Lecture 10: Omitted Variable Bias

ECON 480 - Econometrics - Fall 2018

Ryan Safner

October 23, 2018

HOOD COLLEGE

Omitted Variable Bias

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i + \epsilon_i$$

- Error term, ϵ_i, includes **all other variables that affect** Y_i

Review: ϵ

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i + \epsilon_i$$

- Error term, ϵ_i, includes **all other variables that affect** Y_i
- Every regression has always omitted variables assumed into the error term (ϵ_i)

HOOD COLLEGE

Review: ϵ

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i + \epsilon_i$$

- Error term, ϵ_i, includes **all other variables that affect** Y_i
- Every regression has always omitted variables assumed into the error term (ϵ_i)
    - Often unobservable or hard to measure (e.g. innate ability, the weather at the time, etc.)

HOOD COLLEGE

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i + \epsilon_i$$

- Error term, ϵ_i, includes **all other variables that affect** Y_i
- Every regression has always omitted variables assumed into the error term (ϵ_i)
    - Often unobservable or hard to measure (e.g. innate ability, the weather at the time, etc.)
- Again, we assume ϵ_i is **random** with $E[\epsilon|X] = 0$ and $var(\epsilon) = \sigma_\epsilon^2$

HOOD COLLEGE

Review: ϵ

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i + \epsilon_i$$

- Error term, ϵ_i, includes **all other variables that affect** Y_i
- Every regression has always omitted variables assumed into the error term (ϵ_i)
    - Often unobservable or hard to measure (e.g. innate ability, the weather at the time, etc.)
- Again, we assume ϵ_i is **random** with $E[\epsilon|X] = 0$ and $var(\epsilon) = \sigma_\epsilon^2$
- *Sometimes* the omission of these variables can **bias** OLS estimators ($\hat{\beta}_0$ and $\hat{\beta}_1$)

HOOD COLLEGE

Omitted Variable Bias

- Omitted variable bias for some omitted variable Z exists if two conditions are met:

Omitted Variable Bias

- Omitted variable bias for some omitted variable Z exists if two conditions are met:
  1. Z is a determinant of Y

HOOD COLLEGE

Omitted Variable Bias

- Omitted variable bias for some omitted variable Z exists if two conditions are met:
  1. Z is a determinant of Y
     - i.e. Z is included in the error term, ϵ_i

HOOD COLLEGE

Omitted Variable Bias

- Omitted variable bias for some omitted variable Z exists if two conditions are met:
  1. Z is a determinant of Y
     - i.e. Z is included in the error term, ϵ_i
  2. Z is correlated with the regressor X

HOOD COLLEGE

Omitted Variable Bias

- Omitted variable bias for some omitted variable Z exists if two conditions are met:
  1. Z is a determinant of Y
     - i.e. Z is included in the error term, ϵ_i
  2. Z is correlated with the regressor X
     - i.e. $corr(X, Z) \neq 0$

HOOD COLLEGE

Omitted Variable Bias

- Omitted variable bias for some omitted variable Z exists if two conditions are met:
  1. Z is a determinant of Y
     - i.e. Z is included in the error term, ϵ_i
  2. Z is correlated with the regressor X
     - i.e. $corr(X, Z) \neq 0$
- Omitted variable bias makes X **endogenous**

HOOD COLLEGE

Omitted Variable Bias

- Omitted variable bias for some omitted variable Z exists if two conditions are met:
  1. Z is a determinant of Y
     - i.e. Z is included in the error term, ϵ_i
  2. Z is correlated with the regressor X
     - i.e. $corr(X, Z) \neq 0$
- Omitted variable bias makes X **endogenous**
  - $E(\epsilon_i | X_i) \neq 0 \implies$ knowing X tells you something about ϵ

HOOD COLLEGE

Omitted Variable Bias

- Omitted variable bias for some omitted variable Z exists if two conditions are met:
  1. Z is a determinant of Y
     - i.e. Z is included in the error term, ϵ_i
  2. Z is correlated with the regressor X
     - i.e. $corr(X, Z) \neq 0$
- Omitted variable bias makes X **endogenous**
  - $E(\epsilon_i|X_i) \neq 0 \implies$ knowing X tells you something about ϵ
  - Thus, X tells you something about Y *not* by way of X!

$$E[\hat{\beta}_1] \neq \beta_1$$

HOOD COLLEGE

Omitted Variable Bias

- Omitted variable bias for some omitted variable Z exists if two conditions are met:
  1. Z is a determinant of Y
     - i.e. Z is included in the error term, ϵ_i
  2. Z is correlated with the regressor X
     - i.e. $corr(X, Z) \neq 0$
- Omitted variable bias makes X **endogenous**
  - $E(\epsilon_i|X_i) \neq 0 \implies$ knowing X tells you something about ϵ
  - Thus, X tells you something about Y *not* by way of X!
- Therefore, $\hat{\beta}_1$ is **biased** and systematically over- or under-estimates the true relationship β_1

$$E[\hat{\beta}_1] \neq \beta_1$$

HOOD COLLEGE

Omitted Variable Bias

- Omitted variable bias for some omitted variable Z exists if two conditions are met:
  1. Z is a determinant of Y
     - i.e. Z is included in the error term, ϵ_i
  2. Z is correlated with the regressor X
     - i.e. $corr(X, Z) \neq 0$
- Omitted variable bias makes X **endogenous**
  - $E(\epsilon_i|X_i) \neq 0 \implies$ knowing X tells you something about ϵ
  - Thus, X tells you something about Y *not* by way of X!
- Therefore, $\hat{\beta}_1$ is **biased** and systematically over- or under-estimates the true relationship β_1
  - $\hat{\beta}_1$ "picks up" *both* the effect of $X \rightarrow Y$ and the effect of $Z \rightarrow Y$ through X

$$E[\hat{\beta}_1] \neq \beta_1$$

HOOD COLLEGE

Example

$$\widehat{\text{Test Score}} = \hat{\beta}_0 + \hat{\beta}_1 STR_i + \epsilon_i$$

Example

$$\widehat{\text{Test Score}} = \hat{\beta}_0 + \hat{\beta}_1 STR_i + \epsilon_i$$

- Z_i: Time of Day of the Test (?)

Example

$$\widehat{\text{Test Score}} = \hat{\beta}_0 + \hat{\beta}_1 STR_i + \epsilon_i$$

- Z_i: Time of Day of the Test (?)
- Z_i: Parking Space per Student (?)

Example

$$\widehat{\text{Test Score}} = \hat{\beta}_0 + \hat{\beta}_1 STR_i + \epsilon_i$$

- Z_i: Time of Day of the Test (?)
- Z_i: Parking Space per Student (?)
- Z_i: Percent of ESL Students (?)

HOOD COLLEGE

- The true expected value of $\hat{\beta}_1$ is actually[1]:

$$E[\hat{\beta}_1] = \beta_1 + corr(X, \epsilon)\frac{\sigma_\epsilon}{\sigma_X}$$

[1] See **handout** on unbiasedness for proof

- The true expected value of $\hat{\beta}_1$ is actually[1]:

$$E[\hat{\beta}_1] = \beta_1 + corr(X, \epsilon)\frac{\sigma_\epsilon}{\sigma_X}$$

- Takeaways:

[1] See **handout** on unbiasedness for proof

- The true expected value of $\hat{\beta}_1$ is actually[1]:

$$E[\hat{\beta}_1] = \beta_1 + corr(X, \epsilon)\frac{\sigma_\epsilon}{\sigma_X}$$

- Takeaways:
  - If X is exogenous: $corr(X, \epsilon) = 0$, we're just left with β_1

[1] See **handout** on unbiasedness for proof

Recall: Endogeneity and Bias

- The true expected value of $\hat{\beta}_1$ is actually[1]:

$$E[\hat{\beta}_1] = \beta_1 + corr(X, \epsilon)\frac{\sigma_\epsilon}{\sigma_X}$$

- Takeaways:
  - If X is exogenous: $corr(X, \epsilon) = 0$, we're just left with β_1
  - The larger $corr(X, \epsilon)$ is, larger bias: $\left(E[\hat{\beta}_1] - \beta_1\right)$

[1] See **handout** on unbiasedness for proof

HOOD COLLEGE

Recall: Endogeneity and Bias

- The true expected value of $\hat{\beta}_1$ is actually[1]:

$$E[\hat{\beta}_1] = \beta_1 + corr(X, \epsilon)\frac{\sigma_\epsilon}{\sigma_X}$$

- Takeaways:
  - If X is exogenous: $corr(X, \epsilon) = 0$, we're just left with β_1
  - The larger $corr(X, \epsilon)$ is, larger bias: $\left(E[\hat{\beta}_1] - \beta_1\right)$
  - We can also "sign" the direction of the bias based on $corr(X, \epsilon)$

[1] See **handout** on unbiasedness for proof

HOOD COLLEGE

Recall: Endogeneity and Bias

- The true expected value of $\hat{\beta}_1$ is actually[1]:

$$E[\hat{\beta}_1] = \beta_1 + corr(X, \epsilon)\frac{\sigma_\epsilon}{\sigma_X}$$

- Takeaways:
  - If X is exogenous: $corr(X, \epsilon) = 0$, we're just left with β_1
  - The larger $corr(X, \epsilon)$ is, larger bias: $\left(E[\hat{\beta}_1] - \beta_1\right)$
  - We can also "sign" the direction of the bias based on $corr(X, \epsilon)$
    - **Positive** $corr(X, \epsilon)$ overestimates the true β_1 ($\hat{\beta}_1$ is too high)

[1] See **handout** on unbiasedness for proof

- The true expected value of $\hat{\beta}_1$ is actually[1]:

$$E[\hat{\beta}_1] = \beta_1 + corr(X, \epsilon)\frac{\sigma_\epsilon}{\sigma_X}$$

- Takeaways:
  - If X is exogenous: $corr(X, \epsilon) = 0$, we're just left with β_1
  - The larger $corr(X, \epsilon)$ is, larger bias: $\left(E[\hat{\beta}_1] - \beta_1\right)$
  - We can also "sign" the direction of the bias based on $corr(X, \epsilon)$
    - **Positive** $corr(X, \epsilon)$ overestimates the true β_1 ($\hat{\beta}_1$ is too high)
    - **Negative** $corr(X, \epsilon)$ underestimates the true β_1 ($\hat{\beta}_1$ is too low)

[1] See **handout** on unbiasedness for proof

HOOD COLLEGE

- Here is where checking correlations between variables helps:

```
# Select only the three variables we want (there are many)
CAcorr<-subset(CASchool, select=c("str","testscr","el_pct"))

# Make a correlation table
corr<-cor(CAcorr)
corr
```

```
##                str    testscr     el_pct
## str      1.0000000 -0.2263628  0.1876424
## testscr -0.2263628  1.0000000 -0.6441237
## el_pct   0.1876424 -0.6441237  1.0000000
```

```
library("stargazer")
stargazer(corr, type="latex", header=FALSE, float=FALSE)
```

	str	testscr	el_pct
str	1	-0.226	0.188
testscr	-0.226	1	-0.644
el_pct	0.188	-0.644	1

- %EL is strongly correlated with Test Score (Condition 1)

```
library("stargazer")
stargazer(corr, type="latex", header=FALSE, float=FALSE)
```

	str	testscr	el_pct
str	1	-0.226	0.188
testscr	-0.226	1	-0.644
el_pct	0.188	-0.644	1

- %EL is strongly correlated with Test Score (Condition 1)
- %EL is reasonably correlated with STR (Condition 2)

HOOD COLLEGE

```
summary(CASchool$testscr)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   605.5   640.0   654.5   654.2   666.7   706.8
```

```
# find the median of %EL
summary(CASchool$el_pct)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   0.000   1.941   8.778  15.768  22.970  85.540
```

HOOD COLLEGE

```
# look at test scores for districts with less than median %EL
summary(CASchool$testscr[CASchool$el_pct<8.7])
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   634.0   653.2   663.9   664.4   672.6   706.8
```

```
# look at test scores for districts with median or more %EL
summary(CASchool$testscr[CASchool$el_pct>=8.7])
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   605.5   632.5   643.2   644.2   655.7   691.4
```

- Test scores are *lower* in districts with relatively *high* %EL!

HOOD COLLEGE

```
# look at test scores for districts with less than median %EL
summary(CASchool$testscr[CASchool$el_pct<8.7])
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   634.0   653.2   663.9   664.4   672.6   706.8
```

```
# look at test scores for districts with median or more %EL
summary(CASchool$testscr[CASchool$el_pct>=8.7])
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   605.5   632.5   643.2   644.2   655.7   691.4
```

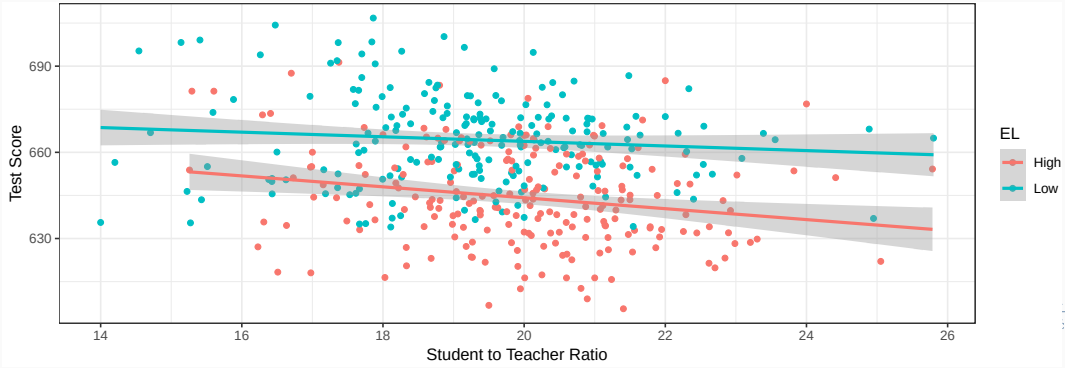
- Test scores are *lower* in districts with relatively *high* %EL!
- Test scores are *higher* in districts with relatively *low* %EL!

```
# Very useful function:
# ifelse(conditions, do.this.if.conditions.are.met, this.if.not)

CASchool$EL<-ifelse(CASchool$el_pct>8.7,"High","Low")

# i.e. I am making a new variable in the dataframe called EL
# defining it to be "High" if el_pct>8.7
# defining it to be "Low" if el_pct is NOT >8.7
```

HOOD COLLEGE

```
library("ggplot2")

ggplot(CASchool, aes(x=testscr,fill=EL))+
  geom_density(alpha=0.5)+theme_bw()+xlab("Test Score")+ylab("Density")
```

```
ggplot(CASchool, aes(x=str,y=testscr,color=EL))+
  geom_point()+geom_smooth(method="lm")+
  theme_bw()+xlab("Student to Teacher Ratio")+ylab("Test Score")
```



Omitted Variable Bias in Class Size Example

$$E[\hat{\beta}_1] = \beta_1 + \underbrace{corr(X, \epsilon)\frac{\sigma_\epsilon}{\sigma_X}}_{\text{Omitted Variable Bias}}$$

[2] Hard to think about...but you'll see when we run the different regressions below!

Omitted Variable Bias in Class Size Example

$$E[\hat{\beta}_1] = \beta_1 + \underbrace{corr(X, \epsilon)\frac{\sigma_\epsilon}{\sigma_X}}_{\text{Omitted Variable Bias}}$$

- $corr(STR, \epsilon)$ is positive (through %EL)

[2] Hard to think about...but you'll see when we run the different regressions below!

Omitted Variable Bias in Class Size Example

$$E[\hat{\beta}_1] = \beta_1 + \underbrace{corr(X, \epsilon)\frac{\sigma_\epsilon}{\sigma_X}}_{\text{Omitted Variable Bias}}$$

- $corr(STR, \epsilon)$ is positive (through %EL)
- $corr(\epsilon, \text{Test Score})$ is negative (through %EL)

[2] Hard to think about...but you'll see when we run the different regressions below!

$$E[\hat{\beta}_1] = \beta_1 + \underbrace{corr(X, \epsilon)\frac{\sigma_\epsilon}{\sigma_X}}_{\text{Omitted Variable Bias}}$$

- $corr(STR, \epsilon)$ is positive (through %EL)
- $corr(\epsilon, \text{Test Score})$ is negative (through %EL)
- β_1 is negative (between Test Score and STR)

[2] Hard to think about...but you'll see when we run the different regressions below!

$$E[\hat{\beta}_1] = \beta_1 + \underbrace{corr(X, \epsilon)\frac{\sigma_\epsilon}{\sigma_X}}_{\text{Omitted Variable Bias}}$$

- $corr(STR, \epsilon)$ is positive (through %EL)
- $corr(\epsilon, \text{Test Score})$ is negative (through %EL)
- β_1 is negative (between Test Score and STR)
- Bias is positive, but since β_1 is negative, it is made a *more* negative number than it should be[2]

[2] Hard to think about...but you'll see when we run the different regressions below!

Omitted Variable Bias in Class Size Example

$$E[\hat{\beta}_1] = \beta_1 + \underbrace{corr(X, \epsilon)\frac{\sigma_\epsilon}{\sigma_X}}_{\text{Omitted Variable Bias}}$$

- $corr(STR, \epsilon)$ is positive (through %EL)
- $corr(\epsilon, \text{Test Score})$ is negative (through %EL)
- β_1 is negative (between Test Score and STR)
- Bias is positive, but since β_1 is negative, it is made a *more* negative number than it should be[2]
  - Implies that β_1 *over*states the effect of reducing STR on improving Test Scores

[2] Hard to think about...but you'll see when we run the different regressions below!

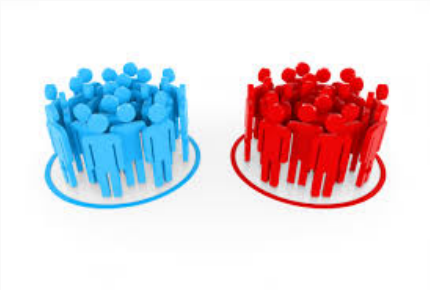
Omitted Variable Bias: Messing with Causality

- If school districts with higher Test Scores happen to have both lower STR **AND** districts with smaller STR sizes tend to have less %EL...

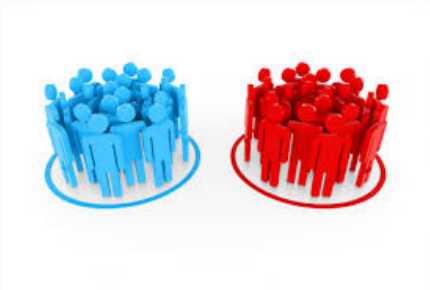
HOOD COLLEGE

- If school districts with higher Test Scores happen to have both lower STR **AND** districts with smaller STR sizes tend to have less %EL...
- How can we say $\hat{\beta}_1$ estimates the **marginal effect** of $\Delta STR \rightarrow \Delta$Test Score?

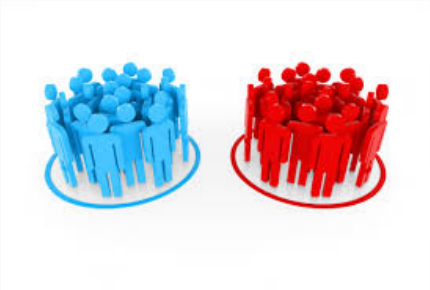
- Recall our best working definition of causality: result of ideal **random controlled trials (RCTs)**



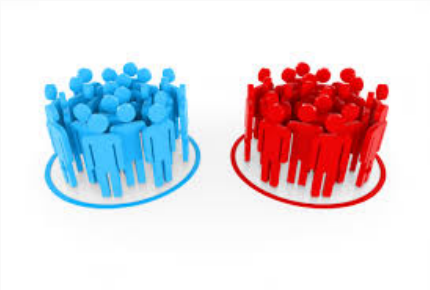
- Recall our best working definition of causality: result of ideal **random controlled trials (RCTs)**
  - **Randomly** assign experimental units (e.g. people, cities, etc) into two (or more) groups:



- Recall our best working definition of causality: result of ideal **random controlled trials (RCTs)**
  - **Randomly** assign experimental units (e.g. people, cities, etc) into two (or more) groups:
    - Treatment group(s): gets a (certain type or level of) treatment

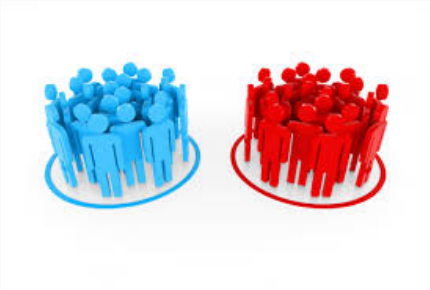


- Recall our best working definition of causality: result of ideal **random controlled trials (RCTs)**
  - **Randomly** assign experimental units (e.g. people, cities, etc) into two (or more) groups:
    - Treatment group(s): gets a (certain type or level of) treatment
    - Control group: gets *no* treatment(s)



Omitted Variable Bias: Messing with Causality II

- Recall our best working definition of causality: result of ideal **random controlled trials (RCTs)**
  - **Randomly** assign experimental units (e.g. people, cities, etc) into two (or more) groups:
    - Treatment group(s): gets a (certain type or level of) treatment
    - Control group: gets *no* treatment(s)
  - Compare results of two groups to get the causal effect of treatment (on average)



HOOD COLLEGE

Example

- Imagine an ideal RCT for measuring the effect of STR on Test Score

Example

- Imagine an ideal RCT for measuring the effect of STR on Test Score
- School districts would be **randomly assigned** a STR

Example

- Imagine an ideal RCT for measuring the effect of STR on Test Score
- School districts would be **randomly assigned** a STR
- With random assignment, all factors in ϵ (parental income, family size, # of siblings, English proficiency, etc) are distributed *independently* of class size

Example

- Imagine an ideal RCT for measuring the effect of STR on Test Score
- School districts would be **randomly assigned** a STR
- With random assignment, all factors in ϵ (parental income, family size, # of siblings, English proficiency, etc) are distributed *independently* of class size
- Thus, $corr(STR, \epsilon) = 0$ and $E(\epsilon_i | STR_i) = 0$: exogenity!

Example

- Imagine an ideal RCT for measuring the effect of STR on Test Score
- School districts would be **randomly assigned** a STR
- With random assignment, all factors in ϵ (parental income, family size, # of siblings, English proficiency, etc) are distributed *independently* of class size
- Thus, $corr(STR, \epsilon) = 0$ and $E(\epsilon_i | STR_i) = 0$: exogenity!
- The resulting β_1 is an unbiased estimate for the marginal effect of $\Delta STR \rightarrow \Delta$Test Score

But We Rarely Have RCTs

- But our data is *not* an RCT, it is observational data!

But We Rarely Have RCTs

- But our data is *not* an RCT, it is observational data!
- "Treatment" of having a large or small class size is **NOT** randomly assigned!

But We Rarely Have RCTs

- But our data is *not* an RCT, it is observational data!
- "Treatment" of having a large or small class size is **NOT** randomly assigned!
- Again consider $\%EL$: plausibly fits the criteria of O.V. bias!

But We Rarely Have RCTs

- But our data is *not* an RCT, it is observational data!
- "Treatment" of having a large or small class size is **NOT** randomly assigned!
- Again consider $\%EL$: plausibly fits the criteria of O.V. bias!
    1. %EL is a determinant of Test Score

But We Rarely Have RCTs

- But our data is *not* an RCT, it is observational data!
- "Treatment" of having a large or small class size is **NOT** randomly assigned!
- Again consider $\%EL$: plausibly fits the criteria of O.V. bias!
    1. %EL is a determinant of Test Score
    2. %EL is correlated with STR

But We Rarely Have RCTs

- But our data is *not* an RCT, it is observational data!
- "Treatment" of having a large or small class size is **NOT** randomly assigned!
- Again consider *%EL*: plausibly fits the criteria of O.V. bias!
    1. %EL is a determinant of Test Score
    2. %EL is correlated with STR
- Thus, "control" group and "treatment" group differs systematically!

But We Rarely Have RCTs

- But our data is *not* an RCT, it is observational data!
- "Treatment" of having a large or small class size is **NOT** randomly assigned!
- Again consider *%EL*: plausibly fits the criteria of O.V. bias!
  1. %EL is a determinant of Test Score
  2. %EL is correlated with STR
- Thus, "control" group and "treatment" group differs systematically!
  - Small STR also tend to have lower %EL; large STR also tend to have higher %EL

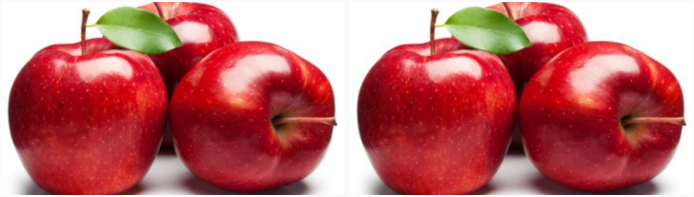
HOOD COLLEGE

But We Rarely Have RCTs

- But our data is *not* an RCT, it is observational data!
- "Treatment" of having a large or small class size is **NOT** randomly assigned!
- Again consider $\%EL$: plausibly fits the criteria of O.V. bias!
  1. %EL is a determinant of Test Score
  2. %EL is correlated with STR
- Thus, "control" group and "treatment" group differs systematically!
  - Small STR also tend to have lower %EL; large STR also tend to have higher %EL
  - **Selection bias**: $corr(STR, \%EL) \neq 0$, $E[\epsilon_i | STR_i] \neq 0$

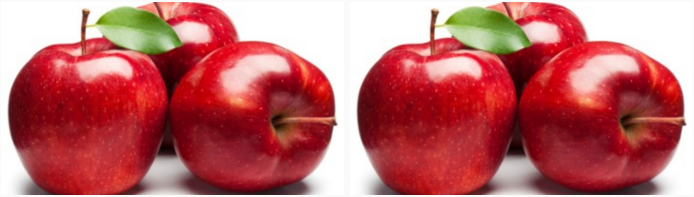
There's Another Way to Neutralize OVB

- Look at effect of STR on Test Score by comparing districts with the **same** %EL.



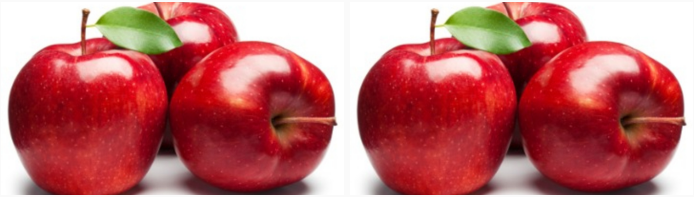
There's Another Way to Neutralize OVB

- Look at effect of STR on Test Score by comparing districts with the **same** %EL.
  - Eliminates differences in %EL between high and low STR classes



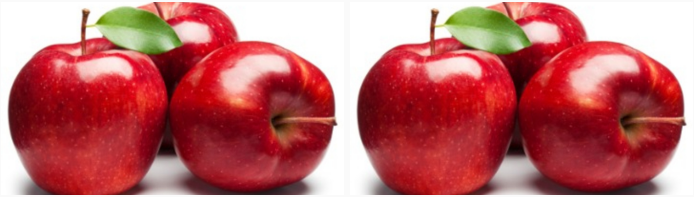
There's Another Way to Neutralize OVB

- Look at effect of STR on Test Score by comparing districts with the **same** %EL.
  - Eliminates differences in %EL between high and low STR classes
  - "As if" we had a control group! Hold %EL constant



There's Another Way to Neutralize OVB

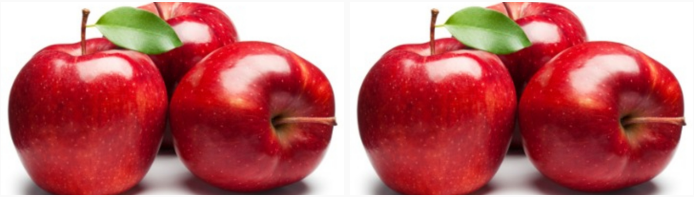
- Look at effect of STR on Test Score by comparing districts with the **same** %EL.
  - Eliminates differences in %EL between high and low STR classes
  - "As if" we had a control group! Hold %EL constant
- The simple fix is just to **not omit %EL**!



HOOD COLLEGE

There's Another Way to Neutralize OVB

- Look at effect of STR on Test Score by comparing districts with the **same** %EL.
  - Eliminates differences in %EL between high and low STR classes
  - "As if" we had a control group! Hold %EL constant
- The simple fix is just to **not omit %EL**!
  - Make it *another* independent variable on the righthand side of the regression



The Multivariate Regression Model

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \epsilon_i$$

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \epsilon_i$$

- X_{1i} and X_{2i} are two independent variables (regressors)

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \epsilon_i$$

- X_{1i} and X_{2i} are two independent variables (regressors)
- (Y_i, X_{1i}, X_{2i}) are the values of variables Y, X_1, and X_2 for individual i

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \epsilon_i$$

- X_{1i} and X_{2i} are two independent variables (regressors)
- (Y_i, X_{1i}, X_{2i}) are the values of variables Y, X_1, and X_2 for individual i
- β_0: a population constant

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \epsilon_i$$

- X_{1i} and X_{2i} are two independent variables (regressors)
- (Y_i, X_{1i}, X_{2i}) are the values of variables Y, X_1, and X_2 for individual i
- β_0: a population constant
- β_1: marginal effect on Y of a change in X_1, holding X_2 constant

HOOD COLLEGE

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \epsilon_i$$

- X_{1i} and X_{2i} are two independent variables (regressors)
- (Y_i, X_{1i}, X_{2i}) are the values of variables Y, X_1, and X_2 for individual i
- β_0: a population constant
- β_1: marginal effect on Y of a change in X_1, holding X_2 constant
- β_2: marginal effect on Y of a change in X_2, holding X_1 constant

HOOD COLLEGE

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \epsilon_i$$

- X_{1i} and X_{2i} are two independent variables (regressors)
- (Y_i, X_{1i}, X_{2i}) are the values of variables Y, X_1, and X_2 for individual i
- β_0: a population constant
- β_1: marginal effect on Y of a change in X_1, holding X_2 constant
- β_2: marginal effect on Y of a change in X_2, holding X_1 constant
- ϵ_i: regression error (omitted variables)

HOOD COLLEGE

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i}$$

- Consider changing X_1 by ΔX_1 while holding X_2 constant:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i}$$

- Consider changing X_1 by ΔX_1 while holding X_2 constant:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 \qquad \text{Before the change}$$

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i}$$

- Consider changing X_1 by ΔX_1 while holding X_2 constant:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 \qquad \text{Before the change}$$

$$Y + \Delta Y = \beta_0 + \beta_1 (X_1 + \Delta X_1) + \beta_2 X_2 \qquad \text{After the change}$$

HOOD COLLEGE

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i}$$

- Consider changing X_1 by ΔX_1 while holding X_2 constant:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 \quad \text{Before the change}$$

$$Y + \Delta Y = \beta_0 + \beta_1 (X_1 + \Delta X_1) + \beta_2 X_2 \quad \text{After the change}$$

$$\Delta Y = \beta_1 \Delta X_1 \quad \text{The difference}$$

HOOD COLLEGE

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i}$$

- Consider changing X_1 by ΔX_1 while holding X_2 constant:

$$\begin{aligned} Y &= \beta_0 + \beta_1 X_1 + \beta_2 X_2 && \text{Before the change} \\ Y + \Delta Y &= \beta_0 + \beta_1 (X_1 + \Delta X_1) + \beta_2 X_2 && \text{After the change} \\ \Delta Y &= \beta_1 \Delta X_1 && \text{The difference} \\ \frac{\Delta Y}{\Delta X_1} &= \beta_1 && \text{Solving for } \beta_1 \end{aligned}$$

HOOD COLLEGE

$$\beta_1 = \frac{\Delta Y}{\Delta X_1} \text{ holding } X_2 \text{ constant}$$

$$\beta_1 = \frac{\Delta Y}{\Delta X_1} \text{ holding } X_2 \text{ constant}$$

Similarly, for β_2:

$$\beta_2 = \frac{\Delta Y}{\Delta X_2} \text{ holding } X_1 \text{ constant}$$

HOOD COLLEGE

$$\beta_1 = \frac{\Delta Y}{\Delta X_1} \text{ holding } X_2 \text{ constant}$$

Similarly, for β_2:

$$\beta_2 = \frac{\Delta Y}{\Delta X_2} \text{ holding } X_1 \text{ constant}$$

And for the constant, β_0:

$$\beta_0 = \text{predicted value of Y when } X_1 = 0,\ X_2 = 0$$

HOOD COLLEGE

If You Like Your Intutions, You Can Keep Them...But They're Wrong Now

- We have been envisioning OLS regressions as the equation of a line through a scatterplot of data on two variables, X and Y

HOOD COLLEGE

If You Like Your Intutions, You Can Keep Them...But They're Wrong Now

- We have been envisioning OLS regressions as the equation of a line through a scatterplot of data on two variables, X and Y
  - β_1: "slope"

HOOD COLLEGE

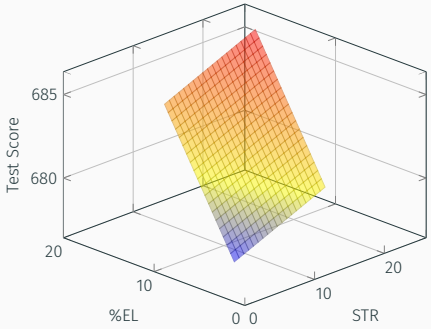
If You Like Your Intutions, You Can Keep Them...But They're Wrong Now

- We have been envisioning OLS regressions as the equation of a line through a scatterplot of data on two variables, X and Y
  - β_1: "slope"
  - β_0: "y-intercept"

HOOD COLLEGE

If You Like Your Intutions, You Can Keep Them...But They're Wrong Now

- We have been envisioning OLS regressions as the equation of a line through a scatterplot of data on two variables, X and Y
  - β_1: "slope"
  - β_0: "y-intercept"
- With 3+ variables, it OLS regressions is no longer a "line" for us to estimate



$$\widehat{\text{Test Score}} = 686 - 1.10STR - 0.65\%EL$$

- Alternatively, we can write the population regression equation as:

$$Y_i = \beta_0 X_{0i} + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i$$

The "Constant"

- Alternatively, we can write the population regression equation as:

$$Y_i = \beta_0 X_{0i} + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i$$

- Here, we added X_{0i} to β_0

HOOD COLLEGE

The "Constant"

- Alternatively, we can write the population regression equation as:

$$Y_i = \beta_0 X_{0i} + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i$$

- Here, we added X_{0i} to β_0
- X_{0i} is a constant regressor, as we define $X_{0i} = 1$ for all i observations

HOOD COLLEGE

The "Constant"

- Alternatively, we can write the population regression equation as:

$$Y_i = \beta_0 X_{0i} + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i$$

- Here, we added X_{0i} to β_0
- X_{0i} is a constant regressor, as we define $X_{0i} = 1$ for all i observations
- Likewise, β_0 is more generally called the constant term in the regression (instead of the "intercept")

HOOD COLLEGE

- Alternatively, we can write the population regression equation as:

$$Y_i = \beta_0 X_{0i} + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i$$

- Here, we added X_{0i} to β_0
- X_{0i} is a constant regressor, as we define $X_{0i} = 1$ for all i observations
- Likewise, β_0 is more generally called the constant term in the regression (instead of the "intercept")
- This may seem silly and trivial, but this will be important soon!

HOOD COLLEGE

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + ... + \beta_k X_{ki} + \epsilon_i$$

- In general, a multivariate model has k regressor variables

[3] Note your Bailey textbook defines k to include both the number of variables plus the constant

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + ... + \beta_k X_{ki} + \epsilon_i$$

- In general, a multivariate model has k regressor variables
- We estimate $k + 1$ parameters (including β_0, the constant, when $\beta_1 = \beta_2 = ... = \beta_k = 0$)[3]

[3] Note your Bailey textbook defines k to include both the number of variables plus the constant

Example

$$\widehat{Consumption}_i = \beta_0 + \beta_1 Price_i + \beta_2 Income_i + \beta_3 CompsPrice_i + \beta_4 SubsPrice + \epsilon_i$$

- Let's see what you remember from micro(econ)!

HOOD COLLEGE

Example

$$\widehat{Consumption}_i = \beta_0 + \beta_1 Price_i + \beta_2 Income_i + \beta_3 CompsPrice_i + \beta_4 SubsPrice + \epsilon_i$$

- Let's see what you remember from micro(econ)!
- What measures the **price effect**? What sign should it have?

Example

$$\widehat{Consumption}_i = \beta_0 + \beta_1 Price_i + \beta_2 Income_i + \beta_3 CompsPrice_i + \beta_4 SubsPrice + \epsilon_i$$

- Let's see what you remember from micro(econ)!
- What measures the **price effect**? What sign should it have?
- What measures the **income effect**? What should inferior, necessities, and luxuries look like?

HOOD COLLEGE

Example

$$\widehat{Consumption}_i = \beta_0 + \beta_1 Price_i + \beta_2 Income_i + \beta_3 CompsPrice_i + \beta_4 SubsPrice + \epsilon_i$$

- Let's see what you remember from micro(econ)!
- What measures the **price effect**? What sign should it have?
- What measures the **income effect**? What should inferior, necessities, and luxuries look like?
- What measures the **cross-price effect**? What should substitutes and complements look like?

HOOD COLLEGE

Example

$$\widehat{BeerCons}_i = 20 - 1.5Price_i + 1.25Income_i - 0.75WingsPrice_i + 1.3WinePrice_i$$

- Interpret each $\hat{\beta}$

```
# syntax: reg.object<-lm(y~x1+x2, data=mydf)
# y goes first, then all of your x's after the ~, separated by +'s
```

Remember, we "regress *Y* on *X*'s"

Multivariate Regression In R II

```r
multireg<-lm(testscr~str+el_pct, data=CASchool)
summary(multireg)
```

```
##
## Call:
## lm(formula = testscr ~ str + el_pct, data = CASchool)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -48.845 -10.240  -0.308   9.815  43.461
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 686.03225    7.41131  92.566  < 2e-16 ***
## str          -1.10130    0.38028  -2.896  0.00398 **
## el_pct       -0.64978    0.03934 -16.516  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 14.46 on 417 degrees of freedom
## Multiple R-squared:  0.4264, Adjusted R-squared:  0.4237
## F-statistic:   155 on 2 and 417 DF,  p-value: < 2.2e-16
```

HOOD COLLEGE

Multivariate Regression In R III

```
library("stargazer")
stargazer(school.regression, multireg, header=FALSE, type="latex",
          float=FALSE, font.size="tiny",
          dep.var.labels = c("Test Score"),
          covariate.labels = c("Student Teacher Ratio",
                               "Pct ESL Students"))
```

	Dependent variable:	
	Test Score	
	(1)	(2)
Student Teacher Ratio	−2.280***	−1.101***
	(0.480)	(0.380)
Pct ESL Students		−0.650***
		(0.039)
Constant	698.933***	686.032***
	(9.467)	(7.411)
Observations	420	420
R^2	0.051	0.426
Adjusted R^2	0.049	0.424
Residual Std. Error	18.581 (df = 418)	14.464 (df = 417)
F Statistic	22.575*** (df = 1; 418)	155.014*** (df = 2; 417)
Note:		*p<0.1; **p<0.05; ***p<0.01