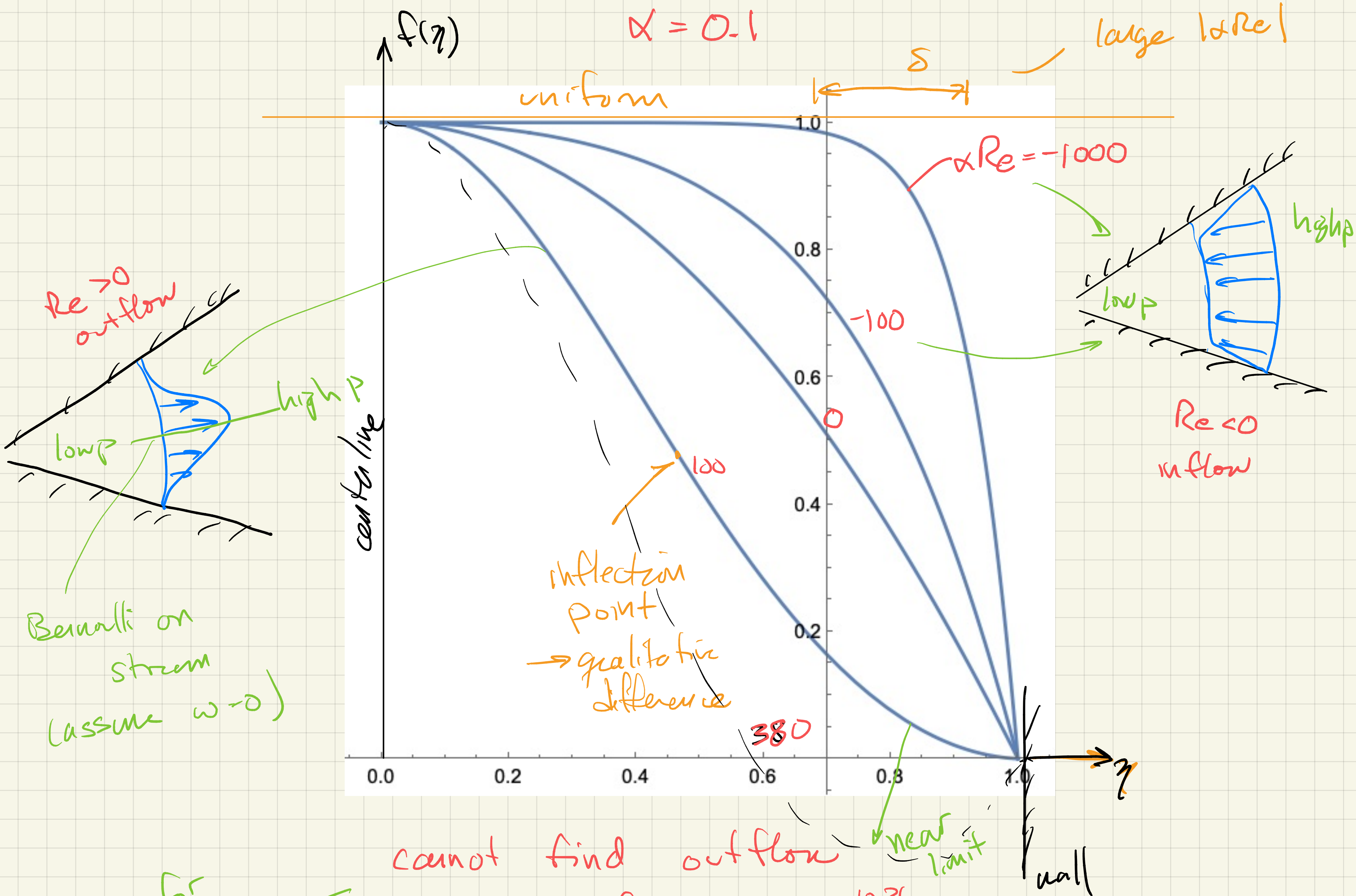
Lecture 06

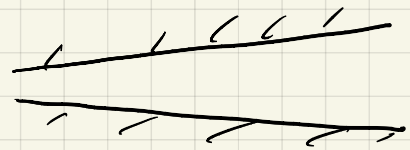
- finish Jeffrey-Hammel
- the jet...



cannot find outflow solutions for $Re > 10.31/\alpha$

"fails" for adverse (decellerating) pressure gradient tractable limits ...

- α small



nearly parallel walls

$|Re|$ small

from before: $f''' + 2\cancel{\alpha}\cancel{Re}\, f f' + 4\cancel{\alpha}^2 f' = 0$

small $\simeq 0$ $\qquad \simeq 0$

$$f''' = 0$$

$$f(\eta) = A\eta^2 + B\eta + C$$

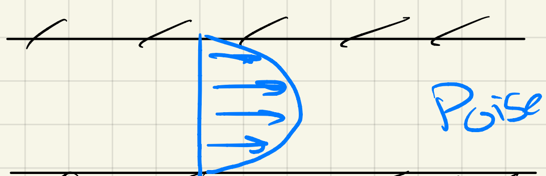
$\dfrac{u_r}{u_{max}} = f(\eta)$

BC's: $f'(0) = 0 \longrightarrow B = 0$

$$f(1) = 0 = A + C$$

$$f(0) = 1 = C$$

$$-1 = A$$



Poiseuille Flow

parabolic

$$f(\eta) = 1 - \eta^2 = \frac{u_r}{u_{max}}$$

- α small (nearly parallel)

  $|Re|$ large (and $Re < 0$)

  $\alpha |Re|$ large $\rightarrow$ matches realite $Re \approx 10^6$

  $\alpha \approx 0.01$ close to parallel

- $$\eta = \int_f^1 \frac{d\phi}{\left[(1-\phi)\left(\frac{2}{3} Re\, \alpha\, (\phi^2+\phi) + 4\cancel{\alpha^2}\phi + C\right)\right]^{1/2}}$$

  small

only one viable C for $\eta \to 1$; $f \to 0$

$f(\eta) = 0$ (no slip)

From Bachelor

$$1 = \int_0^1 \frac{d\phi}{\left[(1-\phi)\left(\frac{2}{3} Re\,\alpha\,(\phi^2+\phi) + C\right)\right]^{1/2}}$$

$$\int_0^1 \frac{1}{r^{1/2}}\, dr$$

$\downarrow$

denominator must dissapear / nearly dissapear to $\int_0^1 \rightarrow 1$

$$\int_0^1 \frac{1}{r}\,dr \qquad \phi = 1 \qquad c = -\frac{2}{3}\mathrm{Re}\,\alpha(1+1) = -\frac{4}{3}\mathrm{Re}\,\alpha$$

$$\log r \Big|_0^1$$

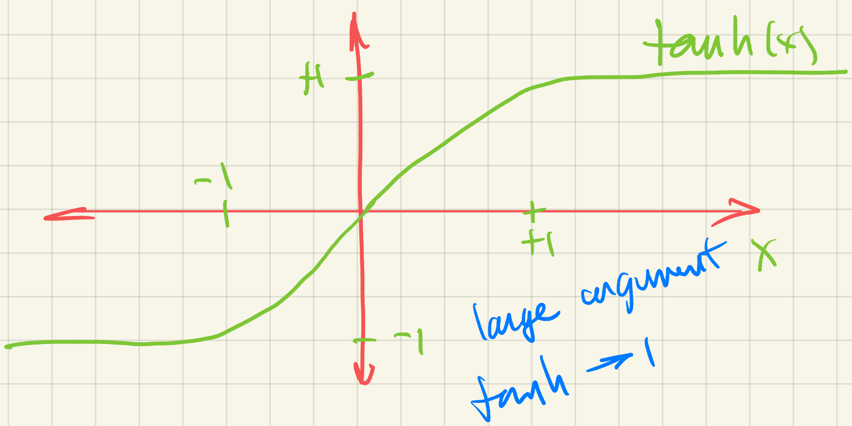
$$\int_0^1 \cancel{\frac{dr}{r^{1/2}(Br+c)^{1/2}}}$$

$$\eta = \frac{1}{\left(-\frac{2}{3}\mathrm{Re}\,\alpha\right)^{1/2}} \int_f^1 \frac{d\phi}{(1-\phi)^{1/2}\underbrace{(-\phi^2-\phi+2)^{1/2}}_{=(1-\phi)(\phi+2)}}$$

$$\int_f^1 \frac{d\phi}{(1-\phi)(\phi+2)^{1/2}}$$

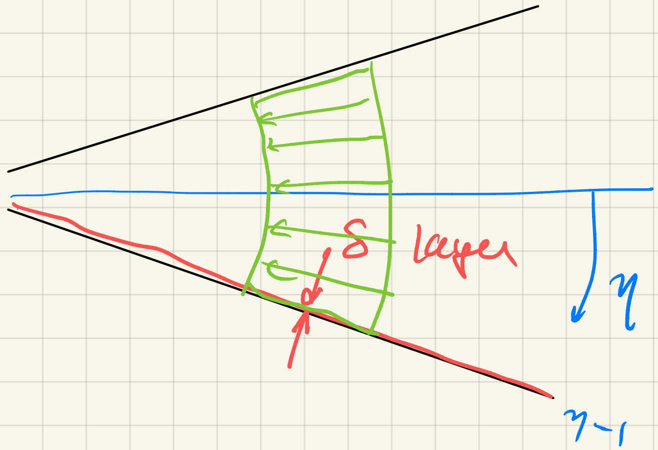
$$\int_0^f \frac{d\phi}{(1-\phi)(\phi+2)^{1/2}} = \frac{2}{\sqrt{3}} \tanh^{-1}\sqrt{\frac{2+\phi}{3}}\,\Bigg|_0^f$$

$$f(\eta) = 3\tanh^2\left[\left(-\tfrac{1}{2}\alpha\,\mathrm{Re}\right)^{1/2}(1-\eta) + \tanh^{-1}\sqrt{2/3}\right] - 2$$



large except when $1-\eta=\delta$ is small

$f(\eta) = 1$ except $1-\eta=\delta$ small



$$\delta = 1-\eta = O\left(\frac{1}{(\alpha Re)^{1/2}}\right)$$

$$Re \approx \frac{1}{\nu}$$

$\delta \sim \sqrt{\nu}$ as in all other cases

Cross out small terms from

α small
$|Re\alpha|$ large

$$\cancel{\underbrace{f'''}_{\sim 1}} + 2\underbrace{Re\alpha}_{\text{large}} f f' + \cancel{\underbrace{4\alpha^2}_{0} f'} = 0$$

$\div Re\alpha$

$$2ff' = 0$$

$$\frac{df^2}{d\eta} = 0 \qquad f^2 = \hat{C}$$

$$f = \sqrt{\hat{C}} \qquad / \qquad \frac{u}{u_{max}} = 1 \ @\ \eta = 0$$

B.C. $f(0) = 1$

→ $f(\eta) = 1$

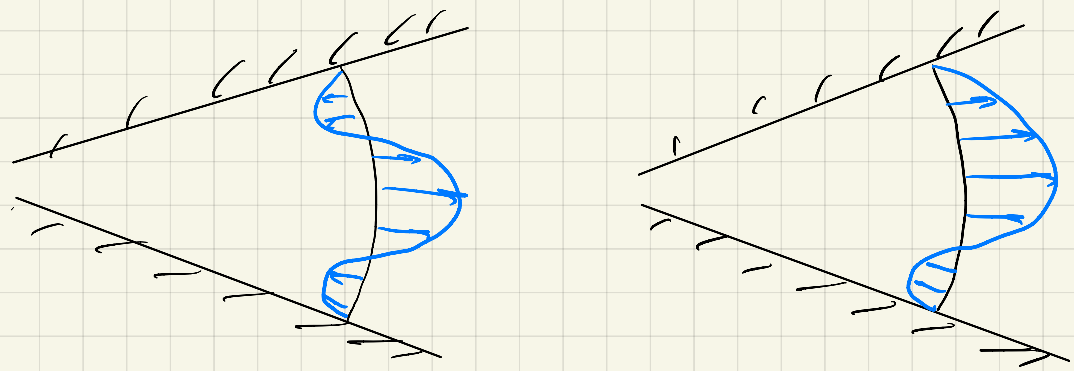
great approximation

$1-\eta$ not small

→ misses boundary layer

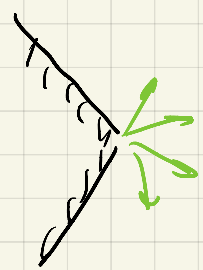
Other results

- any Re, α → there are multiple in/out solutions



- fixed α, all #'s of in/out fail for Re large enough (outflow)

- $\frac{\pi}{2} < \alpha < \pi$



no solutions

- $\alpha < \pi/2$

  - always pure inflow solution
  - always pure outflow $Re < 10.31/\alpha$