

# 1.6 – Income & Substitution Effects

ECON 306 • Microeconomic Analysis • Fall 2021

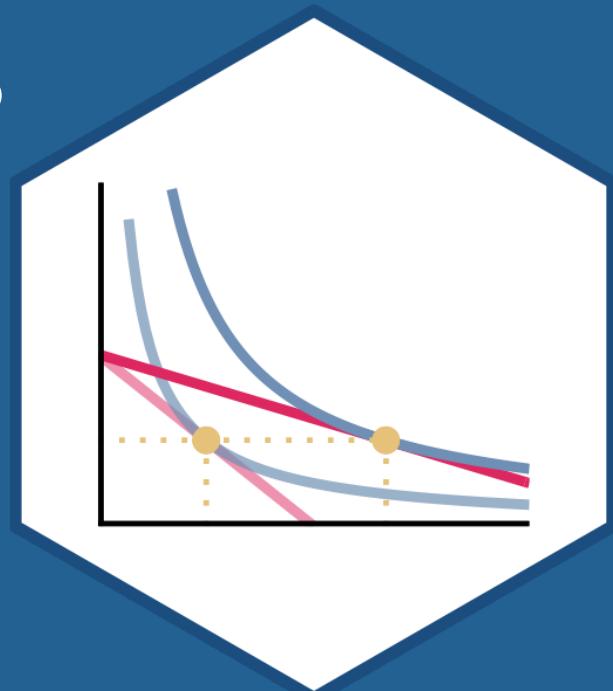
Ryan Safner

Assistant Professor of Economics

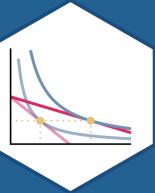
 [safner@hood.edu](mailto:safner@hood.edu)

 [ryansafner/microF21](https://github.com/ryansafner/microF21)

 [microF21.classes.ryansafner.com](http://microF21.classes.ryansafner.com)



# Outline



The (Own) Price Effect

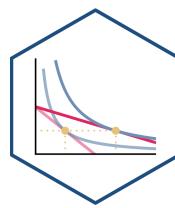
(Real) Income Effect

Substitution Effect

Putting the Effects Together

From Optimal Consumption Points to Demand

# A Demand Function (Again)



- A consumer's **demand** (for good x) depends on current prices & income:

$$q_x^D = q_x^D(m, p_x, p_y)$$

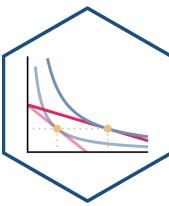
- How does **demand for x** change?
  1. **Income effects**  $\left( \frac{\Delta q_x^D}{\Delta m} \right)$ : how  $q_x^D$  changes with changes in income
  2. **Cross-price effects**  $\left( \frac{\Delta q_x^D}{\Delta p_y} \right)$ : how  $q_x^D$  changes with changes in prices of *other* goods (e.g. y)
  3. **(Own) Price effects**  $\left( \frac{\Delta q_x^D}{\Delta p_x} \right)$ : how  $q_x^D$  changes with changes in price (of x)





# The (Own) Price Effect

# The (Own) Price Effect



- **Price effect:** change in optimal consumption of a good associated with a change in its price, holding income and other prices constant

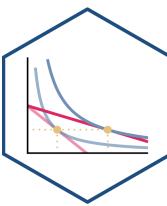
$$\frac{\Delta q_x^D}{\Delta p_x} < 0$$

**The law of demand:** as the price of a good rises, people will tend to buy less of that good (and vice versa)

- i.e. **the price effect is negative!**



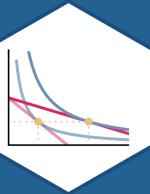
# Decomposing the Price Effect



The **price effect** (law of demand) is actually the **net result of two effects**

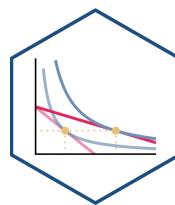
1. **(Real) income effect**: change in consumption due to change in real purchasing power
2. **Substitution effect**: change in consumption due to change in relative prices

$$\text{Price Effect} = \text{Real income effect} + \text{Substitution Effect}$$



# (Real) Income Effect

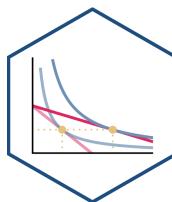
# (Real) Income Effect: Demonstration



- Suppose there is only 1 good to consume,  $x$ . You have a \$100 income, and the price of  $x$  is \$10. You consume 10 units of  $x$
- Suppose the price of  $x$  falls to \$5. You now consume 20 units of  $x$ .
- This is the **real income effect**



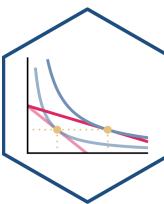
# (Real) Income Effect: Demonstration



- **Real income effect:** your consumption mix changes because of the change in the price of  $x$  changes your **real income** or **purchasing power** (the amount of goods you can buy)
- Note your **actual (nominal) income (\$100) never changed!**



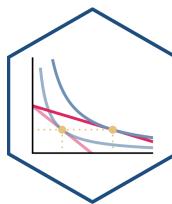
# (Real) Income Effect: Size



- The *size* of the income effect depends on how large a *portion of your budget* you spend on the good
- **Large-budget items:**
  - e.g. Housing/apartment rent, car prices
  - Price increase makes you much poorer
  - Price decrease makes you much wealthier

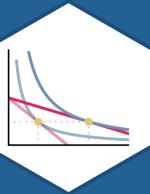


# (Real) Income Effect: Size



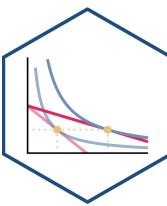
- The *size* of the income effect depends on how large a *portion of your budget* you spend on the good
- **Small-budget items:**
  - e.g. pencils, toothpicks, candy
  - Price changes don't have much of an effect on your wealth or change your behavior much





# Substitution Effect

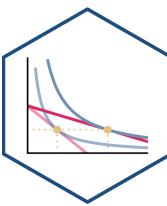
# Substitution Effect: Demonstration



- Suppose there are 1000's of goods, none of them a major part of your budget
  - So real income effect is insignificant
- Suppose the price of one good,  $x$  increases
- You would consume *less* of  $x$  relative to other goods because  $x$  is now *relatively* more expensive
- That's the **substitution effect**

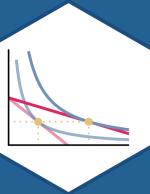


# Substitution Effect: Demonstration



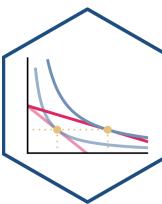
- **Substitution effect:** consumption mix changes because of a change in **relative prices**
- Buy more of the (now) relatively cheaper items
- Buy less of the (now) relatively more expensive item ( $x$ )





# Putting the Effects Together

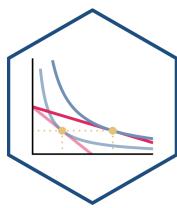
# Putting the Effects Together



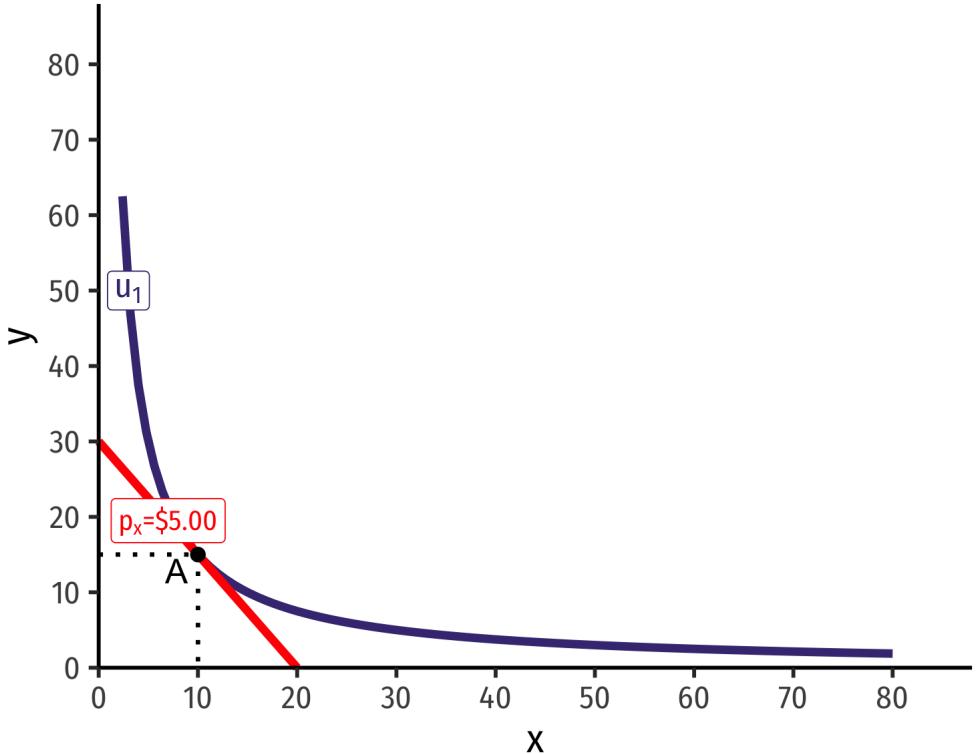
- **Real income effect:** change in consumption due to change in real purchasing power
  - Can be positive (**normal goods**) or negative (**inferior goods**)
  - Lower price of  $x$  means you can buy more  $x$ ,  $y$ , or *both* (depending on your preferences between  $x$  and  $y$ )
- **Substitution effect:** change in consumption due to change in relative prices
  - If  $x$  gets cheaper relative to  $y$ , consume  $\downarrow y$  (and  $\uparrow x$ )
  - This is always the same direction! ( $\downarrow$  relatively expensive goods,  $\uparrow$  relatively cheaper goods)
  - This is why demand curves slope downwards!

$$\text{Price Effect} = \text{Real income effect} + \text{Substitution Effect}$$

# Real Income and Substitution Effects, Graphically I

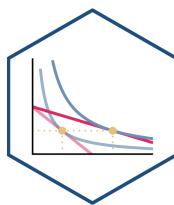


- Original optimal consumption ( $A$ )

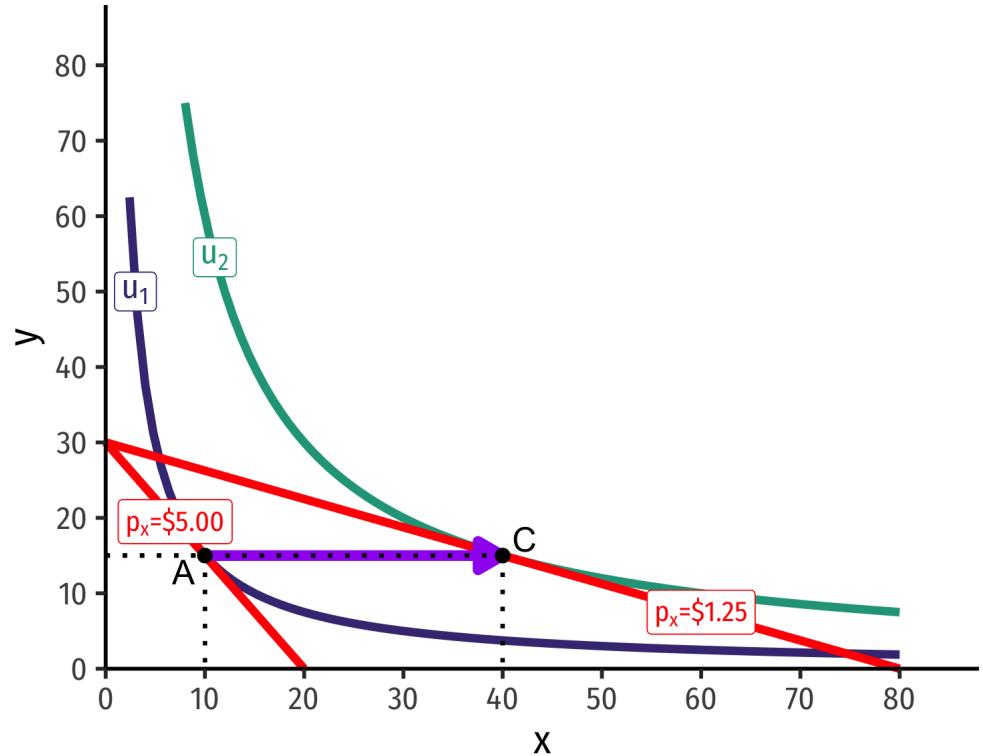


Optima with  $u(x, y) = x^{0.5}y^{0.5}$ ,  $m = 100$ ,  $p_y = 3.33$

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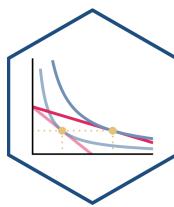


- Original optimal consumption ( $A$ )
- **(Total) price effect:**  $A \rightarrow C$
- Let's decompose this into the two effects

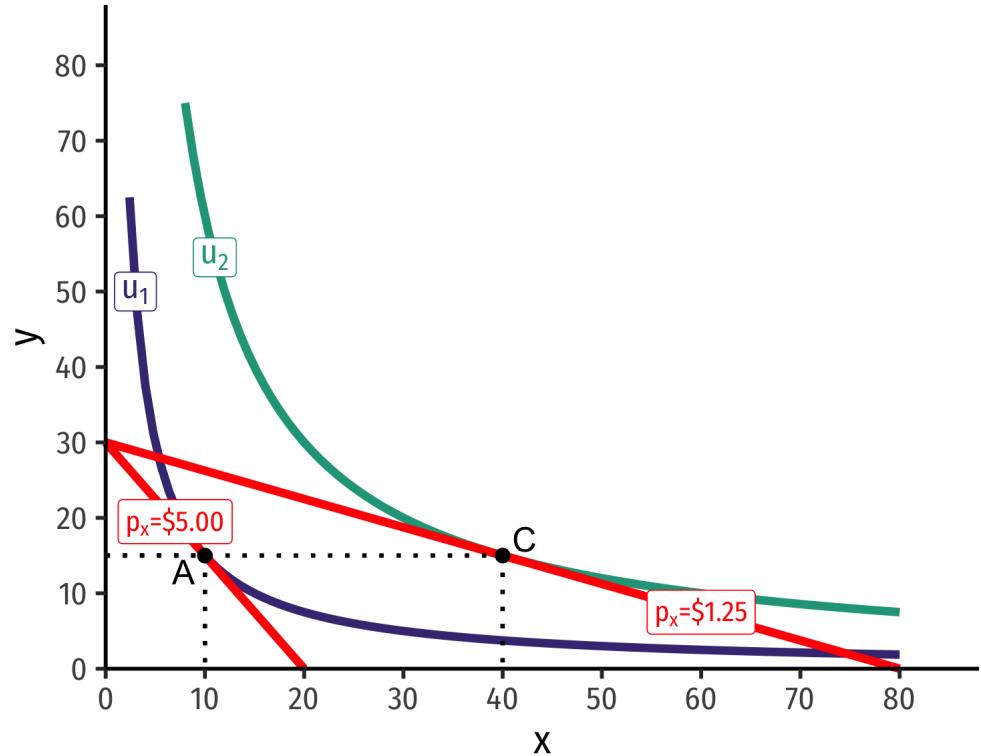


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# Real Income and Substitution Effects, Graphically II

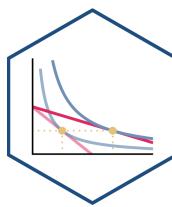


- **Substitution effect:** what you would choose under the **new exchange rate** to **remain indifferent** as before the change

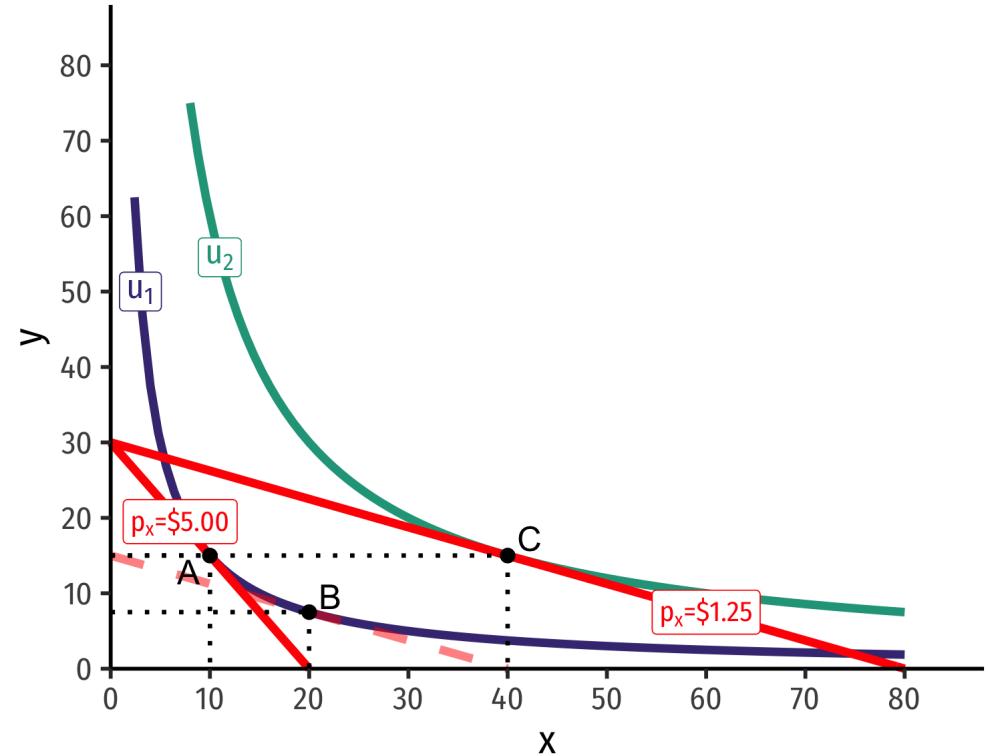


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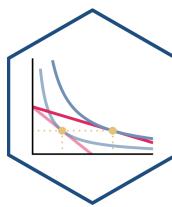


- **Substitution effect:** what you would choose under the **new exchange rate** to **remain indifferent** as before the change
- Graphically: shift *new* budget constraint inwards until tangent with *old* indifference curve
- $A \rightarrow B$  on same I.C. ( $\uparrow x, \downarrow y$ )
  - Point B *must* be a *different* point on the original curve!

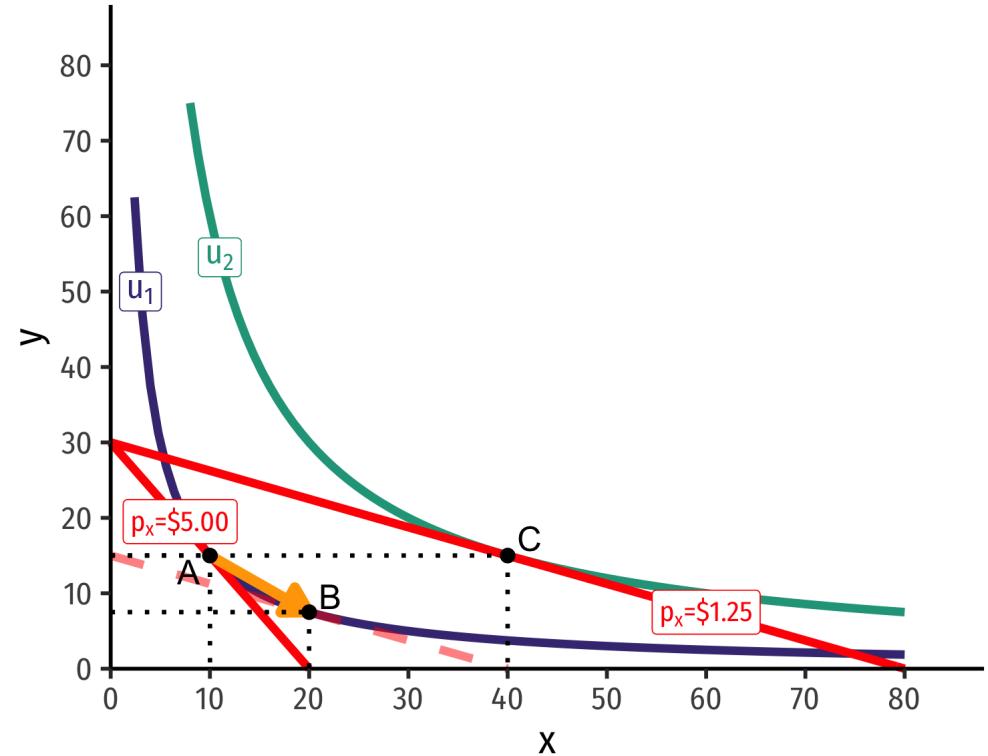


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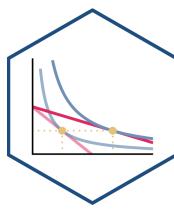


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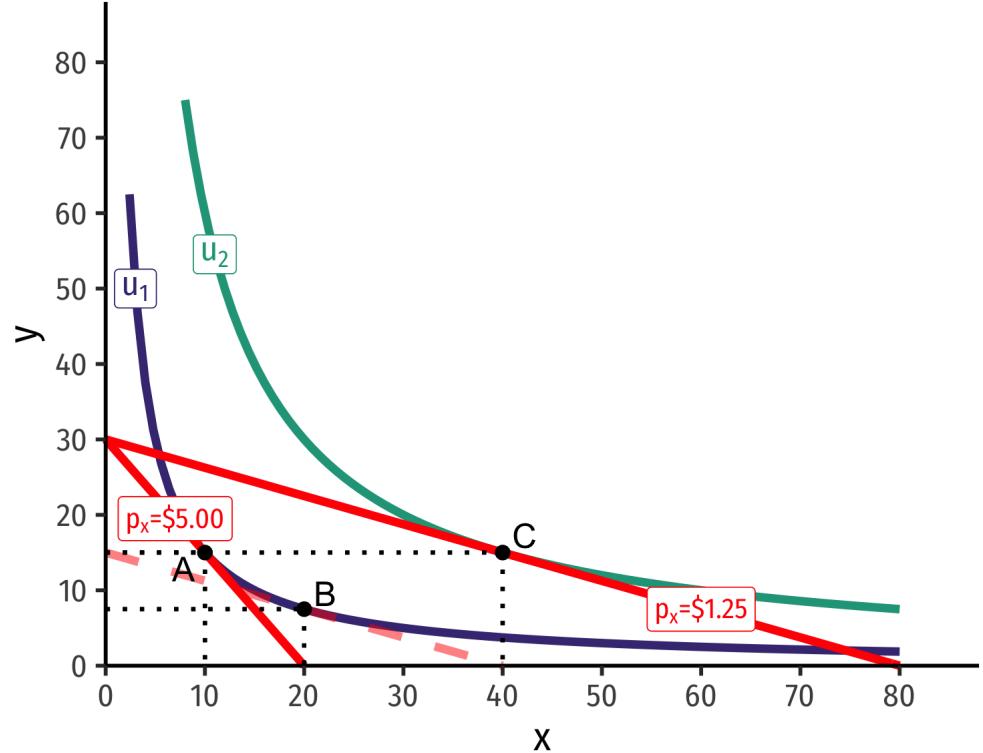


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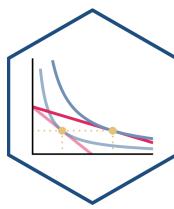


- **(Real) income effect:** change in consumption due to the **change in purchasing power** from the change in price

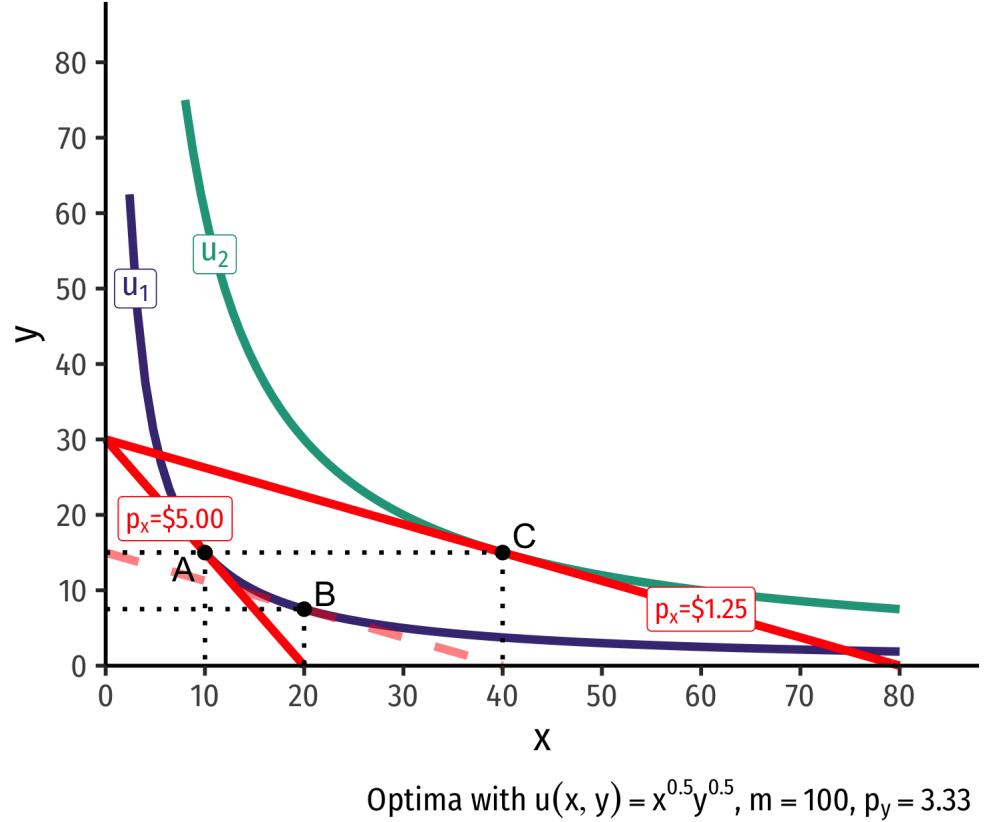


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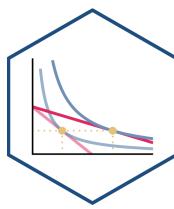
# Real Income and Substitution Effects, Graphically III



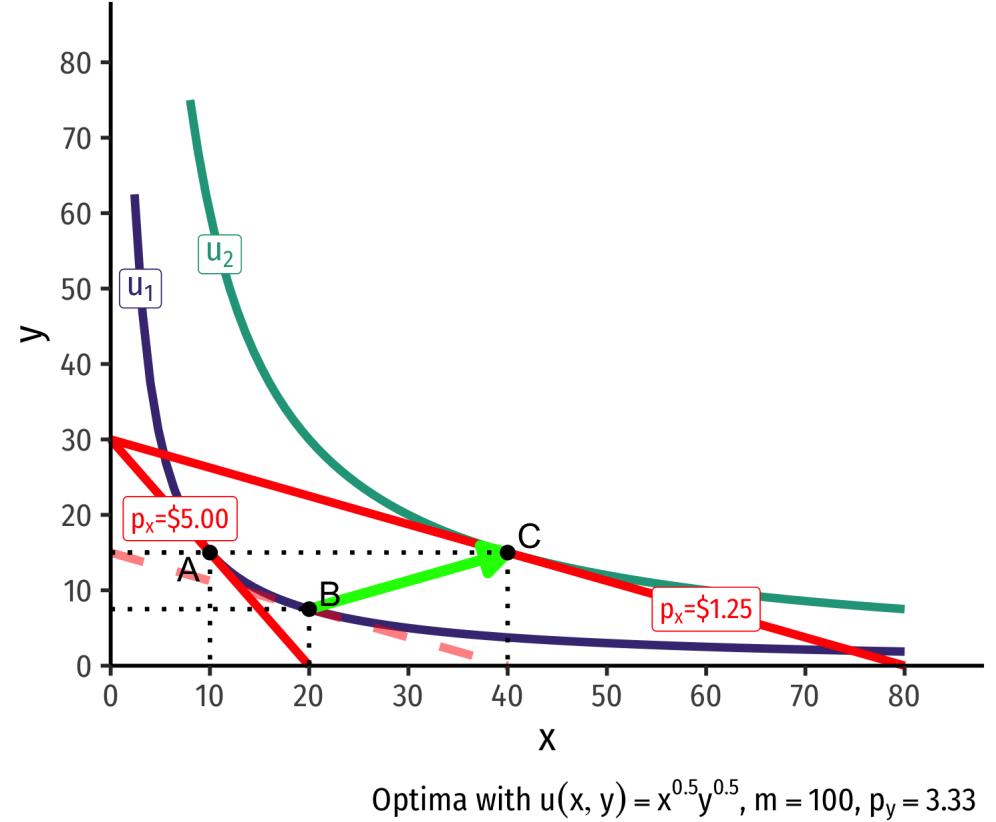
- **(Real) income effect:** change in consumption due to the **change in purchasing power** from the change in price
- $B \rightarrow C$  to new budget constraint (can buy more of  $x$  and/or  $y$ )



# Real Income and Substitution Effects, Graphically III

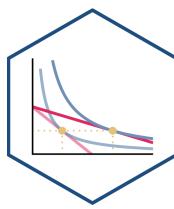


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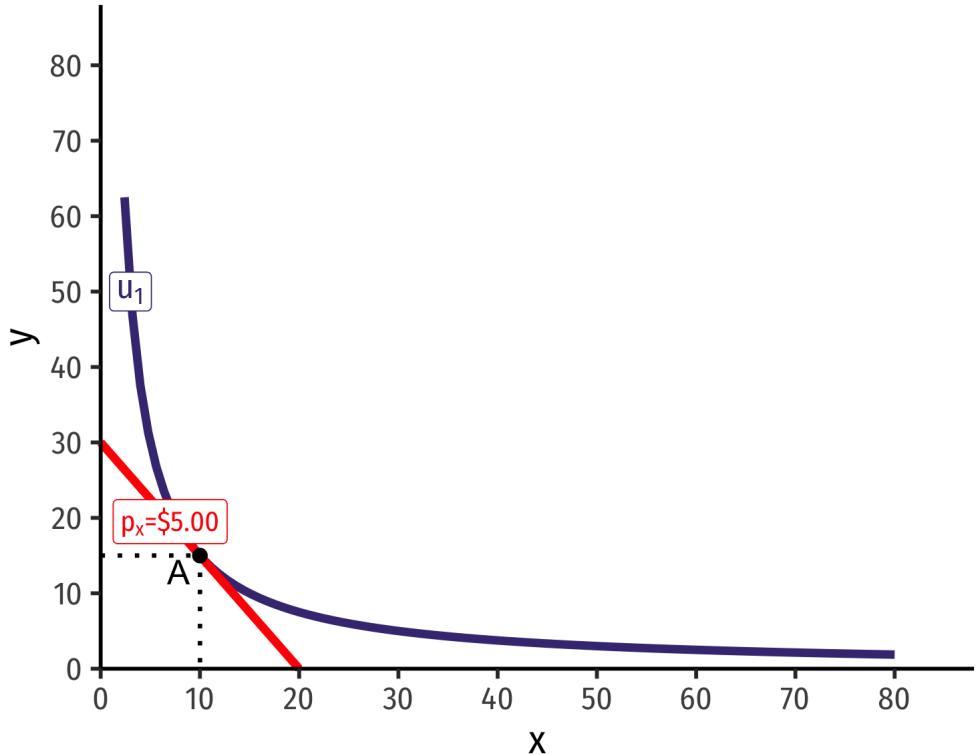


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# Real Income and Substitution Effects, Graphically IV

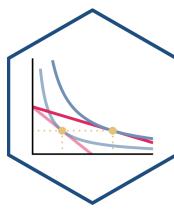


- Original optimal consumption ( $A$ )

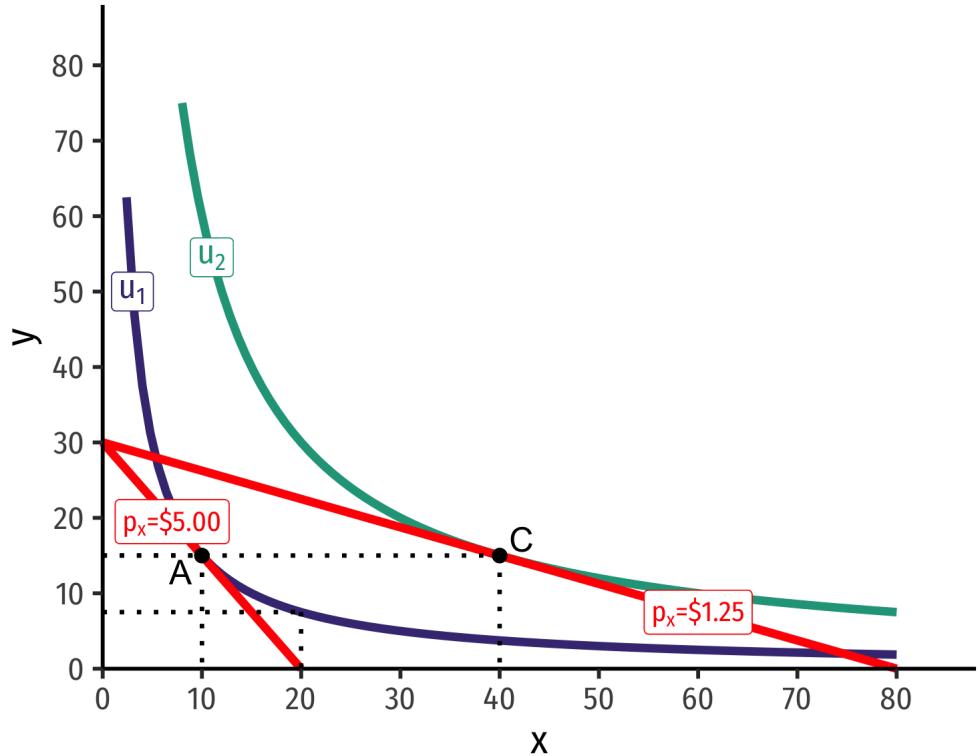


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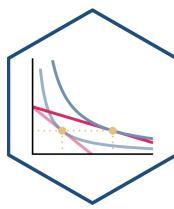


- Original optimal consumption ( $A$ )
- Price of  $x$  falls, new optimal consumption at ( $C$ )

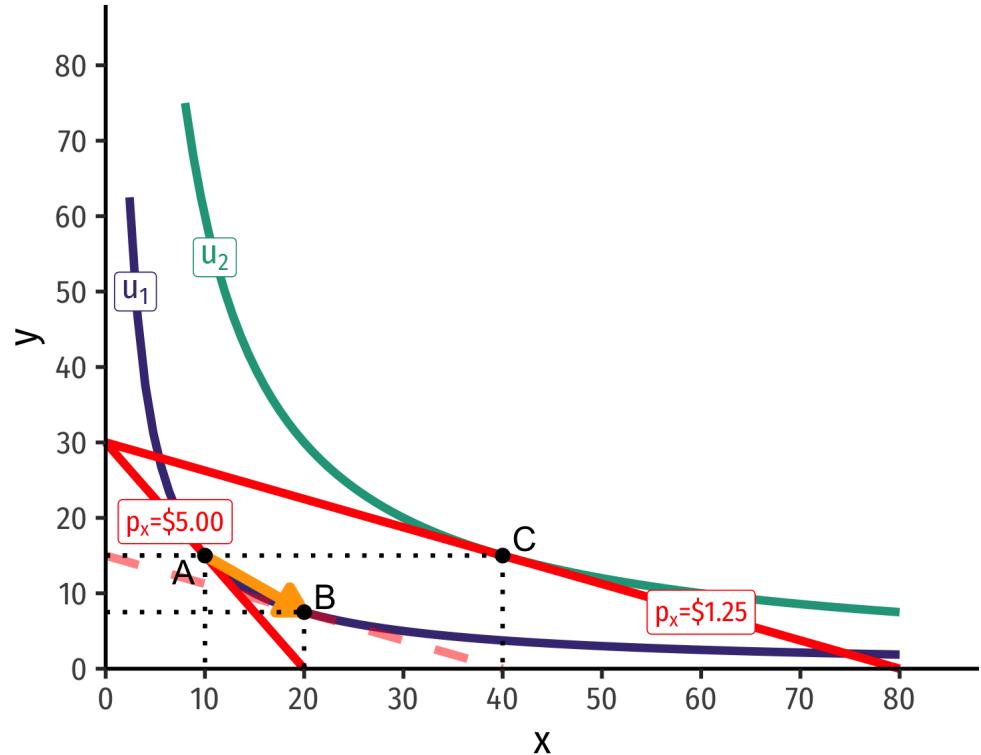


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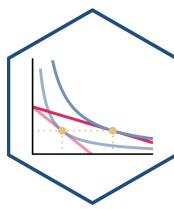


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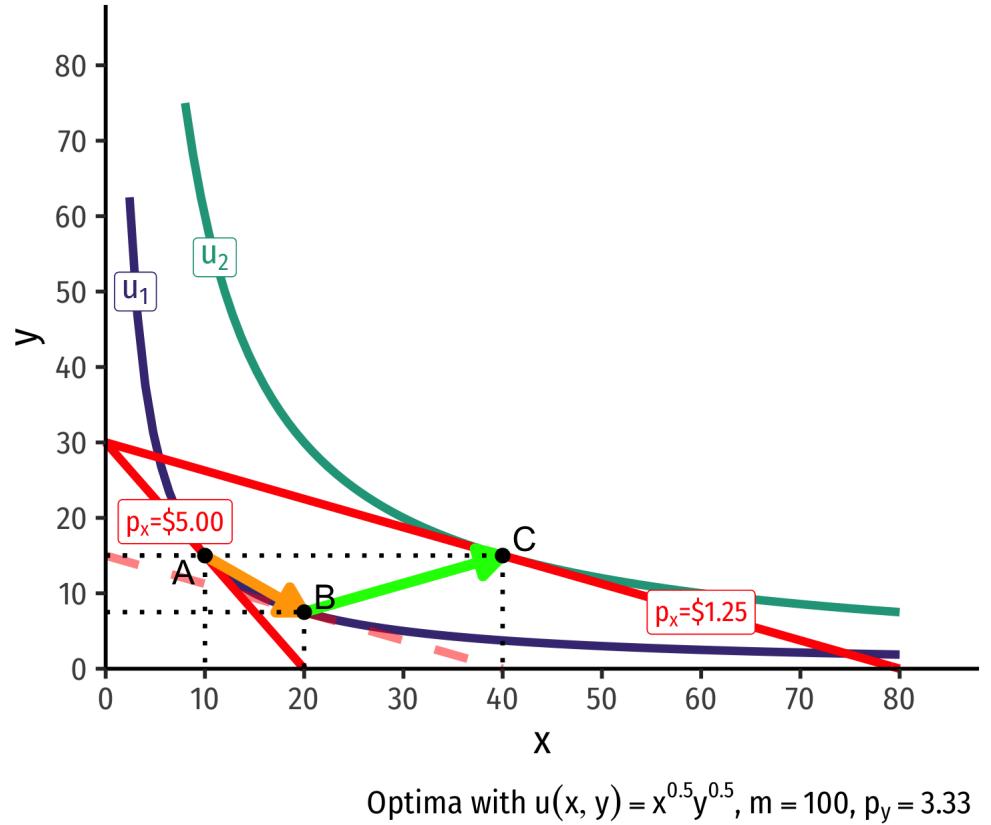


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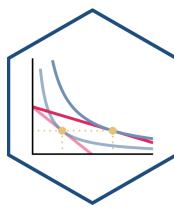
# Real Income and Substitution Effects, Graphically IV



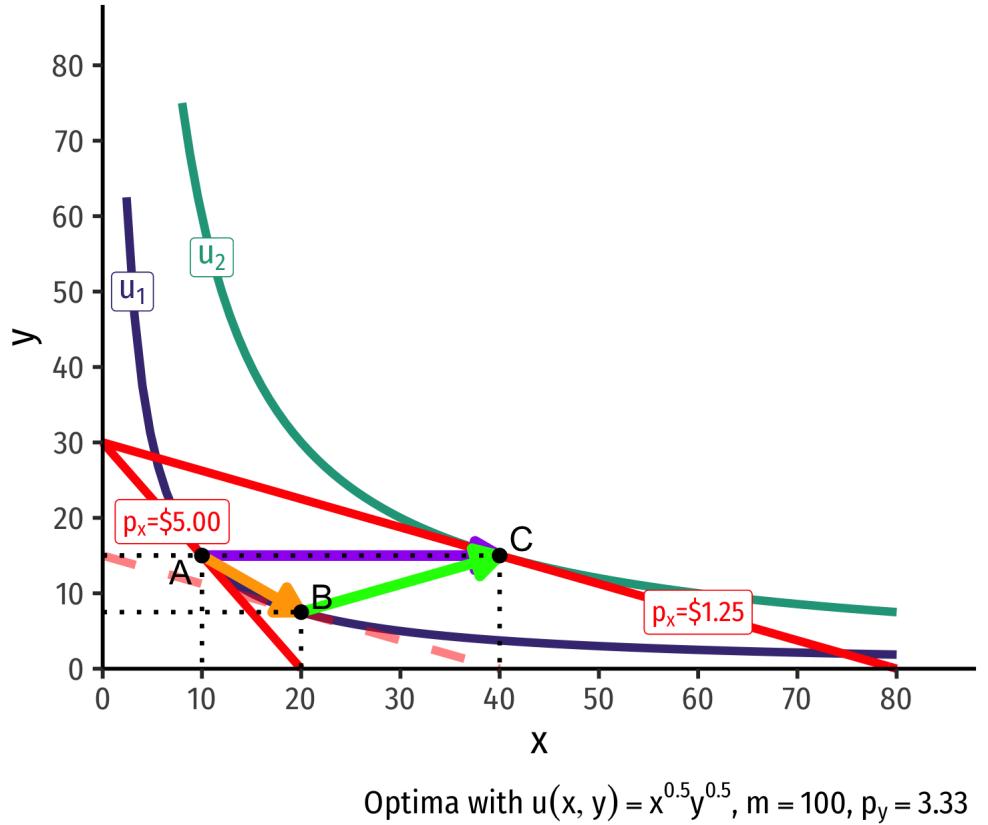
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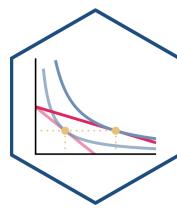
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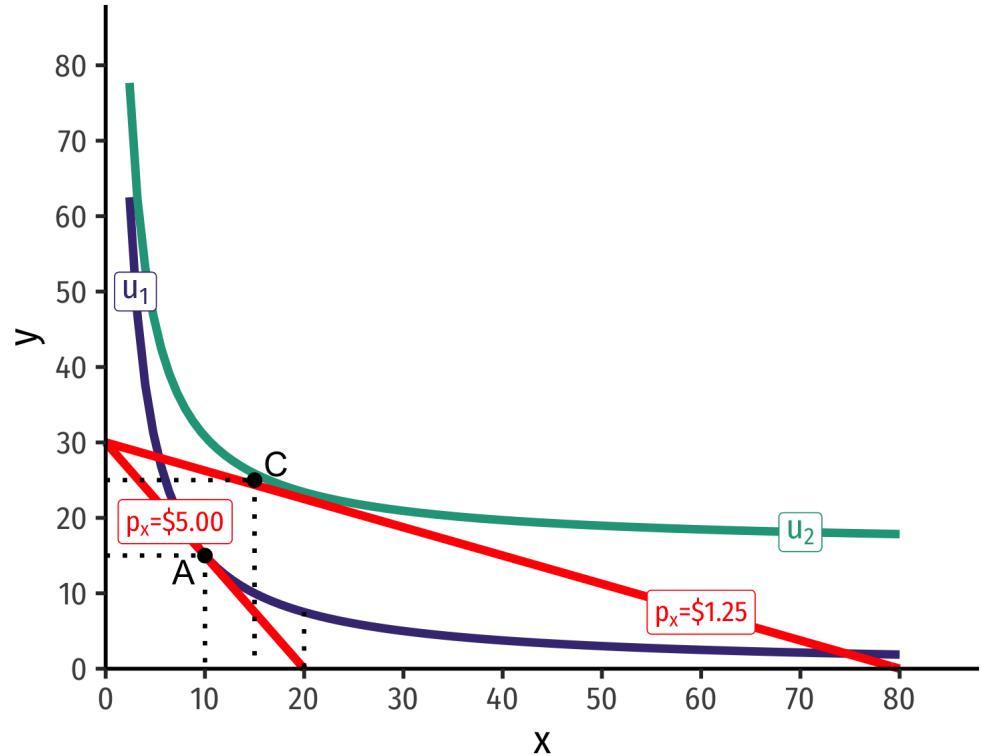
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- **(Total) price effect:**  $A \rightarrow C$



# Real Income and Substitution Effects: Inferior Good

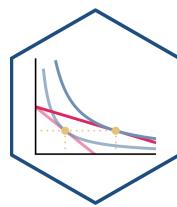


- What about an **inferior** good (Ramen)?

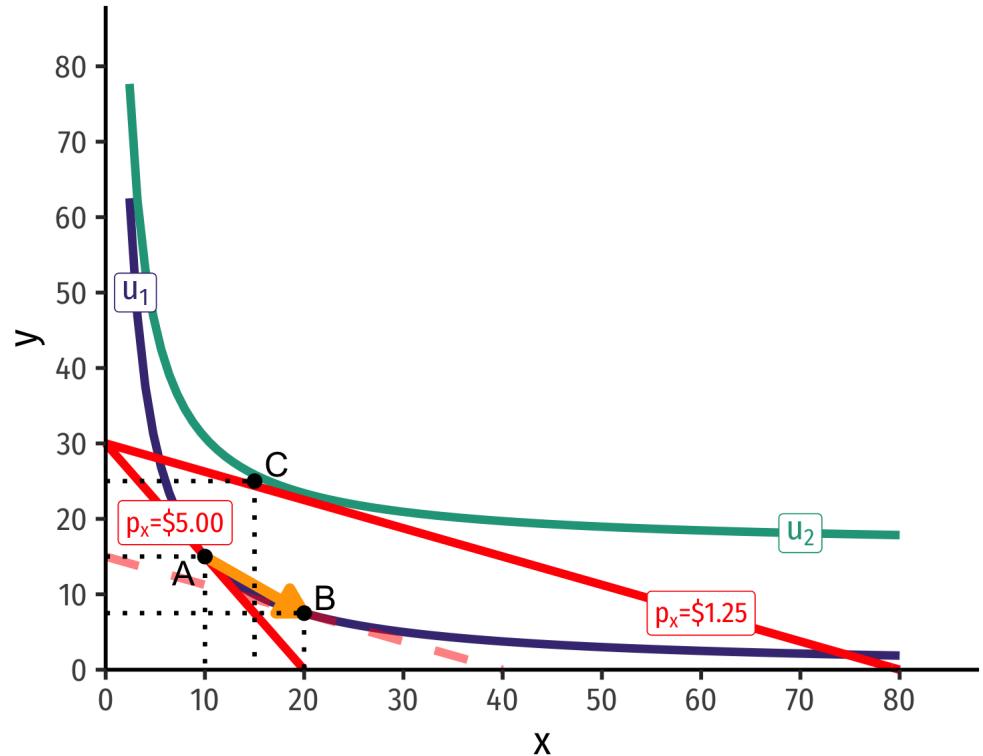


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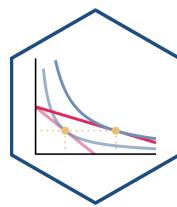


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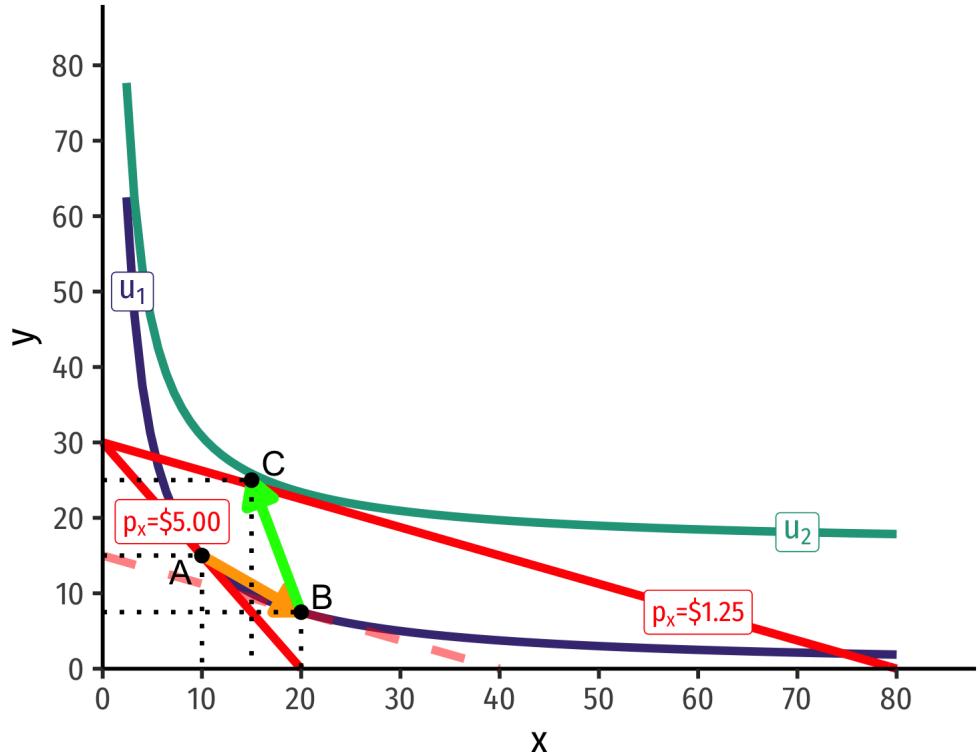


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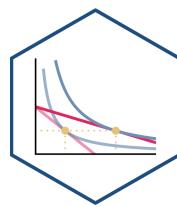


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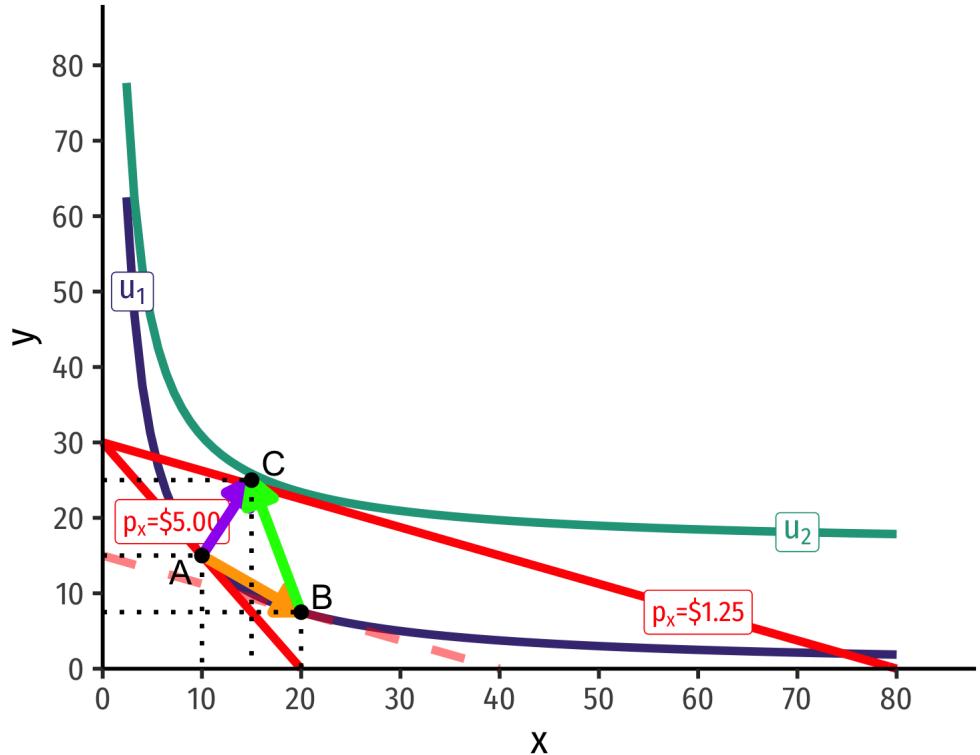


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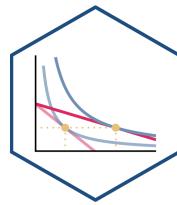


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- **(Total) price effect:**  $A \rightarrow C$

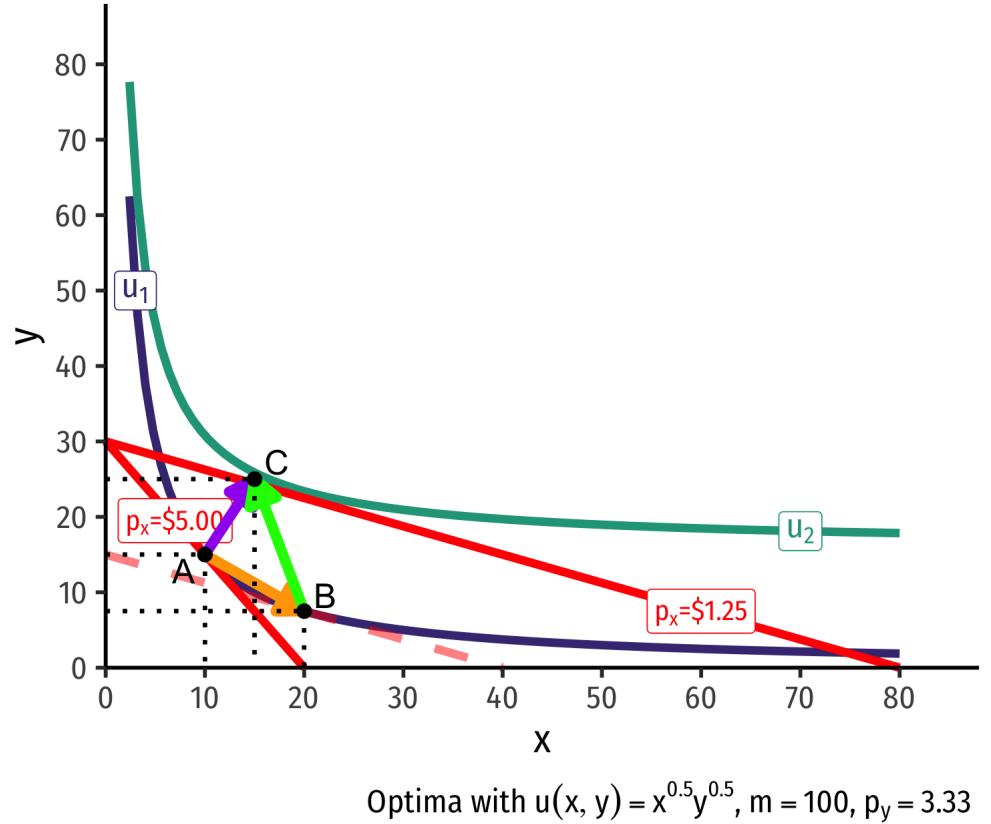


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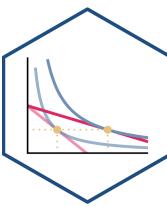
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- **(Real) income effect:**  $B \rightarrow C$  to new budget constraint (can buy more of  $x$  and/or  $y$ ) **is negative**
- **(Total) price effect:**  $A \rightarrow C$
- Price effect is *still* an  $\uparrow x$  from a  $\downarrow p_x$ !
  - Person would just prefer to spend more new purchasing power on other goods

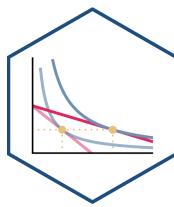


# Violating the Law of Demand



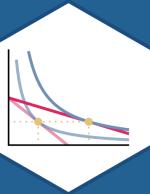
**Example:** What would it take to violate the law of demand?

# Recap: Real Income and Substitution Effects



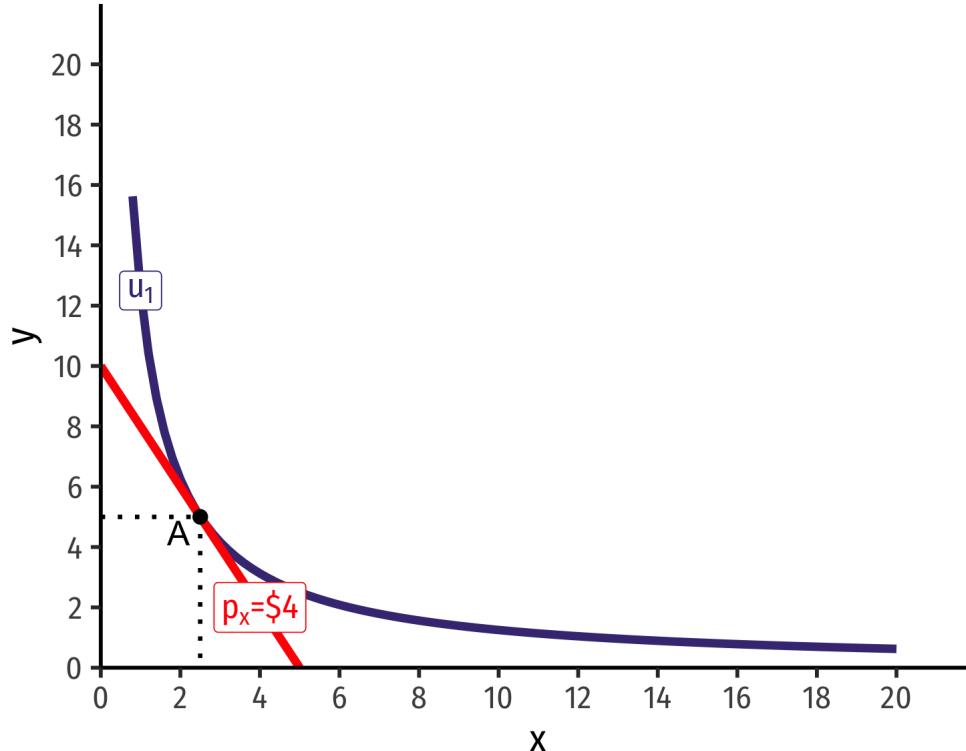
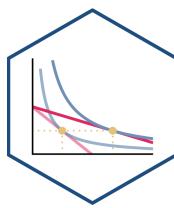
**Price Effect** = **Real income effect** + **Substitution Effect**

- **Substitution effect**: is always in the direction of the cheaper good
- **Real Income effect**: can be positive (normal) or negative (inferior)
- **Law of Demand**/Demand curves slope downwards (**Price effect**) mostly because of the substitution effect
  - Even (inferior) goods with negative real income effects overpowered by substitution effect
- Exception in the theoretical **Giffen good**: negative R.I.E. > S.E.
  - An upward sloping demand curve!

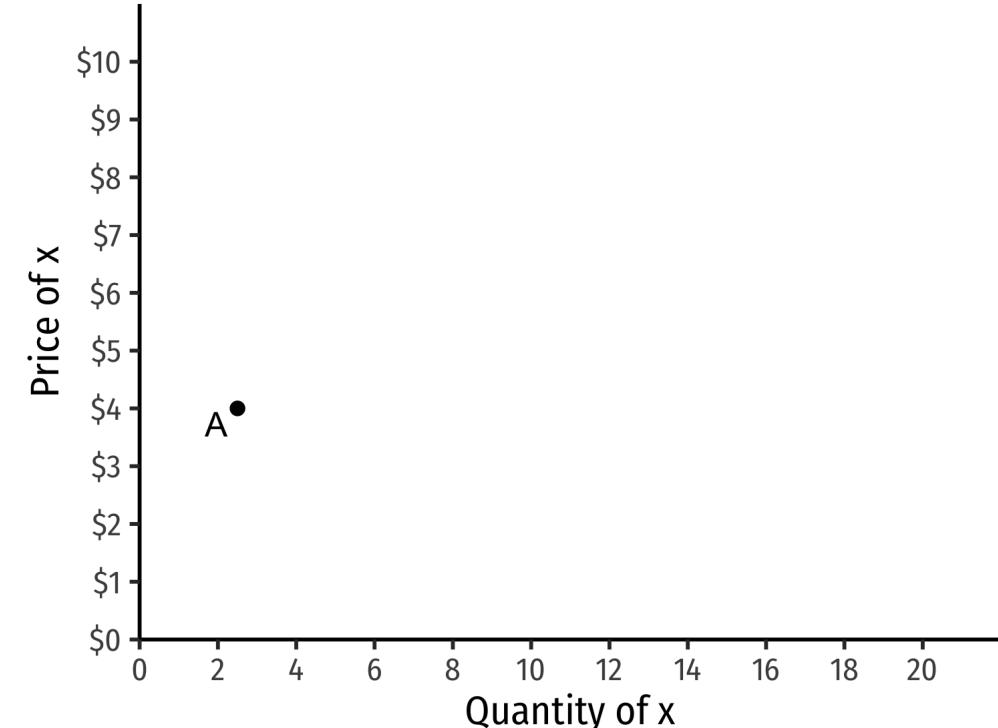


# From Optimal Consumption Points to Demand

# Deriving a Demand Curve Graphically



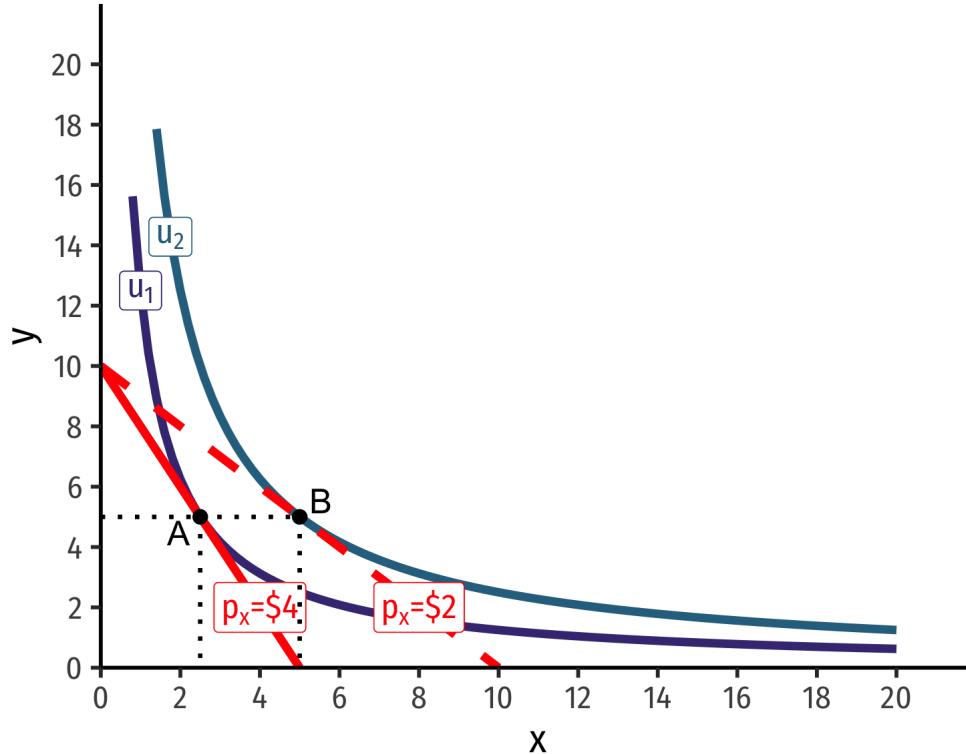
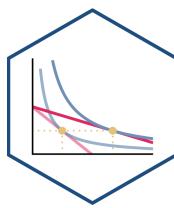
Optima with  $u(x, y) = x^{0.5}y^{0.5}$ ,  $m = 20$ ,  $p_y = 2$



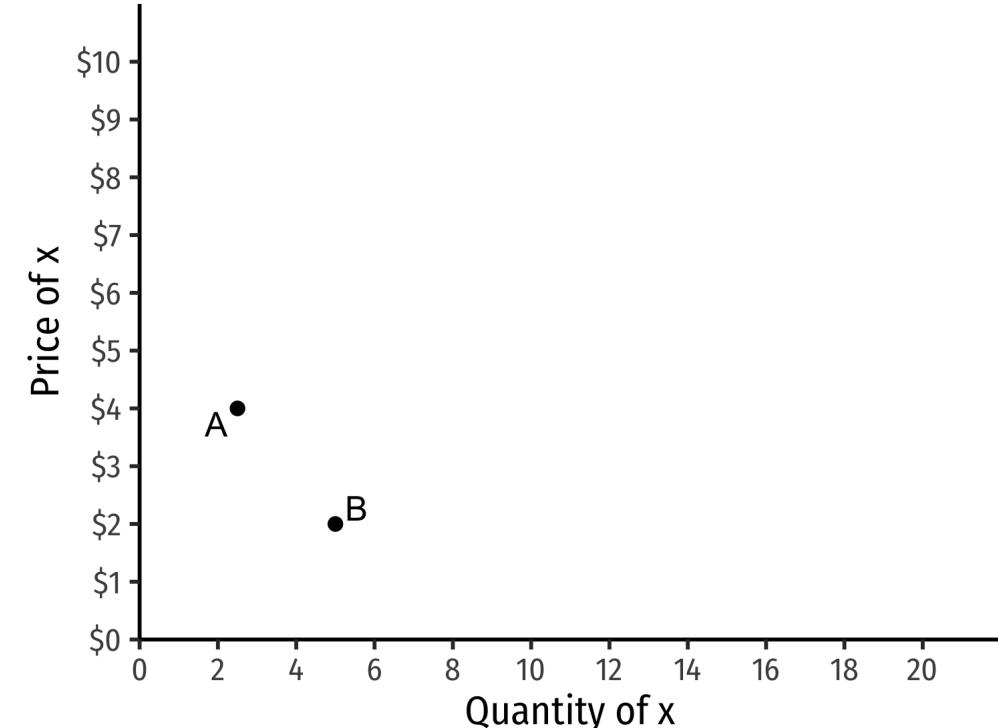
Demand function:  $\frac{m}{2p}$ ; Inverse Demand function:  $p = \frac{m}{2q}$

- Demand curve for  $x$  relates optimal consumption of  $x$  ("quantity") as price of  $x$  changes
- At  $p_x = 4$ , consumer buys  $2x$

# Deriving a Demand Curve Graphically



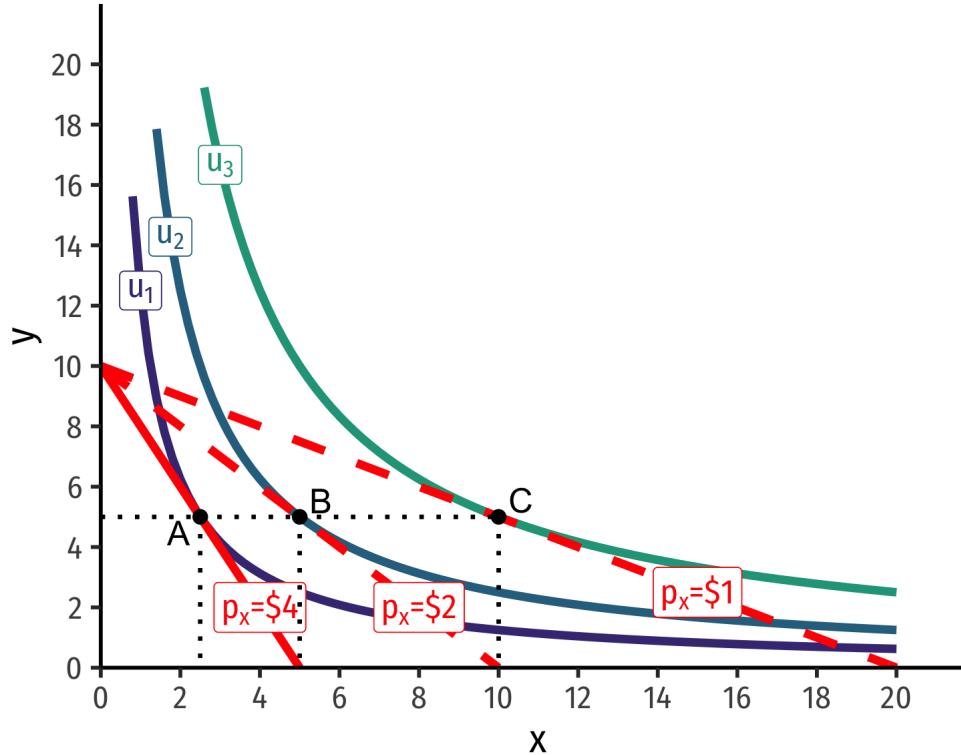
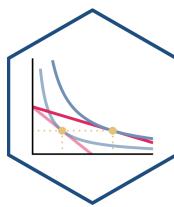
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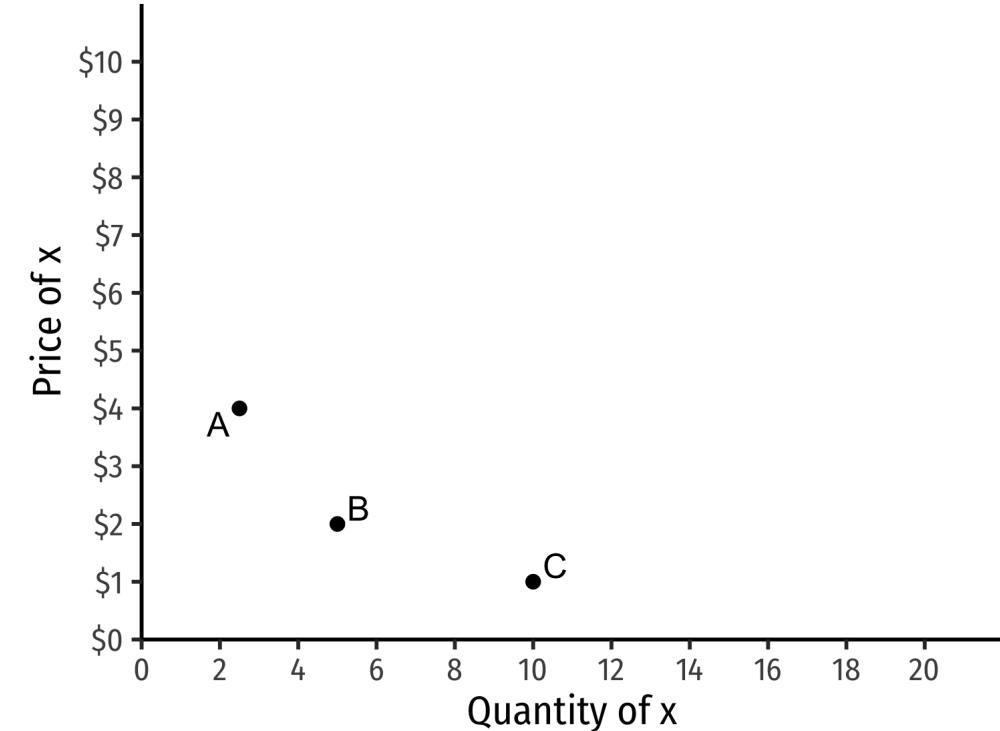
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# Deriving a Demand Curve Graphically



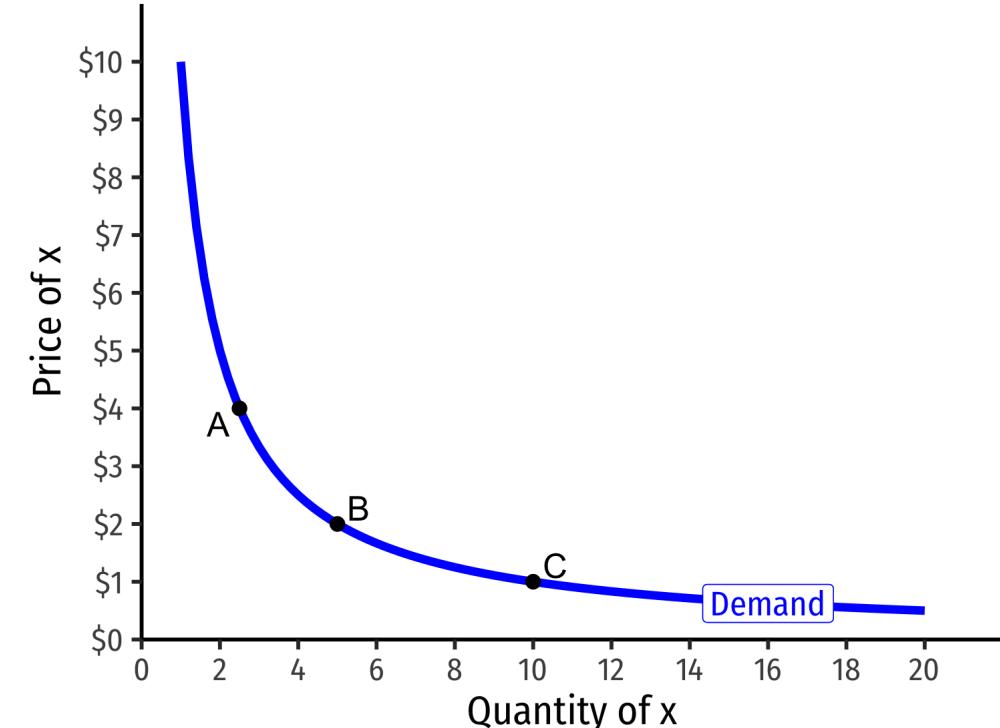
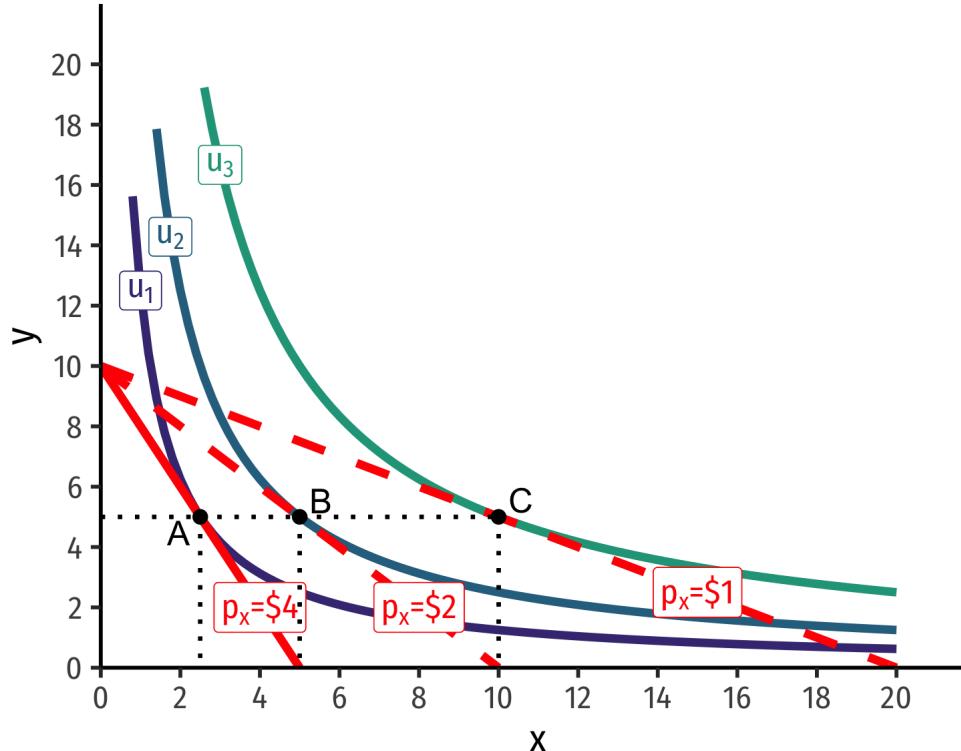
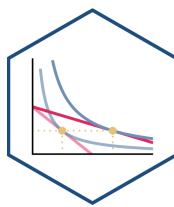
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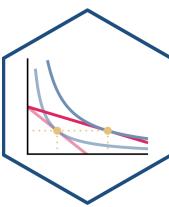
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- At  $p_x = 4$ , consumer buys 2  $x$ ; at  $p_x = 2$ , consumer buys 5  $x$ ; at  $p_x = 1$ , consumer buys 10  $x$

# Deriving a Demand Curve Graphically

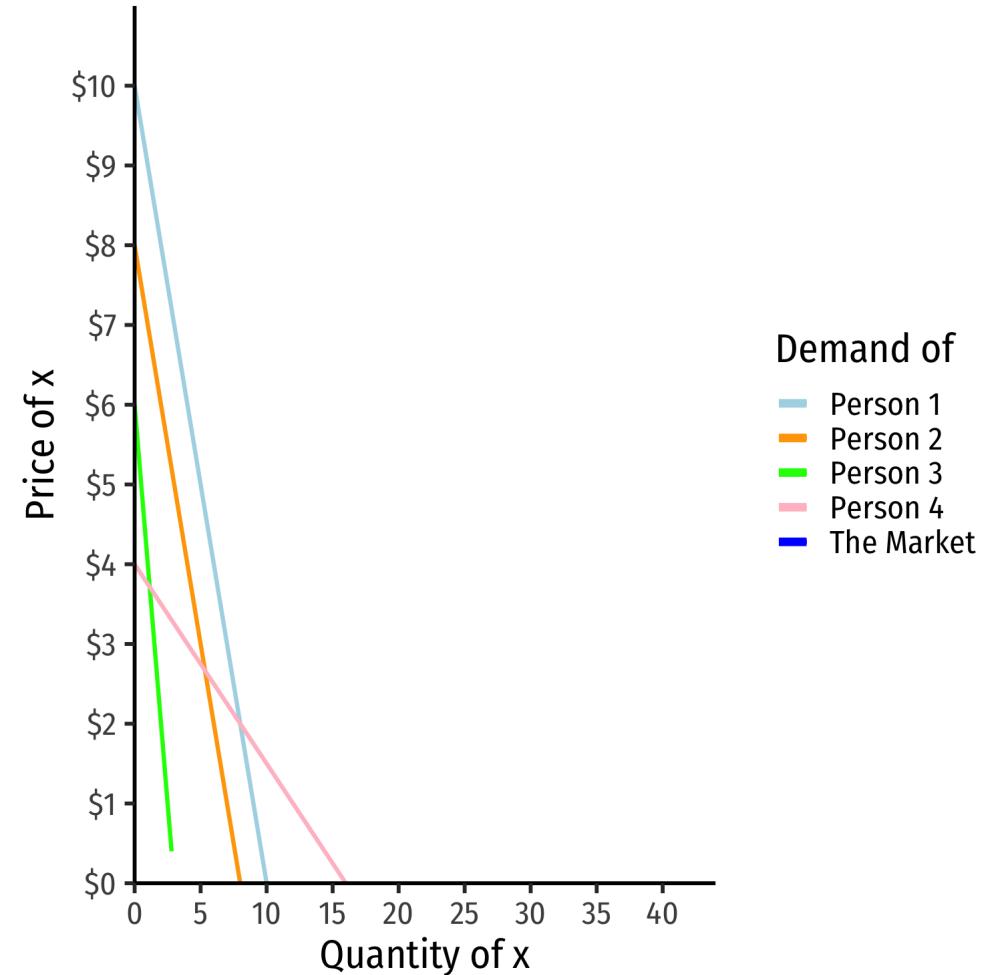


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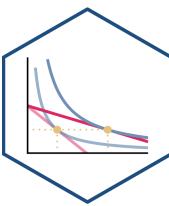
# From Individual Demand to Market Demand



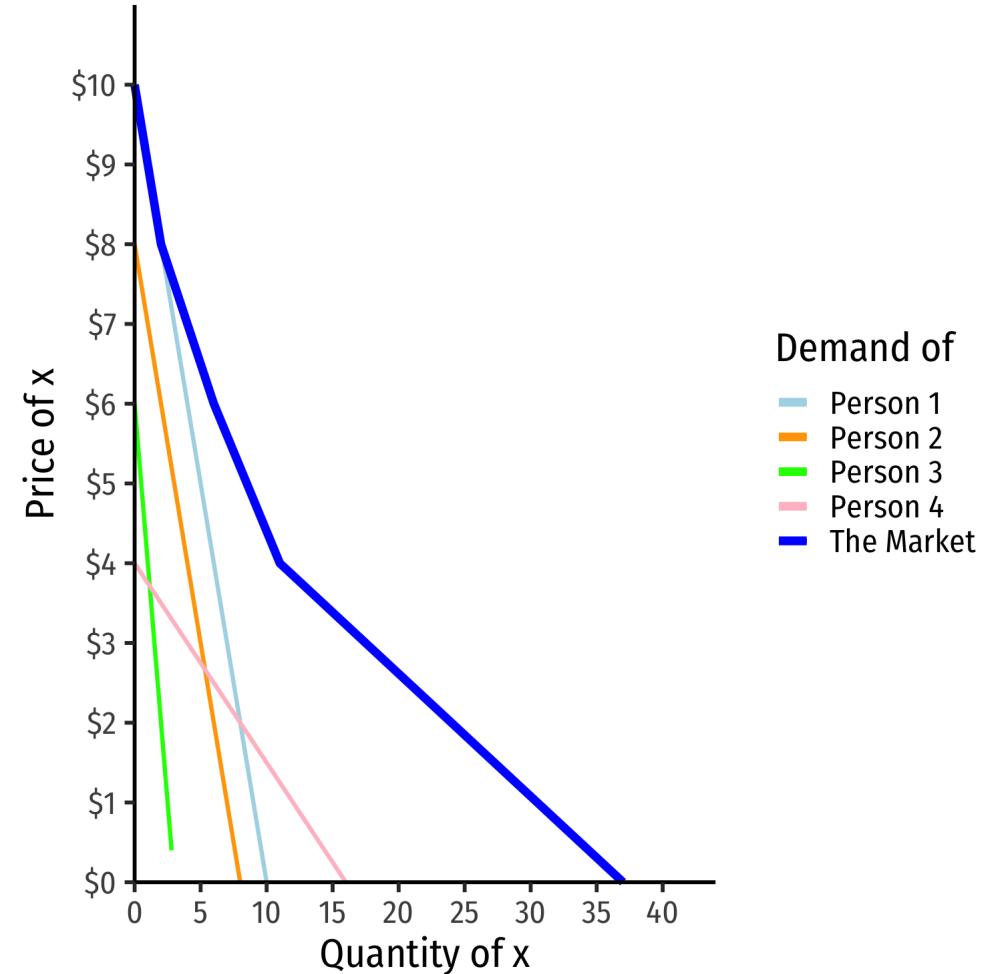
- Note so far we have been talking about *an individual person's demand*
- In principles, you learned about the entire **market demand**



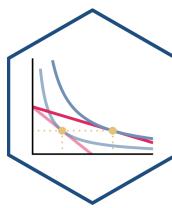
# From Individual Demand to Market Demand



- Note so far we have been talking about *an individual person's demand*
- In principles, you learned about the entire **market demand**
- This is simply the sum of all individuals' demands



# Demand Schedule (For Individual Or Market)



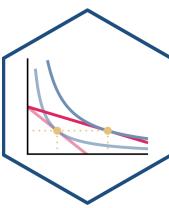
- **Demand schedule** expresses the quantity of good a person(s) would be willing to buy ( $q_D$ ) at any given price ( $p_x$ )

- Holding constant all other prices ( $p_y$ ) and income ( $m$ )! (“**ceterus paribus**”)

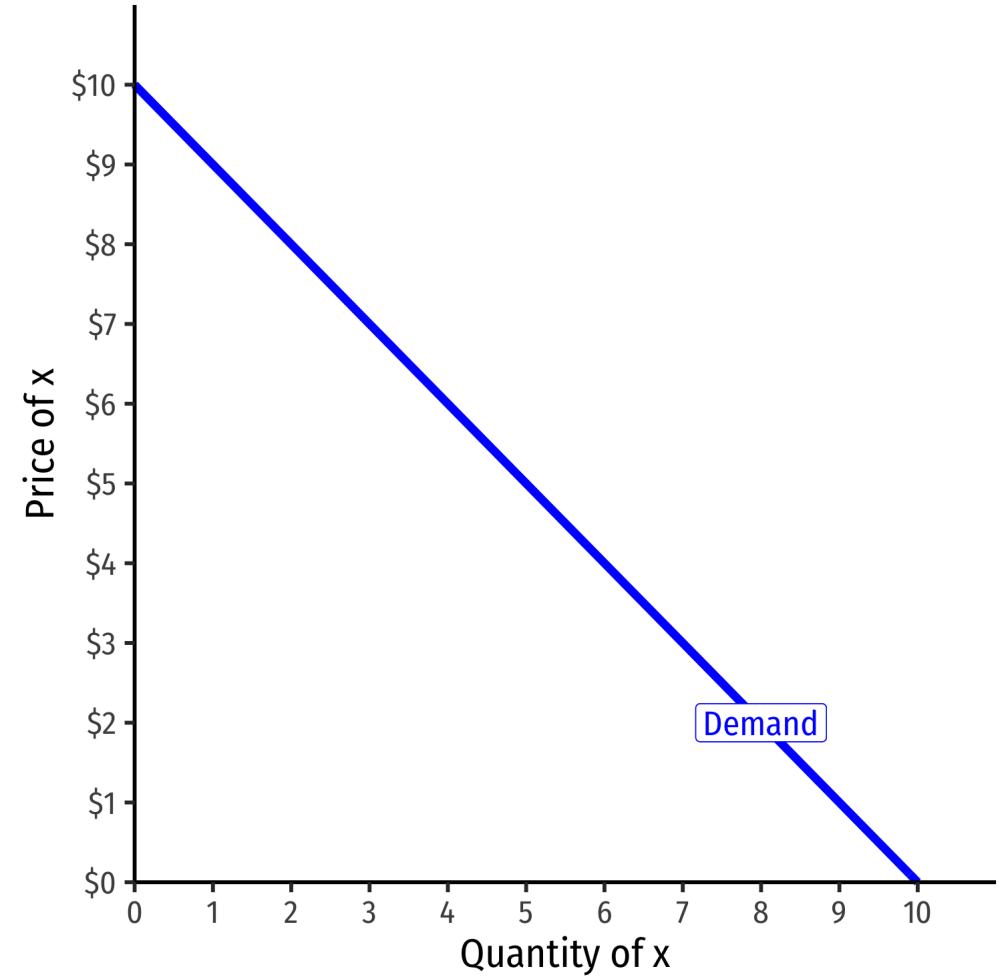
- Note: each of these is a consumer's optimum at a given price!

| price | quantity |
|-------|----------|
| 10    | 0        |
| 9     | 1        |
| 8     | 2        |
| 7     | 3        |
| 6     | 4        |
| 5     | 5        |
| 4     | 6        |
| 3     | 7        |
| 2     | 8        |
| 1     | 9        |
| 0     | 10       |

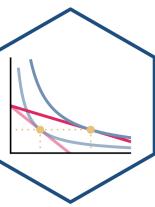
# Demand Curve



- **Demand curve** graphically represents the demand schedule
- Also measures a person's **maximum willingness to pay (WTP)** for a given quantity
- **Law of Demand (price effect)**  $\Rightarrow$  demand curves always slope downwards



# Demand Function



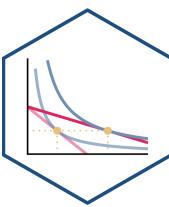
- **Demand function** relates quantity to price

**Example:**

$$q = 10 - p$$

- Not graphable (wrong axes)!

# Inverse Demand Function

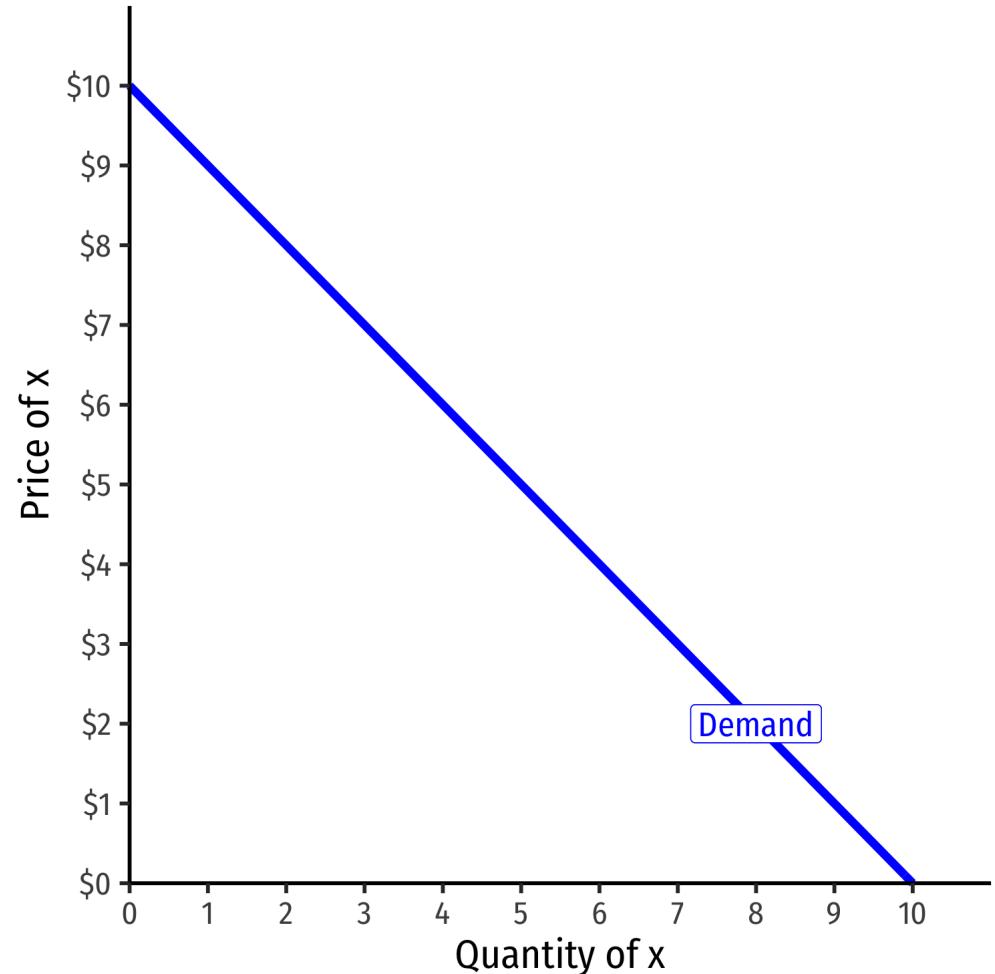


- ***Inverse demand function*** relates price to quantity
  - Take demand function and solve for  $p$

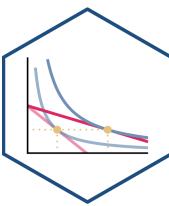
## Example:

$$p = 10 - q$$

- Graphable (price on vertical axis)!



# Inverse Demand Function

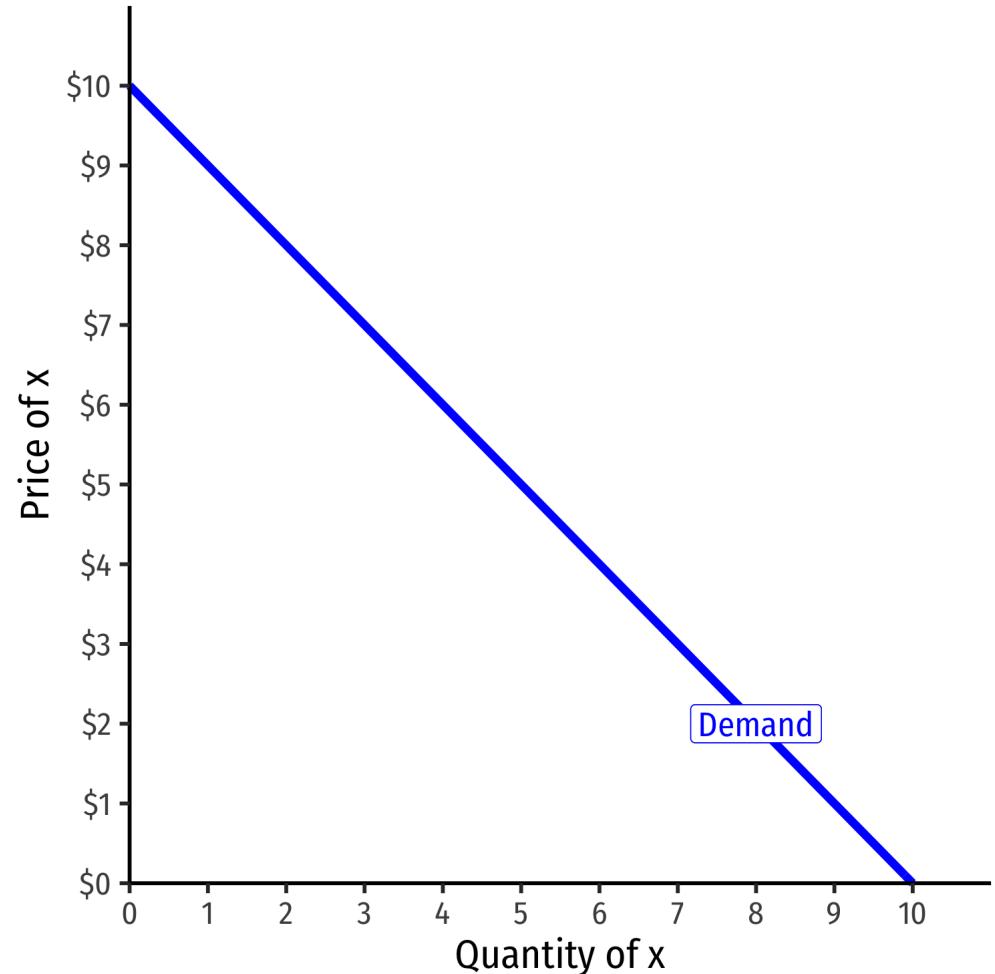


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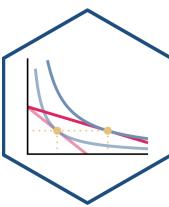
## Example:

$$p = 10 - q$$

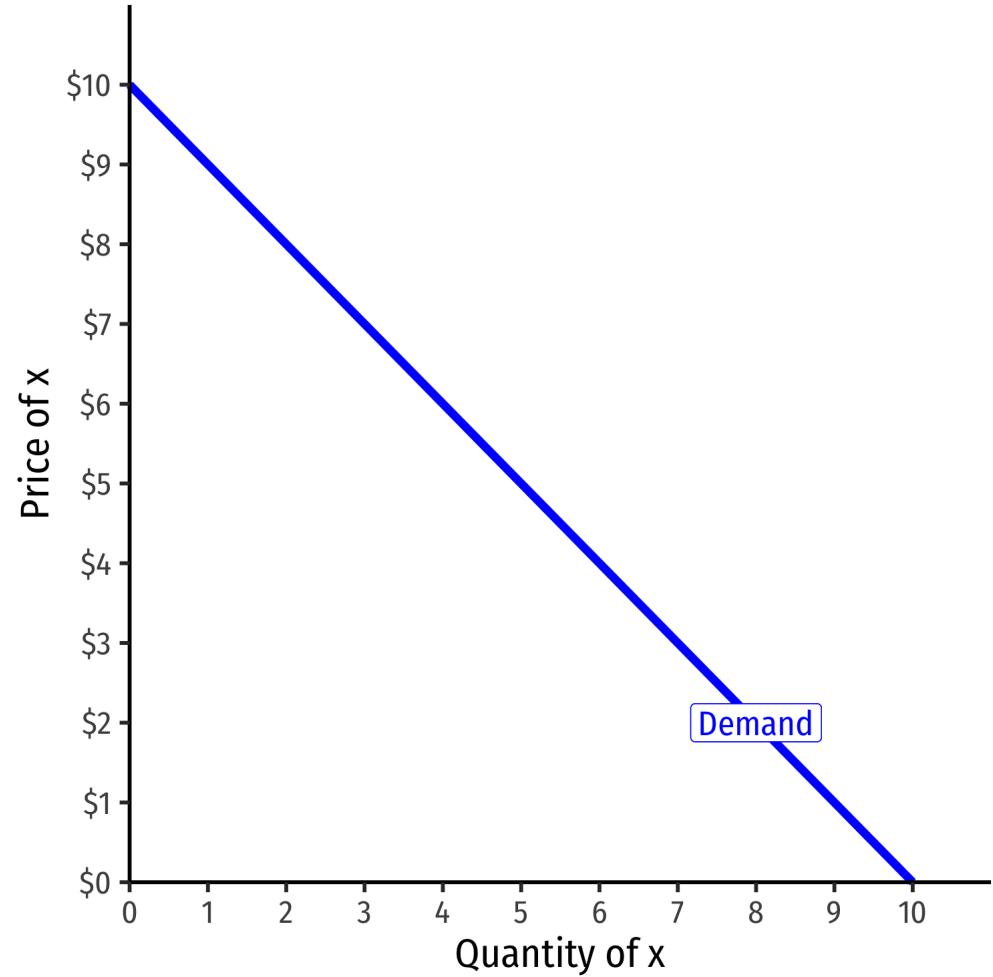
- Vertical intercept ("Choke price"): price where  $q_D = 0$  (\$10), just high enough to discourage *any* purchases



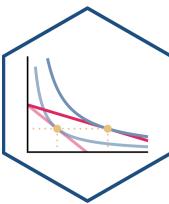
# Inverse Demand Function



- Read two ways:
- Horizontally: at any given price, how many units person wants to buy
- Vertically: at any given quantity, the **maximum willingness to pay (WTP)** for that quantity
  - This way will be very useful later



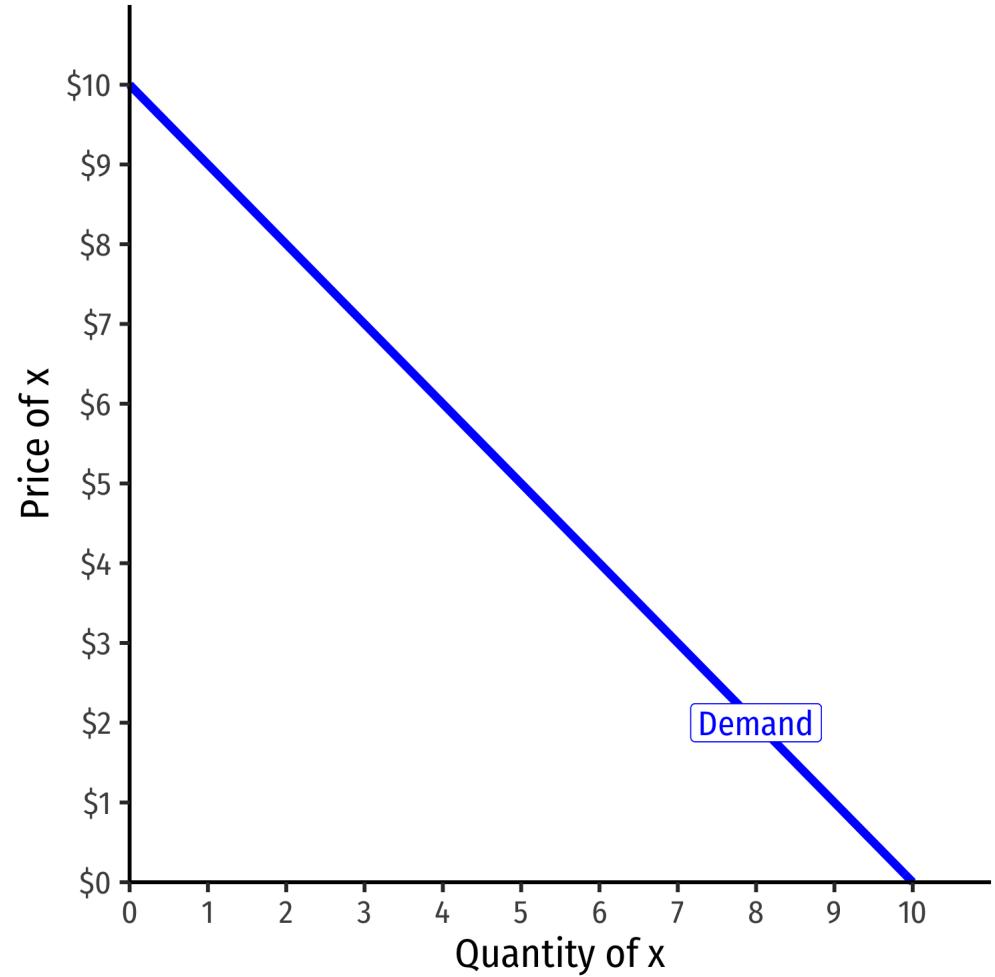
# Shifts in Demand I



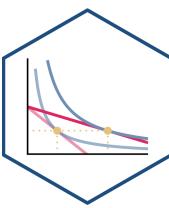
- Note a simple (inverse) demand function only relates (own) **price** and **quantity**

**Example:**  $q = 10 - p$  or  
 $p = 10 - q$

- What about all the other "**determinants of demand**" like income and other prices?



# Shifts in Demand II



- A change in one of the "**determinants of demand**" will **shift** demand curve!
  1. Change in **income**  $m$
  2. Change in **price of other goods**  $p_y$
  3. Change in **preferences** or **expectations** about good  $x$
- Shows up in (inverse) demand function by a **change in intercept (choke price)**!
- See my [Visualizing Demand Shifters](#)

