Lecture 02

history

exact NS

last lecture - Lighthill

this lecture - Schlichting

- 1800's — flow equations thought to be known

  Navier (1822) — Stokes (1845)

$$\frac{m \cdot a}{vol} = \frac{\Sigma F}{vol}$$

$$\rho \frac{D\underline{u}}{Dt} = -\nabla p + \mu \nabla^2 \underline{u}$$

$\mu = const$

$\nabla \cdot \underline{u} = 0$

→ only solvable in a few cases

- Analyse to simplify (to solve / approximate)

  non-dimensional form

  L - length scale (be more specific later)

  ρ - density ($\rho = const$)

  μ - dynamic viscosity

Do this?

$$\frac{D\underline{u}}{Dt} = -\nabla p + \frac{1}{Re}\nabla^2 \underline{u}$$

$$Re = \frac{\rho U L}{\mu}$$

Reynolds (1883)

$$p^* = \frac{p}{\rho U^2} \qquad \frac{p}{\mu U / L}$$

inertia scale

→ a choice

why not? → $= -\frac{1}{Re}\nabla p + \frac{1}{Re}\nabla^2 \underline{u}$

note: for $\nabla \cdot \underline{u} = 0$,

p is not thermo. pressure

→ Lagrange multiplier that enforces $\nabla \cdot \underline{u} = 0$

$T\,ds = de + \cancel{p\,dv}$

if "doing" viscous flow $0 = -\nabla p + \nabla^2 \underline{u}$

→ μ scale makes more sense

$\rho_{air} = 1.2$ kg/m^3

$U = 1$ m/s

$L = 1$ m

$\mu = 2 \times 10^{5}$ Pa-s

$\Rightarrow Re \simeq 10^5$

- Idea! neglect $\frac{1}{Re}$ term

still non linear, but very solvable...

- μ needed to create vorticity $\underline{\omega} = \nabla \times \underline{u}$

- if $\underset{\sim}{\omega} = 0$ everywhere

$$\underline{u} = \nabla \phi \quad \rightarrow \quad \nabla \times \underline{u} = \nabla \times \nabla \phi = 0 \quad \text{vector ID}$$

$$\nabla \cdot \underline{u} = 0 \qquad \nabla \cdot \nabla \phi = \nabla^2 \phi$$

relatively easy

how is it now linear?

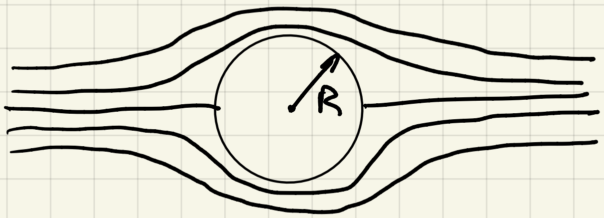
- nonlinearity is in the "post processing" to get p

$$p = p_\infty - \tfrac{1}{2} \rho \left(|\underline{u}|^2 - U_\infty^2 \right) \quad \text{— Bernoulli}$$

does not need to be on a streamline if $\underline{\omega} = 0$

- e.g. flow over a cylinder

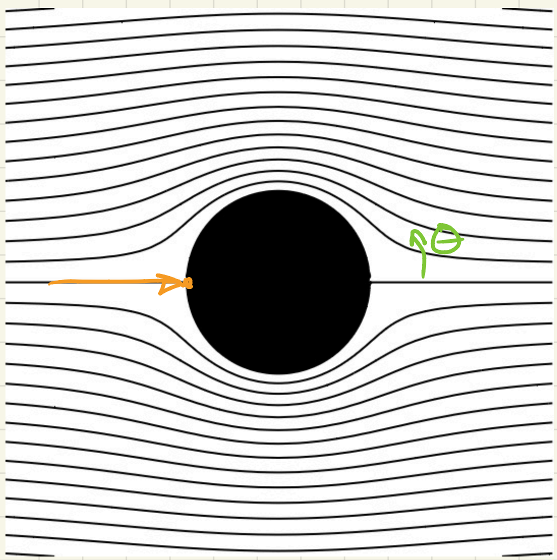
$U \rightarrow$



$$\psi = U\left(r - \frac{R^2}{r}\right)\sin\theta$$

$$v_r = U\left(1 - \frac{R^2}{r^2}\right)\cos\theta$$

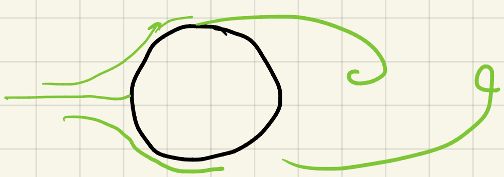
$$v_\theta = -U\left(1 + \frac{R^2}{r^2}\right)\sin\theta$$



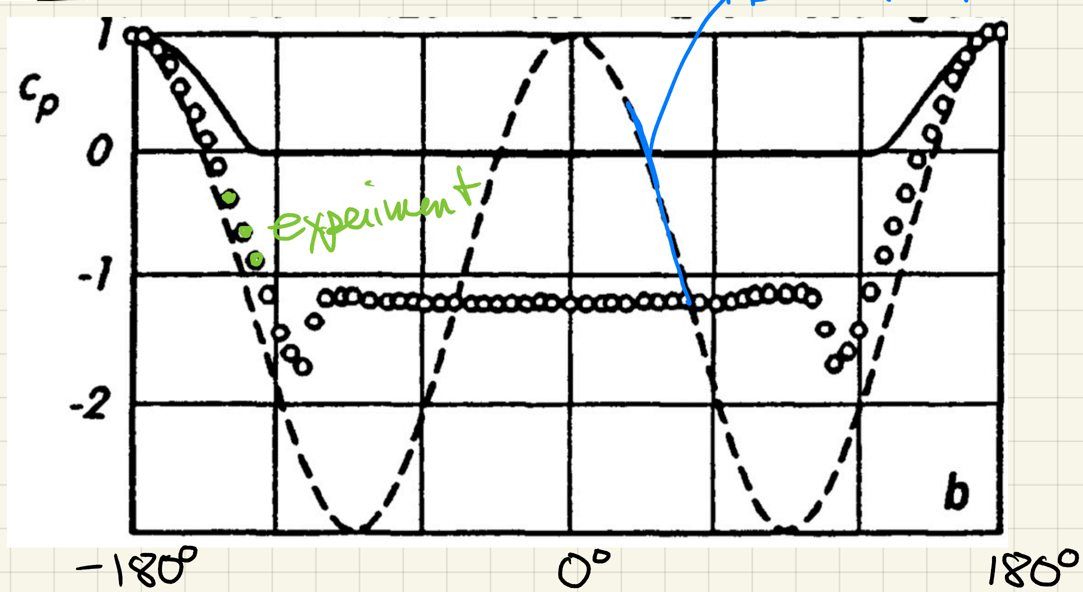
symmetric

$\hookrightarrow C_D = \dfrac{F_D}{\frac{1}{2}\rho U^2 R^2}$ wrong

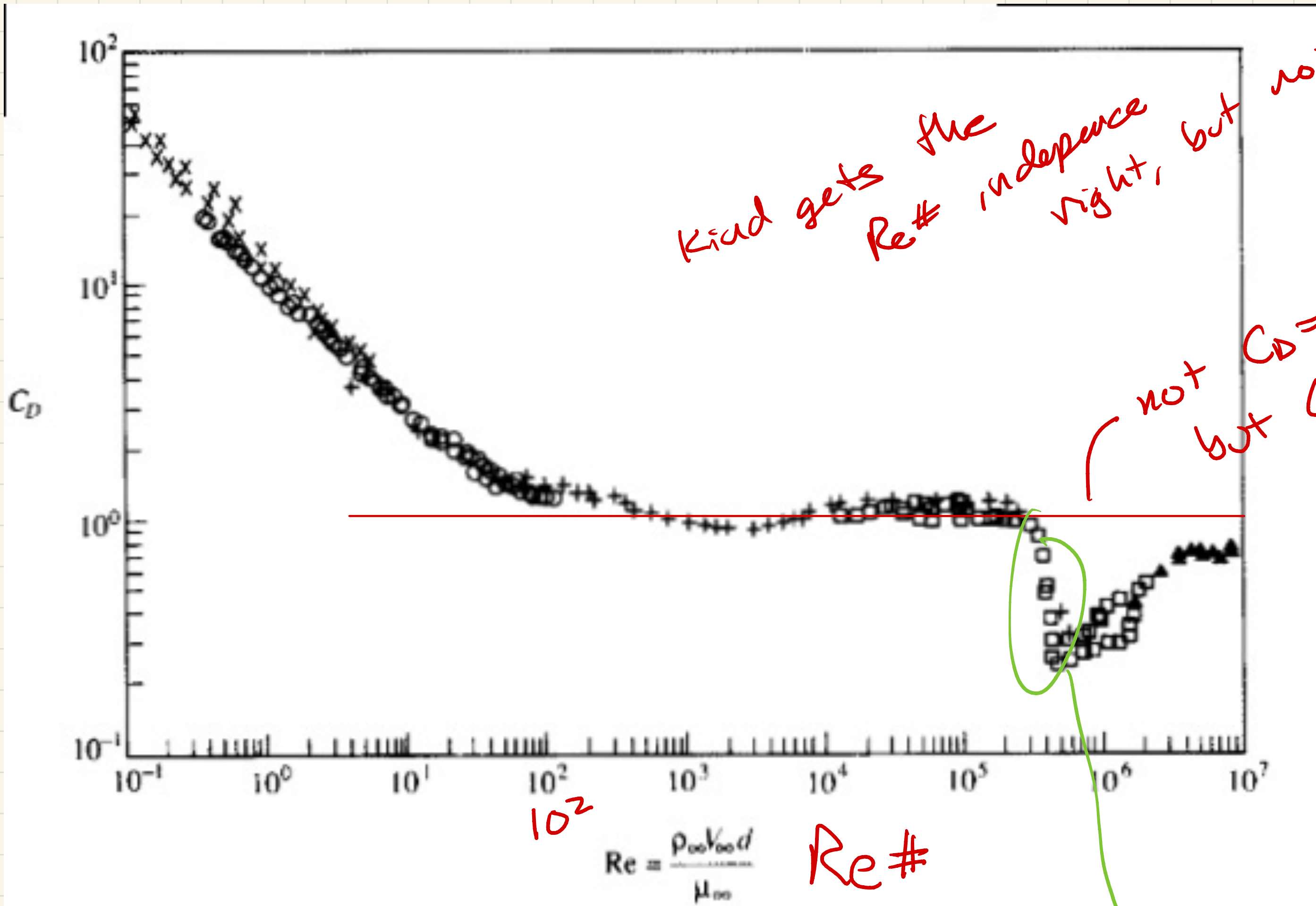
$$C_p = \frac{2(p - p_\infty)}{\rho U^2}$$ pressure coefficient



Bernoulli P for potential solution



Key observations :
- very good near stagnation
- bad elsewhere
- almost const where "bad"



C_D

Kind gets the Re# independence right, but not $C_D = 0$

not $C_D = 0$, but $C_D \approx$ const

10^2

Re#

drag crisis

- Are N-S equations wrong /incomplete?

- 1904 Prandtl   1905 Einsten

  Annus Mirabilis
  - special relativity
  - photo voltaic effect
  - $E = mc^2$
  - Brownian motion

  — recognize 2 interlinked problems

  1. enforcement of $\underline{u} = 0$

     — not of itself a problem

     ↳ C_p around stagnation is "right"

  2. recognizing $\overset{\text{BL theory}}{Re \to \infty}$ not same as $\overset{\text{potential flow}}{\mu = 0}$

     → $Re \to \infty$ a singular limit

- inviscid flow: scalar 2nd order $\nabla^2 \phi = 0$

  or

  vector 1st order $\rho \frac{D\underline{u}}{Dt} = -\nabla p$

→ need 2 b.c.

① flow at ∞

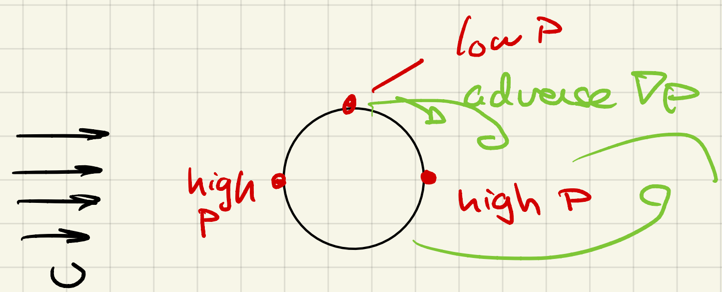
② no penetration $\underset{\sim}{u} \cdot \underline{n} = 0$ on wall

- Prandtl → 3BC model
  - BL equations → fixing singular limit
  - flow adjusts to no slip in a thin layer
    $\underline{u} = 0$ on wall

- Qualitatively different than $\nabla^2 \phi = 0$

  → extra "terms" BL system can fail w/ global impact

- flow separation
- adverse p-gradient



- potential (+ corrections) works great for airfoils w/o separation

- Q: 4^{th} BC?
  - higher order approximations
  - coupling w additional flow complexity

- NOTE: no huge surprise; nonlinear equations can generate their own length scales

BL theory : δ $\quad Re_\delta = \frac{\rho U \delta}{\mu} \simeq 1$

Burger's Eq. $\quad u_t + u u_x = \nu u_{xx}$