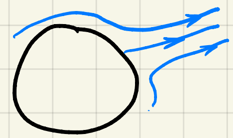
Lecture 18

Separation

BL Separation

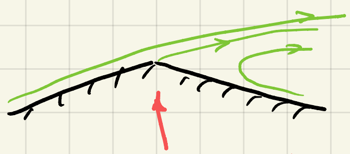
e.g. stage III of the "impulsively started object"



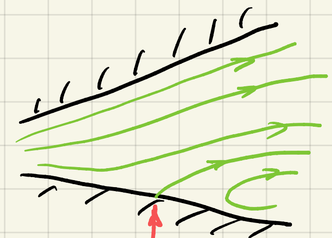
not obvious where it occurs in many cases

- in time, ω diffuses enough that it can be advected away from object

→ break down of potential flow approx. away from body : $\underline{\omega} \neq 0$



can anticipate point of separation – salient point of separation



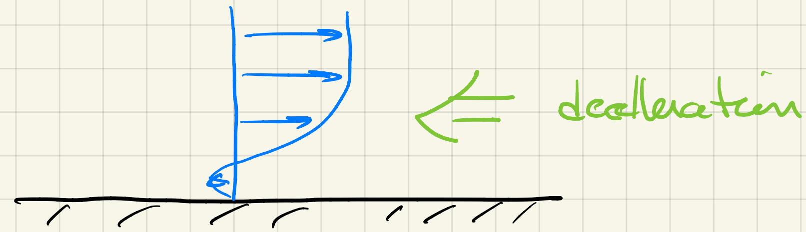
hard to anticipate exactly where, linked to other side

- a halmark of separation in J-H flow (outflow) → seems related to solution "failure"
- a halmark in BL equations in F-S flow $m<0$ → $(m \lesssim -0.1)$ seems to fail to find 1-way flow solutions

⇒ key feature: adverse pressure gradient

↳ decelerating free stream

→ increasing p downstream



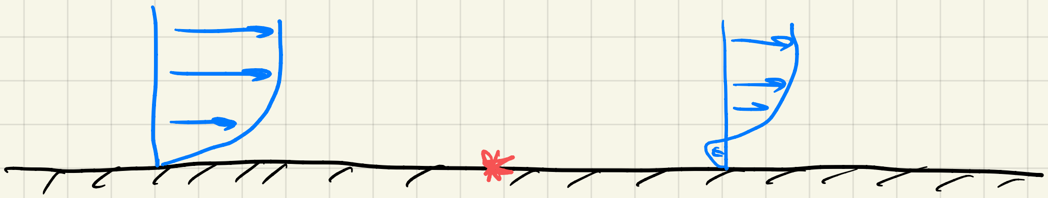
Why? flow closer to wall is slower, has less inertia, easier to reverse

so p same thru BL

NOTES

- reverse flow is supported by BL equations, so long as upstream / downstream BC are set consistently

- transition from uniformly forward to reverse has flow



pt. of reversal in BL limit
is singular $\rightarrow$ BL eq fail
$\rightarrow \frac{\partial^2 u}{\partial x^2}$ not small, e.g.,

$\rightarrow$ can't extend solution across

- observation: modest decelleration is tolerated
(e.g. F-S $m = -0.05$)

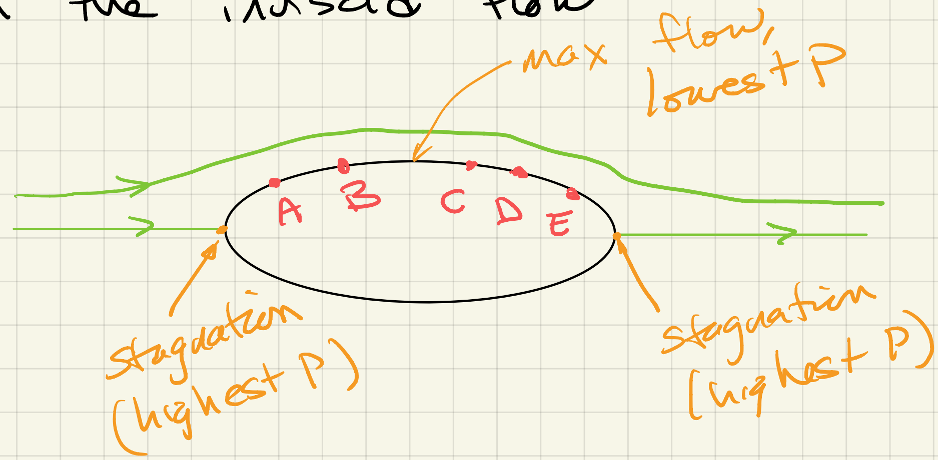
$\rightarrow$ no reversal, unidirectional solutions found

$\rightarrow$ diffusion must keep the near-wall
flow going forward

Appeal to <u>BIG IDEA</u> : advection – diffusion of ω

reference Lighthill

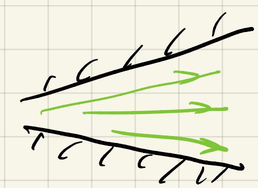
Consider the inviscid flow



<u>$A \to B$</u>

- accelerating
- decreasing p
-  like this
- BL theory w/ inviscid OUTER flow "works"

<u>$C \to D \to E$</u>

- decellerating
- increasing p
- like 
- FS, JH "fail"

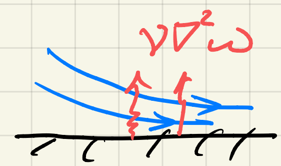
Recall ω facts

- net ω in BL $\propto$ U outside BL $\qquad \Omega = \int_0^\infty \omega \, dy$
- ω generated only at wall : 2D $\quad \dfrac{D\omega}{Dt} = \nu \nabla^2 \omega$

  $\rightarrow$ conservative

  $\rightarrow$ $\Delta\Omega$ must come from wall

- ω diffuses, primarily $\perp$ streamlines



- ω advects, primarily $\parallel$ streamlines

Side discussion

C

$$\Omega \equiv \int_V \omega \, d\underline{x}$$

V = volume

$$\frac{\partial \omega}{\partial t} = -\nabla \cdot \underset{\sim}{F}$$

$$\frac{\partial \Omega}{\partial t} = -\int_V \nabla \cdot \underset{\sim}{F} \, d\underline{x}$$

$$= -\int_S \underline{n} \cdot \underline{F} \, dx$$

S = surface $\leftarrow$ must be wall

A

$$\frac{D\omega}{Dt} = \nu \nabla^2 \omega$$

$$\nabla \cdot \underline{u} = 0 \qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$\times\omega \qquad \omega \dfrac{\partial u}{\partial x} + \omega \dfrac{\partial v}{\partial y} = 0$

$+$

$$\frac{\partial \omega}{\partial t} + u\frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} = \nabla \cdot (\nu \nabla \omega)$$

$$u\frac{\partial \omega}{\partial x} + \omega \frac{\partial u}{\partial x} + v\frac{\partial \omega}{\partial y} + \omega\frac{\partial v}{\partial y}$$

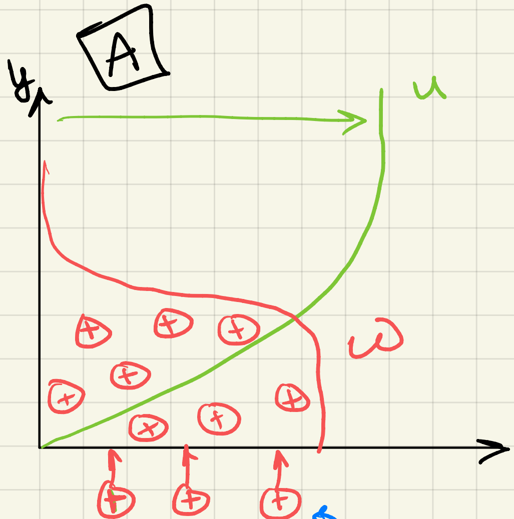
$$\frac{\partial u\omega}{\partial x} + \frac{\partial v\omega}{\partial y}$$

B

$$\rightarrow \frac{\partial \omega}{\partial t} + \nabla \cdot \underbrace{\left[\underline{u}\omega - \nu \nabla \omega\right]}_{\underset{\sim}{F}} = 0$$

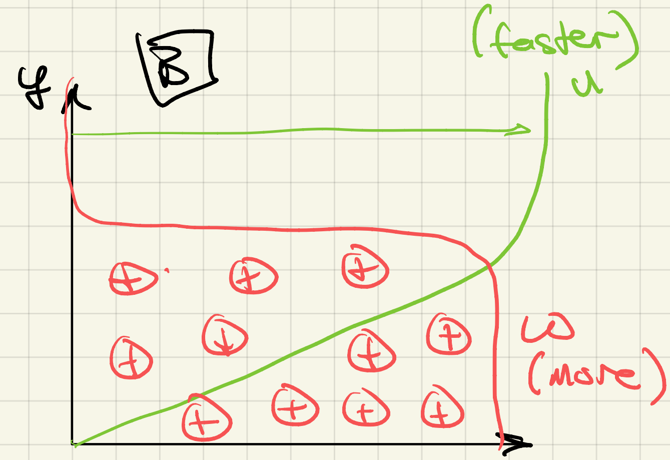
conclusion

A → B



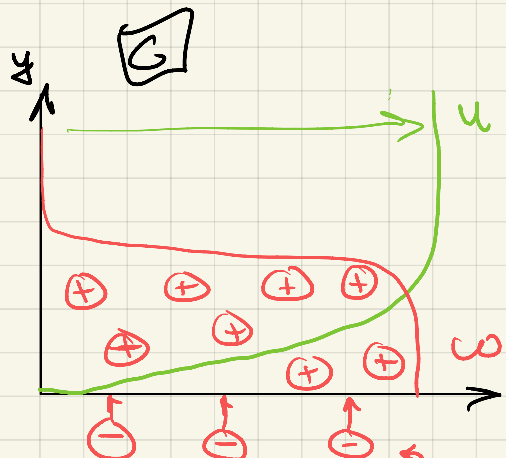
⊕ corotational ω

ACCELERATING ⟹



new ω from wall
— added to ω advected downstream

C → D → E



DECELERATING ⟹

opposite sign generated at wall

→ counters ω ⊕ advecting from upstream

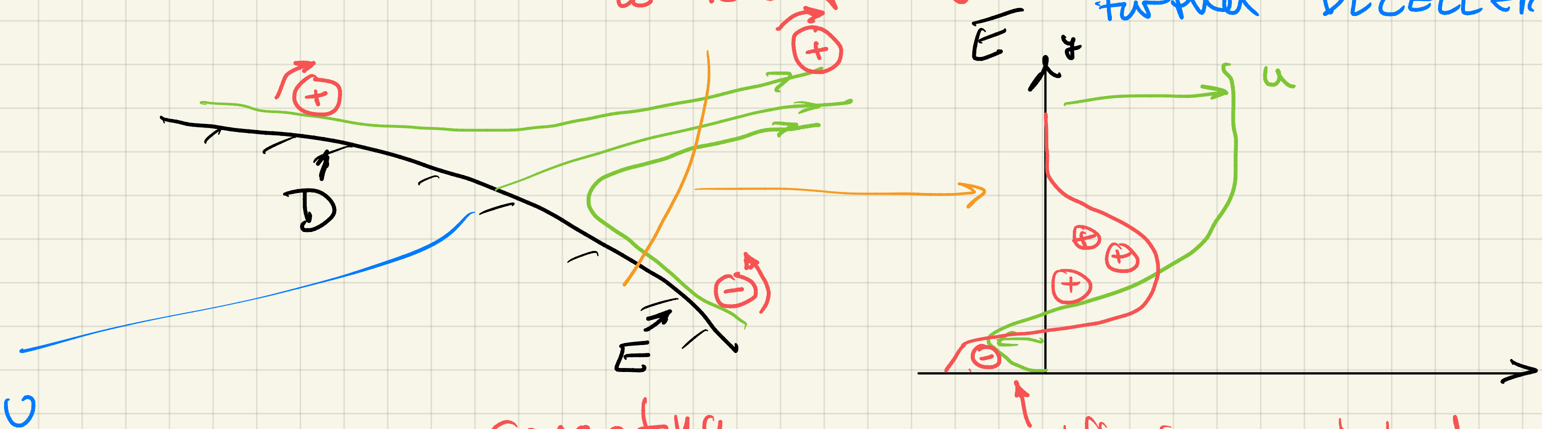
less near wall because negated by these new ⊖

all ⊕ → know that diffusion $\nu \nabla^2 \omega$ must be mixing ⊖ into ⊕

⇓

ω leaving body

further DECELLERATION



separating

diffusion overwhelmed by new ⊖ from wall

$\Omega = -U$

only 100%

for where BL works

← neglect $\frac{\partial v}{\partial x}$