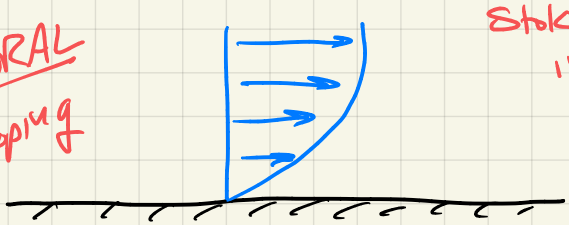
Lecture 11

- Temporal v Spatial
- Falkner-Skan
- Displacement

Blasius v. Stokes 1st

TEMPORAL

t-developing



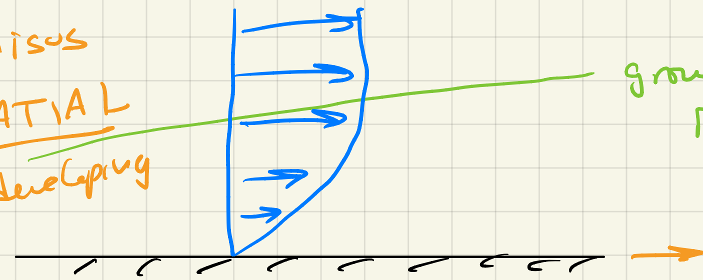
Stokes 1st in frame of plate

∞ plate → stops @ $t=0$

Blaisus

SPATIAL

x-developing



∞ plate, leading edge at $x=0$

$$\tau_w = \mu \frac{\partial u}{\partial y}\Big|_0 = \underbrace{\frac{1}{\pi^{1/2}}}_{0.56} \rho U^2 \left(\frac{\nu}{U^2 t}\right)^{1/2}$$

$$\tau_w = \mu \frac{du}{dy}\Big|_0 = \underbrace{0.33}_{f''(0)} \rho U^2 \left(\frac{\nu}{Ux}\right)^{1/2}$$

if $x = Ut$ → $U^2 t$

Stokes 1st nearly 2x higher

No ∇p

why?

$$\underbrace{\frac{\partial u}{\partial t}}_{\text{TEMPORAL}} + \underbrace{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial x}}_{\text{SPATIAL}} = \nu \frac{\partial^2 u}{\partial y^2}$$

we took $x = Ut$ → $\frac{\partial u}{\partial t} = U \frac{\partial u}{\partial x}$ not $u \frac{\partial u}{\partial x}$

temporal too high in part because $U > u$

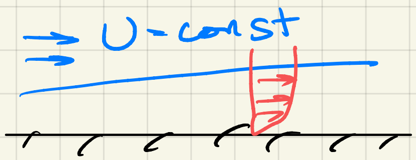
AND $v \neq 0$ for SPATIAL — Blasius has wall $\perp$ velocity $v \sim \sqrt{\nu} \sim \sqrt{1/Re}$

$v = 0$ for TEMPORAL

SMALL but only need to act on δ-scale

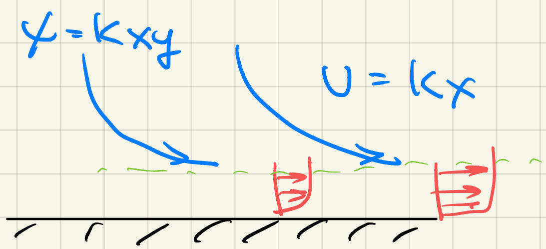
Free Stream "Options"

Blasius



$\delta \sim \sqrt{x}$ $\qquad \tau_w \sim 1/\sqrt{x}$

Stagn. Pt

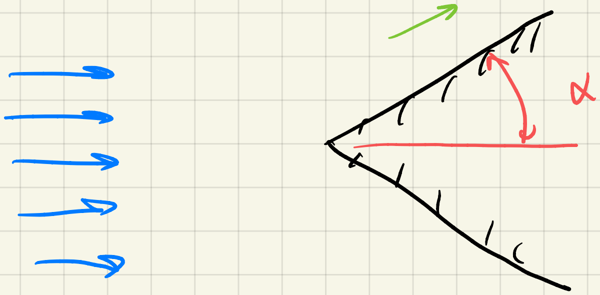


$\delta \sim 1$ (const) $\qquad \tau_w \sim x$

Both part of $U = Cx^m$ family

Faulkner-Skan wedge Flows

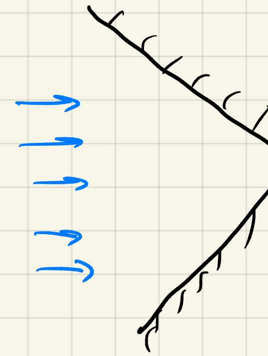
accelerating flow



$\alpha = \frac{\pi m}{m+1}$

$m = 0$ – Blasius

$m = 1$ – Stag Pt



$m > 1$ no proble

Jeffry - Hammel Like, but allows v - wall $\perp$ velocity

$m < 0$ – no problem mathematically

$$\psi = (\nu U x)^{1/2} f(\eta)$$

$$\eta = \left(\frac{U}{\nu x}\right)^{1/2} y \qquad u = U f'(\eta)$$

$\vdots$

$m = 0$ → Blasius

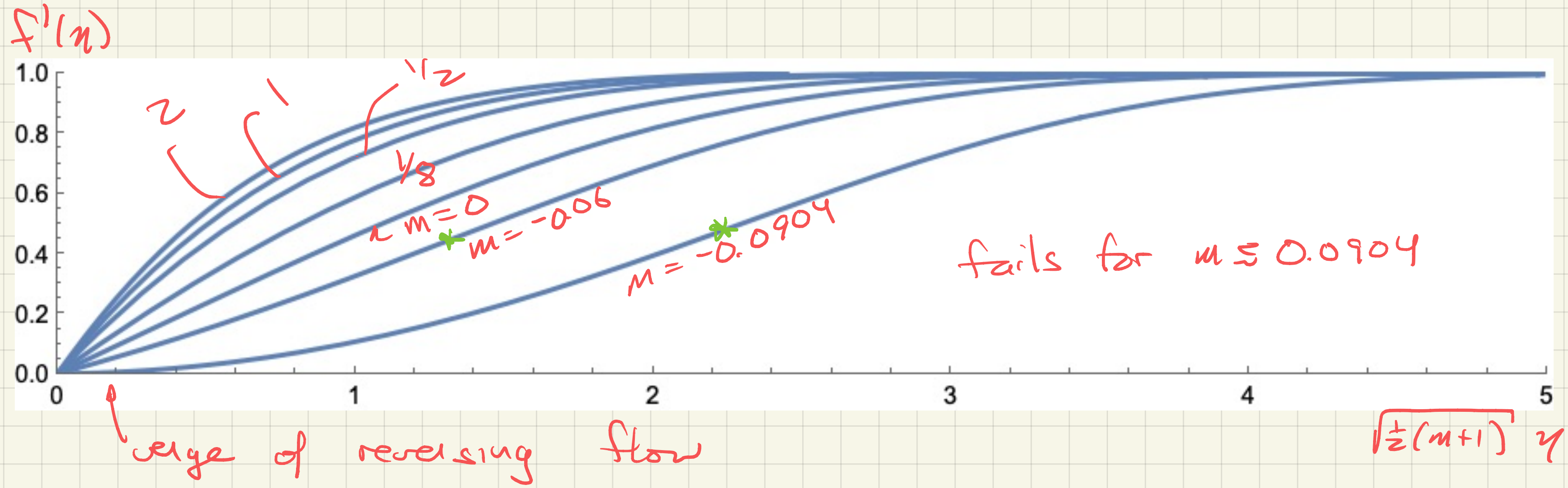
$$f''' + m = m f'^2 - \tfrac{1}{2}(m+1) f f''$$

BC $\quad f(0) = f'(0) = 0 \quad$ no slip $u, v = 0$ @ $y = 0$

$f'(\eta \to \infty) = 1$ – free stream

$y/\delta \to \infty$ matches $U = Cx^m$

Numerical Solutions



→ only a weak reversed flow leads to big changes

- $m < 0$ – has inflection pt

→ inflection pts ⇒ a strong inviscid instability

$$\underset{0}{u}\frac{\partial u}{\partial x} + \underset{0}{v}\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$$

$y=0$

$y = 0 \qquad u = v = 0$

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = \underbrace{\nu \frac{\partial^2 u}{\partial y^2}}$$

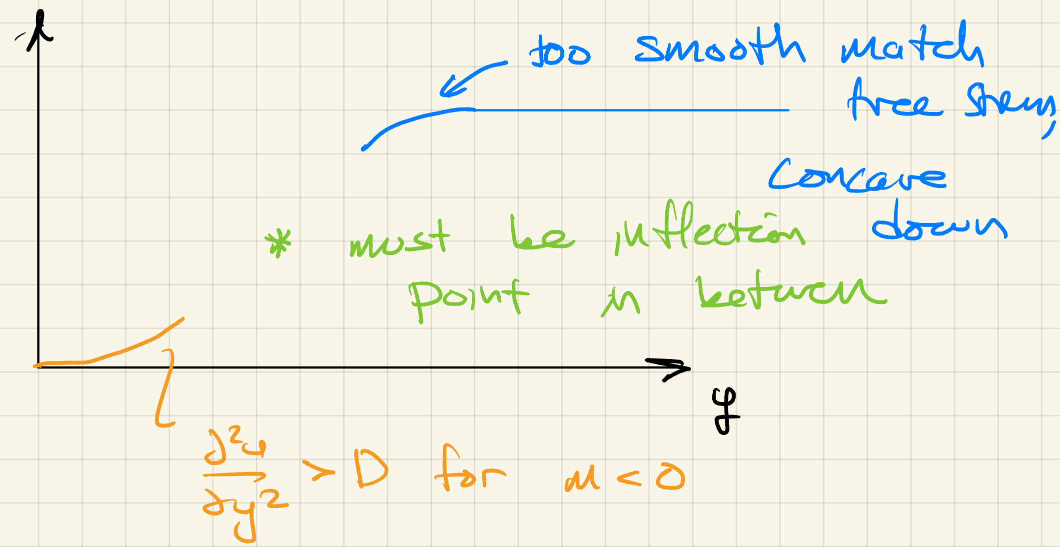
$$\frac{1}{\rho}\frac{\partial p}{\partial x} = -U\frac{dU}{dx} = -C^2 x^m m x^{m-1} = -C m x^{2m-1} = \nu \frac{\partial^2 u}{\partial y^2}$$

changes sign w/ m $\qquad$ concavity

→ edge of BL → inviscid

→ Bernoulli

not $\frac{\partial^2 u}{\partial y^2} = 0$ blasius



* must be inflection point in between

• growth rate?

$$\delta_1 = \int_0^\infty (1 - u/U)\,dy$$

Displacement thickness

$$\delta_1 = \int_0^\infty \left(1 - f'(\eta)\right) \underbrace{\frac{dy}{d\eta}}\, d\eta$$

$$= \left(\frac{x\nu}{U}\right)^{1/2} \underbrace{\int_0^\infty (1 - f'(\eta))\, d\eta}_{\text{some \# for any } m}$$

$U = Cx^m$

$$\sim x^{1/2} \, x^{-m/2}$$

$$\sim x^{\frac{1}{2}(1-m)}$$

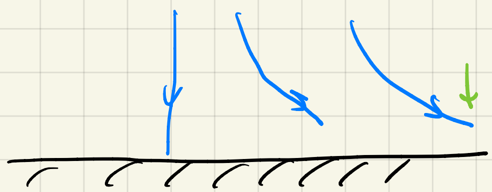
$m = 1 \rightarrow \delta_1 = \text{const}$ (stag. pt.)

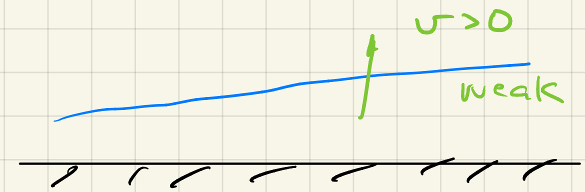
$m > 1 \rightarrow$ thins

$m < 1 \rightarrow$ thickens

$\delta_1 \sim x^{1/2}$ Blasius

• growth (thicken / thinning) related to wallward advection





at BL edge OUTER perspective

$U = Cx^m$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0$$ OUTER cons. mass

$\uparrow$

Cmx^{m-1}

$$\frac{\partial V}{\partial y} = -Cmx^{m-1}$$

change in x $\quad V \propto -mCx^{m-1}$

$$V = -mCx^{m-1} L_y$$

by integrating
some scale

sign

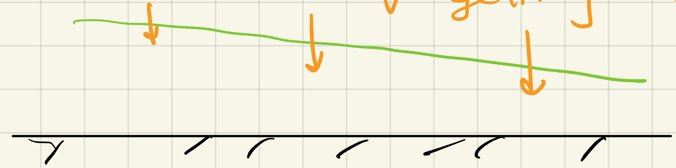
• if $m > 0$

$V < 0$ toward wall —

working to thin BL

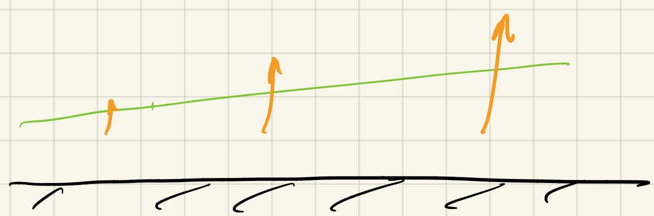
→ advecting vorticity toward wall

- if $m > 1$, wall ward advection increasing in strength downstream



→ a reason why BL thins for $m>1$

- if $m < 0$, $V > 0$ advection away from wall



BL fails when this advection overwhelms diffusion