

Econ 514 Assignment 2

- (1) Four Router Problem (Extensive Game of Imperfect information) In this game, there are four players, C_1 , C_2 , R_1 and R_2 . The first two players are computers who have a single packet to send through a computer network. The last two are routers who observe the set of packets in their queue then send one of them to another router, either R_3 or R_4 .

R_3 is not a player, it just sends one of the packets, if any, it receives on to the internet. The other router R_4 selects one of the packets it receives and tries to send it on to the internet, except that this only succeeds with probability $0 \leq \beta \leq 1$.

R_1 and R_2 simultaneously choose which of the routers R_3 and R_4 to send their packet to (if they have one in their queue). R_1 always sends its packet successfully, but like R_4 , R_2 only succeeds with probability β .

The two computers C_1 and C_2 know all this, and they simultaneously choose whether they send their packet (which they always have) to either R_1 or R_2 . Routers can only see how many packets they have in their queue - if they only have 1 packet, they don't know or care which computer sent it.

Each computer gets a payoff of 1 if their packet makes it through the four routers to the internet. Each router gets a payoff of 1 if its packet gets through one of the routers R_3 or R_4 . Otherwise payoffs are zero.

Suppose that C_1 and C_2 use the same strategy in which π is the probability with which they send their packet to R_1 . The choices of C_1 and C_2 determine the queue, either 0, 1, or 2, at each of the routers. Depending on the outcome of the computers' randomization, each router will have a queue of either 0, 1, or 2 packets. If they have 2 packets, they drop one (each has the same chance of being dropped) and try to send the other packet on to either router 3 or 4. If they have one packet, they try to send it on to 3 or 4. Suppose R_1 sends its packet to R_3 with probability ρ_1 , while R_2 sends its packet to R_3 with probability ρ_2 .

This is a relatively complicated extensive form game of imperfect information since computers and routers move simultaneously. It is possible but awkward to draw the game tree, but it might help you think through the logic.

Use Julia with SymPy to find a mixed strategy equilibrium for this game (i.e., the three mixed strategies) as function of β . Once you have found this equilibrium, use it to compute the expected number of packets that get through to the internet (called anarchy). Then find the strategies that a planner would instruct the players to use in order to maximize the expected number of packets that get to the internet.

Since this all depends on β find the value of β at which the ratio of the expected number of packets with anarchy to the expected number of packets with a planner is lowest.

This is basically the price of anarchy idea. You'll be finding the worst case performance of anarchy relative to the best achievable performance.

- (2) Estimate the distribution of players loss aversion types in the ultimatum game as we did in class using only data from the experiments where no limits were imposed on (you are encouraged to experiment to show that the estimation procedure can be improved) . Given your best estimate, what would you predict the impact of a 10 dollar limit should be on outcomes.