3.7 — Interaction Effects

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Ryan Safner

Assistant Professor of Economics

safner@hood.edu

ryansafner/metricsF21

metricsF21.classes.ryansafner.com

Outline

Sliders and Switches

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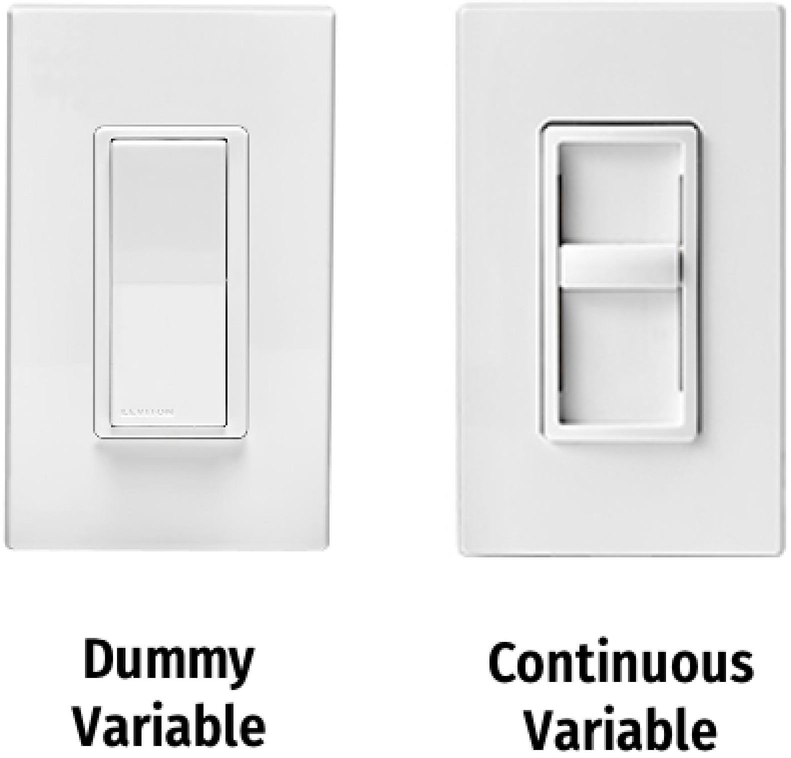


- Marginal effect of dummy variable: effect on Y of going from 0 to 1
- Marginal effect of continuous variable: effect on Y of a 1 unit change in X

Interaction Effects

- Sometimes one X variable might *interact* with another in determining Y

Example: Consider the gender pay gap again.

- *Gender* affects wages
- *Experience* affects wages
- **Does experience affect wages *differently* by gender?**
  - i.e. is there an **interaction effect** between gender and experience?
- **Note this is *NOT the same* as just asking: "do men earn more than women *with the same amount of experience*?"**

$$\widehat{\text{wages}}_i = \beta_0 + \beta_1 \, Gender_i + \beta_2 \, Experience_i$$

Three Types of Interactions

- Depending on the types of variables, there are 3 possible types of interaction effects

- We will look at each in turn

1. Interaction between a **dummy** and a **continuous** variable:

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 D_i + \beta_3 (X_i \times D_i)$$

2. Interaction between a **two dummy** variables:

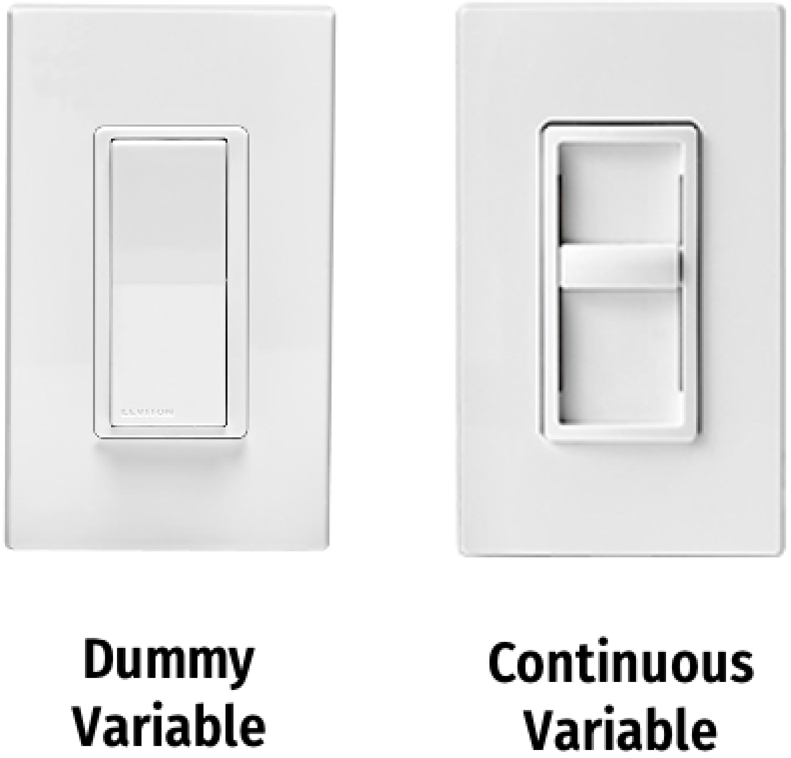
$$Y_i = \beta_0 + \beta_1 D_{1i} + \beta_2 D_{2i} + \beta_3 (D_{1i} \times D_{2i})$$

3. Interaction between a **two continuous** variables:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 (X_{1i} \times X_{2i})$$

Interactions Between a Dummy and Continuous Variable

Interactions: A Dummy & Continuous Variable



- Does the marginal effect of the continuous variable on Y change depending on whether the dummy is “on” or “off”?

Interactions: A Dummy & Continuous Variable I

- We can model an interaction by introducing a variable that is an **interaction term** capturing the interaction between two variables:

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 D_i + \beta_3 (X_i \times D_i) \qquad \text{where } D_i = \{0, 1\}$$

- β_3 estimates the **interaction effect** between X_i and D_i on Y_i

- What do the different coefficients (β)'s tell us?

  - Again, think logically by examining each group $(D_i = 0$ or $D_i = 1)$

Interaction Effects as Two Regressions I

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 D_i + \beta_3 X_i \times D_i$$

- When $D_i = 0$ (Control group):

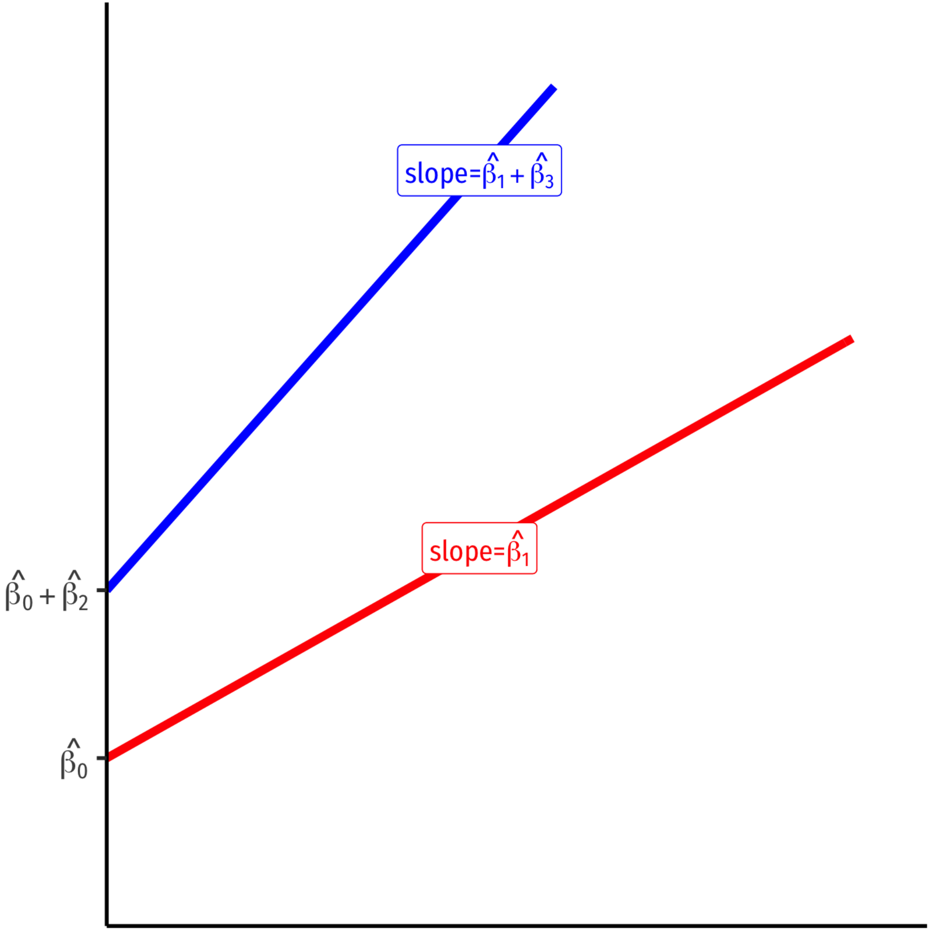
$$\begin{aligned}\hat{Y}_i &= \hat{\beta}_0 + \hat{\beta}_1 X_i + \hat{\beta}_2(0) + \hat{\beta}_3 X_i \times (0) \\ \hat{Y}_i &= \hat{\beta}_0 + \hat{\beta}_1 X_i\end{aligned}$$

- When $D_i = 1$ (Treatment group):

$$\begin{aligned}\hat{Y}_i &= \hat{\beta}_0 + \hat{\beta}_1 X_i + \hat{\beta}_2(1) + \hat{\beta}_3 X_i \times (1) \\ \hat{Y}_i &= (\hat{\beta}_0 + \hat{\beta}_2) + (\hat{\beta}_1 + \hat{\beta}_3) X_i\end{aligned}$$

- So what we really have is *two* regression lines!

Interaction Effects as Two Regressions II



- $D_i = 0$ group:

$$Y_i = \hat{\beta}_0 + \hat{\beta}_1 X_i$$

- $D_i = 1$ group:

$$Y_i = (\hat{\beta}_0 + \hat{\beta}_2) + (\hat{\beta}_1 + \hat{\beta}_3) X_i$$

Interpretting Coefficients I

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 D_i + \beta_3 (X_i \times D_i)$$

- To interpret the coefficients, compare cases after changing X by ΔX:

$$Y_i + \Delta Y_i = \beta_0 + \beta_1 (X_i + \Delta X_i) \beta_2 D_i + \beta_3 \big((X_i + \Delta X_i) D_i\big)$$

- Subtracting these two equations, the difference is:

$$\Delta Y_i = \beta_1 \Delta X_i + \beta_3 D_i \Delta X_i$$
$$\frac{\Delta Y_i}{\Delta X_i} = \beta_1 + \beta_3 D_i$$

- **The effect of $X \rightarrow Y$ depends on the value of D_i!**

- β_3: ***increment* to the effect of $X \rightarrow Y$ when $D_i = 1$ (vs. $D_i = 0$)**

Interpretting Coefficients II

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 D_i + \beta_3 (X_i \times D_i)$$

- $\hat{\beta}_0$: $E[Y_i]$ for $X_i = 0$ and $D_i = 0$
- β_1: Marginal effect of $X_i \rightarrow Y_i$ for $D_i = 0$
- β_2: Marginal effect on Y_i of difference between $D_i = 0$ and $D_i = 1$
- β_3: The **difference** of the marginal effect of $X_i \rightarrow Y_i$ between $D_i = 0$ and $D_i = 1$
- This is a bit awkward, easier to think about the two regression lines:

Interpretting Coefficients III

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 D_i + \beta_3 (X_i \times D_i)$$

For $D_i = 0$ Group: $\hat{Y_i} = \hat{\beta_0} + \hat{\beta_1} X_i$

- Intercept: $\hat{\beta_0}$
- Slope: $\hat{\beta_1}$

For $D_i = 1$ Group:
$\hat{Y_i} = (\hat{\beta_0} + \hat{\beta_2}) + (\hat{\beta_1} + \hat{\beta_3}) X_i$

- Intercept: $\hat{\beta_0} + \hat{\beta_2}$
- Slope: $\hat{\beta_1} + \hat{\beta_3}$

- $\hat{\beta_2}$: difference in intercept between groups
- $\hat{\beta_3}$: difference in slope between groups
- How can we determine if the two lines have the same slope and/or intercept?
  - Same intercept? t-test $H_0: \beta_2 = 0$
  - Same slope? t-test $H_0: \beta_3 = 0$

Example I

Example:

$$\widehat{wage_i} = \hat{\beta}_0 + \hat{\beta}_1 exper_i + \hat{\beta}_2 female_i + \hat{\beta}_3 (exper_i \times female_i)$$

- For males $(female = 0)$:

$$\widehat{wage_i} = \hat{\beta}_0 + \hat{\beta}_1 exper$$

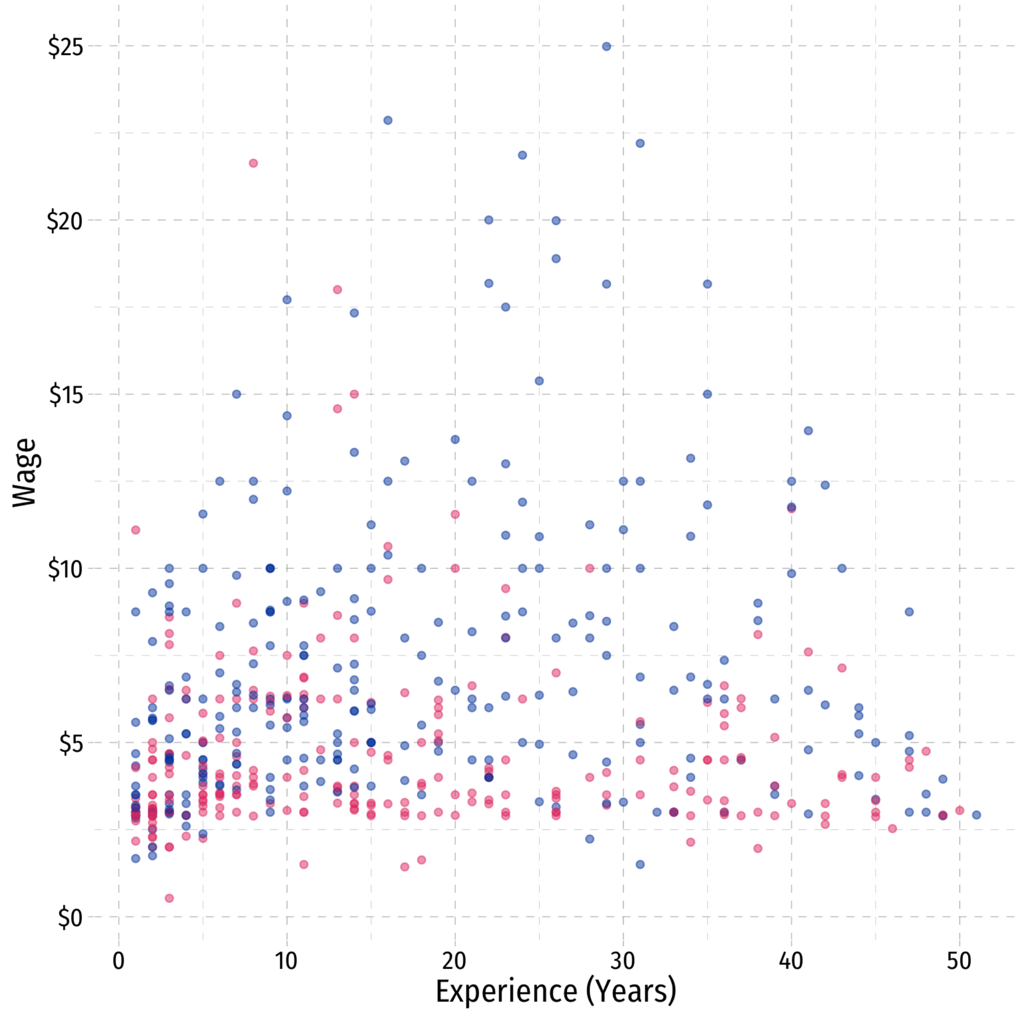
- For females $(female = 1)$:

$$\widehat{wage_i} = \underbrace{(\hat{\beta}_0 + \hat{\beta}_2)}_{\text{intercept}} + \underbrace{(\hat{\beta}_1 + \hat{\beta}_3)}_{\text{slope}} exper$$

Example II

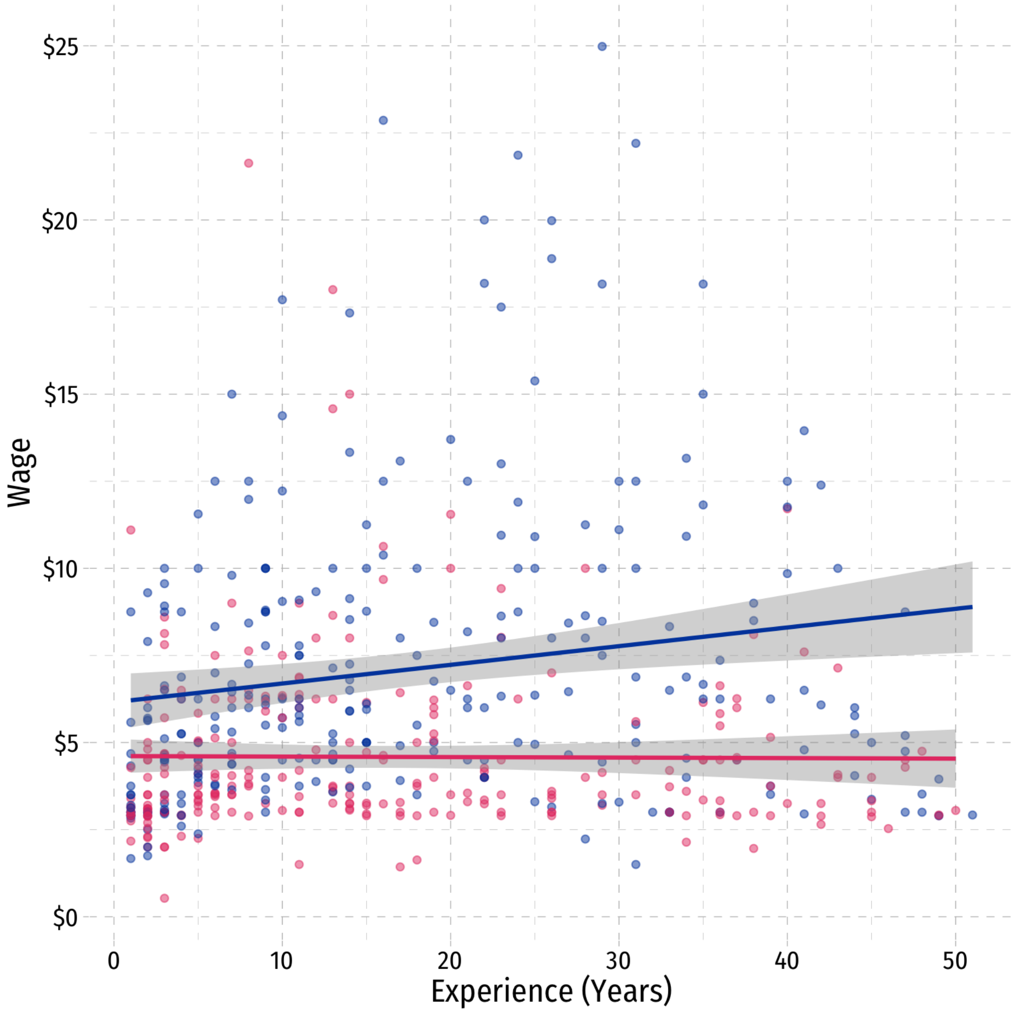
```
interaction_plot <- ggplot(data = wages)+
  aes(x = exper,
      y = wage,
      color = as.factor(Gender))+ # make factor
  geom_point(alpha = 0.5)+
  scale_y_continuous(labels=scales::dollar)+
  labs(x = "Experience (Years)",
       y = "Wage")+
  scale_color_manual(values = c("Female" = "#e64173",
                                "Male" = "#0047AB")
                     )+ # setting custom colors
  guides(color=F)+ # hide legend
  theme_slides
interaction_plot
```

- Need to make sure `color` aesthetic uses a `factor` variable
    - Can just use `as.factor()` in ggplot code



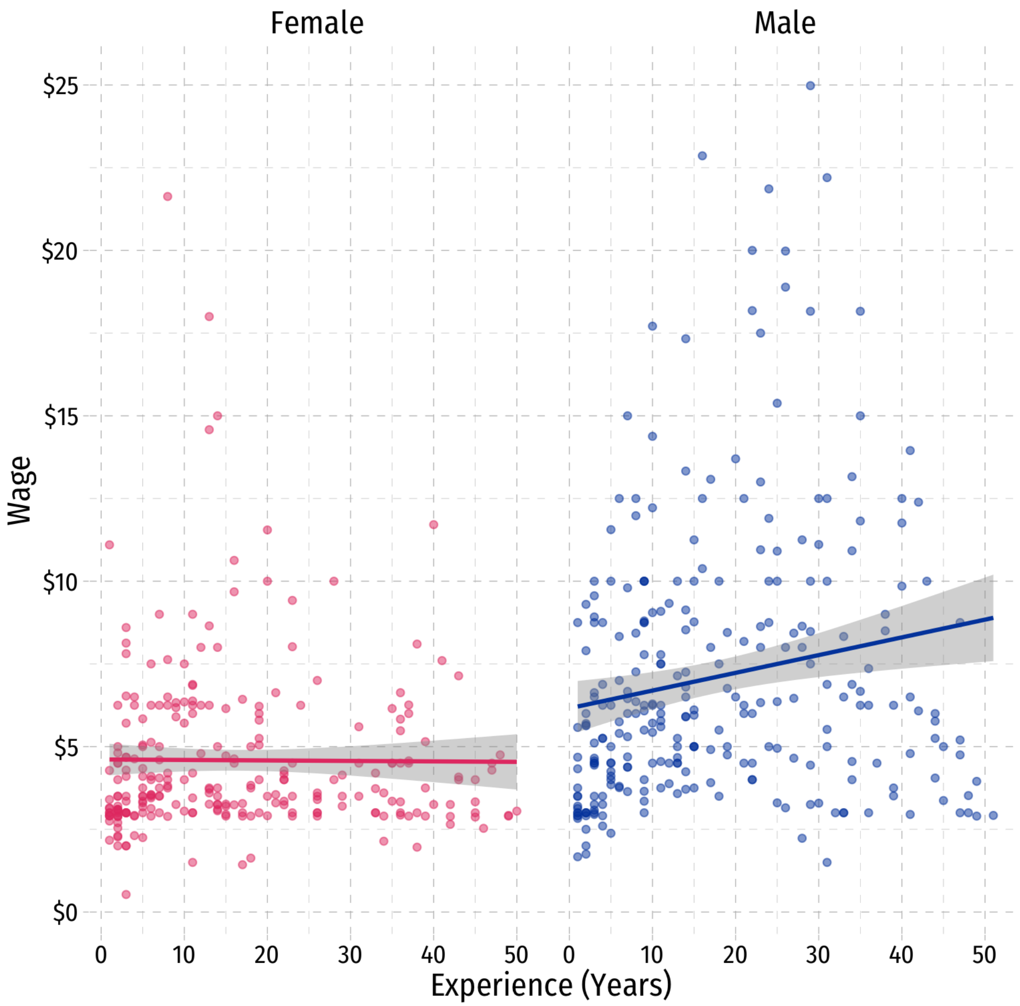
Example II

```
interaction_plot+
  geom_smooth(method="lm")
```



Example II

```r
interaction_plot+
  geom_smooth(method="lm")+
  facet_wrap(~Gender)
```



Example Regression in R I

- Syntax for adding an interaction term is easy in `R`: `var1 * var2`
  - Or could just do `var1 * var2` (multiply)

```r
# both are identical in R
interaction_reg <- lm(wage ~ exper * female, data = wages)
interaction_reg <- lm(wage ~ exper + female + exper * female, data = wages)
```

term	estimate	std.error	statistic	p.value
<chr>	<dbl>	<dbl>	<dbl>	<dbl>
(Intercept)	6.15827549	0.34167408	18.023830	7.998534e-57
exper	0.05360476	0.01543716	3.472450	5.585255e-04
female	-1.54654677	0.48186030	-3.209534	1.411253e-03
exper:female	-0.05506989	0.02217496	-2.483427	1.332533e-02

4 rows

Example Regression in R III

```
library(huxtable)
huxreg(interaction_reg,
       coefs = c("Constant" = "(Intercept)",
                 "Experience" = "exper",
                 "Female" = "female",
                 "Experience * Female" = "exper:female"),
       statistics = c("N" = "nobs",
                      "R-Squared" = "r.squared",
                      "SER" = "sigma"),
       number_format = 2)
```

	(1)
Constant	6.16 ***
	(0.34)
Experience	0.05 ***
	(0.02)
Female	-1.55 **
	(0.48)
Experience * Female	-0.06 *
	(0.02)
N	526
R-Squared	0.14
SER	3.44

Example Regression in R: Interpretting Coefficients

$$\widehat{wage_i} = 6.16 + 0.05\,Experience_i - 1.55\,Female_i - 0.06\,(Experience_i \times Female_i)$$

- $\hat{\beta}_0$:

Example Regression in R: Interpretting Coefficients

$$\widehat{wage_i} = 6.16 + 0.05\,Experience_i - 1.55\,Female_i - 0.06\,(Experience_i \times Female_i)$$

- $\hat{\beta}_0$: **Men** with 0 years of experience earn 6.16

- $\hat{\beta}_1$:

Example Regression in R: Interpretting Coefficients

$$\widehat{wage_i} = 6.16 + 0.05\, Experience_i - 1.55\, Female_i - 0.06\, (Experience_i \times Female_i)$$

- $\hat{\beta}_0$: **Men** with 0 years of experience earn 6.16
- $\hat{\beta}_1$: For every additional year of experience, **men** earn \$0.05
- $\hat{\beta}_2$:

Example Regression in R: Interpretting Coefficients

$$\widehat{wage_i} = 6.16 + 0.05\, Experience_i - 1.55\, Female_i - 0.06\, (Experience_i \times Female_i)$$

- $\hat{\beta}_0$: **Men** with 0 years of experience earn 6.16
- $\hat{\beta}_1$: For every additional year of experience, **men** earn \$0.05
- $\hat{\beta}_2$: **Women** with 0 years of experience earn \$1.55 **less than men**
- $\hat{\beta}_3$:

Example Regression in R: Interpretting Coefficients

$$\widehat{wage_i} = 6.16 + 0.05\,Experience_i - 1.55\,Female_i - 0.06\,(Experience_i \times Female_i)$$

- $\hat{\beta}_0$: **Men** with 0 years of experience earn 6.16
- $\hat{\beta}_1$: For every additional year of experience, **men** earn \$0.05
- $\hat{\beta}_2$: **Women** with 0 years of experience earn \$1.55 **less than men**
- $\hat{\beta}_3$: **Women** earn \$0.06 **less than men** for every additional year of experience

Interpretting Coefficients as 2 Regressions I

$$\widehat{wage_i} = 6.16 + 0.05\, Experience_i - 1.55\, Female_i - 0.06\, (Experience_i \times Female_i)$$

Regression for men $(female = 0)$

$$\widehat{wage_i} = 6.16 + 0.05\, Experience_i$$

- Men with 0 years of experience earn \$6.16 on average
- For every additional year of experience, men earn \$0.05 more on average

Interpretting Coefficients as 2 Regressions II

$$\widehat{wage_i} = 6.16 + 0.05\, Experience_i - 1.55\, Female_i - 0.06\,(Experience_i \times Female_i)$$

Regression for women $(female = 1)$

$$\begin{aligned}\widehat{wage_i} &= 6.16 + 0.05\, Experience_i - 1.55(1) - 0.06\, Experience_i \times (1)\\ &= (6.16 - 1.55) + (0.05 - 0.06)\, Experience_i\\ &= 4.61 - 0.01\, Experience_i\end{aligned}$$

- Women with 0 years of experience earn \$4.61 on average
- For every additional year of experience, women earn \$0.01 *less* on average

Example Regression in R: Hypothesis Testing

- Are slopes & intercepts of the 2 regressions statistically significantly different?

$$
\begin{aligned}
\widehat{wage_i} &= 6.16 + 0.05\, Experience_i - 1.55\, Female_i \\
&\quad - 0.06\, (Experience_i \times Female_i)
\end{aligned}
$$

```
## # A tibble: 4 × 5
##   term          estimate std.error statistic  p.val
##   <chr>            <dbl>     <dbl>     <dbl>    <db
## 1 (Intercept)     6.16      0.342      18.0   8.00e-
## 2 exper           0.0536    0.0154      3.47 5.59e-
## 3 female         -1.55      0.482      -3.21 1.41e-
## 4 exper:female   -0.0551    0.0222     -2.48 1.33e-
```

Example Regression in R: Hypothesis Testing

- Are slopes & intercepts of the 2 regressions statistically significantly different?

- Are intercepts different? $H_0 : \beta_2 = 0$
  - Difference between men vs. women for no experience?
  - Is $\hat{\beta}_2$ significant?
  - Yes (reject) H_0: p-value = 0.00

$$\begin{aligned}\widehat{wage_i} &= 6.16 + 0.05\, Experience_i - 1.55\, Female_i \\ &- 0.06\, (Experience_i \times Female_i)\end{aligned}$$

```
## # A tibble: 4 × 5
##   term          estimate std.error statistic  p.val
##   <chr>            <dbl>     <dbl>     <dbl>    <db
## 1 (Intercept)    6.16      0.342      18.0  8.00e-
## 2 exper          0.0536    0.0154      3.47 5.59e-
## 3 female        -1.55      0.482      -3.21 1.41e-
## 4 exper:female  -0.0551    0.0222     -2.48 1.33e-
```

Example Regression in R: Hypothesis Testing

- Are slopes & intercepts of the 2 regressions statistically significantly different?

- Are intercepts different? $H_0: \beta_2 = 0$
  - Difference between men vs. women for no experience?
  - Is $\hat{\beta}_2$ significant?
  - Yes (reject) H_0: p-value = 0.00

- Are slopes different? $H_0: \beta_3 = 0$
  - Difference between men vs. women for marginal effect of experience?
  - Is $\hat{\beta}_3$ significant?
  - Yes (reject) H_0: p-value = 0.01

$$\begin{aligned}\widehat{wage_i} &= 6.16 + 0.05\, Experience_i - 1.55\, Female_i \\ &\quad - 0.06\,(Experience_i \times Female_i)\end{aligned}$$

```
## # A tibble: 4 × 5
##   term          estimate std.error statistic  p.val
##   <chr>            <dbl>     <dbl>     <dbl>    <db
## 1 (Intercept)     6.16      0.342      18.0  8.00e-
## 2 exper           0.0536    0.0154      3.47 5.59e-
## 3 female         -1.55      0.482      -3.21 1.41e-
## 4 exper:female   -0.0551    0.0222     -2.48 1.33e-
```

Interactions Between Two Dummy Variables

Interactions Between Two Dummy Variables

- Does the marginal effect on Y of one dummy going from “off” to “on” change depending on whether the *other* dummy is “off” or “on”?

Interactions Between Two Dummy Variables

$$Y_i = \beta_0 + \beta_1 D_{1i} + \beta_2 D_{2i} + \beta_3 (D_{1i} \times D_{2i})$$

- D_{1i} and D_{2i} are dummy variables
- $\hat{\beta}_1$: effect on Y of going from $D_{1i} = 0$ to $D_{1i} = 1$ when $D_{2i} = 0$
- $\hat{\beta}_2$: effect on Y of going from $D_{2i} = 0$ to $D_{2i} = 1$ when $D_{1i} = 0$
- $\hat{\beta}_3$: effect on Y of going from $D_{1i} = 0$ to $D_{1i} = 1$ when $D_{2i} = 1$
    - *increment* to the effect of D_{1i} going from 0 to 1 when $D_{2i} = 1$ (vs. 0)
- As always, best to think logically about possibilities (when each dummy $= 0$ or $= 1$)

2 Dummy Interaction: Interpretting Coefficients

$$Y_i = \beta_0 + \beta_1 D_{1i} + \beta_2 D_{2i} + \beta_3 (D_{1i} \times D_{2i})$$

- To interpret coefficients, compare cases:
  - Hold D_{2i} constant (set to some value $D_{2i} = \mathbf{d_2}$)
  - Plug in 0s or 1s for D_{1i}

$$E(Y_i | D_{1i} = 0, D_{2i} = \mathbf{d_2}) = \beta_0 + \beta_2 \mathbf{d_2}$$
$$E(Y_i | D_{1i} = 1, D_{2i} = \mathbf{d_2}) = \beta_0 + \beta_1(1) + \beta_2 \mathbf{d_2} + \beta_3(1)\mathbf{d_2}$$

- Subtracting the two, the difference is:

$$\beta_1 + \beta_3 \mathbf{d_2}$$

- **The marginal effect of $D_{1i} \rightarrow Y_i$ depends on the value of D_{2i}**
  - $\hat{\beta}_3$ is the *increment* to the effect of D_1 on Y when D_2 goes from 0 to 1

Interactions Between 2 Dummy Variables: Example

Example: Does the gender pay gap change if a person is married vs. single?

$$\widehat{wage_i} = \hat{\beta}_0 + \hat{\beta}_1 female_i + \hat{\beta}_2 married_i + \hat{\beta}_3 (female_i \times married_i)$$

- Logically, there are 4 possible combinations of $female_i = \{0, 1\}$ and $married_i = \{0, 1\}$

1) **Unmarried men** ($female_i = 0$, $married_i = 0$)

$$\widehat{wage_i} = \hat{\beta}_0$$

Interactions Between 2 Dummy Variables: Example

Example: Does the gender pay gap change if a person is married vs. single?

$$\widehat{wage_i} = \hat{\beta}_0 + \hat{\beta}_1 female_i + \hat{\beta}_2 married_i + \hat{\beta}_3 (female_i \times married_i)$$

- Logically, there are 4 possible combinations of $female_i = \{0, 1\}$ and $married_i = \{0, 1\}$

1) **Unmarried men** ($female_i = 0$, $married_i = 0$)

$$\widehat{wage_i} = \hat{\beta}_0$$

2) **Married men** ($female_i = 0$, $married_i = 1$)

$$\widehat{wage_i} = \hat{\beta}_0 + \hat{\beta}_2$$

3) **Unmarried women** ($female_i = 1$, $married_i = 0$)

$$\widehat{wage_i} = \hat{\beta}_0 + \hat{\beta}_1$$

Interactions Between 2 Dummy Variables: Example

Example: Does the gender pay gap change if a person is married vs. single?

$$\widehat{wage_i} = \hat{\beta}_0 + \hat{\beta}_1 female_i + \hat{\beta}_2 married_i + \hat{\beta}_3 (female_i \times married_i)$$

- Logically, there are 4 possible combinations of $female_i = \{0, 1\}$ and $married_i = \{0, 1\}$

1) **Unmarried men** ($female_i = 0$, $married_i = 0$)

$$\widehat{wage_i} = \hat{\beta}_0$$

2) **Married men** ($female_i = 0$, $married_i = 1$)

$$\widehat{wage_i} = \hat{\beta}_0 + \hat{\beta}_2$$

3) **Unmarried women** ($female_i = 1$, $married_i = 0$)

$$\widehat{wage_i} = \hat{\beta}_0 + \hat{\beta}_1$$

4) **Married women** ($female_i = 1$, $married_i = 1$)

$$\widehat{wage_i} = \hat{\beta}_0 + \hat{\beta}_1 + \hat{\beta}_2 + \hat{\beta}_3$$

Looking at the Data

```r
# get average wage for unmarried men
wages %>%
  filter(female == 0,
         married == 0) %>%
  summarize(mean = mean(wage))
```

```
##       mean
## 1 5.168023
```

```r
# get average wage for married men
wages %>%
  filter(female == 0,
         married == 1) %>%
  summarize(mean = mean(wage))
```

```
##       mean
## 1 7.983032
```

```r
# get average wage for unmarried women
wages %>%
  filter(female == 1,
         married == 0) %>%
  summarize(mean = mean(wage))
```

```
##       mean
## 1 4.611583
```

```r
# get average wage for married women
wages %>%
  filter(female == 1,
         married == 1) %>%
  summarize(mean = mean(wage))
```

```
##       mean
## 1 4.565909
```

Two Dummies Interaction: Group Means

$$\widehat{wage_i} = \hat{\beta}_0 + \hat{\beta}_1 female_i + \hat{\beta}_2 married_i + \hat{\beta}_3 (female_i \times married_i)$$

	Men	Women
Unmarried	\$5.17	\$4.61
Married	\$7.98	\$4.57

Interactions Between Two Dummy Variables: In R I

```r
reg_dummies <- lm(wage ~ female + married + female:married, data = wages)
reg_dummies %>% tidy()
```

```
## # A tibble: 4 × 5
##   term           estimate std.error statistic  p.value
##   <chr>             <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)       5.17      0.361     14.3  2.26e-39
## 2 female           -0.556     0.474     -1.18 2.41e- 1
## 3 married           2.82      0.436      6.45 2.53e-10
## 4 female:married   -2.86      0.608     -4.71 3.20e- 6
```

Interactions Between Two Dummy Variables: In R II

```
library(huxtable)
huxreg(reg_dummies,
       coefs = c("Constant" = "(Intercept)",
                 "Female" = "female",
                 "Married" = "married",
                 "Female * Married" = "female:marr
       statistics = c("N" = "nobs",
                      "R-Squared" = "r.squared",
                      "SER" = "sigma"),
       number_format = 2)
```

	(1)
Constant	5.17 ***
	(0.36)
Female	-0.56
	(0.47)
Married	2.82 ***
	(0.44)
Female * Married	-2.86 ***
	(0.61)
N	526
R-Squared	0.18
SER	3.35

*** p < 0.001; ** p < 0.01; * p < 0.05.

2 Dummies Interaction: Interpretting Coefficients

$$\widehat{wage_i} = 5.17 - 0.56\,female_i + 2.82\,married_i - 2.86\,(female_i \times married_i)$$

	Men	Women
Unmarried	\$5.17	\$4.61
Married	\$7.98	\$4.57

- Wage for **unmarried** **men**: $\hat{\beta}_0 = 5.17$
- Wage for **married** **men**: $\hat{\beta}_0 + \hat{\beta}_2 = 5.17 + 2.82 = 7.98$
- Wage for **unmarried** **women**: $\hat{\beta}_0 + \hat{\beta}_1 = 5.17 - 0.56 = 4.61$
- Wage for **married** **women**: $\hat{\beta}_0 + \hat{\beta}_1 + \hat{\beta}_2 + \hat{\beta}_3 = 5.17 - 0.56 + 2.82 - 2.86 = 4.57$

2 Dummies Interaction: Interpretting Coefficients

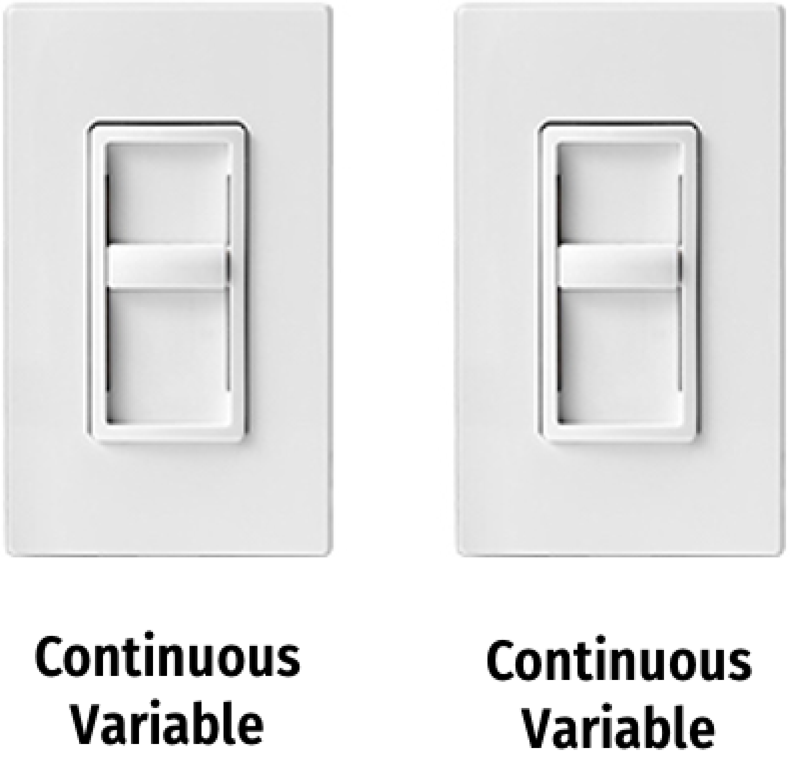
$$\widehat{wage_i} = 5.17 - 0.56\, female_i + 2.82\, married_i - 2.86\, (female_i \times married_i)$$

	Men	Women
Unmarried	\$5.17	\$4.61
Married	\$7.98	\$4.57

- $\hat{\beta}_0$: Wage for **unmarried** **men**
- $\hat{\beta}_2$: Effect of **marriage** on wages for **men**
- $\hat{\beta}_2$: *Difference* in wages between **men** and **women** who are **unmarried**
- $\hat{\beta}_3$: *Difference* in:
  - effect of **Marriage** on wages between **men** and **women**
  - effect of **Gender** on wages between **unmarried** and **married** individuals

Interactions Between Two Continuous Variables

Interactions Between Two Continuous Variables



- Does the marginal effect of X_1 on Y depend on what X_2 is set to?

Interactions Between Two Continuous Variables

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 (X_{1i} \times X_{2i})$$

- To interpret coefficients, compare changes after changing ΔX_{1i} (holding X_2 constant):

$$Y_i + \Delta Y_i = \beta_0 + \beta_1 (X_1 + \Delta X_{1i}) \beta_2 X_{2i} + \beta_3 ((X_{1i} + \Delta X_{1i}) \times X_{2i})$$

- Take the difference to get:

$$\begin{aligned} \Delta Y_i &= \beta_1 \Delta X_{1i} + \beta_3 X_{2i} \Delta X_{1i} \\ \frac{\Delta Y_i}{\Delta X_{1i}} &= \beta_1 + \beta_3 X_{2i} \end{aligned}$$

- **The effect of $X_1 \rightarrow Y_i$ depends on X_2**
    - β_3: *increment* to the effect of $X_1 \rightarrow Y_i$ for every 1 unit change in X_2

Continuous Variables Interaction: Example

Example: Do education and experience interact in their determination of wages?

$$\widehat{wage_i} = \hat{\beta}_0 + \hat{\beta}_1\, educ_i + \hat{\beta}_2\, exper_i + \hat{\beta}_3(educ_i \times exper_i)$$

- Estimated effect of education on wages depends on the amount of experience (and vice versa)!

$$\frac{\Delta wage}{\Delta educ} = \hat{\beta}_1 + \beta_3\, exper_i$$

$$\frac{\Delta wage}{\Delta exper} = \hat{\beta}_2 + \beta_3\, educ_i$$

- This is a type of nonlinearity (we will examine nonlinearities next lesson)

Continuous Variables Interaction: In R I

```
reg_cont <- lm(wage ~ educ + exper + educ:exper, data = wages)
reg_cont %>% tidy()
```

```
## # A tibble: 4 × 5
##   term        estimate std.error statistic  p.value
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept) -2.86      1.18       -2.42  1.58e- 2
## 2 educ         0.602     0.0899      6.69  5.64e-11
## 3 exper        0.0458    0.0426      1.07  2.83e- 1
## 4 educ:exper   0.00206   0.00349     0.591 5.55e- 1
```

Continuous Variables Interaction: In R II

```
library(huxtable)
huxreg(reg_cont,
       coefs = c("Constant" = "(Intercept)",
                 "Education" = "educ",
                 "Experience" = "exper",
                 "Education * Experience" = "educ:exper"),
       statistics = c("N" = "nobs",
                      "R-Squared" = "r.squared",
                      "SER" = "sigma"),
       number_format = 3)
```

	(1)
Constant	-2.860 *
	(1.181)
Education	0.602 ***
	(0.090)
Experience	0.046
	(0.043)
Education * Experience	0.002
	(0.003)
N	526
R-Squared	0.226
SER	3.259

*** p < 0.001; ** p < 0.01; * p < 0.05.

Continuous Variables Interaction: Marginal Effects

$$\widehat{wages}_i = -2.860 + 0.602\, educ_i + 0.047\, exper_i + 0.002\, (educ_i \times exper_i)$$

Marginal Effect of *Education* on Wages by Years of *Experience*:

Experience	$\frac{\Delta wage}{\Delta educ} = \hat{\beta}_1 + \hat{\beta}_3\, exper$
5 years	$0.602 + 0.002(5) = 0.612$
10 years	$0.602 + 0.002(10) = 0.622$
15 years	$0.602 + 0.002(15) = 0.632$

- Marginal effect of education $\rightarrow$ wages **increases** with more experience

Continuous Variables Interaction: Marginal Effects

$$\widehat{wages}_i = -2.860 + 0.602\, educ_i + 0.047\, exper_i + 0.002\, (educ_i \times exper_i)$$

Marginal Effect of *Experience* on Wages by Years of *Education*:

Education	$\frac{\Delta wage}{\Delta exper} = \hat{\beta}_2 + \hat{\beta}_3\, educ$
5 years	$0.047 + 0.002(5) = 0.057$
10 years	$0.047 + 0.002(10) = 0.067$
15 years	$0.047 + 0.002(15) = 0.077$

- Marginal effect of experience $\rightarrow$ wages **increases** with more education
- If you want to estimate the marginal effects more precisely, and graph them, see the appendix in today's class page