Modeling Intensive Longitudinal Data in Discrete and Continuous Time

R group University of Zurich
Day 2

Oisín Ryan

Utrecht University

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Intro

Previous Lecture: *Discrete-Time* (DT) models

- Current observation regressed on previous observation(s)
- *Time series analysis* tradition of economics

Continuous-Time models

- Based on *differential equations*
- Attempt to model dynamics using relationships at the *moment-to-moment* level
- *Dynamical Systems Theory* - Physics, Ecology, Biology
- Different perspective, but many shared concepts and connections

Why care about CT models?

- Flexible modeling approach with a large literature
  - Focus on *qualitative behaviour* of a system
  - Increasing attention on generative/formal models (e.g. Robinaugh et al. 2020, Haslbeck, Ryan, Robinaugh et al, 2021, Borsboom et al 2021)

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- Practical and Conceptual Advantages
  - *The Time-Interval Problem*
  - Unequally spaced measurements
  - New perspective on network structure, direct effects, interventions
  - Can be estimated from the same type of data as DT models

Overview

1. **Basic concepts of CT models**
   - What is a differential equation?
   - Connection with VAR(p) models
   - CT-AR(1) and CT-VAR(1)
2. Why should you care?
3. What can you do in practice?

In many cases, CT models describe the same qualitative behaviour as DT models

CT and DT models

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But CT models use a different perspective to do so, and so, different language

- Differential Equations and Integral Solutions

CT and DT models

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But CT models use a different perspective to do so, and so, different language

- Differential Equations and Integral Solutions

To build intuition for this, let's use the simplest possible example

Impulse Response Functions help us understand qualitative behaviour

1. Pick an interesting value for Y_0
2. Take your model + parameters
3. Calculate expected value of Y_1
4. Repeat and plot to visualize time-evolution of the system

A simple discrete-time model

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AR(1) Model

$$E[Y_1] = \phi Y_0$$

A simple discrete-time model

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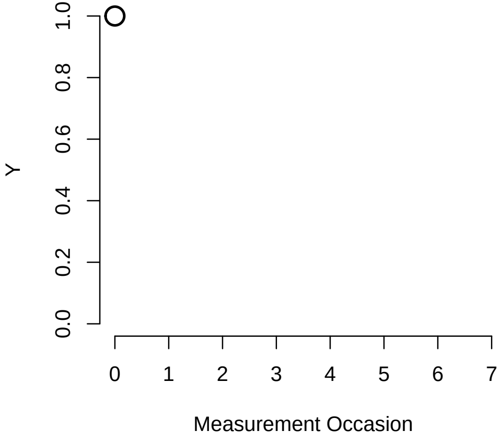
AR(1) Model

$$E[Y_1] = \phi Y_0$$

with

- $\phi = .5$
- $Y_0 = 1$

A simple discrete-time model



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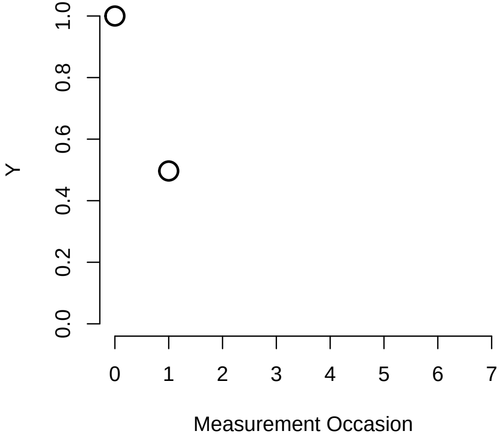

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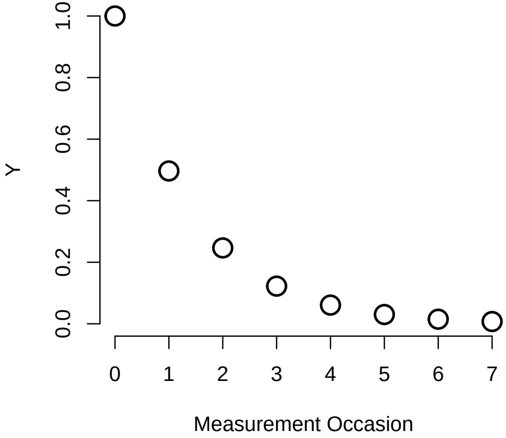

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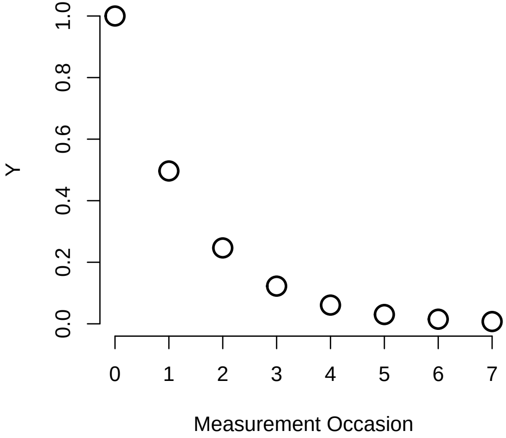

AR(1) Model

$$E[Y_2] = \phi \times \phi Y_0$$

with

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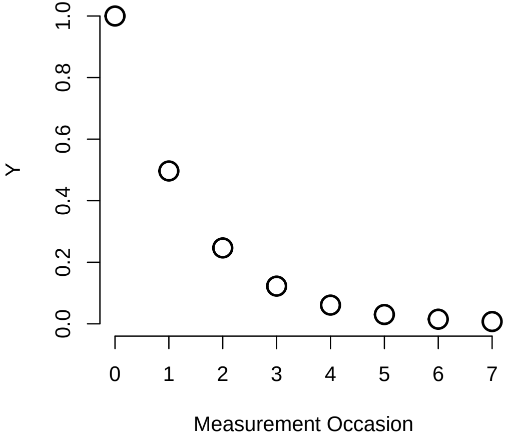
- $Y_0 = 1$

A simple discrete-time model



AR(1) Model

$$E[Y_\tau] = \phi^\tau Y_0$$

with

- $\phi = .5$
- $Y_0 = 1$

The Discrete-Time AR(1) model:

- when $0 < \phi < 1$ describes an *equilibrium reverting* system
- after a *shock* the system gets pulled back towards resting state
- moves there in an *exponential decay*
- Note: We ignore the $-1 < \phi < 0$ case for now

A simple discrete-time model

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Dynamics understood in terms of discrete “jumps" from one *occassion* to the next

A simple discrete-time model

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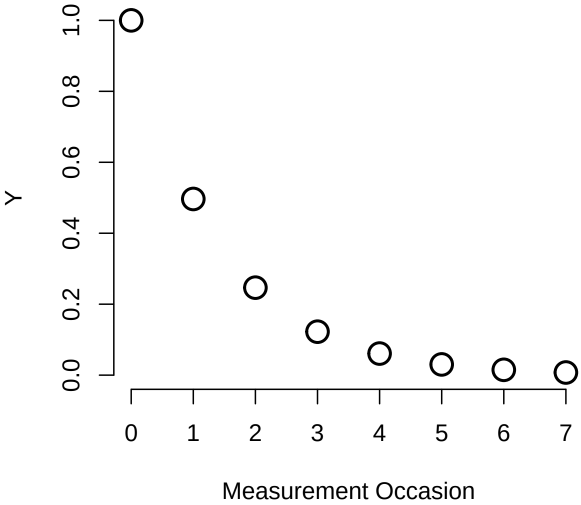
Dynamics understood in terms of discrete “jumps" from one *occassion* to the next

A **continuous-time** model can describe the same *qualitative behaviour*, but treating *time* itself as a continuous dimension


1.0
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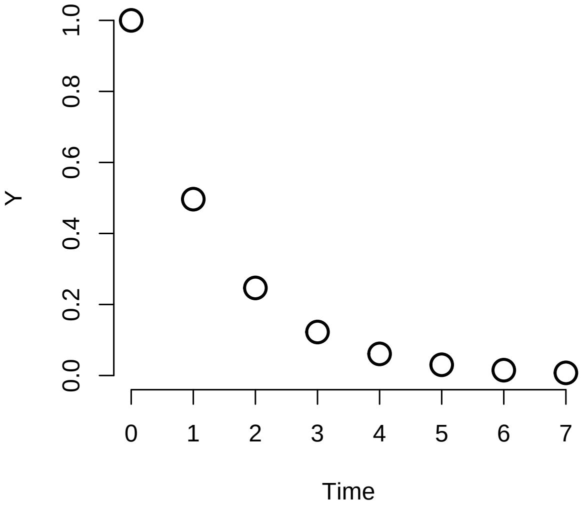
Measurement Occasion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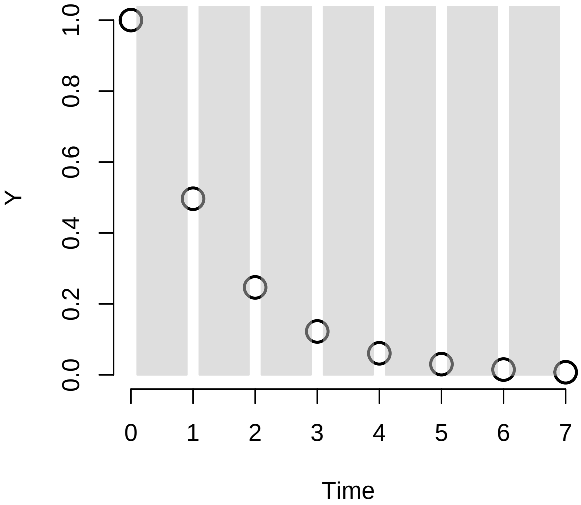
Time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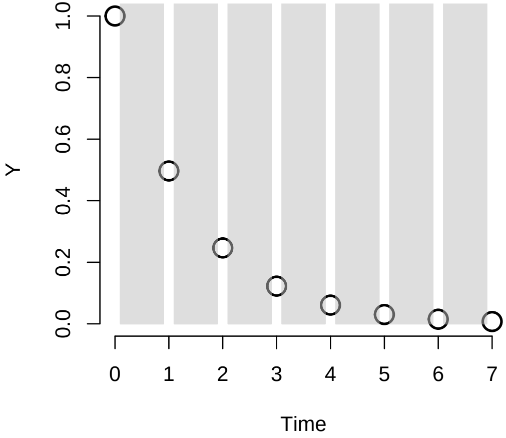
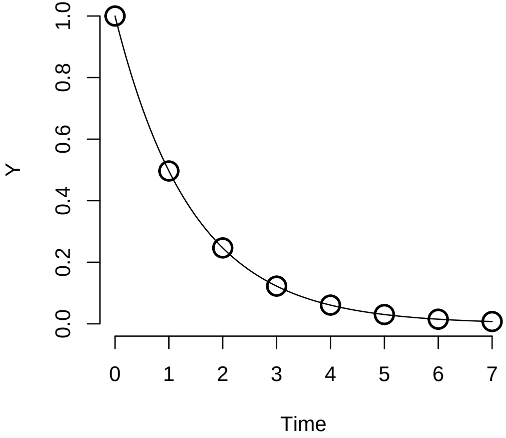
A simple continuous-time model



Continuous-Time Models

- Process takes on *some value* at every moment in time

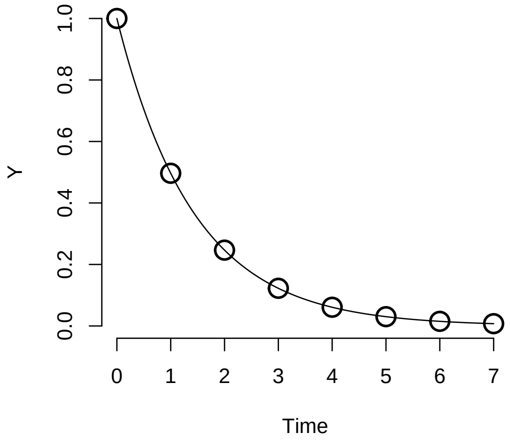
A simple continuous-time model



Continuous-Time Models

- Process takes on *some value* at every moment in time
- System evolves in a smooth and continuous manner

A simple continuous-time model

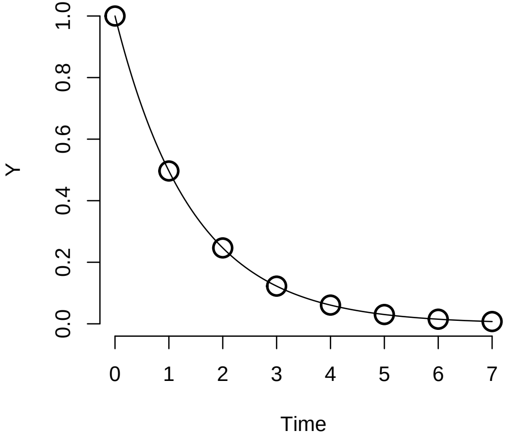


Continuous-Time Models

- Process takes on *some value* at every moment in time
- System evolves in a smooth and continuous manner

We can describe the evolution of the system using *Differential Equations*

A simple continuous-time model

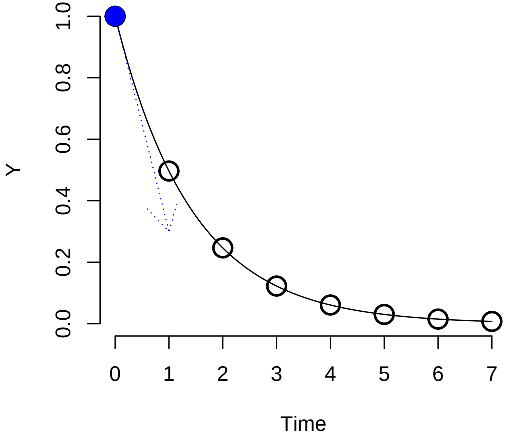


First-order DE

$$\frac{dY(t)}{dt} = A \times Y(t)$$

with

- $A = -.69$



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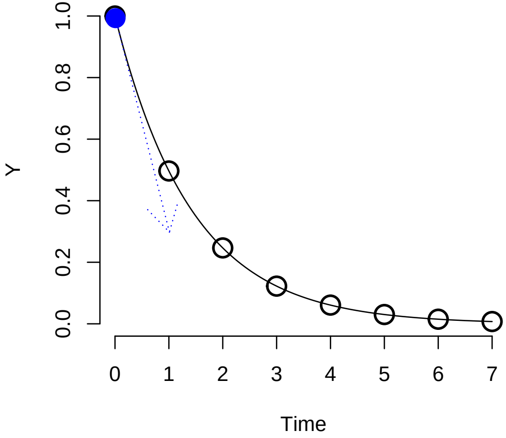

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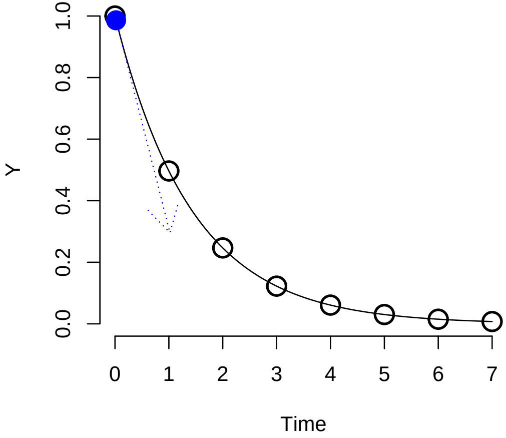

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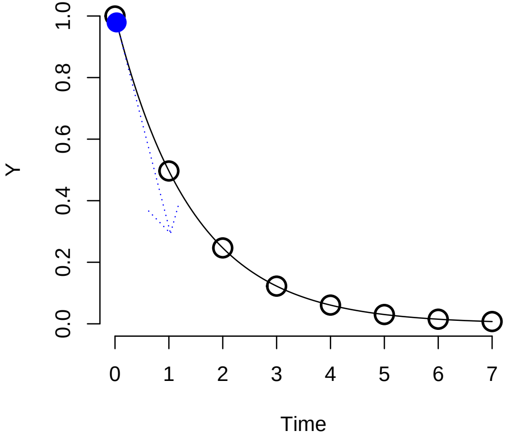

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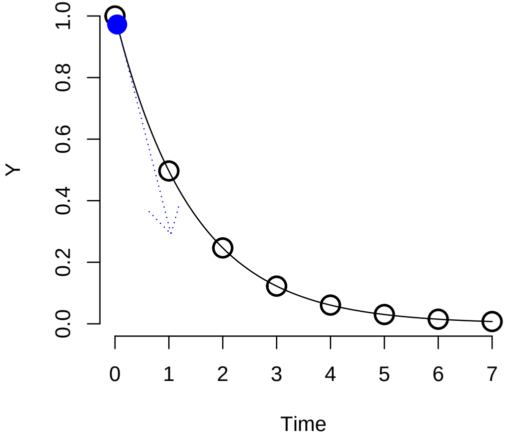

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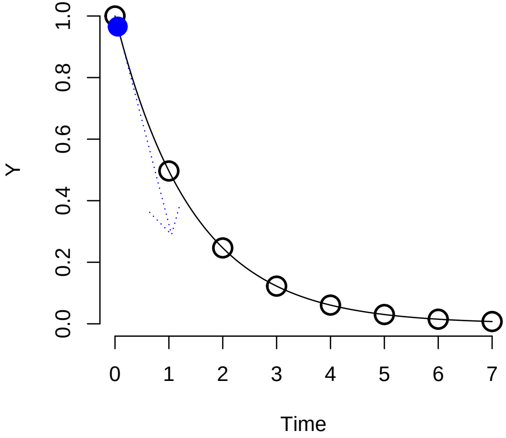

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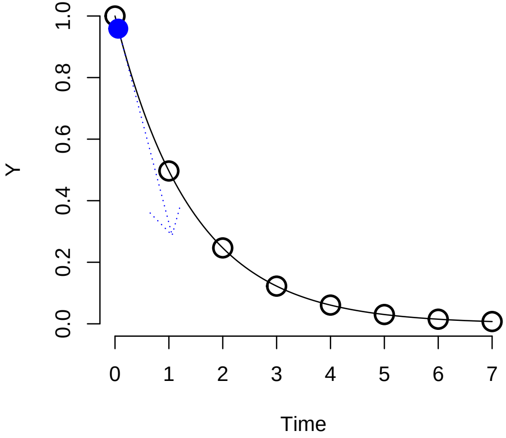

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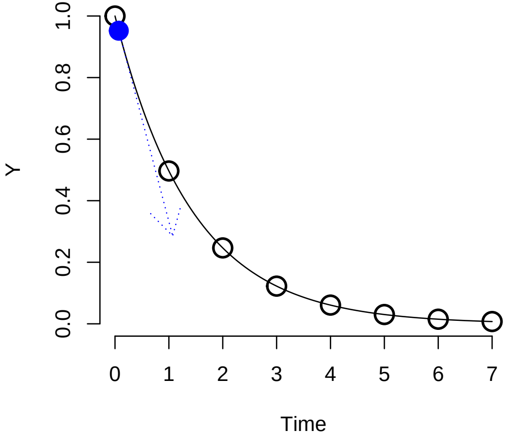

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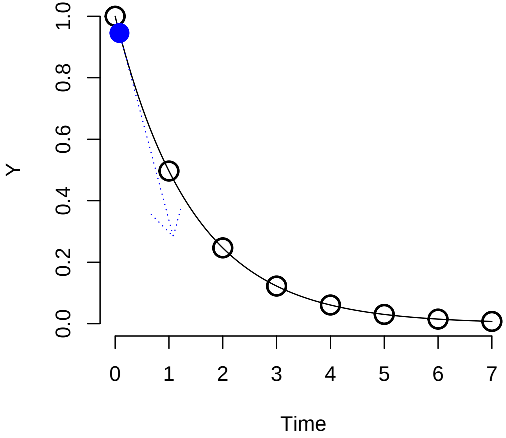

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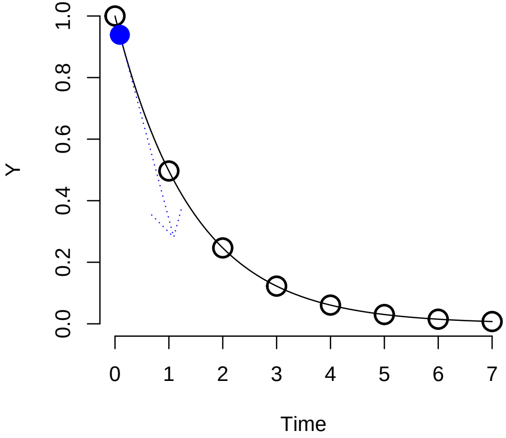

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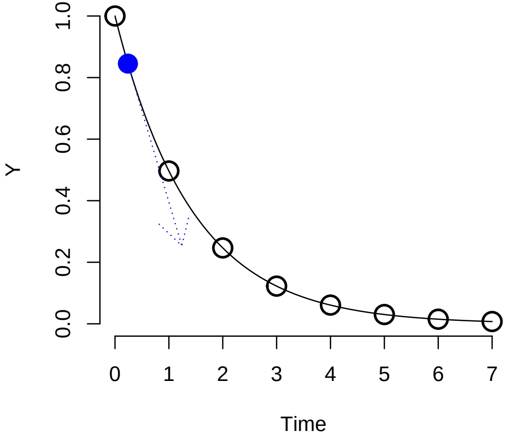

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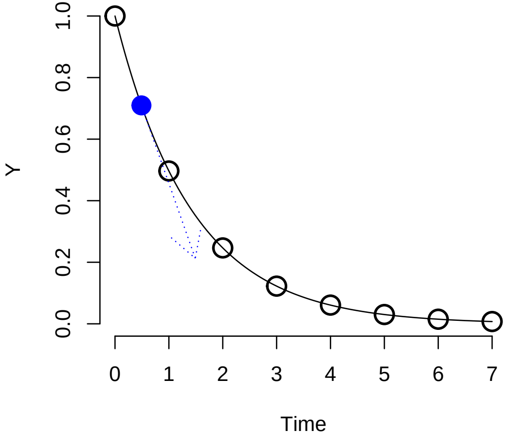

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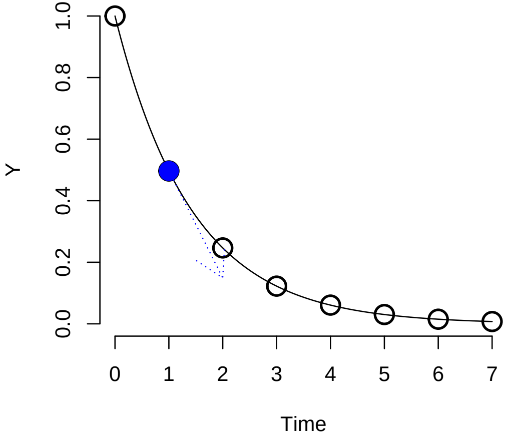

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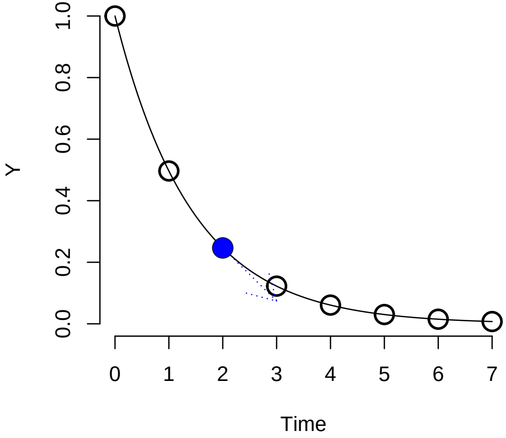

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The same, but different

AR(1) Model

$$E[Y_{\tau+1}] = \phi \times Y_\tau$$

with

- $0 < \phi < 1$
- Equally spaced measurements
- Discrete-time model

First-order DE

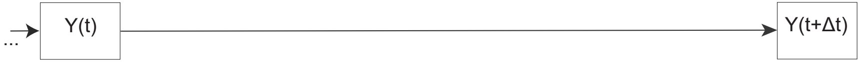
$$\frac{dY(t)}{dt} = A \times Y(t)$$

with

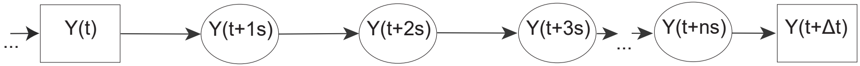
- $A < 0$
- Continuous-time model

Both models are equivalent, in the sense they describe the same qualitative behaviour

Discrete-Time AR(1)



Continuous-Time AR(1)



Model Equivalence

Re-write the DE so that the rhs is “Y now” and the lhs is “Y later”

$$\frac{dY(t)}{dt} = A \times Y(t)$$
$$\lim_{s \to 0} \frac{Y(t+s) - Y(t)}{s} = A \times Y(t)$$

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$$\lim_{s\to 0} \frac{Y(t+s) - Y(t)}{s} = A \times Y(t)$$

$$\lim_{s\to 0} Y(t+s) = (1 + A \lim_{s\to 0} s) Y(t)$$

Re-write the DE so that the rhs is “Y now” and the lhs is “Y later”

$$\begin{aligned}
\frac{dY(t)}{dt} &= A \times Y(t) \\
\lim_{s\to 0} \frac{Y(t+s) - Y(t)}{s} &= A \times Y(t) \\
\lim_{s\to 0} Y(t+s) &= (1 + A \lim_{s\to 0} s) Y(t) \\
Y(t+\Delta t) &
\end{aligned}$$

Re-write the DE so that the rhs is “Y now” and the lhs is “Y later”

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\lim_{s \to 0} Y(t+s) &= (1 + A \lim_{s \to 0} s) Y(t) \\
Y(t + \Delta t) &= \lim_{n \to \infty} (1 + A \frac{\Delta t}{n})^n Y(t)
\end{aligned}
$$

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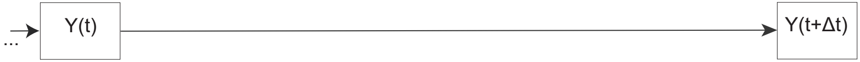
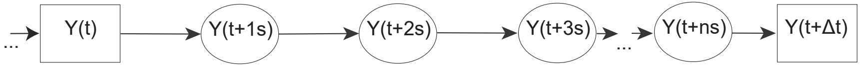
$$Y(t + \Delta t) = \lim_{n \to \infty} (1 + A \frac{\Delta t}{n})^n Y(t)$$

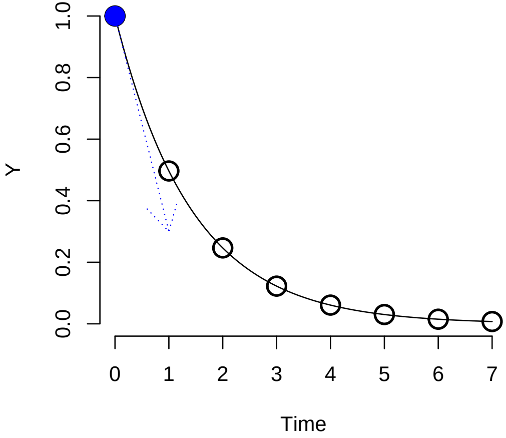
$$Y(t + \Delta t) = e^{A \Delta t} Y(t)$$

Discrete-Time AR(1)



Continuous-Time AR(1)





First-order DE

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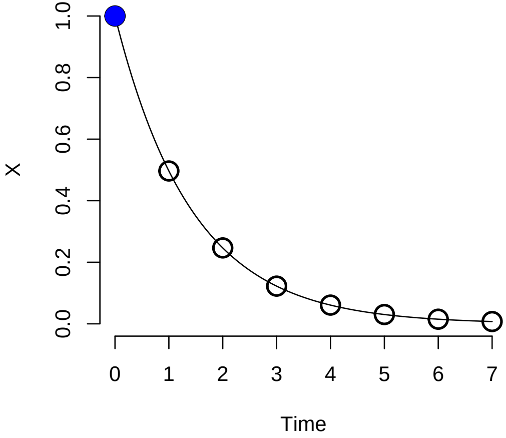
with

- $A = -.69$

A simple continuous-time model



Integral Solution

$$Y(t + \Delta t) = e^{A\Delta t} Y(t)$$

with

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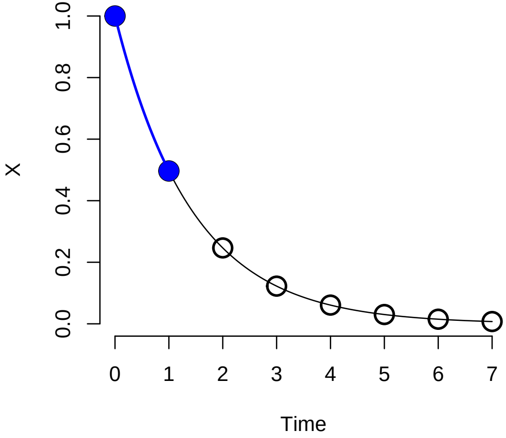
A simple continuous-time model



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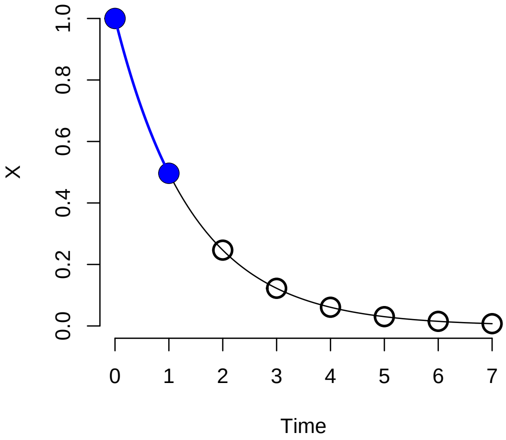


Integral Solution

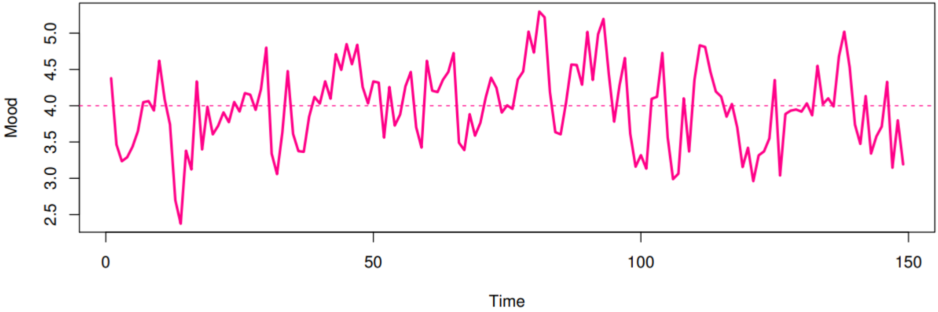
$$Y(t+\Delta t) = e^{A\Delta t}Y(t)$$

with

- $A = -.69$
- $e^{(-.69\times 1)} = \phi = .5$

Adding Noise

So far we only dealt with the *deterministic* part of the DE model. But we typically also want to allow for random noise or innovation variance



AR(1) Model

$$Y_{\tau+1} = \phi Y_\tau + \varepsilon_\tau$$

with

- $0 < \phi < 1$
- Equally spaced measurements
- $\varepsilon_\tau \sim N(0, \sigma)$

AR(1) Model

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First-order Stochastic DE

$$\frac{dY(t)}{dt} = A \times Y(t) + \frac{dW(t)}{dt}$$

with

- $A < 0$
- $\frac{dW(t)}{dt}$ is a *Wiener process*
- A random walk/white noise in CT

Adding Noise

AR(1) Model

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Adding Noise

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- $0 < \phi < 1$
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CT-AR(1)

$$Y(t + \Delta t) = e^{A\Delta t} Y(t) + \varepsilon(\Delta t)$$

with

- $A < 0$
- $\varepsilon(\Delta t)$ is Gaussian noise whose variance scales with Δt
- When $\Delta t \to 0$, $\varepsilon \to 0$
- When $\Delta t \to \infty$, $\varepsilon \sim N(0, \gamma)$

VAR(1) Model

$$\boldsymbol{Y}_{\tau+1} = \boldsymbol{\Phi}\boldsymbol{Y}_{\tau} + \boldsymbol{\epsilon}_{\tau}$$

with

- $0 < \lambda_{\Phi} < 1$ (eigenvalues)
- Equally spaced measurements
- $\varepsilon_{\tau} \sim N(0, \Sigma)$

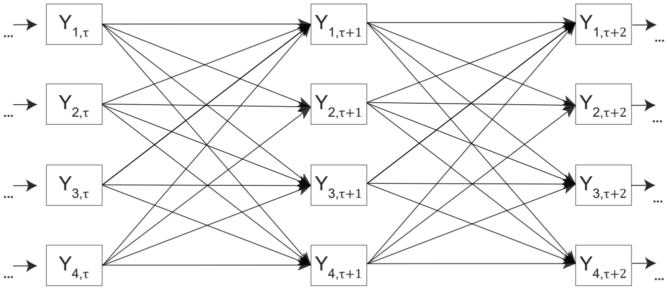
CT-VAR(1)

$$\boldsymbol{Y}(t + \Delta t) = e^{\boldsymbol{A}\Delta t}\boldsymbol{Y}(t) + \boldsymbol{\epsilon}(\Delta t)$$

with

- $\lambda_A < 0$ (eigenvalues)
- $\boldsymbol{\epsilon}(\Delta t)$ is Gaussian noise whose variance scales with Δt
- When $\Delta t \to 0$, $\boldsymbol{\epsilon} \to 0$
- When $\Delta t \to \infty$, $\boldsymbol{\epsilon} \sim N(0, \boldsymbol{\Gamma})$

$$\boldsymbol{Y}_{\tau+1} = \boldsymbol{\Phi}\boldsymbol{Y}_{\tau} + \boldsymbol{\epsilon}_{\tau}$$



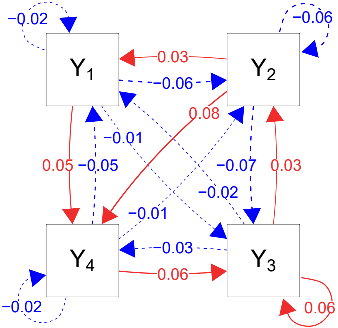
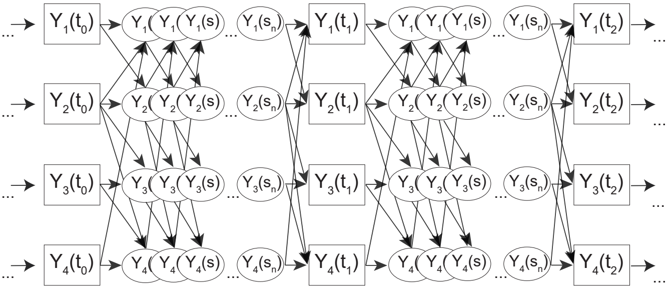


Figure from Ryan & Hamaker (2021)

$$\boldsymbol{Y}(t+\Delta t) = e^{\boldsymbol{A}\Delta t}\boldsymbol{Y}(t) + \boldsymbol{\epsilon}(\Delta t)$$



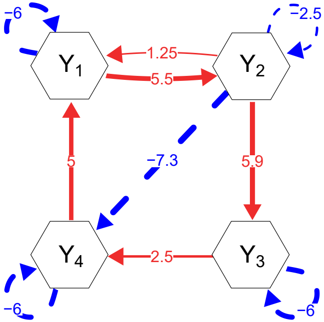


Figure from Ryan & Hamaker (2021)

Higher-Order Models

Just like in a DT model, we can get a broader range of dynamic behaviour by considering higher-order models

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- Discrete-Time: AR(2) model produces "oscillating" behaviour
- Continuous-Time: Second-order derivative $(d^2y(t)/dt)$ to produce this behaviour

Generally: If you can fit a (CT-)VAR(1), you can fit a higher order model, by **re-writing** it as a multivariate first-order model

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Generally: If you can fit a (CT-)VAR(1), you can fit a higher order model, by **re-writing** it as a multivariate first-order model

DT models with **negative eigenvalues** are trickier... need more complicated CT models for this

Overview

1. Basic concepts of CT models
2. **Why should you care?**
   - DT models and the time-interval problem
   - CT mediation and network analysis
3. What can you do in practice?

A continuous-time perspective

Psychological Processes as Continuous-Time Processes (Boker, 2002)

- CT may be a closer approximation to psychological dynamics
- Phenomena such as stress and anxiety can be defined at any point in time, not just at measurement occassions
- Likely to continuously influence one another to some degree

Explicit modeling of the time-interval

- Time-interval is important to our understanding of dynamics
- Aspirin-Headache effect, Stress-Rumination effect

The Time-Interval Problem

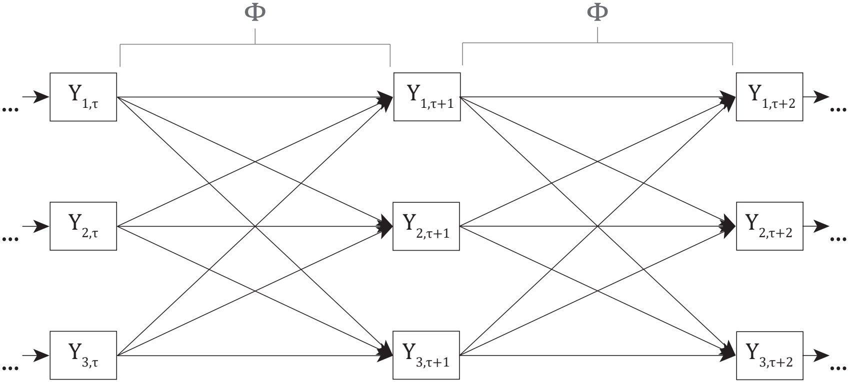
Advantages of CT modeling $\Rightarrow$ Disadvantages of DT modeling

If we think our process is CT, we have to seriously re-think how we fit and interpret DT models

DT lagged relationships critically dependent on the time-interval

$$e^{(A\Delta t)} = \mathbf{\Phi}(\Delta t)$$

$$\boldsymbol{Y}_{\tau} = \boldsymbol{\Phi}(\Delta t)\boldsymbol{Y}_{\tau-1} + \boldsymbol{\epsilon}_{\tau}$$



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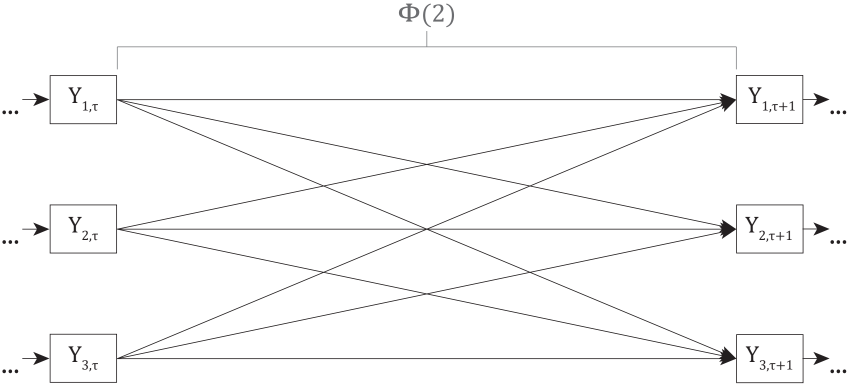

Time-Interval Dependency

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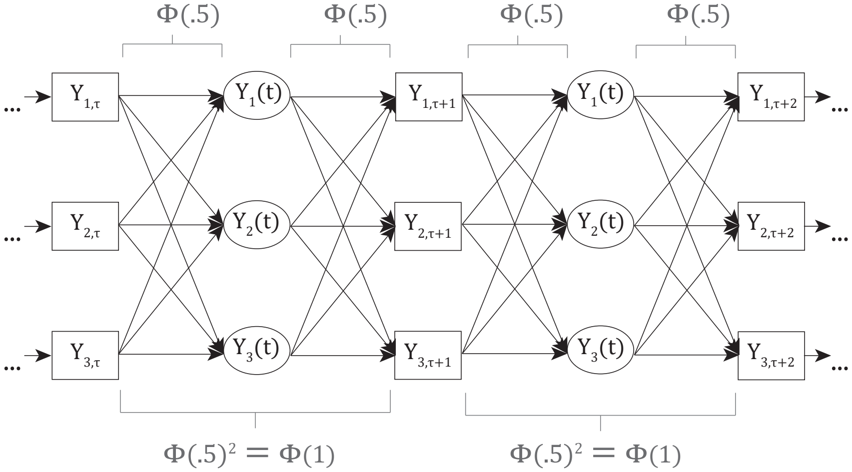
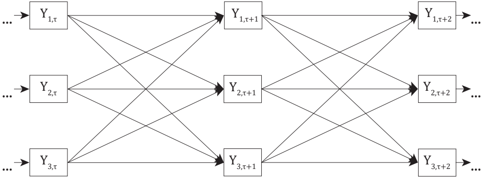


Consequences of time-interval dependency

1. Equal time-intervals: not generalizable
   - $\mathbf{\Phi}(\Delta t = 1) \neq \mathbf{\Phi}(\Delta t = .5)$



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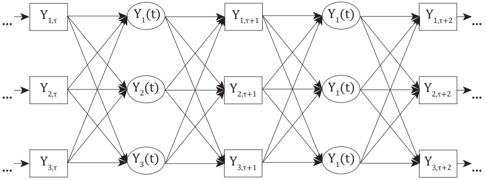

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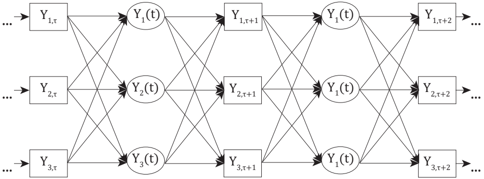

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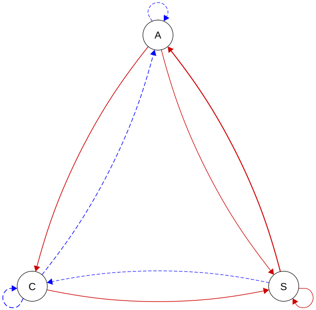

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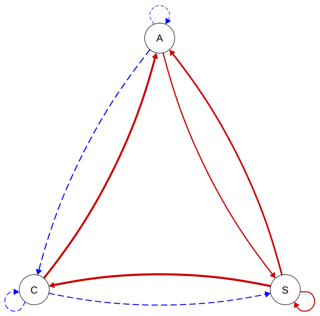

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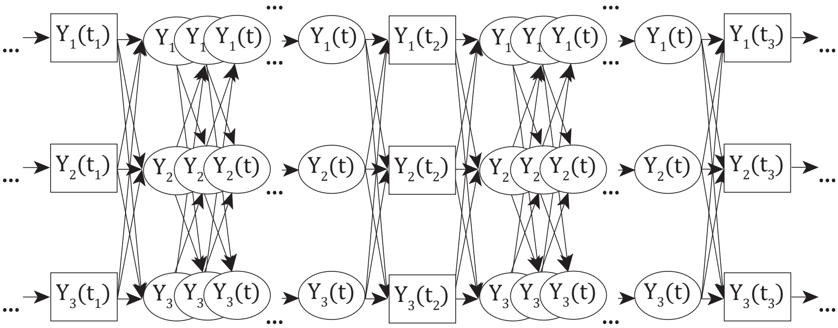

The CT-VAR(1) model

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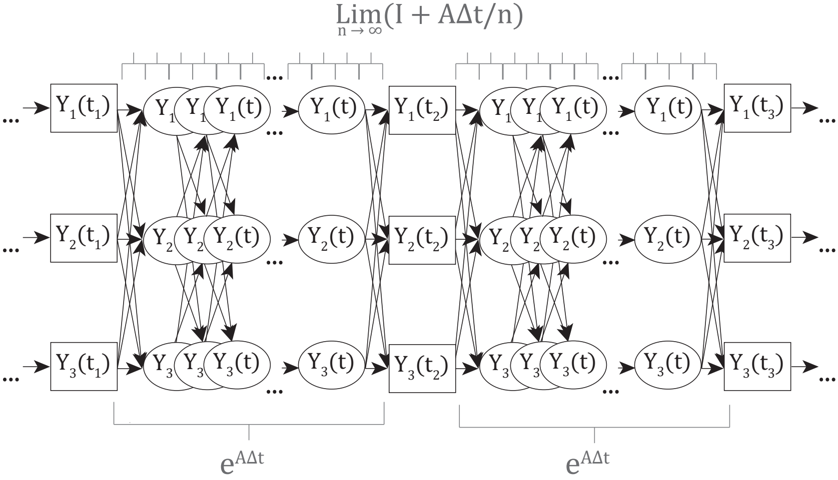
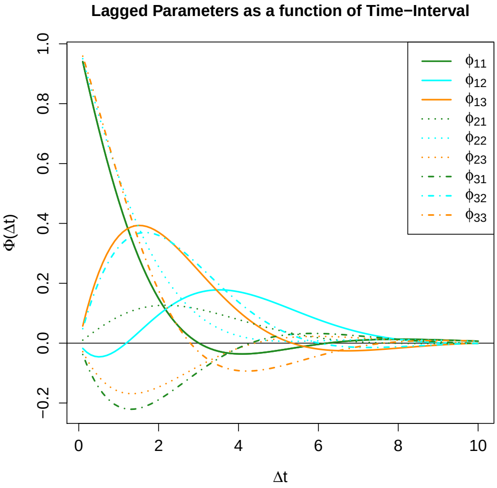


Time-interval dependency of VAR estimates

$$e^{A\Delta t} = \Phi(\Delta t)$$



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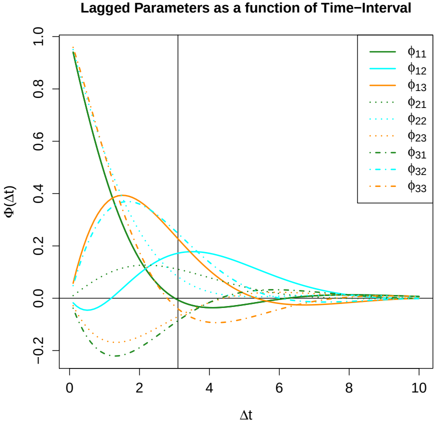

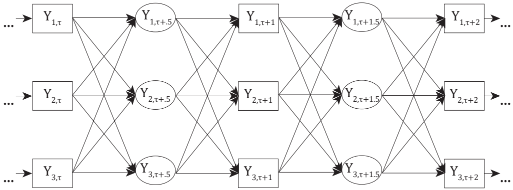

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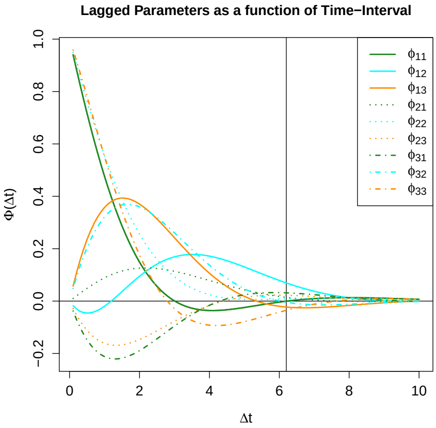

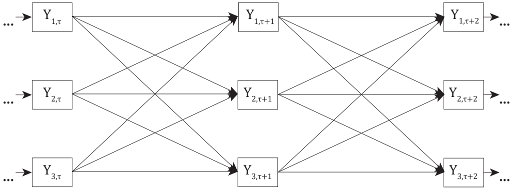

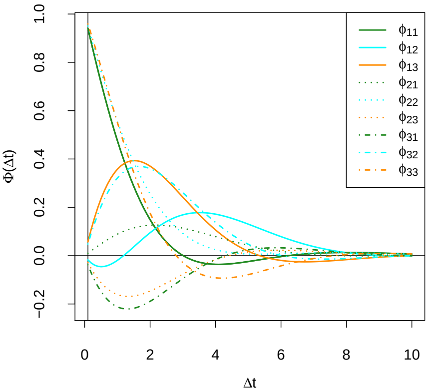

1.0
0.8
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Φ(Δt)
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6
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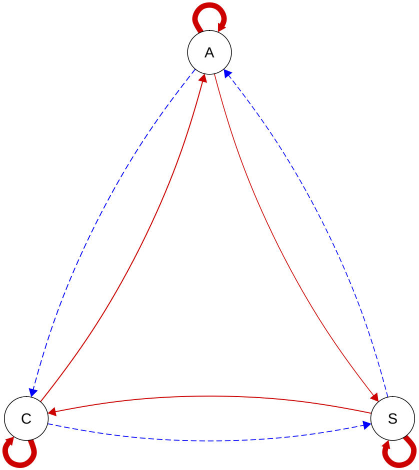

A
C
S

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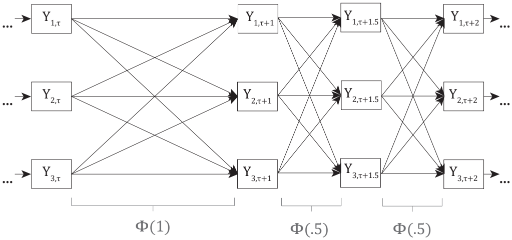

2. Unequal time-intervals: mix of effects
   - $\hat{\mathbf{\Phi}} = ?$
   - If not accounted for, may not reflect effects at *any* time-interval

Consequences of time-interval dependency

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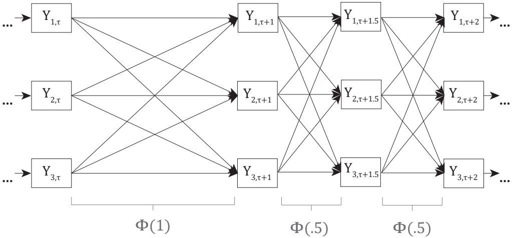
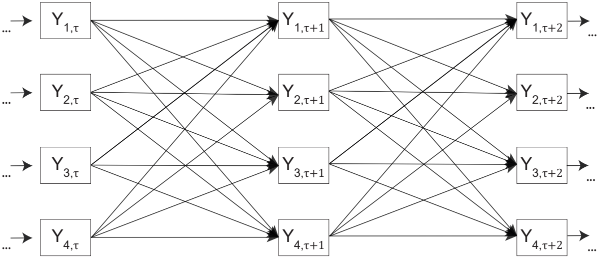


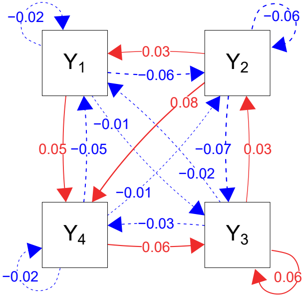
2. Unequal time-intervals: mix of effects
   - $\hat{\Phi} = ?$
   - If not accounted for, may not reflect effects at *any* time-interval

3. $\Phi(\Delta t)$ should not be interpreted as *direct effects*

Use of DT models often based on interpretation of Φ parameters as *direct* effects

- Mediation analysis and path tracing (Cole and Maxwell 2003)
- Network analysis, network structure, centrality, intervention targets (Bringmann et al 2013)





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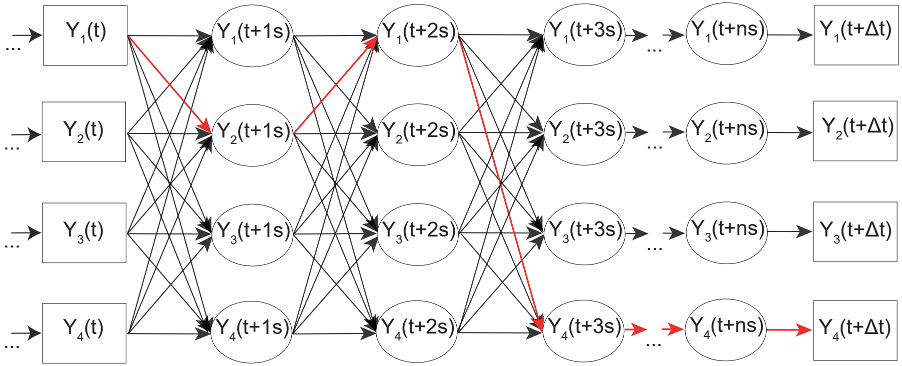
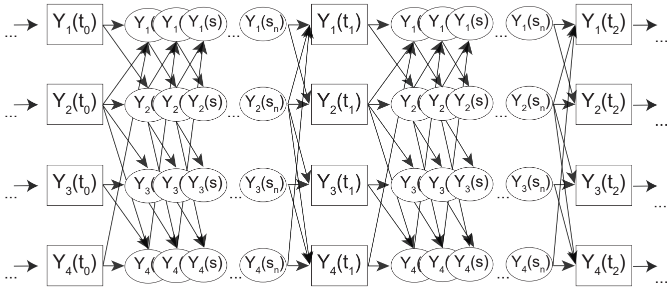
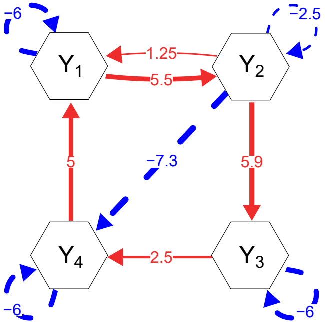
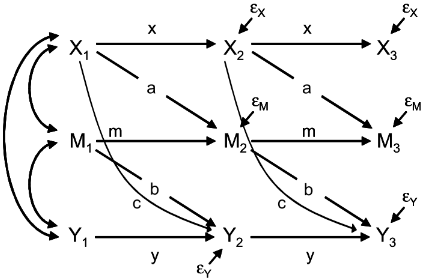


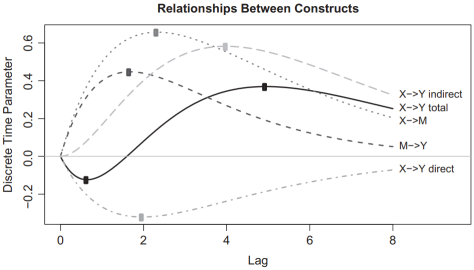
Instead the CT matrix A should be seen as **direct** moment-to-moment effects, and so should be the basis of path-tracing, centrality, network structure (Ryan & Hamaker, 2021)





Concepts like Total and Indirect Effects can also be defined using a path-tracing logic between $\boldsymbol{Y}(t)$ and $\boldsymbol{Y}(t+\Delta t)$ (Deboeck & Preacher 2013)





Recall our interpretation of the matrix exponential as a path-tracing operation:

$$\begin{aligned}
\lim_{s\to 0} Y(t+s) &= (1 + A \lim_{s\to 0} s) Y(t) \\
Y(t+\Delta t) &= \lim_{n\to\infty} (1 + A\frac{\Delta t}{n})^n Y(t) \\
Y(t+\Delta t) &= e^{A\Delta t} Y(t)
\end{aligned}$$

To obtain an indirect effect, we simply alter $\boldsymbol{A}$ to "shut-off" the direct path, i.e., set $a_{ij} = 0$ before taking the matrix exponential (Ryan & Hamaker, 2021)

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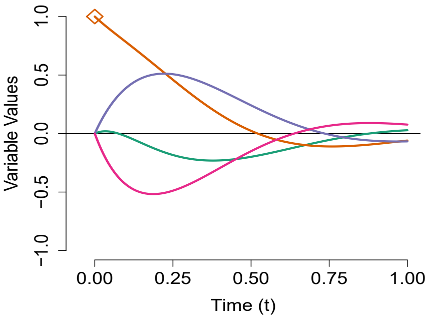
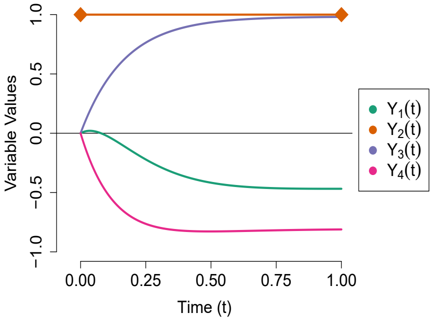
This is fine, but sometimes difficult to interpret. Path-tracing effects and (related) centrality measures often used to **identify intervention targets**. How does this work in principle?

Intervention Targets



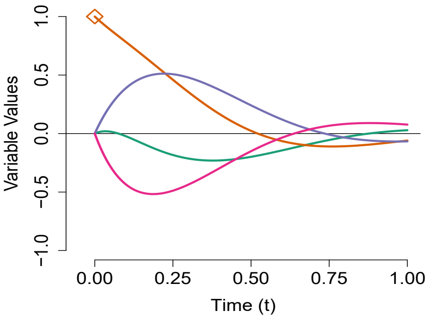
Pulse Intervention



Press Intervention

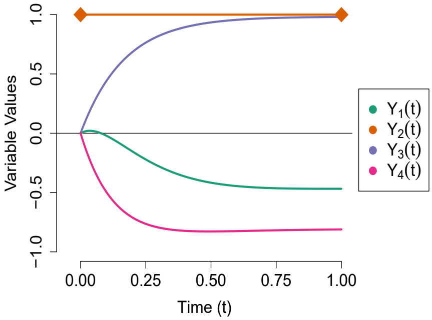
Ryan & Hamaker, 2021

Intervention Targets?



Pulse Intervention

Total Effect Centrality



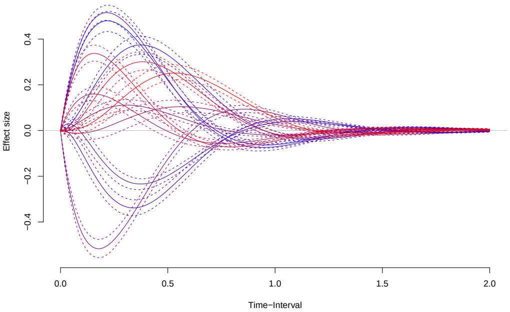
Press Intervention

Indirect Effect Centrality

Ryan & Hamaker, 2021

A Continuous-Time Approach to Network Analysis

- CT network analysis and centrality
  - *ctnet* (github: ryanoisin/ctnet)



Ryan O. & Hamaker, E.L. (2021). Time to Intervene: A Continuous-Time Approach to Network Analysis and Centrality. *Psychometrika*. `https://doi.org/10.1007/s11336-021-09767-0`

Overview

1. Basic concepts of CT models
2. Why should you care?
3. **What can you do in practice?**

CT modeling in Practice

CT models like the CT-VAR(1) can be fit to standard ESM-type data

- *ctsem* (R; Driver et al. 2018) based on *rstan*
- *dynr* (R; Ou, Hunter and Chow, 2018)
- You need information about the spacing of observations
- Many extensions not shown here (Single-subject or multilevel, mean trends, regime-switching, non-linearity, higher-order models)

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Based on this, many developments in CT alternatives to DT practice

- Tools for interpreting CT models (Ryan, Kuiper Hamaker, 2018)
- CT mediation for tri-variate models (Deboeck & Preacher, 2016)
- CT network analysis and path-tracing (Ryan & Hamaker, 2021)
- CT meta-analysis (Kuiper & Ryan, 2020; Dormann, Guthier & Cortina, 2020)

Lab Session

Outstanding Issues

Of course, many of the same issues we saw yesterday also rear their head here

Centering/ Detrending / Stationarity

- We also try to model stable/stationary processes. Check eigenvalues of $\boldsymbol{A} < 0$ instead of eigenvalues $-1 < \boldsymbol{\Phi} < 1$.
- Center data around the equilibrium position
- De-trend first, or model the trend in the differential equation (*ctsem*: CINT)

Model Selection: Still tricky!

Descriptive/Exploratory: In principle, ACF and PACF cannot deal with unequal intervals. Currently working with Nick Jacobson to create a method for exploratory CT analysis.

Outstanding Issues

Other ways of dealing with the time-interval problem: kalman filter or "bayesian imputation" approach

- Implemented in Mplus with TINTERVAL: insert missing values to make observations approximately equally spaced
- You can do this yourself before feeding the model to JAGs
- Conceptually tricky approach: Still only estimating lagged effects at one particular interval

Continuous-Time versions of regime-switching models, such as HMM's also available

- msm package: CT-HMM, but limited multilevel capabilities

Thank you!

o.ryan@uu.nl | oisinryan.org

Thank you!

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For more info, some relevant summer schools at UU:

- Introduction to SEM using Mplus
- Advanced course on Mplus
- Modeling the Dynamics of Intensive Longitudinal Data

Key References

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Additional Slides

Making sense of CT models

- Interpret the estimated parameters directly
  - $\boldsymbol{A}$ describes how position $Y(t)$ is related to the *rate of change* $\frac{dY(t)}{dt}$

- Visualise the predicted behaviour of the system
  - Just like the VAR(1), the CT-VAR(1) describes a system that varies around a stable equilibrium
  - The estimated model parameters describe how changes in one variable result in changes in the others

Example Analysis (Ryan, Kuiper & Hamaker, 2018) [1]

Subset of data from Wichers & Groot (2016)

- Single-subject
- 286 measurements over 42 days
- Modal interval: 1.77 hours (IQR: 1.25 - 3.23 hrs)

Two items selected:

- *Down*: "I feel down"
- *Tired*: "I am tired"
- Both were centered and standardized

How do Down and Tired influence one another over time?

[1] Complete analysis on github.com/ryanoisin/continuous_time-ILD-what-why-how

Example Analysis (Ryan, Kuiper & Hamaker, 2018)

The drift matrix relating the processes Down ($Do(t)$) and Tired ($Ti(t)$) is given by

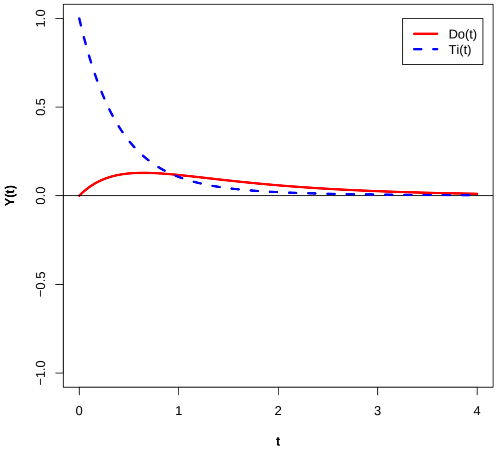
$$\boldsymbol{A} = \begin{bmatrix} -0.995 & 0.573 \\ 0.375 & -2.416 \end{bmatrix}.$$

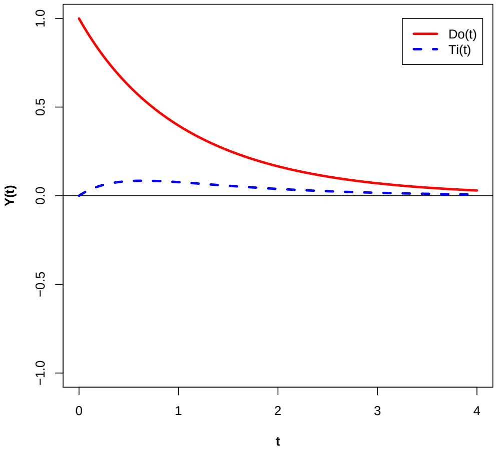
Which means

$$\frac{d\hat{Do}(t)}{dt} = -0.995Do(t) + 0.573Ti(t)$$
$$\frac{d\hat{Ti}(t)}{dt} = 0.375Do(t) - 2.416Ti(t)$$

Visualisation I: Impulse Response Functions

IRFs: How does the system react to a given impulse?





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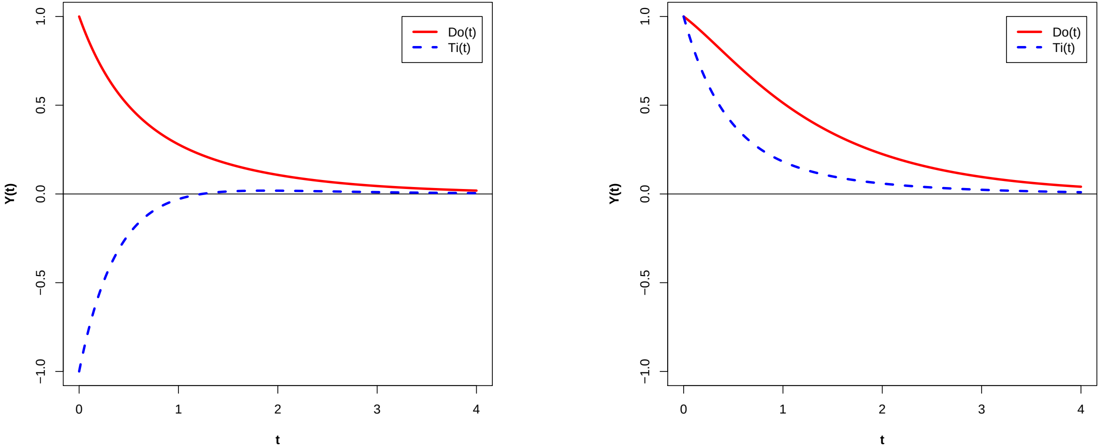
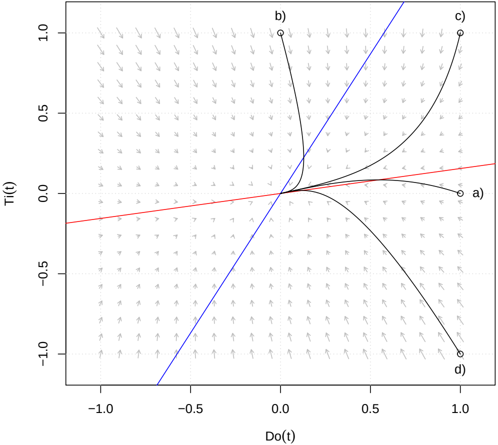
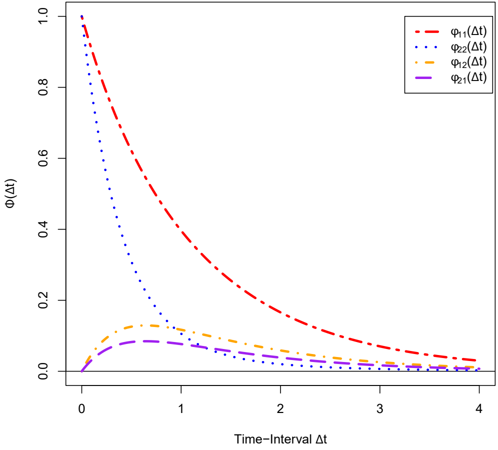


Vector Fields: What trajectories are possible?



Visualisation III: $\Phi(\Delta t)$

How do the lagged parameters depend on time-interval?



Visualisation III: $\Phi(\Delta t)$

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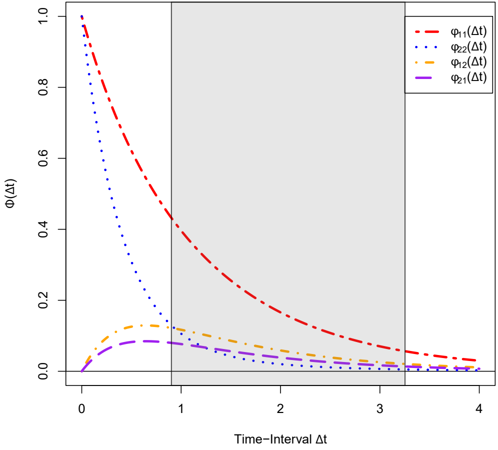

Get in Touch

- ryanoisin.github.io
- o.ryan@uu.nl