1.4 — Utility Maximization

ECON 306 • Microeconomic Analysis • Spring 2022
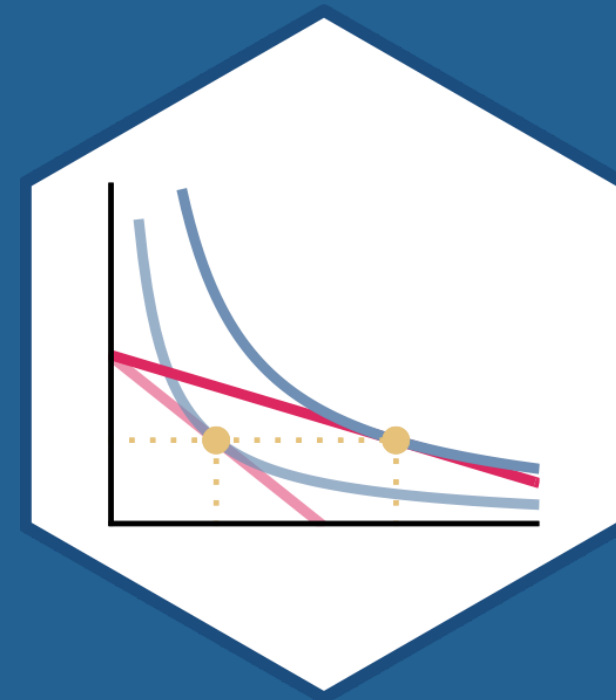
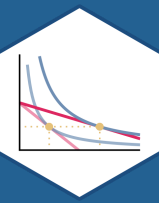Ryan Safner

Assistant Professor of Economics

✈ safner@hood.edu
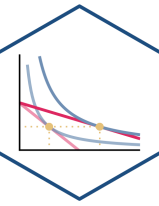
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🌐 microS22.classes.ryansafner.com

Constrained Optimization

Constrained Optimization I
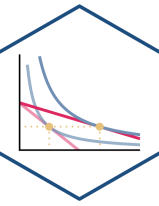
- We model most situations as a **constrained optimization problem**:

- People **optimize**: make tradeoffs to achieve their **objective** *as best as they can*

- Subject to **constraints**: limited resources (income, time, attention, etc)

Constrained Optimization II

- One of the most generally useful mathematical models

- *Endless applications*: how we model nearly every decision-maker

  > consumer, business firm, politician, judge, bureaucrat, voter, dictator, pirate, drug cartel, drug addict, parent, child, etc

- **Key economic skill: recognizing how to apply the model to a situation**

Remember!
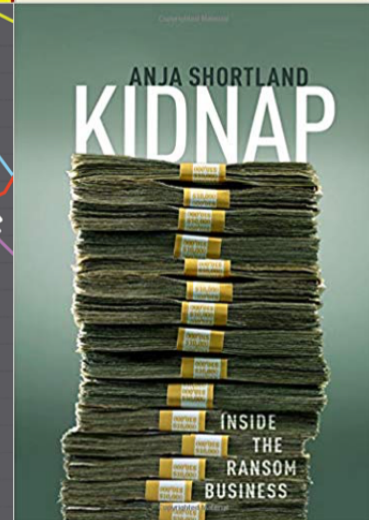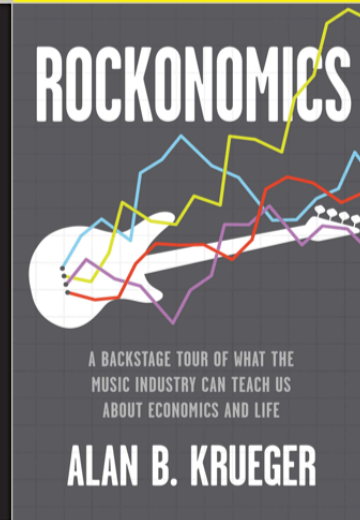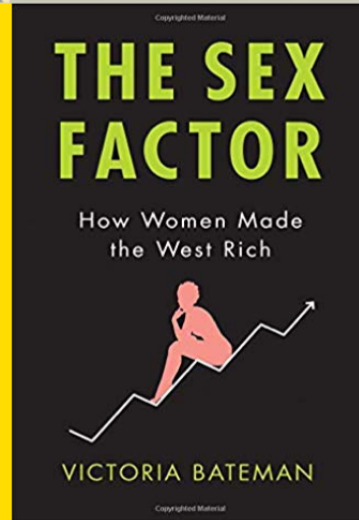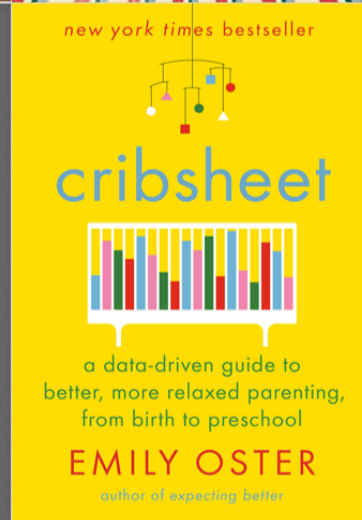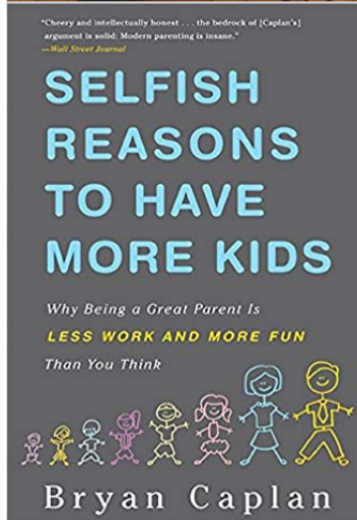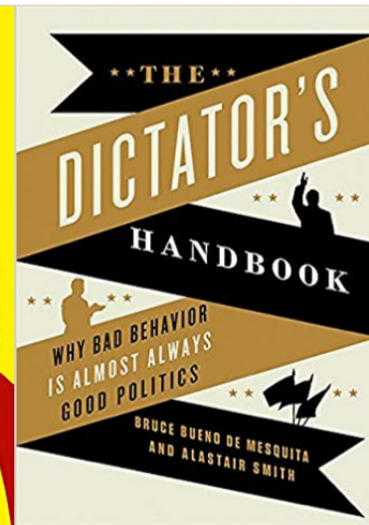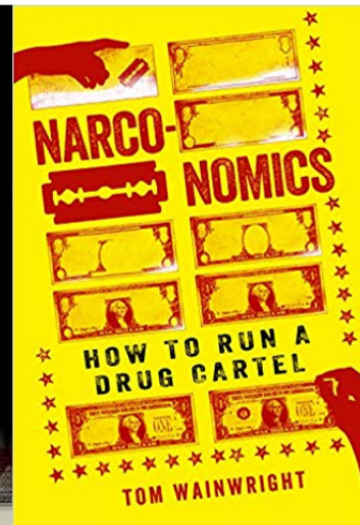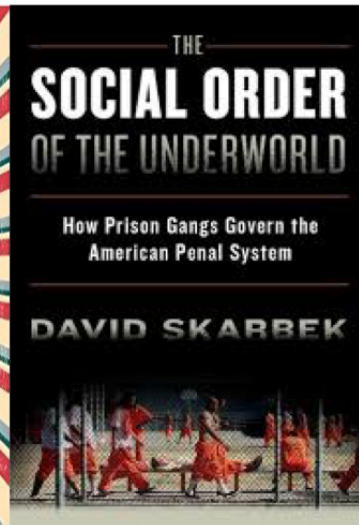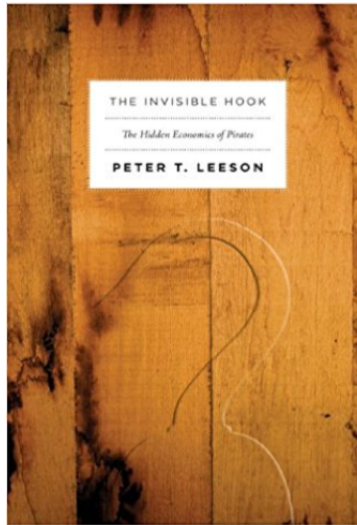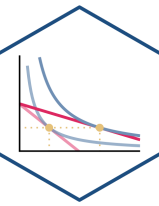
Constrained Optimization III
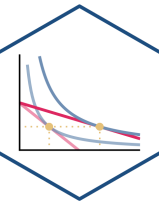
- All constrained optimization models have three moving parts:

Constrained Optimization III
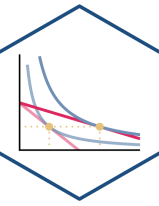
- All constrained optimization models have three moving parts:

1. **Choose:** **< some alternative >**

Constrained Optimization III
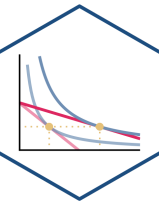
- All constrained optimization models have three moving parts:

1. **Choose:** **< some alternative >**

2. **In order to maximize:** **< some objective >**

Constrained Optimization III
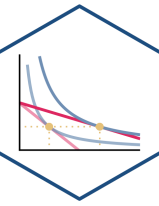
- All constrained optimization models have three moving parts:

1. **Choose:** **< some alternative >**

2. **In order to maximize:** **< some objective >**

3. **Subject to:** **< some constraints >**

Constrained Optimization: Example I
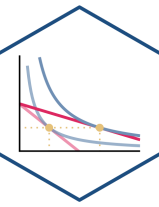
Example: A Hood student picking courses hoping to achieve the highest GPA while getting an Econ major.

1. **Choose:**

2. **In order to maximize:**

3. **Subject to:**

Constrained Optimization: Example II
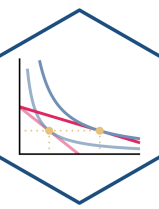
Example: How should FedEx plan its delivery route?

1. **Choose:**

2. **In order to maximize:**
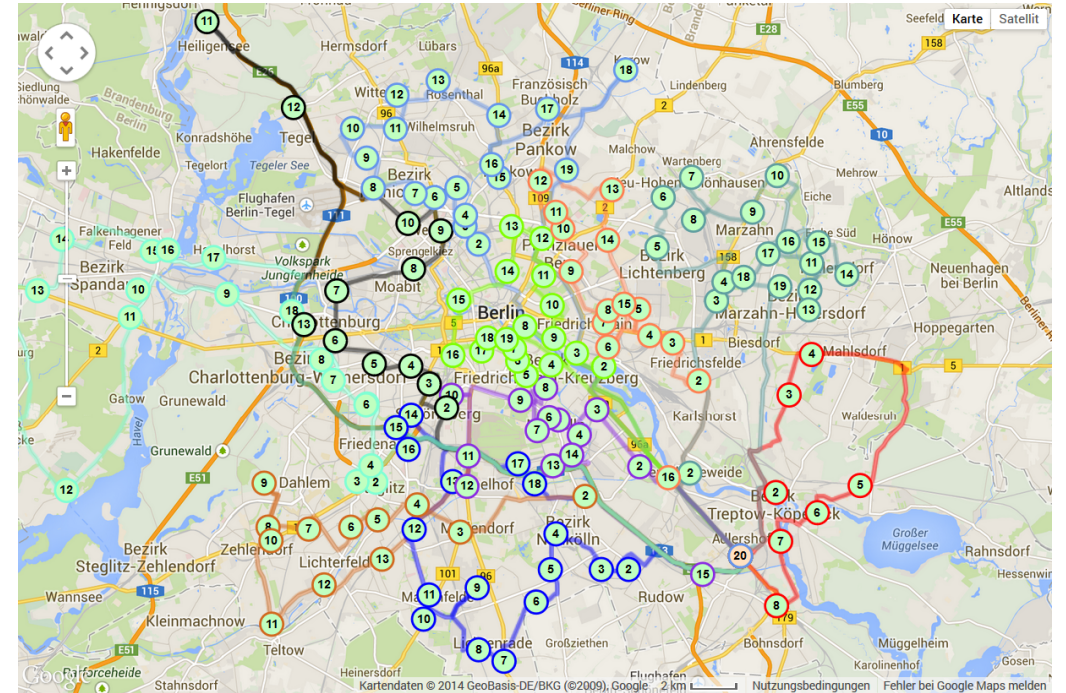
3. **Subject to:**

Constrained Optimization: Example III
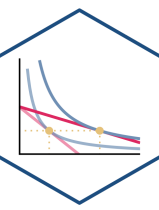
Example: The U.S. government wants to remain economically competitive but reduce emissions by 25%.

1. **Choose:**

2. **In order to maximize:**

3. **Subject to:**

Constrained Optimization: Example IV
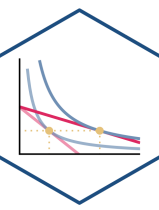
Example: How do elected officials make decisions in politics?

1. **Choose:**

2. **In order to maximize:**

3. **Subject to:**

The Utility Maximization Problem

- The individual's **utility maximization problem** we've been modeling, finally, is:

1. **Choose:** **< a consumption bundle >**

2. **In order to maximize:** **< utility >**

3. **Subject to:** **< income and market prices >**

The Utility Maximization Problem: Tools
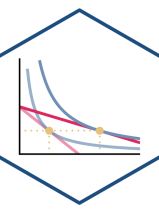
- We now have the tools to understand individual choices:

- **Budget constraint**: individual's **constraints** of income and market prices

  - How market trades off between goods
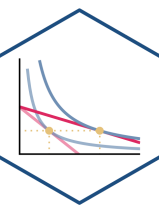 - Marginal cost (of good x, in terms of y)

- **Utility function**: individual's **objective** to maximize, based on their preferences

  - How individual trades off between goods
  - Marginal benefit (of good x, in terms of y)

The Utility Maximization Problem: Verbally
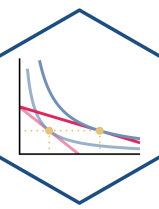
- The **individual's constrained optimization problem**:

  > choose a bundle of goods to maximize utility, subject to income and market prices

The Utility Maximization Problem: Mathematically
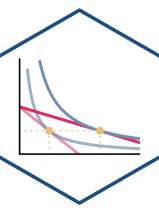
$$\max_{x,y \geq 0} u(x,y)$$

$$s.t.\, p_x x + p_y y = m$$

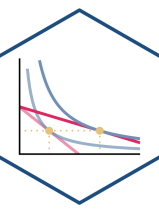- This requires calculus to solve.[†] We will look at **graphs** instead!

[†] See the mathematical appendix in today's class notes on how to solve it with calculus, and an example.

The Individual's Optimum: Graphically

- **Graphical solution**: **Highest indifference curve *tangent* to budget constraint**
  - Bundle A!
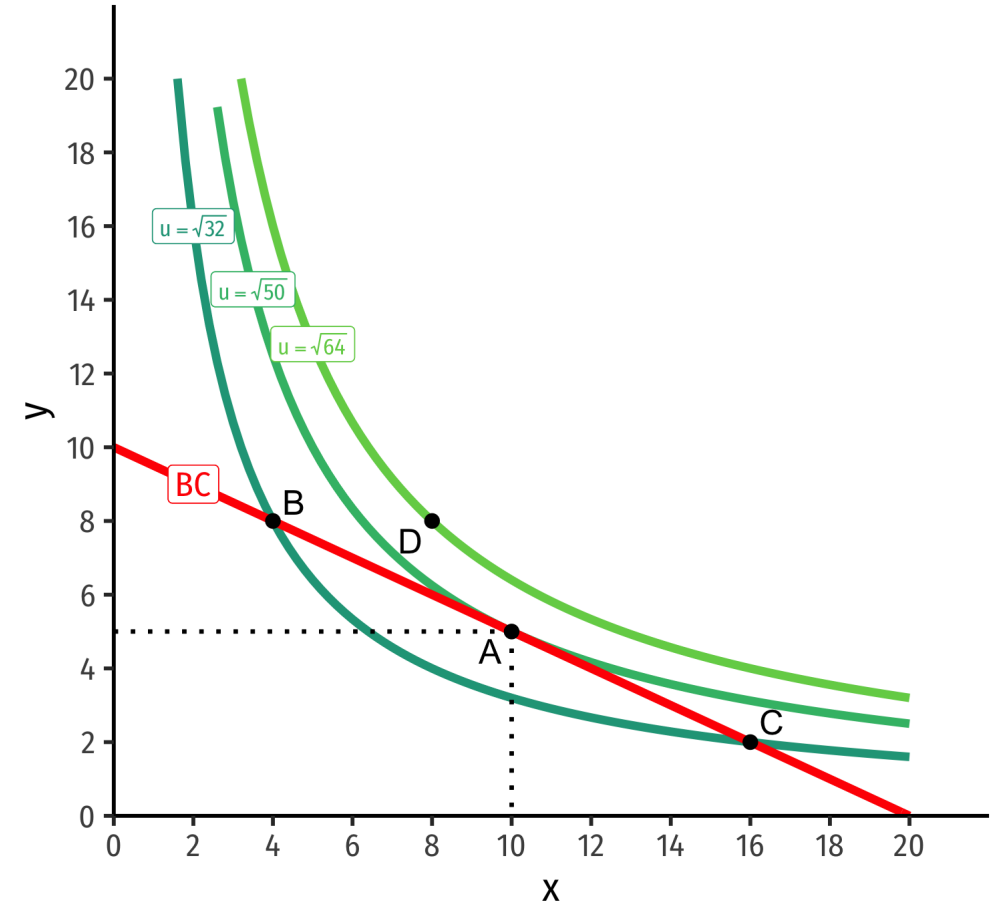
$u(x, y) = \sqrt{xy}$, m = \$20, p_x = \$1, p_y = \$2

The Individual's Optimum: Graphically

- **Graphical solution**: **Highest indifference curve *tangent* to budget constraint**

  ○ Bundle A!

- B or C spend all income, but a better combination exists

$u(x, y) = \sqrt{xy}$, m = \$20, p_x = \$1, p_y = \$2
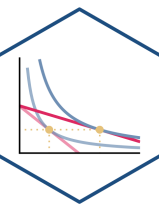
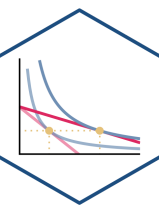# The Individual's Optimum: Graphically

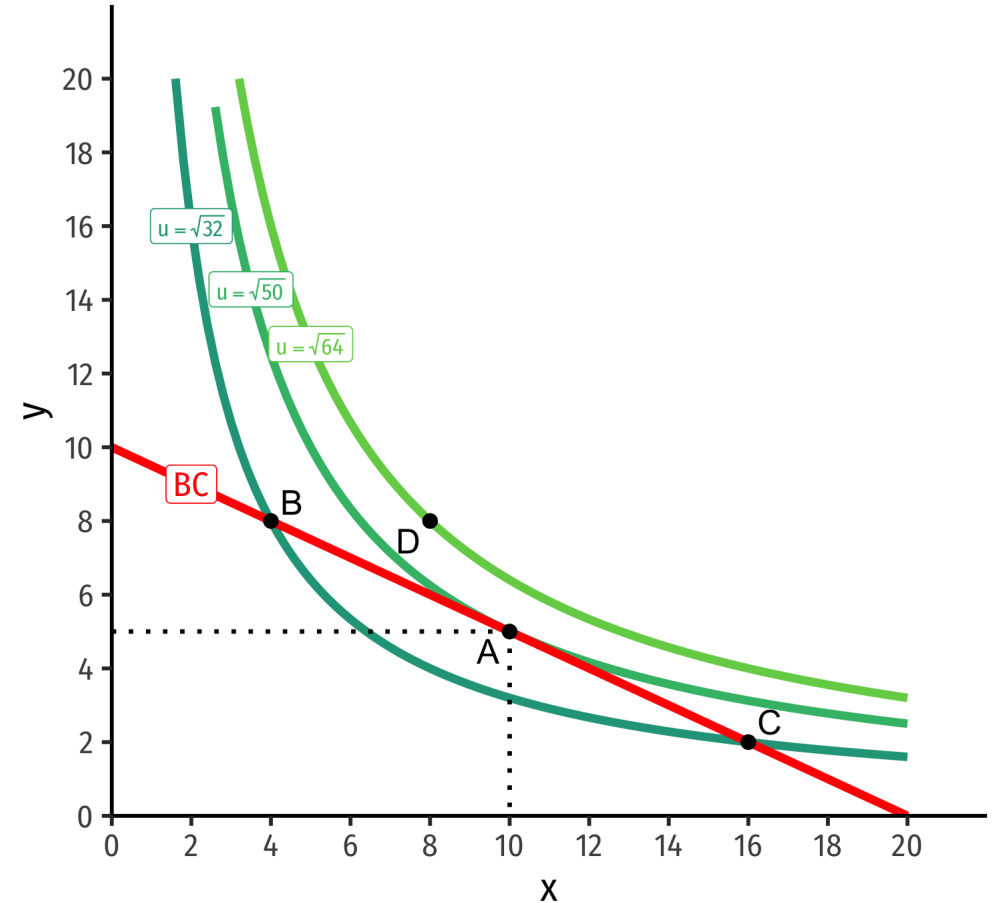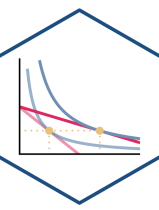- **Graphical solution**: **Highest indifference curve *tangent* to budget constraint**

  - Bundle A!

- B or C spend all income, but a better combination exists

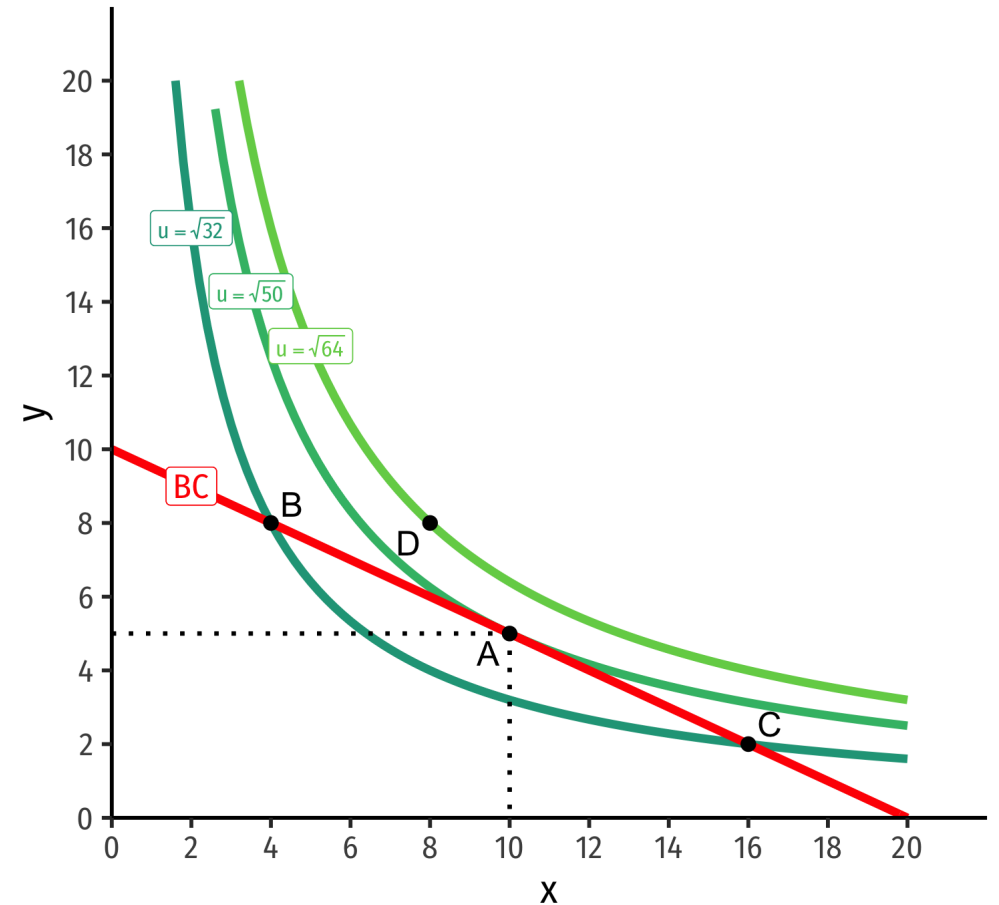- D is higher utility, but *not affordable* at current income & prices

$u(x, y) = \sqrt{xy}$, m = \$20, p_x = \$1, p_y = \$2
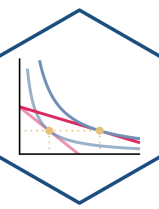
The Individual's Optimum: Why Not B?

indiff. curve slope $>$ budget constr. slope

$u(x, y) = \sqrt{xy}$, m = \$20, p_x = \$1, p_y = \$2

The Individual's Optimum: Why Not B?

indiff. curve slope > budget constr. slope

$$\frac{MU_x}{MU_y} > \frac{p_x}{p_y}$$

$$2 > 0.5$$

- **Consumer** views MB of x is 2 units of y

  - Consumer's "exchange rate:" **2Y:1X**

- **Market**-determined MC of x is 0.5 units of y
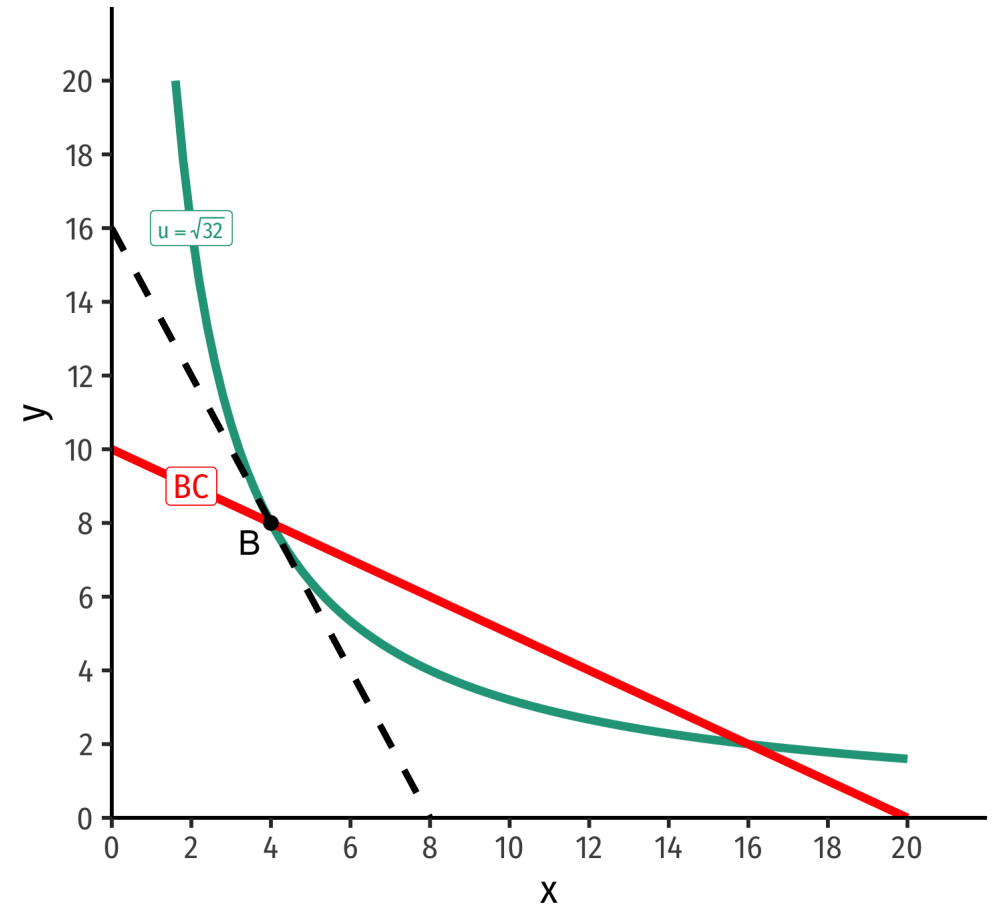
  - Market exchange rate is **0.5Y:1X**

$u(x, y) = \sqrt{xy}$, m = \$20, p_x = \$1, p_y = \$2

The Individual's Optimum: Why Not B?

indiff. curve slope > budget constr. slope

$$\frac{MU_x}{MU_y} > \frac{p_x}{p_y}$$

$$2 > 0.5$$

- **Consumer** views MB of x is 2 units of y

  - Consumer's "exchange rate:" **2Y:1X**

- **Market**-determined MC of x is 0.5 units of y

  - Market exchange rate is **0.5Y:1X**

- Can **spend less on y, more on x** for **more utility!**

$u(x, y) = \sqrt{xy}$, m = $20, p_x = $1, p_y = $2

The Individual's Optimum: Why Not C?

indiff. curve slope $<$ budget constr. slope

u = √32

BC

C

y

x

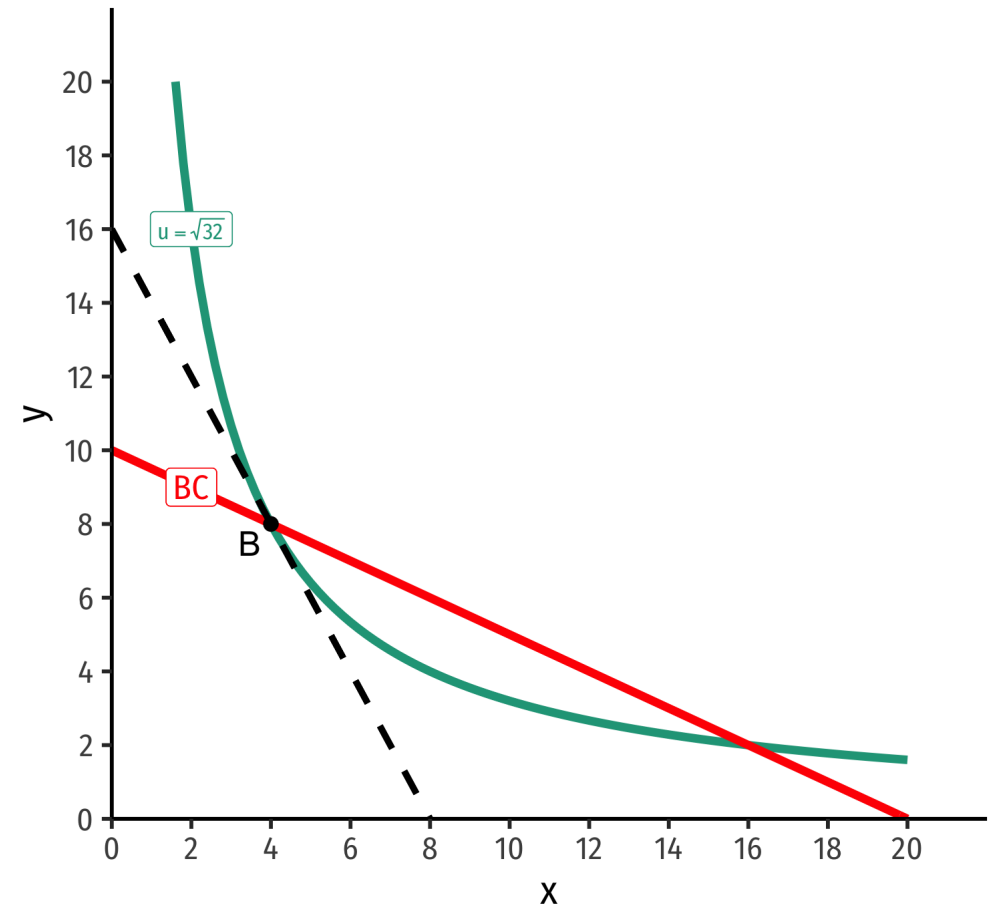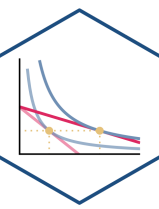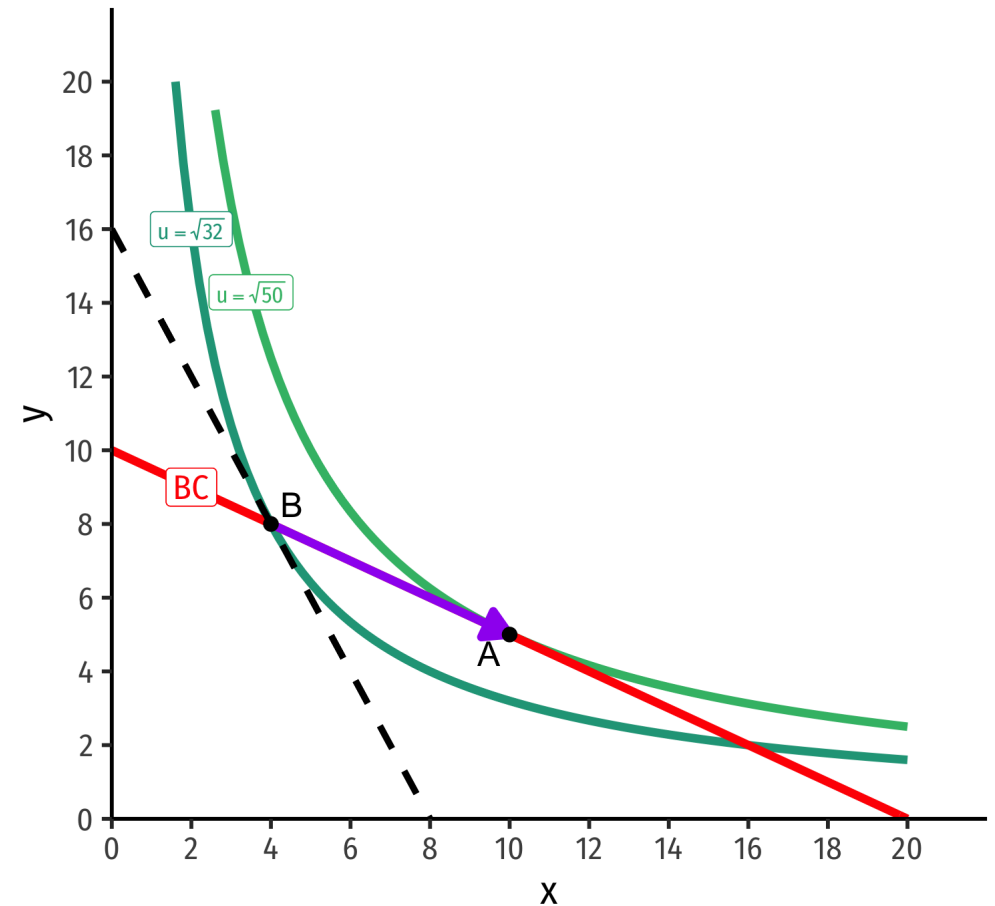$u(x, y) = \sqrt{xy}$, m = \$20, p_x = \$1, p_y = \$2

The Individual's Optimum: Why Not C?

indiff. curve slope $<$ budget constr. slope

$$\frac{MU_x}{MU_y} < \frac{p_x}{p_y}$$

$$0.125 < 0.5$$

- **Consumer** views MB of x is 0.125 units of y

  - Consumer's "exchange rate:" **0.125Y:1X**

- **Market**-determined MC of x is 0.5 units of y

  - Market exchange rate is **0.5Y:1X**

$u = \sqrt{32}$

BC

C

$u(x, y) = \sqrt{xy}$, m = \$20, p_x = \$1, p_y = \$2

The Individual's Optimum: Why Not C?
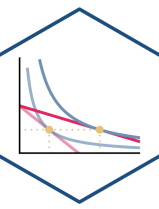
indiff. curve slope $<$ budget constr. slope

$$\frac{MU_x}{MU_y} < \frac{p_x}{p_y}$$

$$0.125 < 0.5$$

- **Consumer** views MB of x is 0.125 units of y

  - Consumer's "exchange rate:" **0.125Y:1X**

- **Market**-determined MC of x is 0.5 units of y

  - Market exchange rate is **0.5Y:1X**

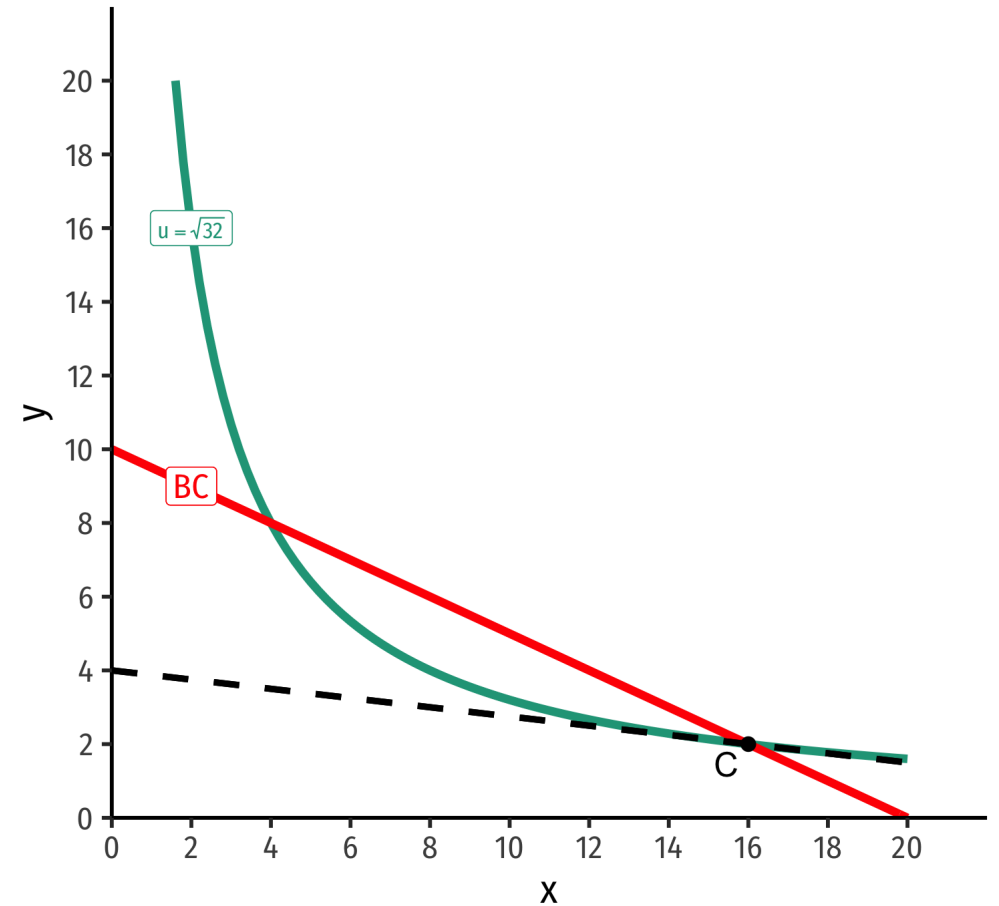- Can **spend less on y, more on x** for **more utility!**
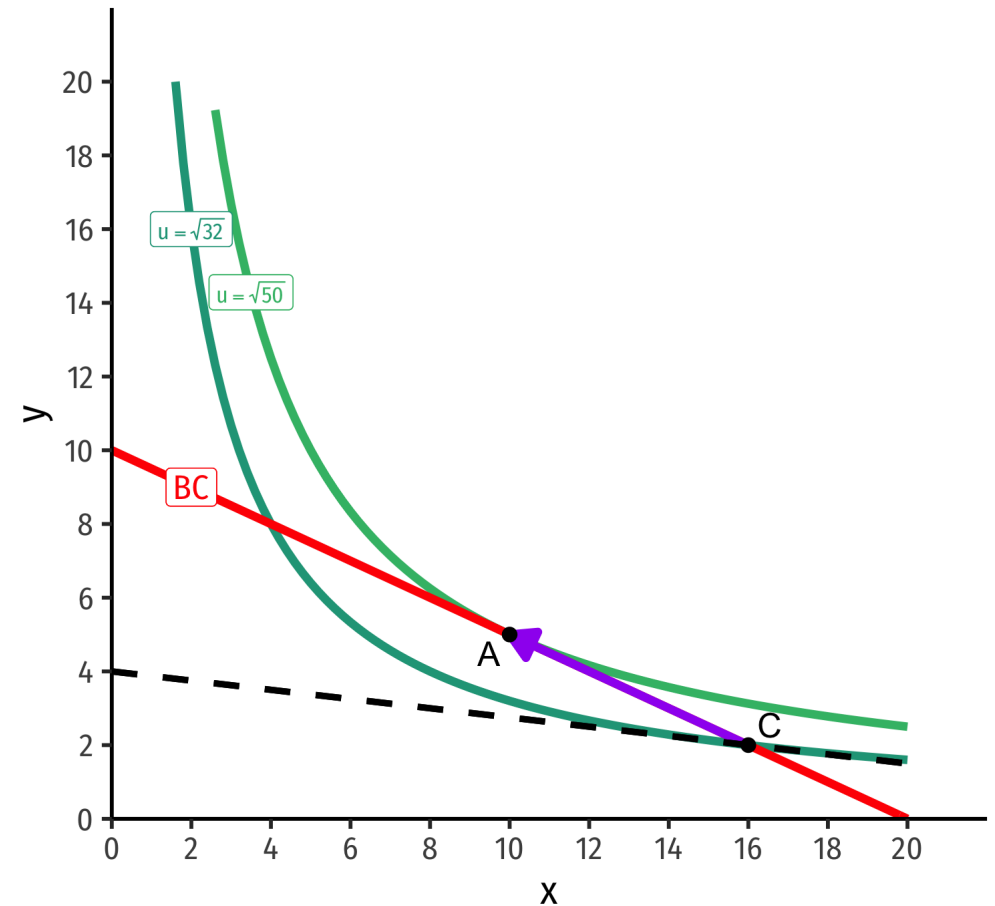
$u(x, y) = \sqrt{xy}$, m = \$20, p_x = \$1, p_y = \$2
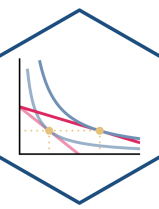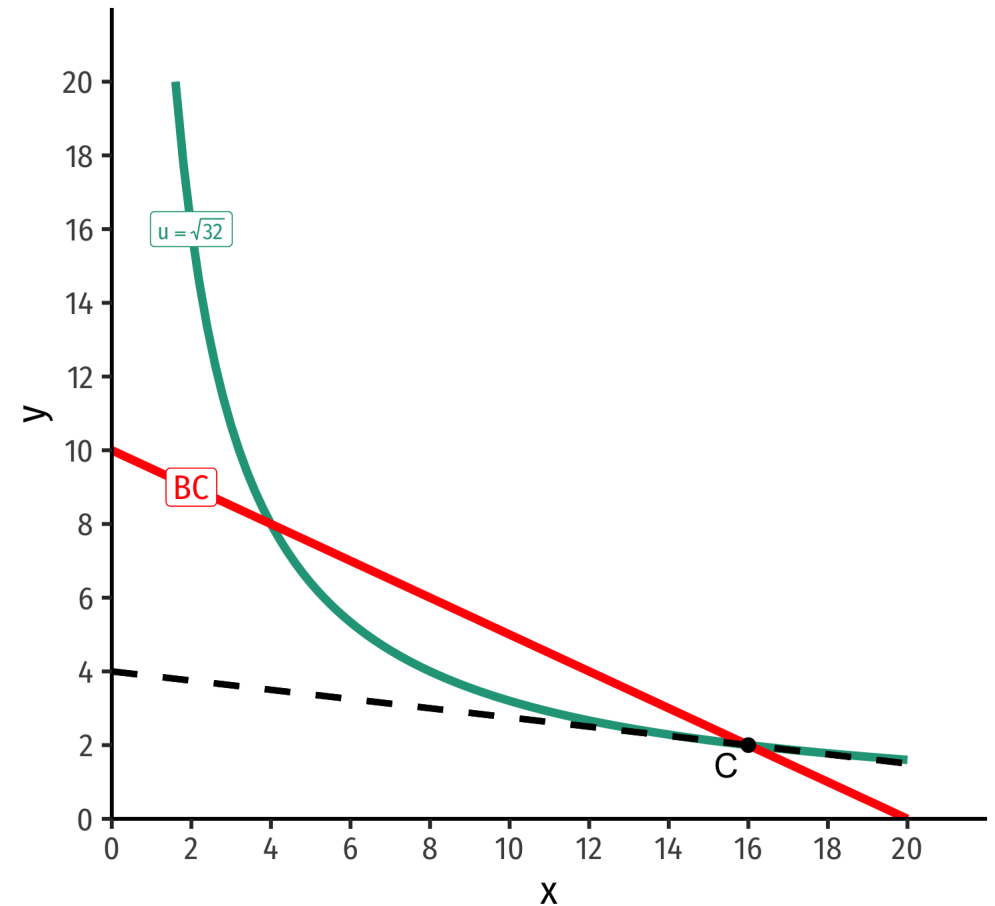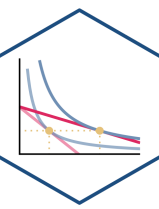
The Individual's Optimum: Why A?

indiff. curve slope = budget constr. slope

$u(x, y) = \sqrt{xy}$, m = \$20, p_x = \$1, p_y = \$2

The Individual's Optimum: Why A?
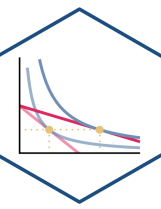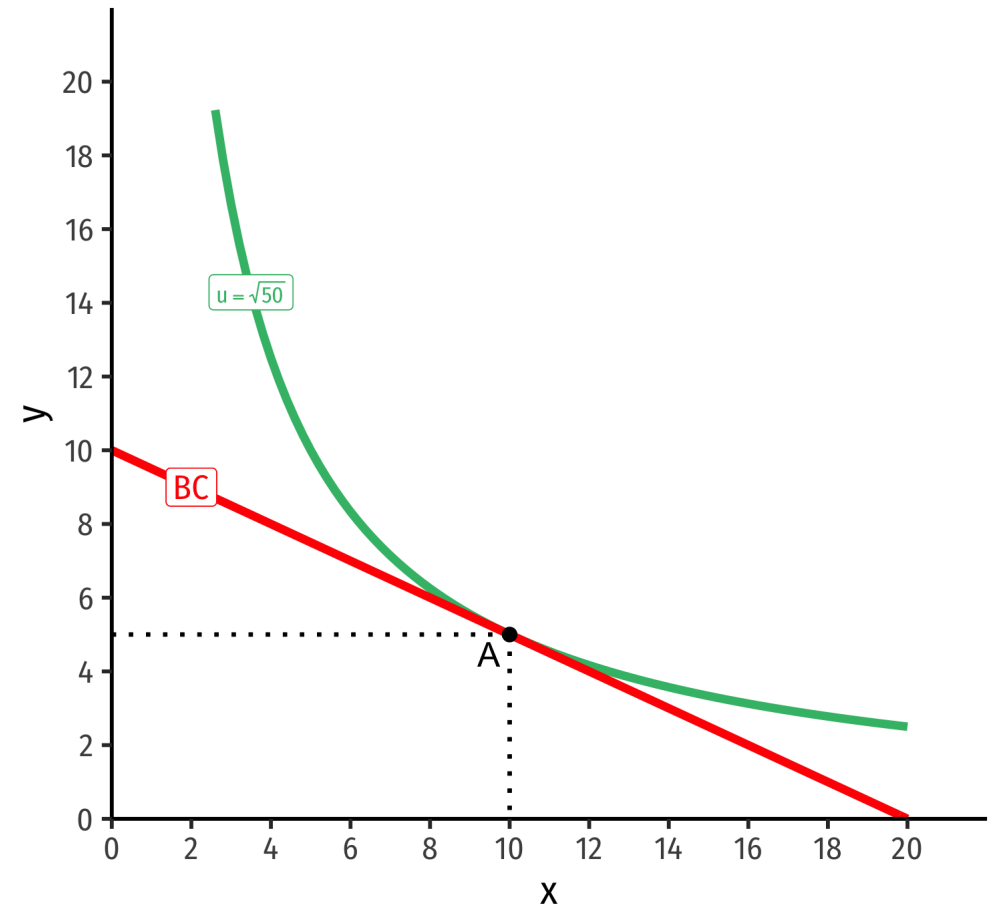
indiff. curve slope $=$ budget constr. slope

$$\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$$

$$0.5 = 0.5$$

- **Marginal benefit** = **Marginal cost**

  - **Consumer** exchanges at same rate as **market**

- *No other combination of (x,y) exists that could increase utility!*[†]

[†] At *current* income and market prices!
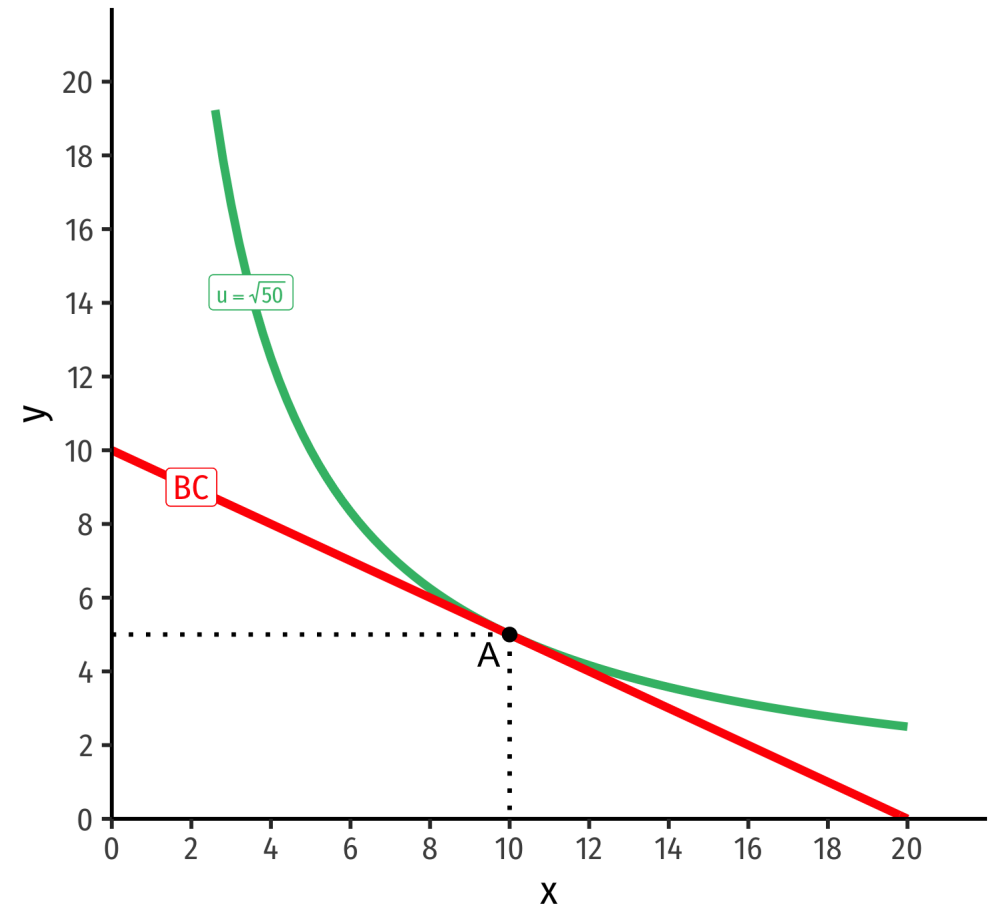
$u(x, y) = \sqrt{xy}$, m = \$20, p_x = \$1, p_y = \$2

The Individual's Optimum: Two Equivalent Rules

Rule 1

$$\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$$

- Easier for calculation (slopes)

$u(x, y) = \sqrt{xy}$, m = $20, p_x = $1, p_y = $2

The Individual's Optimum: Two Equivalent Rules

Rule 1

$$\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$$

- Easier for calculation (slopes)

Rule 2

$$\frac{MU_x}{p_x} = \frac{MU_y}{p_y}$$

- Easier for intuition (next slide)

$u(x, y) = \sqrt{xy}$, m = \$20, p_x = \$1, p_y = \$2
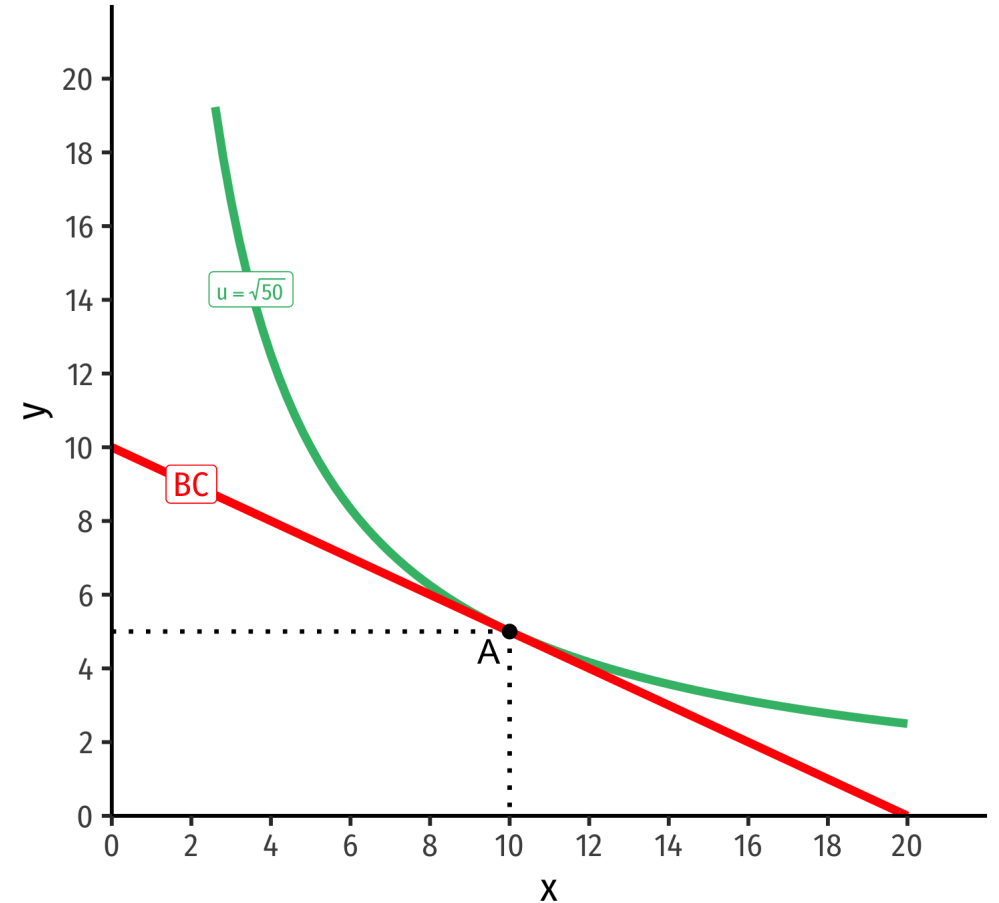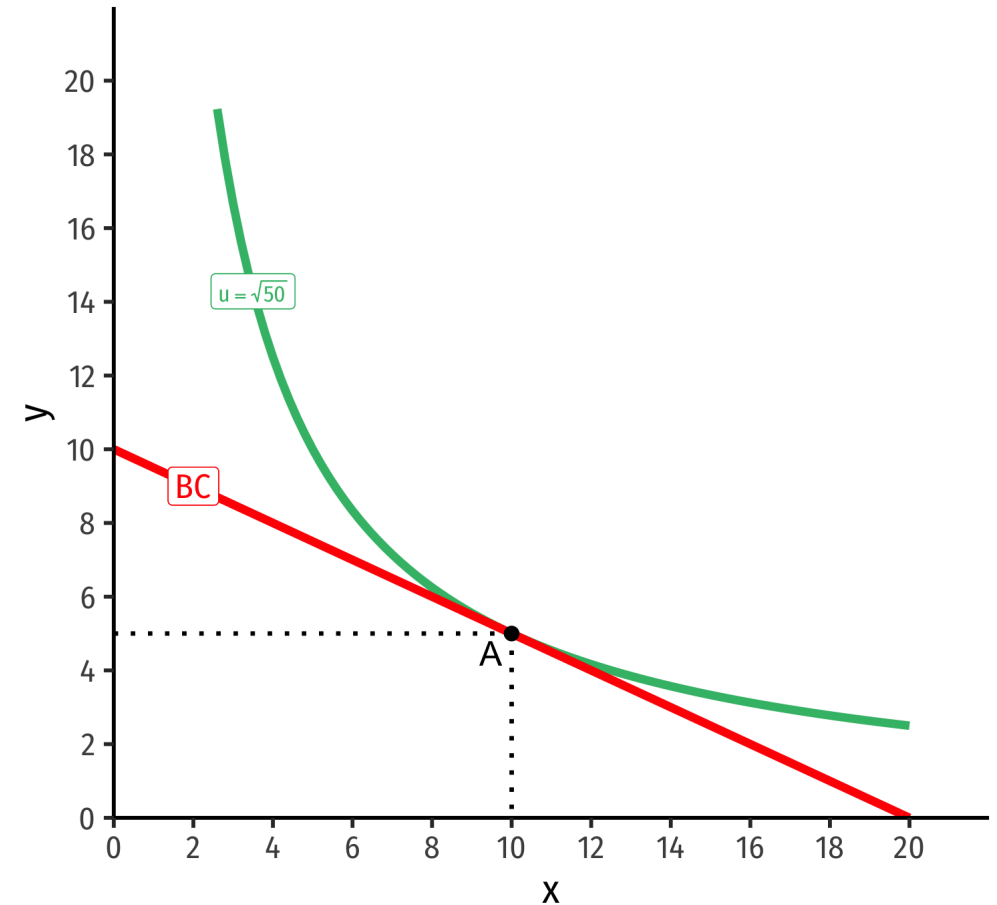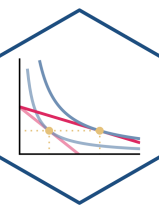
The Individual's Optimum: The Equimarginal Rule

$$\frac{MU_x}{p_x} = \frac{MU_y}{p_y} = \cdots = \frac{MU_n}{p_n}$$

- **Equimarginal Rule**: consumption is optimized where the **marginal utility per dollar spent** is **equalized** across all n possible goods/decisions

- Always choose an option that gives higher marginal utility (e.g. if $MU_x < MU_y$), consume more y!

    - But each option has a different price, so weight each option by its price, hence $\frac{MU_x}{p_x}$

An Optimum, By Definition
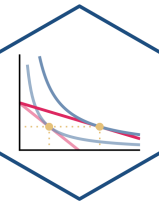
- Any **optimum** in economics: no better alternatives exist under current constraints

- No possible change in your consumption that would increase your utility

Practice I
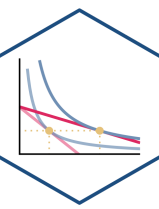
Example: You can get utility from consuming bags of Almonds (a) and bunches of Bananas (b), according to the utility function:
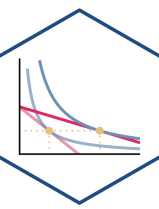
$$u(a, b) = ab$$
$$MU_a = b$$
$$MU_b = a$$

You have an income of $50, the price of Almonds is $10, and the price of Bananas is $2. Put Almonds on the horizontal axis and Bananas on the vertical axis.

1. What is your utility-maximizing bundle of Almonds and Bananas?
2. How much utility does this provide? [Does the answer to this matter?]

Practice II, Cobb-Douglas!

Example: You can get utility from consuming Burgers (b) and Fries (f), according to the utility function:

$$u(b, f) = \sqrt{bf}$$
$$MU_b = 0.5b^{-0.5}f^{0.5}$$
$$MU_f = 0.5b^{0.5}f^{-0.5}$$

You have an income of $20, the price of Burgers is $5, and the price of Fries is $2. Put Burgers on the horizontal axis and Fries on the vertical axis.

1. What is your utility-maximizing bundle of Burgers and Fries?
2. How much utility does this provide?