

1.7 – Income & Substitution Effects

ECON 306 • Microeconomic Analysis • Fall 2020

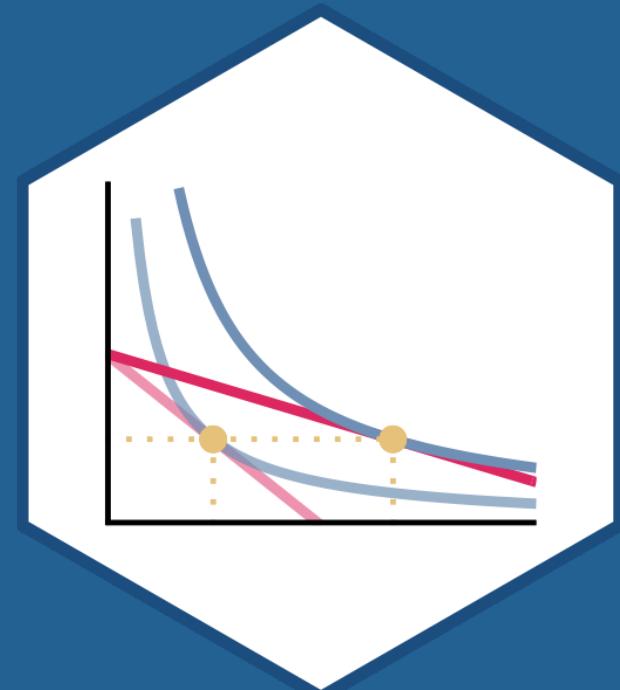
Ryan Safner

Assistant Professor of Economics

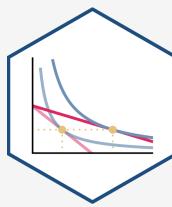
 safner@hood.edu

 [ryansafner/microF20](https://github.com/ryansafner/microF20)

 microF20.classes.ryansafner.com



A Demand Function (Again)

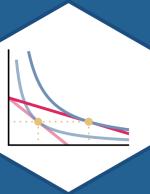


- A consumer's **demand** (for good x) depends on current prices & income:

$$q_x^D = q_x^D(m, p_x, p_y)$$

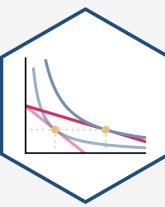
- How does **demand for x** change?
 1. **Income effects** $(\frac{\partial q_x^D}{\partial m})$: how (q_x^D) changes with changes in income
 2. **Cross-price effects** $(\frac{\partial q_x^D}{\partial p_y})$: how (q_x^D) changes with changes in prices of *other* goods (e.g. (y))
 3. **(Own) Price effects** $(\frac{\partial q_x^D}{\partial p_x})$: how (q_x^D) changes with changes in price (of (x))





The (Own) Price Effect

The (Own) Price Effect



- **Price effect:** change in optimal consumption of a good associated with a change in its price, holding income and other prices constant

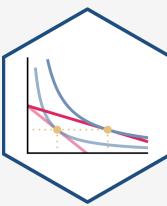
$$\frac{\Delta q_x^D}{\Delta p_x} < 0$$

The law of demand: as the price of a good rises, people will tend to buy less of that good (and vice versa)

- i.e. **the price effect is negative!**



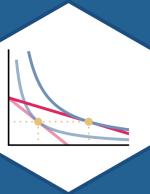
Decomposing the Price Effect



The **price effect** (law of demand) is actually the **net result of two effects**

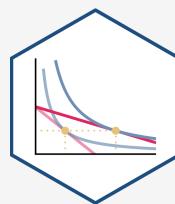
1. **(Real) income effect**: change in consumption due to change in real purchasing power
2. **Substitution effect**: change in consumption due to change in relative prices

Price Effect \((=)\) Real income effect \((+\)\) Substitution Effect



(Real) Income Effect

(Real) Income Effect: Demonstration

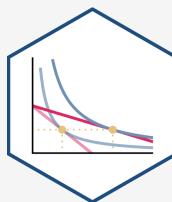


- Suppose there is only 1 good to consume, $\{x\}$. You have a \$100 income, and the price of $\{x\}$ is \$10. You consume 10 units of $\{x\}$
- Suppose the price of $\{x\}$ falls to \$5. You now consume 20 units of $\{x\}$.
- This is the **real income effect**



© BCCL 2014. ALL RIGHTS RESERVED.

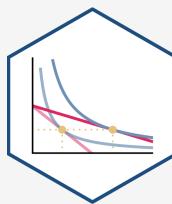
(Real) Income Effect: Demonstration



- **Real income effect:** your consumption mix changes because of the change in the price of $\backslash(x\backslash)$ changes your **real income** or **purchasing power** (the amount of goods you can buy)
- Note your **actual(nominal) income (\$100) never changed!**



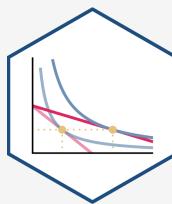
(Real) Income Effect: Size



- The *size* of the income effect depends on how large a *portion of your budget* you spend on the good
- **Large-budget items:**
 - e.g. Housing/apartment rent, car prices
 - Price increase makes you much poorer
 - Price decrease makes you much wealthier

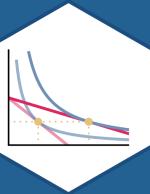


(Real) Income Effect: Size



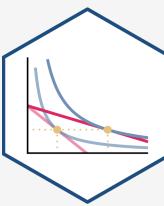
- The *size* of the income effect depends on how large a *portion of your budget* you spend on the good
- **Small-budget items:**
 - e.g. pencils, toothpicks, candy
 - Price changes don't have much of an effect on your wealth or change your behavior much





Substitution Effect

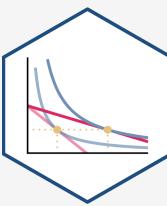
Substitution Effect: Demonstration



- Suppose there are 1000's of goods, none of them a major part of your budget
 - So real income effect is insignificant
- Suppose the price of one good, $\backslash(x\backslash)$ increases
- You would consume *less* of $\backslash(x\backslash)$ relative to other goods because $\backslash(x\backslash)$ is now *relatively* more expensive
- That's the **substitution effect**

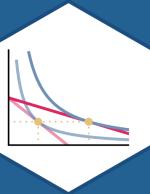


Substitution Effect: Demonstration



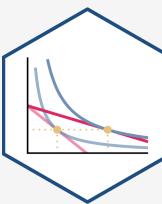
- **Substitution effect:** consumption mix changes because of a change in **relative prices**
- Buy more of the (now) relatively cheaper items
- Buy less of the (now) relatively more expensive item $\backslash((x)\backslash)$





Putting the Effects Together

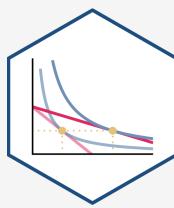
Putting the Effects Together



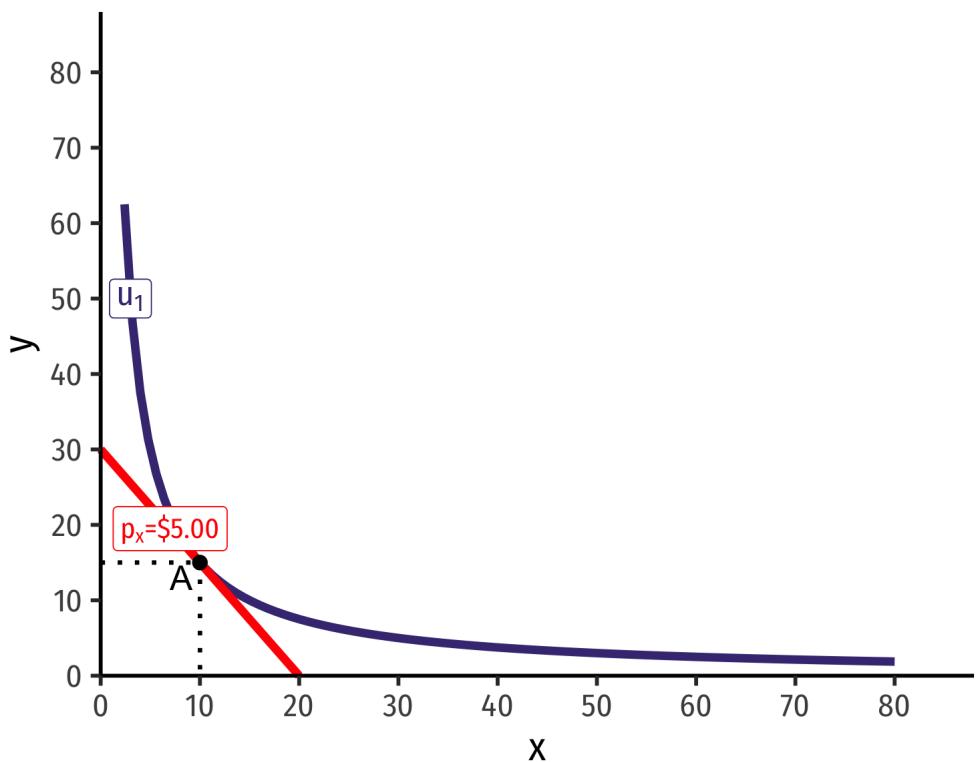
- **Real income effect:** change in consumption due to change in real purchasing power
 - Can be positive (**normal goods**) or negative (**inferior goods**)
 - Lower price of $\backslash(x\backslash)$ means you can buy more $\backslash(x\backslash)$, $\backslash(y\backslash)$, or *both* (depending on your preferences between $\backslash(x\backslash)$ and $\backslash(y\backslash)$)
- **Substitution effect:** change in consumption due to change in relative prices
 - If $\backslash(x\backslash)$ gets cheaper relative to $\backslash(y\backslash)$, consume $\backslash(\downarrow y\backslash)$ (and $\backslash(\uparrow x\backslash)$)
 - This is always the same direction! $\backslash((\downarrow)\backslash$ relatively expensive goods, $\backslash(\uparrow)\backslash$ relatively cheaper goods)
 - This is why demand curves slope downwards!

Price Effect $\backslash(=\backslash)$ Real income effect $\backslash(+\backslash)$ Substitution Effect

Real Income and Substitution Effects, Graphically I

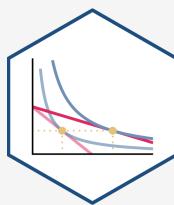


- Original optimal consumption $\backslash((A)\backslash)$

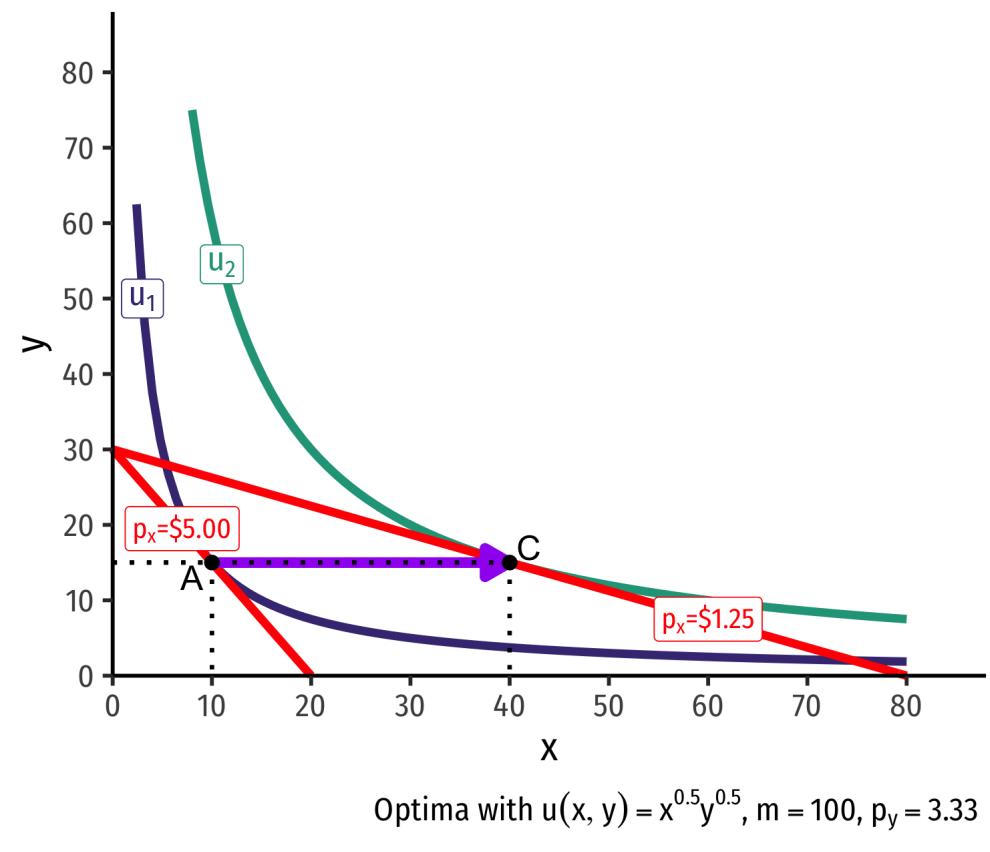


Optima with $u(x, y) = x^{0.5}y^{0.5}$, $m = 100$, $p_y = 3.33$

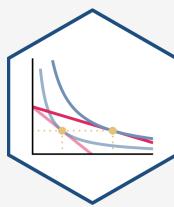
Real Income and Substitution Effects, Graphically I



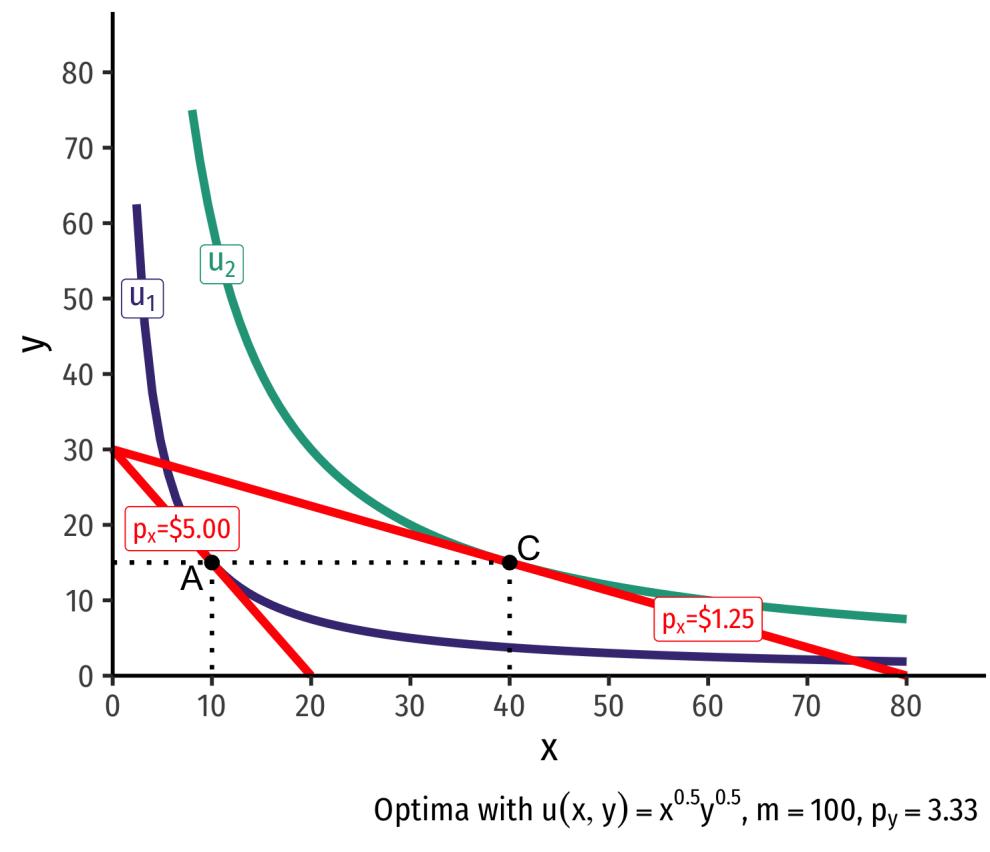
- Original optimal consumption $\langle(A)\rangle$
- **(Total) price effect:** $\langle A \rightarrow C \rangle$
- Let's decompose this into the two effects



Real Income and Substitution Effects, Graphically II

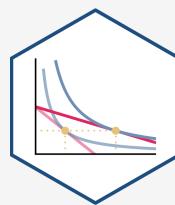


- **Substitution effect:** what you would choose under the **new exchange rate** to **remain indifferent** as before the change

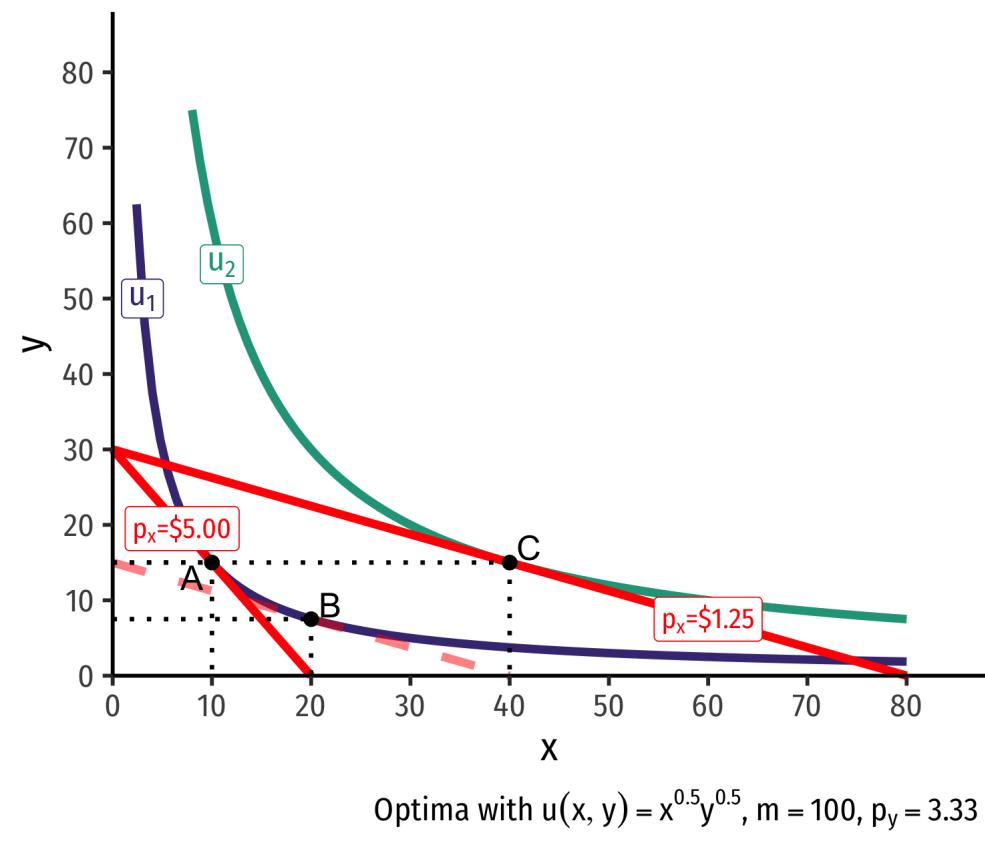


$$\text{Optima with } u(x, y) = x^{0.5}y^{0.5}, m = 100, p_y = 3.33$$

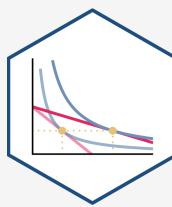
Real Income and Substitution Effects, Graphically II



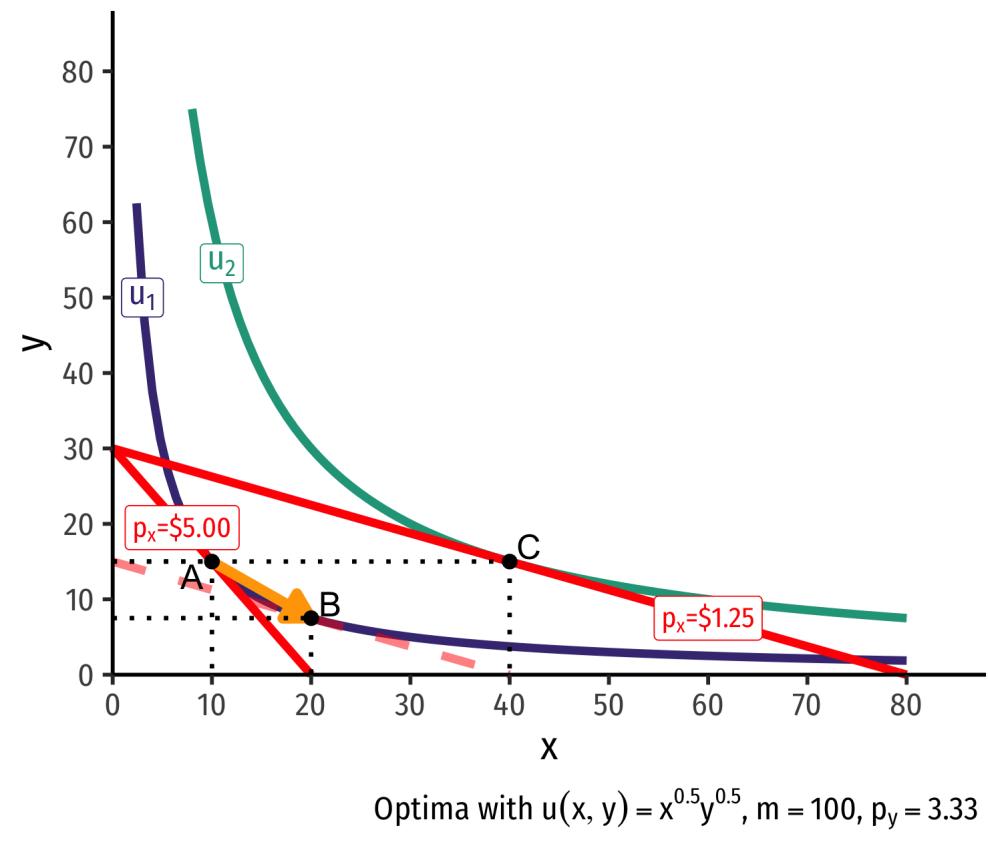
- **Substitution effect:** what you would choose under the **new exchange rate** to **remain indifferent** as before the change
- Graphically: shift *new* budget constraint inwards until tangent with *old* indifference curve
- $\backslash(A \rightarrowtail B)$ on same I.C. $\backslash(\uparrow x), \backslash(\downarrow y)$
 - Point B *must* be a *different* point on the original curve!



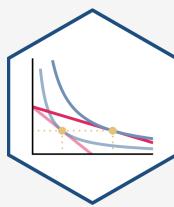
Real Income and Substitution Effects, Graphically II



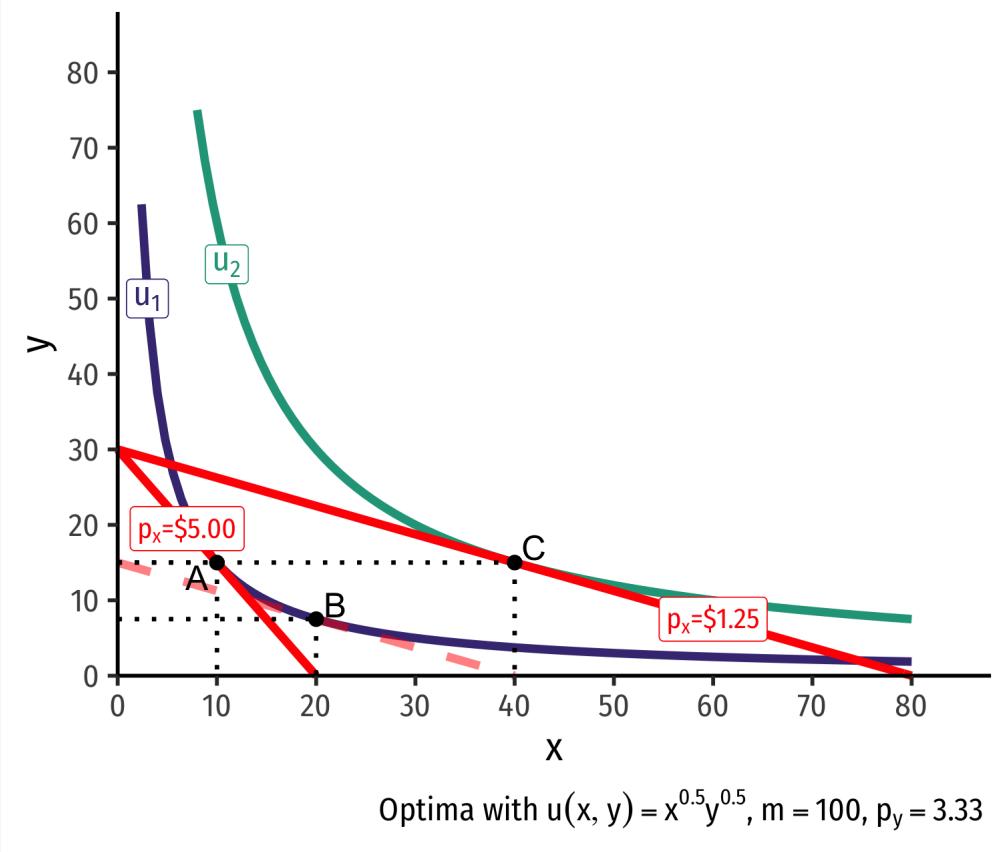
- **Substitution effect:** what you would choose under the **new exchange rate** to **remain indifferent** as before the change
- Graphically: shift *new* budget constraint inwards until tangent with *old* indifference curve
- $\backslash(A \rightarrowtail B)$ on same I.C. $\backslash(\uparrow x), \backslash(\downarrow y)$
 - Point B *must* be a *different* point on the original curve!



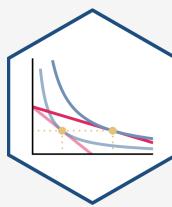
Real Income and Substitution Effects, Graphically III



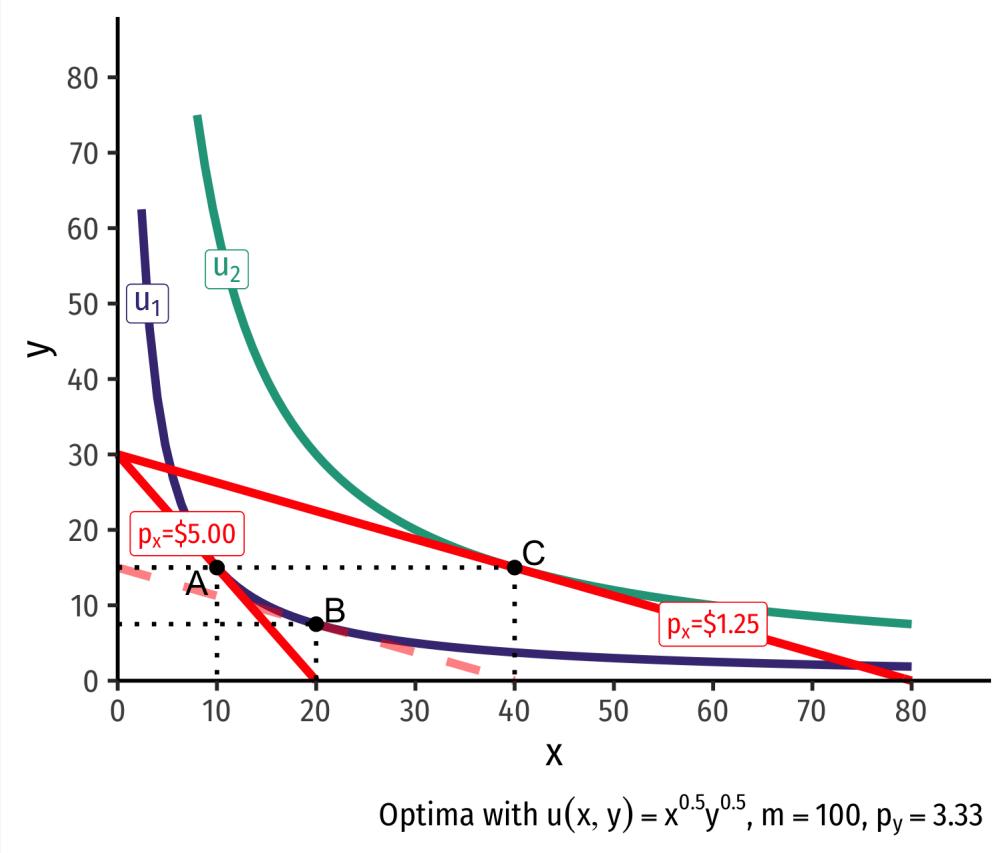
- **(Real) income effect:** change in consumption due to the **change in purchasing power** from the change in price



Real Income and Substitution Effects, Graphically III

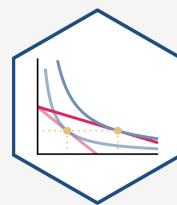


- **(Real) income effect:** change in consumption due to the **change in purchasing power** from the change in price
- $\backslash(B \rightarrow C)$ to new budget constraint (can buy more of $\backslash(x\backslash)$ and/or $\backslash(y\backslash)$)

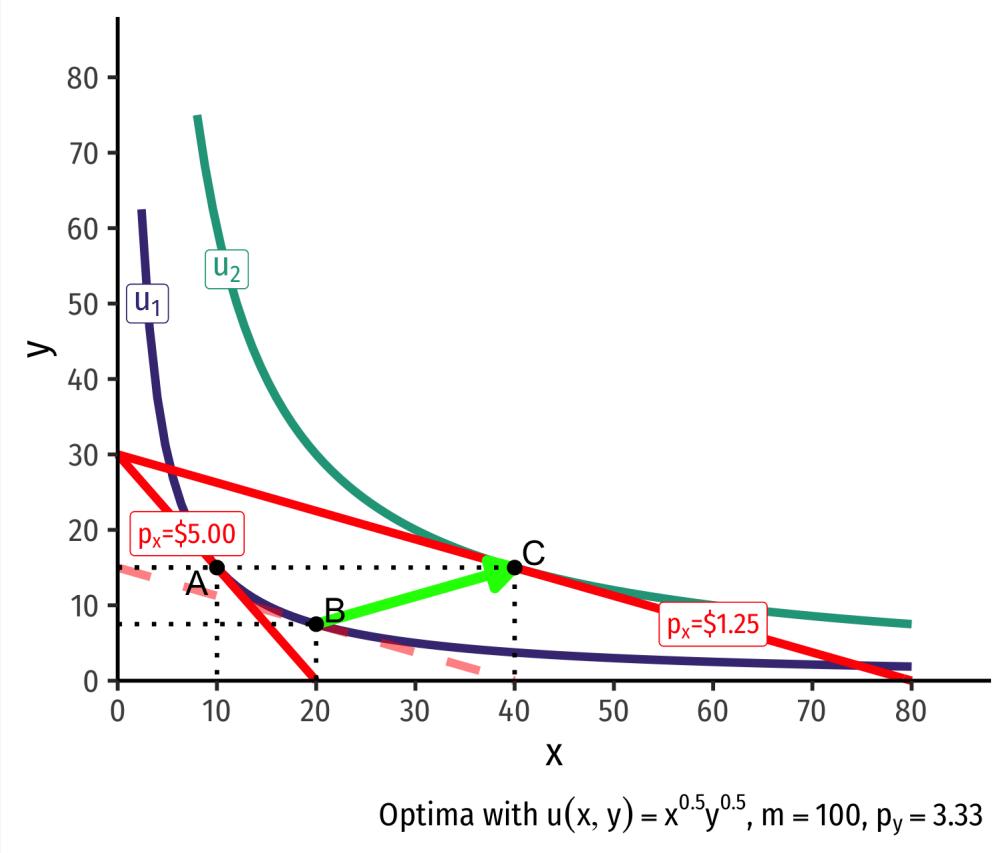


$$\text{Optima with } u(x, y) = x^{0.5}y^{0.5}, m = 100, p_y = 3.33$$

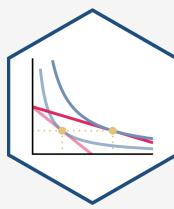
Real Income and Substitution Effects, Graphically III



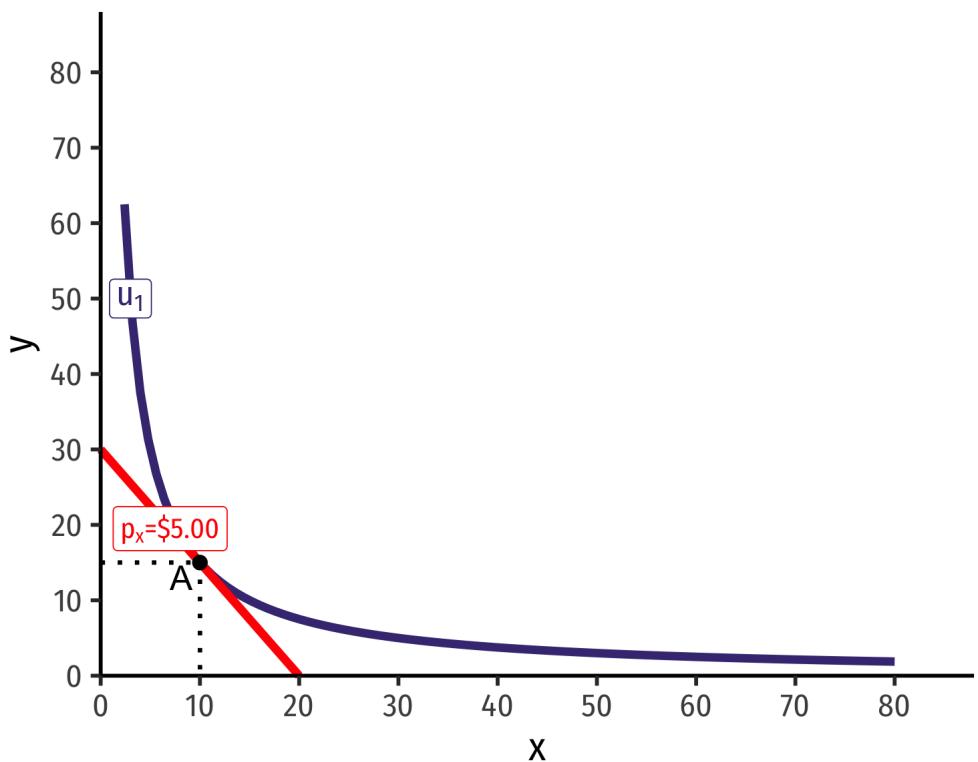
- **(Real) income effect:** change in consumption due to the **change in purchasing power** from the change in price
- $\backslash(B \rightarrow C)$ to new budget constraint (can buy more of $\backslash(x\backslash)$ and/or $\backslash(y\backslash)$)



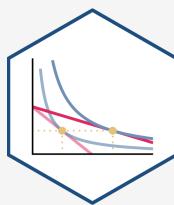
Real Income and Substitution Effects, Graphically IV



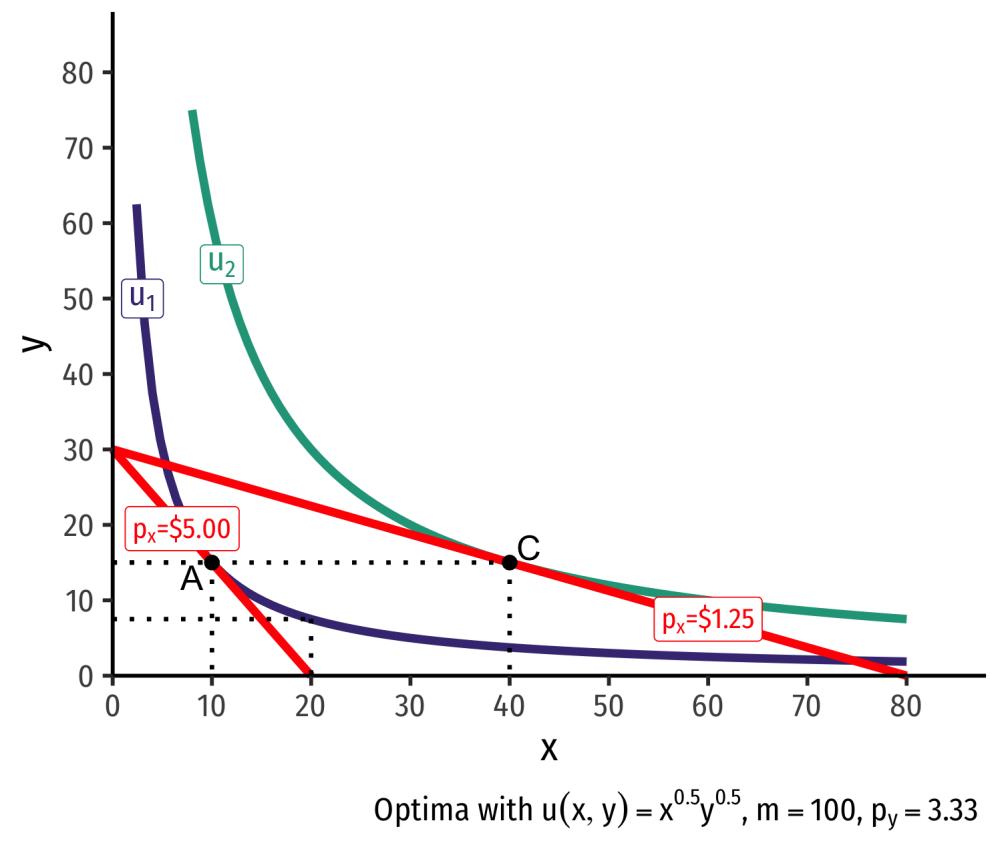
- Original optimal consumption \((A)\)



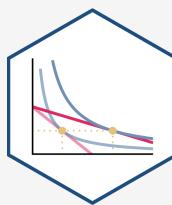
Real Income and Substitution Effects, Graphically IV



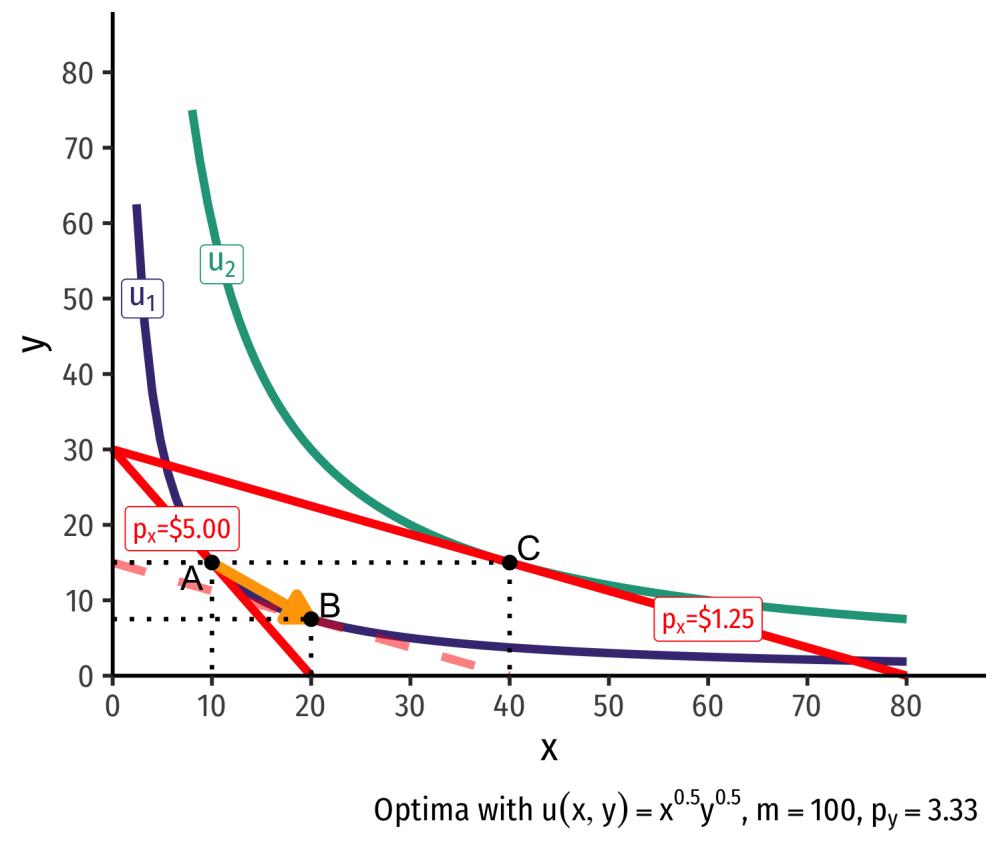
- Original optimal consumption $\mathbf{(A)}$
- Price of $\mathbf{(x)}$ falls, new optimal consumption at $\mathbf{(C)}$



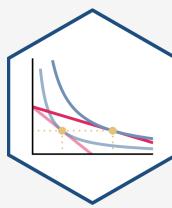
Real Income and Substitution Effects, Graphically IV



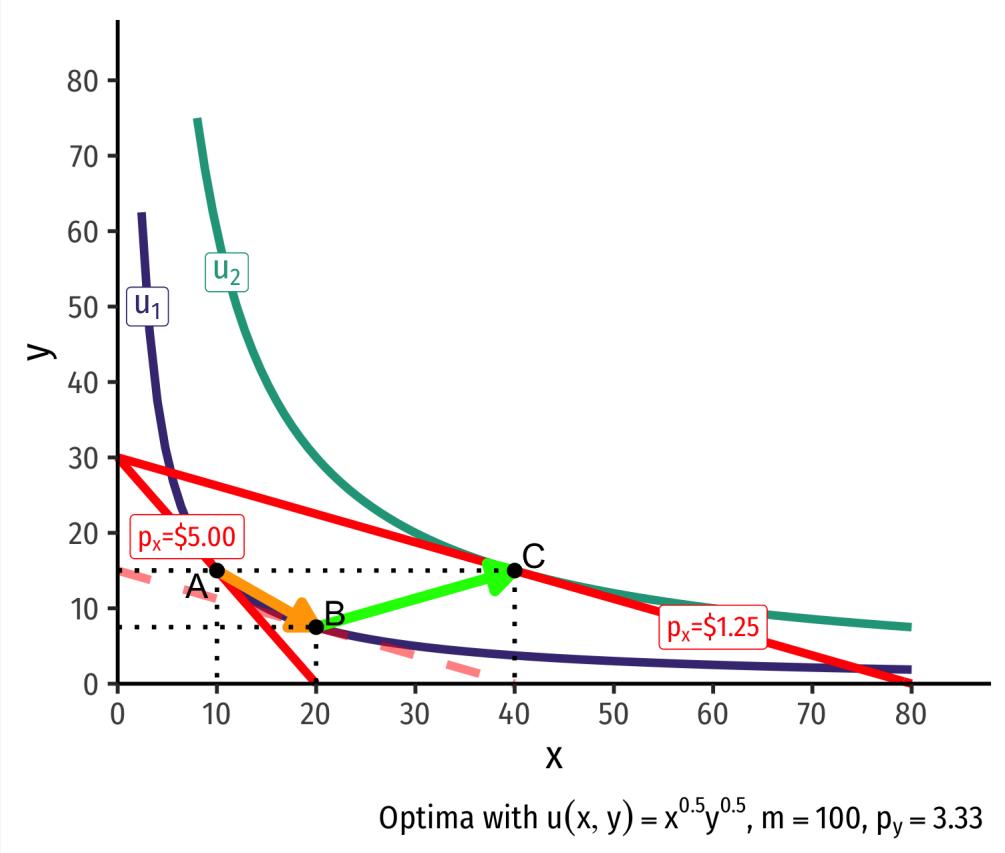
- Original optimal consumption $\langle(A)\rangle$
- Price of $\langle x \rangle$ falls, new optimal consumption at $\langle(C)\rangle$
- **Substitution effect:** $\langle A \rightarrow B \rangle$ on same I.C. $\langle(\uparrow) \rangle$ cheaper $\langle x \rangle$ and $\langle(\downarrow) \rangle \langle y \rangle$



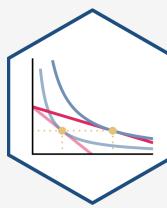
Real Income and Substitution Effects, Graphically IV



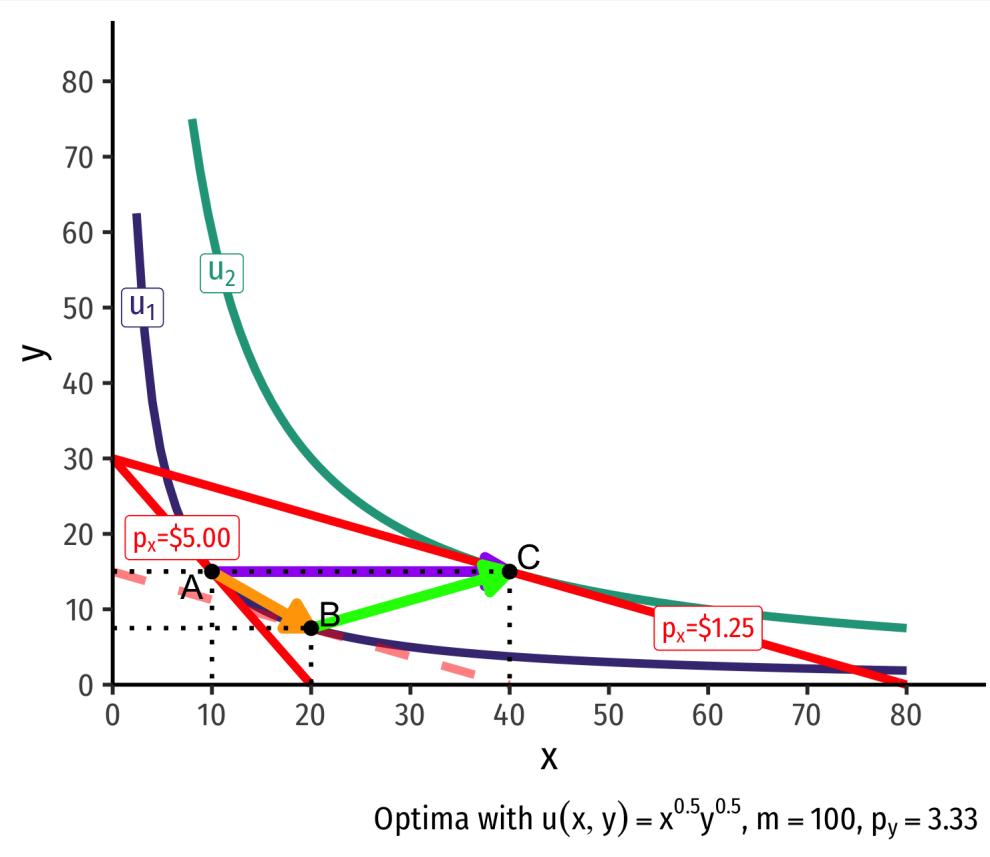
- Original optimal consumption $\mathbf{(A)}$
- Price of $\mathbf{(x)}$ falls, new optimal consumption at $\mathbf{(C)}$
- **Substitution effect:** $\mathbf{(A \rightarrow B)}$ on same I.C. $\mathbf{(\uparrow)}$ cheaper $\mathbf{(x)}$ and $\mathbf{(\downarrow) (y)}$
- **(Real) income effect:** $\mathbf{(B \rightarrow C)}$ to new budget constraint (can buy more of $\mathbf{(x)}$ and/or $\mathbf{(y)}$)



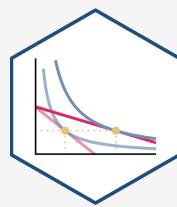
Real Income and Substitution Effects, Graphically IV



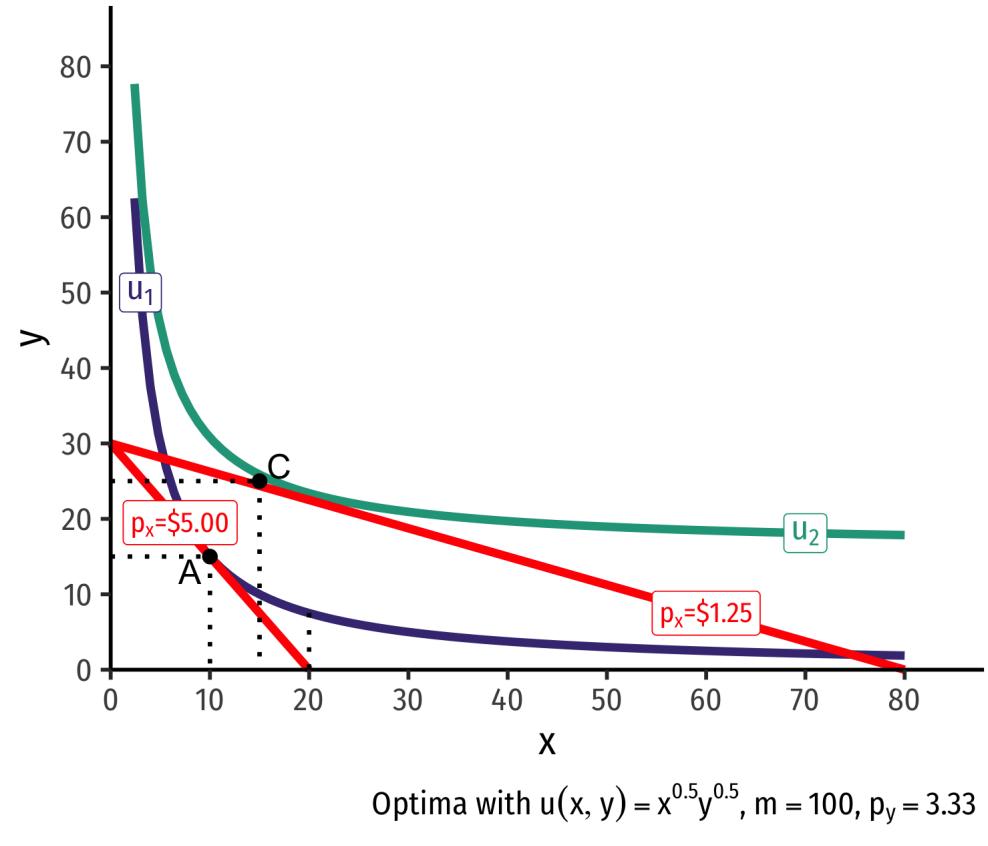
- Original optimal consumption $\mathbf{(A)}$
- Price of $\mathbf{(x)}$ falls, new optimal consumption at $\mathbf{(C)}$
- **Substitution effect:** $\mathbf{(A \rightarrow B)}$ on same I.C. $\mathbf{(\uparrow)}$ cheaper $\mathbf{(x)}$ and $\mathbf{(\downarrow) (y)}$
- **(Real) income effect:** $\mathbf{(B \rightarrow C)}$ to new budget constraint (can buy more of $\mathbf{(x)}$ and/or $\mathbf{(y)}$)
- **(Total) price effect:** $\mathbf{(A \rightarrow C)}$



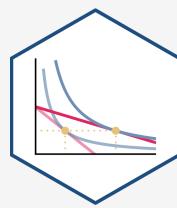
Real Income and Substitution Effects: Inferior Good



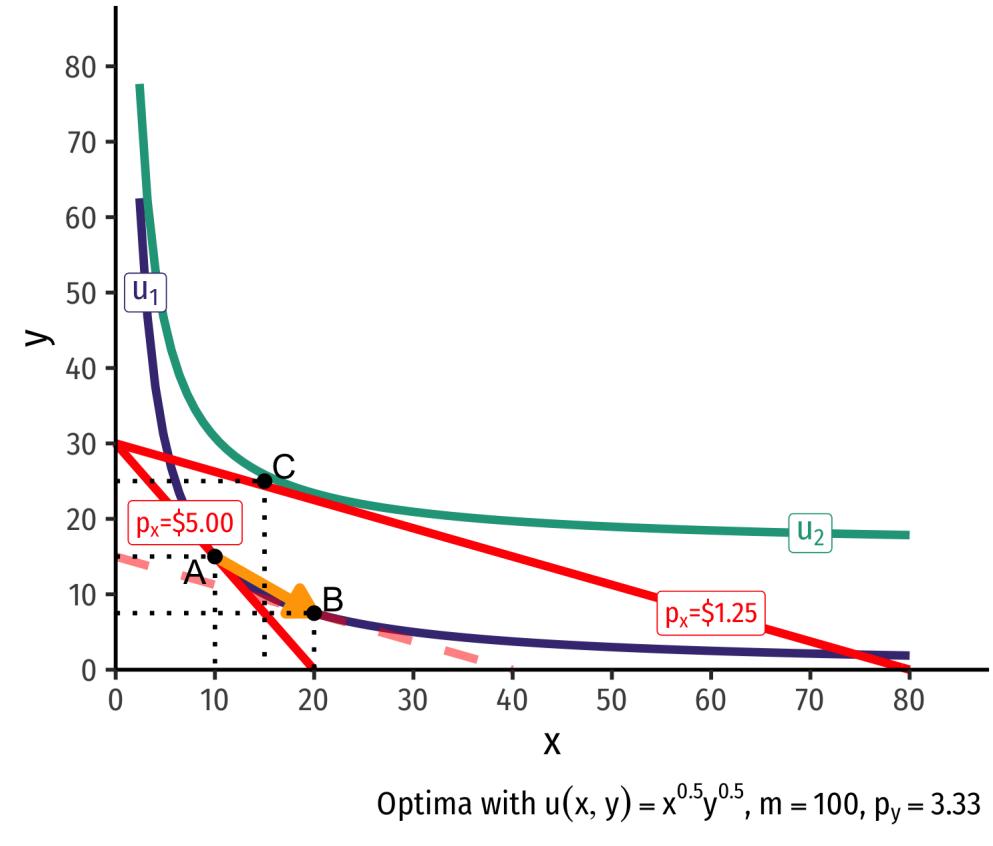
- What about an **inferior** good (Ramen)?



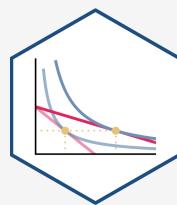
Real Income and Substitution Effects: Inferior Good



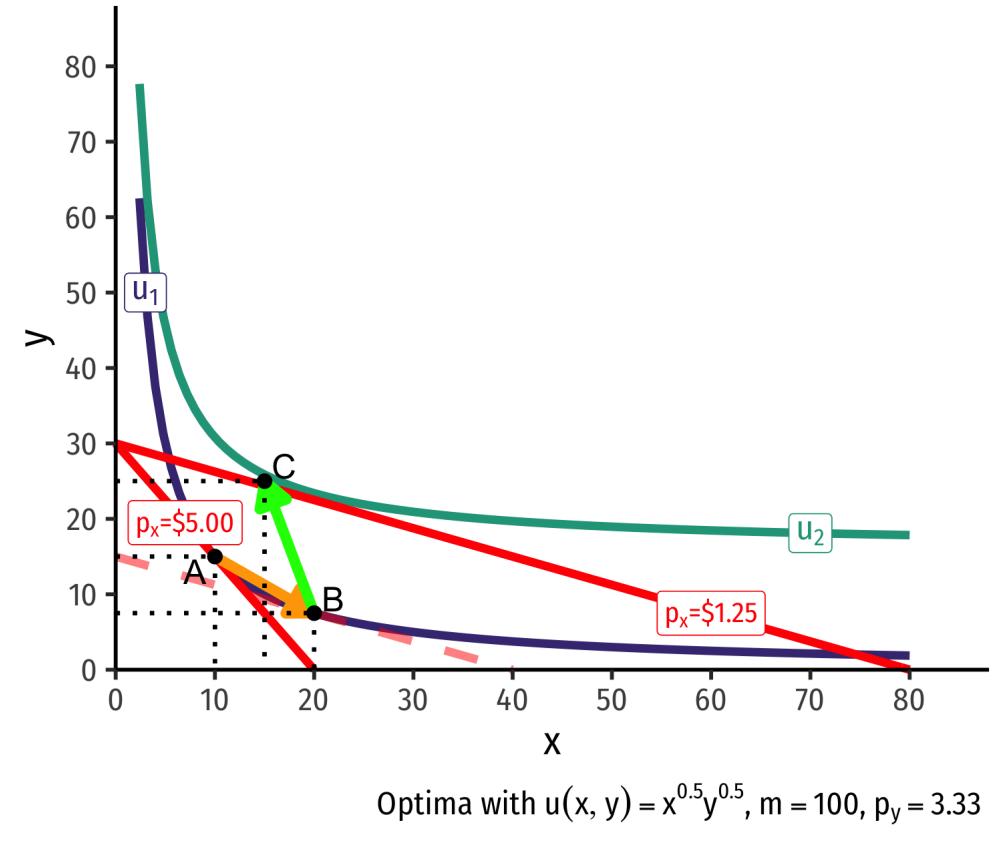
- What about an **inferior** good (Ramen)?
- **Substitution effect:** $A \rightarrow B$ on same I.C. \uparrow cheaper x and \downarrow y



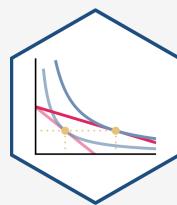
Real Income and Substitution Effects: Inferior Good



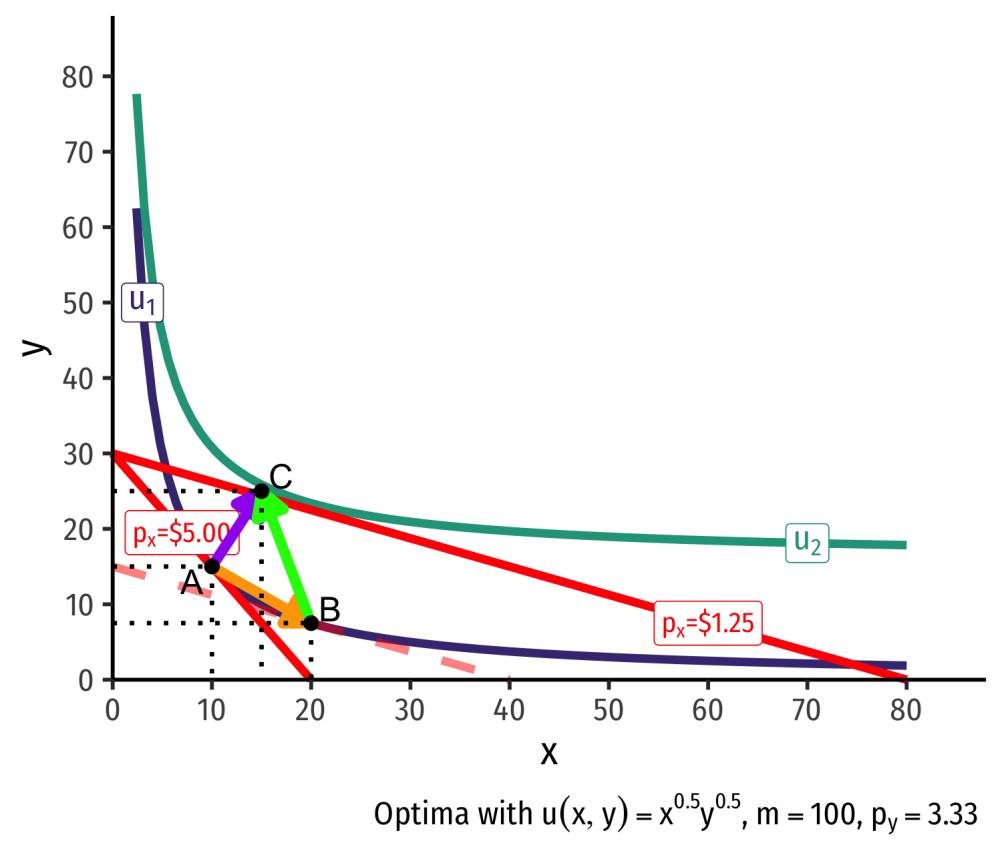
- What about an **inferior** good (Ramen)?
- **Substitution effect:** $(A \rightarrow B)$ on same I.C. \uparrow cheaper (x) and \downarrow (y)
- **(Real) income effect:** $(B \rightarrow C)$ to new budget constraint (can buy more of (x) and/or (y)) **is negative**



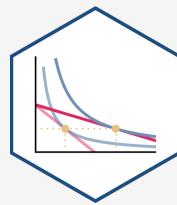
Real Income and Substitution Effects: Inferior Good



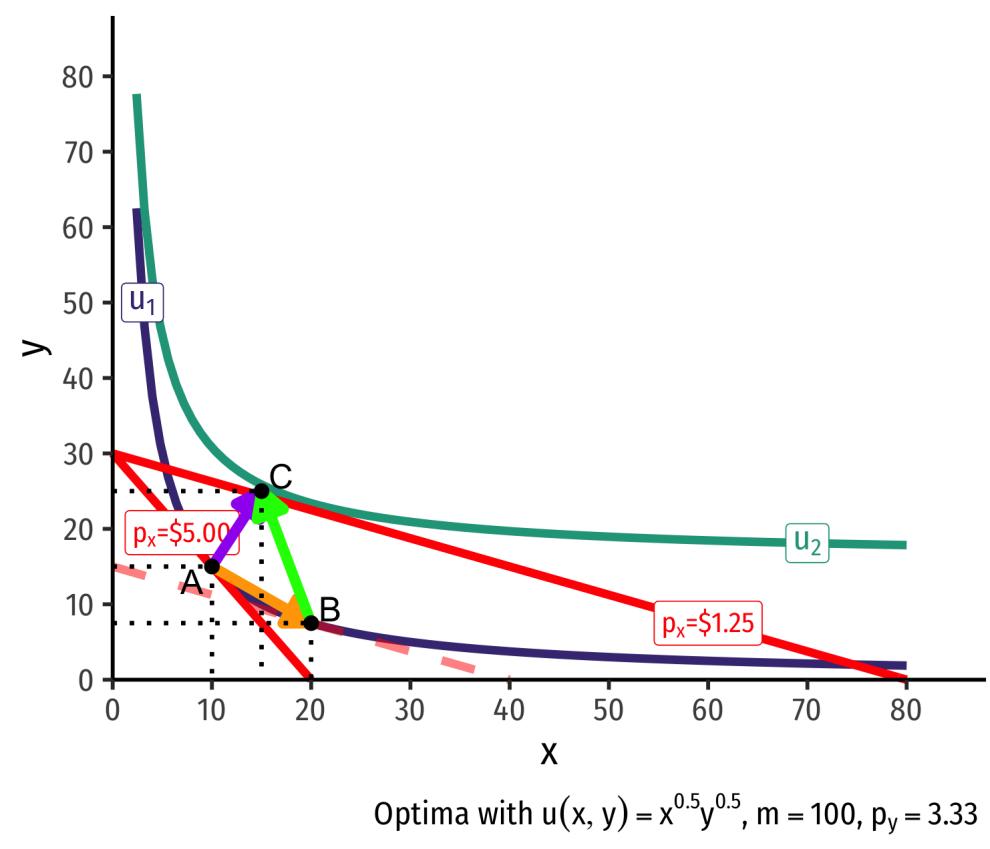
- What about an **inferior** good (Ramen)?
- **Substitution effect:** $(A \rightarrow B)$ on same I.C. \uparrow cheaper (x) and \downarrow (y)
- **(Real) income effect:** $(B \rightarrow C)$ to new budget constraint (can buy more of (x) and/or (y)) **is negative**
- **(Total) price effect:** $(A \rightarrow C)$



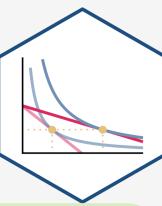
Real Income and Substitution Effects: Inferior Good



- What about an **inferior** good (Ramen)?
- **Substitution effect:** $(A \rightarrow B)$ on same I.C. \uparrow cheaper (x) and \downarrow (y)
- **(Real) income effect:** $(B \rightarrow C)$ to new budget constraint (can buy more of (x) and/or (y)) **is negative**
- **(Total) price effect:** $(A \rightarrow C)$
- Price effect is *still* an $\uparrow x$ from a $\downarrow p_x$!

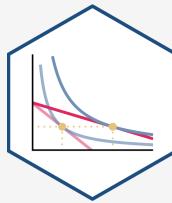


Violating the Law of Demand



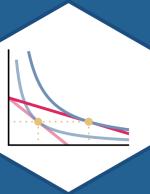
Example: What would it take to violate the law of demand?

Recap: Real Income and Substitution Effects



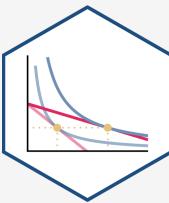
Price Effect \(=\) Real income effect \(+\) Substitution Effect

- **Substitution effect:** is always in the direction of the cheaper good
- **Real Income effect:** can be positive (normal) or negative (inferior)
- **Law of Demand**/Demand curves slope downwards (**Price effect**) mostly because of the substitution effect
 - Even (inferior) goods with negative real income effects overpowered by substitution effect
- Exception in the theoretical **Giffen good**: negative R.I.E. \(>\) S.E.
 - An upward sloping demand curve!



From Optimal Consumption Points to Demand

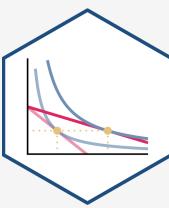
Demand Schedule



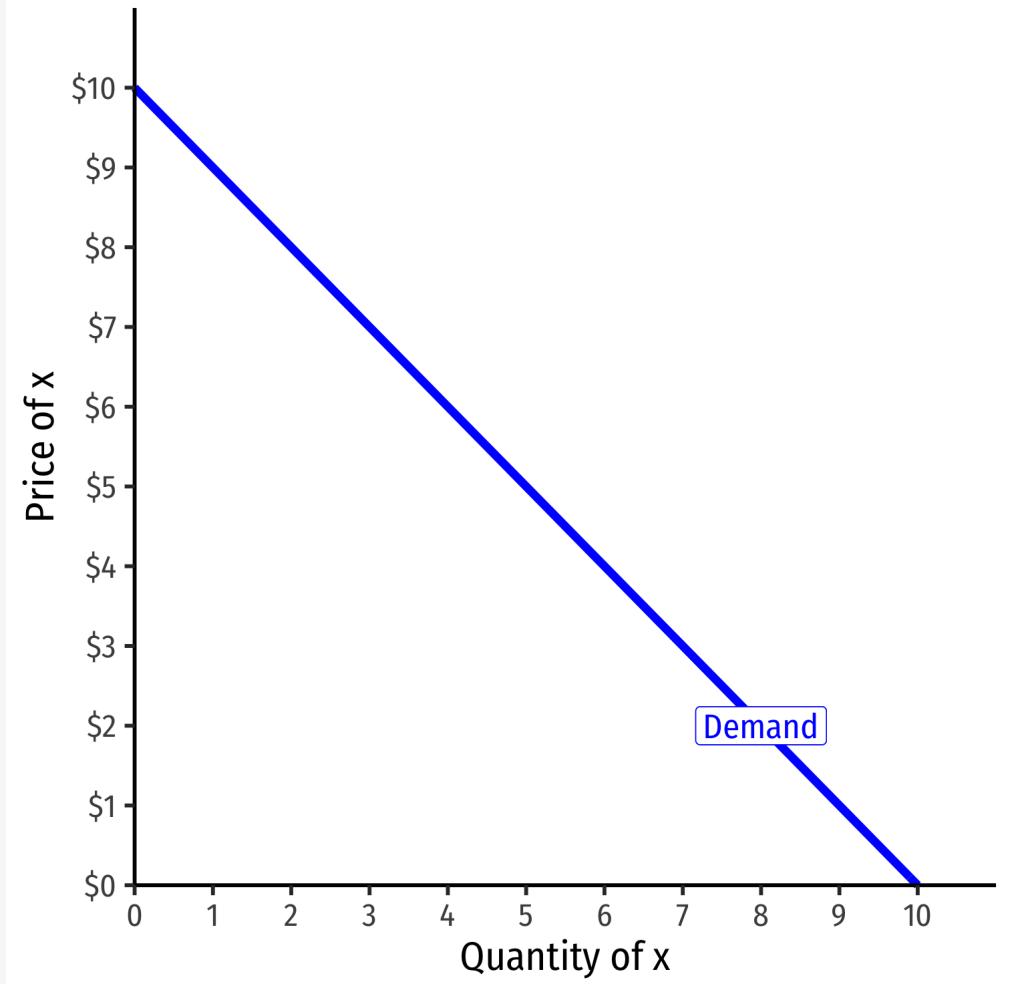
- **Demand schedule** expresses the quantity of good a person would be willing to buy (q_D) at any given price (p_x)
- Note: each of these is a consumer's optimum at a given price!

price	quantity
10	0
9	1
8	2
7	3
6	4
5	5
4	6
3	7
8	2
9	1

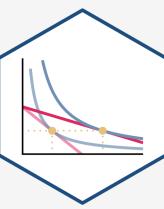
Demand Curve



- **Demand curve** graphically represents the demand schedule
- Also measures a person's **maximum willingness to pay (WTP)** for a given quantity
- Law of Demand (price effect) \(\backslash(\backslash implies\backslash)\)
Demand curves always slope downwards



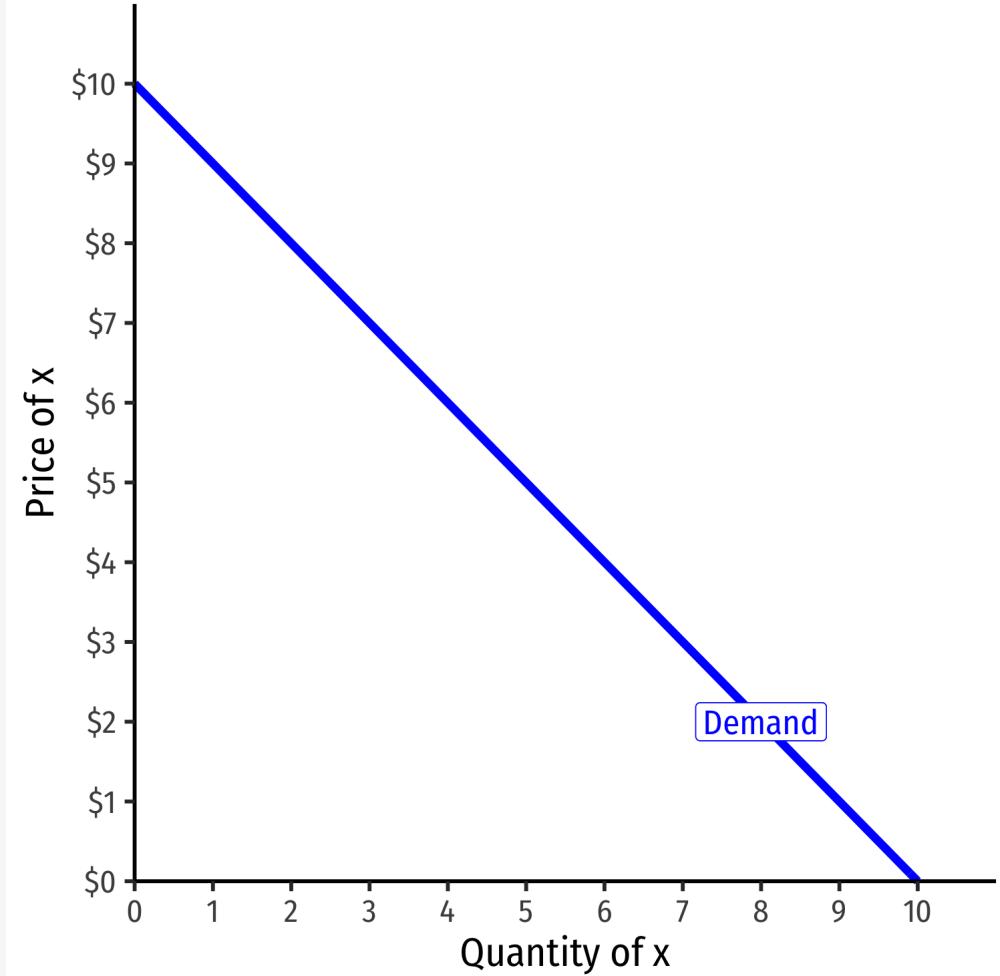
Demand Function



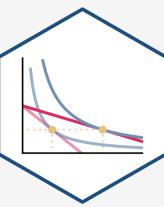
- **Demand function** relates quantity to price

Example: $\$q=10-p\$$

- Not graphable (wrong axes)!



Inverse Demand Function

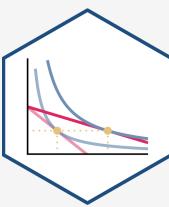


- ***Inverse demand function*** relates price to quantity
 - Take demand function and solve for $p(q)$

Example: $p=10-q$

- Graphable (price on vertical axis)!

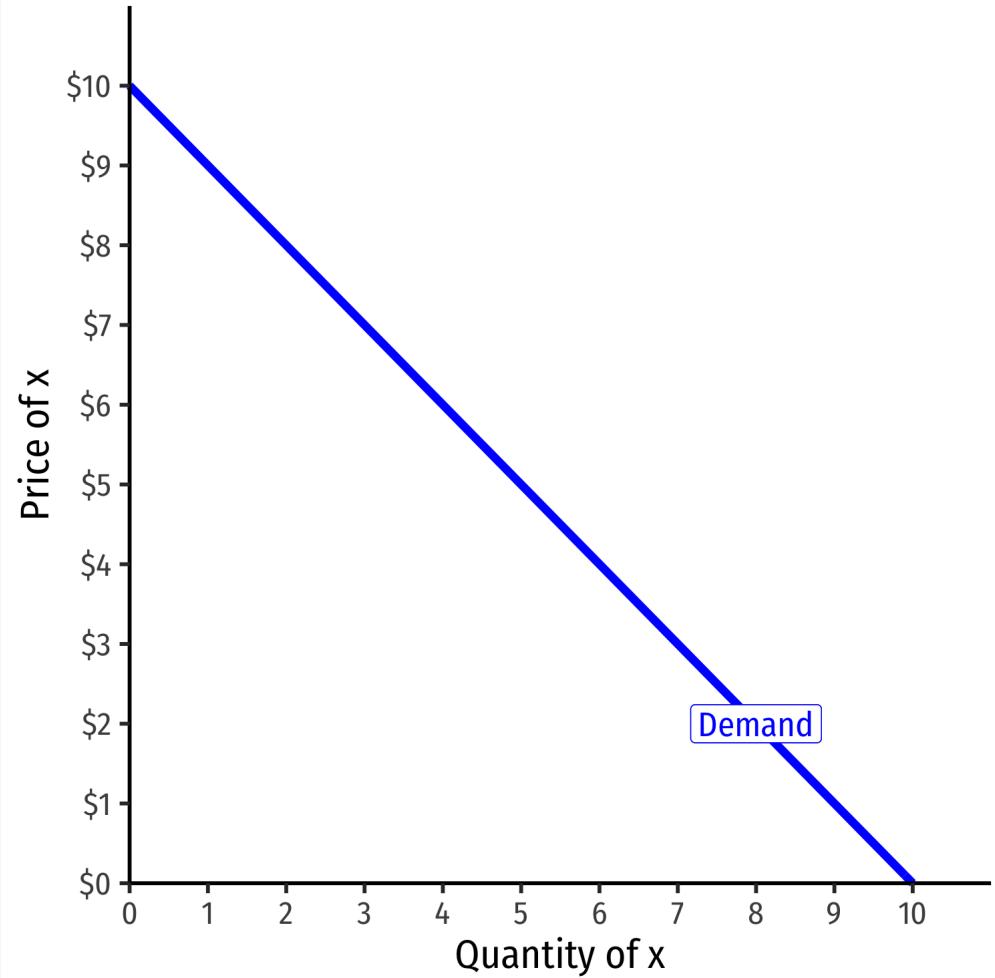
Inverse Demand Function



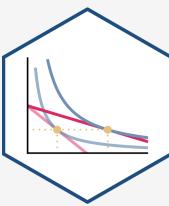
- ***Inverse demand function*** relates price to quantity
 - Take demand function and solve for $\backslash(p\backslash)$

Example: $\$p=10-q\$$

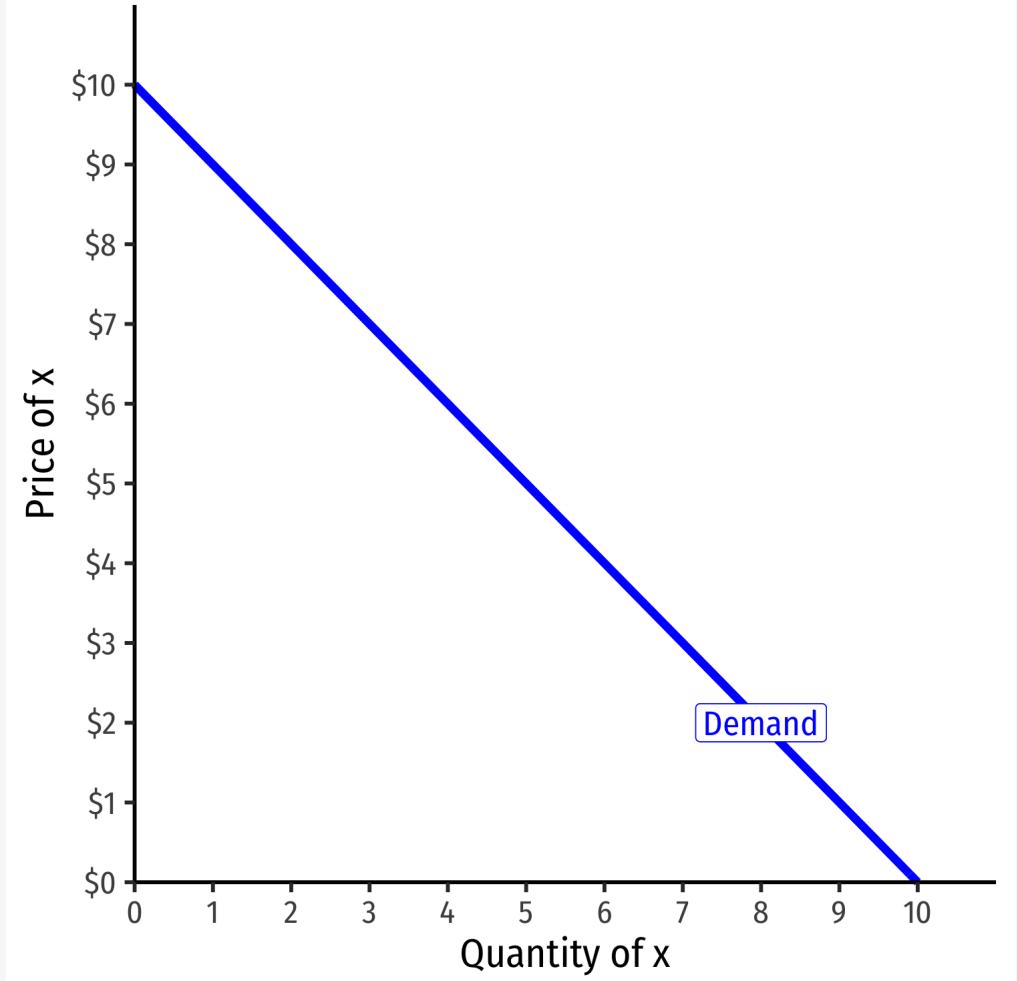
- Vertical intercept ("Choke price"): price where $\backslash(q_D=0\backslash)$ (\$10), just high enough to discourage *any* purchases



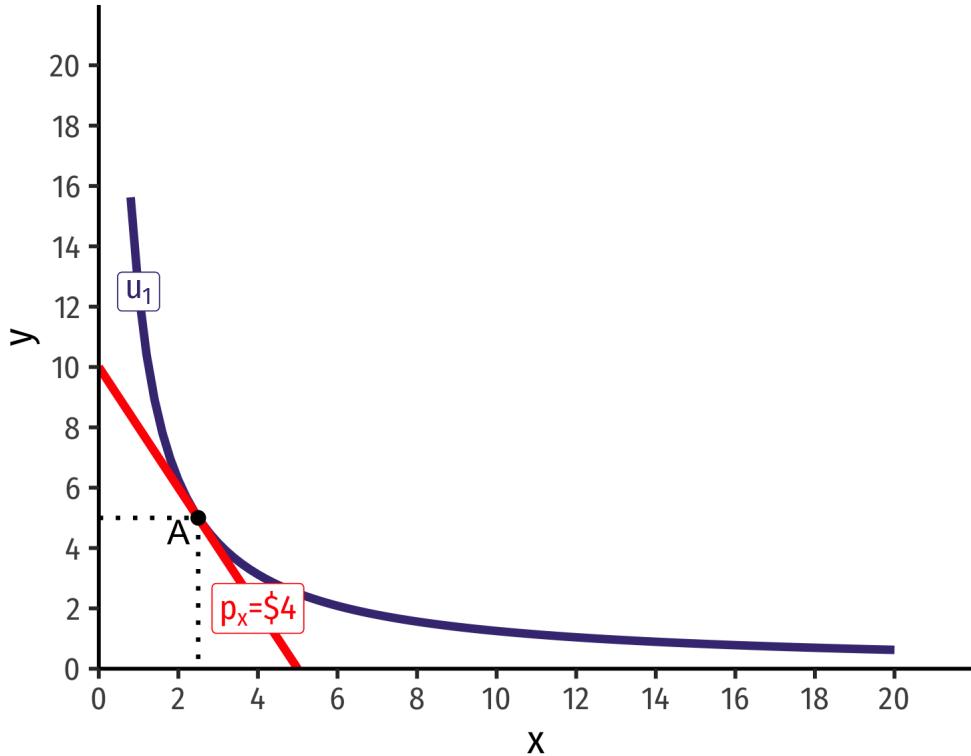
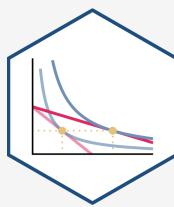
Inverse Demand Function



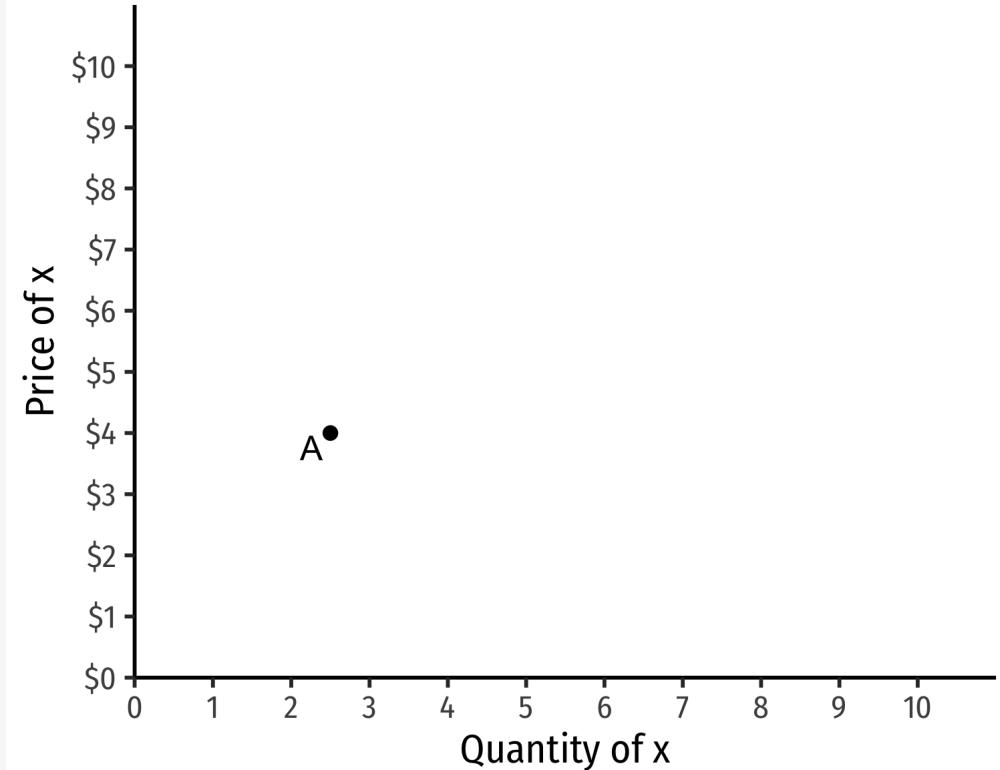
- Read two ways:
- Horizontally: at any given price, how many units person wants to buy
- Vertically: at any given quantity, the **maximum willingness to pay (WTP)** for that quantity
 - This way will be very useful later



Deriving a Demand Curve Graphically

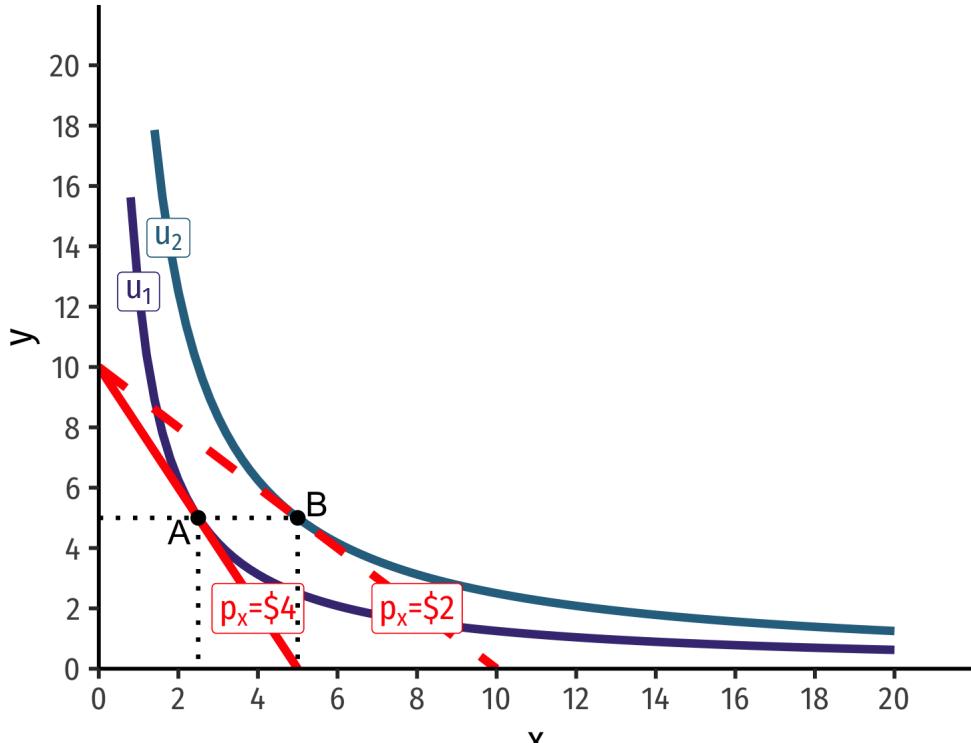
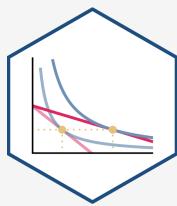


Optima with $u(x, y) = x^{0.5}y^{0.5}$, $m = 20$, $p_y = 2$

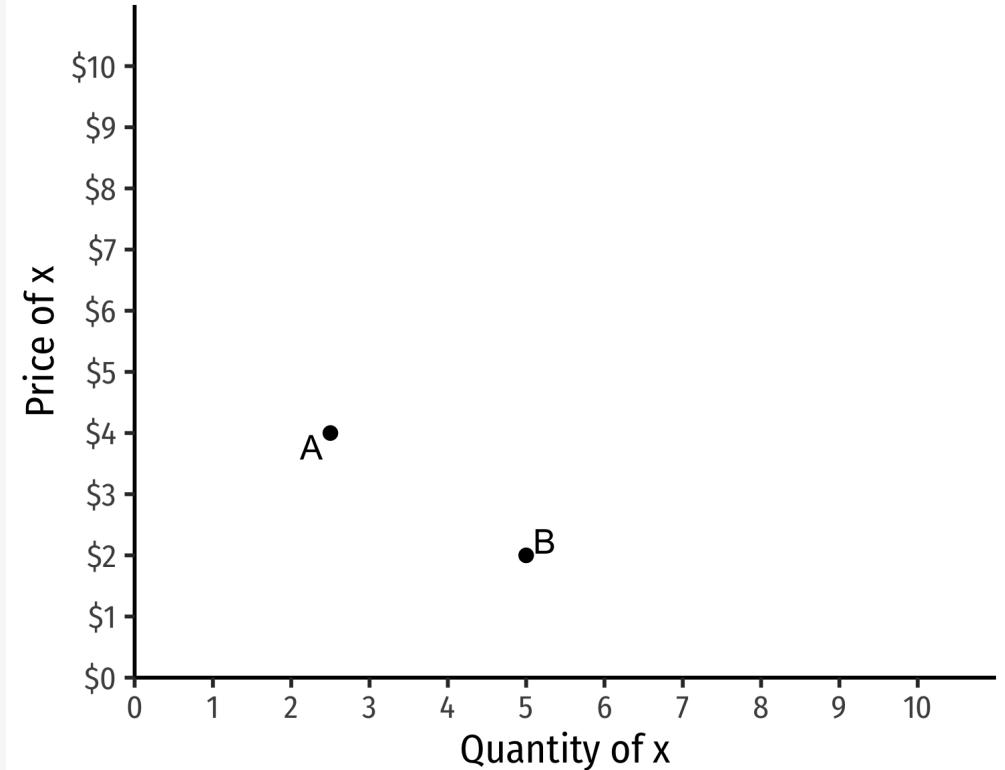


Demand function: $0.5m/p_x$; Inverse Demand function: $p_x = 0.5m/x$

Deriving a Demand Curve Graphically

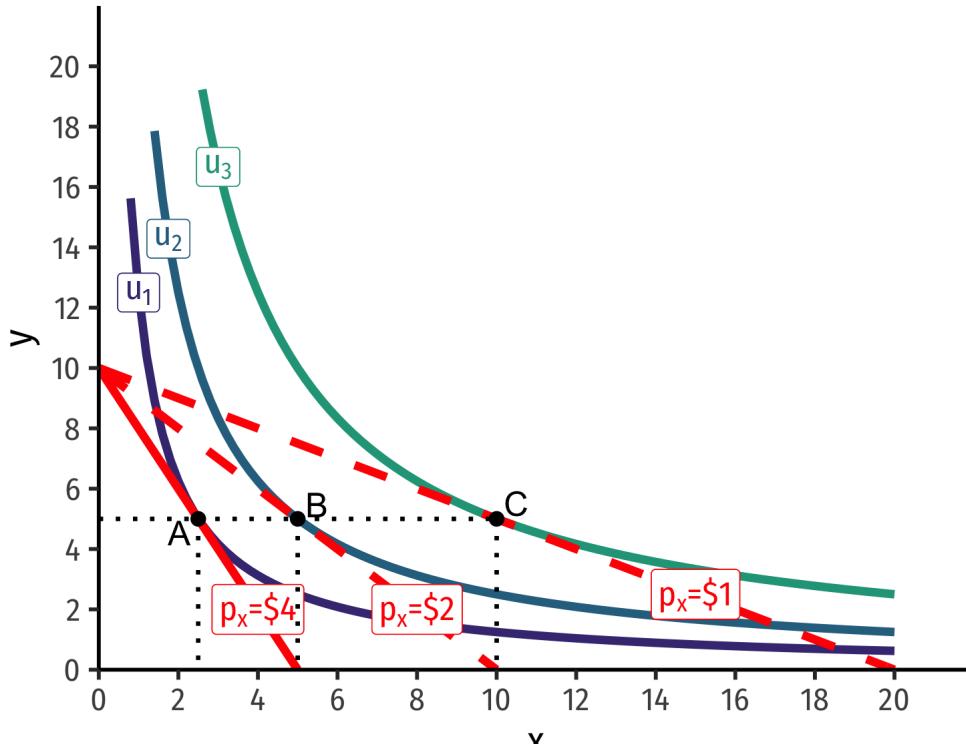
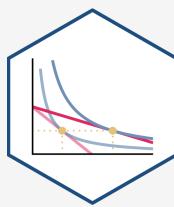


Optima with $u(x, y) = x^{0.5}y^{0.5}$, $m = 20$, $p_y = 2$

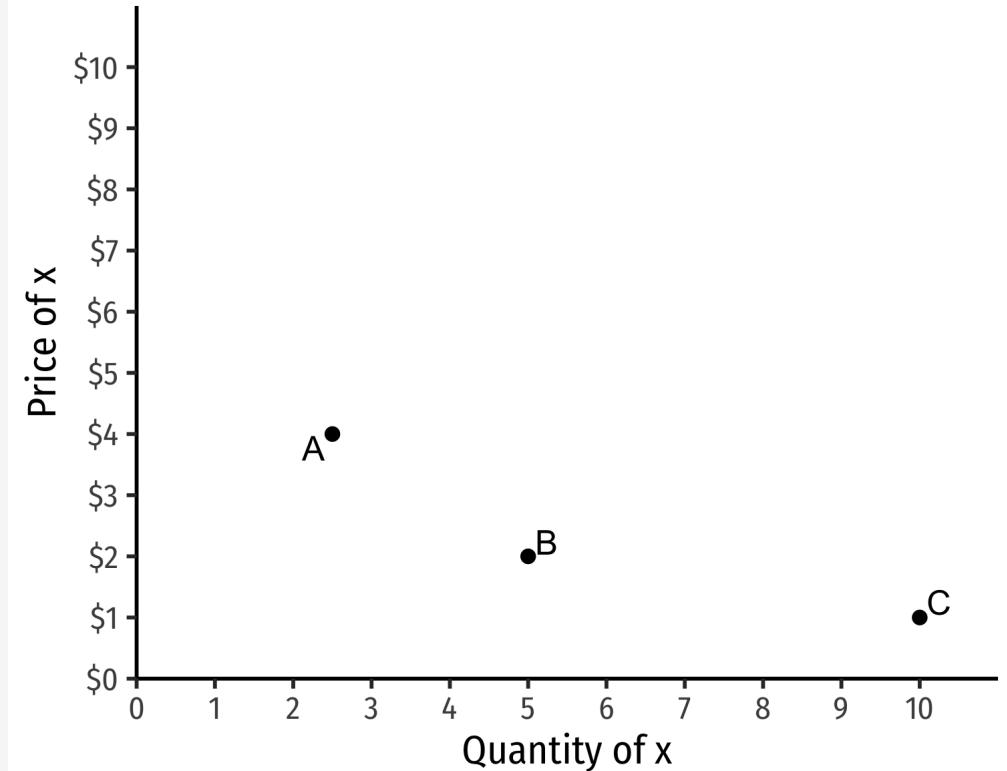


Demand function: $0.5m/p_x$; Inverse Demand function: $p_x = 0.5m/x$

Deriving a Demand Curve Graphically

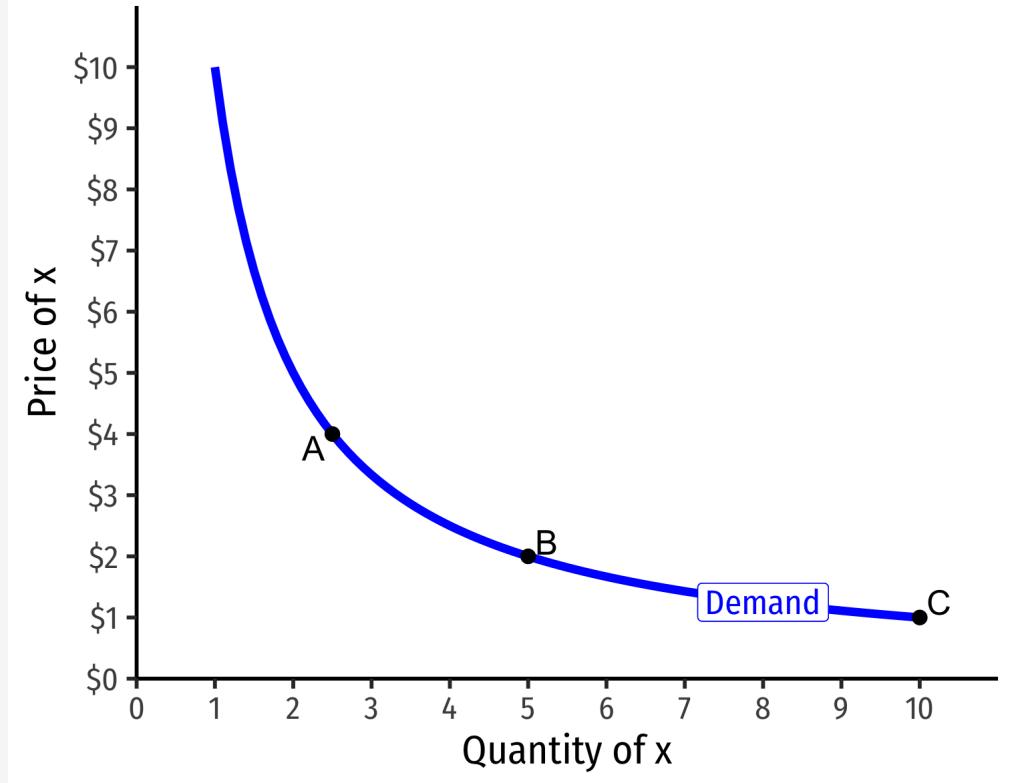
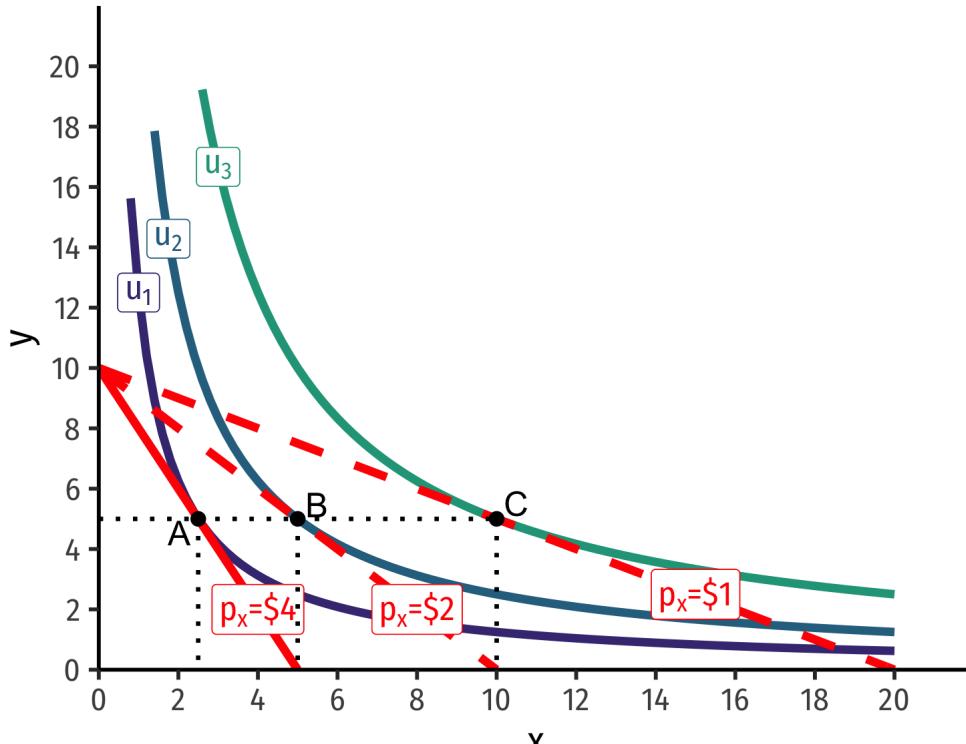
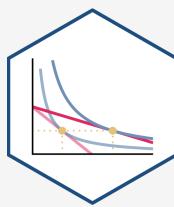


Optima with $u(x, y) = x^{0.5}y^{0.5}$, $m = 20$, $p_y = 2$

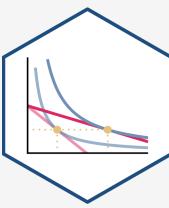


Demand function: $0.5m/p_x$; Inverse Demand function: $p_x = 0.5m/x$

Deriving a Demand Curve Graphically

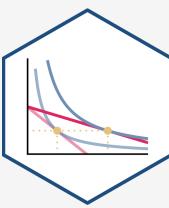


Deriving a Demand *Function* I



- I will always give you a (linear) demand function
- Today's class notes page shows how you can derive actual demand functions from utility functions

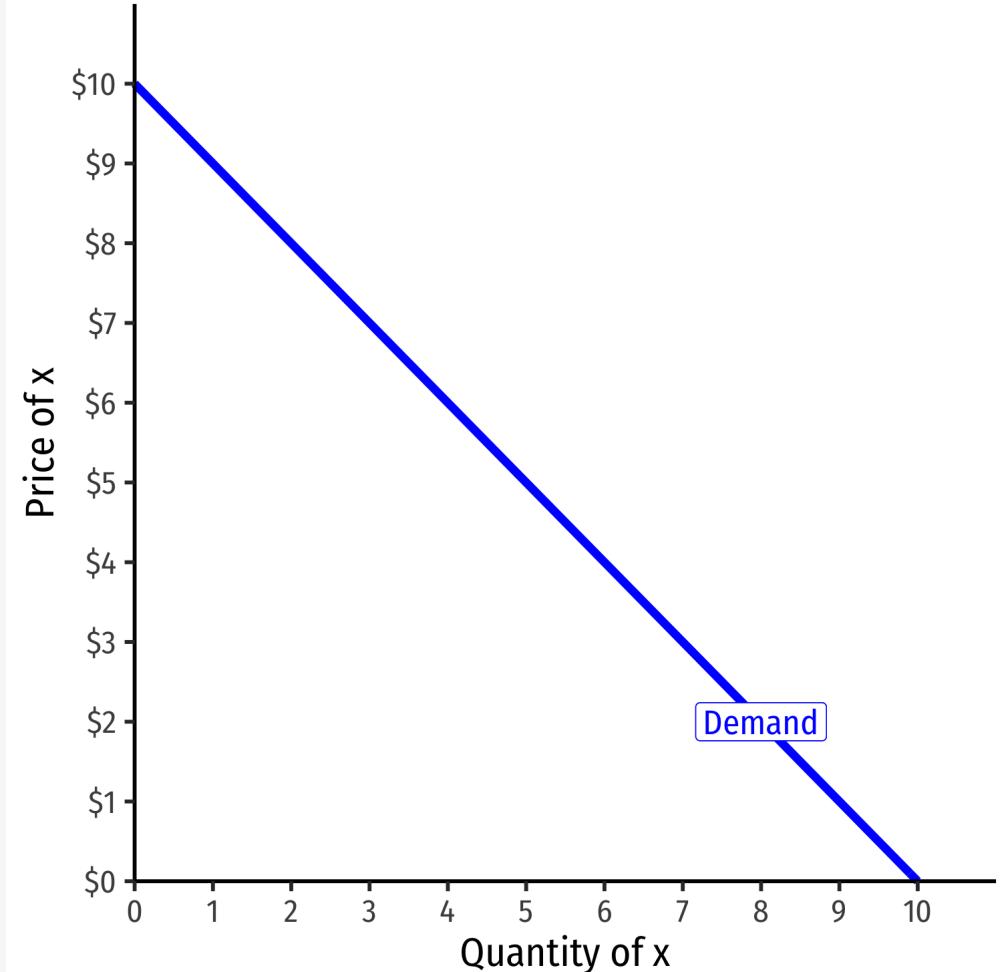
Shifts in Demand I



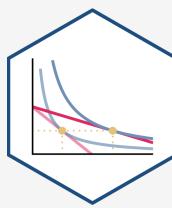
- Note a simple (inverse) demand function only relates (own) **price** and **quantity**

Example: $\{q=10-p\}$ or $\{p=10-q\}$

- What about all the other "**determinants of demand**" like income and other prices?
- They are captured in the vertical intercept (choke price)!



Shifts in Demand II



- A change in one of the "**determinants of demand**" will **shift** demand curve!
 - Change in **income** (m)
 - Change in **price of other goods** (p_y) (substitutes or complements)
 - Change in **preferences** or **expectations** about good (x)
 - Change in **number of buyers**
- Shows up in (inverse) demand function by a **change in intercept (choke price)**!
- See my [Visualizing Demand Shifters](#)

