Exploratory and Confirmatory Modeling of Unequally Spaced Time-Series Data

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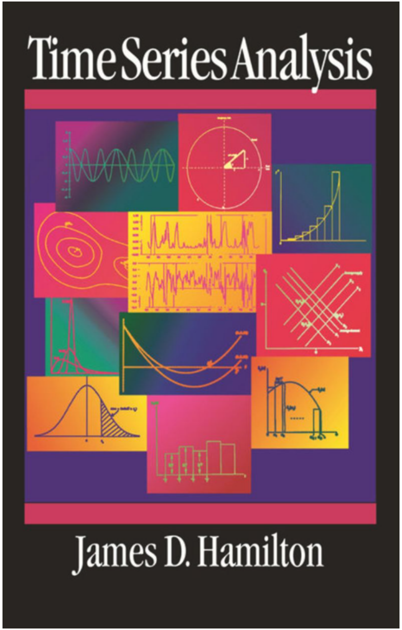
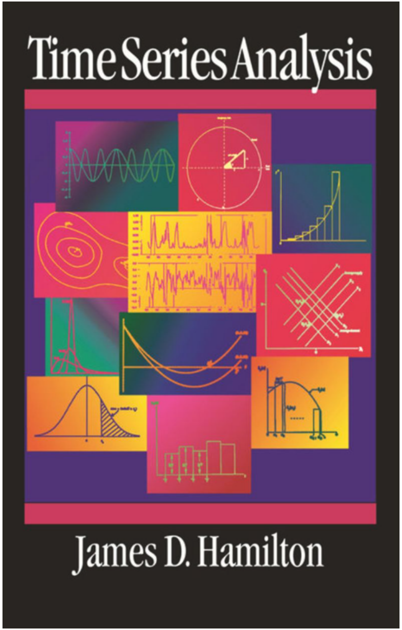
Small is beautiful {once more}
The third international N=1 Symposium
April 2023


POLAR
MEM
START STOP
SET
I feel anxious right now
0
(not at all)
10
(very much)
I want to binge eat
0
(not at all)
10
(very much)



Exploratory (Descriptive) Tools

- What patterns of *lagged* dependency (between past and future values) are present in my data?
- Autocorrelation function (ACF)
- Cross-Correlation function (CCF)



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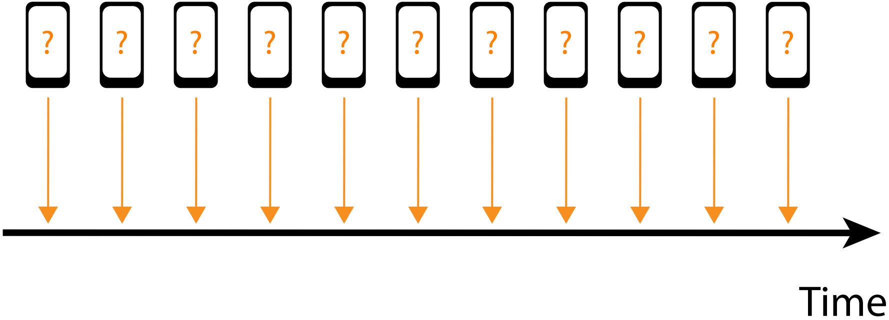
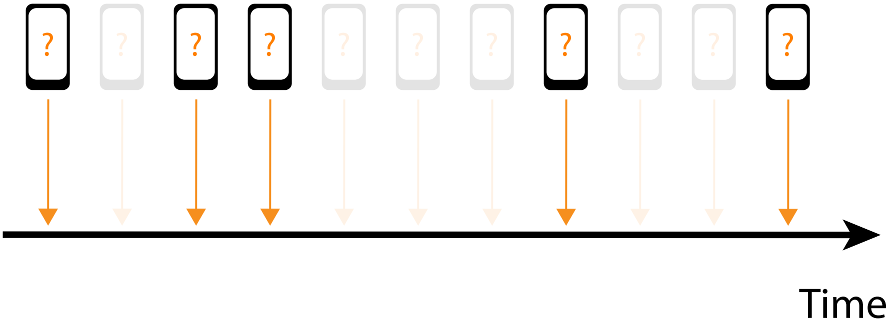
Confirmatory Models

- What model *explains* the lagged dependencies in my data?
- AR(1), AR(2), VAR(1), VAR(p) etc.

Assumption: Equally Spaced Measurements



Reality: Irregularly Spaced Measurements



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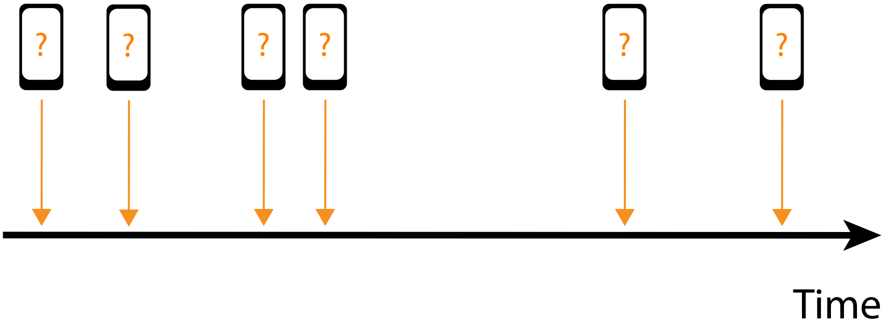
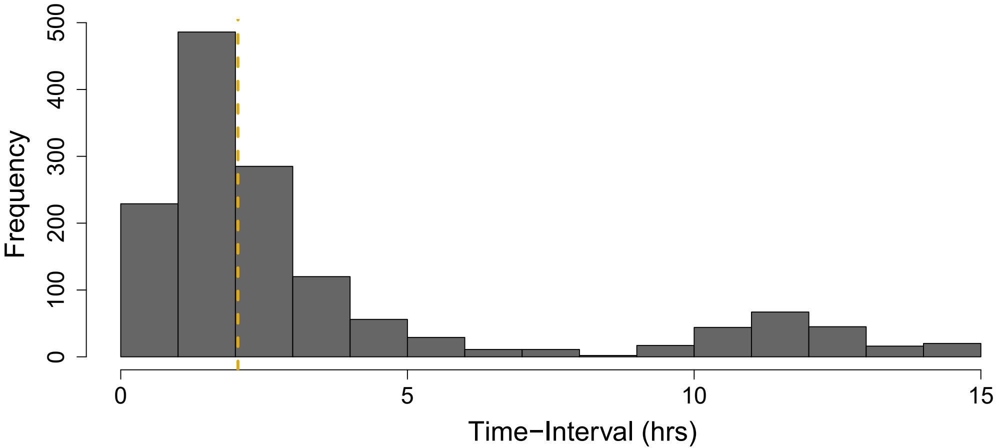


Empirical data



Ryan & Hamaker (2022); Kossakowski, Groot, Haslbeck, Borsboom & Wichers (2017)

Problem

Using standard time-series approaches without accounting for unequal spacing of measurements can lead to **biased estimates**, **inaccurate descriptives**, **misspecified models** and **misleading conclusions** about the process under investigation

Solutions

1. Confirmatory: Continuous-Time Modeling
   - Lecture, Practical in R
2. Exploratory: *expct*
   - Lecture, Practical in R

Materials:
`github.com/ryanoisin/WorkshopExploratoryConfirmatoryCT`

Part 1: Confirmatory (Continuous-Time) Modeling

The AR(1) Model

$$Y_{t+1} = \phi Y_t + \epsilon_t$$

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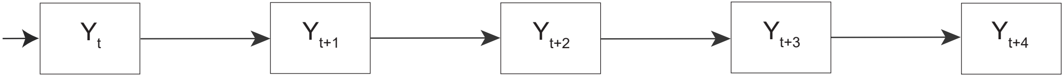

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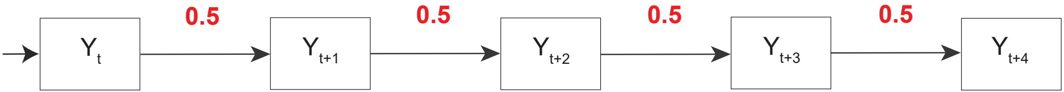

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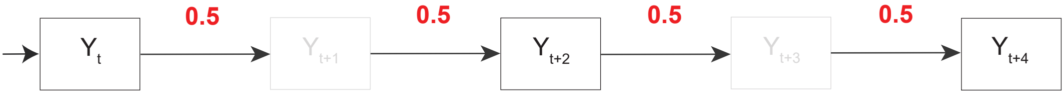

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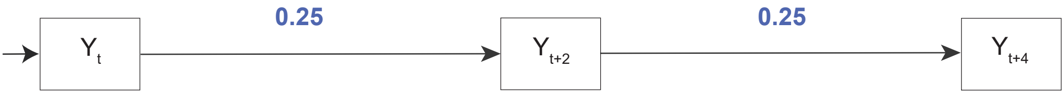

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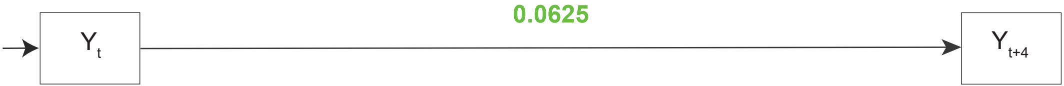

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$$Y_{t+1} = \phi(\Delta t) Y_t + \epsilon_t$$

Continuous Time Models

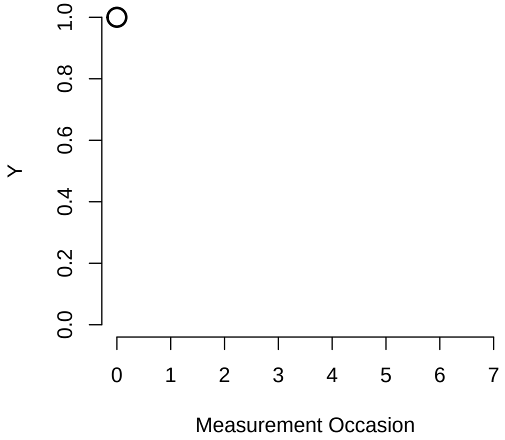
Can be used to describe the **same processes** as autoregressive models but from a different **perspective**, with a different **language**

- *Differential Equations*

Easier to express how and why statistical relations depend on the *time-interval* between measurements

- Fit statistical models that describe and model how relationships depend on the time-interval, to data taken with unequally spaced measurements

Same Process, Different Language



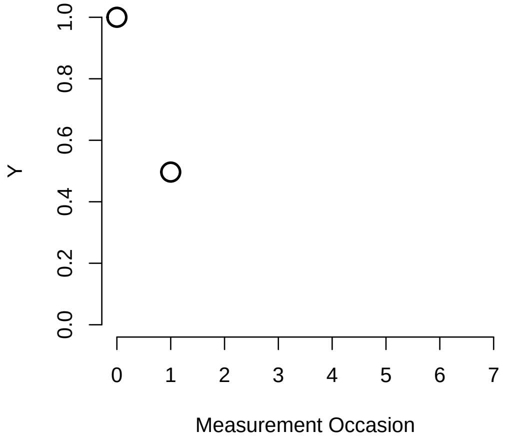
AR(1) Model

$$E[Y_1] = \phi Y_0$$

with

- $\phi = .5$

Same Process, Different Language



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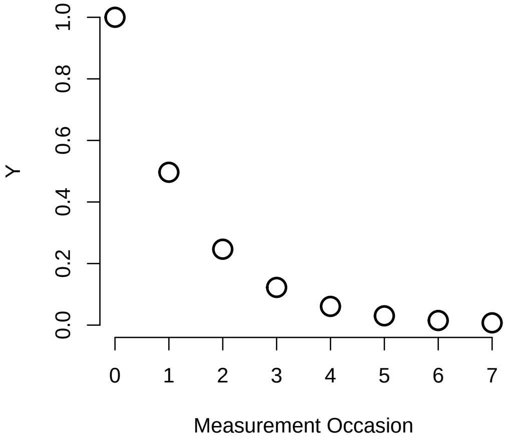

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Process *described* in terms of discrete "jumps" from one *occassion* to the next

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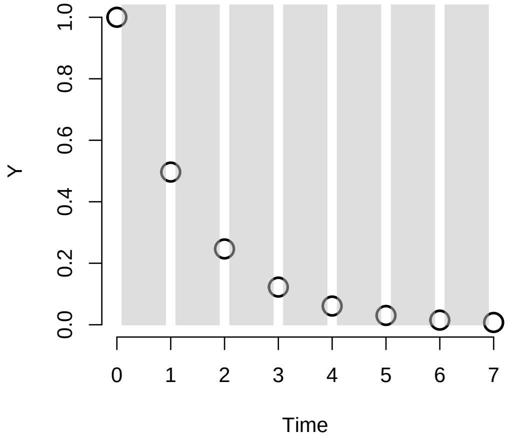
A **continuous-time** model can describe the same *qualitative behaviour*, but treating *time* itself as a continuous dimension

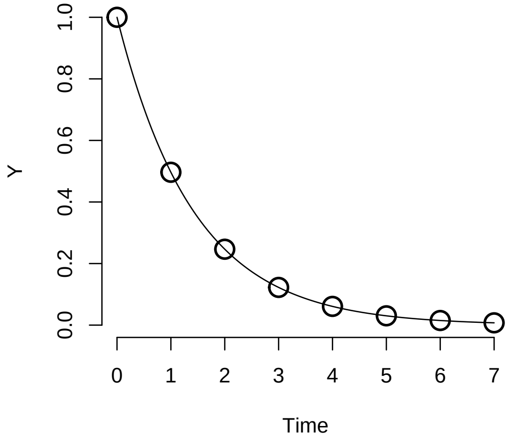
Same Process, Different Language



Continuous-Time Models

- Process takes on *some value* at every moment in time

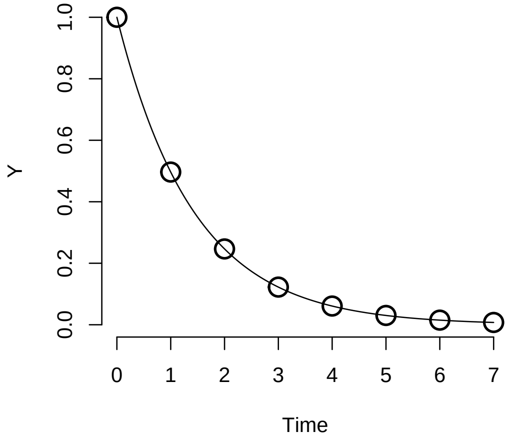
Same Process, Different Language



Continuous-Time Models

- Process takes on *some value* at every moment in time
- System evolves in a smooth and continuous manner

Same Process, Different Language

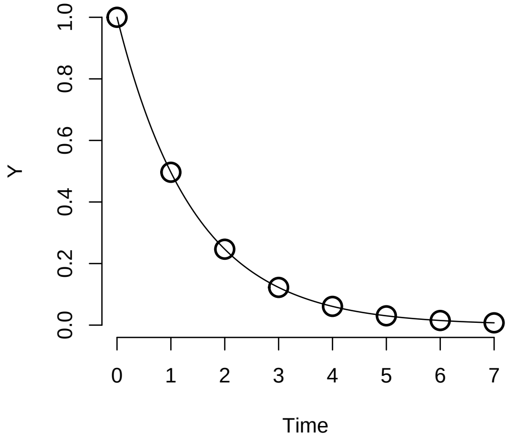


Continuous-Time Models

- Process takes on *some value* at every moment in time
- System evolves in a smooth and continuous manner

We can describe the evolution of the system using *Differential Equations*

Same Process, Different Language



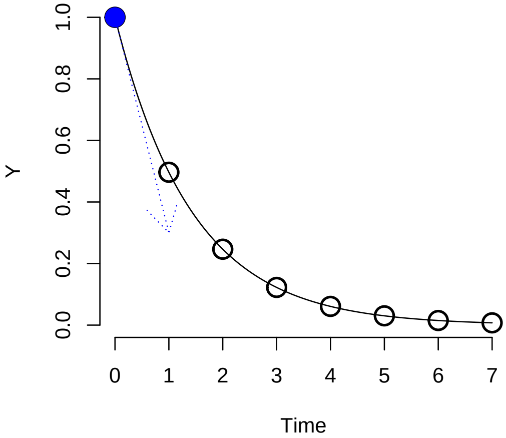
First-order Differential Equation

$$\frac{dY(t)}{dt} = A \times Y(t)$$

with

- $A = -.69$

Same Process, Different Language



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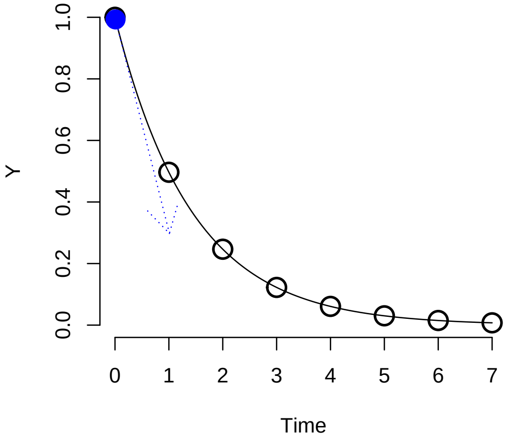

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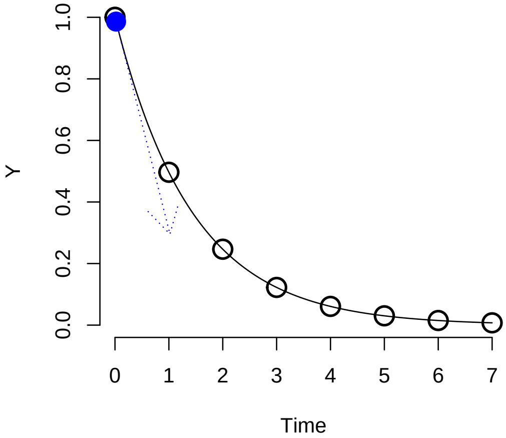

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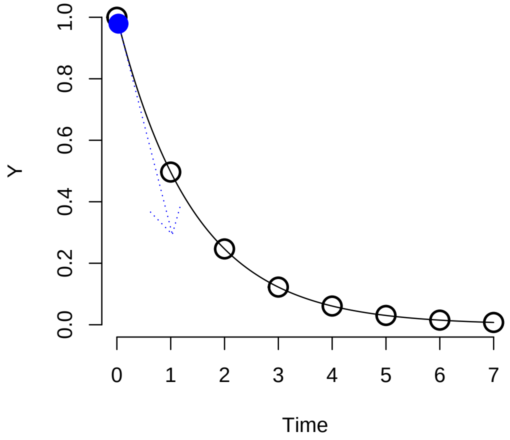

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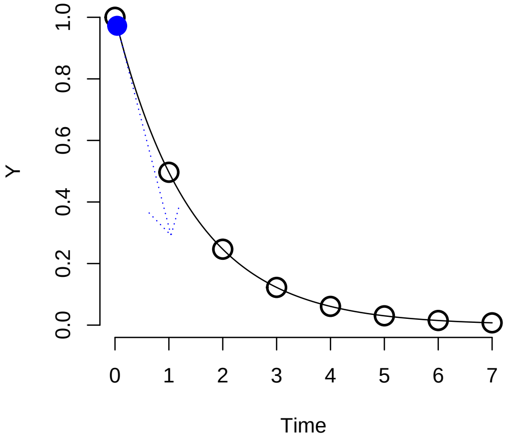

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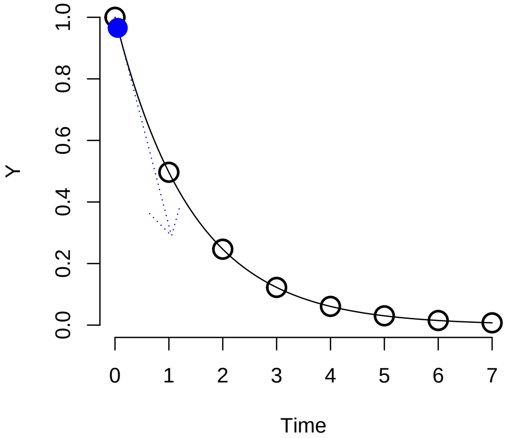

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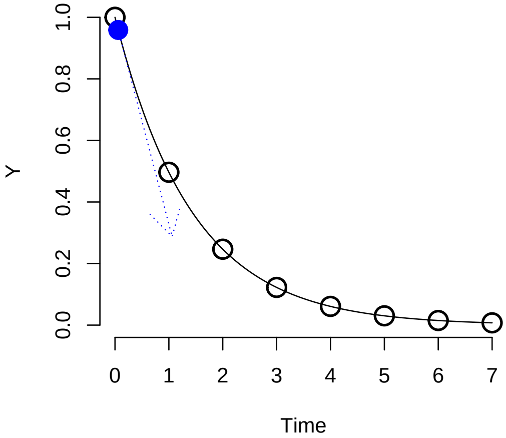

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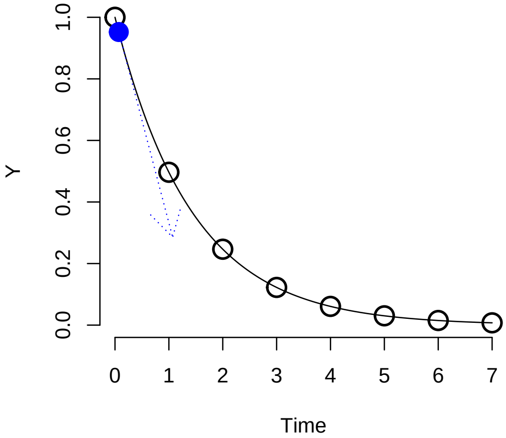

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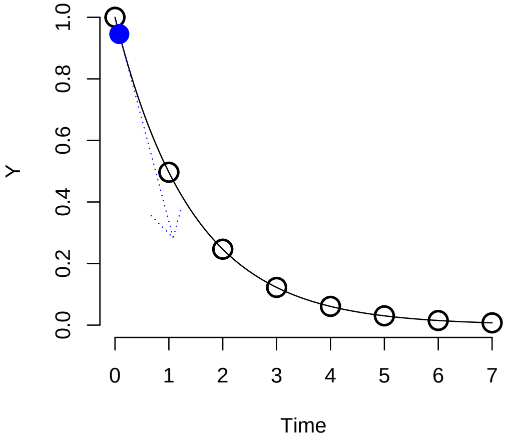

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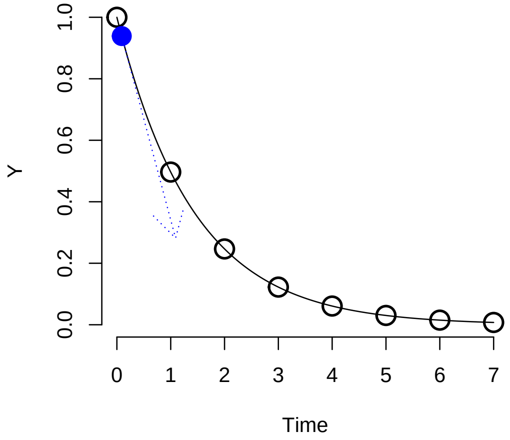

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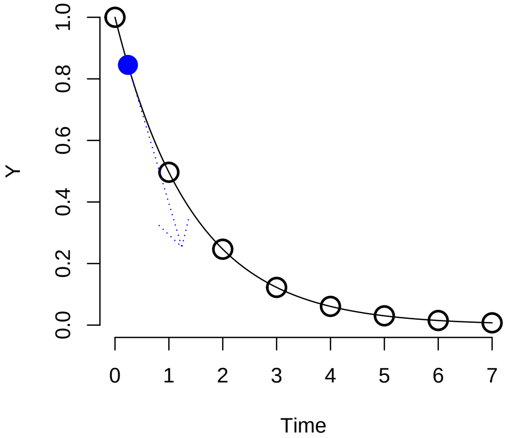

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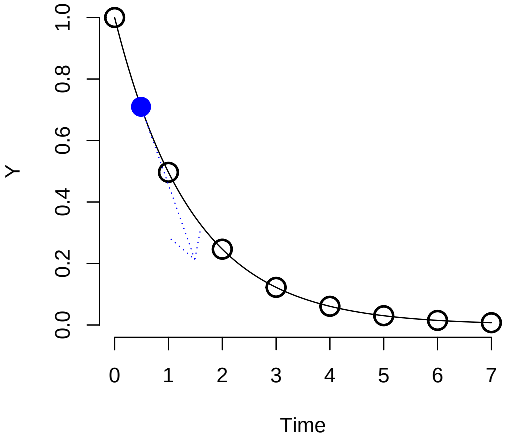

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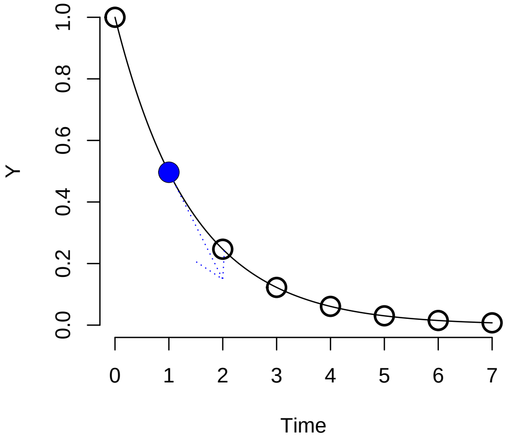

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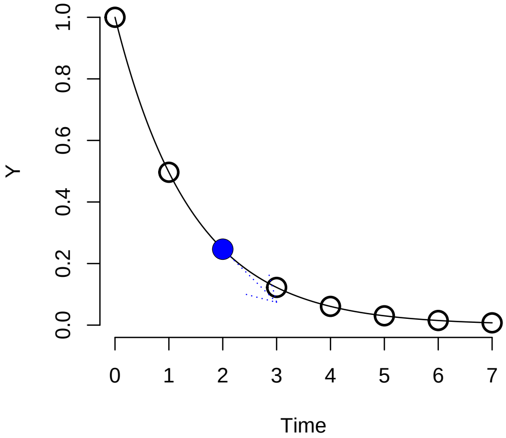

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Continuous-Time Models

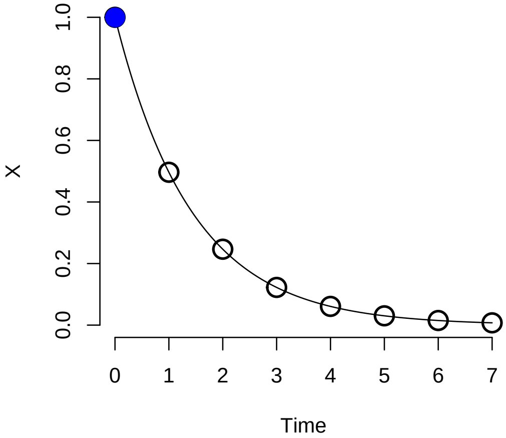
The differential equation is a continuous-time model, but it might not be immediately clear how we can **estimate** this from data

- After all, we don't *observe* the derivative, and we might not be able to *measure* the process frequently enough to calculate it directly

It turns out we can **re-write** the differential equation so that it looks closer to an auto-regresive model

- So that it relates current observations to observations taken *some time in the past*

Same Process, Different Language



Continuous-Time AR(1)

$$Y(t+\Delta t) = e^{A\Delta t}Y(t)$$

with

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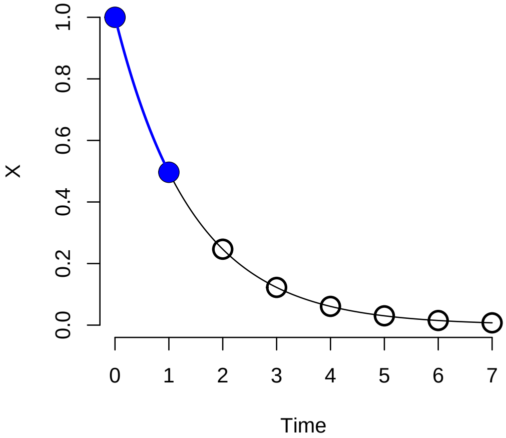
Same Process, Different Language



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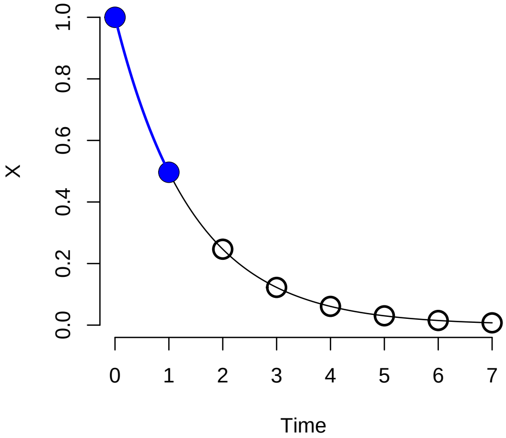

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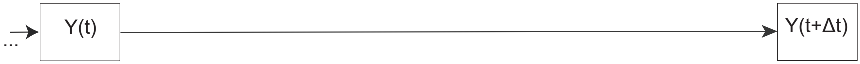
- $e^{(-.69\times 1)} = \phi = .5$

Discrete-Time AR(1)



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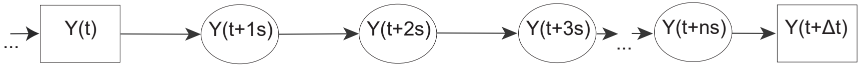


Continuous-Time Models

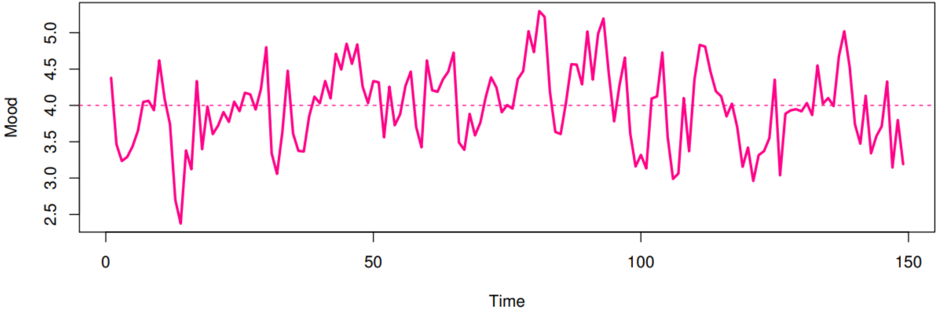
In principle we can estimate the CT-AR(1) model from standard time series data, **as long as we know when measurements were taken**

By using Δt information in estimating the CT-AR(1), we:

- Can estimate the model from **unequally spaced measurements**
- Can model lagged relationships as a non-linear function of *moment-to-moment* effects A and the time-interval Δt

Adding Noise

So far we only dealt with the *deterministic* part of the DE model. But we typically also want to allow for random noise or innovation variance



Adding Noise

AR(1) Model

$$Y_{\tau+1} = \phi Y_{\tau} + \epsilon_{\tau}$$

with

- $0 < \phi < 1$
- Equally spaced measurements
- $\epsilon_{\tau} \sim N(0, \sigma)$

Adding Noise

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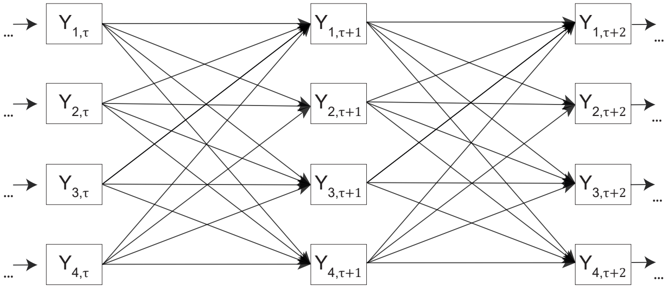
CT-AR(1)

$$Y(t + \Delta t) = e^{A\Delta t} Y(t) + \epsilon(\Delta t)$$

with

- $A < 0$
- $\epsilon(\Delta t)$ is Gaussian noise whose variance scales with Δt
- When $\Delta t \to 0$, $\epsilon \to 0$
- When $\Delta t \to \infty$, $\epsilon \sim N(0, \gamma)$

$$\boldsymbol{Y}_{\tau+1} = \boldsymbol{\Phi}\boldsymbol{Y}_{\tau} + \boldsymbol{\epsilon}_{\tau}$$



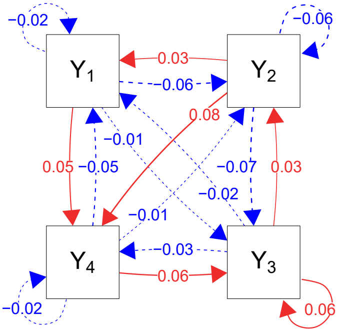


Figure from Ryan & Hamaker (2021)

Multivariate Version

VAR(1) Model

$$\boldsymbol{Y}_{\tau+1} = \boldsymbol{\Phi} \boldsymbol{Y}_{\tau} + \boldsymbol{\epsilon}_{\tau}$$

with

- $0 < \lambda_{\boldsymbol{\Phi}} < 1$ (eigenvalues)
- Equally spaced measurements
- $\epsilon_{\tau} \sim N(0, \Sigma)$

CT-VAR(1)

$$\boldsymbol{Y}(t + \Delta t) = e^{\boldsymbol{A}\Delta t} \boldsymbol{Y}(t) + \boldsymbol{\epsilon}(\Delta t)$$

with

- $\lambda_{A} < 0$ (eigenvalues)
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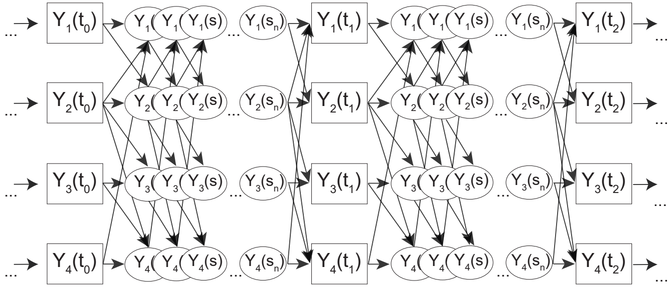


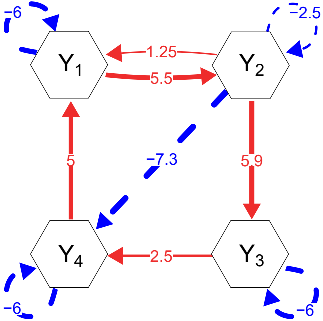
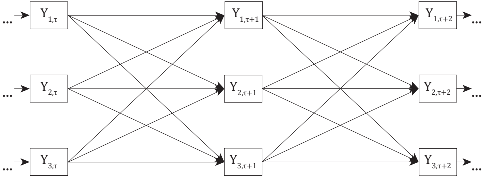


Figure from Ryan & Hamaker (2021)

Consequences of time-interval dependency

1. Equal time-intervals: not generalizable
   - $\mathbf{\Phi}(\Delta t = 1) \neq \mathbf{\Phi}(\Delta t = .5)$



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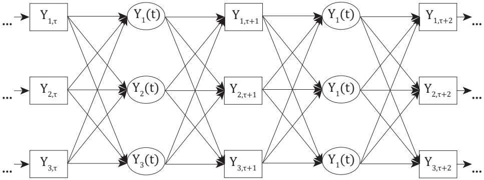

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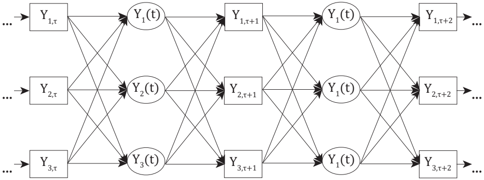

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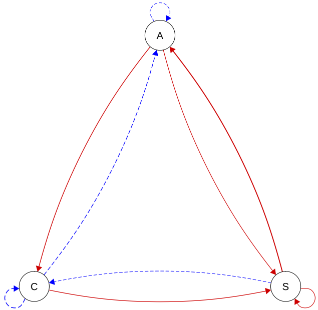

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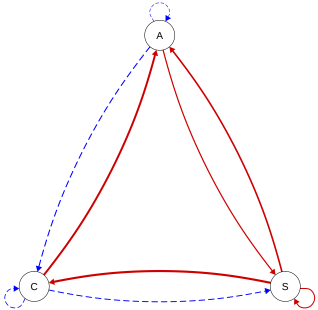
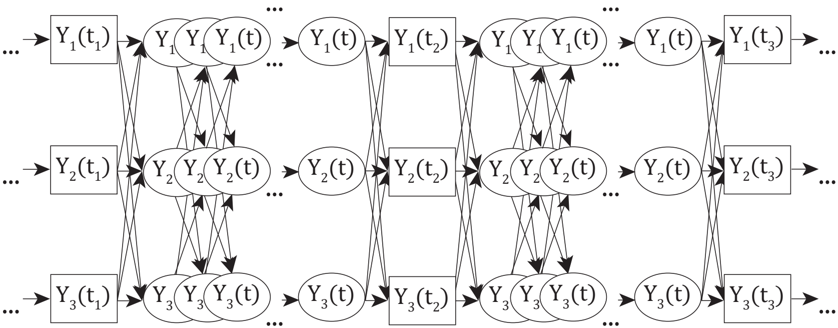
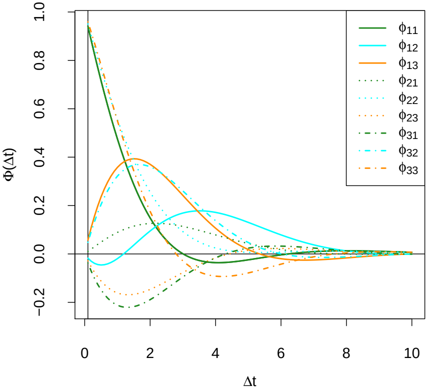


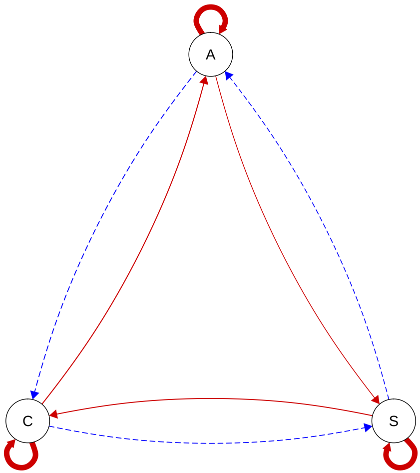
The CT-VAR(1) model

$$\boldsymbol{Y}(t) = \boldsymbol{e}^{\boldsymbol{A}\Delta t}\boldsymbol{Y}(t) + \epsilon(\Delta t)$$



Network structure as a function of time-interval





Consequences of time-interval dependency

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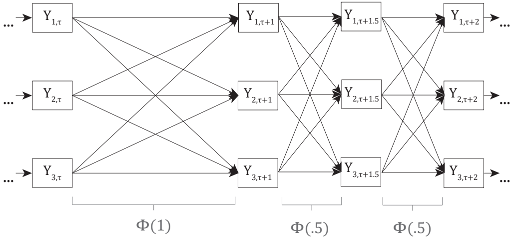
2. Unequal time-intervals: mix of effects
   - $\hat{\boldsymbol{\Phi}} = ?$
   - If not accounted for, may not reflect effects at *any* time-interval



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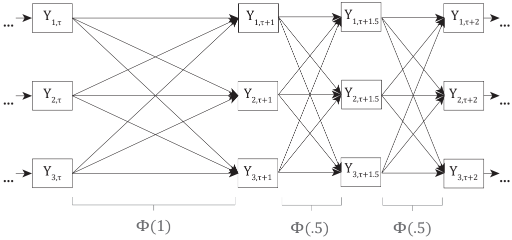

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   - $\hat{\mathbf{\Phi}} = ?$
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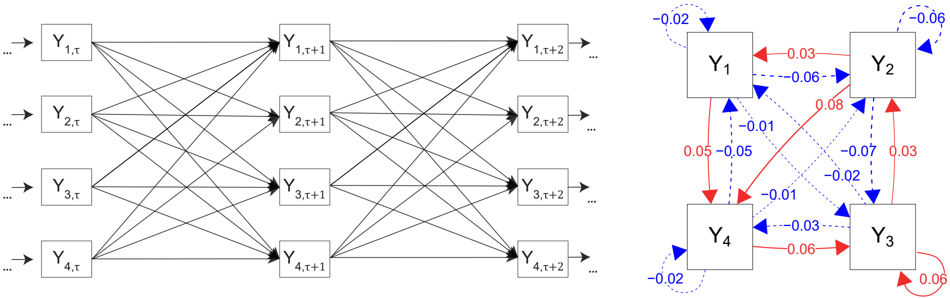
3. $\mathbf{\Phi}(\Delta t)$ should not be interpreted as *direct effects*

Use of DT models often based on interpretation of $\mathbf{\Phi}$ parameters as *direct* effects

- Mediation analysis and path tracing (Cole and Maxwell 2003)
- Network analysis, network structure, centrality, intervention targets (Bringmann et al 2013)



From a CT perspective, $\Phi(\Delta t)$ are *total effects*, including paths through *latent* values of the process in-between measurement occassions

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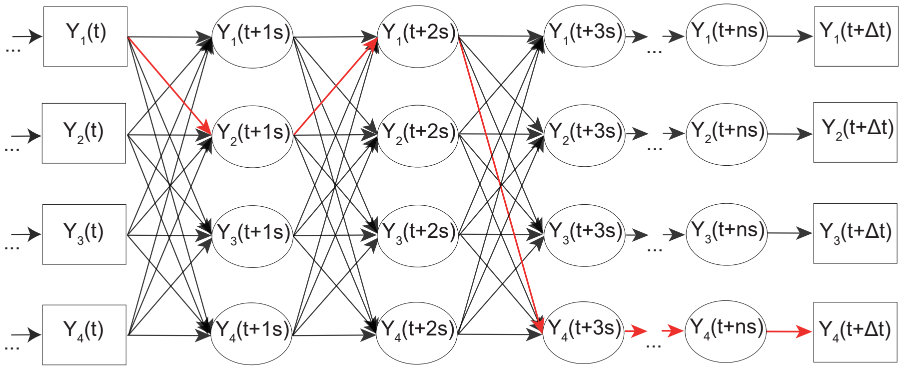
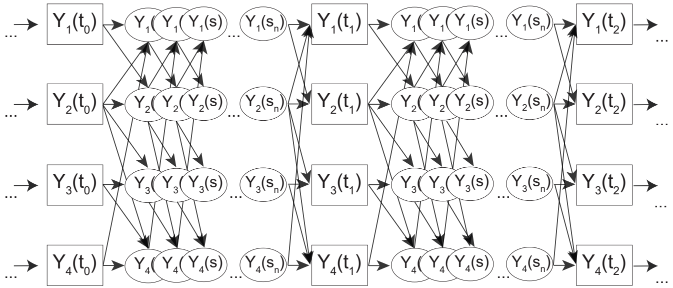
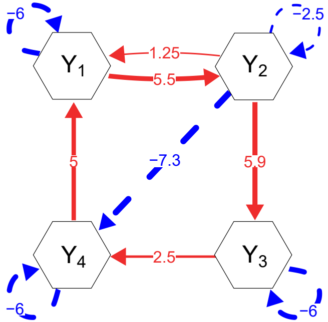


Instead the CT matrix $\boldsymbol{A}$ should be seen as direct moment-to-moment effects, and so should be the basis of path-tracing, centrality, network structure (Ryan & Hamaker, 2021)





CT modeling in Practice

CT models like the CT-VAR(1) can be fit to standard ESM-type data

- *ctsem* (R; Driver et al. 2018) based on *rstan*
- *dynr* (R; Ou, Hunter and Chow, 2018)
- You need information about the spacing of observations
- Many extensions not shown here (multilevel, mean trends, regime-switching, non-linearity, higher-order models)

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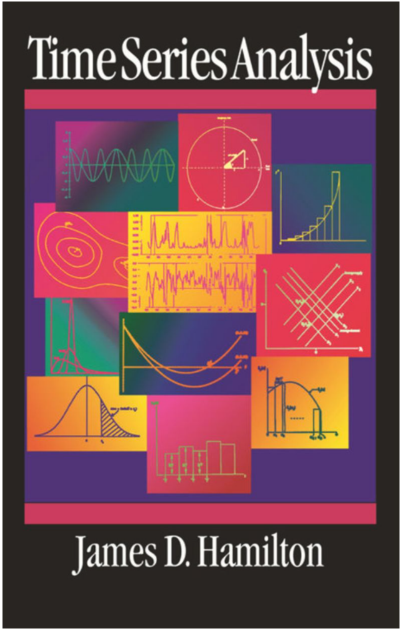
Based on this, many developments in CT alternatives to DT practice

- Tools for interpreting CT models (Ryan, Kuiper & Hamaker, 2018)
- CT mediation for tri-variate models (Deboeck & Preacher, 2016)
- CT network analysis and path-tracing (Ryan & Hamaker, 2021)
- CT meta-analysis (Kuiper & Ryan, 2020; Dormann, Guthier & Cortina, 2020)

Part 2: Exploratory Continuous-Time Modelling

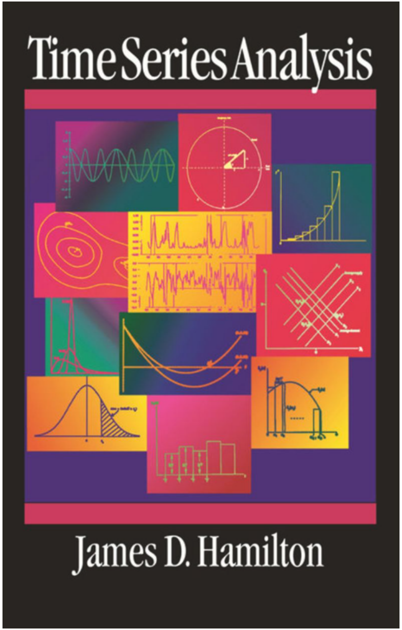


Exploratory (Descriptive) Tools

- What patterns of *lagged* dependency (between past and future values) are present in my data?
- Autocorrelation function (ACF)
- Cross-Correlation function (CCF)

Confirmatory Models

- What model *explains* the lagged dependencies in my data?
- AR(1), AR(2), VAR(1), VAR(p) etc.



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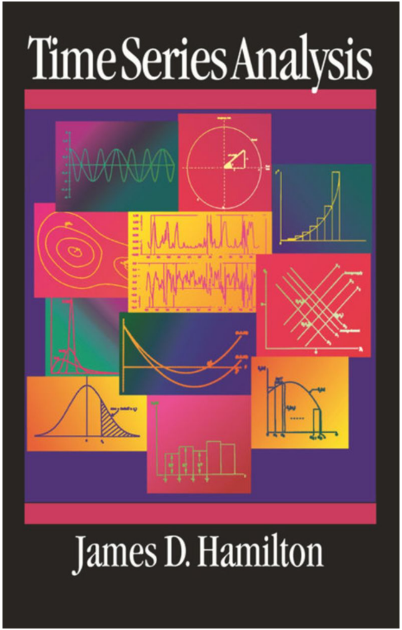

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Y

y_1

y_2

y_3

y_4

y_5

y_6

y_7

y_8

$\ldots$

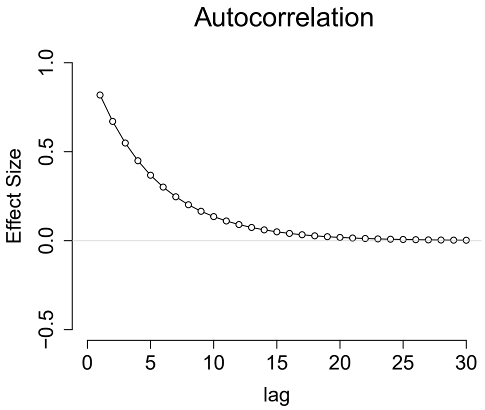
y_T

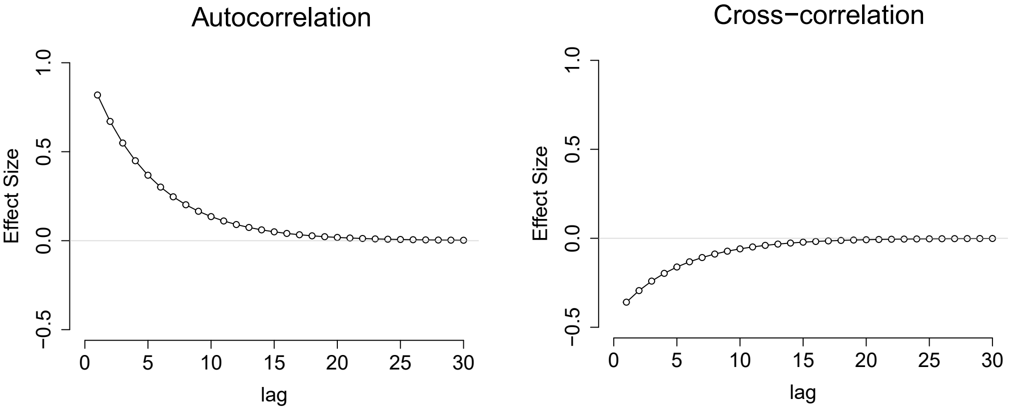
Y	Y at lag 1
y_1	
y_2	y_1
y_3	y_2
y_4	y_3
y_5	y_4
y_6	y_5
y_7	y_6
y_8	y_7
$\dots$	$\dots$
y_T	y_{T-1}
	y_T

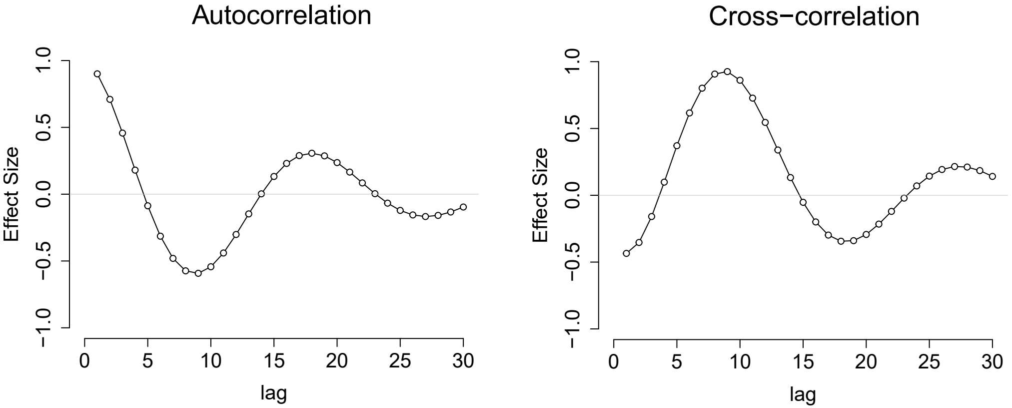
Y	Y at lag 1	Y at lag 2
y_1		
y_2	y_1	
y_3	y_2	y_1
y_4	y_3	y_2
y_5	y_4	y_3
y_6	y_5	y_4
y_7	y_6	y_5
y_8	y_7	y_6
$\dots$	$\dots$	$\dots$
y_T	y_{T-1}	y_{T-2}
	y_T	y_{T-1}
		y_T

Y	Y at lag 1	Y at lag 2
y_1		
y_2	y_1	
y_3	y_2	y_1
y_4	y_3	y_2
y_5	y_4	y_3
y_6	y_5	y_4
y_7	y_6	y_5
y_8	y_7	y_6
$\ldots$	$\ldots$	$\ldots$
y_T	y_{T-1}	y_{T-2}
	y_T	y_{T-1}
		y_T

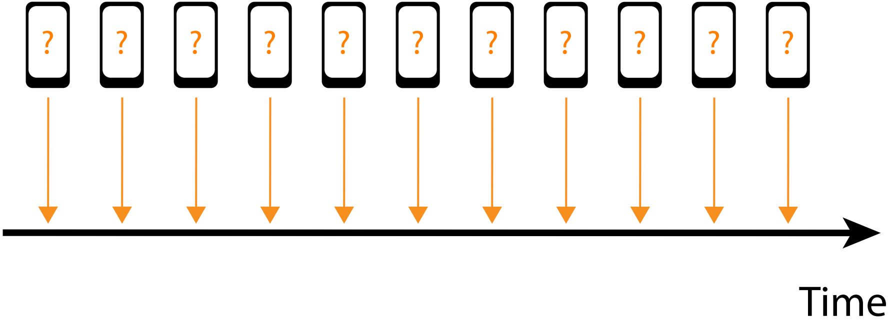
Y	Y at lag 1	Y at lag 2
y_1		
y_2	y_1	
y_3	y_2	y_1
y_4	y_3	y_2
y_5	y_4	y_3
y_6	y_5	y_4
y_7	y_6	y_5
y_8	y_7	y_6
$\dots$	$\dots$	$\dots$
y_T	y_{T-1}	y_{T-2}
	y_T	y_{T-1}
		y_T


Autocorrelation
Effect Size
1.0
0.5
0.0
−0.5
0
5
10
15
20
25
30
lag

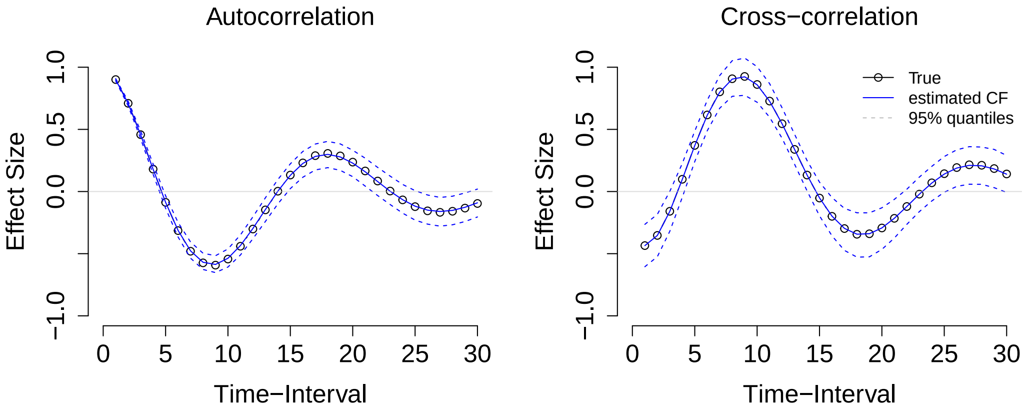

Autocorrelation
Effect Size
lag
Cross−correlation
Effect Size
lag


Autocorrelation
Cross−correlation
Effect Size
lag
1.0
0.5
0.0
−0.5
−1.0
0
5
10
15
20
25
30

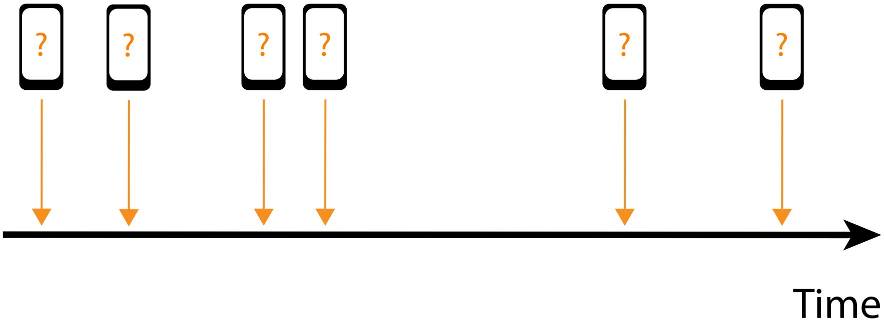
Assumption: Equally Spaced Measurements



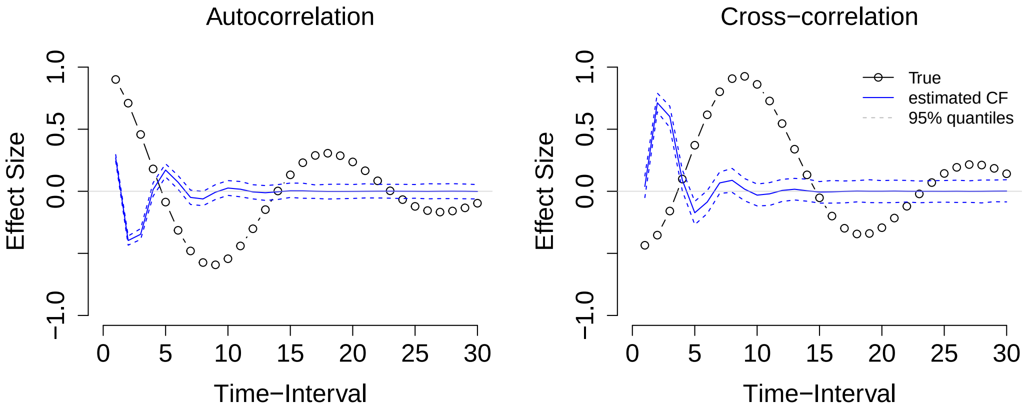
CF estimation: Equally Spaced



Reality: Irregularly Spaced Measurements



CF estimation: Unequally Spaced



Continuous-Time Modeling

At this point you might be thinking: Just fit a continuous-time model with *ctsem*!

$$\boldsymbol{Y}(t + \Delta t) = \mathbf{e}^{\boldsymbol{A}\Delta t}\boldsymbol{Y}(t) + \epsilon(\Delta t)$$

But **remember**: This is a (confirmatory) **model**, not a method of computing descriptive statistics!

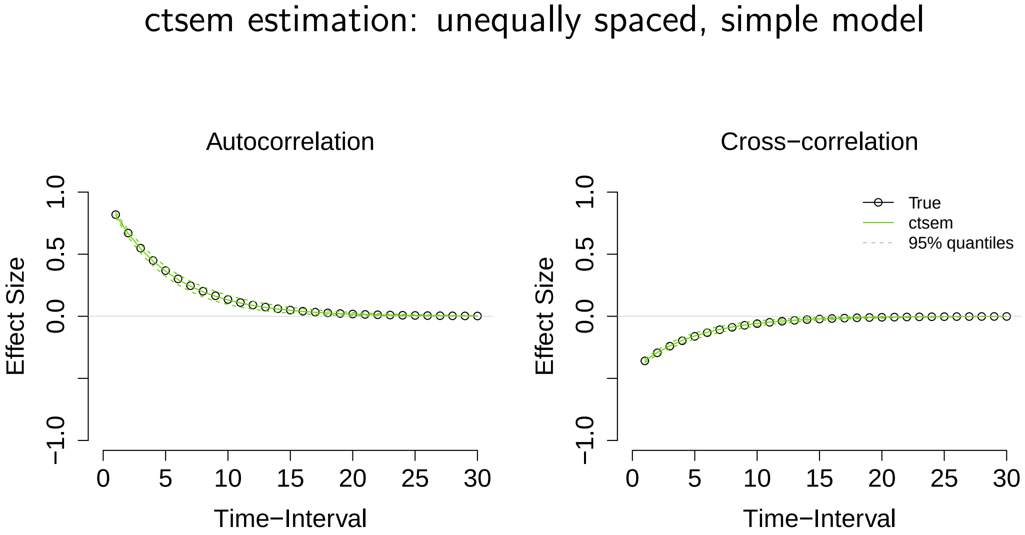
Problem: Model Misspecification

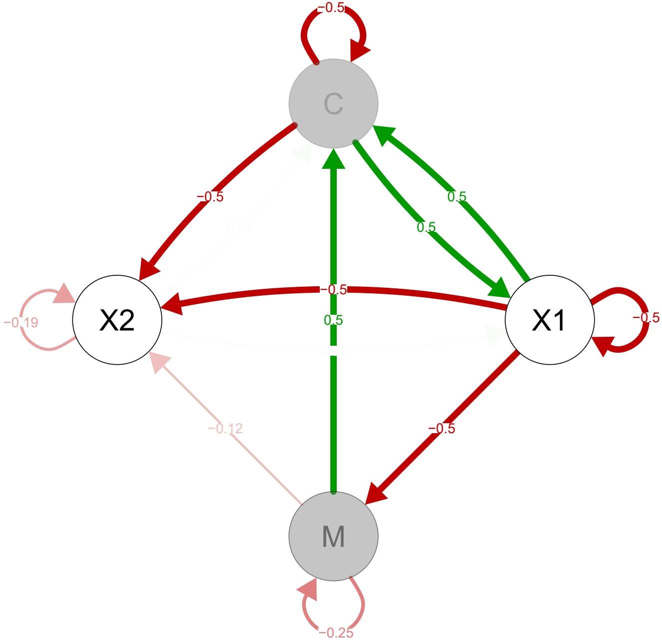
With a CT model, the auto- and cross- relations are **derived** based on the estimated drift matrix $\boldsymbol{A}$.

- Model-based estimate. Not (entirely) data-driven / exploratory

But this will only match **reality** if the (simple, low-D, linear) model is **correctly specified**.

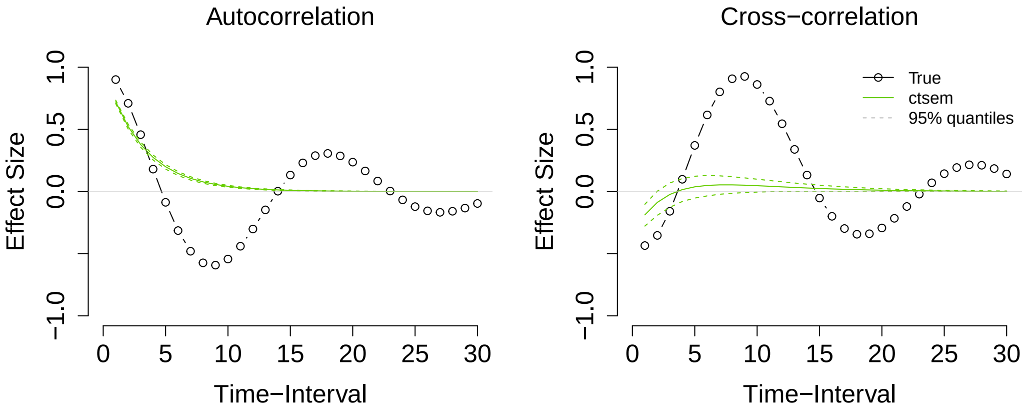
- If the **order** of the model (first vs second) is wrong, or if we have **unobserved confounding** these will be incorrect
- Think about the contrast between an AR(1) and an AR(2); You can also fit higher-order, more complex CT-AR models!


ctsem estimation: unequally spaced, simple model
Autocorrelation
Cross−correlation
Effect Size
Time−Interval
1.0
0.5
0.0
−1.0
0
5
10
15
20
25
30
True
ctsem
95% quantiles



−0.5
C
−0.5
0.5
0.5
0.5
−0.5
X2
X1
−0.19
−0.5
−0.12
−0.5
M
−0.25

ctsem estimation: model misspecified



Traditional ACF and CCF estimation:

- **Data-driven and exploratory** (relatively model-free) method for exploring dynamic features
- Does not perform well with irregularly spaced data

CT model estimation:

- Can be estimated from irregularly spaced data and in principle capture non-linear patterns of correlations
- But we need a way to check our model specification, and to know what patterns in the data the model is trying to explain!

expct: Exploratory Continuous Time Modeling

Estimate ACF and CCF functions from data taken with any arbitrary sampling scheme

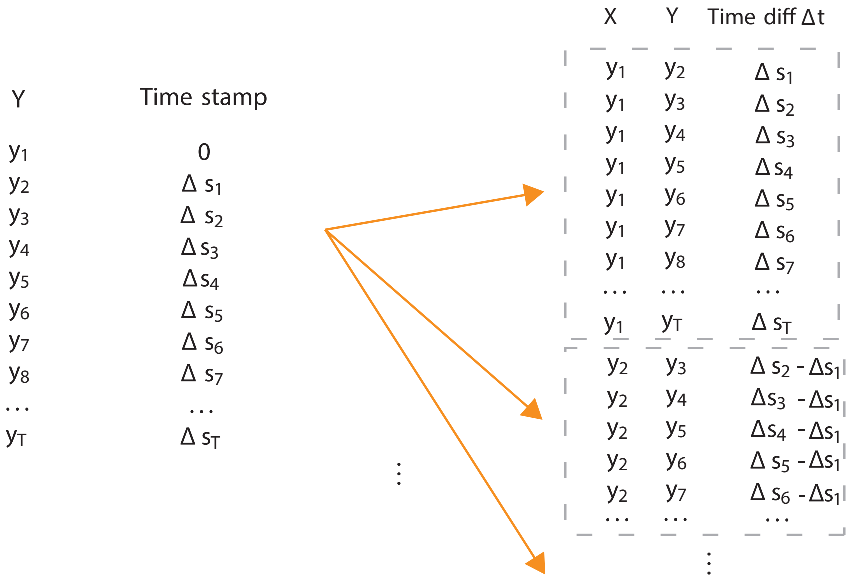
- Development version available github: ryanoisin/expct

Two-step procedure

1. Create a "stacked" data frame: Every observation acts as a predictor for every future observation, with the time-interval Δt as additional variable

Y	Y at lag 1	Y at lag 2
y_1		
y_2	y_1	
y_3	y_2	y_1
y_4	y_3	y_2
y_5	y_4	y_3
y_6	y_5	y_4
y_7	y_6	y_5
y_8	y_7	y_6
$\dots$	$\dots$	$\dots$
y_T	y_{T-1}	y_{T-2}
	y_T	y_{T-1}
		y_T

Y	Time stamp
y_1	0
y_2	Δs_1
y_3	Δs_2
y_4	Δs_3
y_5	Δs_4
y_6	Δs_5
y_7	Δs_6
y_8	Δs_7
$\ldots$	$\ldots$
y_T	Δs_T



Y
Time stamp
y_1 0
y_2 Δs_1
y_3 Δs_2
y_4 Δs_3
y_5 Δs_4
y_6 Δs_5
y_7 Δs_6
y_8 Δs_7
$\cdots$ $\cdots$
y_T Δs_T
X
Y
Time diff Δt
y_1 y_2 Δs_1
y_1 y_3 Δs_2
y_1 y_4 Δs_3
y_1 y_5 Δs_4
y_1 y_6 Δs_5
y_1 y_7 Δs_6
y_1 y_8 Δs_7
$\cdots$ $\cdots$ $\cdots$
y_1 y_T Δs_T
y_2 y_3 $\Delta s_2 - \Delta s_1$
y_2 y_4 $\Delta s_3 - \Delta s_1$
y_2 y_5 $\Delta s_4 - \Delta s_1$
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$$Y = f(\Delta t)X + \epsilon$$

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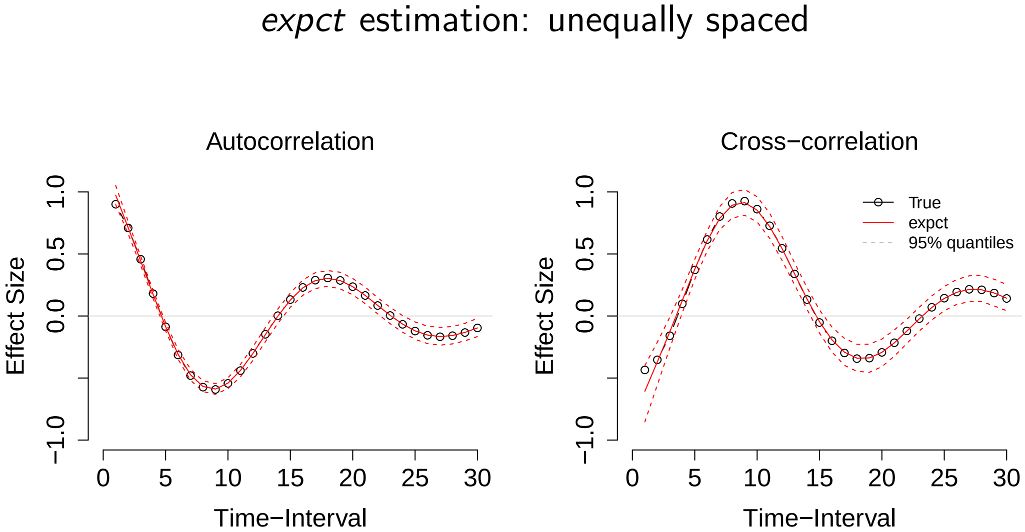
and rescale to correlations $g(\Delta t)$. Auto and cross-correlation functions are estimated for arbitrary Δt by fitting seperate bivariate GAMMs. In this way we approximate

$$\hat{cor}(Y_t, Y_{t+\Delta t}) = \hat{g}(\Delta t)$$



expct estimation: unequally spaced
Autocorrelation
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Effect Size
1.0
0.5
0.0
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0
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10
15
20
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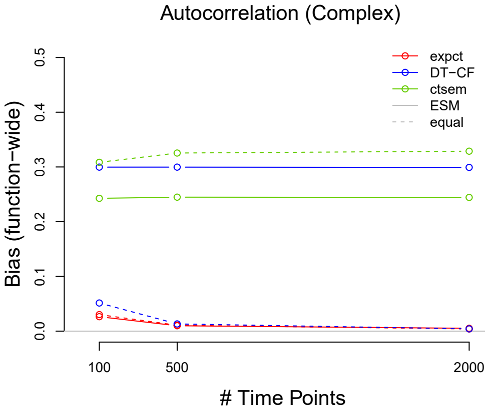
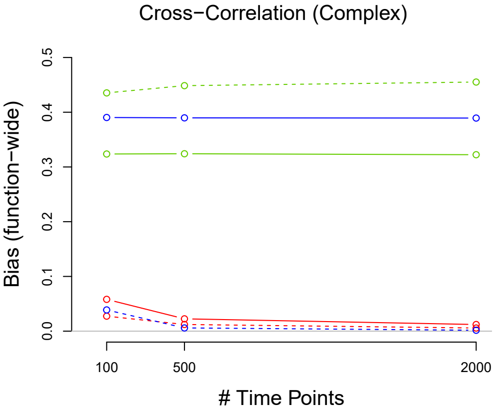
expct
95% quantiles

Simulation Results





Extracting Dynamic Features from Irregularly Spaced Time Series

expct: Exploratory continuous-time modeling

- Available as an R package github: ryanoisin/expct
- Overcomes equal-interval limitation of traditional ACF/CCF estimation
- Avoids reliance on correct lagged model specification in confirmatory continuous-time models
- GAMM-based, unbiased, good coverage with novel llci method
- Can potentially be used for variables collected at *very different sampling frequencies*
- Still in development (TBD: Partial ACFs)

Ryan O., Wu, K., & Jacobson, N.K. (in preparation). Exploratory Continuous-Time Modeling (expct): Extracting Dynamic Features from Irregularly Spaced Time Series

Conclusion

Summary

Using standard time-series approaches without accounting for unequal spacing of measurements can lead to **biased estimates**, **inaccurate descriptives**, **misspecified models** and **misleading conclusions** about the process under investigation

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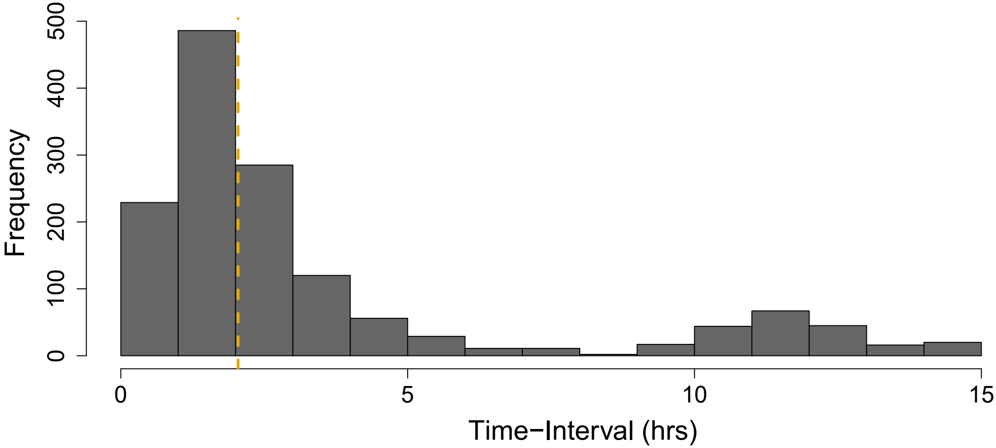
Exploratory Continuous-Time Modeling using the *expct* package is a relatively new and complimentary approach. Potentially key tool for descriptive analysis and model checking, still in development (so let me know if you use it and like it / run into problems!)

At a minimum, if you're going to collect and analyze time-series data, **think carefully** about your spacing, and always check what spacing you ended up with!



Ryan & Hamaker (2022); Kossakowski, Groot, Haslbeck, Borsboom & Wichers (2017)

Thanks!

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