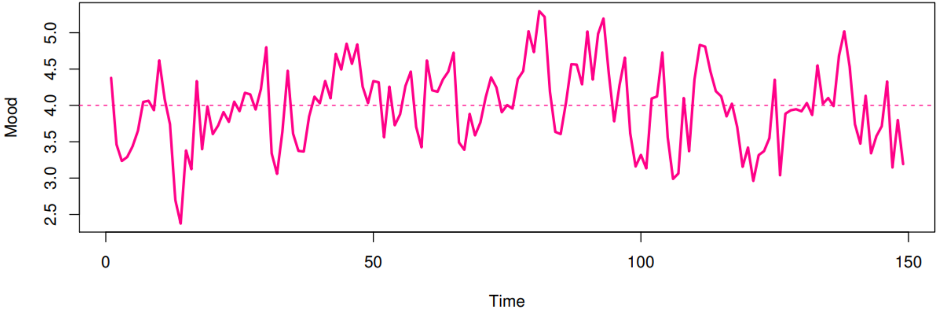
$$Y(t + \Delta t) = e^{A\Delta t} Y(t)$$

with

- $A = -.69$
- $e^{(-.69\times 1)} = \phi = .5$

Adding Noise

So far we only dealt with the *deterministic* part of the DE model. But we typically also want to allow for random noise or innovation variance



Adding Noise

AR(1) Model

$$Y_{\tau+1} = \phi Y_\tau + \epsilon_\tau$$

with

- $0 < \phi < 1$
- Equally spaced measurements
- $\epsilon_\tau \sim N(0, \sigma)$

Adding Noise

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First-order Stochastic DE

$$\frac{dY(t)}{dt} = A \times Y(t) + \frac{dW(t)}{dt}$$

with

- $A < 0$
- $\frac{dW(t)}{dt}$ is a *Wiener process*
- A random walk/white noise in CT

Adding Noise

AR(1) Model

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CT-AR(1)

$$Y(t + \Delta t) = e^{A\Delta t} Y(t) + \epsilon(\Delta t)$$

with

- $A < 0$
- $\epsilon(\Delta t)$ is Gaussian noise whose variance scales with Δt
- When $\Delta t \to 0$, $\epsilon \to 0$
- When $\Delta t \to \infty$, $\epsilon \sim N(0, \gamma)$

Multivariate Version

VAR(1) Model

$$\boldsymbol{Y}_{\tau+1} = \Phi \boldsymbol{Y}_{\tau} + \boldsymbol{\epsilon}_{\tau}$$

with

- $0 < \lambda_{\Phi} < 1$ (eigenvalues)
- Equally spaced measurements
- $\epsilon_{\tau} \sim N(0, \Sigma)$

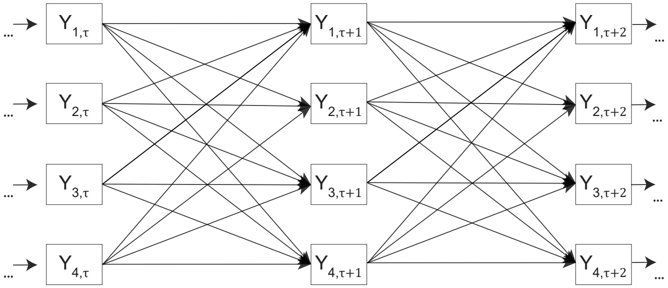
CT-VAR(1)

$$\boldsymbol{Y}(t + \Delta t) = e^{\boldsymbol{A}\Delta t}\boldsymbol{Y}(t) + \boldsymbol{\epsilon}(\Delta t)$$

with

- $\lambda_A < 0$ (eigenvalues)
- $\boldsymbol{\epsilon}(\Delta t)$ is Gaussian noise whose variance scales with Δt
- When $\Delta t \to 0$, $\boldsymbol{\epsilon} \to 0$
- When $\Delta t \to \infty$, $\boldsymbol{\epsilon} \sim N(0, \Gamma)$

$$\boldsymbol{Y}_{\tau+1} = \Phi \boldsymbol{Y}_{\tau} + \epsilon_{\tau}$$



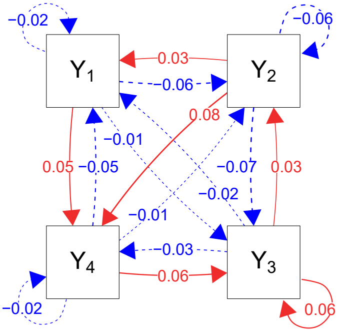


Figure from Ryan & Hamaker (2021)

$$\boldsymbol{Y}(t+\Delta t) = e^{\boldsymbol{A}\Delta t}\boldsymbol{Y}(t) + \epsilon(\Delta t)$$

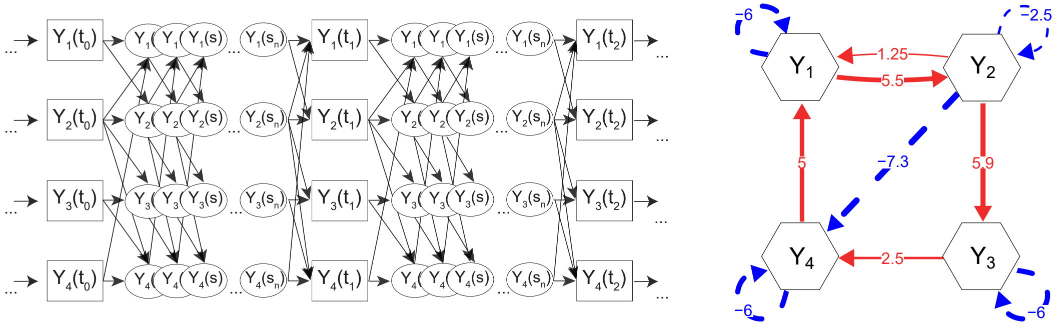


Figure from Ryan & Hamaker (2021)

Overview

1. Basic concepts of CT models
2. **Why should you care?**
3. What can you do in practice?

A continuous-time perspective

Psychological Processes as Continuous-Time Processes (Boker, 2002)

- CT may be a closer approximation to psychological dynamics
- Phenomena such as stress and anxiety can be defined at any point in time, not just at measurement occassions
- Likely to continuously influence one another to some degree

Explicit modeling of the time-interval

- Time-interval is important to our understanding of dynamics
- Aspirin-Headache effect, Stress-Rumination effect

The Time-Interval Problem

Advantages of CT modeling $\Rightarrow$ Disadvantages of DT modeling

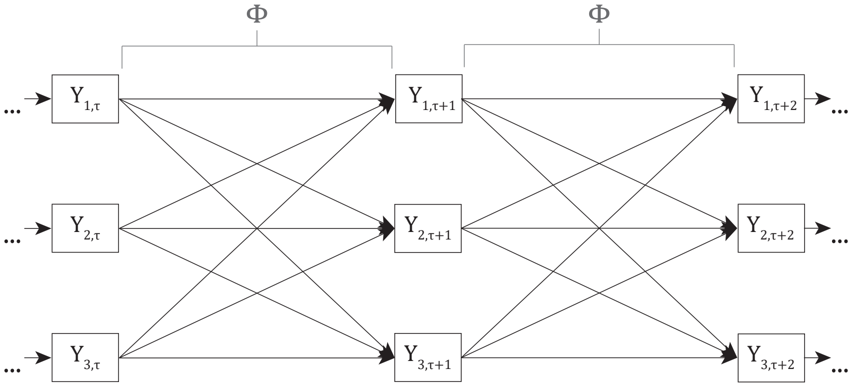
If we think our process is CT, we have to seriously re-think how we fit and interpret DT models

DT lagged relationships critically dependent on the time-interval

$$e^{(A\Delta t)} = \Phi(\Delta t)$$

Time-Interval Dependency

$$\boldsymbol{Y}_\tau = \Phi(\Delta t)\boldsymbol{Y}_{\tau-1} + \boldsymbol{\epsilon}_\tau$$



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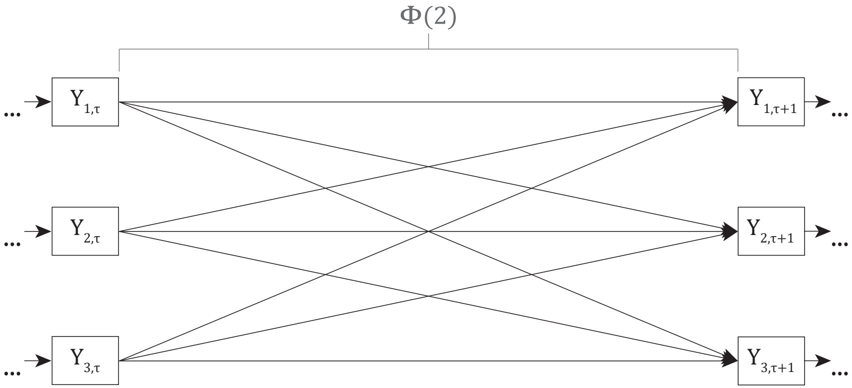

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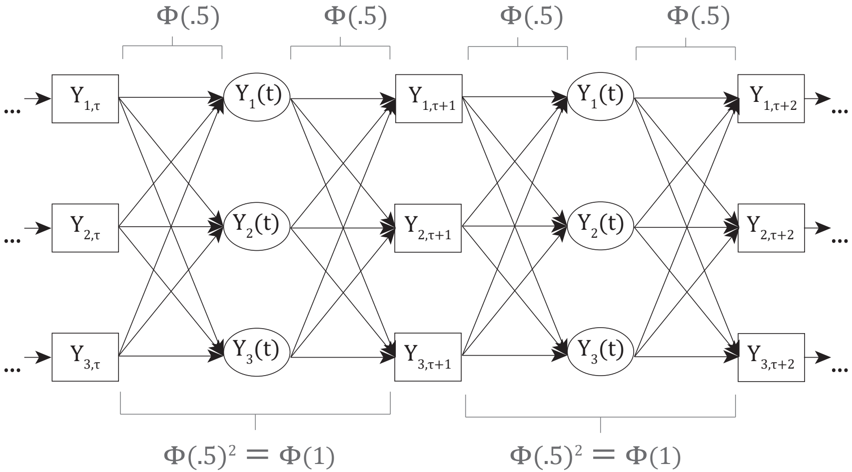
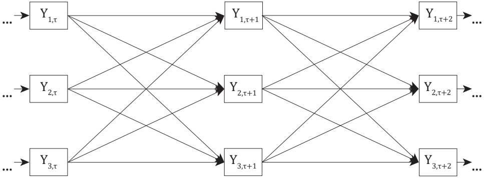


Consequences of time-interval dependency

1. Equal time-intervals: not generalizable
   - $\Phi(\Delta t = 1) \neq \Phi(\Delta t = .5)$



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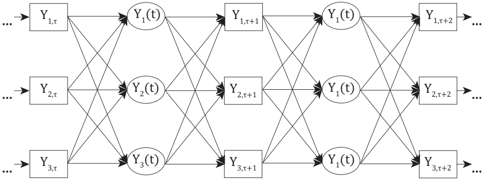

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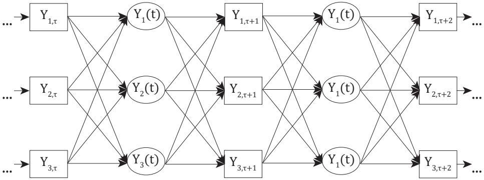

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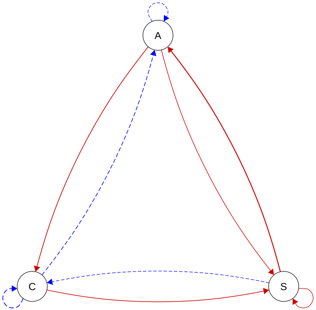

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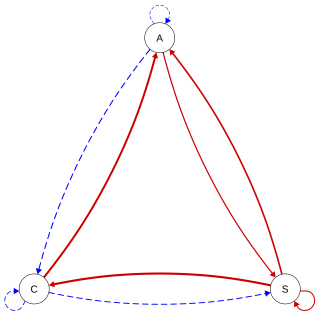
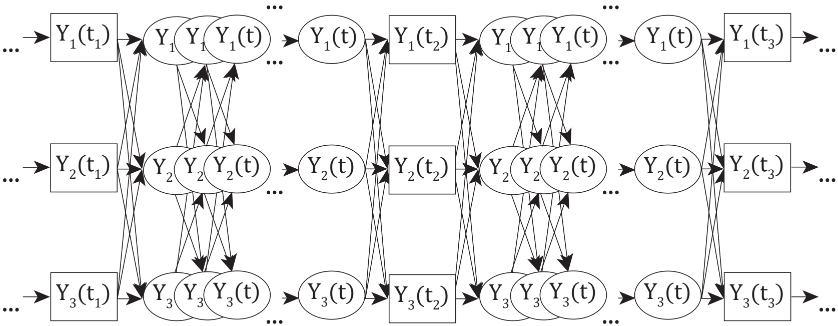


The CT-VAR(1) model

$$\boldsymbol{Y}(t) = \boldsymbol{e}^{\boldsymbol{A}\Delta t}\boldsymbol{Y}(t) + \epsilon(\Delta t)$$



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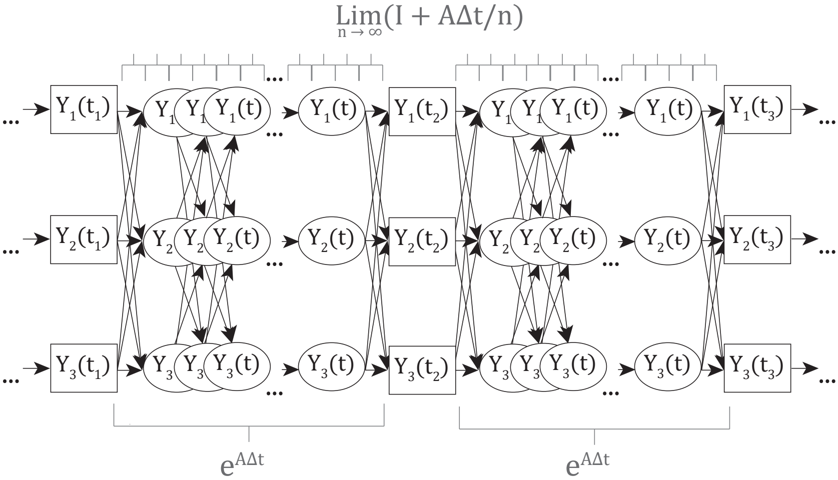
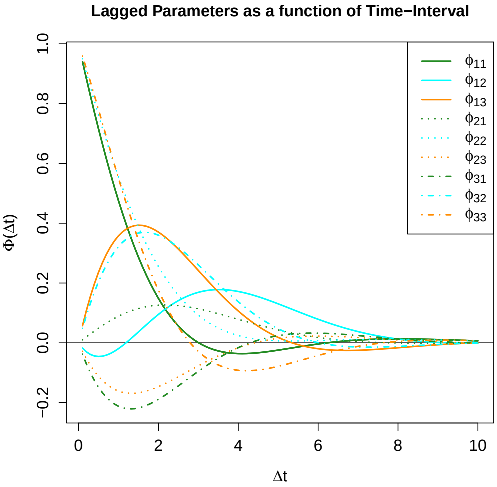


Time-interval dependency of VAR estimates

$$e^{\boldsymbol{A}\Delta t} = \Phi(\Delta t)$$



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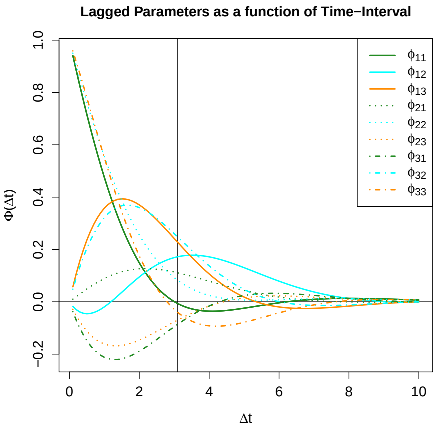

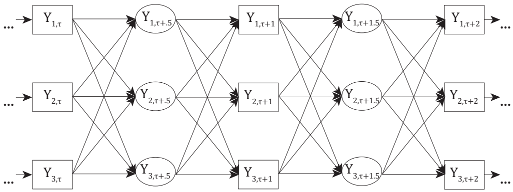

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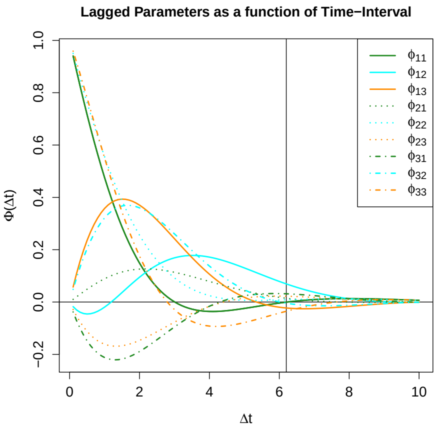
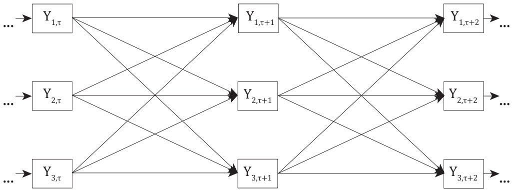
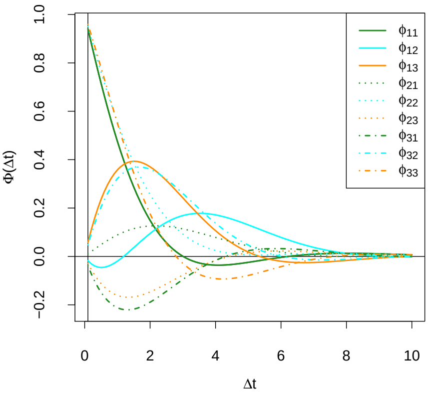
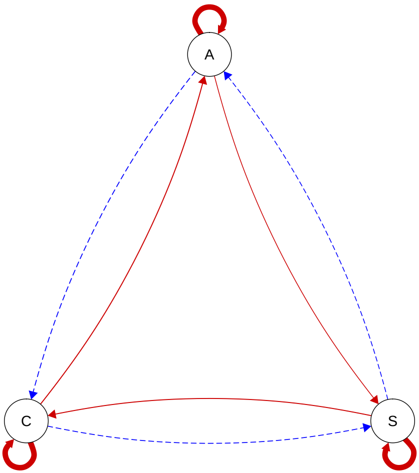




Network structure as a function of time-interval





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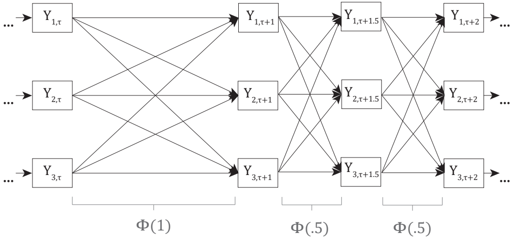

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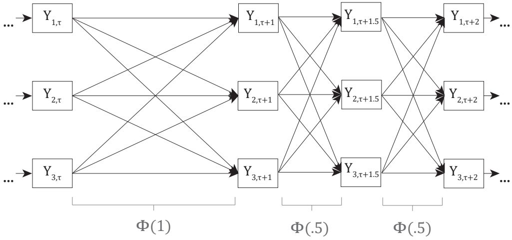

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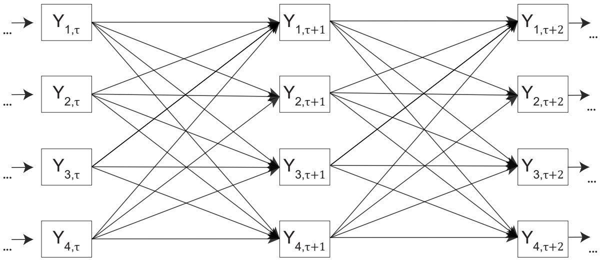
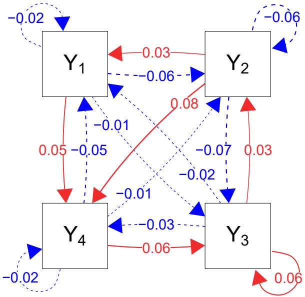
3. $\Phi(\Delta t)$ should not be interpreted as *direct effects*

Use of DT models often based on interpretation of Φ parameters as *direct* effects

- Mediation analysis and path tracing (Cole and Maxwell 2003)
- Network analysis, network structure, centrality, intervention targets (Bringmann et al 2013)





From a CT perspective, $\Phi(\Delta t)$ are *total effects*, including paths through *latent* values of the process in-between measurement occassions

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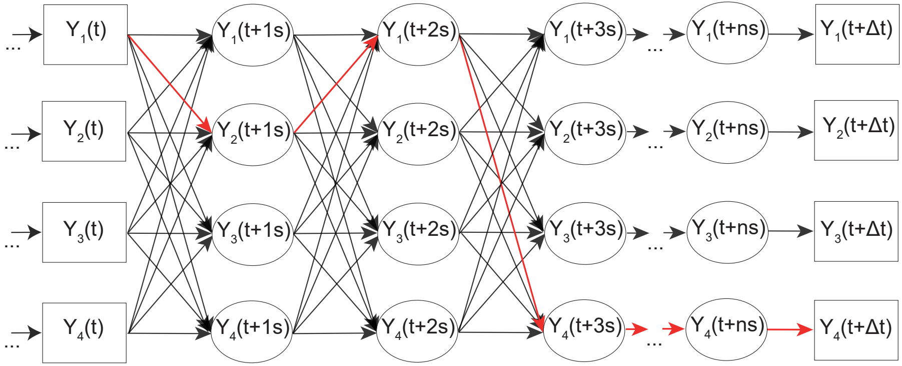
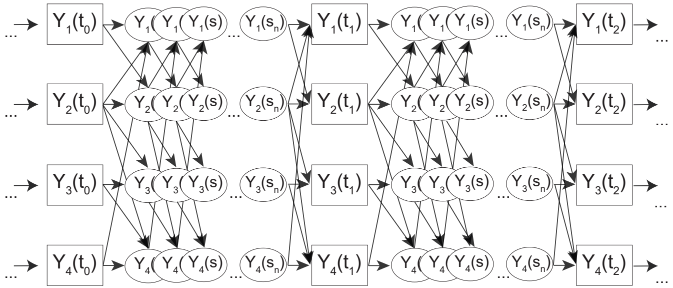
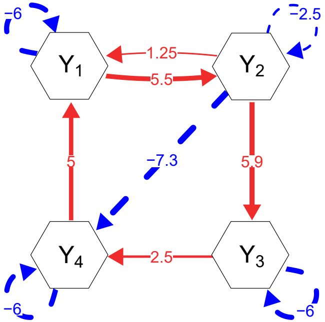


Instead the CT matrix $\boldsymbol{A}$ should be seen as direct moment-to-moment effects, and so should be the basis of path-tracing, centrality, network structure (Ryan & Hamaker, 2021)





Overview

1. Basic concepts of CT models
2. Why should you care?
3. **What can you do in practice?**

CT modeling in Practice

CT models like the CT-VAR(1) can be fit to standard ESM-type data

- *ctsem* (R; Driver et al. 2018) based on *rstan*
- *dynr* (R; Ou, Hunter and Chow, 2018)
- You need information about the spacing of observations
- Many extensions not shown here (Single-subject or multilevel, mean trends, regime-switching, non-linearity, higher-order models)

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Based on this, many developments in CT alternatives to DT practice

- Tools for interpreting CT models (Ryan, Kuiper Hamaker, 2018)
- CT mediation for tri-variate models (Deboeck & Preacher, 2016)
- CT network analysis and path-tracing (Ryan & Hamaker, 2021)
- CT meta-analysis (Kuiper & Ryan, 2020; Dormann, Guthier & Cortina, 2020)

Lab Session

Key References

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Additional Slides

Making sense of CT models

- Interpret the estimated parameters directly
  - $\boldsymbol{A}$ describes how position $Y(t)$ is related to the *rate of change* $\frac{dY(t)}{dt}$
- Visualise the predicted behaviour of the system
  - Just like the VAR(1), the CT-VAR(1) describes a system that varies around a stable equilibrium
  - The estimated model parameters describe how changes in one variable result in changes in the others

Example Analysis (Ryan, Kuiper & Hamaker, 2018) [1]

Subset of data from Wichers & Groot (2016)

- Single-subject
- 286 measurements over 42 days
- Modal interval: 1.77 hours (IQR: 1.25 - 3.23 hrs)

Two items selected:

- *Down*: "I feel down"
- *Tired*: "I am tired"
- Both were centered and standardized

How do Down and Tired influence one another over time?

[1] Complete analysis on github.com/ryanoisin/continuous_time-ILD-what-why-how

Example Analysis (Ryan, Kuiper & Hamaker, 2018)

The drift matrix relating the processes Down ($Do(t)$) and Tired ($Ti(t)$) is given by

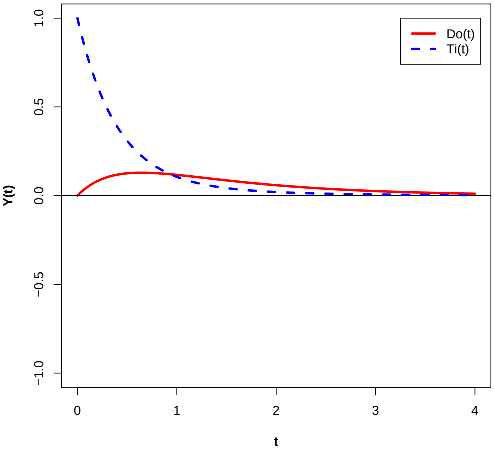
$$\boldsymbol{A} = \begin{bmatrix} -0.995 & 0.573 \\ 0.375 & -2.416 \end{bmatrix}.$$

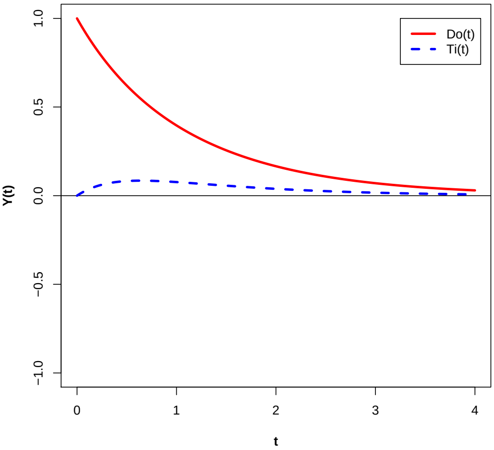
Which means

$$\frac{d\hat{Do}(t)}{dt} = -0.995 Do(t) + 0.573 Ti(t)$$
$$\frac{d\hat{Ti}(t)}{dt} = 0.375 Do(t) - 2.416 Ti(t)$$

Visualisation I: Impulse Response Functions

IRFs: How does the system react to a given impulse?





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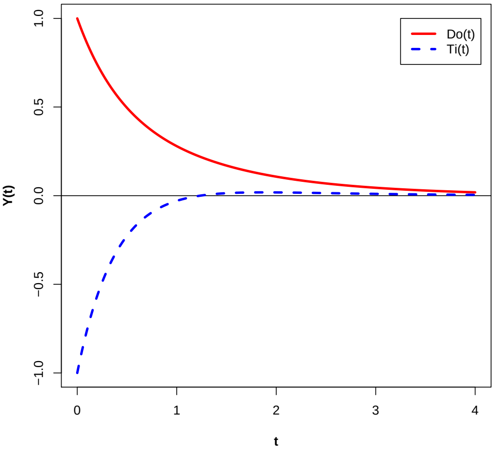
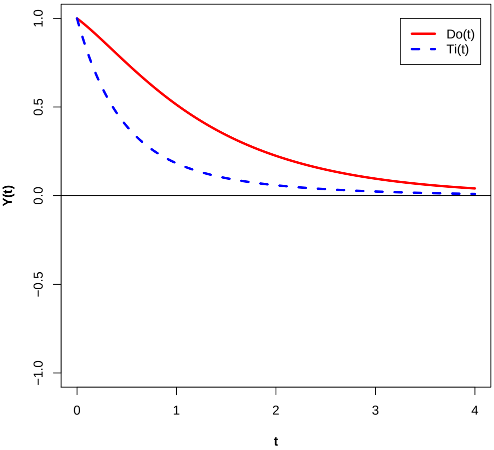
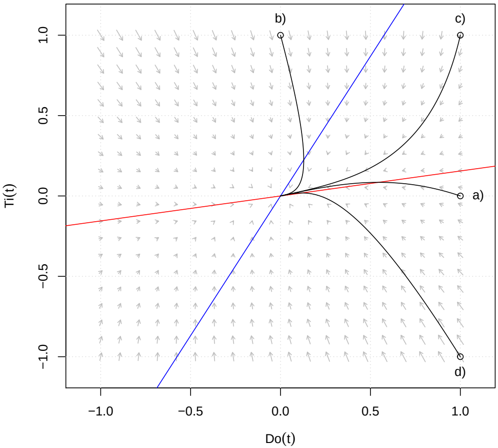




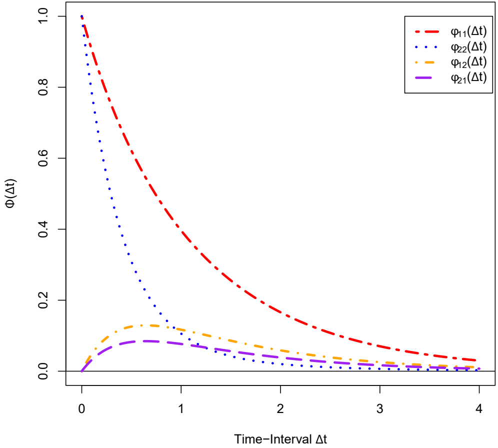
Visualisation II: Vector Fields

Vector Fields: What trajectories are possible?



Visualisation III: $\Phi(\Delta t)$

How do the lagged parameters depend on time-interval?



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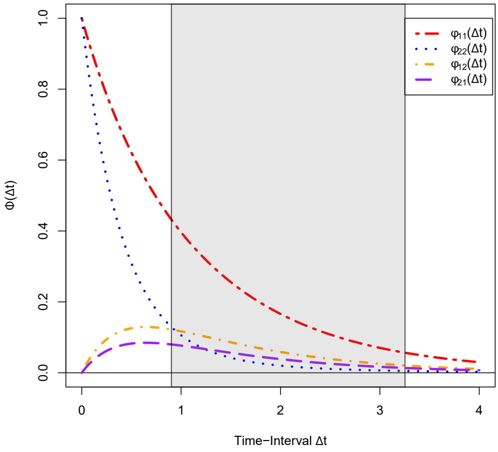

Get in Touch

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