Modeling Intensive Longitudinal Data in Discrete and Continuous Time

R group University of Zurich
Day 1

Oisín Ryan

Utrecht University

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Postdoc at the Department of Methodology & Statistics, Utrecht University, The Netherlands

Psychology (Bachelors) and Methods & Stats (Masters, PhD)

Originally from Ireland (hence, “Oisín” $\rightarrow$ [Uh-sheen])

Research focuses on Dynamical Systems and Causal Modeling in (Clinical) Psychology

PhD supervisor: Ellen Hamaker

Universiteit Utrecht

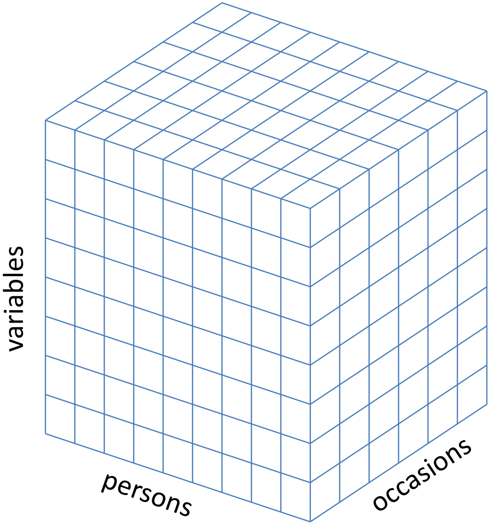
This Workshop

Aim: Introduce you to the basic principles in modeling *intensive longitudinal data*

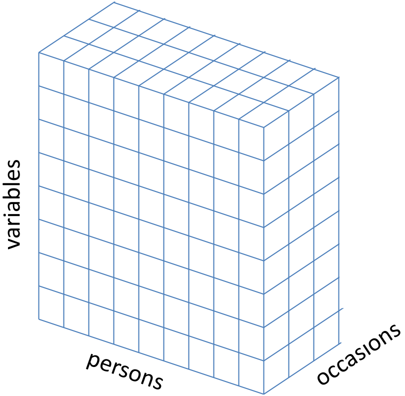
Broadly:

- Two related frameworks: Discrete-Time and Continuous-Time modeling
- Both model change over time using longitudinal data / repeated measures of the same variables
- Models built on *lagged regression*: X now predicts X later
- Can be viewed through the lens of SEM
- Primarily *single subject* approaches which can be generalized to a *multi-level* setting
- Focus on conceptual understanding with practical parts in *R*

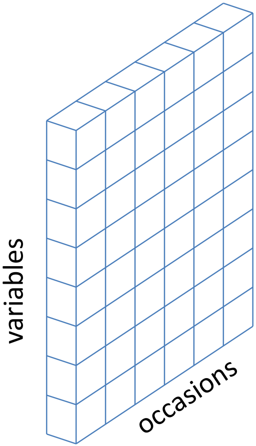
Cattell's data box



Panel Data: N = large and T is small (e.g. 2 -5 measurements)



Intensive Longitudinal Data: N=1 or many, T is large



Intensive Longitudinal Data

- Single (N=1) or multiple subjects (N = many). Many repeated measurements
- Time-points spaced closely together
- Attempts to directly capture short time-scale processes
- Focus primarily on individual within-person (idiographic) structure
- Sometimes called “time series” data
- “Long” format data

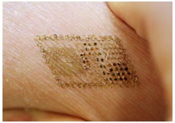
New technology

Smart phones

Smart glasses

Implants

Smart tattoo



Secure continuous remote alcohol monitor (SCRAM)

Smart watches

Activity trackers

Intensive longitudinal data

Different forms of intensive longitudinal data:

- daily diary (DD); self-report end-of-day
- experience sampling method (ESM); self-report of subjective experience
- ecological momentary assessment (EMA); healthcare related self-report
- ambulatory assessment (AA); physiological measurements
- event-based measurements; self-report after a particular event
- observational measurements; expert rater

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For more info on **methodology**, check out:

- Seminar of Tamlin Conner and Joshua Smyth on YouTube (https://www.youtube.com/watch?v=nQBBVp9vBIQ)
- Society for Ambulatory Assessment (http://www.saa2009.org/)
- Life Data (https://www.lifedatacorp.com/)
- Quantified Self (http://quantifiedself.com/)

Characteristics of these kind of data

Data structure:

- one or more measurements per day
- typically for multiple days
- sometimes multiple waves (i.e., Nesselroade's measurement-burst design)

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Proposed Advantages of ESM, EMA and AA

- no recall bias (in comparison to retrospective questionnaires)
- high ecological validity (in comparison to lab experiments)
- physiological measures over a large time span
- monitoring of symptoms and behavior, with new possibilities for feedback and intervention (e-Health and m-Health)
- window into the dynamics of processes

Discrete and Continuous-Time Models

Discrete-Time (DT) models

- Current observation regressed on previous observation(s)
- *Time series analysis* tradition of economics

Continuous-Time models

- Based on *differential equations*: instantaneous change as a function of current value
- *Dynamical Systems Theory* - Physics, Ecology, Biology

Different perspectives/emphases, but many shared concepts and connections

- Qualitative behaviour of the systems being modeled!

Outline

- **Time series analysis: Univariate**
- Time series analysis: Multivariate
    - Practical: Exercises 1, 2 and 3
- Multilevel time series analysis
    - Practical: Exercise 4 & 5
- Extensions and Advanced Issues
- Discussion

What is time series analysis?

Time series analysis is a class of techniques that is used in econometrics, seismology, meteorology, control engineering, and signal processing.

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Main characteristics:

- N=1 technique
- T is large (say $>$50)
- concerned with *trends*, *cycles* and *autocorrelation structure* (i.e., **lagged** relationships)
- traditional emphasis on models for *forecasting*

TSA in the social and medical sciences

In **sociology**:

- quarterly unemployment numbers
- effect of alcohol consumption per capita on criminal violence rates
- effect of suicide news on suicide rates

In **medical research**:

- effect of safety warnings on antidepressants use
- effects of pain control strategies
- effect of 9/11 attacks on weekly psychiatric patient admissions

In **psychology**:

- network of symptoms in depressive patient
- effect of feedback on academic performance
- effect of an intervention on the relationship between stress and affect

Time-Series Models typically take the form

$$Y_t = f_1(T) + f_2(S) + f_3(Y_{t-1}, \dots Y_{t-s}) + \varepsilon_t$$

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  - Typically limit ourselves to considering *linear* and *quadratic* terms (e.g. $B_1 T + B_2 T^2$

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  - Repeating regular patterns. For instance: Ice-cream sales are always highest in the summer months and lowest in the winter months

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  - **Auto-correlation** structure
- A random / noise / residual / innovation term ε_t

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Focus: picking an appropriate model for the *auto-correlation structure*. Linear relationships, so this amounts to picking the appropriate **lag** or **order** of the model

- Should current Y be predicted by Y an hour ago, or should we also include the effect of Y two hours ago? Three hours ago? 24 hours ago?

Y

y_1
y_2
y_3
y_4
y_5
y_6
y_7
y_8
$\ldots$
y_T

Lags

Y	Y at lag 1
y_1	
y_2	y_1
y_3	y_2
y_4	y_3
y_5	y_4
y_6	y_5
y_7	y_6
y_8	y_7
$\dots$	$\dots$
y_T	y_{T-1}
	y_T

Lags

Y	Y at lag 1	Y at lag 2
y_1		
y_2	y_1	
y_3	y_2	y_1
y_4	y_3	y_2
y_5	y_4	y_3
y_6	y_5	y_4
y_7	y_6	y_5
y_8	y_7	y_6
$\ldots$	$\ldots$	$\ldots$
y_T	y_{T-1}	y_{T-2}
	y_T	y_{T-1}
		y_T

Box-Jenkins Method (1970). Aim: Obtain a model with **white noise** residuals (error term contains no information about the future)

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Model Components:

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3. Choose order/lag of model

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- **Autocorrelation function (ACF)**. Raw/marginal correlation between lagged versions of our variable y_t and y_{t-k}

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Diagnostic Tools:

- **Autocorrelation function (ACF)**. Raw/marginal correlation between lagged versions of our variable y_t and y_{t-k}
- **Partial autocorrelation functions (PACF).** Partial correlation between y_t and y_{t-k} when controlling for the effect of intermediate observations y_{t-1} to y_{t-k+1}.

ACF and PACF explained

ACF Lag-1

$cor(Y_t, Y_{t-1})$

Y	Y at lag 1	Y at lag 2
y_1		
y_2	y_1	
y_3	y_2	y_1
y_4	y_3	y_2
y_5	y_4	y_3
y_6	y_5	y_4
y_7	y_6	y_5
y_8	y_7	y_6
$\dots$	$\dots$	$\dots$
y_T	y_{T-1}	y_{T-2}
	y_T	y_{T-1}
		y_T

ACF and PACF explained

ACF Lag-2

$cor(Y_t, Y_{t-2})$

Y	Y at lag 1	Y at lag 2
y_1		
y_2	y_1	
y_3	y_2	y_1
y_4	y_3	y_2
y_5	y_4	y_3
y_6	y_5	y_4
y_7	y_6	y_5
y_8	y_7	y_6
$\dots$	$\dots$	$\dots$
y_T	y_{T-1}	y_{T-2}
	y_T	y_{T-1}
		y_T

ACF and PACF explained

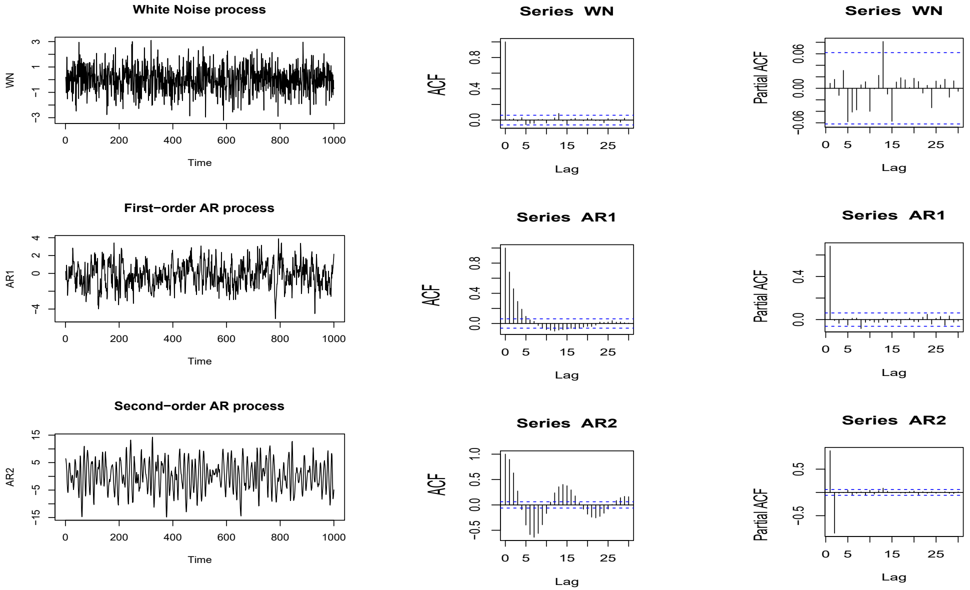
PACF Lag-2

$$cor(Y_t, Y_{t-2} \mid Y_{t-1})$$

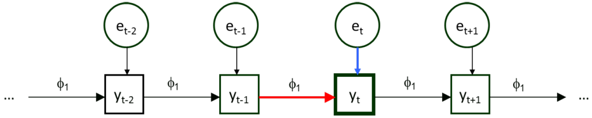
Y	Y at lag 1	Y at lag 2
y_1		
y_2	y_1	
y_3	y_2	y_1
y_4	y_3	y_2
y_5	y_4	y_3
y_6	y_5	y_4
y_7	y_6	y_5
y_8	y_7	y_6
$\dots$	$\dots$	$\dots$
y_T	y_{T-1}	y_{T-2}
	y_T	y_{T-1}
		y_T

Similar to a regression coefficient – controlling for Y_{t-1}

Sequence, ACF and PACF

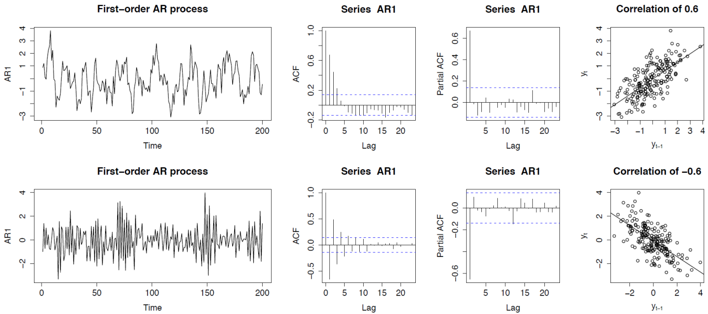


AR(1) Model

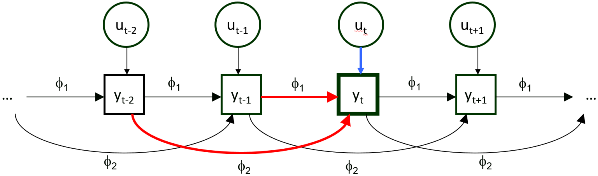


$$y_t = \phi_1 y_{t-1} + e_t$$

$$\phi_1 = \{0.6; -.6\}$$

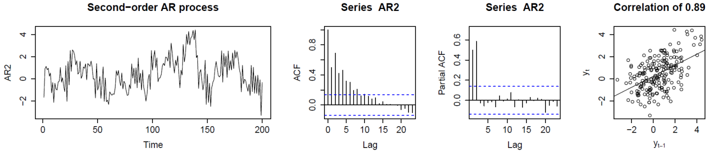


AR(2) Model



$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + e_t$$

$$\phi_1 = 0.2; \ \phi_2 = 0.6$$



Box-Jenkins in a nutshell

General approach:

1. Check for trends and seasonal components. Remove them if present.
2. Plot ACF and PACF. Choose an appropriate autocorrelation structure. Obtain residuals
3. Plot ACF and PACF of residuals. Check if white noise. If not, repeat step 2.

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Sidenote: Sometimes referred to as AR(I)MA modeling. We focus here on autoregressive (AR) terms, but moving average (MA) terms also possible

MA(1) model:

$$y_t = c + \varepsilon_t + \theta \varepsilon_{t-1}$$

Substantive Interpretation of AR processes

Granger and Morris (1976): An AR process is a **momentum effect in a random variable that varies smoothly over time**

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We think about a variable with a stable **equilibrium position.** The errors represent "shocks" that push the system away from equilibrium. The AR parameter describes the "carry-over" or "regulation" of those shocks from one moment to the next.

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AR(1) model: The **auto-regressive parameter** pulls the system **back** to equilibrium. The bigger the **absolute value**, the slower the system is to return to baseline

A simple discrete-time model

Impulse Response Functions help us understand qualitative behaviour

1. Pick an interesting value for Y_0
2. Take your model + parameters
3. Calculate expected value of Y_1
4. Repeat and plot to visualize time-evolution of the system

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AR(1) Model

$$E[Y_1] = \phi Y_0$$

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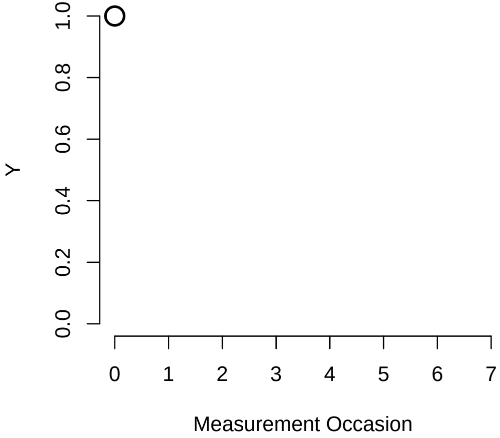
AR(1) Model

$$E[Y_1] = \phi Y_0$$

with

- $\phi = .5$
- $Y_0 = 1$



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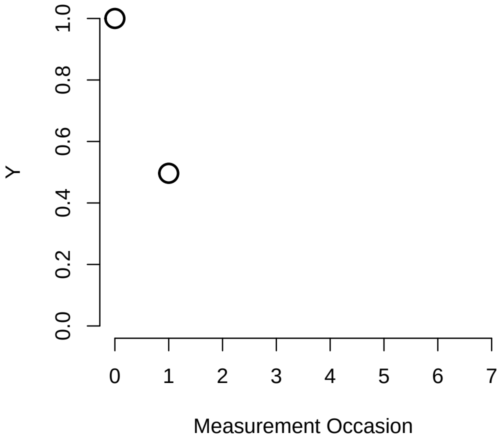

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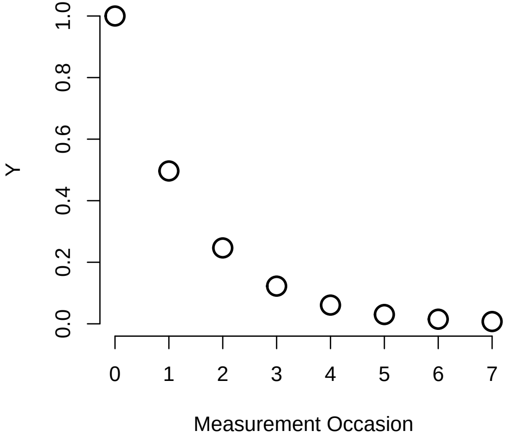

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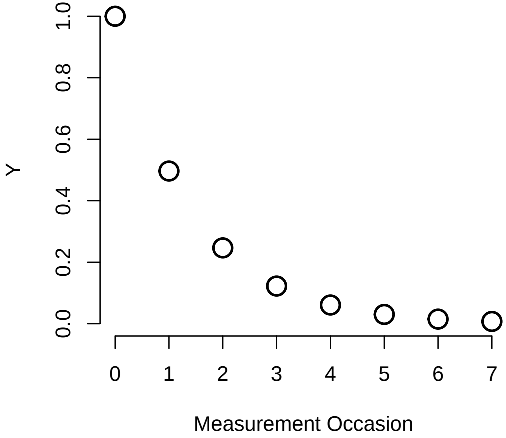
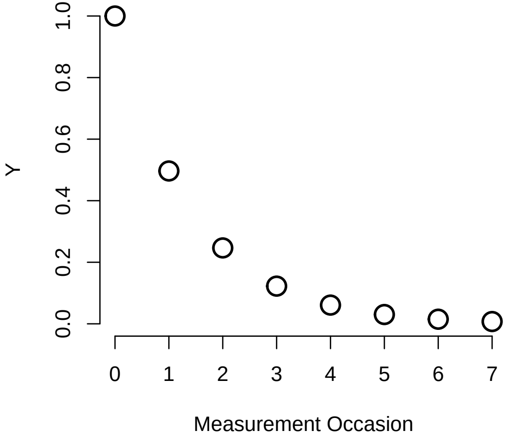


AR(1) Model

$$E[Y_2] = \phi \times \phi Y_0$$

with

- $\phi = .5$
- $Y_0 = 1$



AR(1) Model

$$E[Y_\tau] = \phi^\tau Y_0$$

with

- $\phi = .5$
- $Y_0 = 1$

Stationarity

Time-series analysis focuses on modeling *stationary* processes.

Stationary processes are those for which the mean, variance, and auto-correlation structure **stay the same over our window of observation.**

- Trends can produce non stationarity (changes in mean over time), so have to be removed
  - Failing to remove a trend if present produces bias... but more on that later!
- Lagged regression parameters stay fixed across waves
  - Always necessary in some form - otherwise impossible to estimate!
- Only certain parameter values allowed
  - For AR(1) $-1 < \phi < 1$

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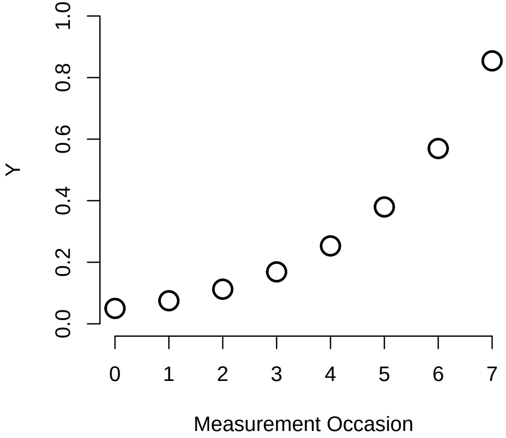
    - For AR(1) $-1 < \phi < 1$
    - AR(1) interpretation: system that gets pushed away from equilibrium by shocks but always returns to equilibrium

Impulse Response Function: Non-Stationary



AR(1) Model

$$E[Y_\tau] = \phi^\tau Y_0$$

with

- $\phi = 1.5$
- $Y_0 = 0.05$

Summary: Univariate Auto-regressive models

After removing of modeling trends and stationarity, we can fit an AR(p) model

$$Y_t = c + \phi_1 Y_{t-1} + \cdots + \phi_p Y_{t-p} + \varepsilon_t$$

where:

- c is the intercept term
- ϕ_p is the auto-regressive effect of order p
- ε_t is a white-noise Gaussian error term $\varepsilon_t \sim N(0, \sigma_\varepsilon)$
- We try to choose an appropriate p using the ACF and PACF

A stationary AR(p) process is one which fluctuates around a *stable equilibrium* i.e. long-run mean value μ

- For an AR(1), the mean is given by $\mu = \frac{c}{1-\phi}$

Outline

- Time series analysis: Univariate
- **Time series analysis: Multivariate**
- Multilevel time series analysis
- Extensions and Advanced Issues
- Discussion

Typically we have ILD that consist of **more than one** variable:

- mother's depression and child's disruptive behaviour
- stress and anxiety
- physical activity and happiness
- sleep quality and mood
- ...

In that situation, we might be interested in what time-series researchers call "**lead-lag relations**" between these variables

Multivariate time-series model

To analyze these relationships, we can use the multivariate generalization of the AR(p)

Known as the **Vector Autoregressive Model** or VAR(p)

$$\boldsymbol{Y}_t = \boldsymbol{c} + \boldsymbol{\Phi}_1 \boldsymbol{Y}_{t-1} + \cdots + \boldsymbol{\Phi}_p \boldsymbol{Y}_{t-p} + \boldsymbol{\epsilon}_t$$

where:

- $\boldsymbol{c}$ is a $q \times 1$ vector of intercepts
- $\boldsymbol{\Phi}_p$ is a $q \times q$ matrix of auto-regressive and cross-lagged effects
- $\boldsymbol{\epsilon}_t$ is a vector of multivariate Gaussian erros $\boldsymbol{\epsilon}_t \sim N(0, \Sigma_\varepsilon)$

A simple multivariate time-series model

The VAR(1) model is a multivariate version of the popular AR(1) model. We can write it as a set of equations:

$$
\begin{aligned}
A_t &= c_A + \phi_{11} A_{t-1} + \phi_{12} S_{t-1} + e_{A_t} \\
S_t &= c_S + \phi_{12} A_{t-1} + \phi_{22} S_{t-1} + e_{S_t}
\end{aligned}
$$

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$$S_t = c_S + \phi_{12} A_{t-1} + \phi_{22} S_{t-1} + e_{S_t}$$

where:
ϕ_{11} and ϕ_{22} are the **autoregressive coefficients**
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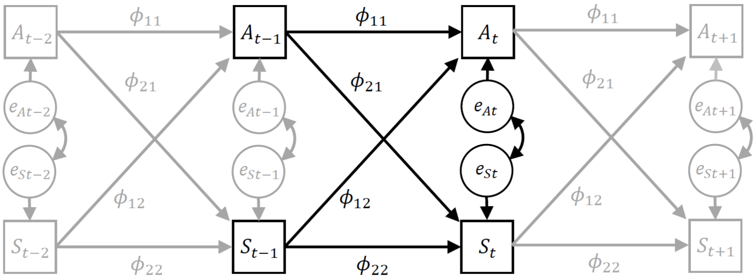
where:
ϕ_{11} and ϕ_{22} are the **autoregressive coefficients**
ϕ_{12} and ϕ_{21} are the **cross-lagged coefficients**

or in **matrix form**

$$\begin{bmatrix} A_t \\ S_t \end{bmatrix} = \begin{bmatrix} c_A \\ c_S \end{bmatrix} + \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{12} & \phi_{22} \end{bmatrix} \begin{bmatrix} A_{t-1} \\ S_{t-1} \end{bmatrix} + \begin{bmatrix} e_{A_t} \\ e_{S_t} \end{bmatrix}$$

Bivariate VAR(1)



Researchers often interpret the cross-lagged parameters in terms of **causal dominance** or **Granger causal** relationships. Note: the **standardized cross-lagged regression coefficients** should be used!

VAR(1) networks

VAR(1) models very popular: easy to fit and visualize the output

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VAR(1) models very popular: easy to fit and visualize the output

Become an increasingly popular tool since the popularity of the **network theory** of psychopathology (Borsboom, 2017)

- Idea: Mental disorders arise due to direct causal connections between themselves, not due to some latent "disease" variable
- Typical Empirical Application: Attempt to learn about network structure using statistical models which encode direct conditional dependencies

VAR(1) networks

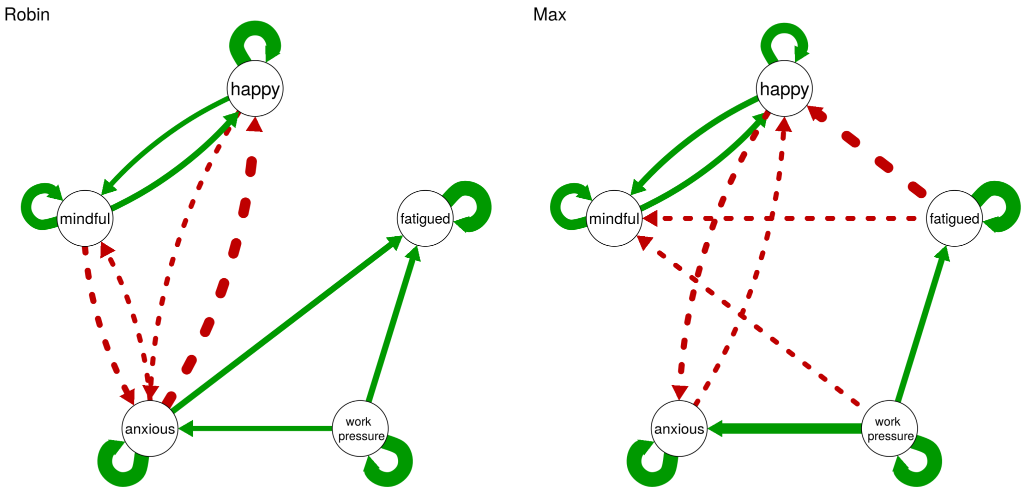
VAR(1) models very popular: easy to fit and visualize the output

Become an increasingly popular tool since the popularity of the **network theory** of psychopathology (Borsboom, 2017)

- Idea: Mental disorders arise due to direct causal connections between themselves, not due to some latent "disease" variable
- Typical Empirical Application: Attempt to learn about network structure using statistical models which encode direct conditional dependencies
- VAR(1) gives the simplest possible directed "network structure" from time-series data

General approach popular in other fields, special case of "Dynamic Bayesian Networks"

VAR(1) networks

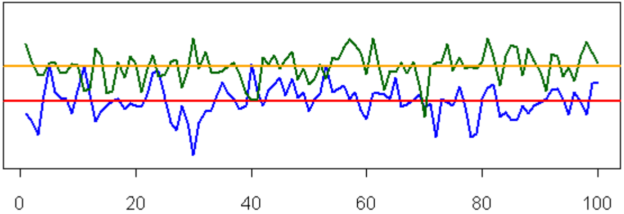


VAR(1) networks

Almost always a "dynamical network" in psychology is just a VAR(1) model with it's lagged parameters plotted

Remember: Qualitative dynamics same as the AR(1)

- Single resting state, which we either always return to or always explode away from



Outline

- Time series analysis: Univariate
- Time series analysis: Multivariate
    - **Practical: Exercises 1, 2 and 3**
- Multilevel time series analysis
    - Practical: Exercise 4 & 5
- Extensions and Advanced Issues
- Discussion

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- Time series analysis: Univariate
- Time series analysis: Multivariate
- **Multilevel time series analysis**
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Single-subject time series data allows us to study (for a *stationary process*):

- Lagged relationships between a variable and itself: **autoregression**
- Lagged relationships between different variables: **cross-lagged relationships**

Lagged relationships in multilevel data

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If we have **time series data from multiple individuals**, we may want to study:

- individual differences in **autoregression**
- individual differences in **cross-lagged relationships**

Lagged relationships in multilevel data

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- individual differences in **cross-lagged relationships**

If we use multilevel modeling for this, we could refer to it as **multilevel time series analysis**, or **dynamic multilevel modeling.**

Creating lagged predictors

ID
1
1
1
1
1
2
2
2
2
2
$\dots$
N
N
N

Creating lagged predictors

ID	y_{it}
1	y_{11}
1	y_{12}
1	y_{13}
1	$\dots$
1	y_{1T}
2	y_{21}
2	y_{22}
2	y_{23}
2	$\dots$
2	y_{2T}
$\dots$	$\dots$
N	y_{N1}
N	y_{N2}
N	y_{N3}

Creating lagged predictors

ID	y_{it}	y_{it-1}
1	y_{11}	
1	y_{12}	y_{11}
1	y_{13}	y_{12}
1	$\dots$	$\dots$
1	y_{1T}	y_{1T-1}
2	y_{21}	
2	y_{22}	y_{21}
2	y_{23}	y_{22}
2	$\dots$	$\dots$
2	y_{2T}	y_{2T-1}
$\dots$	$\dots$	$\dots$
N	y_{N1}	
N	y_{N2}	y_{N1}
N	y_{N3}	y_{N2}

Creating lagged predictors

ID	y_{it}	y_{it-1}	x_{it-1}
1	y_{11}		
1	y_{12}	y_{11}	x_{11}
1	y_{13}	y_{12}	x_{12}
1	$\dots$	$\dots$	$\dots$
1	y_{1T}	y_{1T-1}	x_{1T-1}
2	y_{21}		
2	y_{22}	y_{21}	x_{21}
2	y_{23}	y_{22}	x_{22}
2	$\dots$	$\dots$	$\dots$
2	y_{2T}	y_{2T-1}	x_{2T-1}
$\dots$	$\dots$	$\dots$	$\dots$
N	y_{N1}		
N	y_{N2}	y_{N1}	x_{N1}
N	y_{N3}	y_{N2}	x_{N2}

Inertia research based on multilevel AR(1) models

Level 1 model:

$$NA_{it} = c_i + \phi_i NA_{i,t-1} + \zeta_{it}$$

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Level 2 model:

$$c_i = \gamma_{00} + u_{0i}$$
$$\phi_i = \gamma_{01} + u_{1i}$$

Inertia research based on multilevel AR(1) models

Level 1 model:

$$NA_{it} = c_i + \phi_i NA_{i,t-1} + \zeta_{it}$$

Level 2 model:

$$c_i = \gamma_{00} + u_{0i}$$
$$\phi_i = \gamma_{01} + u_{1i}$$

This research line was initiated by **Suls, Green and Hillis (1998)**, and continued by the group of **Kuppens.**

The focus is on individual differences in the **autoregressive parameter** ϕ_i (=inertia, carry-over, regulatory weakness), which is shown to be:

- positively related to current depression, neuroticism, and being female
- predictive of later depression (Kuppens and Koval)

Dynamic networks based on multilevel VAR(1) models

Level 1 model:

$$
\begin{aligned}
y_{1it} &= c_{1i} + \phi_{11i} y_{1it-1} + \cdots + \phi_{1ki} y_{kit-1} + \zeta_{1it} \\
y_{2it} &= c_{2i} + \phi_{21i} y_{1it-1} + \cdots + \phi_{2ki} y_{kit-1} + \zeta_{2it} \\
&\dots \\
y_{kit} &= c_{ki} + \phi_{k1i} y_{1it-1} + \cdots + \phi_{kki} y_{kit-1} + \zeta_{kit}
\end{aligned}
$$

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\end{aligned}
$$

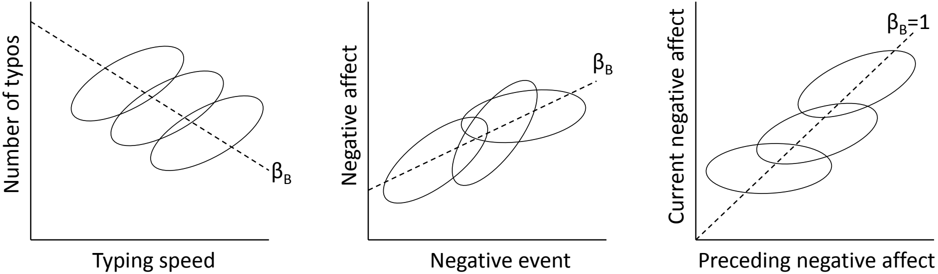
Initiated by **Bringmann et al. (2013)**, and further popularized by the software from **Sacha Epskamp**.

The focus is on **cross-lagged parameters** between variables (=nodes; typically symptoms), and on measures based on these (e.g., centrality).

Main idea is that **stronger connections** lead to an **increased risk** of developing and maintaining psychopathology.

Between-person differences in within-person slopes

- Studying a collection of within-person processes
- We want to know the average within-person process (fixed effects) and how these processes differ between people (random parts)
- "Decomposition" of data into within- and between- person variance components



Taken from Hamaker and Grasman (2014).

Decomposition

Decomposition into a between part and a within part

$$PA_{it} = \mu_{PA,i} + PA^*_{it}$$
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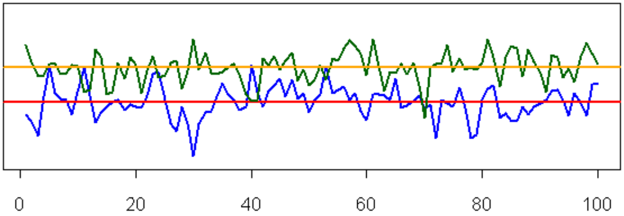
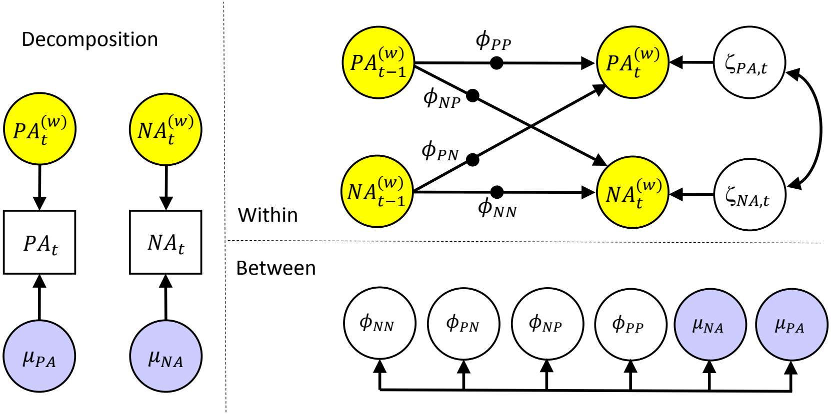
$$PA_{it} = \mu_{PA,i} + PA^*_{it}$$
$$NA_{it} = \mu_{NA,i} + NA^*_{it}$$

where

- $\mu_{PA,i}$ and $\mu_{NA,i}$ are the individual's **means** on PA and NA (i.e., baseline, trait, or equilibrium scores) $\Rightarrow$ between-person part
- PA^*_{it} and NA^*_{it} are the **within-person centered** (cluster-mean centered) scores $\Rightarrow$ within-person part



Bivariate model: Multilevel VAR(1) model



Within-person level model

Lagged within-person model:

$$PA_{it}^{*} = \phi_{PP,i} PA_{i,t-1}^{*} + \phi_{PN,i} NA_{i,t-1}^{*} + \zeta_{PA,it}$$
$$NA_{it}^{*} = \phi_{NN,i} NA_{i,t-1}^{*} + \phi_{NP,i} PA_{i,t-1}^{*} + \zeta_{NA,it}$$

where

- $\phi_{PP,i}$ is the **autoregressive parameter** for PA (i.e., inertia, carry-over)
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- $\zeta_{PA,it}$ is the **innovation** for PA (residual, disturbance, dynamic error)
- $\zeta_{NA,it}$ is the **innovation** for NA (residual, disturbance, dynamic error)

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Residual variances and covariances:

$$\begin{bmatrix} \zeta_{PA,it} \\ \zeta_{NA,it} \end{bmatrix} \sim MN \left[\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \theta_{11} & \\ \theta_{21} & \theta_{22} \end{bmatrix} \right]$$

Between-person level model

Between level: fixed and random effects

$$\begin{bmatrix} \mu_{PA,i} \\ \mu_{NA,i} \\ \phi_{PP,i} \\ \phi_{PN,i} \\ \phi_{NP,i} \\ \phi_{NN,i} \end{bmatrix} = \begin{bmatrix} \gamma_{P} \\ \gamma_{N} \\ \gamma_{PP} \\ \gamma_{PN} \\ \gamma_{NP} \\ \gamma_{NN} \end{bmatrix} + \begin{bmatrix} u_{P,i} \\ u_{N,i} \\ u_{PP,i} \\ u_{PN,i} \\ u_{NP,i} \\ u_{NN,i} \end{bmatrix} \qquad \boldsymbol{u}_i \sim MN(\mathbf{0}, \boldsymbol{\Psi})$$

Where:

- γ_P to $\gamma_{NN} \Rightarrow$ fixed effects
- $u_{P,i}$ to $u_{NN,i} \Rightarrow$ random effects

Between-person level model

Between level: fixed and random effects

$$\begin{bmatrix} \mu_{PA,i} \\ \mu_{NA,i} \\ \phi_{PP,i} \\ \phi_{PN,i} \\ \phi_{NP,i} \\ \phi_{NN,i} \end{bmatrix} = \begin{bmatrix} \gamma_P \\ \gamma_N \\ \gamma_{PP} \\ \gamma_{PN} \\ \gamma_{NP} \\ \gamma_{NN} \end{bmatrix} + \begin{bmatrix} u_{P,i} \\ u_{N,i} \\ u_{PP,i} \\ u_{PN,i} \\ u_{NP,i} \\ u_{NN,i} \end{bmatrix} \qquad \boldsymbol{u}_i \sim MN(\mathbf{0}, \boldsymbol{\Psi})$$

Where:

- γ_P to γ_{NN} $\Rightarrow$ fixed effects
- $u_{P,i}$ to $u_{NN,i}$ $\Rightarrow$ random effects

Parameters estimated at this level are:

- 6 fixed effects (i.e., γ's)
- 6 variances for random effects (i.e., diagonal elements of $\boldsymbol{\Psi}$)
- 15 covariances between the random effects (i.e., off-diagonal elements in $\boldsymbol{\Psi}$)

Why Bayesian Estimation?

Bayesian approaches to fitting multilevel time-series models have become increasingly popular, for a number of reasons:

- In principle can handle missing observations in a tidier way
- Simultaneous estimation of entire model, rather than variable-by-variable approach
- Can incorporate measurement models / latent variables
- Can more easily output within-person standardized cross-lagged parameters

- Bayesian approaches can be implemented in BUGS/JAGS, STAN, and have been implemented in DSEM in Mplus

Bayesian analysis is based on combining the **density of the data** with a **prior distribution** for the unknown parameters, to get a **posterior distribution** of these parameters.

Intermezzo on Bayesian analysis

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Posterior distribution of θ

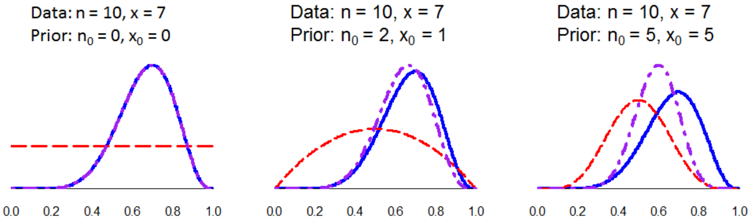
$$p(\theta|y) = \frac{f(y|\theta)p(\theta)}{f(y)}$$

where:

- $f(y|\theta)$ be the **density of the data** y given the parameters θ (also referred to as the likelihood)
- $p(\theta)$ be the **prior distribution** of the parameter(s) θ, which the user needs to specify
- $\int f(y,\theta)d\theta = f(y)$ is the **marginal density**, which can be ignored (because it is a constant)

Intermezzo on Bayesian analysis

Density (blue), prior (red), and posterior (purple):



When the prior is flat (no information), the posterior is identical to the likelihood.

If you have prior knowledge, you can add this to the equation by specifying a prior that reflects this.

For each to be estimated parameter, a prior needs to be specified. In the lab we'll aim to specify uninformative priors.

Intermezzo on Bayesian analysis: Convergence

Bayesian analysis is (often) based on using an **MCMC algorithm** which iteratively **samples** the parameters from their conditional posteriors.

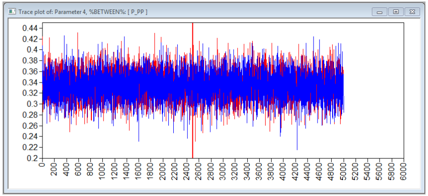
We have to check whether the analysis has **converged** (or: whether there are signs it did **not** converge).

Tools we use for this are:

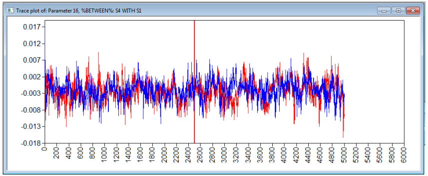
- Multiple chains; multiple runs of the analysis with different starting values.
- These chains should end up at approximately the same estimates.
- Burnin: Part of the iterations (before convergence) are discarded, leaving only 'converged' samples.
- Plots of the chains (fat hairy caterpillars), density plots (should look smooth and normal-ish), gelman rubin statistic: should be very close to 1.

Intermezzo on Bayesian analysis: Trace plots

This looks good (lazy, fat caterpillar):



This looks less good but not really bad; just needs more samples:



	N=1	multilevel
uni-variate	- arima in R - State Space Modeling software - Openmx - Bayesian modeling software	- any multilevel software - MLvar package in R - Bayesian modeling software
some-what multi-variate	- VARS package in R - State Space Modeling Software - Openmx - Bayesian modeling software	- any multilevel software - MLVar package in R - Bayesian modeling software
multi-variate	- State Space Modeling Software (mkfm6; Ox; fkf, dlm, KFAS, and MARSS in R) - Bayesian software (Winbugs, Openbugs, JAGS, STAN)	- Bayesian software (Winbugs, Openbugs, JAGS, STAN)

Multilevel vs Single-Subject Time Series

Advantages of Multi-Level Time Series Models:

- Allow to model within-person and between-person structure simultaneously
- Regularizes estimates of within-person parameters
  - Person-specific parameters get “pulled” towards the mean. Estimates have lower variance across samples
- The more similar people are, and the more people we have, the less observations we need per person

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Disadvantages of Multi-Level Time Series Models:

- Need to assume the same type of time series model (or a very general model) for everyone
- Regularizes estimates of within-person parameters
  - If we have enough time points, and we really care about the individual-specific parameters, we don’t want this to happen
  - If people are not that similar, we might get a very strange/uninformative “average” model

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Conclusion: last form is most meaningful, as it **parallels standardizing when N=1**.

Standardized fixed effect should be the **average standardized within-person effect**.

Within the model, variables are **within-person centered**

Decomposition into a between part and a within part

$$PA_{it} = \mu_{PA,i} + PA^*_{it}$$
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If we have a trend, i.e., some linear or quadratic function of time t, then we typically want to **de-trend per person**.

Be careful: not always implemented in multilevel software, and if you de-trend before fitting the model, the means for every person will be zero!

Detrending and Stationarity

There are actually two distinct mechanisms which can produce **non-stationarity**

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Deterministic Trend: variables are a direct function of time, which we treat as an exogenous variable

$$y_t = c + \phi y_{t-1} + bT + e_t$$

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$$y_t = c + 1.5 y_{t-1} + e_t$$

Solution: *differencing*. $y* = y_t - y_{t-1}$, then $y*$ is stationary.

Detrending and Stationarity

Failing to de-trend when there's a **deterministic trend** present results in bias

But **erroneously de-trending** when actually there is a **stochastic trend** can also lead to bias!

You can test for the presence of a stochastric trend using the *Augmented Dickey-Fuller* test. More details in Ryan & Haslbeck (in preparation)

Time-Varying VAR(1) models:

- Haslbeck, Bringmann & Waldorp (2020)
- Assumes “local stationarity” - all parameters can be smooth functions of time
- Mostly single-subject, needs even more time-points

Regime-Switching Models:

- Time-series models where the system “switches” between different states with different parameters
- (Hidden) Markov Models, Threshold VAR models
- See Hamaker, Grasman, Kamphuis (2010); Haslbeck* & Ryan* (2021); dynr (Ou, Hunter, Chow, 2017)

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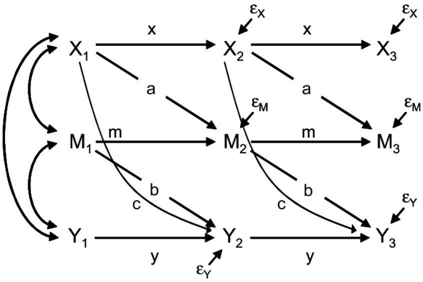
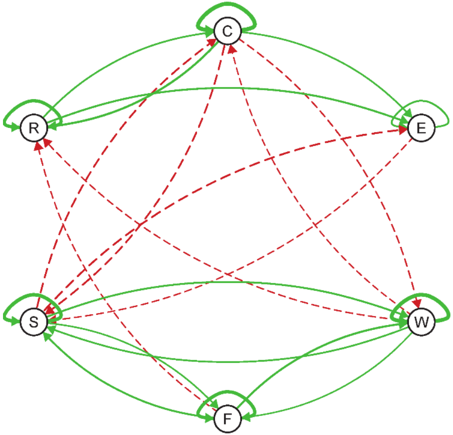
Very much a work in progress. Personal tip: You can always use out-of-sample prediction error to compare models!

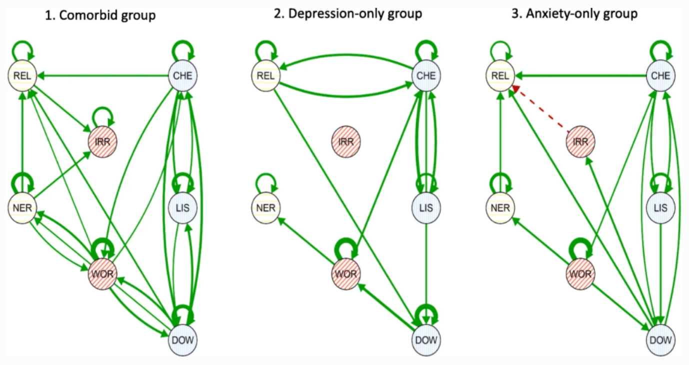
We may also be interested in *direct*, *indirect* and *total effects* **across lags.** Often the interest in *psychological network analysis*





Level-1 Mediation and Network Analysis

Groen, Ryan, Wigman et al (2020): Average within-person VAR(1) network structure of different “symptom-state” items in ESM data, across three groups: Comorbid, Anxiety-only and Depression-only.



No evidence that the comorbid group displayed higher indirect effects through “bridge symptoms” than the other groups. All analyses in DSEM

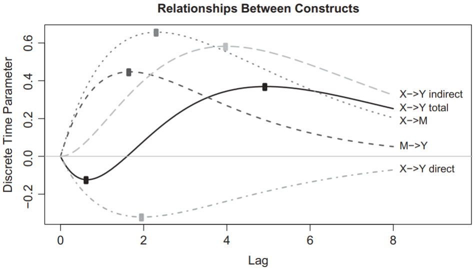
The Time-Interval Problem

VAR(1) models are **discrete time**: Do not account in any way for time-interval information

All lagged regression models (potentially) suffer from the time-interval problem.

Effects can change sign, size and relative ordering depending on how measurements are spaced in time (Kuiper & Ryan, 2018)

More on this **tomorrow!**



Outline

- Time series analysis
- Multilevel time series analysis
- DSEM application: Multilevel VAR(1) model
- Extensions and Issues
- **Discussion**

Summary

Time-series analysis aims to capture the structure of within-person variation over time when we have many repeated measures

When we have ILD from multiple individuals, **multi-level time series models** can be used to try and explore how within-person structure varies across people

The VAR(1) is just one type of time-series model which happens to be very popular - simple, lag-1 linear relationships.

The qualitative behaviour of this system is the same as the AR(1), but multivariate: fluctuating around a single equilibrium position.

Discussion

Of course, big issue is: which time-series models are substantively interesting?

- Which captures/reproduces patterns that are interesting?
- How can this be driven by theory?

Approach direct (and causal) interpretation with care

Tip: Pick a model and simulate time-series data from it (e.g. in R) - get a feel for what types of patterns it produces

These models are just tools. Proper use must be informed by theory (Haslbeck*, Ryan*, Robinaugh* et al, in press)

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