1.4 — Utility Maximization

ECON 306 • Microeconomic Analysis • Spring 2022

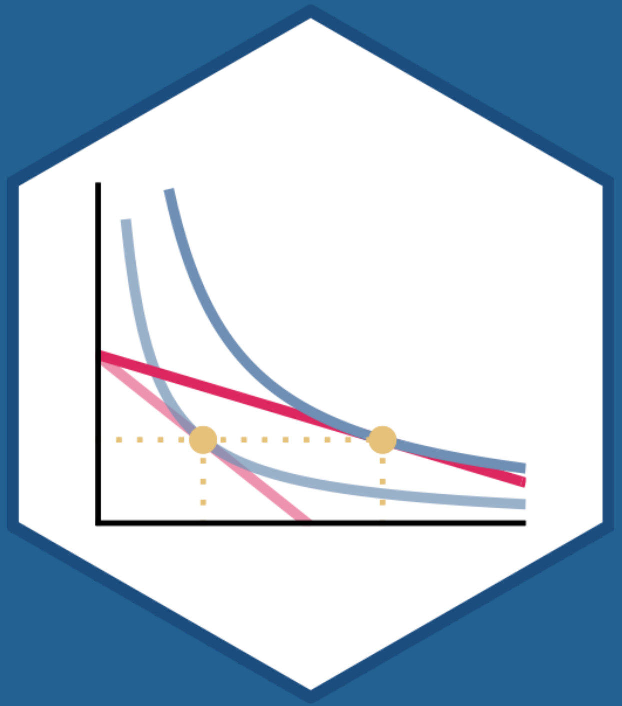
Ryan Safner

Assistant Professor of Economics

safner@hood.edu

[ryansafner/microS22](https://github.com/ryansafner/microS22)

microS22.classes.ryansafner.com



Constrained Optimization

Constrained Optimization I

- We model most situations as a **constrained optimization problem**:

- People **optimize**: make tradeoffs to achieve their **objective** *as best as they can*

- Subject to **constraints**: limited resources (income, time, attention, etc)

Constrained Optimization II

- One of the most generally useful mathematical models

- *Endless applications*: how we model nearly every decision-maker

> consumer, business firm, politician, judge, bureaucrat, voter, dictator, pirate, drug cartel, drug addict, parent, child, etc

- **Key economic skill: recognizing how to apply the model to a situation**

Remember!

THE INVISIBLE HOOK
The Hidden Economics of Pirates
PETER T. LEESON
"WTF?!' IS LIKE 'FREAKONOMICS' ON STEROIDS." —STEVEN D. LEVITT
AN ECONOMIC TOUR OF THE WEIRD
WTF?!
PETER T. LEESON
THE SOCIAL ORDER OF THE UNDERWORLD
How Prison Gangs Govern the American Penal System
DAVID SKARBEK
NARCO-NOMICS
HOW TO RUN A DRUG CARTEL
TOM WAINWRIGHT
THE DICTATOR'S HANDBOOK
WHY BAD BEHAVIOR IS ALMOST ALWAYS GOOD POLITICS
BRUCE BUENO DE MESQUITA AND ALASTAIR SMITH
SELFISH REASONS TO HAVE MORE KIDS
Why Being a Great Parent Is LESS WORK AND MORE FUN Than You Think
Bryan Caplan
new york times bestseller
cribsheet
a data-driven guide to better, more relaxed parenting, from birth to preschool
EMILY OSTER
author of expecting better
THE SEX FACTOR
How Women Made the West Rich
VICTORIA BATEMAN
ROCKONOMICS
A BACKSTAGE TOUR OF WHAT THE MUSIC INDUSTRY CAN TEACH US ABOUT ECONOMICS AND LIFE
ALAN B. KRUEGER
ANJA SHORTLAND
KIDNAP
INSIDE THE RANSOM BUSINESS

Constrained Optimization III

- All constrained optimization models have three moving parts:

Constrained Optimization III

- All constrained optimization models have three moving parts:

1. **Choose:** **< some alternative >**

Constrained Optimization III

- All constrained optimization models have three moving parts:

1. **Choose:** < some alternative >

2. **In order to maximize:** < some objective >

Constrained Optimization III

- All constrained optimization models have three moving parts:

1. **Choose:** < some alternative >

2. **In order to maximize:** < some objective >

3. **Subject to:** < some constraints >

Constrained Optimization: Example I

Example: A Hood student picking courses hoping to achieve the highest GPA while getting an Econ major.

1. **Choose:**

2. **In order to maximize:**

3. **Subject to:**

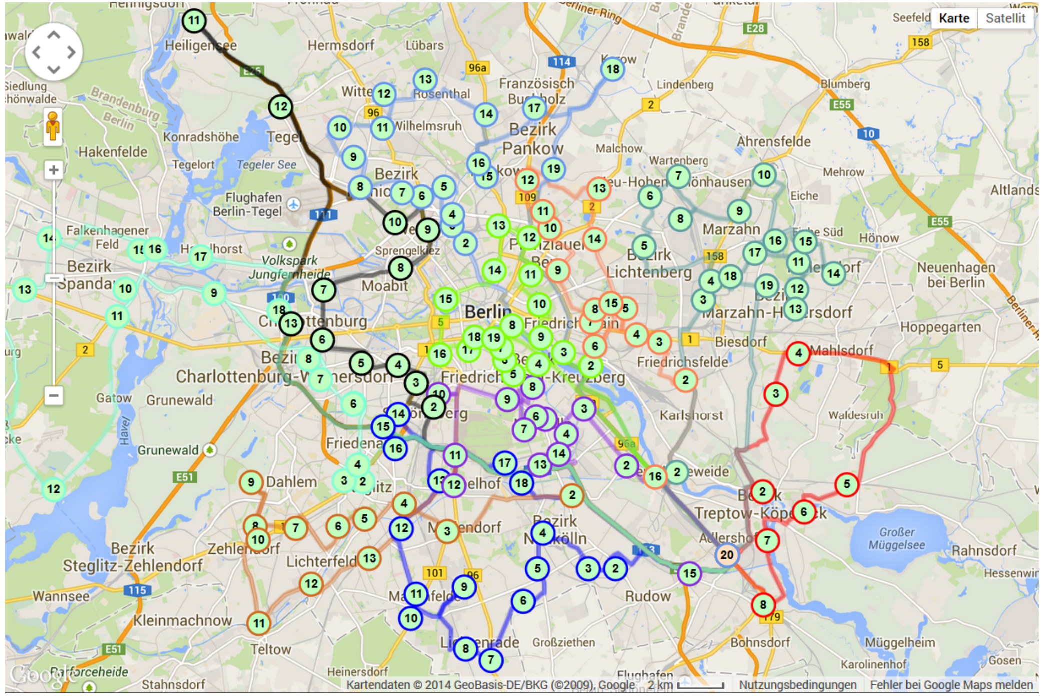
Constrained Optimization: Example II

Example: How should FedEx plan its delivery route?

1. **Choose:**

2. **In order to maximize:**

3. **Subject to:**



Constrained Optimization: Example III

Example: The U.S. government wants to remain economically competitive but reduce emissions by 25%.

1. **Choose:**

2. **In order to maximize:**

3. **Subject to:**

Constrained Optimization: Example IV

Example: How do elected officials make decisions in politics?

1. **Choose:**

2. **In order to maximize:**

3. **Subject to:**

The Utility Maximization Problem

- The individual's **utility maximization problem** we've been modeling, finally, is:

1. **Choose:** < a consumption bundle >

2. **In order to maximize:** < utility >

3. **Subject to:** < income and market prices >

The Utility Maximization Problem: Tools

- We now have the tools to understand individual choices:

- **Budget constraint**: individual's **constraints** of income and market prices
    - How market trades off between goods
    - Marginal cost (of good x, in terms of y)

- **Utility function**: individual's **objective** to maximize, based on their preferences
    - How individual trades off between goods
    - Marginal benefit (of good x, in terms of y)

The Utility Maximization Problem: Verbally

- The **individual's constrained optimization problem**:

> choose a bundle of goods to maximize utility, subject to income and market prices

The Utility Maximization Problem: Mathematically

$$\max_{x,y \geq 0} u(x,y)$$

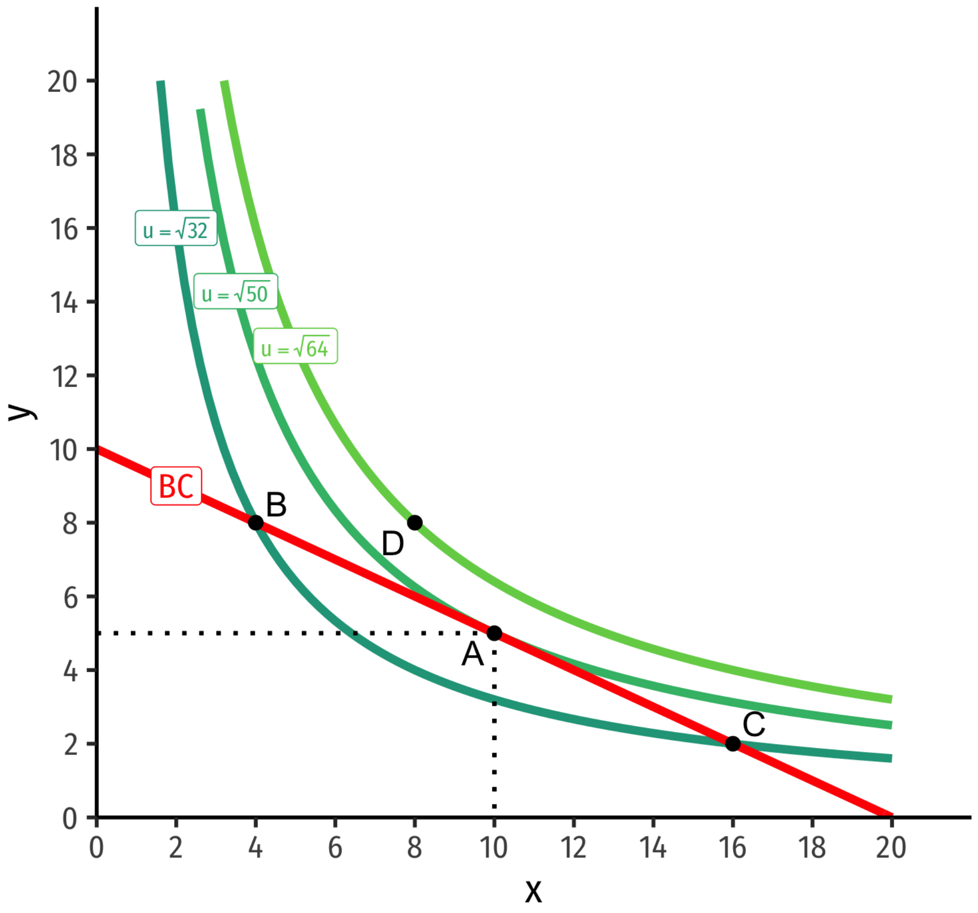
$$s.t. \, p_x x + p_y y = m$$

- This requires calculus to solve.[†] We will look at **graphs** instead!

[†] See the mathematical appendix in today's class notes on how to solve it with calculus, and an example.

The Individual's Optimum: Graphically

- **Graphical solution: Highest indifference curve *tangent* to budget constraint**
  - Bundle A!

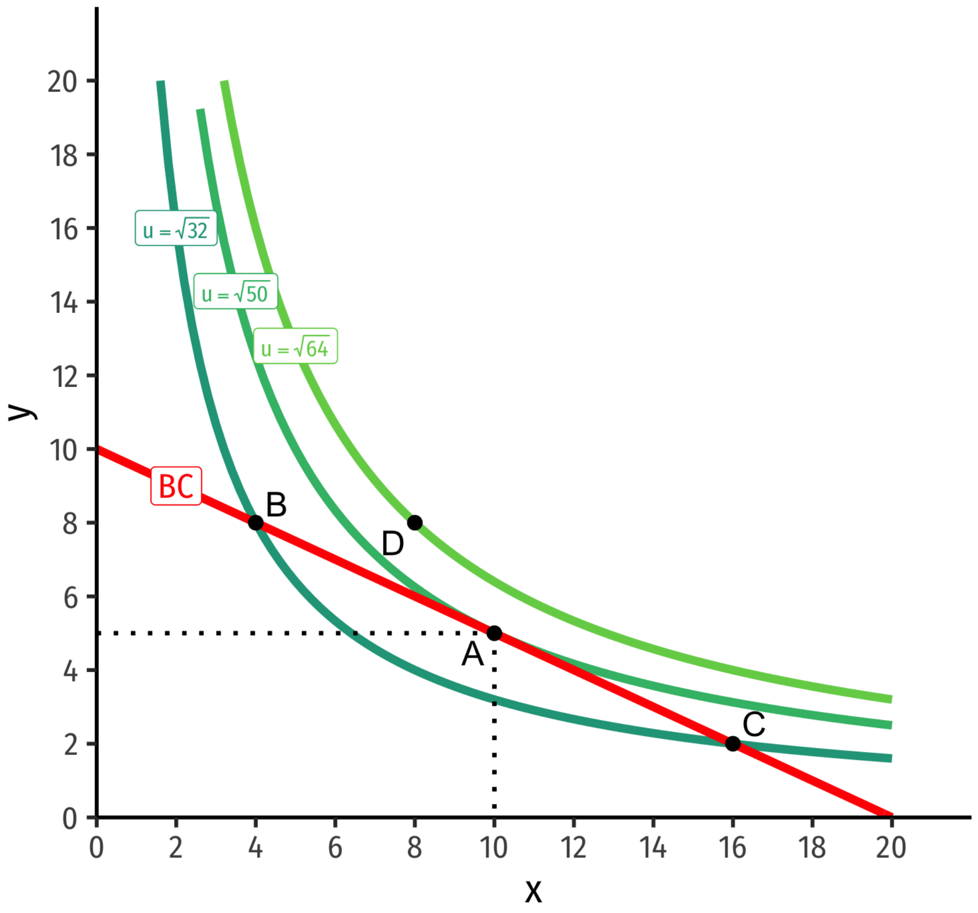


$u(x, y) = \sqrt{xy}$, $m = \$20$, $p_x = \$1$, $p_y = \$2$

The Individual's Optimum: Graphically

- Graphical solution: **Highest indifference curve *tangent* to budget constraint**
  - Bundle A!

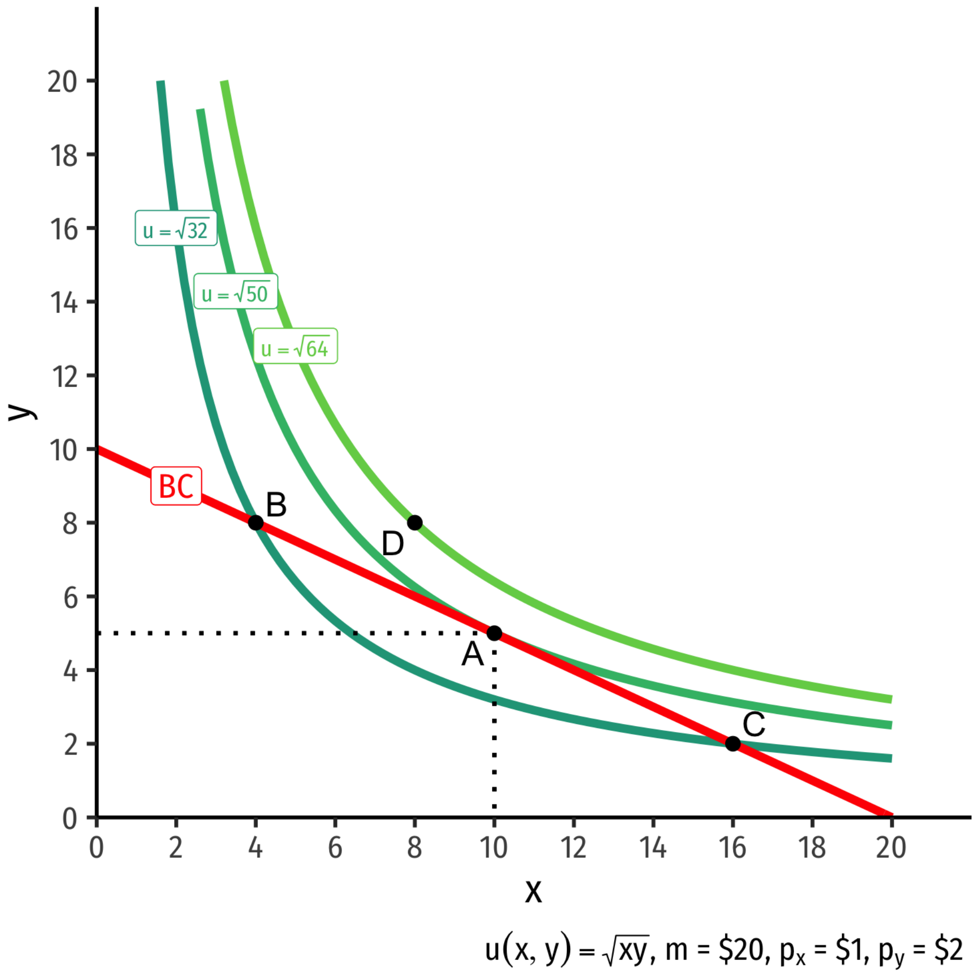
- B or C spend all income, but a better combination exists



$u(x, y) = \sqrt{xy}$, $m = \$20$, $p_x = \$1$, $p_y = \$2$

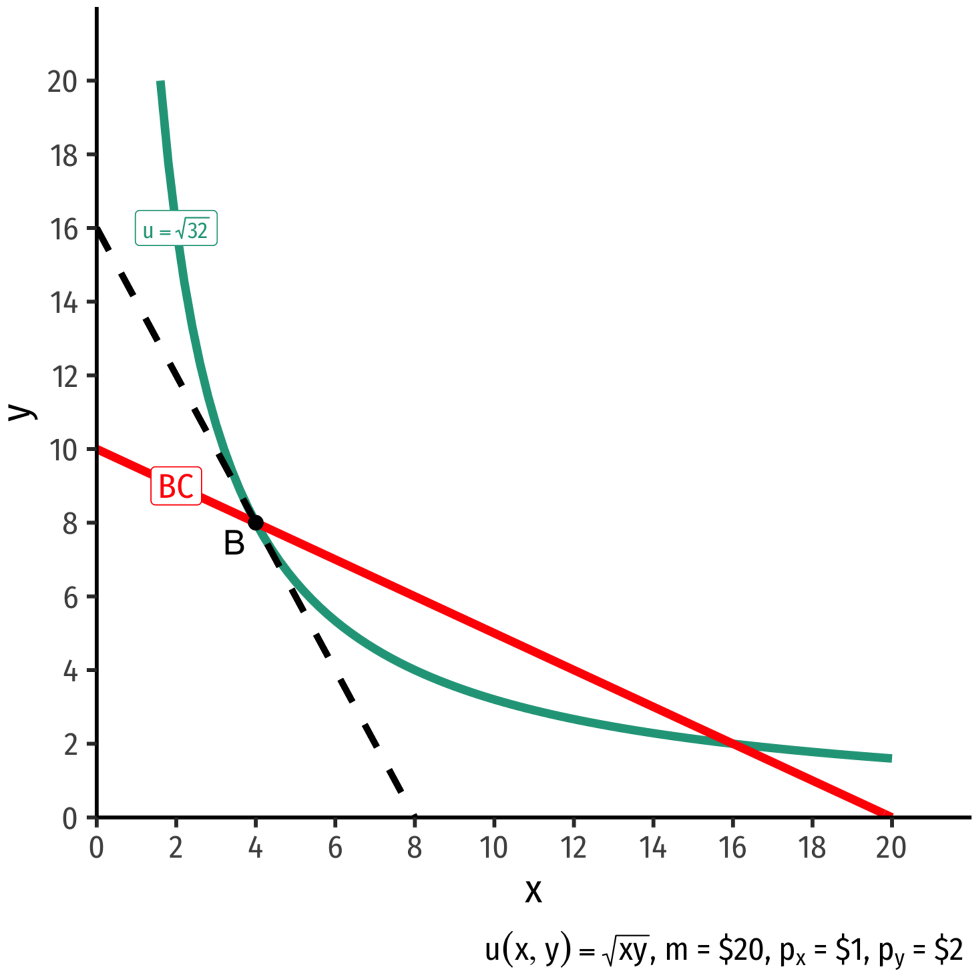
The Individual's Optimum: Graphically

- **Graphical solution**: **Highest indifference curve *tangent* to budget constraint**
    - Bundle A!
- B or C spend all income, but a better combination exists
- D is higher utility, but *not affordable* at current income & prices



The Individual's Optimum: Why Not B?

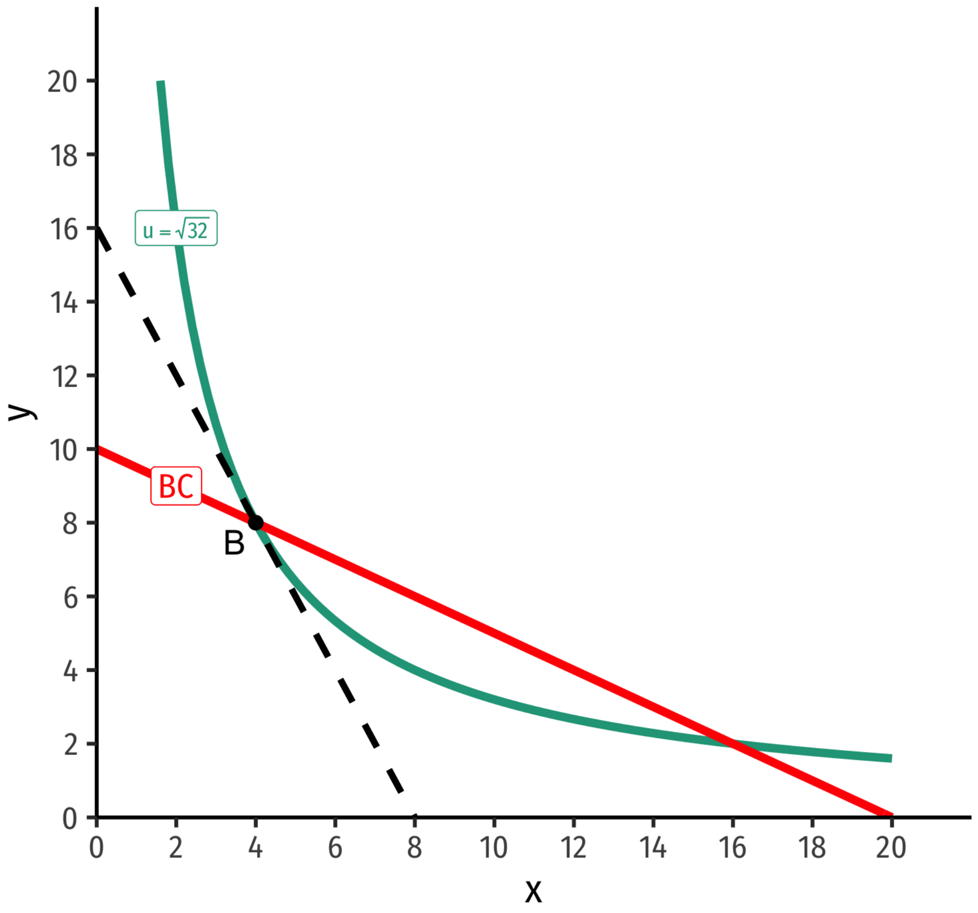
$\color{green}{\text{indiff. curve slope}} > \color{red}{\text{budget constr. slope}}$



The Individual's Optimum: Why Not B?

$$\text{indiff. curve slope} > \text{budget constr. slope}$$
$$\frac{MU_x}{MU_y} > \frac{p_x}{p_y}$$
$$2 > 0.5$$

- **Consumer** views MB of x is 2 units of y
    - Consumer's "exchange rate:" **2Y:1X**
- **Market**-determined MC of x is 0.5 units of y
    - Market exchange rate is **0.5Y:1X**

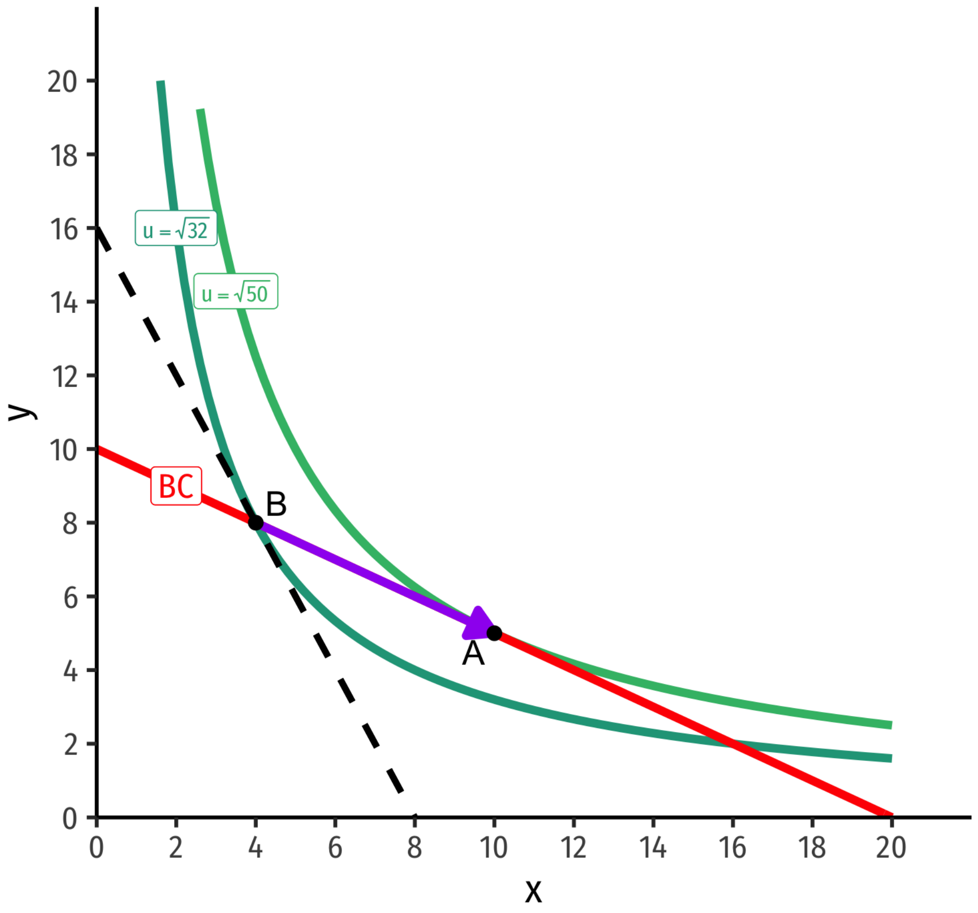


$u(x, y) = \sqrt{xy}$, m = \$20, $p_x = \$1$, $p_y = \$2$

The Individual's Optimum: Why Not B?

$$\text{indiff. curve slope} > \text{budget constr. slope}$$
$$\frac{MU_x}{MU_y} > \frac{p_x}{p_y}$$
$$2 > 0.5$$

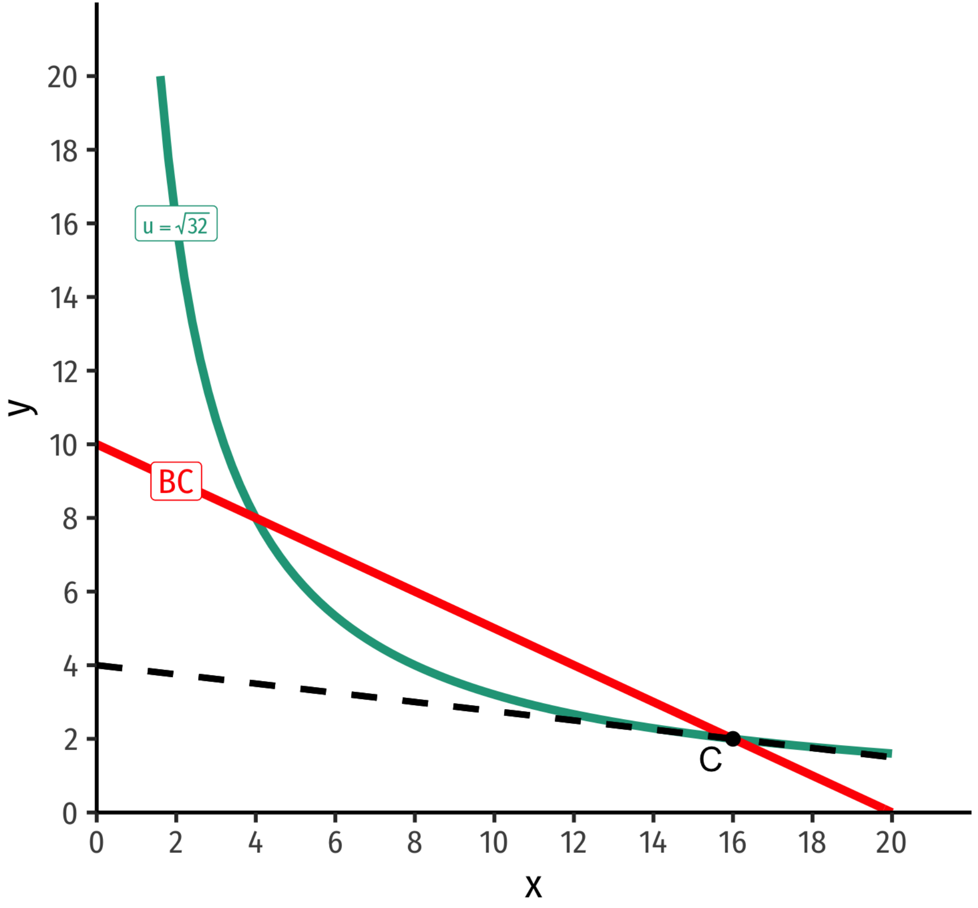
- **Consumer** views MB of x is 2 units of y
  - Consumer's "exchange rate:" **2Y:1X**
- **Market**-determined MC of x is 0.5 units of y
  - Market exchange rate is **0.5Y:1X**
- Can **spend less on y, more on x** for **more utility!**



$u(x, y) = \sqrt{xy}$, $m = \$20$, $p_x = \$1$, $p_y = \$2$

The Individual's Optimum: Why Not C?

$\text{indiff. curve slope} < \text{budget constr. slope}$

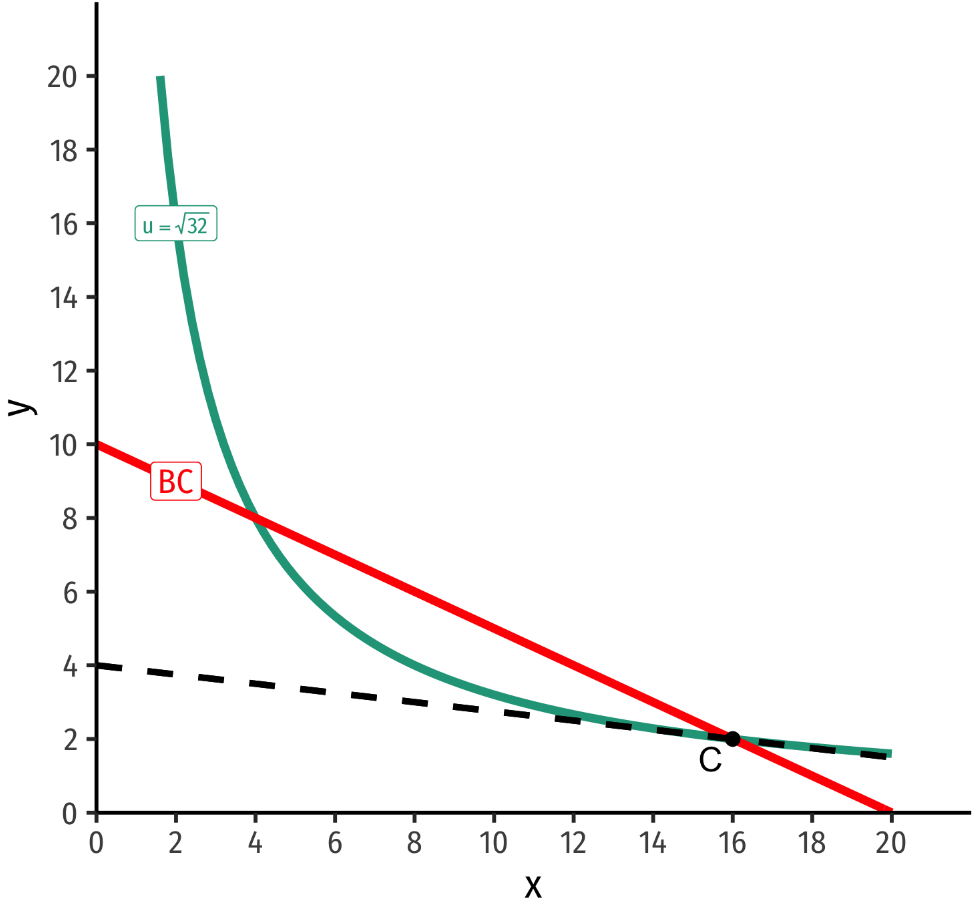


$u(x, y) = \sqrt{xy}$, $m = \$20$, $p_x = \$1$, $p_y = \$2$

The Individual's Optimum: Why Not C?

$$\text{indiff. curve slope} < \text{budget constr. slope}$$
$$\frac{MU_x}{MU_y} < \frac{p_x}{p_y}$$
$$0.125 < 0.5$$

- **Consumer** views MB of x is 0.125 units of y
  - Consumer's "exchange rate:" **0.125Y:1X**
- **Market**-determined MC of x is 0.5 units of y
  - Market exchange rate is **0.5Y:1X**



$u(x, y) = \sqrt{xy}$, m = \$20, p_x = \$1, p_y = \$2

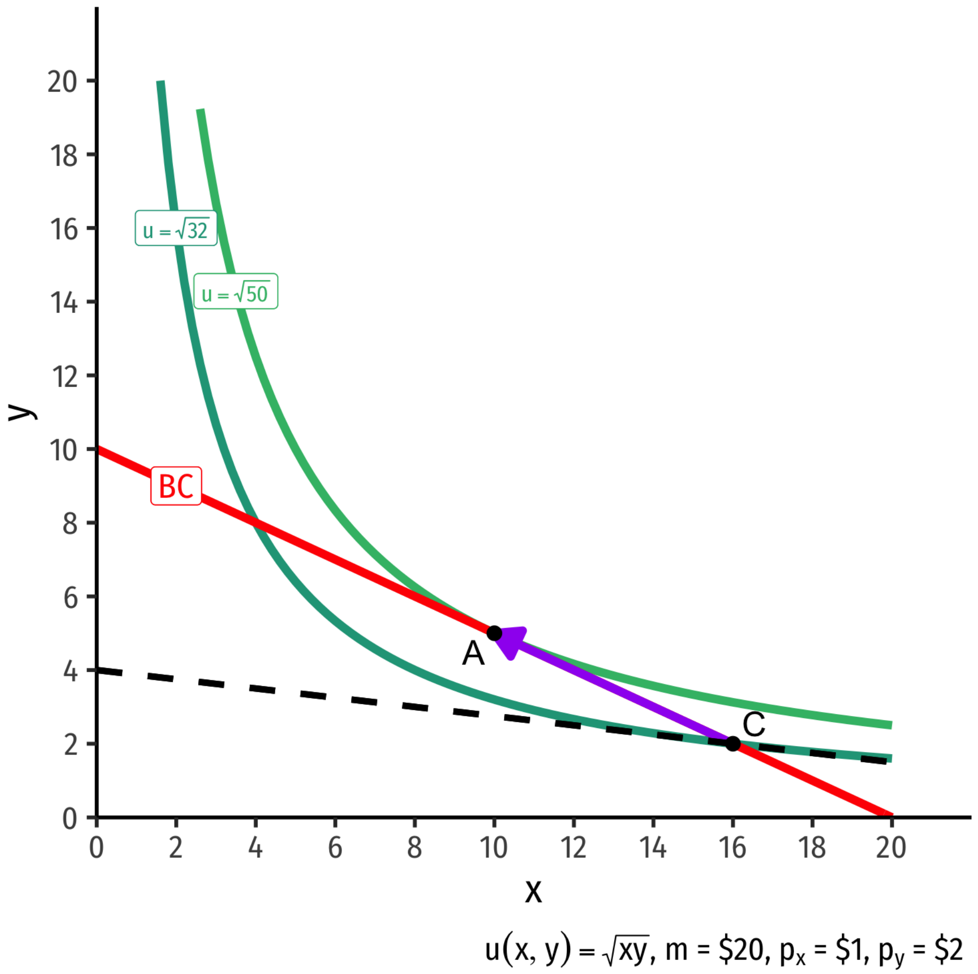
The Individual's Optimum: Why Not C?

$$\text{indiff. curve slope} < \text{budget constr. slope}$$

$$\frac{MU_x}{MU_y} < \frac{p_x}{p_y}$$

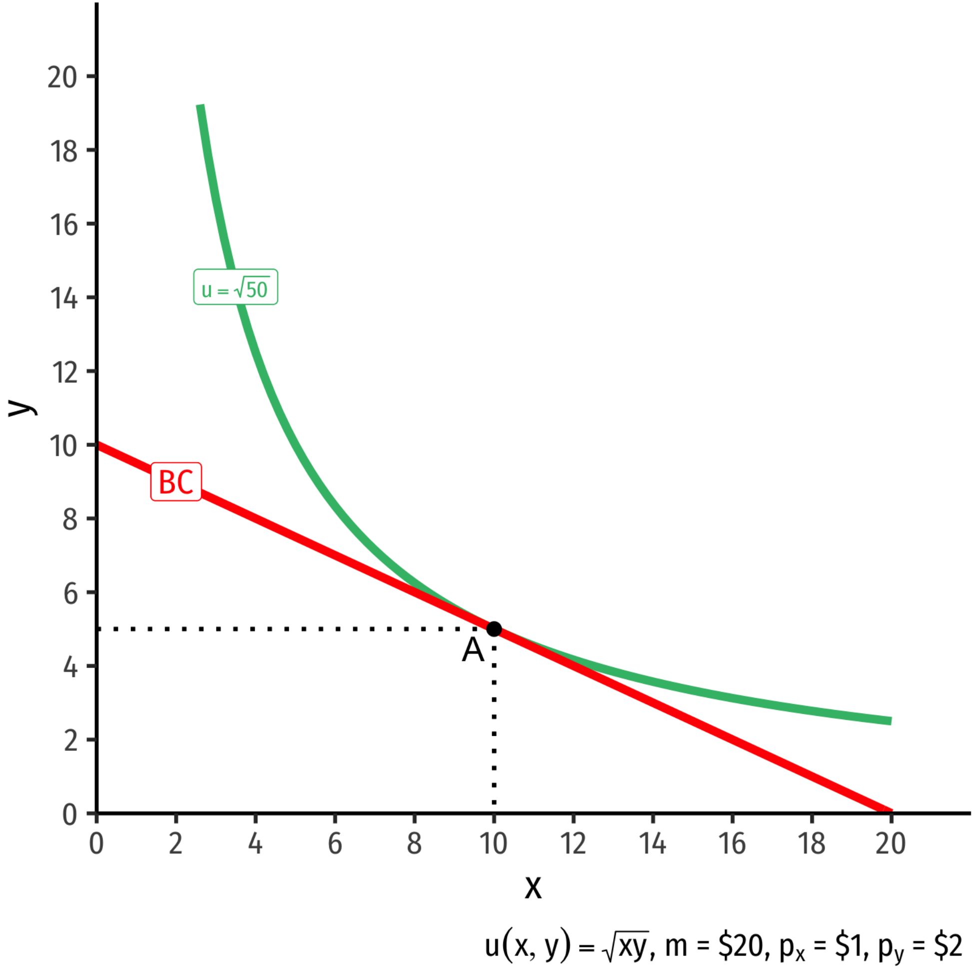
$$0.125 < 0.5$$

- **Consumer** views MB of x is 0.125 units of y
  - Consumer's "exchange rate:" **0.125Y:1X**
- **Market**-determined MC of x is 0.5 units of y
  - Market exchange rate is **0.5Y:1X**
- Can **spend less on y, more on x** for **more utility!**



The Individual's Optimum: Why A?

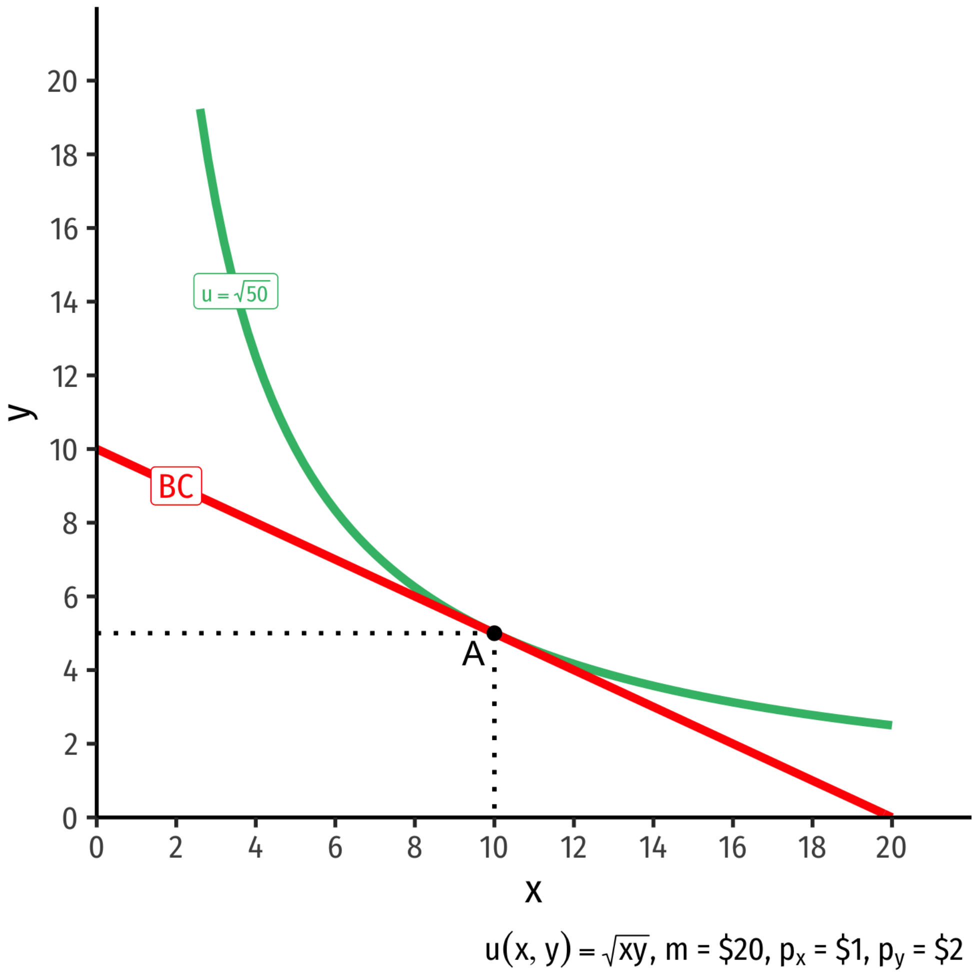
$$\text{indiff. curve slope} = \text{budget constr. slope}$$



The Individual's Optimum: Why A?

$$\text{indiff. curve slope} = \text{budget constr. slope}$$
$$\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$$
$$0.5 = 0.5$$

- **Marginal benefit** = **Marginal cost**
  - **Consumer** exchanges at same rate as **market**
- *No other combination* of (x,y) *exists* that could increase utility![†]



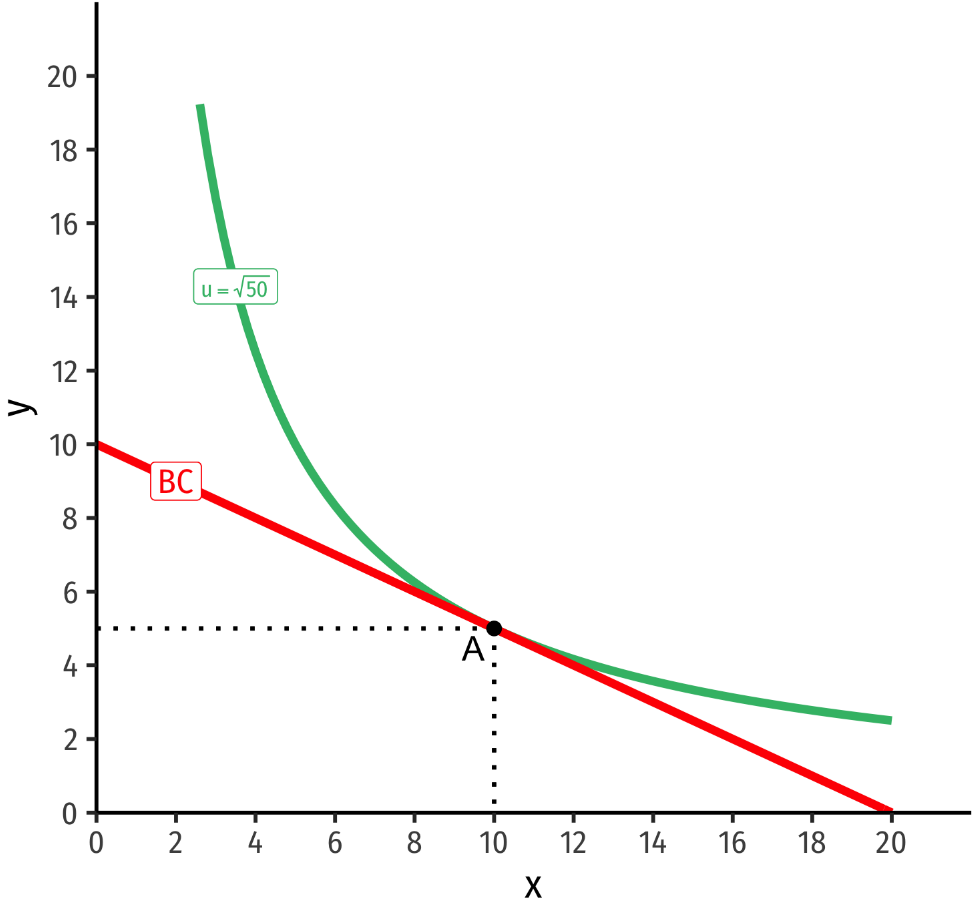
[†] At *current* income and market prices!

The Individual's Optimum: Two Equivalent Rules

Rule 1

$$\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$$

- Easier for calculation (slopes)



$u(x, y) = \sqrt{xy}$, m = \$20, p_x = \$1, p_y = \$2

The Individual's Optimum: Two Equivalent Rules

Rule 1

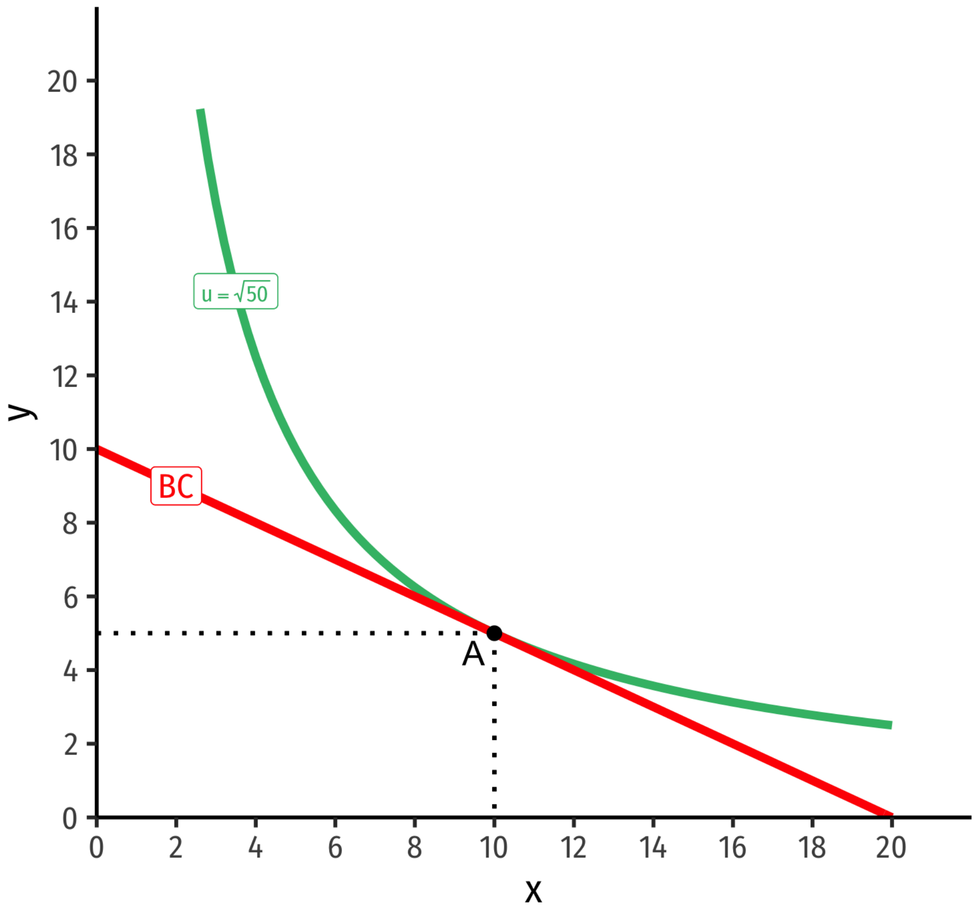
$$\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$$

- Easier for calculation (slopes)

Rule 2

$$\frac{MU_x}{p_x} = \frac{MU_y}{p_y}$$

- Easier for intuition (next slide)



$u(x, y) = \sqrt{xy}$, $m = \$20$, $p_x = \$1$, $p_y = \$2$

The Individual's Optimum: The Equimarginal Rule

$$\frac{MU_x}{p_x} = \frac{MU_y}{p_y} = \cdots = \frac{MU_n}{p_n}$$

- **Equimarginal Rule**: consumption is optimized where the **marginal utility per dollar spent** is **equalized** across all n possible goods/decisions

- Always choose an option that gives higher marginal utility (e.g. if $MU_x < MU_y$), consume more y!

  - But each option has a different price, so weight each option by its price, hence $\frac{MU_x}{p_x}$

An Optimum, By Definition

- Any **optimum** in economics: no better alternatives exist under current constraints

- No possible change in your consumption that would increase your utility

Practice I

Example: You can get utility from consuming bags of Almonds (a) and bunches of Bananas (b), according to the utility function:

$$u(a,b) = ab$$
$$MU_a = b$$
$$MU_b = a$$

You have an income of $50, the price of Almonds is $10, and the price of Bananas is $2. Put Almonds on the horizontal axis and Bananas on the vertical axis.

1. What is your utility-maximizing bundle of Almonds and Bananas?
2. How much utility does this provide? [Does the answer to this matter?]

Practice II, Cobb-Douglas!

Example: You can get utility from consuming Burgers (b) and Fries (f), according to the utility function:

$$\begin{aligned} u(b,f) &= \sqrt{bf} \\ MU_b &= 0.5b^{-0.5}f^{0.5} \\ MU_f &= 0.5b^{0.5}f^{-0.5} \end{aligned}$$

You have an income of $20, the price of Burgers is $5, and the price of Fries is $2. Put Burgers on the horizontal axis and Fries on the vertical axis.

1. What is your utility-maximizing bundle of Burgers and Fries?
2. How much utility does this provide?