Lecture 10

Blasius BL

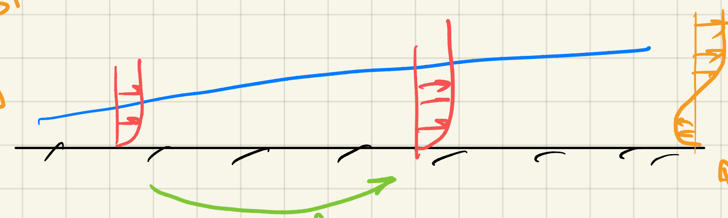
• no Re # in BL equation : Re → ∞ model for Re larger

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} \qquad (x\text{-mom})$$

• no $\frac{\partial^2}{\partial x^2}$ term anymore → changes mathematical character...

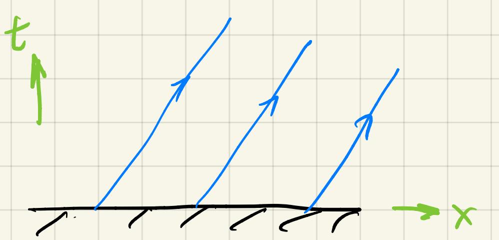
⇒ parabolic in x ... → "information" only influence "things" downstream

why we only needed upstream BC

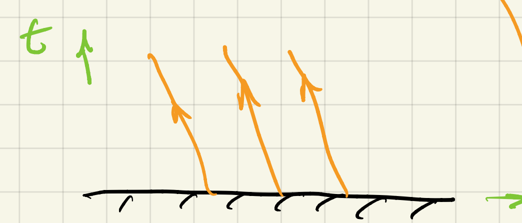


change of sign

$$\underbrace{\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}}_{\text{downstream "tilted" characteristics}} + \cdots$$



makes solutions much easier



left going

→ needs BC on right

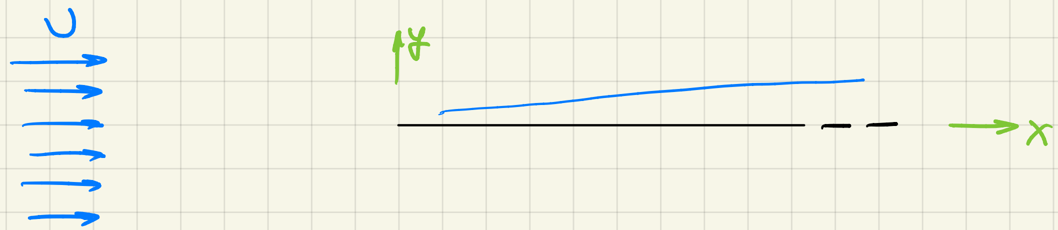
If both something must "break"

→ a singularity in BL

⇒ SEPARATION

BL On A Flat Plate — Blasius BL

$U = \text{const}$, p uniform



assume: δ is so thin that it does not disturb the OUTER flow

$$\delta \sim 1/Re^{1/2}$$

steady $\frac{\partial}{\partial t}$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

assume: b.l flow "forgets" how it started at the tip

- small region, ∞ thin
- "worked" for our $(0+0)$ T-problem

→ ignoring tip region only have x, y as lengths → can hope they "collapse" via self-similarity ⇒ ODE

attempt $\quad \eta = \dfrac{Ay}{x^\alpha} \qquad \psi = B x^\beta f(\eta)$

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x}$$

$\nabla \cdot \underline{u} = 0$

⋮

$$\eta = y\sqrt{\frac{U}{\nu x}} \qquad \psi = \sqrt{\nu U x}\, f(\eta)$$

$u = 0$ $\quad v = 0$

no slip, no penetration

$u = U f'$

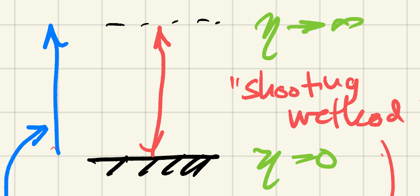
$v = \sqrt{\dfrac{\nu U}{x}}\,(\eta f' - f)$

$y = 0 \Rightarrow \eta = 0 \qquad f'(0) = 0 \qquad f(0) = 0$

$y/\delta \to \infty \Rightarrow \eta \to \infty \qquad f'(\infty) = 1$

ODE: $\quad f''' + \frac{1}{2} f f'' = 0$

needs numerical solve



→ easier to have

$f(0) = 0, \; f'(0) = 0, \; f''(0) = C$

guess C to "hit" $f'(\infty) = 1$

to avoid shooting ...

$$f(\eta) = \alpha F(\zeta) \qquad \zeta = \alpha\eta$$

$$f' = \frac{df}{d\eta} = \alpha \frac{dF}{d\eta} = \alpha \frac{dF}{d\zeta}\frac{d\zeta}{d\eta} = \alpha^2 F'$$

$$f'' = \alpha^3 F''$$

$$f''' = \alpha^4 F'''$$

$$f''' + \tfrac{1}{2} f f'' = 0 \qquad \text{Blasius}$$

$$\alpha^4 F''' + \tfrac{1}{2}\alpha F \alpha^3 F'' = 0$$

$$F''' + \tfrac{1}{2} F F'' = 0$$

$$\eta = 0 \Rightarrow \zeta = 0 \qquad F(0) = F'(0) = 0$$

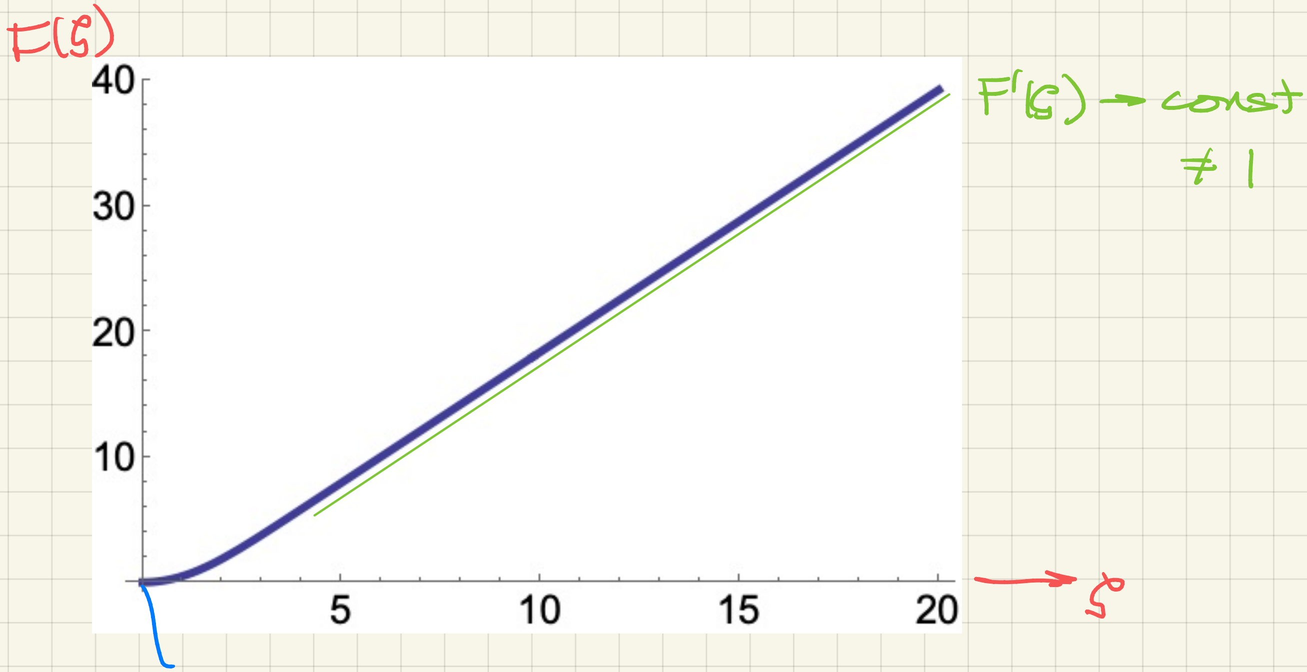
$$\eta \to \infty \Rightarrow \zeta \to \infty \qquad f'(\infty) = 1 \Rightarrow \alpha^2 F'(\infty) = 1$$

$$F'(\infty) = 1/\alpha^2$$

What to do? solve $F''' + \tfrac{1}{2} F F'' = 0$

with $F(0)=0 \quad F'(0)=0 \quad F''(0)=1$

"easy" → march from $\zeta = 0$ to ζ large



$F(0)=0$
$F'(0)=0$
$F''(0)=1$

optional vale

$\rightarrow$ any would work

set α: $\alpha^2 = \dfrac{f'(\infty)}{F'(\infty)}$ — must be 1 Blasius BC

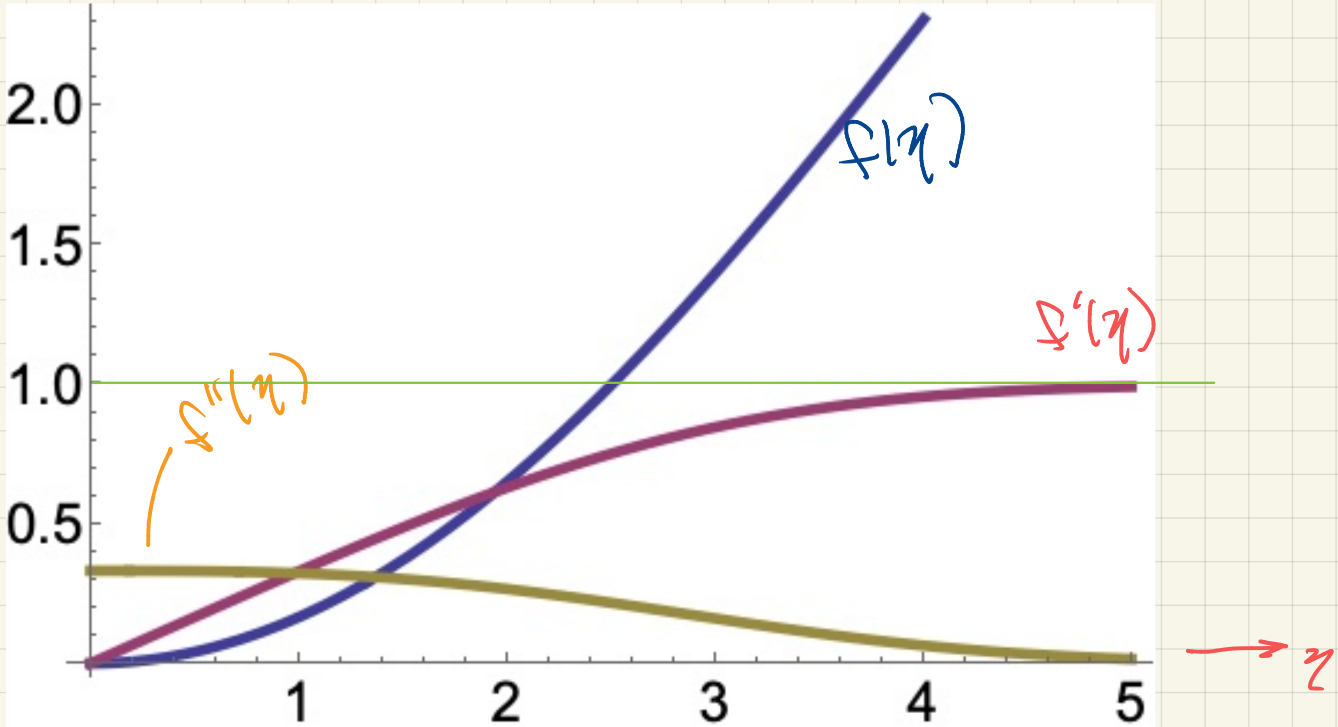
} just solved for...

$$\alpha = \frac{1}{\sqrt{F'(\infty)}}$$

then $f(\eta) = \alpha F(\zeta)$ solves flow

$\alpha = 0.692475$

$\downarrow$ $f''(0) = 0.332057$ $\leftarrow$ would have needed to iterate to this #

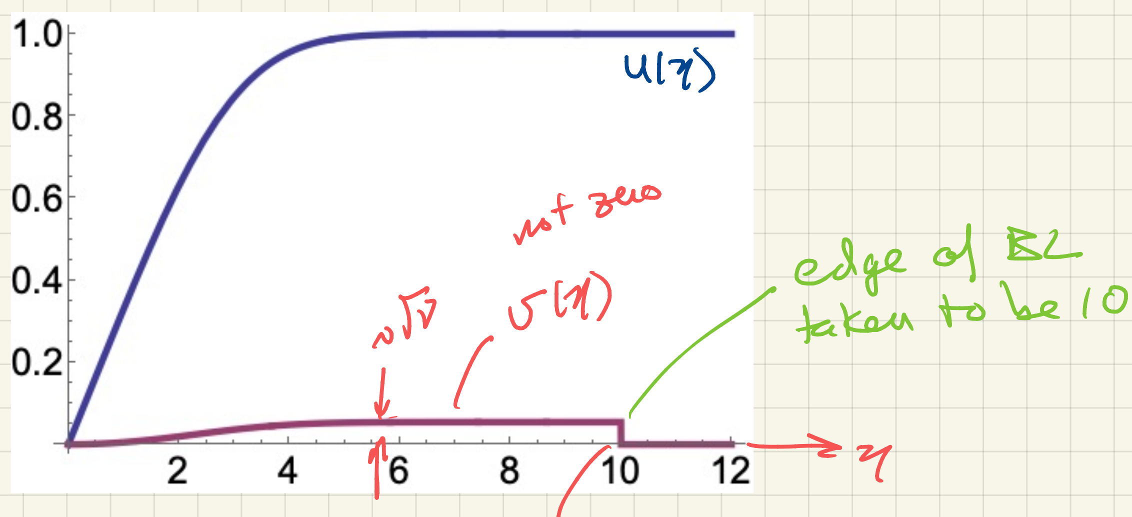


$u = U f'(\eta)$ no ν factor for u

$v = \sqrt{\dfrac{\nu U}{x}}\,(\eta f' - f)$ $\sqrt{\nu}$ for v

$\eta = y\sqrt{\dfrac{U}{\nu x}}$

take $U=1$
$x=10$
$\nu = 10^{-2}$



weird jump

- we assumed BL did not affect OUTER flow

- recall inviscid → was missing no slip condition

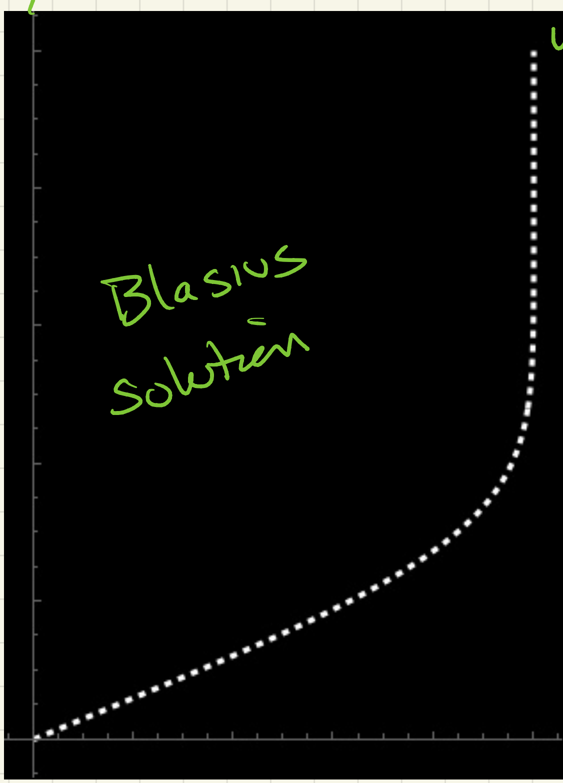
  = BL equation

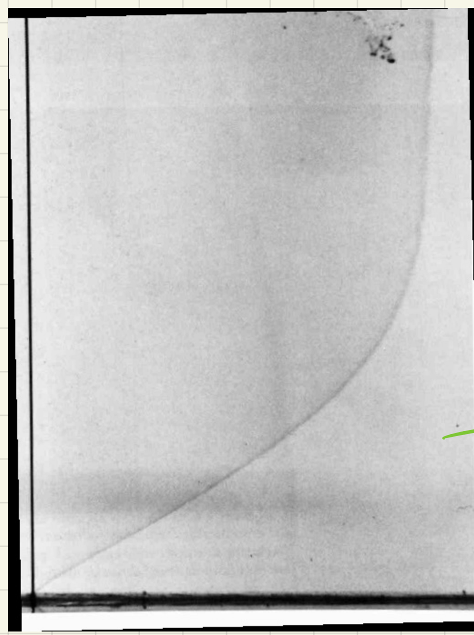
  ↳ 3 B.C.

- now, come to next "missing" BC ... small, but notable

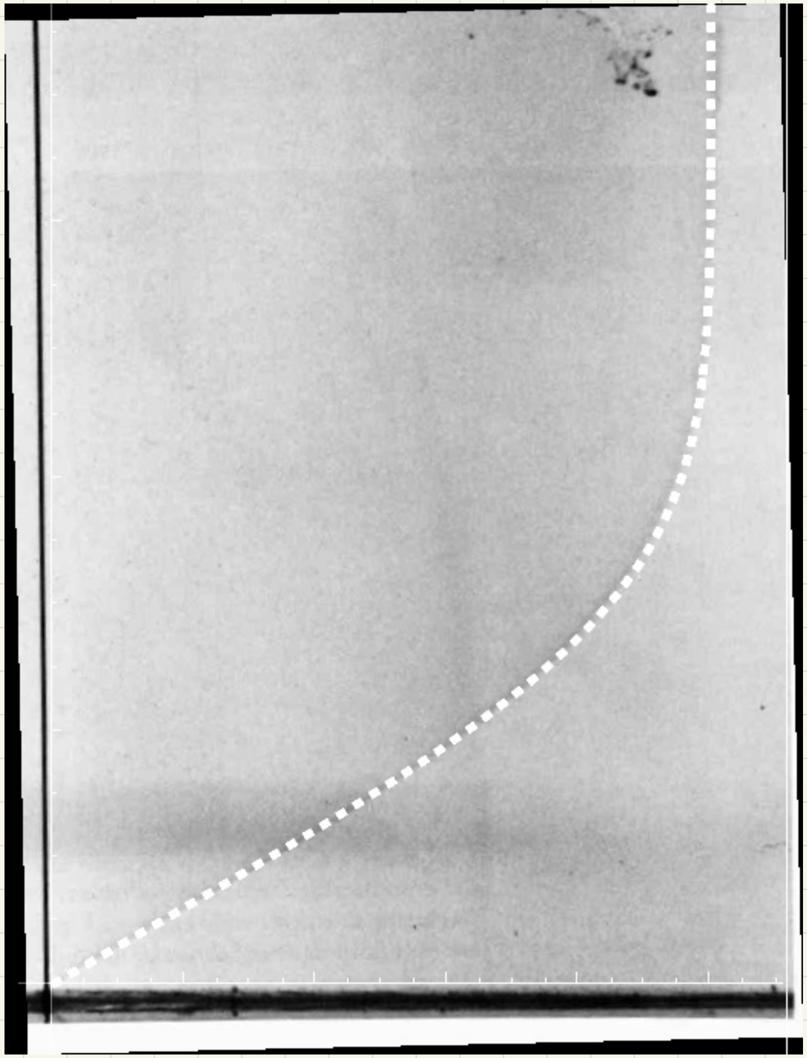
concern? kept $v \frac{\partial u}{\partial y}$ ...

↳ but something "wrong" with v





experiment



<u>Drag</u>

$$\tau_w = \mu \frac{\partial u}{\partial y}\Big|_{wall} \qquad \underline{t} \cdot \underline{\underline{\tau}} \cdot \underline{n}$$

$$= \mu U \frac{df}{dy}\Big|_{wall}$$

$$\vdots$$

$$= \rho \nu U \sqrt{\frac{U}{\nu x}}\, f''(0)$$

decreases downstream

$$= \underbrace{\rho U^2}_{\text{inertia/pressure scale}} \underbrace{\sqrt{\frac{\nu}{Ux}}}_{1/Re_x^{1/2}} \underbrace{f''(0)}_{0.33}$$

net drag on a plate of length ℓ

$$D = \overset{\text{both sides}}{2}\int_0^\ell \rho U^2 \sqrt{\frac{\nu}{Ux}}\, f''(0)\, dx$$

$$= 0.33\, \rho U^2 \ell \sqrt{\frac{\nu}{U\ell}} \quad \sim \ell^{1/2}$$

→ no obvious problem at $x = 0$

$$\delta_{99} = 4.9\sqrt{\frac{\nu x}{U}} = \frac{4.9\, x}{\sqrt{Re_x}}$$