

$$1. m(x) = \frac{1}{N} \sum_{i=1}^N x_i$$

$$m(a+bx) = \frac{1}{N} \sum_{i=1}^N (a+bx_i) = \frac{1}{N} \sum_{i=1}^N a + \underbrace{\sum_{i=1}^N b x_i}$$

$$= \frac{1}{N} \left(Na + b \sum_{i=1}^N x_i \right) = a + b \left(\frac{1}{N} \sum_{i=1}^N x_i \right)$$

$$2. \text{cov}(x, a+bx) = b \text{cov}(x, y)$$

$$\text{cov}(x, y) = \frac{1}{N} \sum_{i=1}^N (x_i - m(x))(y_i - m(y))$$

$$\#1 m(a+bx) = a + b m(x)$$

$$\begin{aligned} & \frac{1}{N} \sum_{i=1}^N (x_i - m(x))(a+bx_i - (a+bx_i)) \\ &= \frac{1}{N} \sum_{i=1}^N (x_i - m(x)) b(y_i - m(y)) \\ &= b \cdot \frac{1}{N} \sum_{i=1}^N (x_i - m(x))(y_i - m(y)) \\ &= b \cdot \text{cov}(x, y) \end{aligned}$$

$$3. \text{cov}(a+bx), a+bx) = b^2 \text{cov}(x, y), \text{cov}(x, x) = \sigma^2$$

$$\frac{1}{N} \sum_{i=1}^N (a+bx_i - m(a+bx))^2$$

$$= \frac{1}{N} \sum_{i=1}^N (a+bx_i - (a+b m(x)))^2 = \frac{1}{N} \sum_{i=1}^N b(x_i - m(x))^2$$

$$= b^2 \cdot \frac{1}{N} \sum_{i=1}^N (x_i - m(x))^2$$

$$\text{cov}(x, x) = \frac{1}{N} \sum_{i=1}^N (x_i - m(x))^2$$

$$= \sigma^2$$