CSci 384: Artificial Intelligence

Assignment 2: CSP (150 points + 15 (optional))

Submission of the implementations only without answering Q2.(1)-(4) and Q3.(1)-(2) won't get any points.
Please read the submission instructions in BB and comply with them.

Q1. [40] Formulation of CSP

Consider the graph with 8 nodes A_1, A_2, A_3, A_4, H, T, F_1, F_2. A_i is connected to A_{i+1} for all i, each A_i is connected to H, H is connected to T, and T is connected to each F_i. These 8 nodes will be colored by {R, G, B} satisfying the constraints.

1) [10] **Formulate** a problem in terms of the variables, domains, and constraints in the *formal description* of Constraint Satisfaction Problem (CSP).

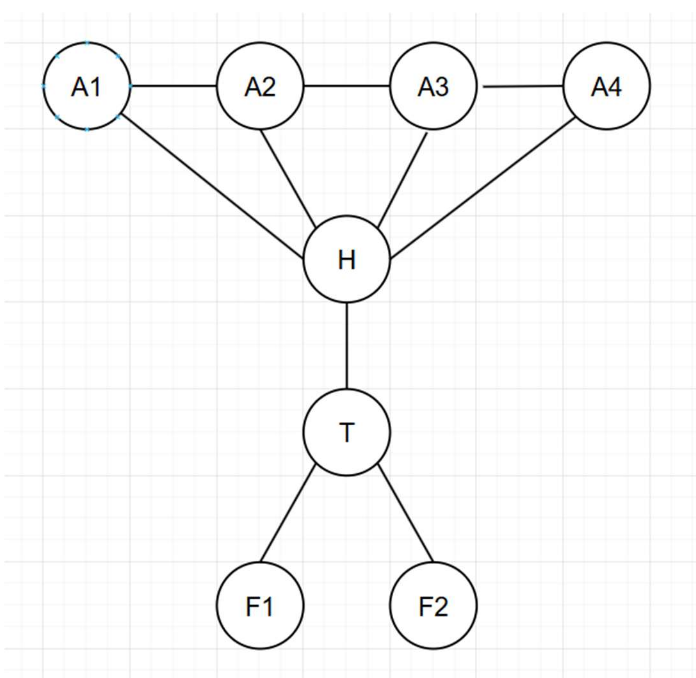
   Variables = $\{A1, A2, A3, A4, H, T, F1, F2\}$

   Domain for each variable = $\{R, G, B\}$

   Constraints:

   - Chain constraint: $A1 \neq A2, A2 \neq A3, A3 \neq A4$
   - Hub constraints: $A1 \neq H, A2 \neq H, A3 \neq H, A4 \neq H$
   - Tail constraints: $H \neq T, T \neq F1, T \neq F2$

2) [5] Draw the Constraint Graph of the problem.



3) [5] In the initial empty assignment, which variable is the best to select in order to solve it by backtracking with inference? Justify your answer.

H would be the best to start with, as it is the most constrained. It has a degree of 5 while every other node has a degree 3 or less. Its assignment will remove many options via inference.

4) [20] Nearly Tree-Structured Problem(s)

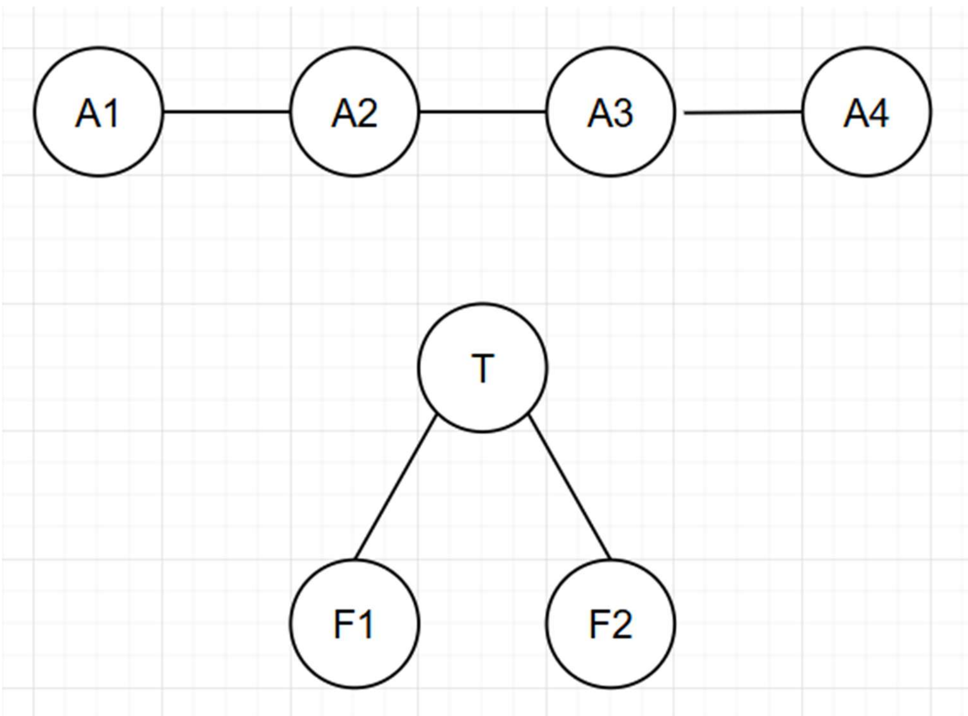
a) [5] In the problem of 1) – 2), find a **cycle cutset** to reduce a problem to a tree-structured problem.

A cycle cutset is a set of variables whose removal renders the constraint graph a tree. In this case, removing the node H will leave us with two trees, $\{T1, F1, F2\}$ and $\{A1, A2, A3, A4\}$

b) [5] With the cycle cut set, how many tree-structured subproblems do you get?

As seen in my response above, we are left with 2 separate tree-structured subproblems.

c) [5] Draw the tree-structured subproblem(s).



d) [5 +(10, optional)] Solve it as a nearly tree-structured problem. You must show its process step by step.

Cycle cutset variable $= H$, which we'll set to R

For the chain $A1$ through $A4$, our remaining domain is $\{G, B\}$. We must then simply alternate G and B to comply with their respective constraints, so $A1 = G, A2 = B, A3 = G, A4 = B$.

For the smaller tree, we again know T cannot be R, so let's just assume its G. That means that that both $F1 = B$ and $F2 = B$.

Q3. [45] Cryptarithmetic Problem

For the given cryptarithmetic problem,

CP + IS + FUN = TRUE

1) [10] Formulate a problem in terms of the variables, domains, and constraints in the formal description of Constraint Satisfaction Problem (CSP).

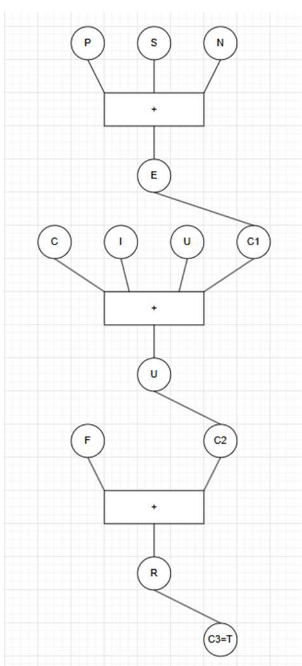
   Variables = $\{C, P, I, S, F, U, N, T, R, E\}$

   Domains = The domain for each letter is $\{0:9\}$ with the exception of the first letter of CP, IS, and FUN, which each have a domain of $\{1:9\}$, as they cannot be 0.

   Constraints:

   - Each letter must have a different digit
   - Arithmetic constraints:
     - $CP = 10 * C + P$
     - $IS = 10 * I + S$
     - $FUN = 100 * F + 10 * U + N$
     - $TRUE = 1000 * T + 100 * R + 10 * U + E$

2) [10] Draw the Hyper-Constraint Graph of the problem.



3) [25] Solve it by the ***Min-Conflict*** algorithm.

Start with the initial state (C=2, P = 3, I = 5, S = 4, F = 9, U = 6, N = 8, T = 1, R = 0, E = 7).
The max number of steps is 500.
In selecting a conflicted variable, choose *the most constrained* (or *constraining*) variable, NOT a random selection.
Implement it in Python (or in your preferred language) to find the solution.

See `csci384-hw2.py` file for implementation. Unfortunately, no matter how I wrote it, the algorithm was unable to solve this problem, at least in 500 steps. My best analysis for this issue is that 500 steps simply isn't enough steps to make it to a functional state from the initial state. This could mean that the given initial state doesn't converge to a valid solution.

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Q3. Cryptarithmetic Puzzle - Min-Conflict Algorithm
No solution found after 500 steps.
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Q2. [65] Logic Puzzle

The Zebra Puzzle is a well-known logic puzzle invented by Albert Einstein as a boy.
It's often called Einstein's Puzzle or Einstein's Riddle. Some claim that Einstein said "only 2 percent of the world's population can solve it". It is also sometimes attributed to Lewis Carroll. However, there is no known evidence for Einstein's or Carroll's authorship; and the original puzzle cited in the textbook (pg. 231) mentions brands of cigarette, such as Kools, that did not exist during Carroll's lifetime or Einstein's boyhood. - Wikipedia.

There are several versions of this puzzle. The original version in the Ex 6.7 (pg. 231) of the textbook is quoted from the first known publication in Life International magazine on December 17, 1962. The March 25, 1963, issue contained the solution given below and the names of several hundred solvers from around the world.

This puzzle is a Constraint Satisfaction Problem which has to satisfy the given constraints below.

A. The Swedish lives in the green house.
B. The Danish keeps dogs as pets.
C. The Norwegian drinks tea.
D. The white house is on the left of the yellow house.
E. The white homeowner drinks coffee.
F. The person who smokes Pall Mall rears birds.
G. The owner of the blue house smokes Dunhill.

H. The man living in the center house drinks milk.

I. The German lives in the first house.

J. The man who smokes Blend lives next to the one who keeps cats.

K. The man who keeps the horse lives next to the man who smokes Dunhill.

L. The owner who smokes Bluemaster drinks beer.

M. The British smokes prince.

N. The German lives next to the red house.

O. The man who smokes Blend has a neighbor who drinks water.

Each of the five houses is painted a different color, and their inhabitants are of different national extractions, own different pets, drink different beverages and smoke different brands of cigarettes. The goal of problem is to characterize a resident of each house, H1, …, H5 in terms of her/his nationality, pet, the house color, cigarette brand, beverage.

Domains of color, nationality, pets, beverage and cigarette brands are given below:
Domain of Color = {red, green, white, yellow, blue}
Domain of Nationality = {British, Swedish, Danish, Norwegian, German}
Domain of Pet = {dog, bird, horse, cat, raccoon}
Domain of beverage = {tea, coffee, milk, beer, water}
Domain of cigarette brand = {Prince, Pall Mall, Dunhill, Blend, Bluemaster}

1) a) [10] Formulate a problem in terms of the variables, domains and constraints in the formal description of Constraint Satisfaction Problem (CSP). You should explicitly formulate how to represent a state.

Variables = $H_i = (Color_i, Nationality_i, Pet_i, Beverage_i, Cigarette_i)$ for $i = 1,2,3,4,5$

Domains:
- $Colors\ \{red, green, white, yellow, blue\}$
- $Nationality\ \{British, Swedish, Danish, Norwegian, German\}$
- $Pet\ \{dog, bird, horse, cat, raccoon\}$
- $Beverage\ \{tea, coffee, milk, beer, water\}$
- $Cigarette\ \{Prince, Pall\ Mall, Dunhill, Blend, Bluemaster\}$

Constraints:

- All houses must have different values for each attribute
- The other constraints listed earlier in the problem definition
    - For each "next-to" constraint, the assigned houses must be adjacent

b) [5] What is the number of possible assignments in a)?

$$(5!)^5 = 120^5 = 24883200000$$

2) a) [10] Discuss a different representation of this problem as a CSP. Clearly define its variables, domains, constraints and others if necessary.
We could have each variable's domain have all 5! permutations of the possible values instead of having the variables be tuples. The constraints would then be specified as relations among the permutations. For

example, if the permutation for nationality assigns Swedish to house i, then the color permutation must also assign green to house i. Each state is now a 5-tuple of permutations, one for each attribute.

b) [5] What is the number of possible assignments in a)?

The number of possible assignments would be the same as before.

3) [5, optional] Over 1) and 2), which representation is more efficient in terms of the number of possible assignments?

I would imagine the first representation is more efficient, as there are less variables & thus less of a search space.

4) [10] Choose either 1) or 2) for your representation of state. Set the *initial* state be an *empty assignment*. ***Manually solve*** the problem by backtracking algorithm with the various heuristics, forward checking, arc-consistency (AC-3) and any intelligent techniques/heuristics. Justify each step by specifying the applied heuristic, forward-checking, AC-3 checks for the available values and/or whether to backtrack, etc.

Using representation 1, we see:

- Initial state is $H1 = \{\quad\}, H2 = \{\quad\}, etc.$
- We first apply the unary constraints
  - $H1's$ nationality becomes $German$
  - $H3's$ beverage becomes $Milk$
- We then follow up with other constraints based on our current state:
  - $H2's$ house must be red, as the German lives next to the red house
- We then use forward checking to see if the current state causes any issues with the remaining constraints, which they don't
- We know that $H4$ and $H5$ must be the $white/yellow$ pair due to their positions, so we can assign $H4 = white$ and $H5 = yellow$
  - Additionally, we can assign $H4's$ beverage as $coffee$ knowing the color
  - Blue is also the only remaining color, so $H1's$ color is $blue$
    - This also means that $H1's$ cigarette is $Dunhill$
- Utilizing the remaining cigarette constraints & the nationality/drink relations, we can see that:
  - $H2$ is British and smokes Prince
  - $H4$ is Norwegian and drinks tea
  - H5 is Danish and keeps dogs
    - We can forward check this section against the other constraints & see that this does not cause issues, so we don't need to test other layouts of the nationalities
- Finalizing the remaining constraints given what we know now, we see that:
  - $H1$: $German, Blue, Dunhill, Water, Cat$
  - $H2$: $Norwegian, Red, Blend, Tea, Horse$
  - $H3$: $Swedish, Green, Pall\ Mall, Milk, Birds$
  - $H4$: $British, White, Prince, Coffee, Raccoon$
  - $H5$: $Danish, Yellow, Bluemaster, Beer, Dogs$

5) [25] Implementation

Implement Q2.(4) in Python (or in your preferred language). You have to implement the heuristics to

choose a variable and a value, backtracking algorithm, AC-3 algorithm, forward checking (if necessary), and any intelligent techniques/heuristics.
Your output must give
a) a list of variables in the order of their selection to assign a value.
b) the final solution, i.e. a complete and consistent assignment.

See `csci384-hw2.py` file for implementation.

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Q2. Zebra Logic Puzzle
Solution:
Color:
-- red: House 2
-- green: House 3
-- white: House 4
-- yellow: House 5
-- blue: House 1
Nationality:
-- British: House 4
-- Swedish: House 3
-- Danish: House 5
-- Norwegian: House 2
-- German: House 1
Pet:
-- dog: House 5
-- bird: House 3
-- horse: House 2
-- cat: House 1
-- raccoon: House 4
Beverage:
-- tea: House 2
-- coffee: House 4
-- milk: House 3
-- beer: House 5
-- water: House 1
Cigarette:
-- Prince: House 4
-- Pall Mall: House 3
-- Dunhill: House 1
-- Blend: House 2
-- Bluemaster: House 5
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