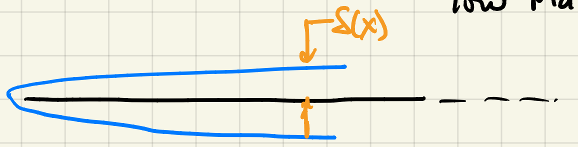
Lecture 25

- BL - shock interaction
- Shock tube BL

Semi-∞ flat plate

low Ma_∞ ~ essentially incompressible

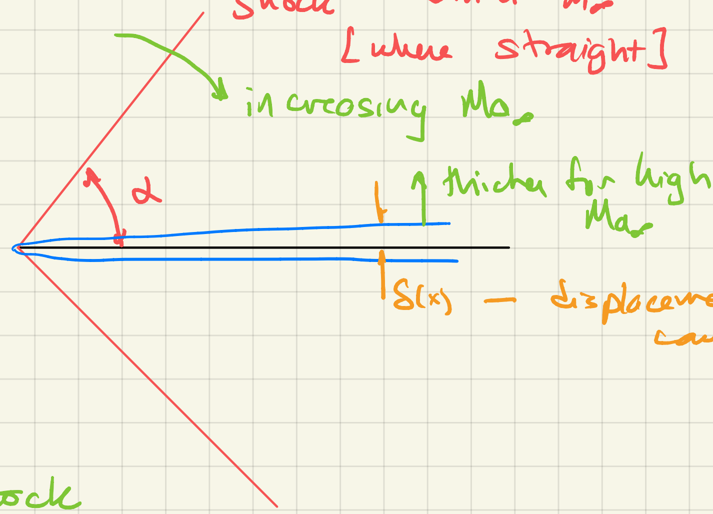


white

BL has small effect on outer flow

→ $\nabla \cdot u$ ⇒ 2^{nd} order → flow over a narrow parabola

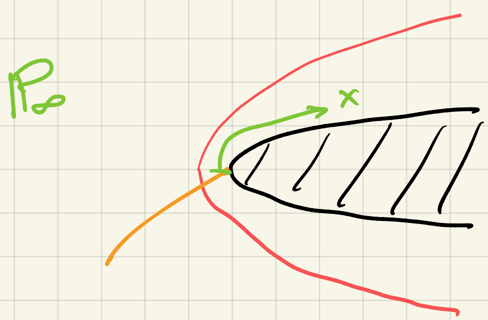
→ "everything" stays close to wall



— displacement that causes shock for $\emptyset$ thickness .... or extra displacement for finite thickness

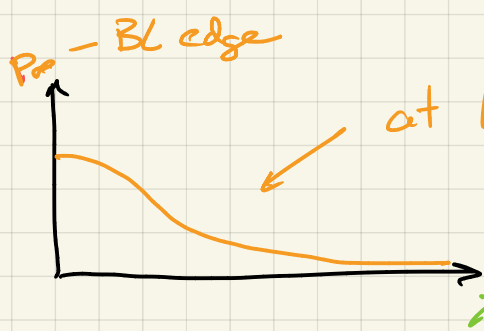
Consider shock - BL interaction

Q: when does the extra δ-displacement matter?



max pressure

- normal shock
- stagnation point



at least qualitatively ... due to shock + shape

semi-empirical formula

$$\frac{P_0}{P_\infty} = 1 + \frac{\gamma}{4}(\gamma+1)K^2 + \gamma K\left\{1+\left[\frac{(\gamma+1)K}{4}\right]\right\}^{1/2}$$

change in displacement $\frac{d\delta}{dx} \simeq \phi$ deflection angle

$\phi \lesssim 20°$

$K = Ma_\infty \phi$

flow deflection by body / body + BL

→ flat plate → just BL*

* for our thinking

$K = 0 \rightarrow P_\infty \simeq P_\infty$

K large $P_\infty > P_\infty$

BL shape → "body" shape → flow deflection ϕ → via $\frac{\partial P_\infty}{\partial x}$ ...

local shock angle/shape → P_∞ (post shock) → BL shape

strong interaction ... or not?

Q: how thick $\delta(x)$? empirical fit $\frac{\delta}{x} \propto Ma^2 \left(\frac{C_w}{Re_x}\right)^{1/2}$

fit of full solutions from last lecture

$Re_x = \frac{\rho U x}{\mu}$ $C_w = \frac{\rho_w \mu_w}{\rho_\infty \mu_\infty}$

$$\delta \propto Ma^2 \left(\frac{C_w}{\frac{\rho_\infty U}{\mu_\infty}}\right)^{1/2} x^{1/2}$$

$$\frac{d\delta}{dx} \propto Ma^2 \left(\frac{C_w}{\sqrt{Re_x}}\right)^{1/2}$$

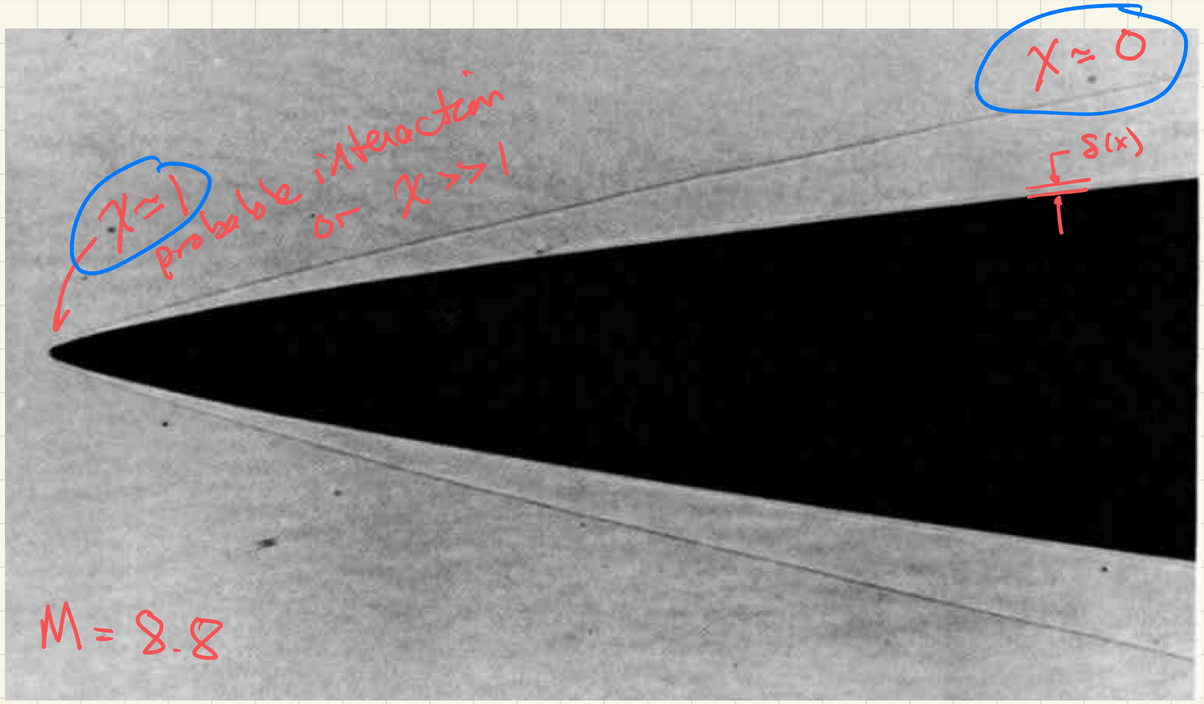
$$K = Ma_\infty \phi \propto \underbrace{Ma_\infty^3 \left(\frac{C_w}{\sqrt{Re_x}}\right)^{1/2}}_{\chi}$$

χ - interaction parameter

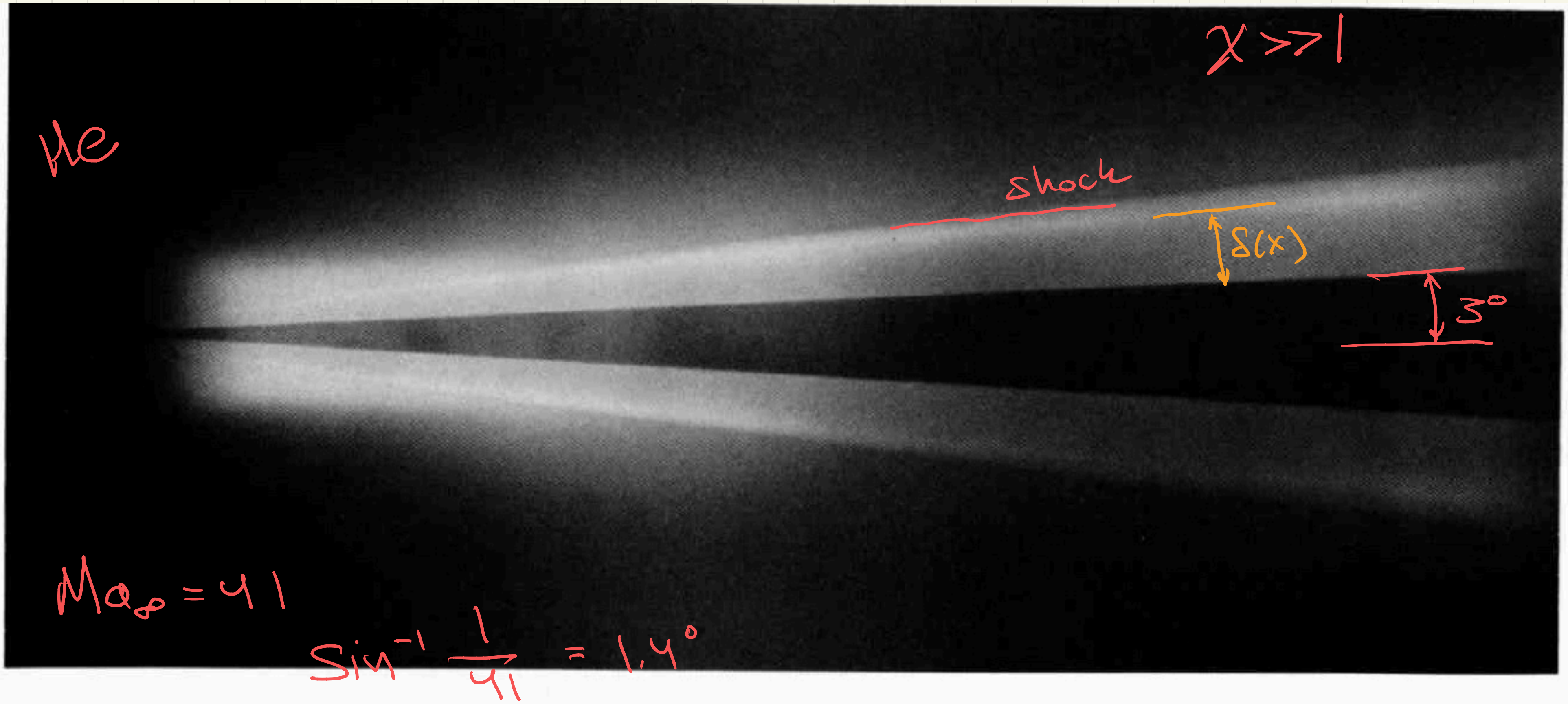
$\chi \ll 1 \rightarrow \frac{p_\infty}{\mathbb{P}_\infty} = 1$ no / little interaction

$\chi \simeq 1 \rightarrow$ flow over body shape + $\delta(x)$

$\chi \gg 1 \rightarrow$ strong interaction $\rightarrow$ BL not a distinct "idea" from gas dynamics

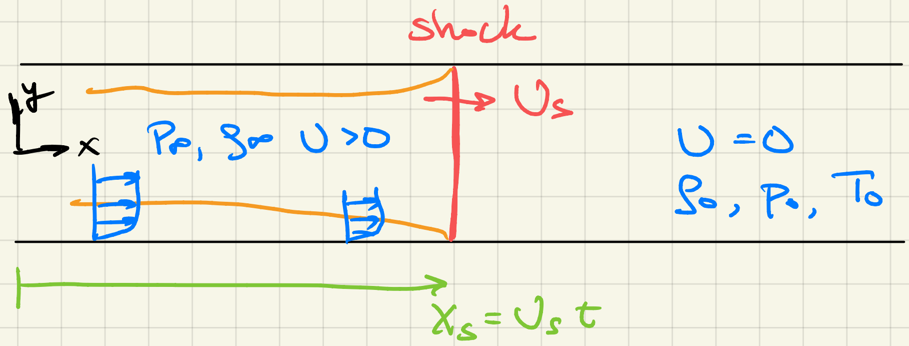

$\chi \simeq 1$
probable interaction
or $\chi >> 1$
$\chi \simeq 0$
$\delta(x)$
$M = 8.8$

van Dyke



274. Hypersonic flow past a slender cone. A cone of 3° semi-vertex angle is shown by the glow-discharge method in helium at Mach number 41 and Reynolds number 560,000 based on length. In this strong-interaction regime the boundary layer is seen to extend about four-fifths of the distance to the shock wave. *Horstman & Kussoy 1968*

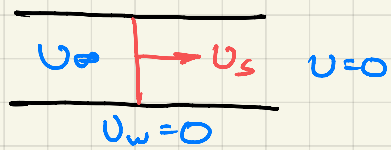
Schlichting



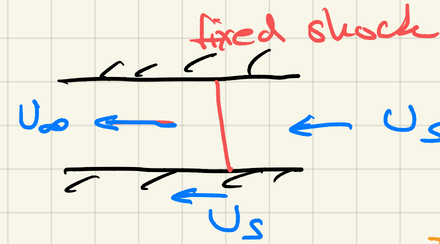
and
t

t dependent

Move to shock's frame



$\Longrightarrow$



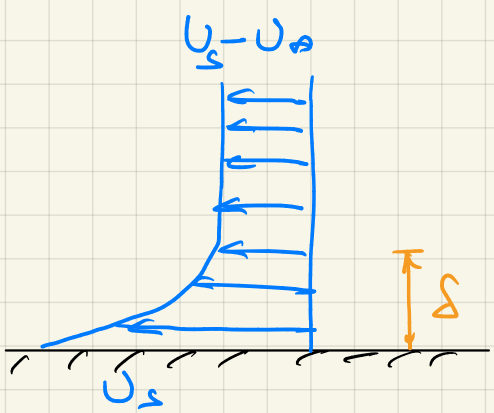
t independent

mass : $\rho_\infty U_\infty = \rho_0 U_s$

$$U_\infty = \frac{\rho_0}{\rho_\infty} U_s$$

$\llcorner$ $\rho_\infty > \rho_0$ (compression)

$\rightarrow U_\infty$ — flow to the left $\rightarrow$ slower than the wall



incompressible $\quad \eta = \dfrac{y}{\sqrt{\nu x}}$

$$Y = \int_0^y \rho/\rho_\infty \, dy$$

now $\quad \eta = \dfrac{Y}{2\sqrt{\nu_\infty (t - x/U_s)}}$

stream function: $\quad \psi = 2\rho_\infty U_\infty \sqrt{\nu_\infty (t - x/U_s)}\; f(\eta)$

$$\cancel{\rho} u = \frac{\partial \psi}{\partial y} = \cancel{2}\,\cancel{\rho_\infty} U_\infty \cancel{\sqrt{\nu (t - x/U_s)}}\, f' \frac{\partial \eta}{\partial y}$$

$$\frac{\partial \eta}{\partial y} = \frac{1}{\cancel{2}\cancel{\sqrt{\nu (t - x/U_s)}}} \underbrace{\frac{\partial}{\partial y} \int_0^y \frac{\rho}{\rho_\infty} dy}_{\cancel{\rho/\rho_\infty}}$$

$$u = U_\infty f'$$

ansatz: $$\frac{T}{T_\infty} = 1 + \frac{\gamma - 1}{2} Ma_\infty^2\, r(\eta) + \frac{T_w - T_{ad}}{T_\infty} s(\eta)$$

$\vdots$

$$T_{ad} = T_\infty \left[1 + \frac{\gamma - 1}{2} Ma_\infty^2\, r_w \right]$$

$$f''' + 2\left(\eta - \frac{U_\infty}{U_s} f\right) f'' = 0$$

$$\frac{1}{Pr} r'' + 2\left(\eta - \frac{U_\infty}{U_s} f\right) r' = -2 f''^2$$

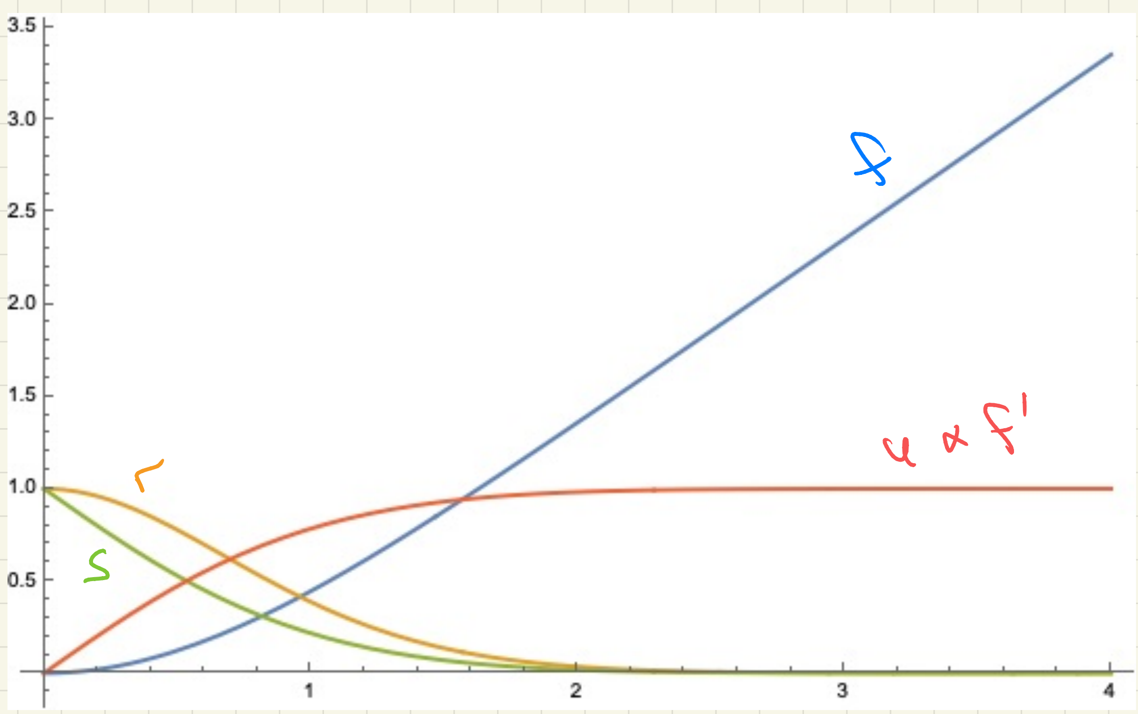
recovery factor

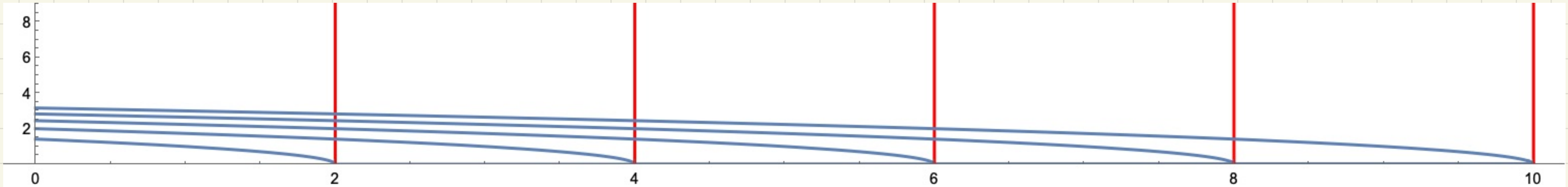
($r_w \to 1$, isentropic)

$$\frac{1}{Pr} s'' + 2\left(\eta - \frac{U_\infty}{U_s} f\right) s' = 0$$

$\eta = 0$: $f = 0$ $f' = 0$ $r' = 0$ $s = 1$

$\eta \to \infty$: $f' = 1$ $r = 0$ $s = 0$





STOP: Meeting #2 material