

$$1. m(x) = \frac{1}{N} \sum_{i=1}^N a_i$$

$$\begin{aligned} m(a+bx) &= \frac{1}{N} \sum_{i=1}^N (a+bx_i) = \frac{1}{N} \sum_{i=1}^N a + \sum_{i=1}^N bx_i \\ &= \frac{1}{N} \left(Na + b \sum_{i=1}^N x_i \right) = a + b \left(\frac{1}{N} \sum_{i=1}^N x_i \right) \end{aligned}$$

$$2. \text{cov}(x, a+bY) = b \text{cov}(x, Y)$$

$$\text{cov}(X, Y) = \frac{1}{N} \sum_{i=1}^N (x_i - m(x))(y_i - m(y))$$

$$\#1 \quad m(a+bY) = a + b m(y)$$

$$\begin{aligned} \frac{1}{N} \sum_{i=1}^N (x_i - m(x))(a + by_i - (a + b m(y))) \\ &= \frac{1}{N} \sum_{i=1}^N (x_i - m(x))b(y_i - m(y)) \\ &= b \cdot \frac{1}{N} \sum_{i=1}^N (x_i - m(x))(y_i - m(y)) \\ &= b \text{cov}(x, y) \end{aligned}$$

$$3. \text{cov}(a+bX), a+bX) = b^2 \text{cov}(X, X), \quad \text{cov}(X, X) = \sigma^2$$

$$\begin{aligned} &\Rightarrow \frac{1}{N} \sum_{i=1}^N (a + bx_i - m(a + bX))^2 \\ &= \frac{1}{N} \sum_{i=1}^N (a + bx_i - (a + b m(x)))^2 = \frac{1}{N} \sum_{i=1}^N b(x_i - m(x))^2 \\ &= b^2 \cdot \frac{1}{N} \sum_{i=1}^N (x_i - m(x))^2 \\ &\text{cov}(X, X) = \frac{1}{N} \sum_{i=1}^N (x_i - m(x))^2 \\ &= \sigma^2 \end{aligned}$$