Lecture 19

Separation - triple deck

3D BL

- Eckmann
- Karman disk

Asymptotic Description of Separation (outline only)

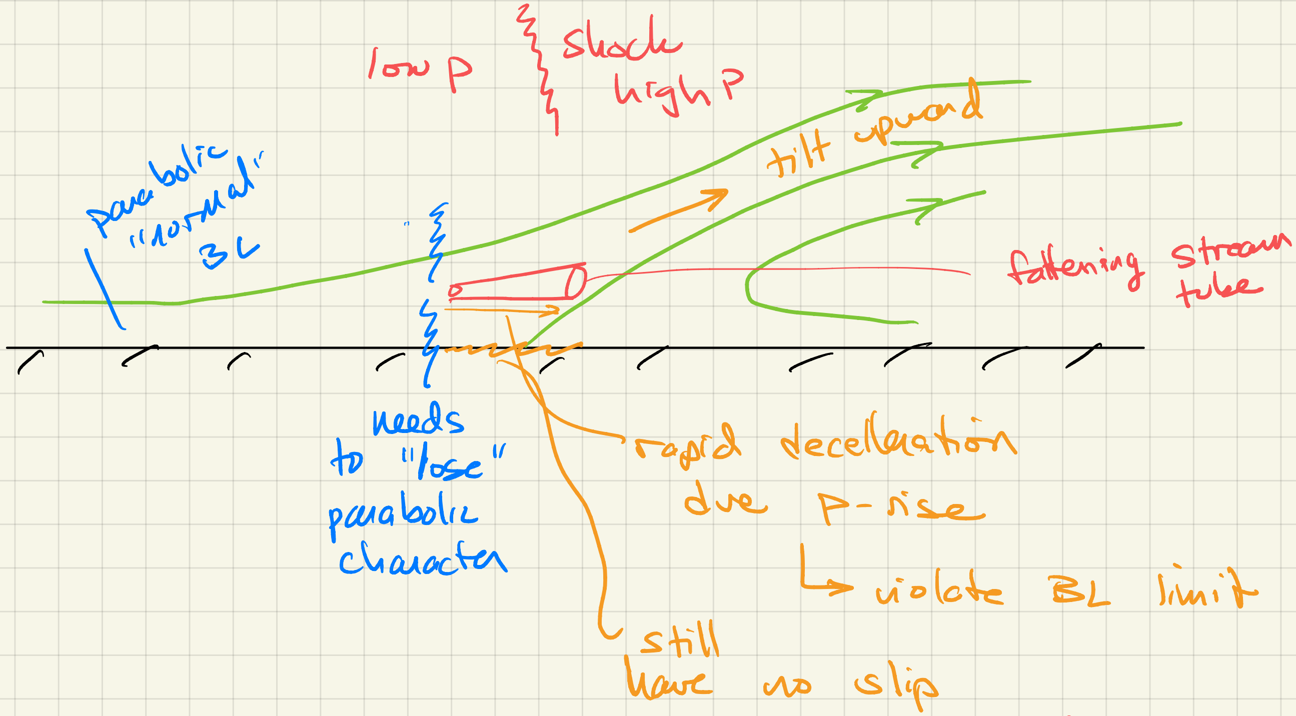
Ockendon + Ockendon

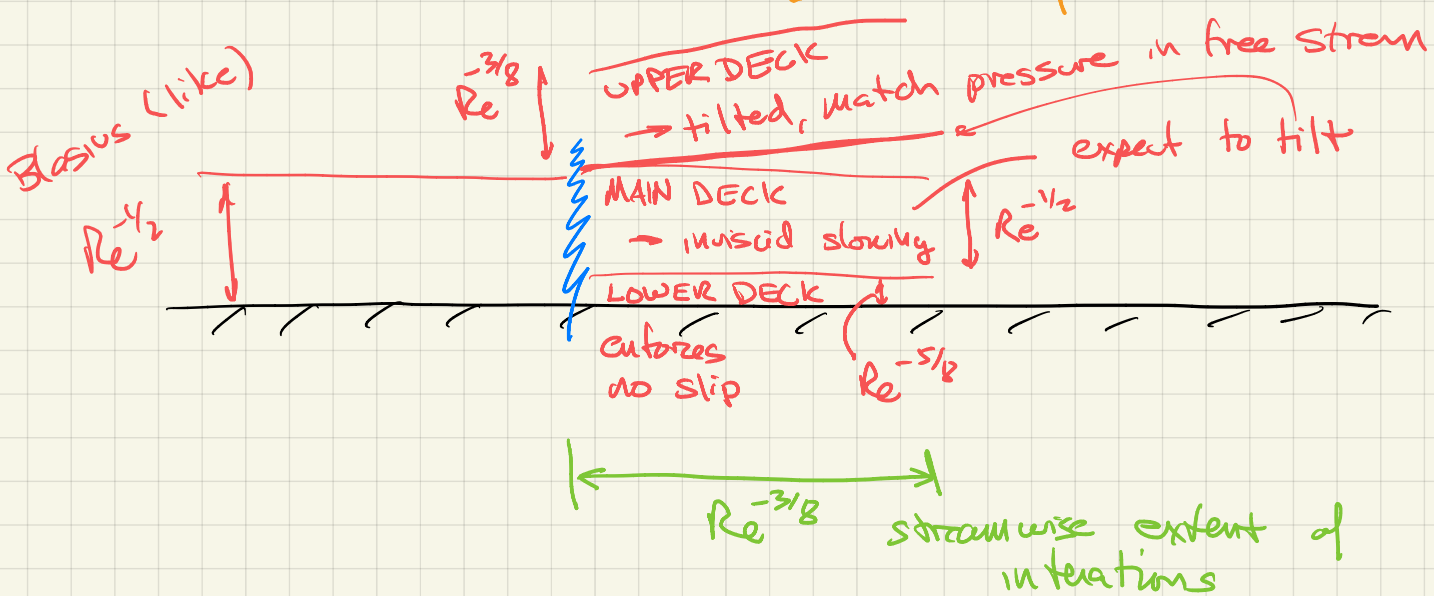
Stewartson + Brown (1969)

Neiland (1969)

⋮

Lighthill (1953)





$\varepsilon \equiv Re^{-1/8}$ $\qquad Re = \varepsilon^{-8}$

UPSTREAM

$x = x_p + LX$

$y = \varepsilon^4 L Y$

$Re^{-1/2}$

SEPARATION REGION

UPPER $\quad y = \varepsilon^3 L Y_u \quad x = x_p + \varepsilon^3 L X$

MAIN $\quad y = \varepsilon^4 L Y_M \quad x = x_p + \varepsilon^3 L X$

LOWER $\quad y = \varepsilon^5 L Y_L \quad x = x_p + \varepsilon^3 L X$

→ sub into N-S, scale, solve parts, connect parts at "bnd" ...

- includes rapid decelleration (MAIN)
- displacement (tilt) (UPPER)
- no slip (LOWR)
- matches some Δp
- matches upstream BL

TRIPLE-DECK THEORY

3D BL

- Basic BL is 2D
  - rapid variation in y
  - slow variation in x
  - $O(1)$ streamwise velocity u
  - $O(Re^{-1/2})$ transverse velocity v

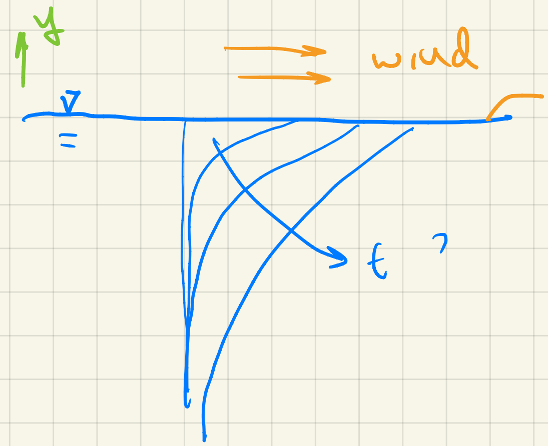
- Additional 3D effects
  - swept wing ← model flow
  - turbulence modeling for 3D flows
  - geophysical flows ← model

OUTLINE

- Eckman layer → 3D BL flow "thinking"
  → v. important geophysical flow
- Karman rotating flow/disk → a main 3D BL effect
  → explain demo

Eckman layer ...   Bachelor

warmup in 1D ... variant of Stokes 1^{st}



exerts stress μS starting at $t=0$

$u(y,0)=0$ IC

BC $\begin{cases} \mu \left.\dfrac{\partial u}{\partial y}\right|_{y=0} = \mu S \Rightarrow \left.\dfrac{\partial u}{\partial y}\right|_0 = S \\ u \to 0, \ y \to -\infty \end{cases}$

reduce NS as for Stokes #1   ... $\dfrac{\partial u}{\partial t} = \nu \dfrac{\partial^2 u}{\partial y^2}$ x-mom

$v=0$

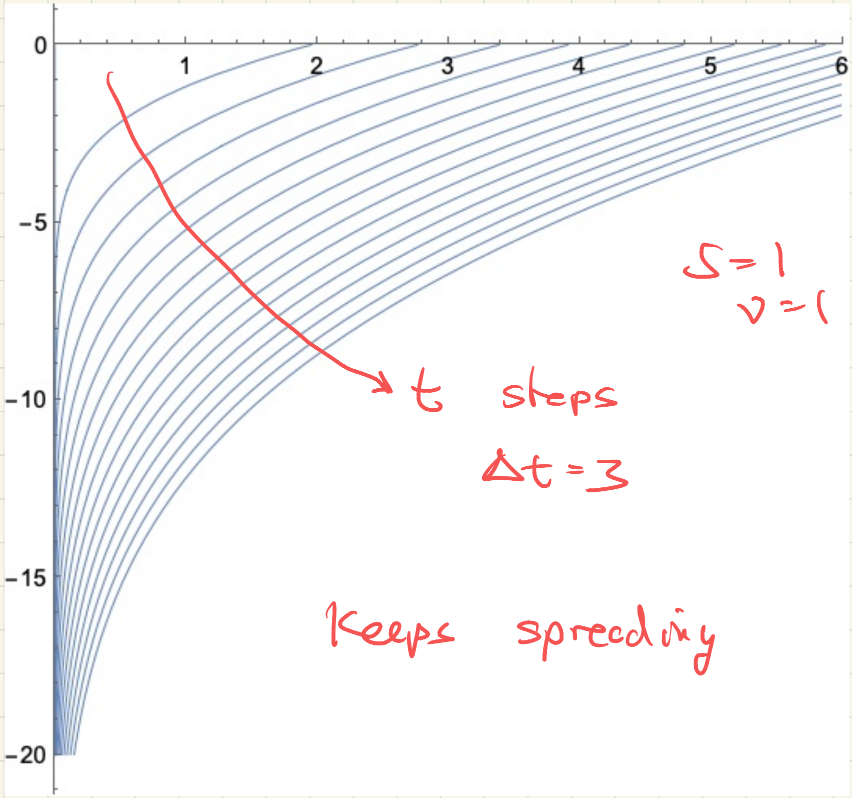
$\dfrac{\partial}{\partial x} \to 0$

$\vdots$

expect same similarity $\dfrac{y}{(\nu t)^{1/2}}$

$\vdots$

$$u(y,t) = Sy + Sy \, \mathrm{erf} \frac{y}{(\nu t)^{1/2}} + 2S\left(\frac{\nu t}{\pi}\right)^{1/2} e^{-y^2/4\nu t}$$



But winds blow for days — earth rotates on day timescale

momentum in rotating frame

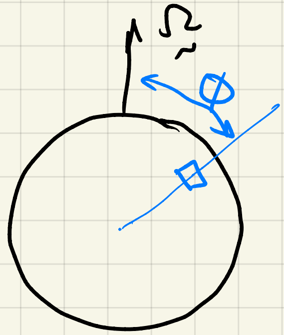
$$\frac{D\underline{u}}{Dt} = -\nabla p - \underline{\Omega} \times \underline{u} + \nu \nabla^2 \underline{u}$$

Coriolis term

absorb into p

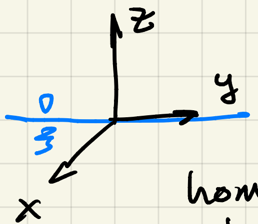
left out centrifugal term

→ local area of ocean



local $\Omega_3 = |\Omega| \cos\phi$

$$\Omega = (0, \Omega_2, \Omega_3)$$



homogeneous locally

w - z velocity $= 0$

$u, v \to 0$ for $z \to -\infty$

$$\frac{du}{\partial z} = S \qquad \frac{\partial v}{\partial z} = 0$$

"wind" in x direction

steady

$$-2\Omega \times u$$

appears in x, y, z mom

$$\begin{vmatrix} e_x & e_y & e_z \\ \Omega_1 & \Omega_2 & \Omega_3 \\ u & v & w \end{vmatrix}$$

x-mom

$$0 = 2v\Omega_3 + \nu \frac{\partial^2 u}{dz^2}$$

$$0 = -2u\Omega_3 + \nu \frac{\partial^2 v}{\partial z^2}$$

$\cdot + i \times \cdot$

$$\nu \frac{d^2(u+iv)}{dz^2} = 2i(u+iv)\Omega_3$$

$$u + iv = A \exp\left[\sqrt{\frac{\Omega_3}{\nu}}\sqrt{2i}\, z\right]$$

pick solution that decay in $-z$

$$\sqrt{i} = \frac{1+i}{\sqrt{2}}$$

$$u + iv = A \exp\left[\sqrt{\frac{\Omega_3}{\nu}}\,(i+1)\, z\right]$$

BC $$S = \frac{du}{dz} + i\frac{dv}{dz} = A\sqrt{\frac{\Omega_3}{\nu}}\,(i+1)\, e^{\sqrt{\Omega_3/\nu}\,(i+1)\,\overset{0}{\cancel{z}}}$$

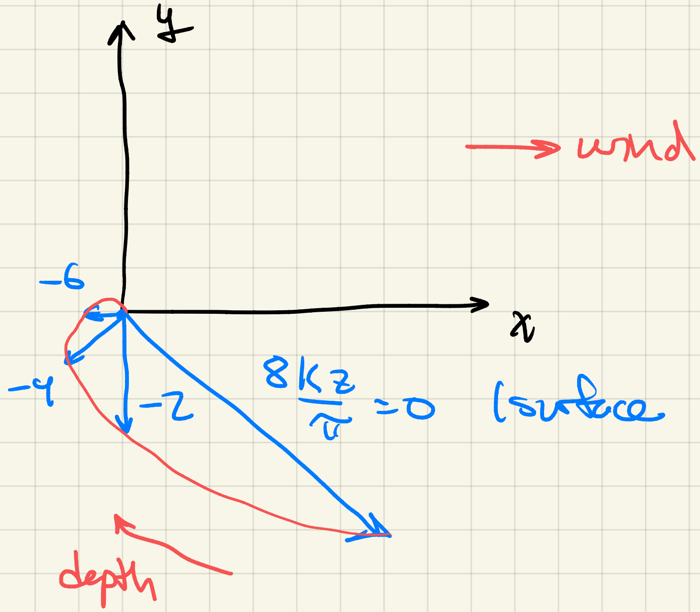
$$A = \frac{S}{i+1}\sqrt{\frac{\nu}{\Omega_3}} = \sqrt{\frac{\nu}{\Omega_3}}\;\frac{1-i}{2}\,S$$

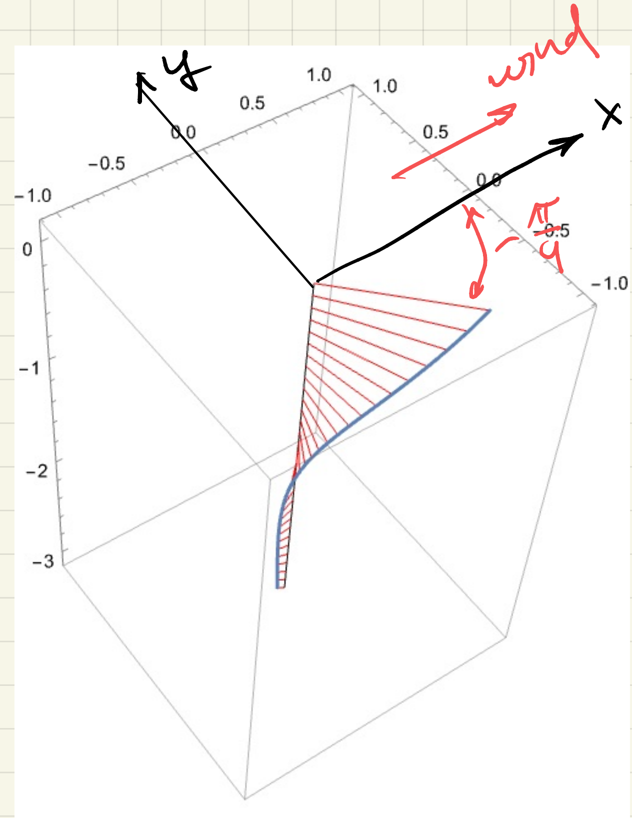
$$u + iv = S\sqrt{\frac{\nu}{\Omega_3}}\;\frac{i-1}{2}\; e^{\sqrt{\frac{\Omega_3}{\nu}}\, z}\; e^{i\sqrt{\frac{\Omega_3}{\nu}}\, z}$$

$$\frac{1}{\sqrt{2}}\, e^{-i\frac{\pi}{4}}$$

$$u = S\sqrt{\frac{\nu}{2\Omega_3}}\; e^{\sqrt{\Omega_3/\nu}\, z}\; \cos\left[\sqrt{\Omega_3/\nu}\; z - \pi/4\right]$$

$$v = S\sqrt{\frac{\nu}{2\Omega_3}}\; e^{\sqrt{\Omega_3/\nu}\, z}\; \sin\left[\sqrt{\Omega_3/\nu}\; z - \frac{\pi}{4}\right]$$





- Rotation confines flow near BND

- Ekman layer