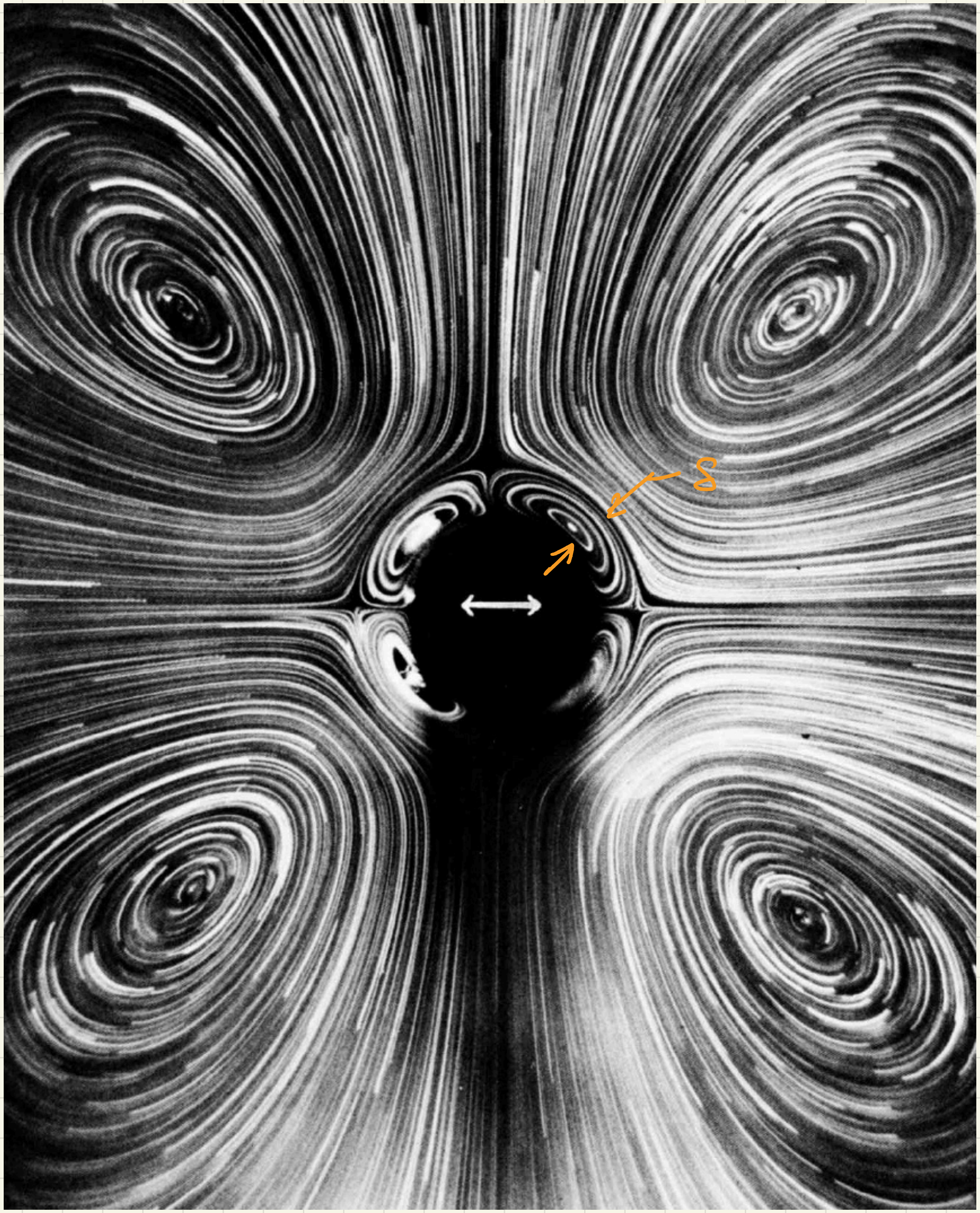
Lecture 26

- Streaming

No office hours today, tomorrow...

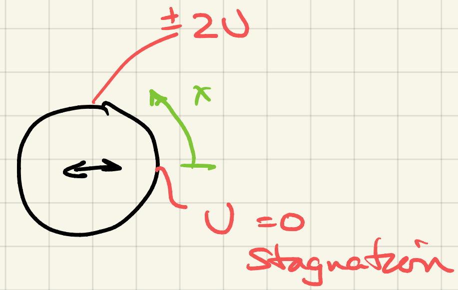
Streaming



Oscillating cylinder

→ net transport of momentum from object outward

Potential flow



outer flow

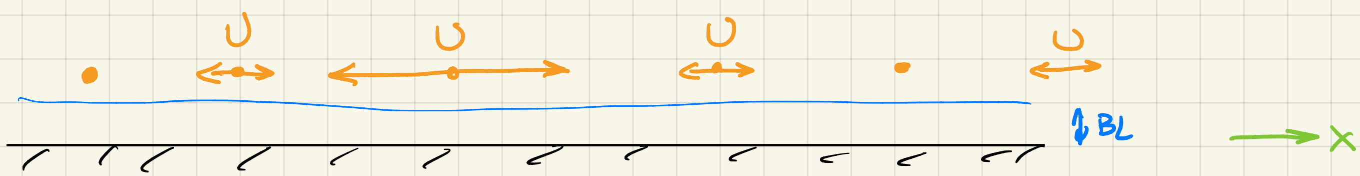
- same phase $\phi(x) = \text{const}$
- different amplitude $A(x)$

$\pm 2U \to 0$

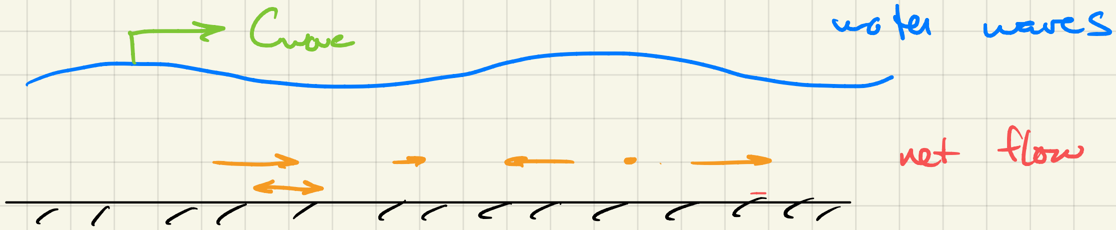
Kundt tube

https://instructional-resources.physics.uiowa.edu/demos/3d3060-kundts-tube

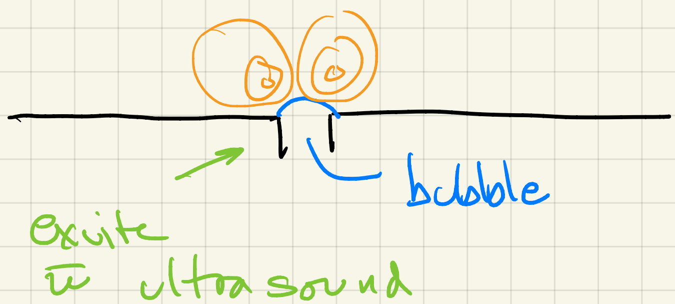
- standing acoustic wave



- same phase $\phi(x) = \text{const}$
- varying amplitude $A(x)$ varies



- vary phase $\phi(x)$ varies
- amplitude const $A(x) =$ cons



https://www.youtube.com/watch?v=M6GvUo5Lmk0

Batchelor + JBL

<u>Streaming</u> → BL in a predominantly oscillating flow can drive a net fluid motion

→ flows are slow... but persistent with a unidirectional component

- assume:
  - high-frequency
  - low amplitude

  } specifics TBD

$$\left|\frac{\partial \underline{u}}{\partial t}\right| \gg \left|\underline{u}\cdot\nabla\underline{u}\right|$$

high frequency $\sim nU$

small – nonlinear part of advection $\sim U^2/L$

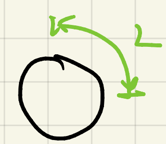
$nU \gg U^2/L$

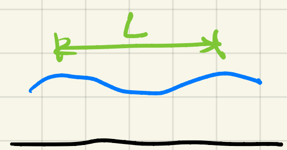
$nL \gg U$

n – frequency (angular)

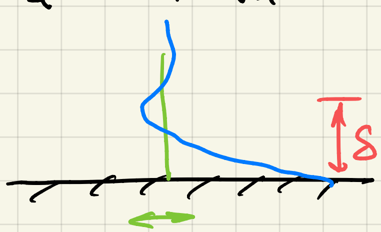
U – amplitude of varying velocity

L – scale over which U varies





expect a thin BL – like Stokes #2



$$\frac{\delta}{(\nu t)^{1/2}} \approx 1$$

$t \sim \frac{2\pi}{n}$ – period of oscillation

$\delta \sim (\nu/n)^{1/2}$

- assume $\delta \ll L$ → high Re#

so $\frac{\delta U}{\nu} \simeq 1$ → a BL

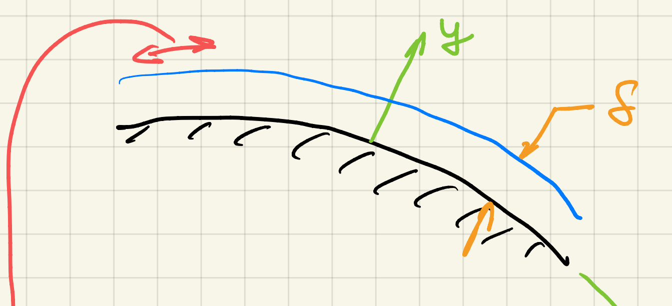
$\frac{LU}{\nu} \gg 1$ → Re#

have $nL \gg U$ or $n \gg U/L$

high frequency

$\frac{nL^2}{\nu} >>> 1$

- consider the oscillating BL under U flow



external osillating flow

edge of BL

$U(x,t) = Re\{\hat{U}(x) e^{int}\}$

will later break into $A(x) e^{i\theta(x)}$

• x-mom BL $\quad \underbrace{\frac{\partial u}{\partial t}}_{\sim n U} + \underbrace{\cancel{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}}}_{\sim U^2/L} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2}$

neglect

from outer flow as usual

$nL >> U$

neglects (same reason)

$$\frac{\partial U}{\partial t} + \cancel{U\frac{\partial U}{\partial x}} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{\partial}{\partial t}\left(\hat{U}(x)e^{int}\right)$$

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial t}\left(\hat{U}e^{int}\right) + \nu\frac{\partial^2 u}{\partial y^2}$$

BC's: $\quad u(y\to 0) = 0 \quad$ (no slip)

$\quad u(y\to\infty) = \hat{U}e^{int} \quad$ (edge of BL)

take $\quad w = u - \hat{U}e^{int} \equiv \hat{w}e^{int} \quad \hat{w}(x,y)$

$$\frac{\partial}{\partial t}\overbrace{\left(u - \hat{U}e^{int}\right)}^{\hat{w}e^{int}} = \nu\frac{\partial^2 u}{\partial y^2}$$

$$in\,\hat{w} = \nu\frac{\partial^2 \hat{w}}{\partial y^2}$$

$$\frac{\partial^2 \hat{w}}{\partial y^2} - \frac{in}{\nu}\hat{w} = 0$$

$$\hat{w}(x,y) = W_+ e^{+\sqrt{\frac{in}{\nu}}\,y} + W_- e^{-\sqrt{\frac{in}{\nu}}\,y}$$

$$= w_+ e^{+(i+1)\sqrt{\frac{n}{2\nu}}\, y} + w_- e^{-(i+1)\sqrt{n/2\nu}\, y}$$

$w_+ = 0$

to decay in y

$$\delta \equiv \sqrt{\frac{2\nu}{n}}$$ define specific δ

$$\hat{w} = w_- e^{-(i+1) y/\delta}$$

$$u(x,y,t) = \hat{U} e^{int} + w_- e^{-(1+i) y/\delta} e^{inT}$$

BC $u(y=0) = 0 \Rightarrow w_- = -\hat{U}$

$$u(x,y,t) = \hat{U}(x)\left(1 - e^{-(1+i) y/\delta}\right) e^{int}$$

- think of as a local BL solution given local $\hat{U}(x)$
  - → slow dependence on x
  - → no persistent advection from upstream

- still just an oscillating flow → no streaming ... get

time average $\overline{u(x,y,t)} = \hat{U} (\quad) \overline{e^{int}}$

$\emptyset$ (zero)

- some effect/mechanism needed to carry momentum from BL out to "driven" outer flow

* Note: if U varies in x (due to A, ϕ), then there is a small v velocity

* that v might carry some x-momentum

* $v = ?$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$v = -\int_0^y \frac{\partial u}{\partial x}\, dy$$

$$= -e^{int} \frac{d\hat{U}}{dx} \int_0^y \left(1 - e^{-(i+1)y'/\delta}\right) dy'$$

$$= -e^{int} \frac{d\hat{U}}{dx} \left[y' + \frac{\delta}{i+1} e^{-(i+1)y'/\delta} \right]_0^y$$

$$v(x,y,t) = -e^{int} \frac{d\hat{U}}{dx} \left(y - \frac{\delta}{i+1} + \frac{\delta}{i+1} e^{-(i+1) y/\delta} \right)$$

- does it transport x-momentum outward

→ net wall normal momentum flux?

averaged over a period of oscillation

$$\int_0^{2\pi/n} u\, v \; dt \neq 0 \;?$$

x-momentum

wall normal advection

- are u, v in / out of phase for $\neq 0$?

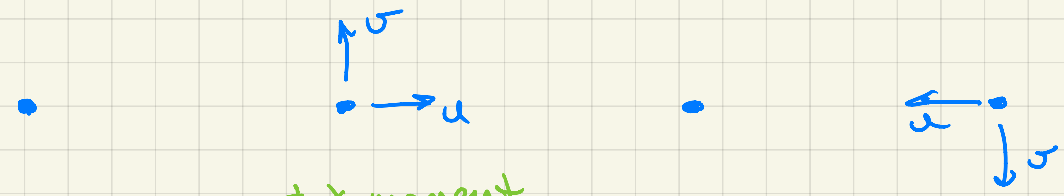
point model, in phase

$$u = \hat{U} \sin nt$$
$$v = \hat{V} \sin nt$$

constants here

period T

$t =$	0	$\frac{1}{4}T$	$\frac{1}{2}T$	$\frac{3}{4}T$	T



+ x moment upward

− x mom downward

net transfer

$$\int_0^T uv \, dt$$

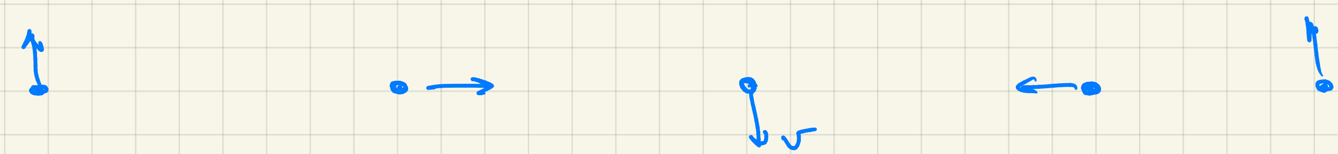
$$\int_0^{\frac{2\pi}{n}} \hat{u}\hat{v} \sin^2 nt \, dt = \frac{\pi}{n} \hat{u}\hat{v} \neq 0$$

out of phase

$u = \hat{u} \sin nt$
$v = \hat{v} \cos nt$

$\pi/2$ out of phase

$t =$	0	$\frac{1}{4}T$	$\frac{1}{2}T$	$\frac{3}{4}T$	T



$\frac{\pi}{2}$ out of phase $\rightarrow$ transport

no net transport

$$\int_0^{\frac{2\pi}{n}} \hat{u}\hat{v} \sin nt \cos nt \, dt = 0$$

- A non $\pi/2$ phasing will come from $O(u^2)$ BL solutions

- Next:

$$\left.\begin{array}{l} u \to u_1 \\ U \to U_1 \\ v \to v_1 \end{array}\right\} O(\varepsilon)$$

$u_1 + u_2$

$v_1 + v_2$

$O(\varepsilon^2)$

$\Rightarrow$ full BL equation