Deforestation and Program Fusion: Applying Equational Transforms to Automatically Simplify Programs

github.com/ryanorendorff/xxxxxxxxxxxxx

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Common way to process a list: map and fold!

As an example, say we want to square all the elements in a list and then sum the result.

$$process :: [Int] \to Int$$
$$process\ xs = sum \circ map\ sq\ \$\ xs$$

Where we have defined the functions as follows.

$$\begin{aligned} map\ \_\ [\,] &= [\,] \\ map\ f\ (x : xs) &= f\ x : map\ f\ xs \end{aligned}$$

$$sq\ x = x * x$$

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$$\begin{aligned} foldr\ \_\ b\ [\,] &= b \\ foldr\ f\ b\ (a : as) &= foldr\ f\ (f\ a\ b)\ as \end{aligned}$$

$$sum = foldr\ (+)\ 0$$

How fast is $process$?

So now that we have our process function, how fast does it run?

$$process :: [Int] \rightarrow Int$$
$$process\ xs = sum \circ map\ sq\ \$\ xs$$

Let's try to process a million elements with our $process$ and $process'$, which uses the standard Prelude sum and map.

$$process\ [0\,..\,1{,}000{,}000]; process'\ [0\,..\,1{,}000{,}000]$$

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How does the Prelude do so much better with the same functions?

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$$\quad \mathbf{where}$$
$$\qquad loop\ n\ [\,] \qquad = n$$
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This mirrors the version of the program one would write imperatively.

How well does our manually fused version do?

$$process\ xs = sum \circ map\ sq\ \$\ xs$$

$$process'\ xs = Prelude.sum \circ Prelude.map\ sq\ \$\ xs$$

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process	220.0	265.26
process′	5.1	80.00
processFused	4.7	80.00
process.c	2.6	8×10^{-5}

GHC generated the loop version automatically

Our manual version $processFused$.

$$
\begin{aligned}
&processFused :: [Int] \to Int\\
&processFused = loop\ 0\\
&\quad \mathbf{where}\\
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&\qquad loop\ n\ (x : xs) = loop\ (n + x * x)\ xs
\end{aligned}
$$

and when we compile the Prelude defined $process'$, GHC produces

$$
\begin{aligned}
&process' :: [Int] \to Int\\
&process'\ xs = loop'\ xs\ 0
\end{aligned}
$$

$$
\begin{aligned}
&loop' :: [Int] \to Int \to Int\\
&loop'\ [\,] \qquad\quad n = n\\
&loop'\ (x : xs)\ n = loop'\ xs\ (n + x * x)
\end{aligned}
$$

We want simpe/readible programs that are also performant

Our original version was simple to understand, but not fast.

$$process\ xs = sum \circ map\ sq\ \$\ xs$$

Our accumulator one is fast but not simple.

$$\begin{aligned}
&processFused :: [Int] \to Int \\
&processFused = loop\ 0 \\
&\quad \textbf{where} \\
&\qquad loop\ n\ [\,] \qquad\quad = n \\
&\qquad loop\ n\ (x : xs) = loop\ (n + x * x)\ xs
\end{aligned}$$

How can we leverage the compile to write simple code that is fast?

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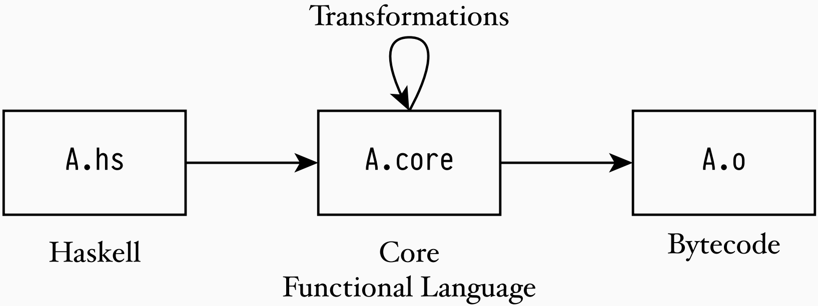
The GHC Compilation Pipeline converts Haskell into an intermediate language and then bytecode

When GHC compiles a Haskell program, it converts the code into an intermediate language called "Core", which is then (eventually) turned into byte code.



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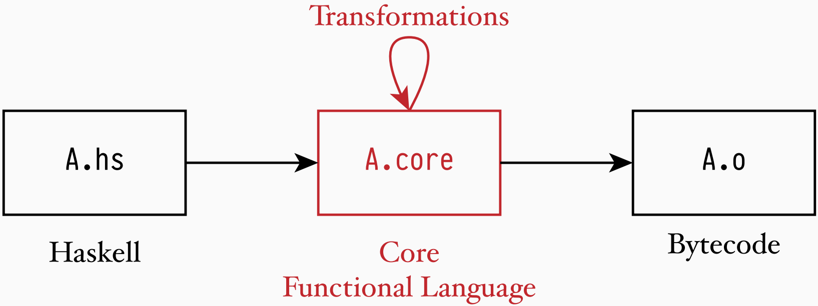

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- *Applying rewrite rules*

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- Combining type casts
- *Applying rewrite rules*
- ...

Rewrite Rules allow us to say two expressions are equivalent

Rewrite rules allow us to replace terms in the program with equivalent terms.

```
{-# RULES "name" forall x. id x = x #-}
```

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Rewrite rules allow us to replace terms in the program with equivalent terms.

{-# RULES "name" forall x. id x = x #-}

"Any time we see the term $id\ x$, replace it with x".

Rules have some restrictions

Rewrite rules have some gotchas.

- Rules doesn't prevent you from doing something silly

```
{-# RULES "id5" forall x. id x = 5 #-}
```

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  $$\{\text{-\# RULES "id5" forall x. id x} = 5 \text{ \#-}\}$$

- The left hand side is only substituted for the right, not the other way around.

  $$\{\text{-\# RULES "id" forall x. id x} = \text{x \#-}\}$$

  $$x \not\Rightarrow id\ x$$

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- You can make the compiler go into an infinite loop.

  {-# RULES "fxy" forall x y. f x y = f y x #-}

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  {-# RULES "fxy" forall x y. f x y = f y x #-}

- If multiple rules are possible, GHC will randomly choose one.

We can combine maps to traverse a list once

Let us introduce the following rule about maps.

$$\{\text{-\# RULES "map}_{\mathsf{f}}\text{/map}_{\mathsf{f}}\text{" forall f g xs. map}_{\mathsf{f}}\text{ f (map}_{\mathsf{f}}\text{ g xs)} = \text{map}_{\mathsf{f}}$$

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{-# RULES "$\mathsf{map_f}$/$\mathsf{map_f}$" forall f g xs. $\mathsf{map_f}$ f ($\mathsf{map_f}$ g xs) = $\mathsf{map_f}$

$$mapTest0 :: [Int] \to [Int]$$
$$mapTest0\ xs = map\ (+1)\ (map\ (*2)\ xs)$$

$$mapTest1 :: [Int] \to [Int]$$
$$mapTest1\ xs = map_f\ (+1)\ (map_f\ (*2)\ xs)$$

Our map fusion performs (a bit) better!

We can test our functions on a million elements

$$mapTest0\ xs = map\ (+1)\ (map\ (*2)\ xs)$$
$$mapTest0\ [1\,..\,1{,}000{,}000]$$

$$mapTest1\ xs = map_f\ (+1)\ (map_f\ (*2)\ xs)$$
$$mapTest1\ [1\,..\,1{,}000{,}000]$$

and find we get a bit better time and space performance.

Function	Time (ms)	Memory (MB)
$mapTest0$	26.4	256.00
$mapTest1$	17.6	184.00

Through rules, GHC performs fusion

Some of the rules work together to perform *fusion*: to combine terms in such a way as to pass over a data structure once.

In our $process$ function, we create an intermediate list

$$process\ xs = sum \circ map\ sq\ \$\ xs$$

whereas our "fused" form did not make any intermediate structure, and used an accumulator instead.

$$\begin{aligned}
&processFused :: [Int] \to Int \\
&processFused = loop\ 0 \\
&\quad \textbf{where} \\
&\qquad loop\ n\ [\,] \qquad\quad = n \\
&\qquad loop\ n\ (x : xs) = loop\ (n + x * x)\ xs
\end{aligned}$$

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foldr / *build* fusion is used to simplify list computations

GHC accomplishes fusion with two functions: foldr and build.

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$foldr$ combines the elements of a list

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foldr\ \_\ b\ [\,] &= b \\
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\end{aligned}
$$

while $build$ builds up a list from a generating function.

$$
\begin{aligned}
&build :: (forall\ \ b \circ (a \to b \to b) \to b \to b) \to [\,a\,] \\
&build\ \ g = g\ \ (:)\ [\,]
\end{aligned}
$$

$$
\begin{aligned}
&build\ \ l \equiv [1, 2, 3] \\
&\quad \textbf{where} \\
&\qquad l\ \ cons\ \ nil = 1\ `cons`\ (2\ `cons`\ (3\ `cons`\ \ nil))
\end{aligned}
$$

The $foldr/build$ rule removes intermediate fold/build pairs

To remove intermediate data structures (those created by $build$), we eliminate $foldr/build$ pairs with a rule.

{-# RULES "$build/foldr$" forall f z (g :: forall b. (a -¿ b -¿ b) -¿

$$
\begin{aligned}
&foldr\ (+)\ 0\ (build\ l) \equiv l\ (+)\ 0 \equiv 1 + (2 + (3 + 0))\\
&\quad \mathbf{where}\\
&\qquad l\ cons\ nil = 1\ `cons`\ (2\ `cons`\ (3\ `cons`\ nil))
\end{aligned}
$$

We need a few extra rules to convert maps into fold/builds

To convert our definition of maps into a fold/build pair, we need the following helper function.

$$map_{fb} :: (b \to l \to l) \to (a \to b) \to a \to l \to l$$
$$map_{fb}\ cons\ f = \lambda x\ ys \to f\ x\ \text{`}cons\text{`}\ ys$$

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With that, we have all we to convert map into build/fold.

{-# RULES "map" forall f xs. map f xs $=$ $build$ (ņ -¿ $foldr$ (map

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{-# RULES "map" forall f xs. map f xs $= build$ (ņ -¿ $foldr\ (map$

We also provide a way to combine sequential map_{fb} functions.

{-# RULES "mapFB" forall c f g. mapFB (mapFB c f) g = mapFI

This is where the example is expanded completely

Will show how the map sum example is converted into the loop we saw.

Will need

$$
\begin{aligned}
&foldr :: (a \to b \to b) \to b \to [a] \to b \\
&foldr\ f\ z = go \\
&\quad \textbf{where} \\
&\qquad go\ [\,] = z \\
&\qquad go\ (y : ys) = y\ `f`\ go\ ys
\end{aligned}
$$

{-# INLINE [0] foldr2 #-}

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We can make $process$ even faster with $Data.Vector$

The $Data.Vector$ package uses stream fusion and many other rewrite rules behind the scenes in order to optimize array based computations.

$$process\ xs = sum \circ map\ sq\ \$\ xs$$

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$$process\ xs = sum \circ map\ sq\ \$\ xs$$

The vector version looks very similar.

$$\textbf{import}\ qualified\ Data.Vector\ as\ V$$

$$processVec\ n = V.sum\ \$\ V.map\ sq\ \$\ V.enumFromTo\ 1\ (n :: Int)$$

We can make $process$ even faster with $Data.Vector$

But has incredible performance!

Function	Time (ms)	Memory (MB)
$process$	220.0	265.26
$process'$	5.1	80.00
$processFused$	4.7	80.00
process.c	2.6	8×10^{-5}
processVec	0.7	16×10^{-5}

What code does $Data.Vector$ generate?

While we wrote this in our program

$$processVec\ n = V.sum\ \$\ V.map\ sq\ \$\ V.enumFromTo\ 1\ (n :: Int)$$

GHC ended up generating the following Core code.

$$
\begin{array}{l}
val\_\$s\$wfoldlM'\_loop \\
val\_\$s\$wfoldlM'\_loop = \\
\quad \lambda sc\ sc1 \rightarrow \\
\qquad \textbf{case}\ tagToEnum\#\ (<=\#sc\ 100000000\#)\ \textbf{of}\ \_\ \{ \\
\qquad\quad False \rightarrow sc1; \\
\qquad\quad True \rightarrow val\_\$s\$wfoldlM'\_loop\ (+\#sc\ 1\#)\ (+\#sc1\ (*\#sc \\
\qquad \}
\end{array}
$$

simplify this core, ignore unboxing

Repa: A numerical Haskell Library using Fusion

Repa also uses fusion in order to handle array operations.

Data Parallel Haskell: Nested Data Parallelism made easy

$$processDPH :: [:Int:] \to Int$$
$$processDPH = sumDPH \circ mapDPH\ sq \,\$\, xs$$

Does dispatch by MPI.