

Fusion: Applying Equational Transforms to Simplify Programs

github.com/ryanorendorff/lc-2017-fusion

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Outline

Motivation: Simple Programs versus Performance

A brief introduction to GHC

List fusion with *foldr/build*

Stream Fusion

Applications of Fusion

Common way to process a list: map and fold!

As an example, say we want to square all the elements in a list and then sum the result. [1–4]

process :: [Int] → Int

process xs = sum ∘ map sq \\$ xs

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$$map [] = []$$
$$map f (x : xs) = f x : map f xs$$
$$sq x = x * x$$
$$foldr :: (a \rightarrow b \rightarrow b) \rightarrow b \rightarrow [a] \rightarrow b$$
$$foldr z [] = z$$
$$foldr f z (x : xs) = f x (foldr f z xs)$$
$$sum = foldr (+) 0$$

How fast is *process*?

So now that we have our *process* function, how fast does it run?

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Let's try to process a million elements with our *process* and *process'*, which uses the standard Prelude *foldr* and *map*.

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How does the Prelude do better with the same functions?

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It seems we have matched GHC's performance!

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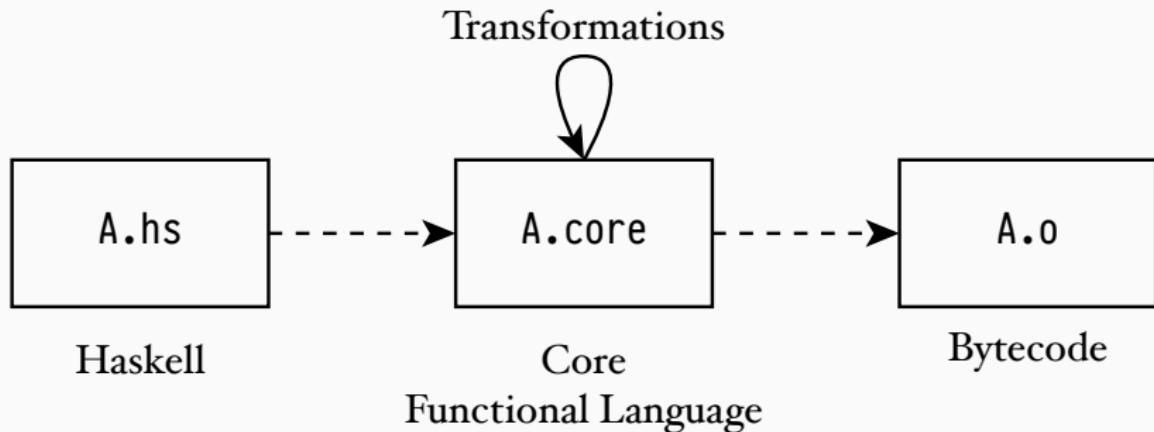
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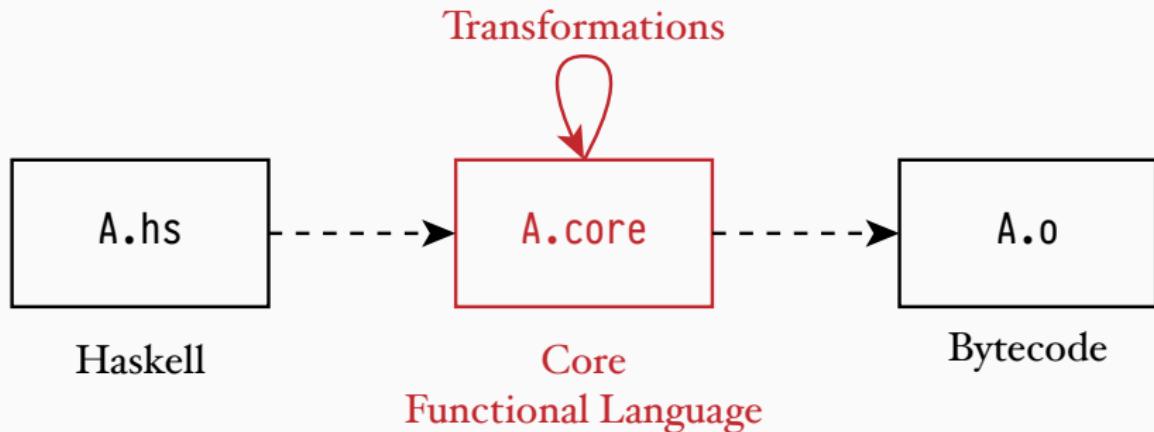
The GHC Compilation Pipeline converts Haskell into an intermediate language and then bytecode

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- ...

Rewrite Rules allow us to say two expressions are equivalent

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- The `forall` brings a variable into scope. Sometimes written \forall .
- After the period are the equivalent statements.

Rules have some restrictions

Rewrite rules have some gotchas. [8]

- Rules don't prevent you from doing something silly

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{-# RULES "fxy" forall x y. f x y = f y x #-}
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- If multiple rules are possible, GHC arbitrarily chooses one.

We can combine maps to traverse a list once

Let us introduce the following rule about maps. [4]

```
{-# RULES "map/map" forall f g xs.  
  mapfuse f (mapfuse g xs) = mapfuse (f . g) xs #-}
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{-# RULES "map/map" forall f g xs.  
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```

$mapTest :: [Int] \rightarrow [Int]$

$mapTest xs = map (+1) (map (*2) xs)$

$mapTest_{\text{fuse}} :: [Int] \rightarrow [Int]$

$mapTest_{\text{fuse}} xs = mapfuse (+1) (mapfuse (*2) xs)$

Our map fusion performs (a bit) better!

We can test our functions on a million elements

$$mapTest \ xs = map \ (+1) \ (map \ (*2) \ xs)$$
$$mapTest_{\text{fuse}} \ xs = map_{\text{fuse}} \ (+1) \ (map_{\text{fuse}} \ (*2) \ xs)$$

and find we get a bit better time and space performance.

Function	Time (ms)	Memory (MB)
$mapTest$	26.4	256.00
$mapTest_{\text{fuse}}$	17.6	184.00

Through rules, GHC performs fusion

Rules allow us to perform *fusion*, where we remove intermediate data structures from the computation.

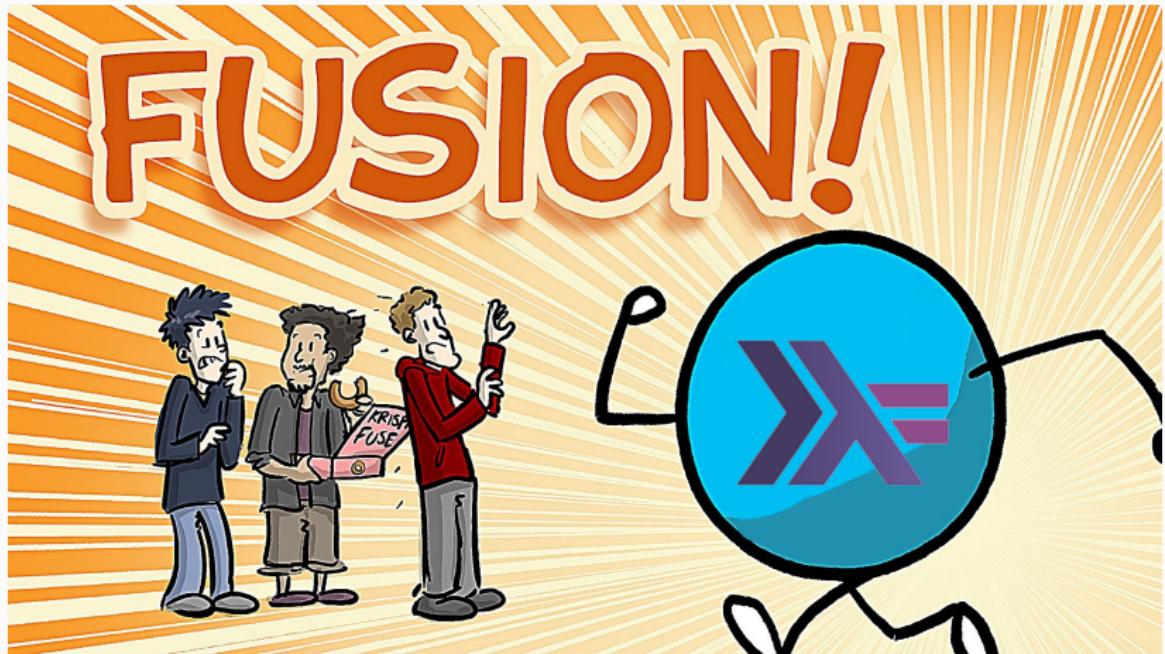
In our *process* function, we create an intermediate list

$$process :: [Int] \rightarrow Int$$
$$process xs = sum \circ map sq \$ xs$$

whereas our "fused" form did not make any intermediate structure, and used an accumulator instead.

$$process_{\text{hand}} :: [Int] \rightarrow Int$$
$$process_{\text{hand}} [] = 0$$
$$process_{\text{hand}} (x : xs) = x * x + process_{\text{hand}} xs$$

FUSION!



From PhD Comics

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foldr combines the elements of a list

foldr :: $(a \rightarrow b \rightarrow b) \rightarrow b \rightarrow [a] \rightarrow b$

foldr f z [] = z

foldr f z (x : xs) = f x (foldr f z xs)

foldr / build **fusion is used to simplify list computations**

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$$\textit{foldr} :: (a \rightarrow b \rightarrow b) \rightarrow b \rightarrow [a] \rightarrow b$$
$$\textit{foldr } f \ z \ [] = z$$
$$\textit{foldr } f \ z \ (x : xs) = f \ x \ (\textit{foldr } f \ z \ xs)$$

while *build* builds up a list from a generating function.

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$$\text{build} :: (\forall b. (a \rightarrow b \rightarrow b) \rightarrow b \rightarrow b) \rightarrow [a]$$
$$\text{build } g = g \ (:) \ []$$
$$\text{build1 } l \equiv [1, 2, 3]$$

where

$$l \text{ cons } nil = 1 \text{ `cons' } (2 \text{ `cons' } (3 \text{ `cons' } nil))$$

The *foldr/build* rule removes intermediate fold/build pairs

To remove intermediate data structures (those created by *build*), we eliminate *foldr/build* pairs with a rule.

```
{-# RULES
"foldr/build"
\ f z (g :: \ b. (a -> b -> b) -> b -> b).
foldr f z (build g) = g f z #-}
```

$$\text{foldr } (+) \ 0 \ (\text{build } l) \equiv l \ (+) \ 0 \equiv 1 + (2 + (3 + 0))$$

where

$$l \ \text{cons} \ \text{nil} = 1 \text{ `cons`} (2 \text{ `cons`} (3 \text{ `cons`} \text{ nil}))$$

We need a few extra rules to convert maps into fold/builds

To convert our definition of maps into a fold/build pair, we need the following helper function. [8, 9]

$$\begin{aligned}mapFB :: (elt \rightarrow lst \rightarrow lst) \rightarrow (a \rightarrow elt) \rightarrow a \rightarrow lst \rightarrow lst \\mapFB\ c\ f = \lambda x\ ys \rightarrow c\ (f\ x)\ ys\end{aligned}$$

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As an example, lets apply the list cons $c = (:)$ and $f = sq$

$$\lambda x \ ys \rightarrow sq \ x : ys$$

We need a few extra rules to convert maps into fold/builds

With that, we have all we need to convert map into build/fold.

```
{-# RULES "map" [~1] ∀ f xs. map f xs =  
    build (\c n -> foldr (mapFB c f) n xs) #-}
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We also provide a way to cancel failed fusion by converting back to a map.

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build ( $\lambda c n \rightarrow \text{foldr } \text{mapFB } c f) n xs)$   
≡ {-inline def of build -}  
 $(\lambda c n \rightarrow \text{foldr } \text{mapFB } c f) n xs) () []$   
≡ {-remove lambda -}  
 $\text{foldr} (\text{mapFB} (:)) f [] xs$ 
```

Manual rewrite rule application

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$$\begin{aligned} \text{process } (x : xs) &= \text{foldr } (\lambda x \text{ } ys \rightarrow \text{sq } x + ys) \text{ } 0 \text{ } (x : xs) \\ &\equiv \{\text{-expand foldr case: } \text{foldr } f \text{ } z \text{ } (x : xs) = f \text{ } x \text{ } (\text{foldr } f \text{ } z \text{ } xs)\text{ -}\} \\ &\quad (\lambda x \text{ } ys \rightarrow \text{sq } x + ys) \text{ } x \text{ } (\text{foldr } (\lambda x \text{ } ys \rightarrow \text{sq } x + ys) \text{ } 0 \text{ } xs) \\ &\equiv \{\text{-use def of process}_\text{fuse: } \text{foldr } f \text{ } 0 \text{ } xs = \text{process}_\text{fuse} \text{ } xs \text{ -}\} \\ &\quad (\lambda x \text{ } ys \rightarrow \text{sq } x + ys) \text{ } x \text{ } (\text{process}_\text{fuse} \text{ } xs) \end{aligned}$$

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Bringing both cases back together

If we now combine our two cases, we have the following

$$process_{fuse} [] = 0$$

$$process_{fuse} (x : xs) = x * x + process_{fuse} xs$$

This is the same as what we had originally written manually!

$$process_{hand} [] = 0$$

$$process_{hand} (x : xs) = x * x + process_{hand} xs$$

We achieved list fusion using *foldr* / *build* with rewrite rules

We managed to fuse *process* using our rewrite rules. We can look at the output of the compiler and it confirms what we expected.

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$$process_{\text{fuse}} [] = 0$$

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As expected, we get the same performance after performing the fusion rules.

Function	Time (ms)	Memory (MB)
<i>process</i>	41.86	265.26
<i>process'</i>	26.60	96.65
<i>process_{hand}</i>	28.80	96.65
<i>process_{fuse}</i>	27.08	96.65

There are many types of fusion concepts out there

While *foldr* / *build* works well, it can have problems fusing *zip* and *foldl*.

There are a few other systems out there. [2, 3]

- *unbuild* / *unfoldr*, where *unfoldr* builds a list and *unbuild* consumes a list. It can have problems fusing *filter*.

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While *foldr* / *build* works well, it can have problems fusing *zip* and *foldl*.

There are a few other systems out there. [2, 3]

- *unbuild* / *unfoldr*, where *unfoldr* builds a list and *unbuild* consumes a list. It can have problems fusing *filter*.
- stream fusion, which works by defining a *Stream* data type that acts like an iterator.

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Introduction to Stream

The Stream fusion system attempts to do something similar, by defining a list as an iterator. [2, 3]

```
data Stream a where
  Stream :: (s → Step a s) → s → Stream a
```

Introduction to Stream

The Stream fusion system attempts to do something similar, by defining a list as an iterator. [2, 3]

data *Stream a where*

Stream :: $(s \rightarrow Step\ a\ s) \rightarrow s \rightarrow Stream\ a$

where *Step a s* informs us how to keep processing the stream.

data *Step a s = Done*

- | *Skip* *s*
- | *Yield a s*

Streams have little helpers to make lists: stream

To work on standard lists, we introduce the following two functions to convert between lists and streams.

stream :: [a] → Stream a

stream xs = Stream uncons xs

where

uncons [] = Done

uncons (y : ys) = Yield y ys

Streams have little helpers to make lists: `unstream`

To work on standard lists, we introduce the following two functions to convert between lists and streams.

unstream :: Stream a → [a]

unstream (Stream next s0) = unfold next s0

where

unfold next s = case next s of

Done → []

Skip s' → unfold next s'

Yield x s' → x : unfold next s'

Let's define *map* for Streams

We can define some standard list processing functions on *Streams*.

Let's try *map*.

$$map_s :: (a \rightarrow b) \rightarrow Stream\ a \rightarrow Stream\ b$$

$$map_s\ f\ (Stream\ next0\ s0) = Stream\ next\ s0$$

where

$$next\ s = \mathbf{case}\ next0\ s\ \mathbf{of}$$

$$Done \quad \rightarrow Done$$

$$Skip\ s' \quad \rightarrow Skip \quad s'$$

$$Yield\ x\ s' \rightarrow Yield\ (f\ x)\ s'$$

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$$Yield\ x\ s' \rightarrow Yield\ (f\ x)\ s'$$

$$map_{[a]} :: (a \rightarrow b) \rightarrow [a] \rightarrow [b]$$

$$map_{[a]}\ f = unstream \circ map_s\ f \circ stream$$

Fusion on Streams

Fusion on streams only has one rewrite rule, and it is pretty simple.

```
{-# RULES "stream" ∀ (s :: Stream a).  
    stream (unstream s) = s #-}
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mapTestStream :: [Int] → [Int]

mapTestStream xs = *map*_[a] (+1) ∘ *map*_[a] (*2) \$ *xs*

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$map_{[a]} (+1) \circ map_{[a]} (*2)$

$\equiv \{-\text{expand mapl -}\}$

$unstream \circ map_s (+1) \circ stream \circ unstream \circ map_s (*2) \circ stream$

$\equiv \{-\text{apply "stream/unstream" -}\}$

$unstream \circ map_s (+1) \circ map_s (*2) \circ stream$

Map fused by Stream Fusion

Our map example

$\text{mapTestStream} :: [\text{Int}] \rightarrow [\text{Int}]$

$\text{mapTestStream } xs = \text{map}_{[a]} (+1) \circ \text{map}_{[a]} (*2) \$ xs$

gets fused into this result.

$\text{mapTestStreamCompiled} :: [\text{Int}] \rightarrow [\text{Int}]$

$\text{mapTestStreamCompiled } [] = []$

$\text{mapTestStreamCompiled } (x : xs) =$

$1 + (x * 2) : \text{mapTestStreamCompiled } xs$

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We can make *process* even faster with *Data.Vector*

The *Data.Vector* package uses stream fusion and many other rewrite rules behind the scenes in order to optimize array based computations.

```
process xs = sum ∘ map sq \$ xs
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We can make *process* even faster with *Data.Vector*

The *Data.Vector* package uses stream fusion and many other rewrite rules behind the scenes in order to optimize array based computations.

```
process xs = sum ∘ map sq \$ xs
```

The vector version looks very similar.

```
import qualified Data.Vector as V
```

```
processVec n = V.sum \$ V.map sq \$ V.enumFromTo 1 (n :: Int)
```

We can make *process* even faster with *Data.Vector*

```
processVec n = V.sum $ V.map sq $ V.enumFromTo 1 (n :: Int)
```

But has awesome performance!

Function	Time (ms)	Memory (MB)
<i>process</i>	41.86	265.26
<i>processfuse</i>	25.31	96.65
<i>processVec</i>	0.7	16×10^{-5}

What code does *Data.Vector* generate?

The *processVec* function is pretty simple in Haskell itself.

```
processVec n = V.sum $ V.map sq $ V.enumFromTo 1 (n :: Int)
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When compiling, GHC fires 202 rules!

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Specifically, this appears when using the debug flag
-ddump-rule-firings. [10]

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What code does *Data.Vector* generate?

The *processVec* function is pretty simple in Haskell itself.

$$\text{processVec } n = V.\text{sum} \$ V.\text{map } \text{sq} \$ V.\text{enumFromTo } 1 (n :: \text{Int})$$

And the final code generated is the following.

$$\text{processVecGHC } n = \text{loop } 1 \ 0$$

where

$$\text{loop } count \ acc = \text{case } count \leq n \ \text{of}$$
$$\text{False} \rightarrow acc$$
$$\text{True} \rightarrow \text{loop } (count + 1) (acc + (count * count))$$

Other use cases for fusion

Besides vector, fusion is used in a few other places.

- Repa, a parallel list processing library [11]

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Besides vector, fusion is used in a few other places.

- Repa, a parallel list processing library [11]
- Vector instructions by SIMD [12]
- Pipes, a stream processing library [13]

Wrap up

What did we talk about today?

- Goal: simple code that performed as well as a optimized version.
- A brief introduction to compilation in GHC and rewrite rules.
- *foldr / build* fusion.
- Showed a second type of fusion: stream fusion.
- Went through some libraries using fusion.

Thanks!

Questions?



References I

- [1] P. Wadler, "Deforestation: Transforming programs to eliminate trees," *Theoretical computer science*, vol. 73, no. 2, pp. 231–248, 1990.
- [2] D. Coutts, R. Leshchinskiy, and D. Stewart, "Stream fusion: From lists to streams to nothing at all," *ACM SIGPLAN Notices*, 2007.
- [3] D. Coutts, "Stream Fusion," Ph.D. dissertation, University of Oxford, Oct. 2010.
- [4] M. Karpov. (2016, Nov.) GHC Optimization and Fusion. [Online]. Available: <https://www.stackbuilders.com/tutorials/haskell/ghc-optimization-and-fusion/>

References II

- [5] G. Team. Compiling one module: HscMain. [Online]. Available: <https://ghc.haskell.org/trac/ghc/wiki/Commentary/Compiler/HscMain>
- [6] ——. Core-to-Core optimization pipeline. [Online]. Available: <https://ghc.haskell.org/trac/ghc/wiki/Commentary/Compiler/Core2CorePipeline>
- [7] (2010, Jan.) Playing by the Rules. [Online]. Available: https://wiki.haskell.org/Playing_by_the_rules#Fast_ByteString_construction
- [8] G. Team. Using Rules. [Online]. Available: https://wiki.haskell.org/GHC/Using_rules

References III

- [9] ——. GHC.Base. [Online]. Available: <https://hackage.haskell.org/package/base-4.9.1.0/docs/src/GHC.Base.html#foldr>
- [10] R. Leshchinskiy. Vector. [Online]. Available: <https://github.com/haskell/vector/blob/master/Data/Vector/Generic.hs#L2025>
- [11] B. Lippmeier, M. M. T. Chakravarty, G. Keller, and A. Robinson, *Data flow fusion with series expressions in Haskell*. ACM, Jan. 2014, vol. 48.
- [12] G. Mainland, R. Leshchinskiy, and S. Peyton Jones, “Exploiting vector instructions with generalized stream fusion,” in *the 18th ACM SIGPLAN international conference*. New York, New York, USA: ACM Press, 2013, pp. 37–12.

References IV

- [13] G. Gonzalez. (2014, Jan.) Shortcut fusion for pipes. [Online]. Available: [http://www.haskellforall.com/2014/01/
stream-fusion-for-pipes.html](http://www.haskellforall.com/2014/01/stream-fusion-for-pipes.html)