

1.5 – Coordination Games & Multiple Equilibria

ECON 316 • Game Theory • Fall 2021

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Outline



Coordination Games

Multiple Equilibria

Rationalizability and the Role of Beliefs



Coordination Games

Coordination Games



- This semester, we are dealing with **non-cooperative games** where each player acts independently
- In **coordination games**, players don't necessarily have conflicting interests
 - Often **positive-sum games**
 - Often have more than one, or zero, Pure Strategy Nash equilibria (PSNE)



Pure Coordination Game



- **Pure coordination game:** does not matter which strategy players choose, so long as they choose the same!

		Sally
	Whitaker	Starbucks
Harry	Whitaker	0
	1	0
	1	0
Starbucks	0	1
	0	1
		1

Pure Coordination Game



- **Pure coordination game:** does not matter which strategy players choose, so long as they choose the same!
- Two Pure Strategy Nash Equilibria:
 1. (**Whitaker, Whitaker**)
 2. (**Starbucks, Starbucks**)

		Sally
	Whitaker	Starbucks
Harry	Whitaker	Starbucks

A 3x3 matrix game between Harry (rows) and Sally (columns). The payoffs are as follows:

		Sally	
		Whitaker	Starbucks
Harry	Whitaker	1 1	0 0
	Starbucks	0 0	1 1

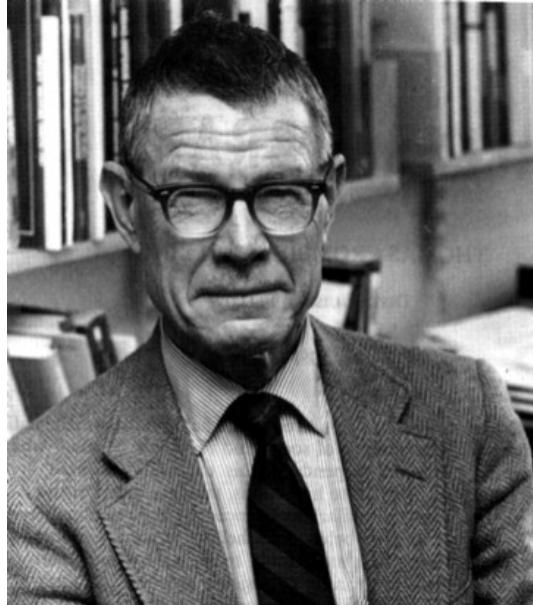
Pure Coordination Game



- The flat tire game from before is also a pure coordination game
- Four PSNE:
 1. (Front L, Front L)
 2. (Front R, Front R)
 3. (Rear L, Rear L)
 4. (Rear R, Rear R)

		Friend			
		Front L	Front R	Rear L	Rear R
You	Front L	1 1	0 0	0 0	0 0
	Front R	0 0	1 1	0 1	0 0
	Rear L	0 0	0 0	1 0	0 0
	Rear R	0 0	0 0	0 1	1 1

Coordination Games: Focal Points



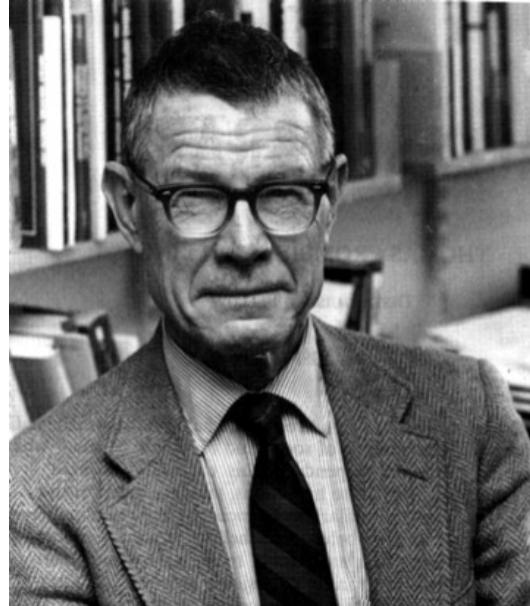
- Without pre-game communication, expectations must converge on a **focal point**
- A major idea in Thomas Schelling's work, we often call them **“Schelling points”**

Thomas Schelling

1921–2016

Economics Nobel 2005

Coordination Games: Focal Points



Thomas Schelling

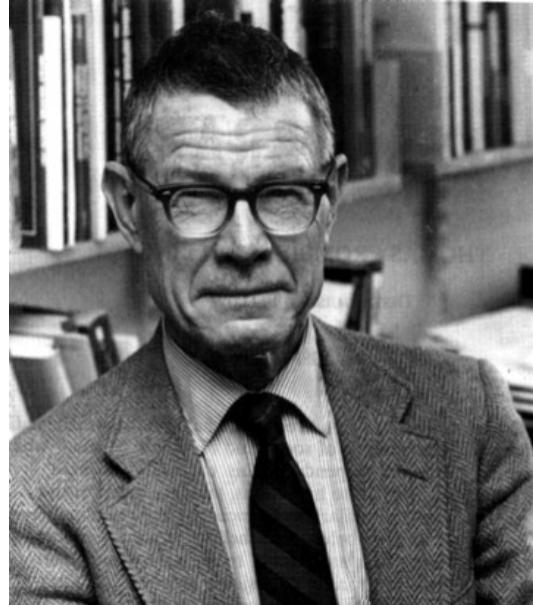
1921–2016

Economics Nobel 2005

“[I]t is instructive to begin with the...case in which two or more parties have identical interests and face the problem not of reconciling interests but only of coordinating their actions for their mutual benefit, when communication is impossible.”

“When a man loses his wife in a department store without any prior understanding on where to meet if they get separated, the chances are good that they will find each other. It is likely that each will think of some obvious place to meet, so obvious that each will be sure that the other is sure that it is ‘obvious’ to both of them. One does not simply predict where the other will go, since the other will go where he predicts the first to go, which is wherever the first predicts the second to predict the first to go, and so ad infinitum.”

Coordination Games: Focal Points



Thomas Schelling

1921–2016

Economics Nobel 2005

“What is necessary is to coordinate predictions, to read the same message in the common situation, to identify the one course of action that their expectations of each other can converge on. They must ‘mutually recognize’ some unique signal that coordinates their expectations of each other. We cannot be sure that they will meet, nor would all couples read the same signal; but the chances are certainly a great deal better than if they pursued a random course of search.” (p.54).

Schelling, Thomas, 1960, *The Strategy of Conflict*

Coordination Games: Focal Points



Example

- If we both pick the same square (without communicating), we each get \$100
- Which one would (should?) you choose?

	1	2	3	4	5	6
A	Green	Cyan	Green	Cyan	Cyan	Red
B	Purple	Purple	Cyan	Green	Purple	Cyan
C	Cyan	Green	Purple	Cyan	Cyan	Green
D	Cyan	Cyan	Cyan	Purple	Purple	Green
E	Cyan	Green	Purple	Cyan	Purple	Cyan
F	Purple	Cyan	Cyan	Cyan	Green	Green

Coordination Games: Focal Points



Example

- If we both pick the same square (without communicating), we each get \$100
- Which one would (should?) you choose?
- Culture and informal norms (“unwritten laws”) play an enormous role!

	1	2	3	4	5	6
A	Green	Cyan	Green	Cyan	Cyan	Red
B	Purple	Purple	Cyan	Green	Purple	Cyan
C	Cyan	Green	Purple	Cyan	Cyan	Green
D	Cyan	Cyan	Cyan	Purple	Purple	Green
E	Cyan	Green	Purple	Cyan	Purple	Cyan
F	Purple	Cyan	Cyan	Cyan	Green	Green

Assurance Games



- “**Assurance**” game: a special case of coordination game where one equilibrium is universally preferred
- Here, both prefer (**Whit**, **Whit**) over (**SB**, **SB**)

		Sally
	Whitaker	Starbucks
Harry	Whitaker	0
	2	0
	2	0
Starbucks	Starbucks	1
	0	1
	0	1

Assurance Games



- “**Assurance**” game: a special case of coordination game where one equilibrium is universally preferred
- Here, both prefer (**Whit**, **Whit**) over (**SB**, **SB**)
- Still two PSNE
 - 1. (**Whit**, **Whit**)
 - 2. (**SB**, **SB**)
- Players get their preferred outcome only if each has enough **assurance** the other

		Sally
	Whitaker	Starbucks
Harry	2 2 0	0 1 1

Assurance Games: A Famed Example



~	!	@	#	\$	%	^	&	*	()	-	=	←	Backspace
`	1	2	3	4	5	6	7	8	9	0	-	=	←	Backspace
Tab ↲	Q	W	E	R	T	Y	U	I	O	P	{	}		\
Caps Lock	A	S	D	F	G	H	J	K	L	:	"	Enter	←	
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~	!	@	#	\$	%	^	&	*	()	{	}	←	Backspace
`	1	2	3	4	5	6	7	8	9	0	[]	←	Backspace
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Shift ↗	:	Q	J	K	X	B	M	W	V	Z	Shift ↗	↑		
Ctrl	Win Key	Alt								Alt Gr	Win Key	Menu	Ctrl	

Assurance Games: Path Dependence & Lock-In



- Suppose all agree Dvorak is superior
 - But not guaranteed to be the outcome!
- **Path Dependence**: early choices may affect later ability to choose or switch
- **Lock-in**: the switching cost of moving from one equilibrium to another becomes prohibitive

		Column	
		Dvorak	QWERTY
Row	Dvorak	2 0	0 2
	QWERTY	0 0	1 1

David, Paul A, 1985, "Clio and the Economics of QWERTY," *American Economic Review*, 75(2):332-337

Assurance Games: Path Dependence & Lock-In



Clio and the Economics of QWERTY

By PAUL A. DAVID*

Cicero demands of historians, first, that we tell true stories. I intend fully to perform my duty on this occasion, by giving you a homely piece of narrative economic history in which “one damn thing follows another.” The main point of the story will become plain enough: it is sometimes not possible to uncover the logic (or illogic) of the world around us except by understanding how it got that way. A *path-dependent* sequence of economic changes is one of which important influences upon the eventual outcome can be exerted by temporally remote events, including happenings dominated by chance elements rather than systematic forces. Stochastic processes like that do not converge automatically to a fixed-point distribution of outcomes, and are called *non-ergodic*. In such circumstances “historical accidents” can neither be ignored, nor neatly quarantined for the purpose of economic analysis; the dynamic process itself takes on an *essentially historical* character. Standing alone, my story will be simply illustrative and does not establish how much of the world works this way. That is an open empirical issue and I would be presumptuous to claim to have settled it, or to instruct you in what to do about it. Let us just hope the tale proves mildly diverting for those waiting to be told if and why the study of economic history is a necessity in the making of economists.

I. The Story of QWERTY

Why does the topmost row of letters on your personal computer keyboard spell out QWERTYUIOP, rather than something else? We know that nothing in the engineering of computer terminals requires the awkward keyboard layout known today as “QWERTY,” and we all are old enough to remember that QWERTY somehow has been handed down to us from the Age of Typewriters. Clearly nobody has been persuaded by the exhortations to discard QWERTY, which apostles of DSK (the Dvorak Simplified Keyboard) were issuing in trade publications such as *Computers and Automation* during the early 1970’s. Why not? Devotees of the keyboard arrangement patented in 1932 by August Dvorak and W. L. Dealey have long held most of the world’s records for speed typing. Moreover, during the 1940’s U.S. Navy experiments had shown that the increased efficiency obtained with DSK would amortize the cost of retraining a group of typists within the first ten days of their subsequent full-time employment. Dvorak’s death in 1975 released him from forty years of frustration with the world’s stubborn rejection of his contribution; it came too soon for him to be solaced by the Apple IIC computer’s built-in switch, which instantly converts its keyboard from QWERTY to virtual DSK, or to be further aggravated by doubts that the switch would not often be flicked.

Assurance Games: Path Dependence & Lock-In



- "First-degree" path dependency:

- Sensitivity to initial conditions
 - But no inefficiency

- Examples:

- language
 - driving on left vs. right side of road

		Column	
		Dvorak	QWERTY
Row	Dvorak	2	0
	QWERTY	2	0
Row	Dvorak	0	1
	QWERTY	0	1

Liebowitz, Stan J and Stephen E Margolis, 1990, "The Fable of the Keys," *Journal of Law and Economics*, 33(1):1-25

Assurance Games: Path Dependence & Lock-In



- "Second-degree" path dependency:

- Sensitivity to initial conditions
- Imperfect information at time of choice
- Outcomes are regrettable *ex post*

		Column	
		Dvorak	QWERTY
Row	Dvorak	2 0	0 2
	QWERTY	0 1	1 0

- Not inefficient: no better decision could have been made *at the time*

Liebowitz, Stan J and Stephen E Margolis, 1990, "The Fable of the Keys," *Journal of Law and Economics*, 33(1):1-25

Assurance Games: Path Dependence & Lock-In



- "Third-degree" path dependency:
 - Sensitivity to initial conditions
 - Worse choice made
 - Avoidable mistake at the time
- Inefficient lock-in

		Column	
		Dvorak	QWERTY
Row	Dvorak	2 0	0 2
	QWERTY	0 1	1 0

Liebowitz, Stan J and Stephen E Margolis, 1990, "The Fable of the Keys," *Journal of Law and Economics*, 33(1):1-25

Assurance Games: Path Dependence & Lock-In



Table 2
An Example: Adoption Payoffs for Homogeneous Agents

Number of previous adoptions	0	10	20	30	40	50	60	70	80	90	100
Technology A	10	11	12	13	14	15	16	17	18	19	20
Technology B	4	7	10	13	16	19	22	25	28	31	34

Arthur, W. Brian, 1989, "Competing Technologies, Increasing Returns, and Lock-In by

Historical Events," *Economic Journal* 99(394): 116-131

- In the long-run, Technology B is superior
- But in the short-run, Technology A has higher payoffs
- Inefficient lock-in
- But what about uncertainty?
 - What set of institutions will choose best under uncertainty?

Assurance Games: Path Dependence & Lock-In



Table 2
An Example: Adoption Payoffs for Homogeneous Agents

Number of previous adoptions	0	10	20	30	40	50	60	70	80	90	100
Technology A	10	11	12	13	14	15	16	17	18	19	20
Technology B	4	7	10	13	16	19	22	25	28	31	34

Arthur, W. Brian, 1989, "Competing Technologies, Increasing Returns, and Lock-In by

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- Role for **entrepreneurial judgment** and "**championing**" a standard
 - Someone who "owns" a standard has strong incentive to see it adopted
- Champions who forecast higher long-term payoffs can subsidize adoption in the short run

Assurance Games: Path Dependence & Lock-In



- September 3, 1967, “H day” in Sweden
 - Högertrafikomläggningen: “right-hand traffic diversion”
- Sweden switched from driving on the left side of the road to the right
 - Both of Sweden’s neighbors drove on the right, 5 million vehicles/year crossing borders



Assurance Games: Stag Hunt



- Famous variant: the “**Stag Hunt**”

“If it was a matter of hunting a deer, everyone well realized that he must remain faithful to his post; but if a hare happened to pass within reach of one of them, we cannot doubt that he would have gone off in pursuit without scruple.”



Assurance Games: Stag Hunt



- Often invoked to discuss public goods, free rider problems
- Two PSNE, and $(\text{Stag}, \text{Stag}) > (\text{Hare}, \text{Hare})$
- Can't take down a Stag alone, need to rely on a group to work together
 - But unlike prisoners' dilemma, no incentive to overtly “screw over” the group

		Column	
		Stag	Hare
Row	Stag	2 2	0 1
	Hare	1 0	1

Prisoners' Dilemma vs. Assurance/Stag Hunt



		Column	
		Cooperate	Defect
		Cooperate	Defect
Row	Cooperate	3 3	1 4
	Defect	4 1	2 2

		Column	
		Cooperate	Defect
		Cooperate	Defect
Row	Cooperate	2 2	0 1
	Defect	1 0	1 1

- Dominant strategy to always **Defect**
- Nash equilibrium: (**Defect**, **Defect**)
- $(\text{Coop}, \text{Coop}) > (\text{Defect}, \text{Defect})$
- $(\text{Coop}, \text{Coop})$ **not** a Nash equilibrium

- No dominant or dominated strategies
- 2 NE: (**Coop**, **Coop**) and (**Defect**, **Defect**)
- $(\text{Coop}, \text{Coop}) > (\text{Defect}, \text{Defect})$
- Can get stuck in (**Defect**, **Defect**) but (**Coop**, **Coop**) is stable & possible

Battle of the Sexes



- Each player prefers a different Nash equilibrium over another
- But coordinating is better than not-coordinating, for both!

		Sally
	Hockey	Ballet
Harry	Hockey	2 0 1 0
	Ballet	0 1 0 2

Battle of the Sexes



- Each player prefers a different Nash equilibrium over another

- But coordinating is better than not-coordinating, for both!

- Two PSNE:

1. (**Hockey**, **Hockey**) – **Harry's** preference
2. (**Ballet**, **Ballet**) – **Sally's** preference

		Sally
	Hockey	Ballet
Hockey	2 1	0 0
Ballet	0 0	1 2

Chicken



- Two strategies per player: act tough/cool vs. weak
- Each prefers to act tough and have the other player act weak
 - But if both act tough, the worst outcome for both
- Often called an “anti-coordination” game

		Column	
		Weak	Tough
Row	Weak	0	-1
	Tough	1	0
Row	Weak	-1	1
	Tough	-2	-2

Chicken



- A common example in movies
- Two cars aimed at each other, or racing furthest to edge of cliff

		Column	
		Swerve	Straight
		0	-1
Row	Swerve	0	1
	Straight	1	-2
		-1	-2

Chicken



- A common example in movies
- Two cars aimed at each other, or racing furthest to edge of cliff
- Two PSNE:
 1. (**Straight**, **Swerve**) – **Row**'s preference
 2. (**Swerve**, **Straight**) – **Column**'s preference
- So long as both choose *different* strategies, avoids worst outcome

		Column	
		Swerve	Straight
		Swerve	0
Row	Swerve	0	-1
	Straight	1	1
Column	Swerve	1	-2
	Straight	-1	-2

Chicken



Chicken



Chicken and Commitment



- Each player may try to influence the game beforehand
- Project and signal “toughness” (or that they are “crazy”) before the game
- Find a commitment strategy so you have **no choice** but to play tough
 - e.g. rip out the steering wheel!
- Schelling: “If you're invited to play chicken and you decline, you've already played [and lost]”

		Column	
		Weak	Tough
Row	Weak	0	-1
	Tough	1	-2

Chicken: Hawk Dove



- One variant of chicken is also famous:
Hawk-Dove game
 - (actually, chicken is just a special case of hawk dove!)
- Evolutionary biology, political science, bargaining

		Column	
		Dove	Hawk
		1	0
Row	Dove	1	0
	Hawk	2	-1

Game Types



Prisoners' Dilemma

		Column	
		A	B
Row	A	3 3	1 4
	B	4 1	2 2

Assurance

		Column	
		A	B
Row	A	2 2	0 0
	B	0 0	1 1

Stag Hunt

		Column	
		A	B
Row	A	2 2	0 1
	B	1 0	1 1

Coordination

		Column	
		A	B
Row	A	1 1	0 0
	B	0 0	1 1

Battle of the Sexes

		Column	
		A	B
Row	A	2 1	0 0
	B	0 0	1 2

Chicken

		Column	
		A	B
Row	A	0 0	-1 1
	B	1 -1	-2 -2

Modeling Social Interactions



- Can *all* players potentially benefit from the interaction?
 - No: chicken
- Do all players prefer one outcome over another?
 - Yes: assurance game
- Does the players prefer *different* outcomes?
 - Yes: battle of the sexes
- Is there a Pareto improvement from Nash equilibrium?
 - Yes: assurance game
 - Yes, but it's not a NE: prisoners' dilemma



Multiple Equilibria

Multiple Equilibria: What to Do?



- Nash equilibrium is the most well known **solution concept** in game theory
 - Method of predicting the outcome of a game
- Suppose we have a coordination game with multiple equilibria
- What can we say about behavior of players?



Multiple Equilibria: What to Do?



- One answer: nothing!
 - Both equilibria are mutual best responses
 - Coordination problem on which strategy to jointly select
 - Two sides of the road to drive on, no one side better than the other



Multiple Equilibria: What to Do?



- Another answer: we must confront multiple equilibria in economics
 - still want to predict *which* outcome will occur
- We need to consider **multiple criteria** beyond best responses to select a plausible equilibrium
 - Focalness/salience
 - Fairness/envy-free-ness
 - Efficiency/payoff dominance
 - Risk dominance



Multiple Equilibria: Efficiency



- Which equilibrium is most (**Pareto efficient**)?
 - Must be no other equilibrium where at least one player earns a higher payoff and no player earns a lower payoff
- Stag Hunt:
 - Both (**Stag, Stag**) and (**Hare, Hare**) are Nash equilibria
 - (**Stag, Stag**) is Pareto superior to (**Hare, Hare**)

		Column	
		Stag	Hare
Row	Stag	2	0
	Hare	2	1
Row	Stag	1	1
	Hare	0	1

Multiple Equilibria: Efficiency



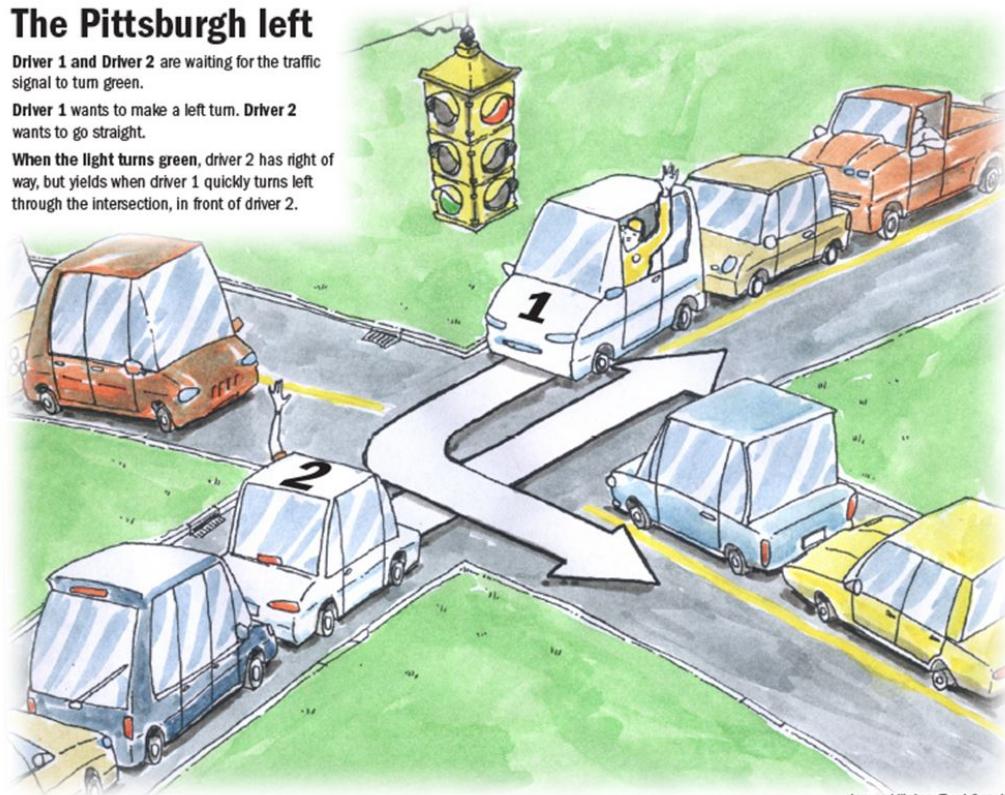
- Consider the “Pittsburgh Left” game

The Pittsburgh left

Driver 1 and Driver 2 are waiting for the traffic signal to turn green.

Driver 1 wants to make a left turn. Driver 2 wants to go straight.

When the light turns green, driver 2 has right of way, but yields when driver 1 quickly turns left through the intersection, in front of driver 2.



James Histon/Post-Gazette

Row

		Column	
		Straight	Yield
		Left	-10
Row	Straight	-10	1
	Yield	-1	-2

Multiple Equilibria: Efficiency



- Consider the “Pittsburgh Left” game
- Two PSNE: (**Left**, Yield) and (**Yield**, **Straight**)
 - Each driver prefers that the other yield
- This is just a variant of **Chicken**
- Both equilibria are Pareto efficient!

		Column	
		Straight	Yield
		-10	1
Row	Left	-10	1
	Yield	-1	-2
		1	-2

Multiple Equilibria: Efficiency



- We often face multiple Pareto efficient equilibria
- Sometimes institutions are created to select and enforce a particular equilibrium



Row

		Column	
		Straight	Yield
		Left	-10
Row	Straight	-10	1
	Yield	-1	-2

Multiple Equilibria: Risk Dominance



- Consider a Stag Hunt
- $(\text{Stag}, \text{Stag})$ is **efficient** and “**payoff dominant**”
 - Highest payoff for each player, no possible Pareto improvement
- $(\text{Hare}, \text{Hare})$ is “**risk dominant**”
 - A less-risky equilibrium
 - By playing **Hare**, each player guarantees themselves 1 regardless of other player's strategy

		Column	
		Stag	Hare
Row	Stag	2 2	0 1
	Hare	1 0	1 1



Rationalizability & the Role of Beliefs

The Role of Beliefs



- Consider the following game

		Column	
		Left	Right
		Up	9
Row	Up	10	9.99
	Down	10	-1000
		10	9.99

The Role of Beliefs



- Consider the following game
- **Column** has a dominant strategy to always play **Left**
- Given this, **Row** should play **Down**
- Unique **Nash equilibrium**: (**Down**, **Left**)

		Column	
		Left	Right
Row	Up	9	8
	Down	<u>10</u>	9.99

		Column	
		Left	Right
Row	Up	9	8
	Down	<u>10</u>	-1000

The Role of Beliefs



- If you were playing as **Row**, would you risk playing **Down** if you believed there was the slightest chance that **Column** would play **Right**?

		Column	
		Left	Right
		Up	9 8
Row	Up	9	8
	Down	10 10	-1000 9.99

Nash Equilibrium and Beliefs



- **Nash equilibrium** requires players to have accurate beliefs about each others' actions
 1. Each player should choose the strategy with the highest-payoff given their beliefs about the other player's (choice of) strategy
 2. These beliefs should be **correct**, i.e. match what the other players **actually do**



Nash Equilibrium and Beliefs



- **Rationalizable** game outcomes are a more general **solution concept** than Nash equilibrium
 - Allows for variations in beliefs
- Nash equilibria are a subset of rationalizable outcomes
 - Where players' maximize their payoff and their beliefs happen to be correct



Rationalizability



- Consider the following game

		Column		
		Left	Middle	Right
Row	Left	0	2	7
	Middle	7	5	0
	Right	5	3	5
Row	Left	2	3	2
	Middle	5	3	2
	Right	7	0	5

Rationalizability



- Consider the following game
- Solved using best response analysis, we see a unique **Nash equilibrium**: (**Middle, Middle**)

		Column		
		Left	Middle	Right
Row	Left	0	2	7
	Middle	7	5	0
	Right	5	3	5
Row	Left	2	3	2
	Middle	5	3	2
	Right	7	2	0
Row	Left	0	5	7
	Middle	7	5	0
	Right	2	0	7

Rationalizability



- Row plays Middle because she **believes** Column will rationally play Middle (who plays that because he believes that Row will play Middle)...
- But players can also **rationalize** other possibilities

		Column		
		Left	Middle	Right
Row	Left	0	2	7
	Middle	7	5	0
	Right	5	3	5
Column	Left	2	3	2
	Middle	5	0	5
	Right	7	5	7

Rationalizability



- For example, **Row** can rationalize playing **Left**
 - If she thinks **Column** will play **Right**, then playing **Left** is her best response
- **Column** can rationalize playing **Right**
 - If he thinks **Row** will play **Right**, then playing **Right** is his best response
- Similarly, we can **rationalize** many game outcomes under certain **beliefs** that players have

		Column		
		Left	Middle	Right
Row	Left	0	2	7
	Middle	7	5	0
	Right	5	3	5
Column	Left	2	3	2
	Middle	5	3	2
	Right	7	0	5

Rationalizability



- In this particular game (i.e. not every game!), all 9 outcomes are rationalizable!

(1) (**Left**, **Left**): **Row** will play **Left** if she believes **Column** will play **Right**; **Column** will play **Left** if he believes **Row** will play **Left**

(2) (**Left**, **Middle**): **Row** will play **Left** if she believes **Column** will play **Right**; **Column** will play **Middle** if he believes **Row** will play **Middle**

(3) (**Left**, **Right**): **Row** will play **Left** if she believes **Column** will play **Right**; **Column** will play **Right** if he believes **Row** will play **Right**

		Column		
		Left	Middle	Right
Row	Left	0	2	7
	Middle	7	5	0
	Right	5	3	5
Column	Left	2	3	2
	Middle	5	2	0
	Right	7	0	5

Rationalizability



- In this particular game (i.e. not every game!), all 9 outcomes are rationalizable!

(4) (**Middle, Left**): **Row** will play **Middle** if she believes **Column** will play **Middle**; **Column** will play **Left** if he believes **Row** will play **Left**

(5) (**Middle, Middle**): **Row** will play **Middle** if she believes **Column** will play **Middle**; **Column** will play **Middle** if he believes **Row** will play **Middle**

(6) (**Middle, Right**): **Row** will play **Middle** if she believes **Column** will play **Middle**; **Column** will play **Right** if he believes **Row** will play **Right**

		Column		
		Left	Middle	Right
Row	Left	0	2	7
	Middle	7	5	0
	Right	5	3	5
Column	Left	2	3	2
	Middle	5	2	0
	Right	0	5	7

Rationalizability



- In this particular game (i.e. not every game!), all 9 outcomes are rationalizable!

(7) (**Right**, **Left**): **Row** will play **Right** if she believes **Column** will play **Left**; **Column** will play **Left** if he believes **Row** will play **Left**

(8) (**Right**, **Middle**): **Row** will play **Right** if she believes **Column** will play **Left**; **Column** will play **Middle** if he believes **Row** will play **Middle**

(9) (**Right**, **Right**): **Row** will play **Right** if she believes **Column** will play **Left**; **Column** will play **Right** if he believes **Row** will play **Right**

		Column		
		Left	Middle	Right
Row	Left	0	2	7
	Middle	7	5	0
	Right	5	3	5
Column	Left	2	3	2
	Middle	3	2	5
	Right	2	0	7

Rationalizability and Best Reponses



- What is key here is that players can rationalize playing a strategy if it is a **best response to at least one strategy**
- Inversely, **if a strategy is never a best response, playing it is not rationalizable**
- For this game, since each strategy is *sometimes* a best-response, for both players, all 9 outcomes are rationalizable

		Column		
		Left	Middle	Right
Row	Left	0	2	7
	Middle	7	5	0
	Right	5	3	5
Row	Left	2	3	2
	Middle	5	3	2
	Right	7	0	5

Rationalizability and Best Responses



- Rationalizability can *sometimes* find us the Nash equilibrium
- Consider the game with some *different* payoffs

		Column		
		Left	Middle	Right
Row	Left	3 2	0 3	2 0
	Middle	1 3	2 0	1 2
	Right	2 1	4 3	0 2

Rationalizability and Best Responses



- Rationalizability can *sometimes* find us the Nash equilibrium
- Consider the game with some *different* payoffs
- First, find all best responses

		Column		
		Left	Middle	Right
Row	Left	3 2	0 3	2 0
	Middle	1 3	2 0	1 2
Right	Left	2 1	4 3	0 2
	Middle	1 3	2 0	2 2

Rationalizability and Best Responses



- Rationalizability can *sometimes* find us the Nash equilibrium
- Consider the game with some *different* payoffs
- First, find all best responses, and next **delete all strategies that are never a best response**

		Column		
		Left	Middle	Right
Row	Left	3 2	0 3	2 0
	Middle	1 3	2 0	1 2
Right	Left	2 1	4 3	0 2
	Middle	1 3	2 0	2 2

Rationalizability and Best Responses



- Rationalizability can *sometimes* find us the Nash equilibrium
- Consider the game with some *different* payoffs
- First, find all best responses, and next **delete all strategies that are never a best response**
 - Note here there are no strictly dominated strategies!

		Column		
		Left	Middle	Right
Row	Left	3 2	0 3	2 0
	Middle	1 3	2 0	1 2
Right	Left	2 1	4 3	0 2
	Middle	1 3	2 0	2 2

Rationalizability and Best Responses



- Rationalizability can *sometimes* find us the Nash equilibrium
- Consider the game with some *different* payoffs
- First, find all best responses, and next **delete all strategies that are never a best response**
 - Note here there are no strictly dominated strategies!
 - For **Row**, playing **Middle** is never a best response

		Column		
		Left	Middle	Right
Row	Left	3 2	0 3	2 0
	Middle	1 3	2 0	1 2
Right	Left	2 1	4 3	0 2
	Middle	1 3	2 0	2 2

Rationalizability and Best Responses



- Now we see **Column** will not play **Left**

		Column	
		Left	Middle
		Left	0
Row	Left	3	2
	Middle	2	3
Right	Left	2	4
	Middle	1	3

Rationalizability and Best Responses



- Now we see **Row** will not play **Left**

		Column
		Middle
Row	Left	0
	Right	4
		3

Rationalizability and Best Responses



- This brings us to the outcome that is the **Nash equilibrium**: (Right, Middle)

	Column Middle
Row Right	4 3

Rationalizability and Best Responses



- This brings us to the outcome that is the **Nash equilibrium**: (Right, Middle)

		Column		
		Left	Middle	Right
Row	Left	3 2	0 3	2 0
	Middle	1 3	2 0	1 2
	Right	2 1	4 3	0 2

Rationalizability and Best Responses



- We will examine the role of beliefs much more rigorously later in the semester when we consider games with **incomplete information** and **Bayesian games** (and a whole separate set of solution concepts!)

