

2.5 – Short Run Profit Maximization

ECON 306 • Microeconomic Analysis • Spring 2022

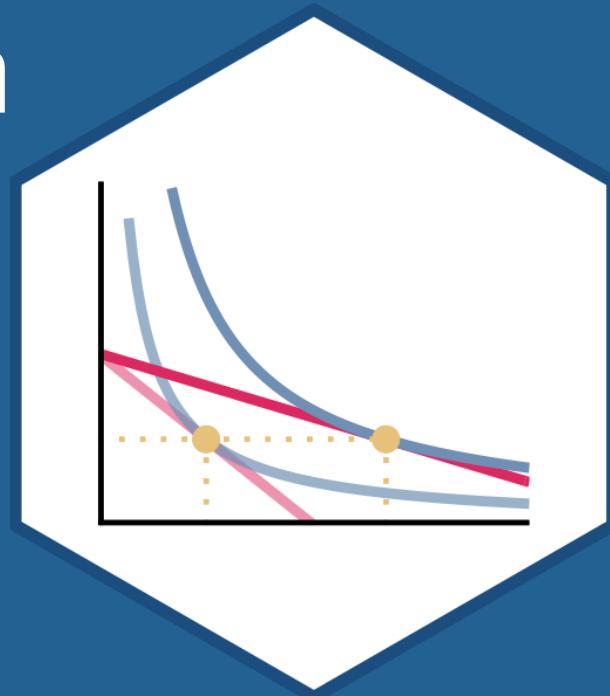
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 safner@hood.edu

 [ryansafner/microS22](https://github.com/ryansafner/microS22)

 microS22.classes.ryansafner.com



Outline



Revenues

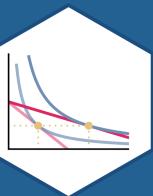
Profits

Comparative Statics

Calculating Profit

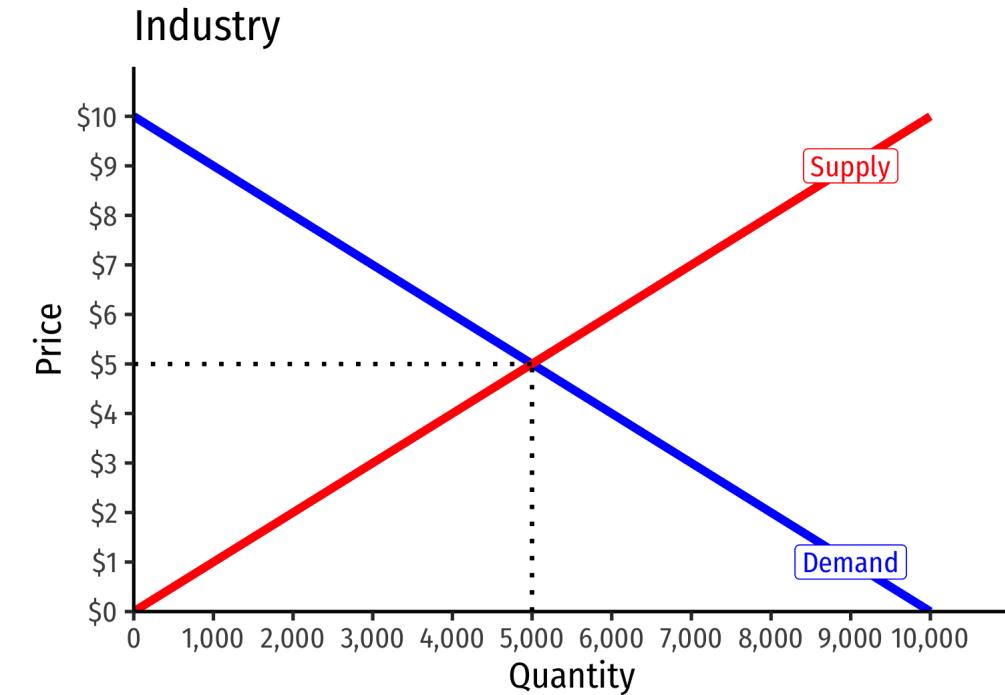
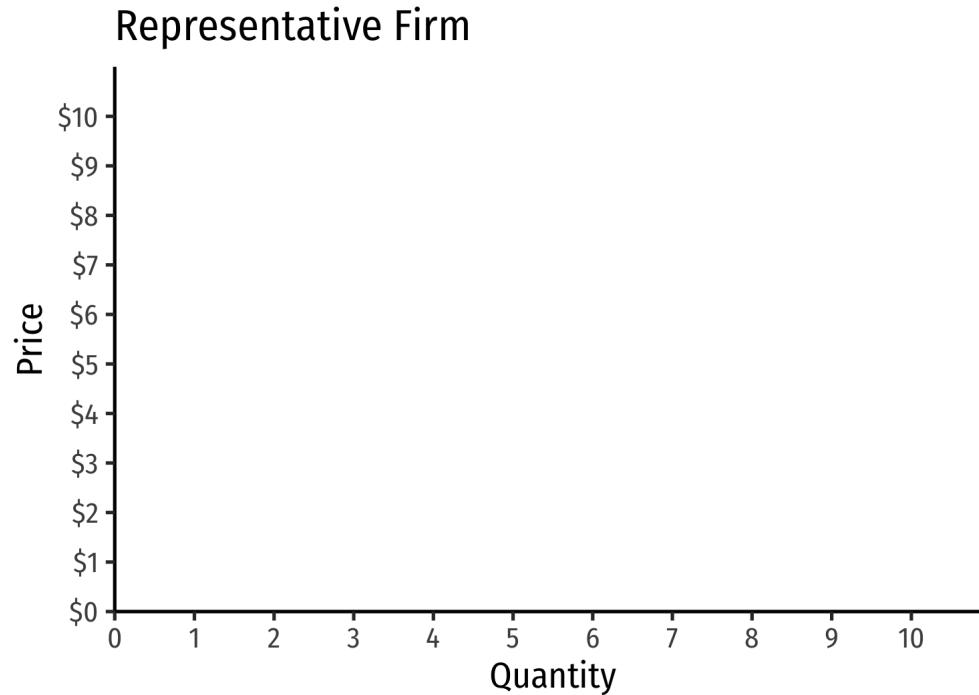
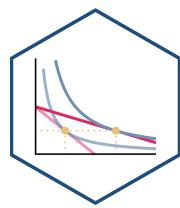
Short-Run Shut-Down Decisions

The Firm's Short-Run Supply Decision

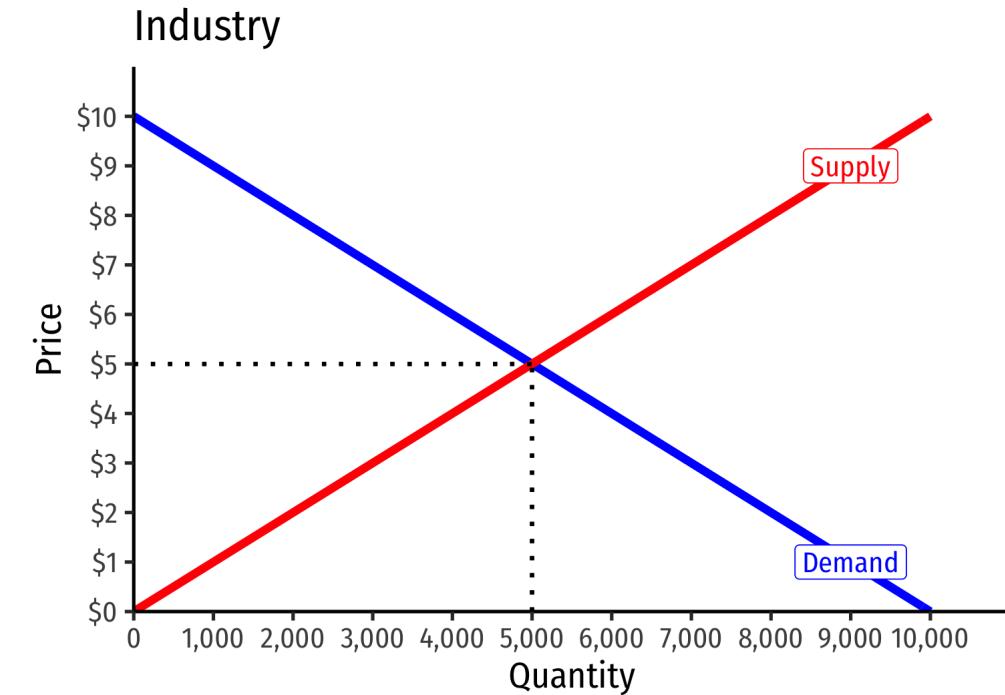
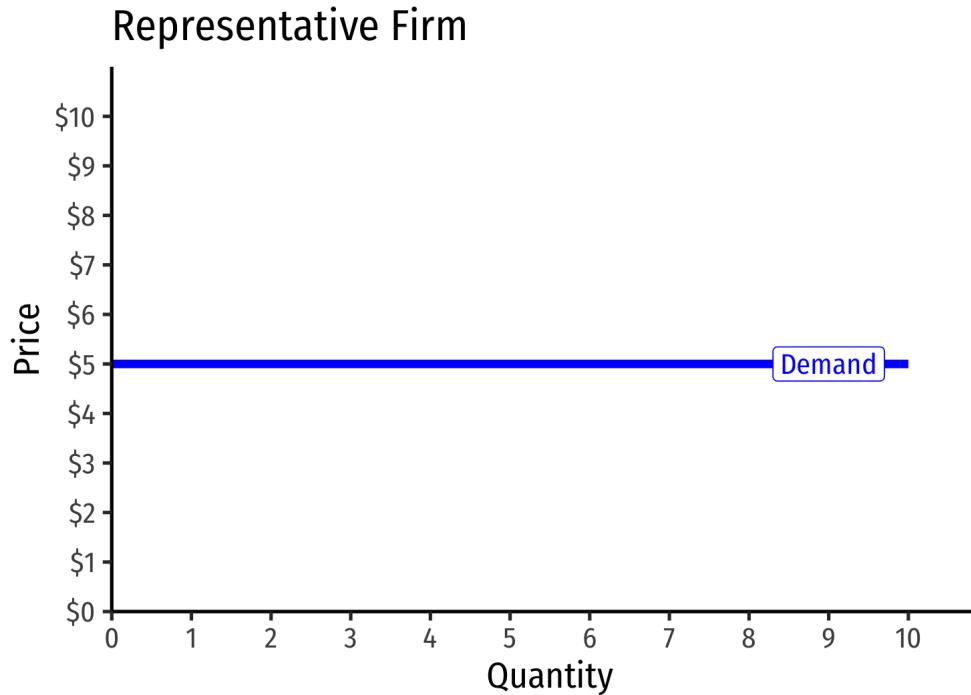
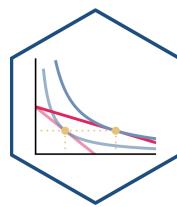


Revenues

Revenues for Firms in *Competitive* Industries I

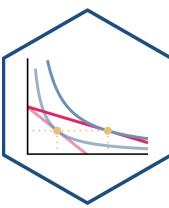


Revenues for Firms in *Competitive* Industries I

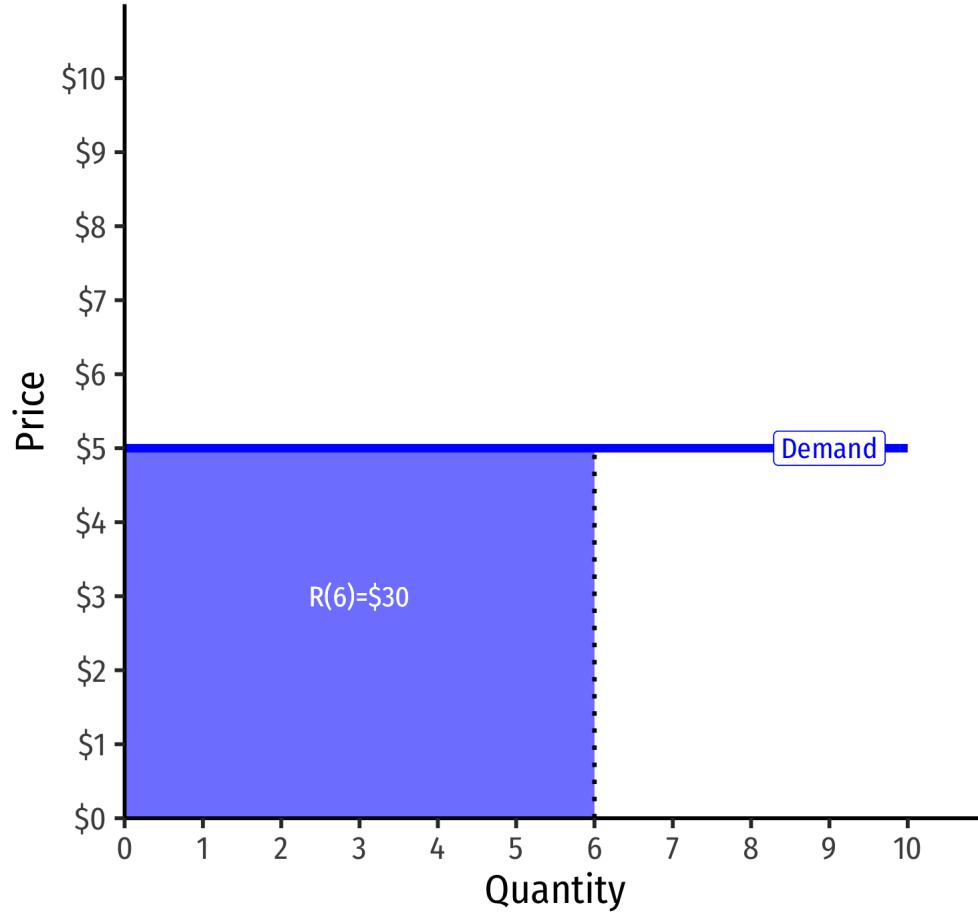


- Demand for a firm's product is **perfectly elastic** at the market price
- Where did the **supply curve** come from? You'll know today

Revenues for Firms in *Competitive* Industries II

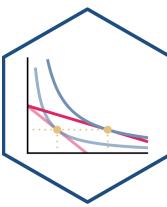


Representative Firm



- **Total Revenue** $R(q) = pq$

Average and Marginal Revenues



- **Average Revenue:** revenue per unit of output

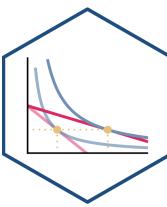
$$AR(q) = \frac{R}{q}$$

- $AR(q)$ is **by definition** equal to the price! (Why?)
- **Marginal Revenue:** change in revenues for each additional unit of output sold:

$$MR(q) = \frac{\Delta R(q)}{\Delta q} \approx \frac{R_2 - R_1}{q_2 - q_1}$$

- Calculus: first derivative of the revenues function
- **For a competitive firm (only), $MR(q) = p$, i.e. the price!**

Average and Marginal Revenues: Example



Example: A firm sells bushels of wheat in a very competitive market. The current market price is \$10/bushel.

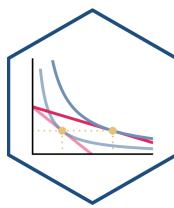
For the 1st bushel sold:

- What is the total revenue?
- What is the average revenue?

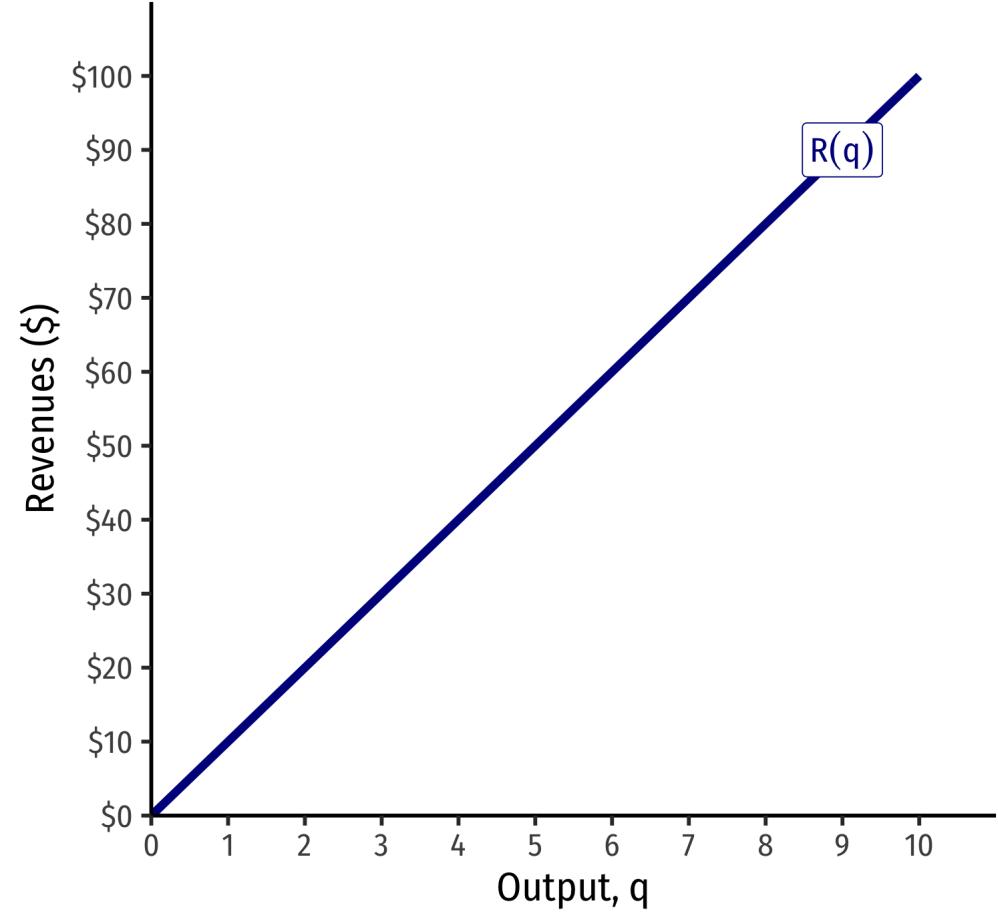
For the 2nd bushel sold:

- What is the total revenue?
- What is the average revenue?
- What is the marginal revenue?

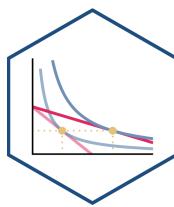
Total Revenue, Example: Visualized



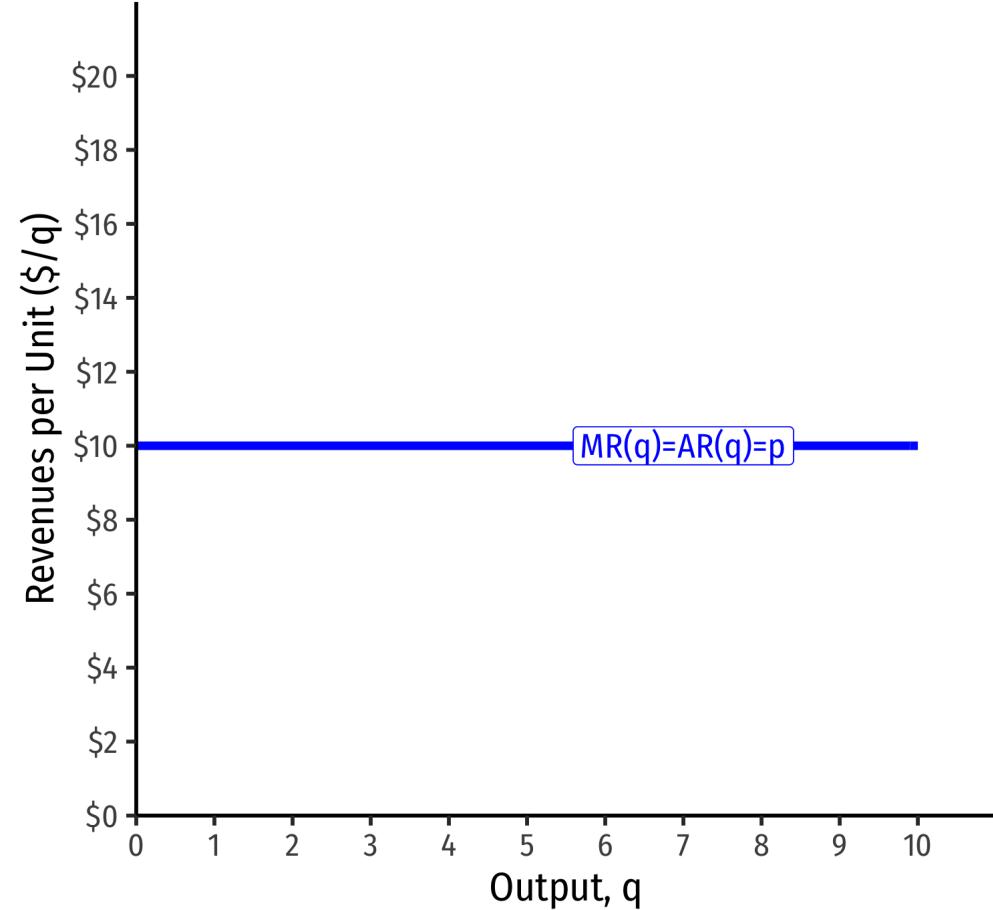
q	$R(q)$
0	0
1	10
2	20
3	30
4	40
5	50
6	60
7	70
8	80
9	90
10	100



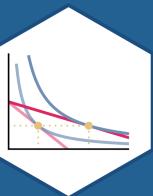
Average and Marginal Revenue, Example: Visualized



q	$R(q)$	$AR(q)$	$MR(q)$
0	0	—	—
1	10	10	10
2	20	10	10
3	30	10	10
4	40	10	10
5	50	10	10
6	60	10	10
7	70	10	10
8	80	10	10
9	90	10	10
10	100	10	10

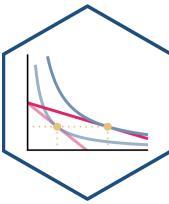


$$R(q) = 10q$$



Profits

Recall: The Firm's Two Problems



1st Stage: firm's profit maximization problem:

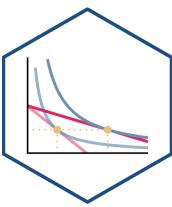
1. Choose: < output >
2. In order to maximize: < profits >

2nd Stage: firm's cost minimization problem:

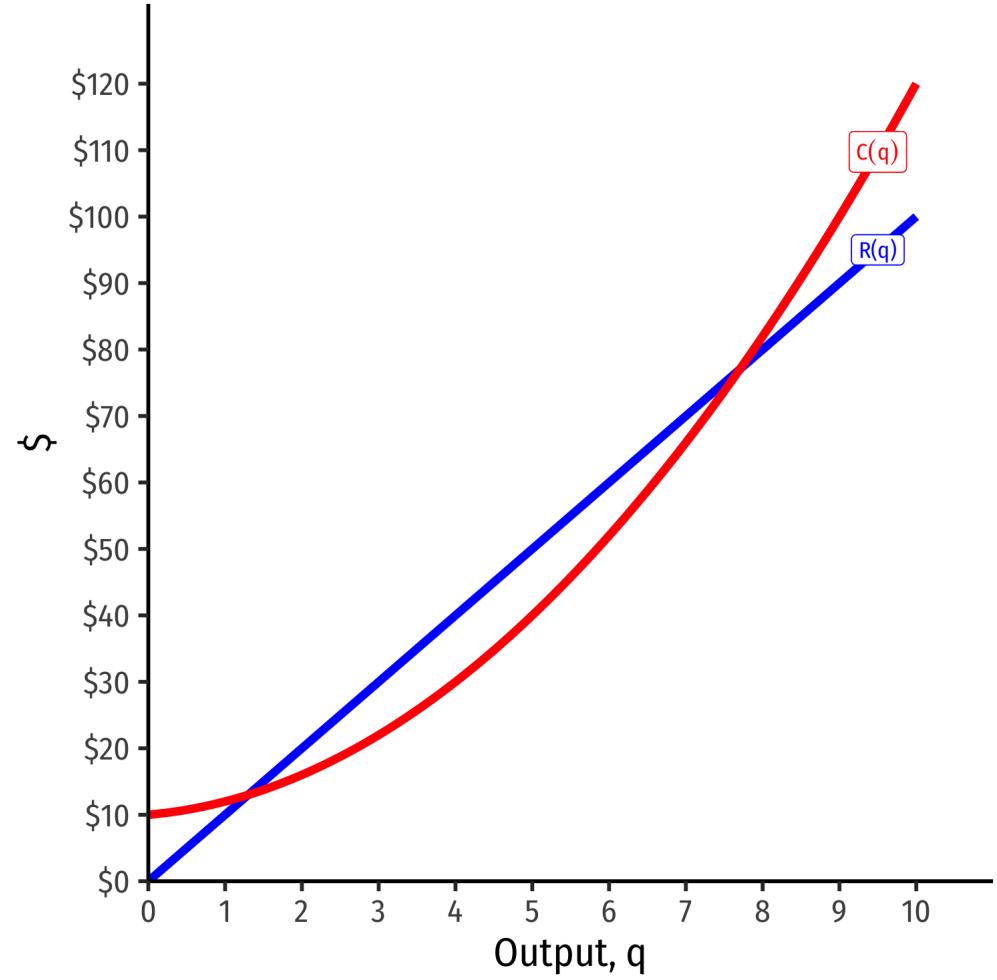
1. Choose: < inputs >
2. In order to minimize: < cost >
3. Subject to: < producing the optimal output >
 - Minimizing costs \iff maximizing profits



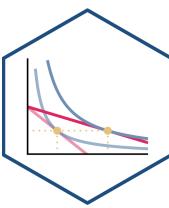
Visualizing Total Profit As $R(q) - C(q)$



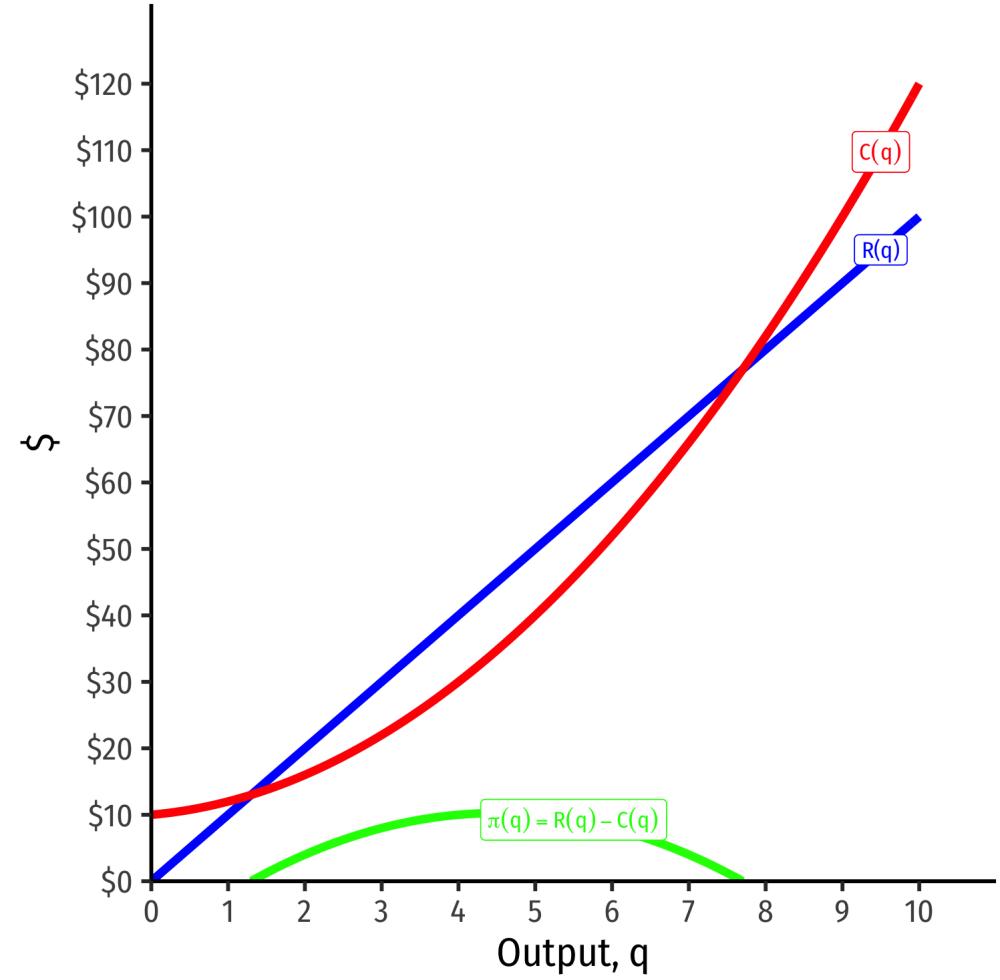
- $\pi(q) = R(q) - C(q)$



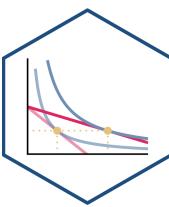
Visualizing Total Profit As $R(q) - C(q)$



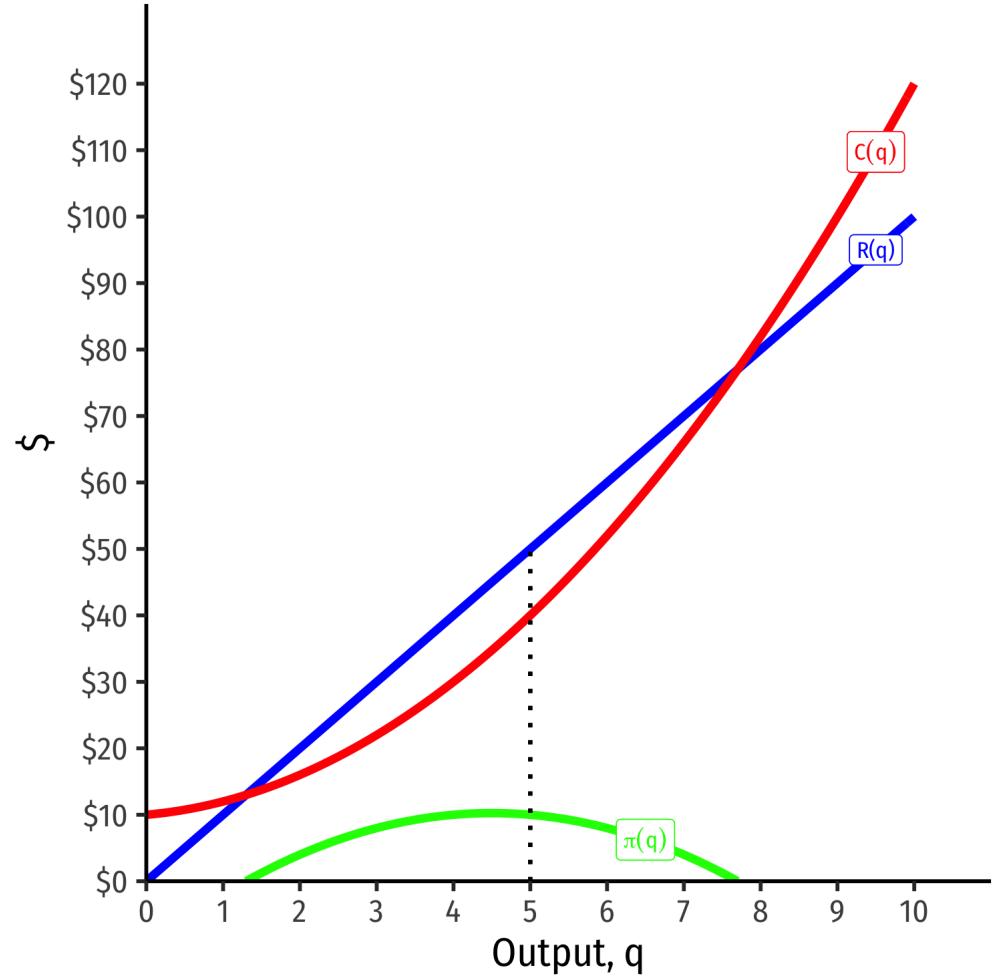
- $\pi(q) = R(q) - C(q)$



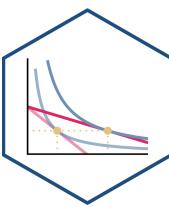
Visualizing Total Profit As $R(q) - C(q)$



- $\pi(q) = R(q) - C(q)$
- Graph: find q^* to max $\pi \implies q^*$ where
max distance between $R(q)$ and $C(q)$

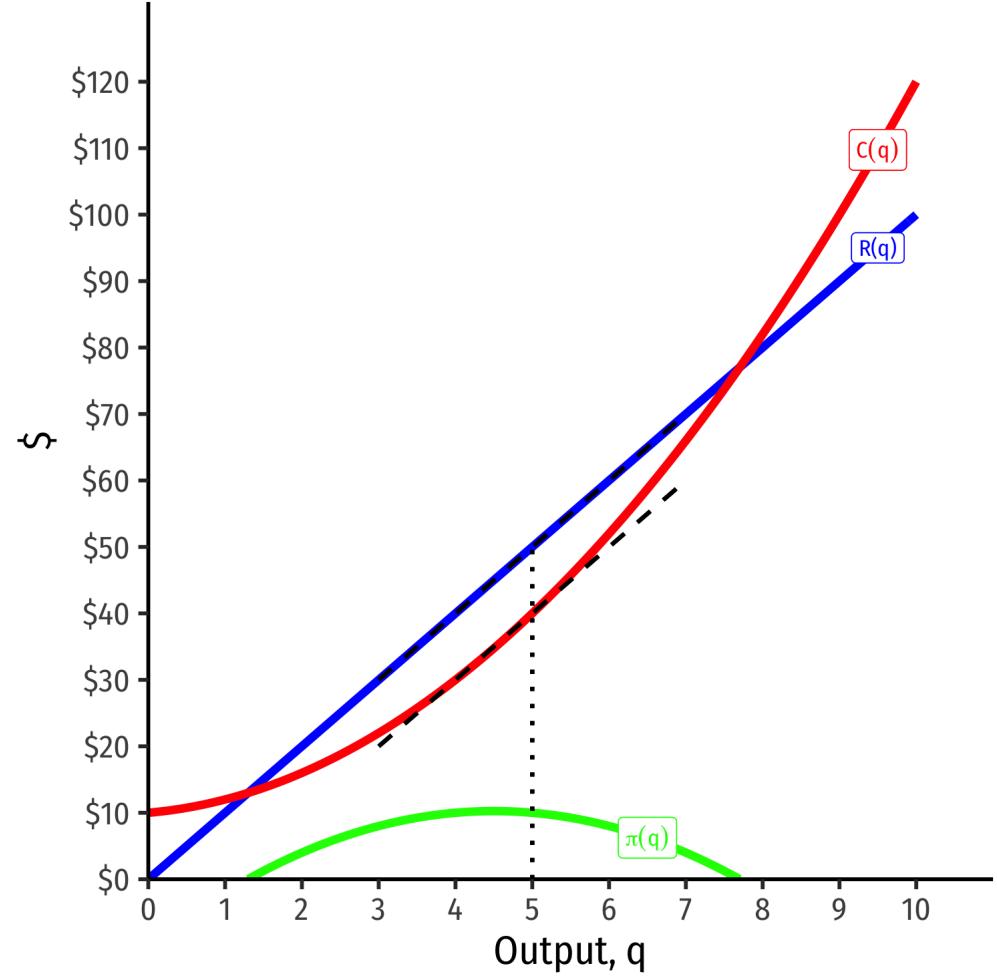


Visualizing Total Profit As $R(q) - C(q)$

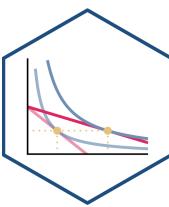


- $\pi(q) = R(q) - C(q)$
- Graph: find q^* to max $\pi \implies q^*$ where max distance between $R(q)$ and $C(q)$
- Slopes must be equal:

$$MR(q) = MC(q)$$



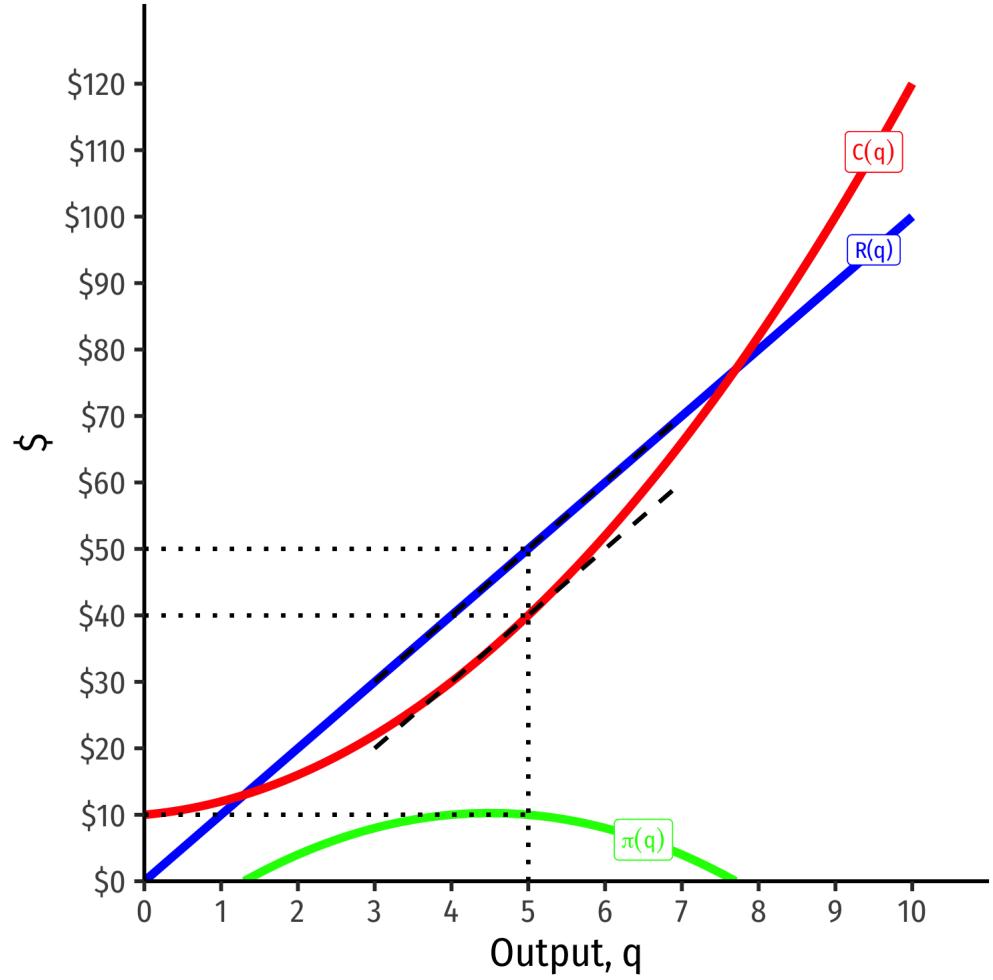
Visualizing Total Profit As $R(q) - C(q)$



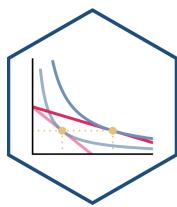
- $\pi(q) = R(q) - C(q)$
- Graph: find q^* to max $\pi \implies q^*$ where max distance between $R(q)$ and $C(q)$
- Slopes must be equal:

$$MR(q) = MC(q)$$

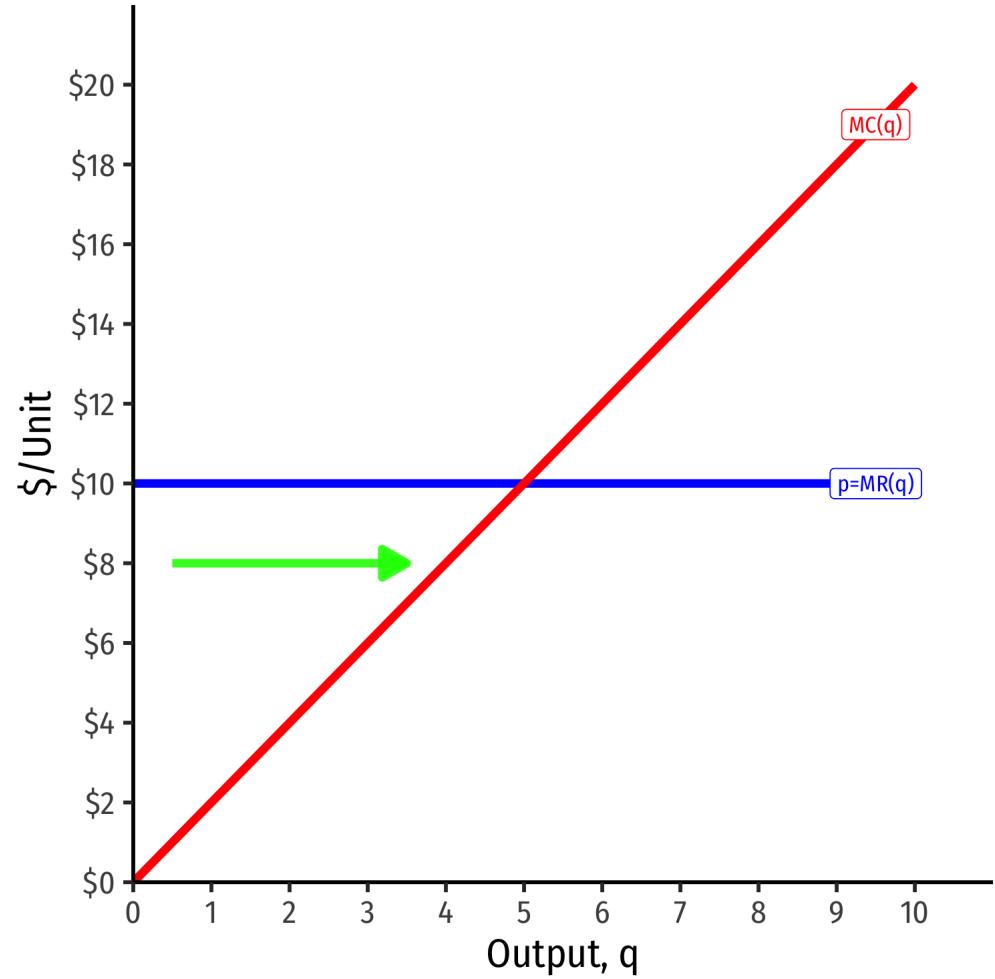
- At $q^* = 5$:
 - $R(q) = 50$
 - $C(q) = 40$
 - $\pi(q) = 10$



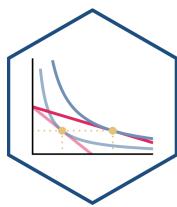
Visualizing Profit Per Unit As $MR(q)$ and $MC(q)$



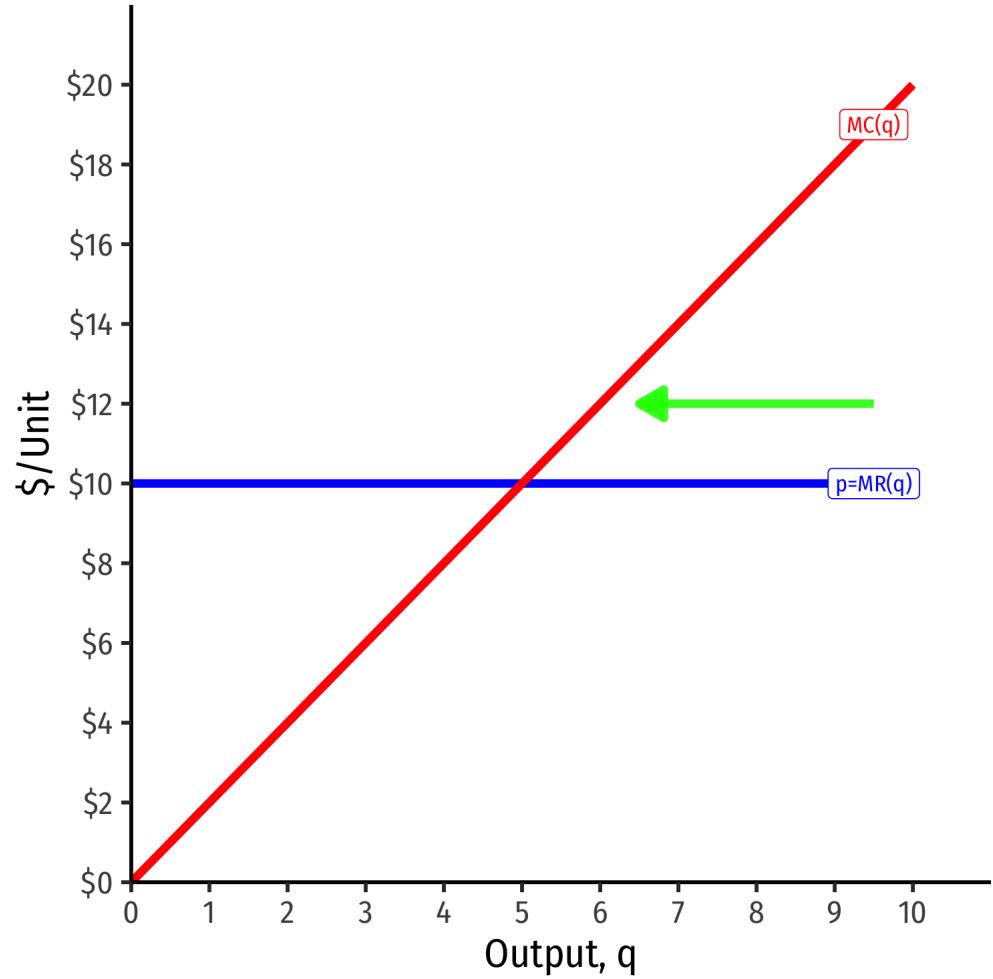
- At low output $q < q^*$, can increase π by producing *more*: $MR(q) > MC(q)$



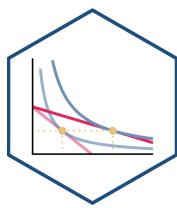
Visualizing Profit Per Unit As $MR(q)$ and $MC(q)$



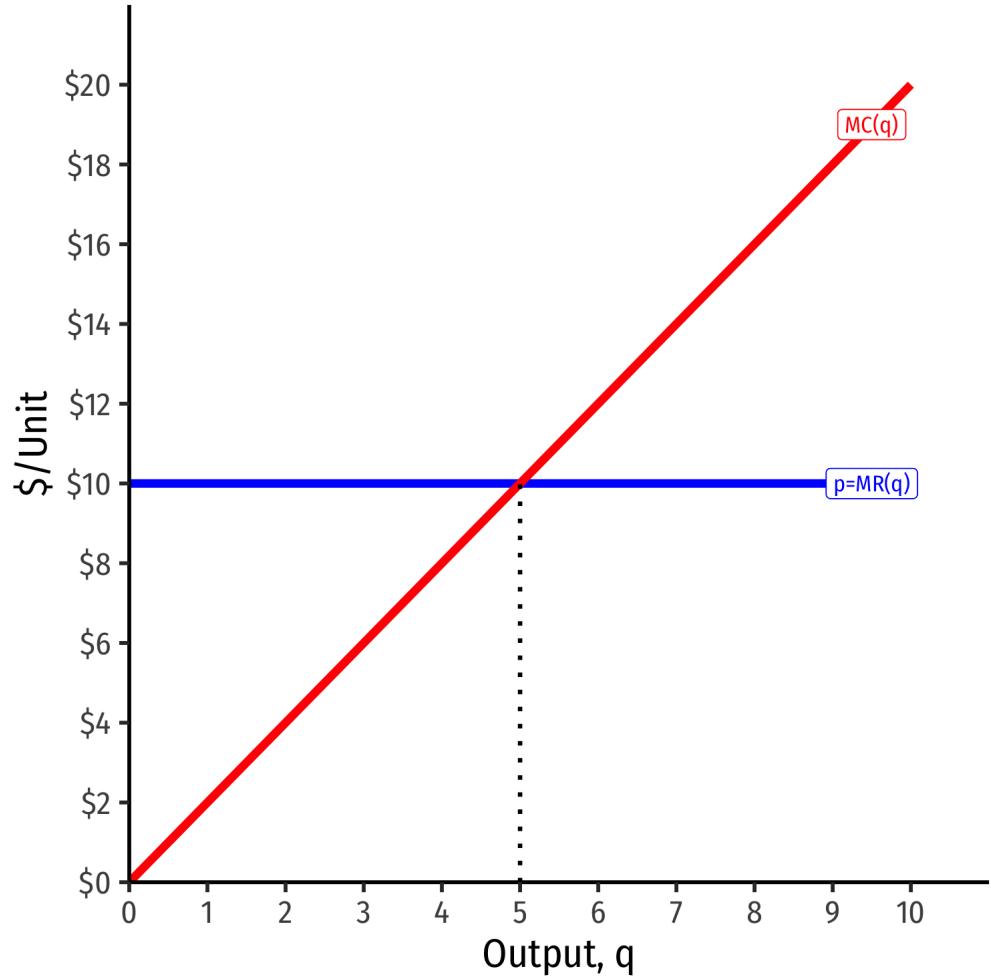
- At high output $q > q^*$, can increase π by producing less: $MR(q) < MC(q)$

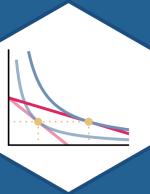


Visualizing Profit Per Unit As $MR(q)$ and $MC(q)$



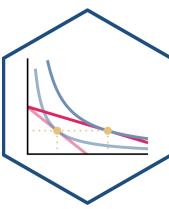
- π is *maximized* where
 $MR(q) = MC(q)$



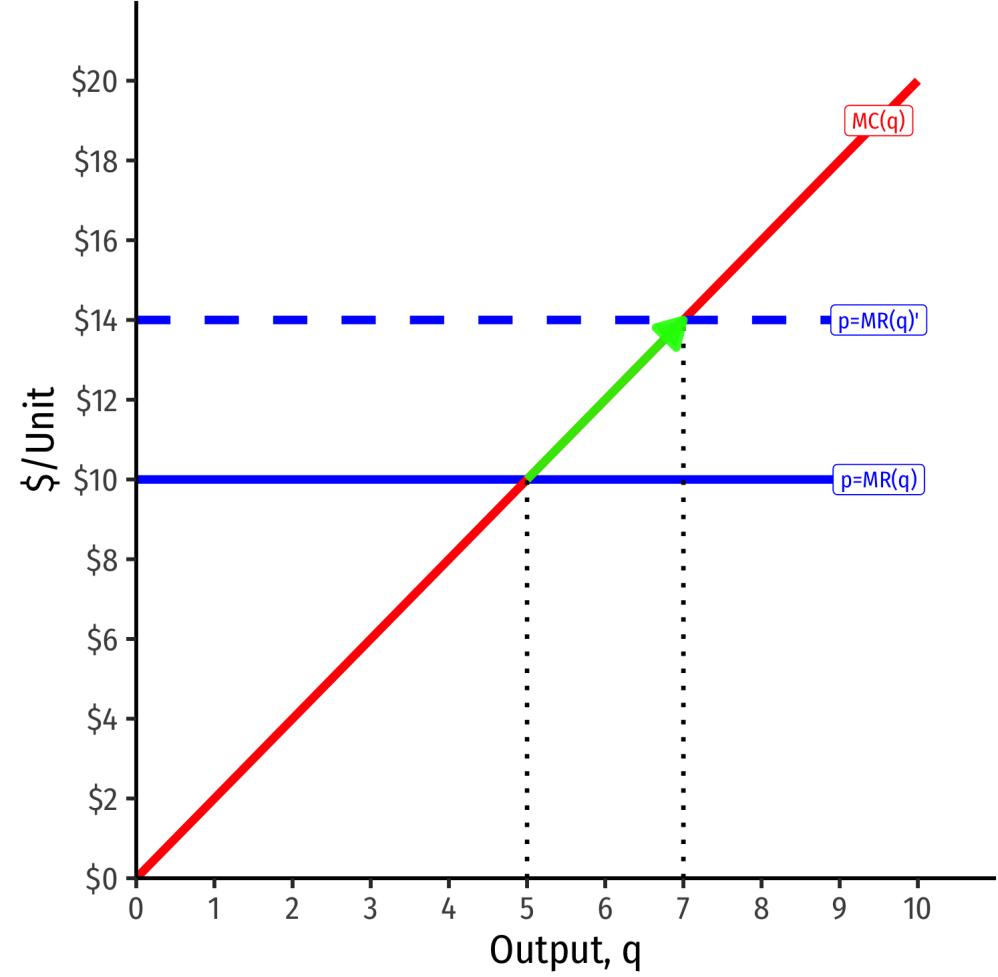


Comparative Statics

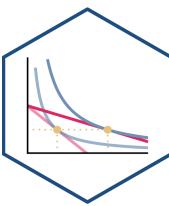
If Market Price Changes I



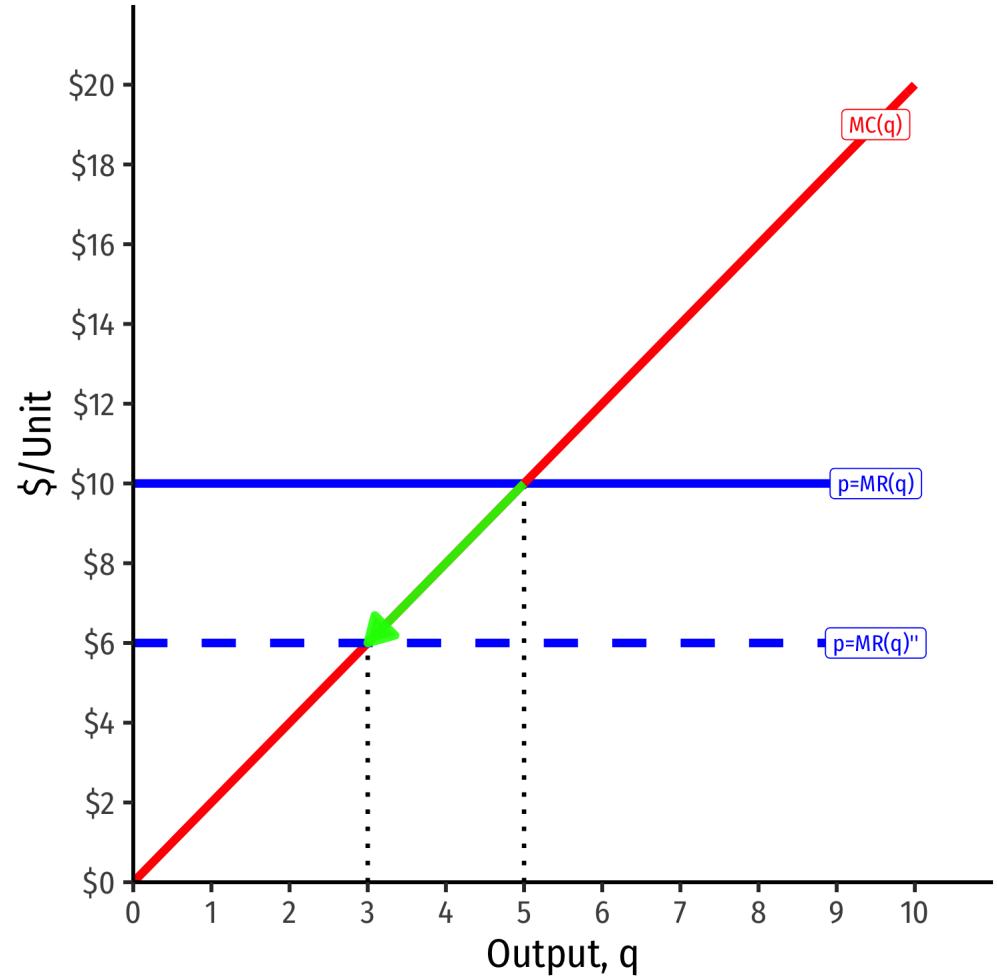
- Suppose the market price **increases**
- Firm (always setting $MR=MC$) will respond by **producing more**



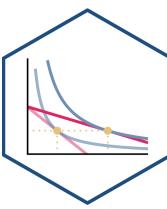
If Market Price Changes II



- Suppose the market price **decreases**
- Firm (always setting $MR=MC$) will respond by **producing less**



The Firm's Supply Curve

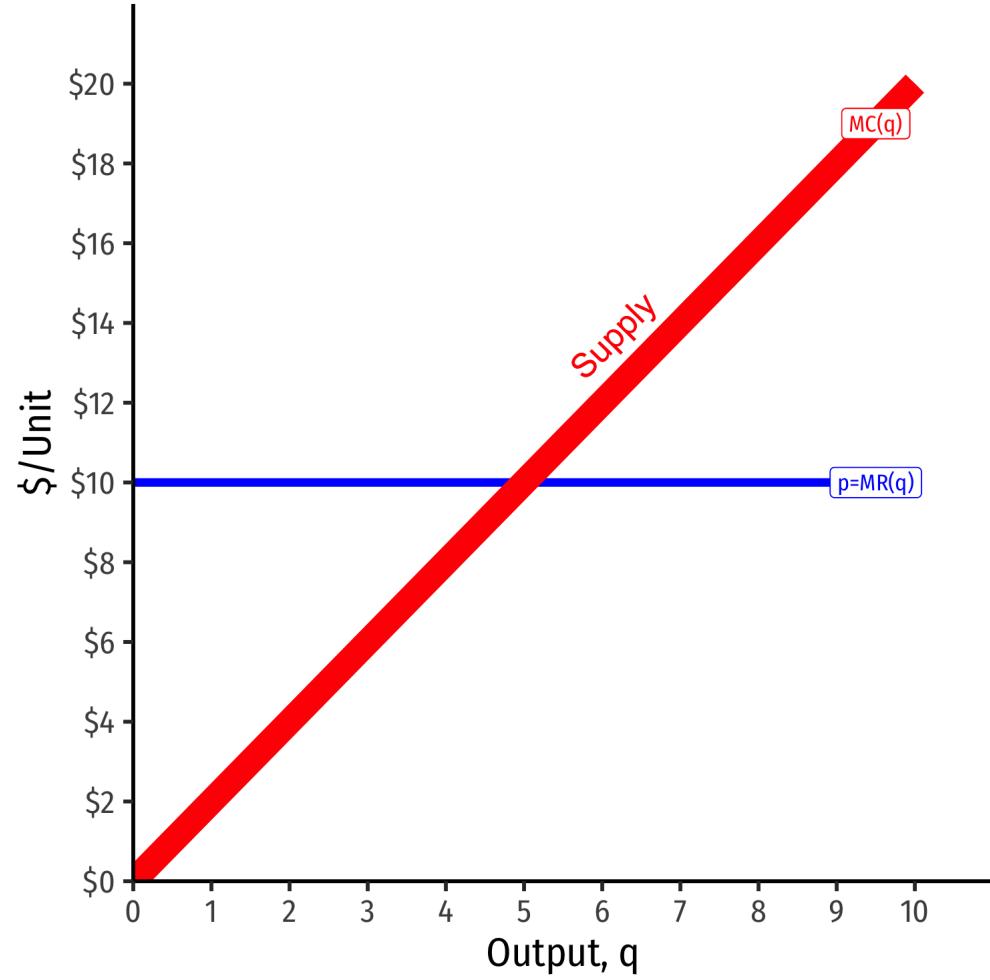


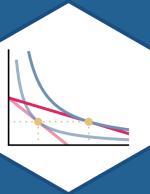
- The firm's marginal cost curve is its supply curve[†]

$$p = MC(q)$$

- How it will supply the optimal amount of output in response to the market price
- Firm always sets its price equal to its marginal cost

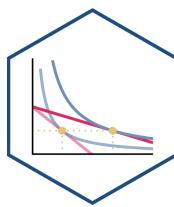
[†] Mostly...there is an important **exception** we will see shortly!





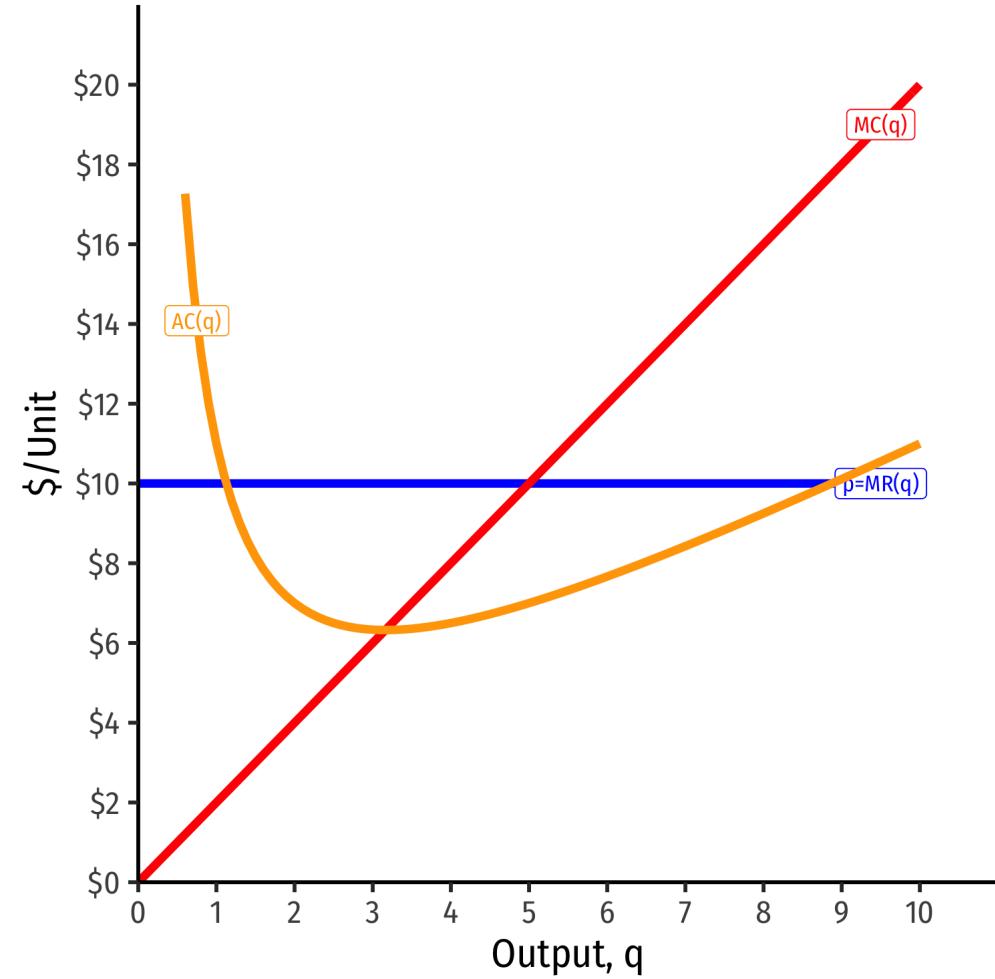
Calculating Profit

Calculating (Average) Profit as AR(q)-AC(q)

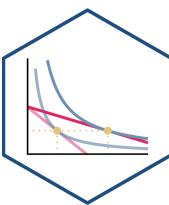


- Profit is

$$\pi(q) = R(q) - C(q)$$



Calculating (Average) Profit as AR(q)-AC(q)

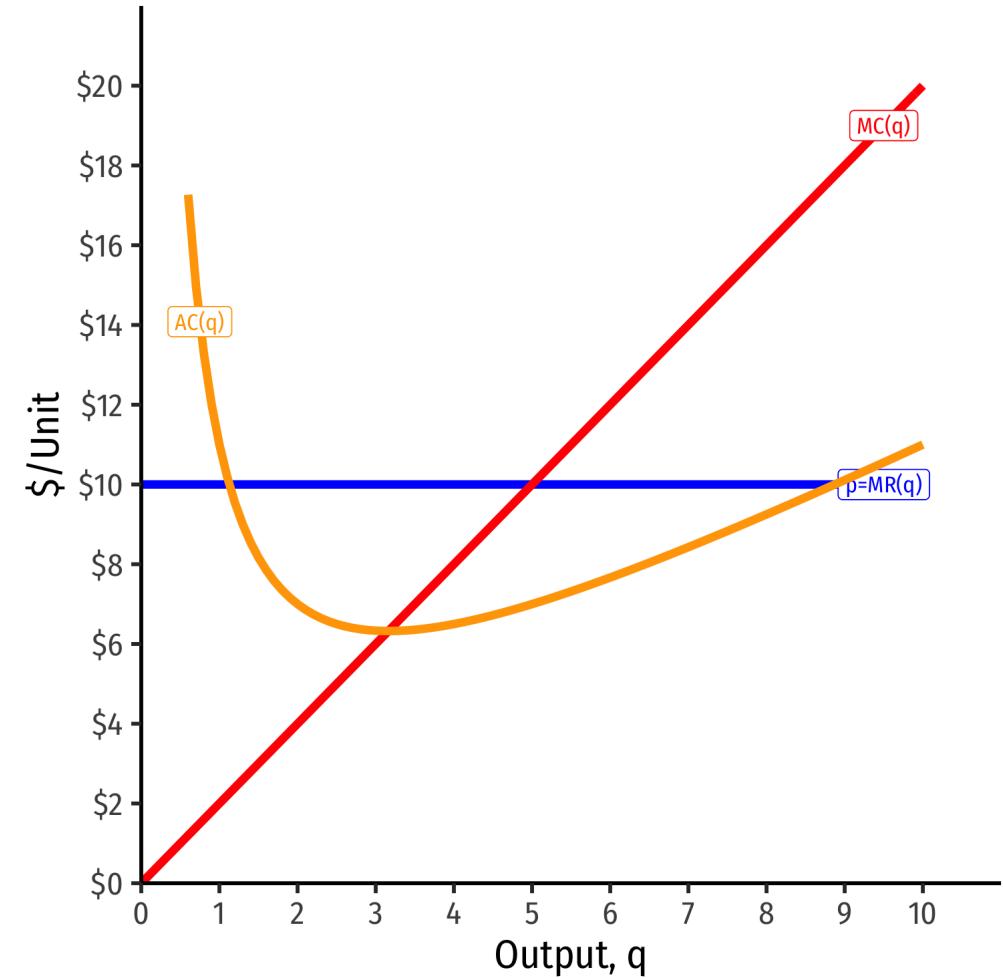


- Profit is

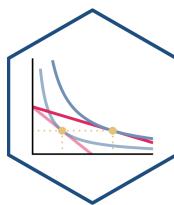
$$\pi(q) = R(q) - C(q)$$

- Profit per unit can be calculated as:

$$\begin{aligned}\frac{\pi(q)}{q} &= AR(q) - AC(q) \\ &= p - AC(q)\end{aligned}$$



Calculating (Average) Profit as AR(q)-AC(q)



- Profit is

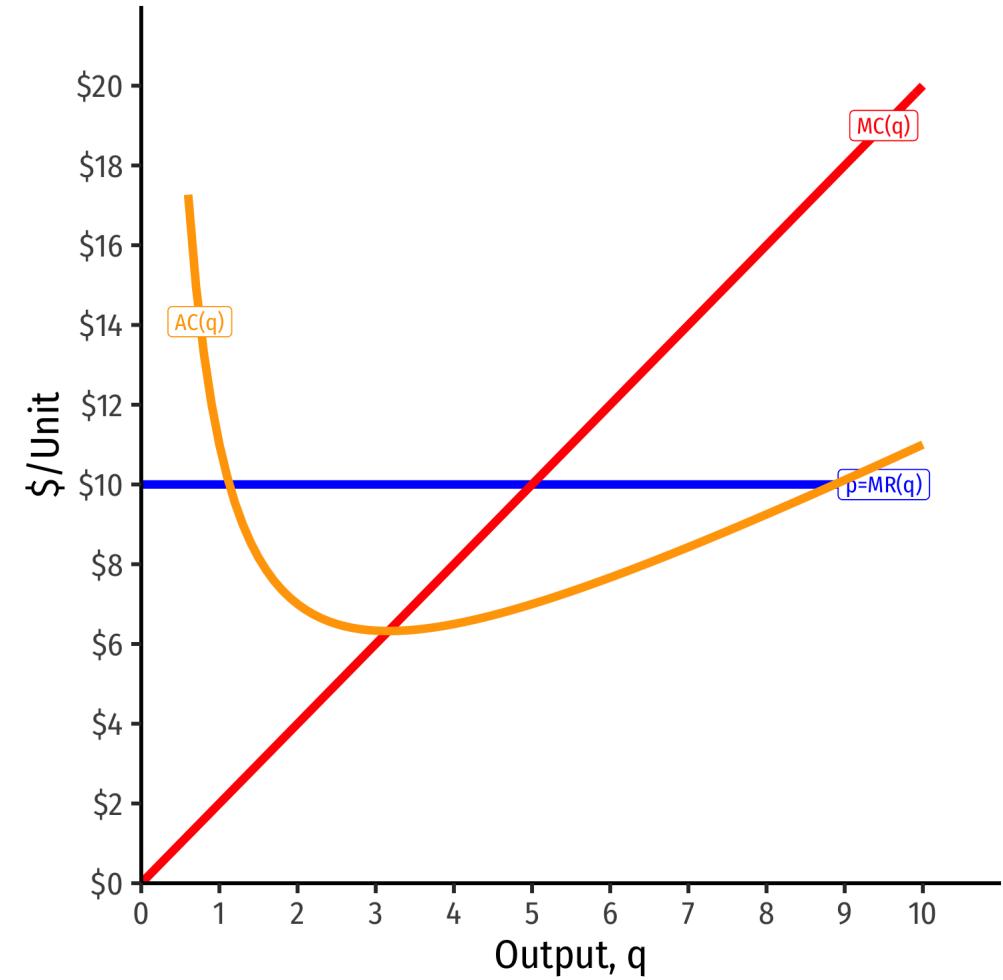
$$\pi(q) = R(q) - C(q)$$

- Profit per unit can be calculated as:

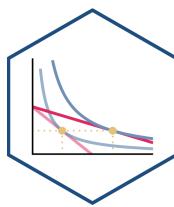
$$\begin{aligned}\frac{\pi(q)}{q} &= AR(q) - AC(q) \\ &= p - AC(q)\end{aligned}$$

- Multiply by q to get total profit:

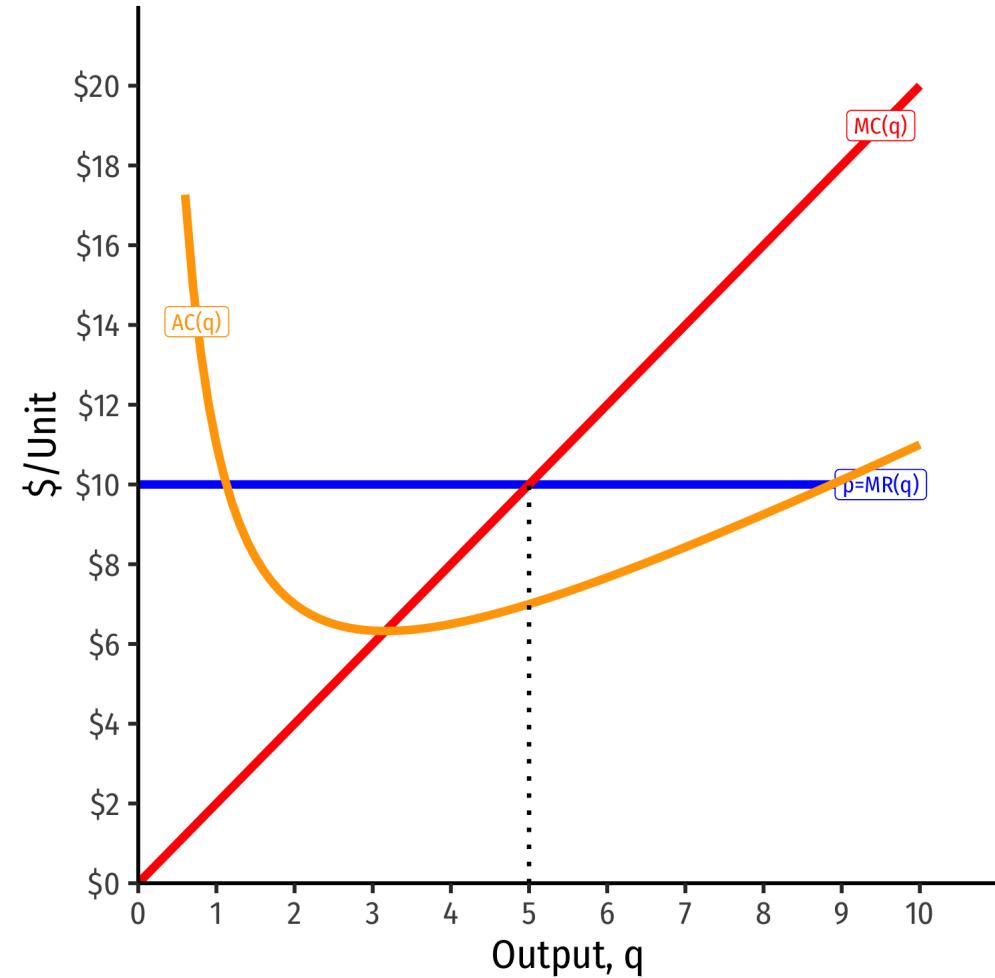
$$\pi(q) = q [p - AC(q)]$$



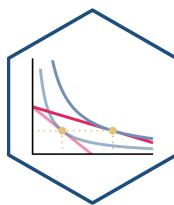
Calculating (Average) Profit as AR(q)-AC(q)



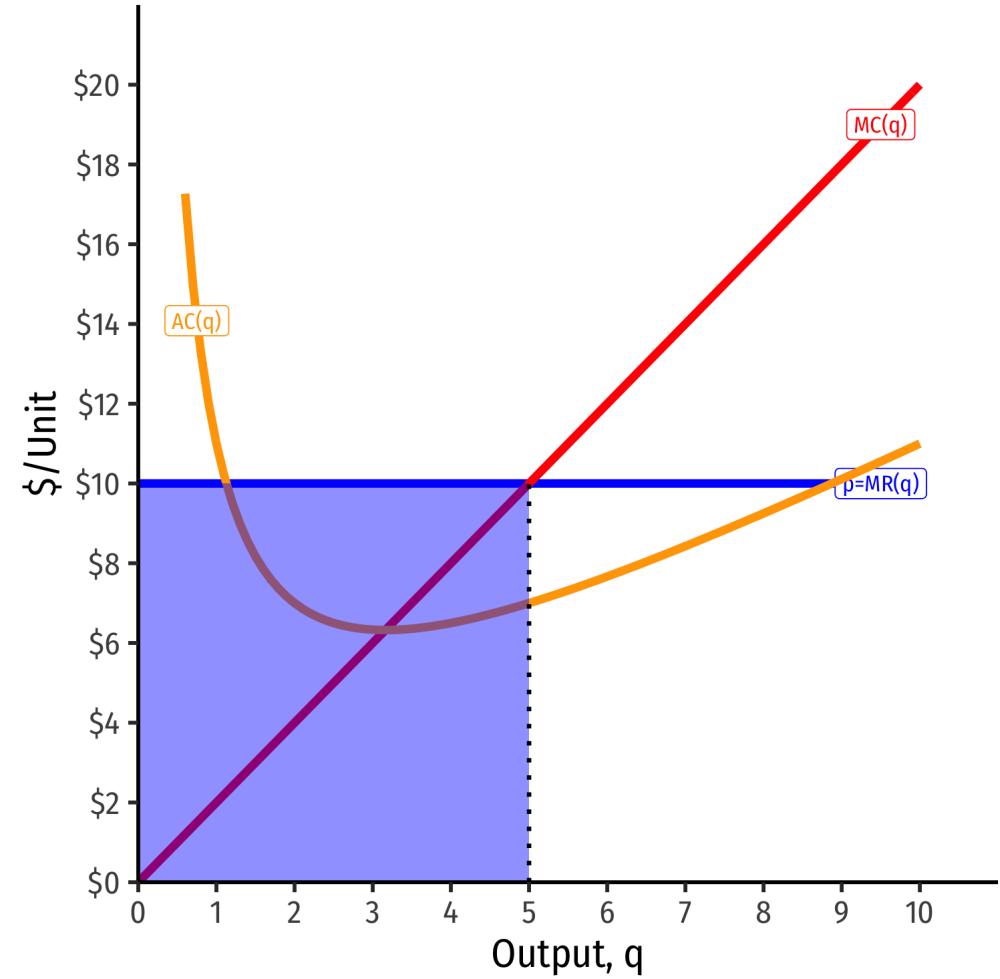
- At market price of $p^* = \$10$
- At $q^* = 5$ (per unit):
- At $q^* = 5$ (totals):



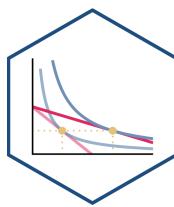
Calculating (Average) Profit as AR(q)-AC(q)



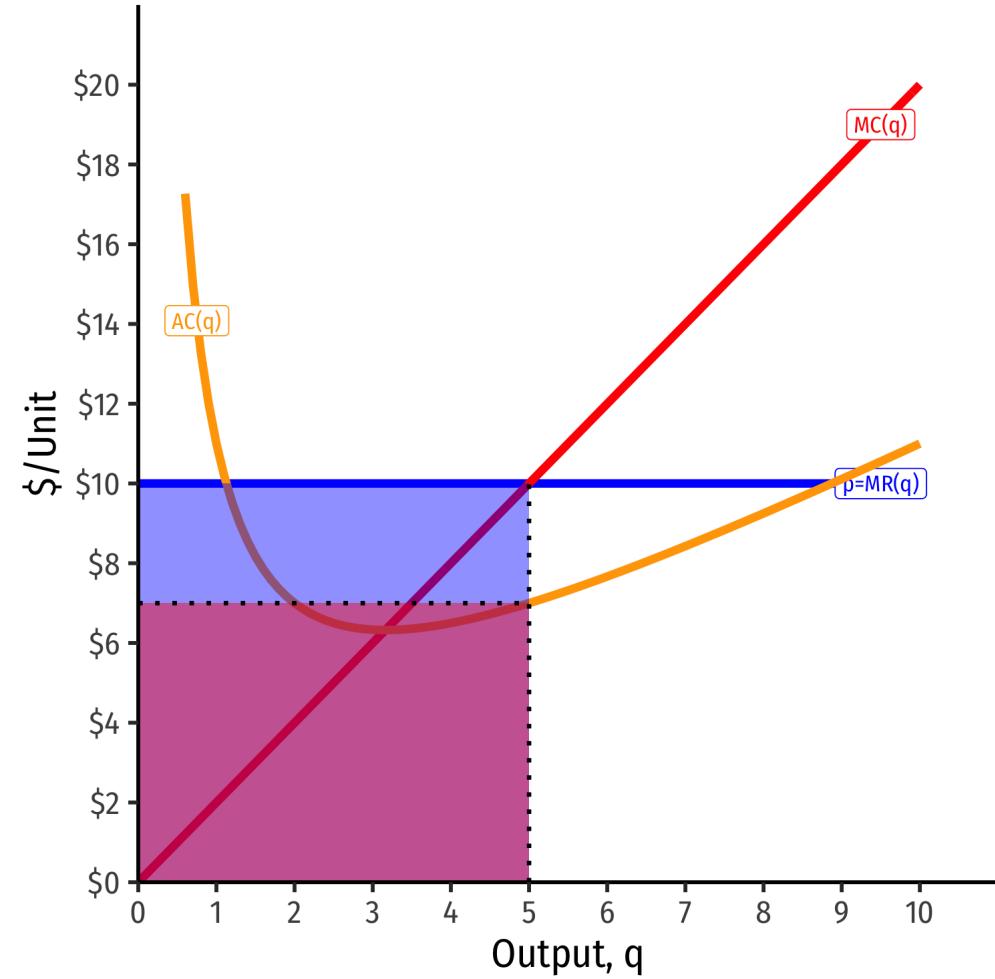
- At market price of $p^* = \$10$
- At $q^* = 5$ (per unit):
 - $AR(5) = \$10/\text{unit}$
- At $q^* = 5$ (totals):
 - $R(5) = \$50$



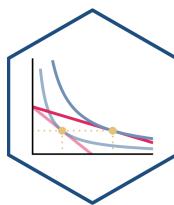
Calculating (Average) Profit as AR(q)-AC(q)



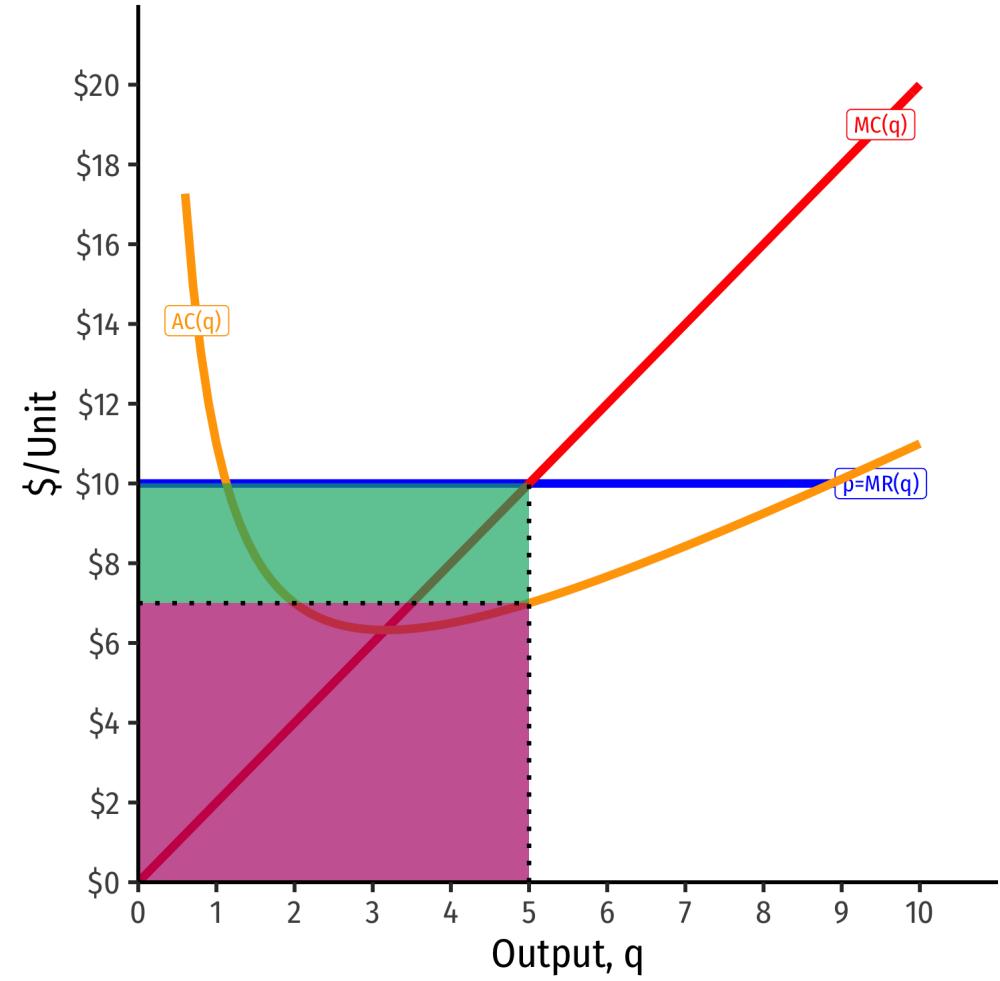
- At market price of $p^* = \$10$
- At $q^* = 5$ (per unit):
 - $AR(5) = \$10/\text{unit}$
 - $AC(5) = \$7/\text{unit}$
- At $q^* = 5$ (totals):
 - $R(5) = \$50$
 - $C(5) = \$35$



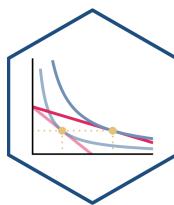
Calculating (Average) Profit as AR(q)-AC(q)



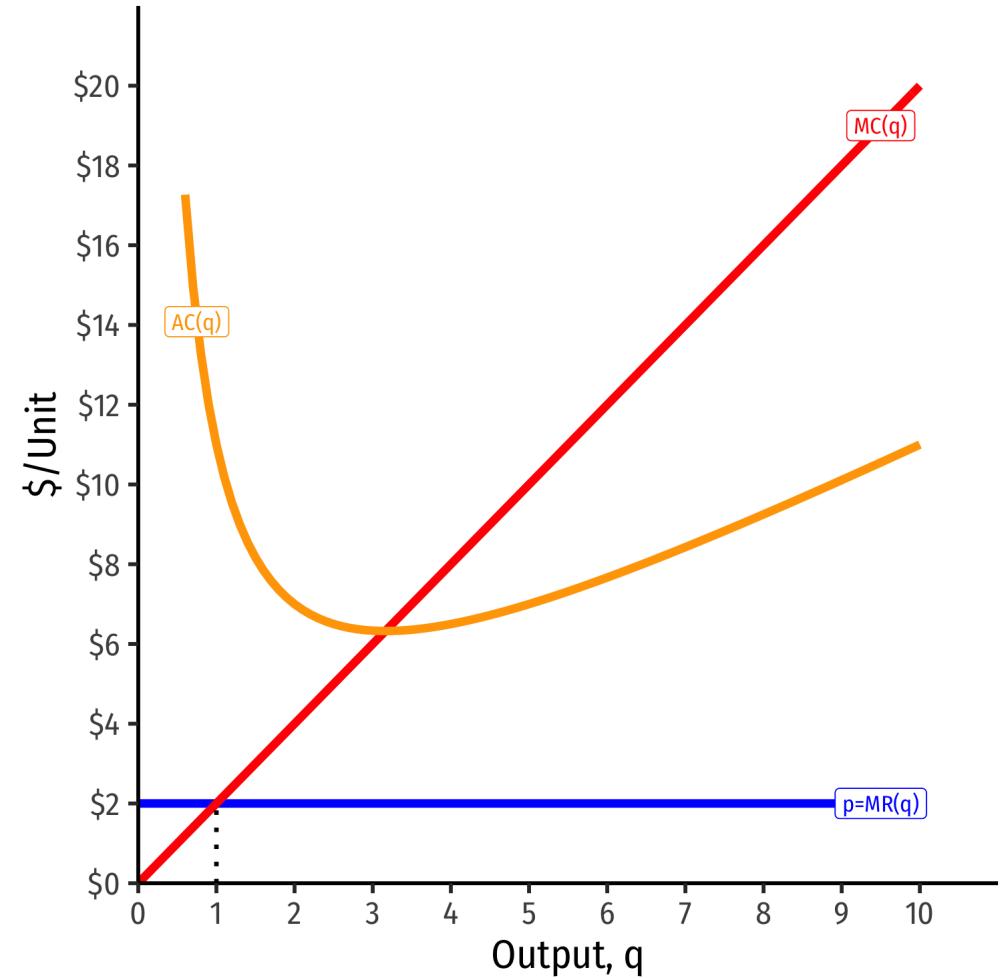
- At market price of $p^* = \$10$
- At $q^* = 5$ (per unit):
 - $AR(5) = \$10/\text{unit}$
 - $AC(5) = \$7/\text{unit}$
 - $A\pi(5) = \$3/\text{unit}$
- At $q^* = 5$ (totals):
 - $R(5) = \$50$
 - $C(5) = \$35$
 - $\pi = \$15$



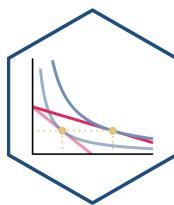
Calculating (Average) Profit as AR(q)-AC(q)



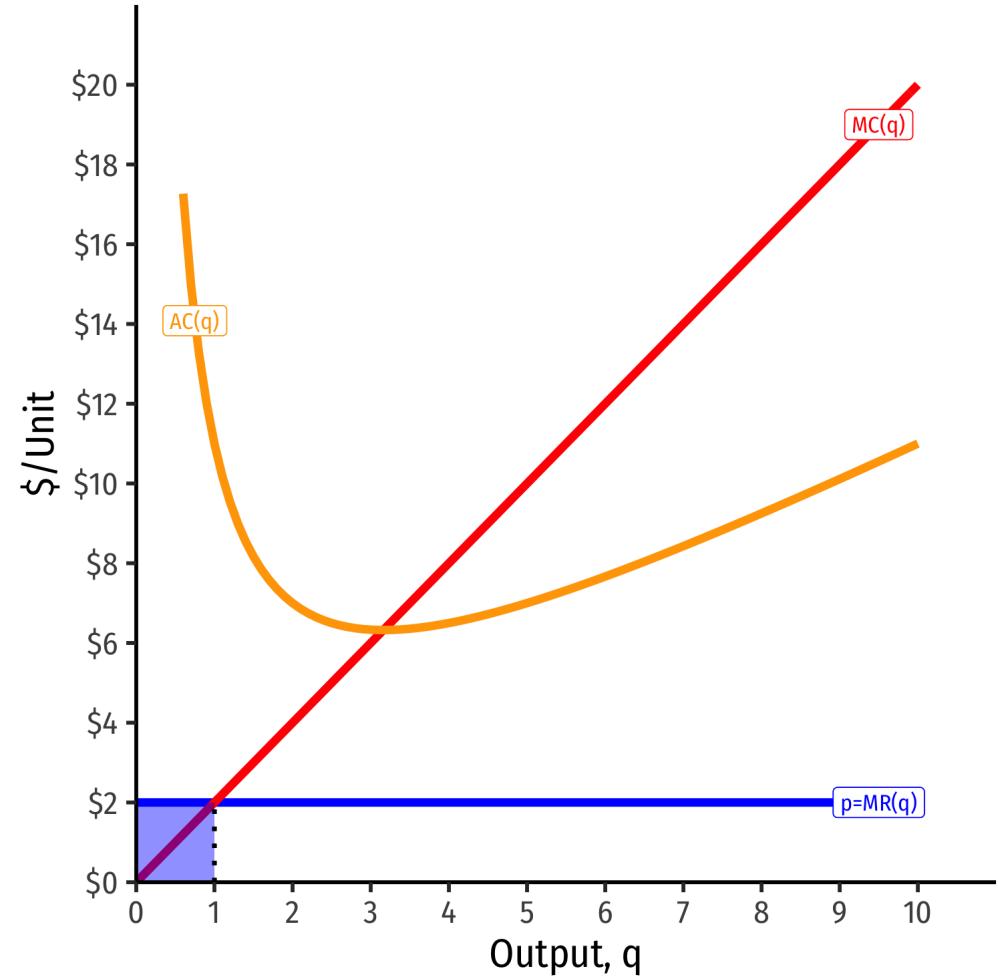
- At market price of $p^* = \$2$
- At $q^* = 1$ (per unit):
- At $q^* = 1$ (totals):



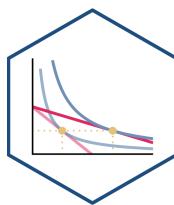
Calculating (Average) Profit as AR(q)-AC(q)



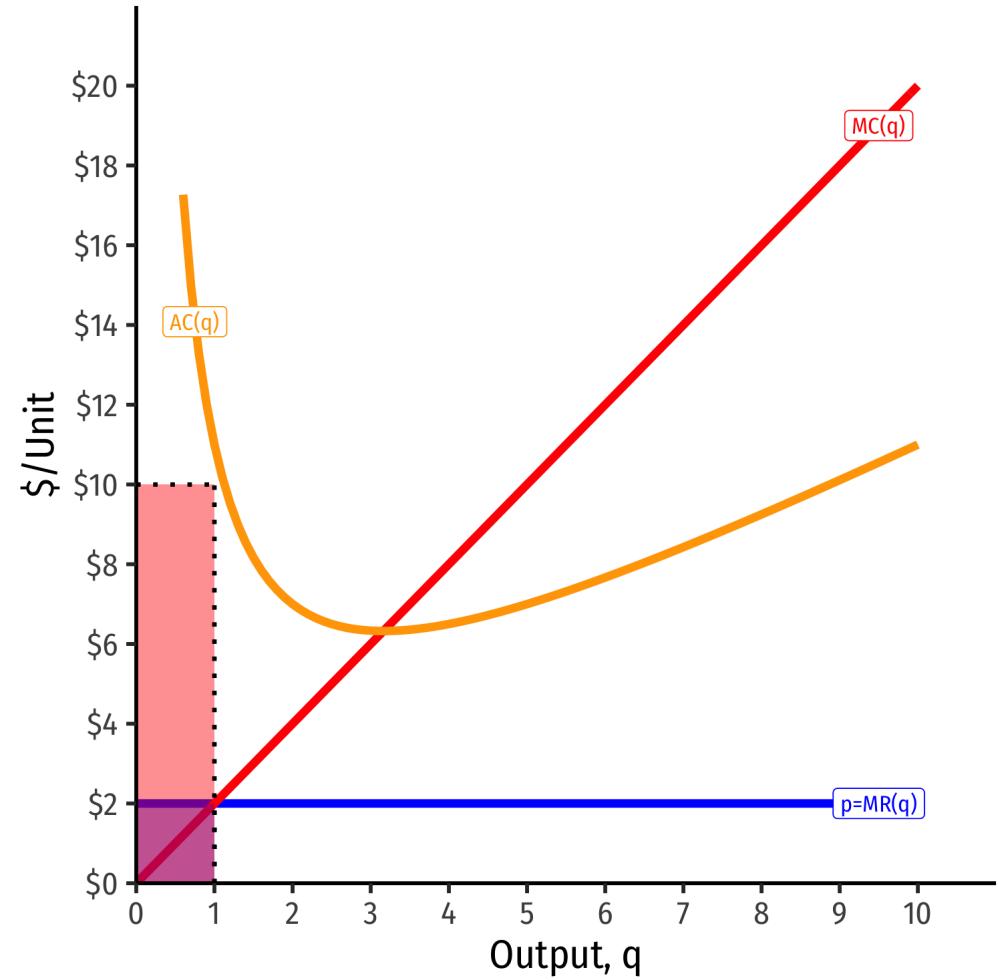
- At market price of $p^* = \$2$
- At $q^* = 1$ (per unit):
 - $AR(1) = \$2/\text{unit}$
- At $q^* = 1$ (totals):
 - $R(1) = \$2$



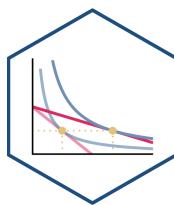
Calculating (Average) Profit as AR(q)-AC(q)



- At market price of $p^* = \$2$
- At $q^* = 1$ (per unit):
 - $AR(1) = \$2/\text{unit}$
 - $AC(1) = \$10/\text{unit}$
- At $q^* = 1$ (totals):
 - $R(1) = \$2$
 - $C(1) = \$10$



Calculating (Average) Profit as AR(q)-AC(q)



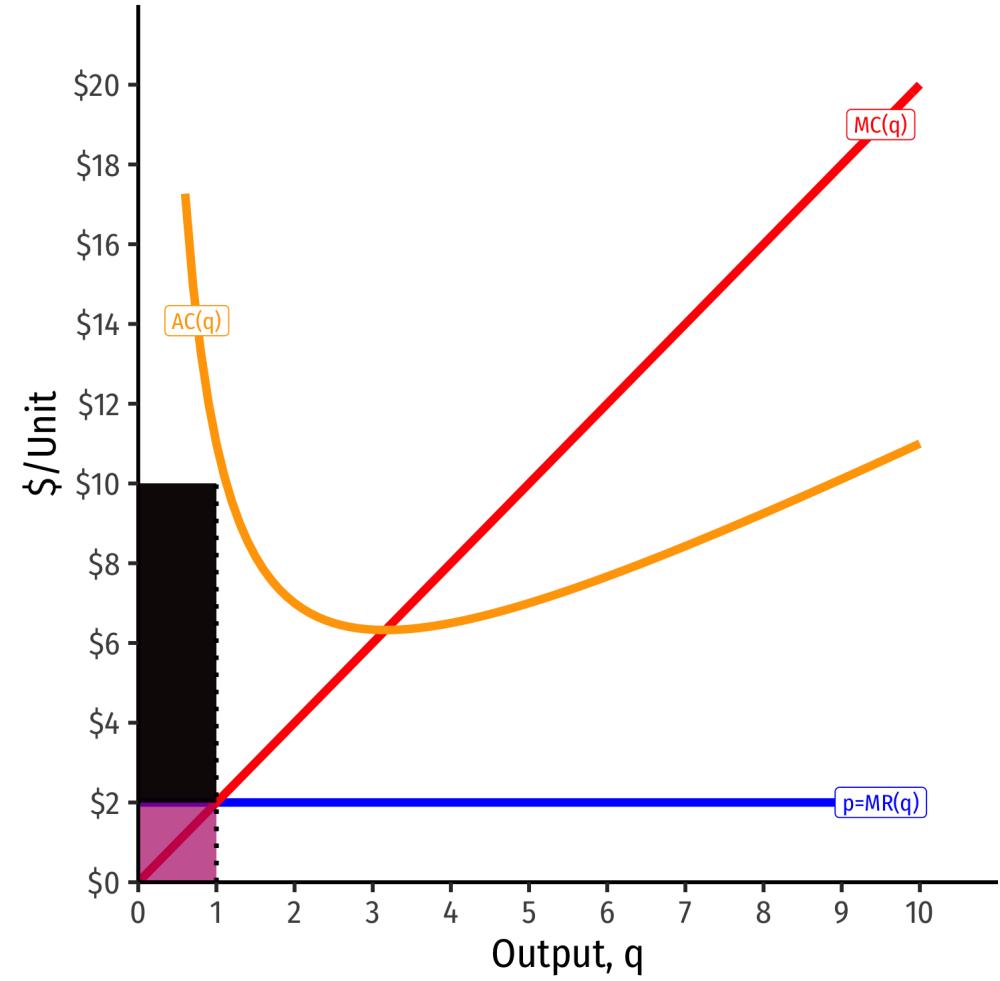
- At market price of $p^* = \$2$

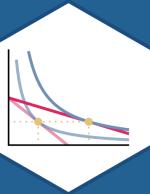
- At $q^* = 1$ (per unit):

- $AR(1) = \$2/\text{unit}$
- $AC(1) = \$10/\text{unit}$
- $A\pi(1) = -\$8/\text{unit}$

- At $q^* = 1$ (totals):

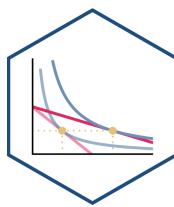
- $R(1) = \$2$
- $C(1) = \$10$
- $\pi(1) = -\$8$





Short-Run Shut-Down Decisions

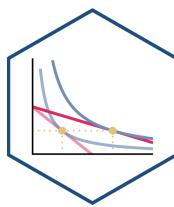
Short-Run Shut-Down Decisions



- What if a firm's profits at q^* are **negative** (i.e. it earns **losses**)?
- Should it produce at all?



Short-Run Shut-Down Decisions

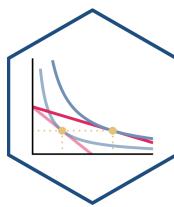


- Suppose firm chooses to produce **nothing** ($q = 0$):
- If it has **fixed costs** ($f > 0$), its profits are:

$$\pi(q) = pq - C(q)$$



Short-Run Shut-Down Decisions



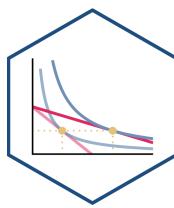
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Short-Run Shut-Down Decisions



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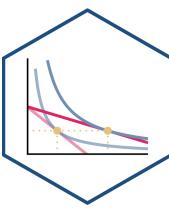
$$\pi(q) = pq - f - VC(q)$$

$$\pi(0) = -f$$

i.e. it (still) pays its fixed costs



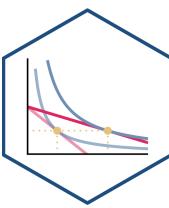
Short-Run Shut-Down Decisions



- A firm should choose to produce **no output** ($q = 0$) only when:

π from producing $<$ π from not producing

Short-Run Shut-Down Decisions

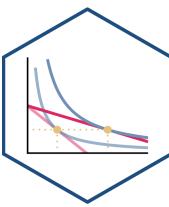


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Short-Run Shut-Down Decisions



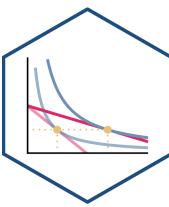
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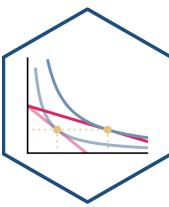
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Short-Run Shut-Down Decisions



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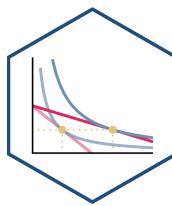
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Short-Run Shut-Down Decisions



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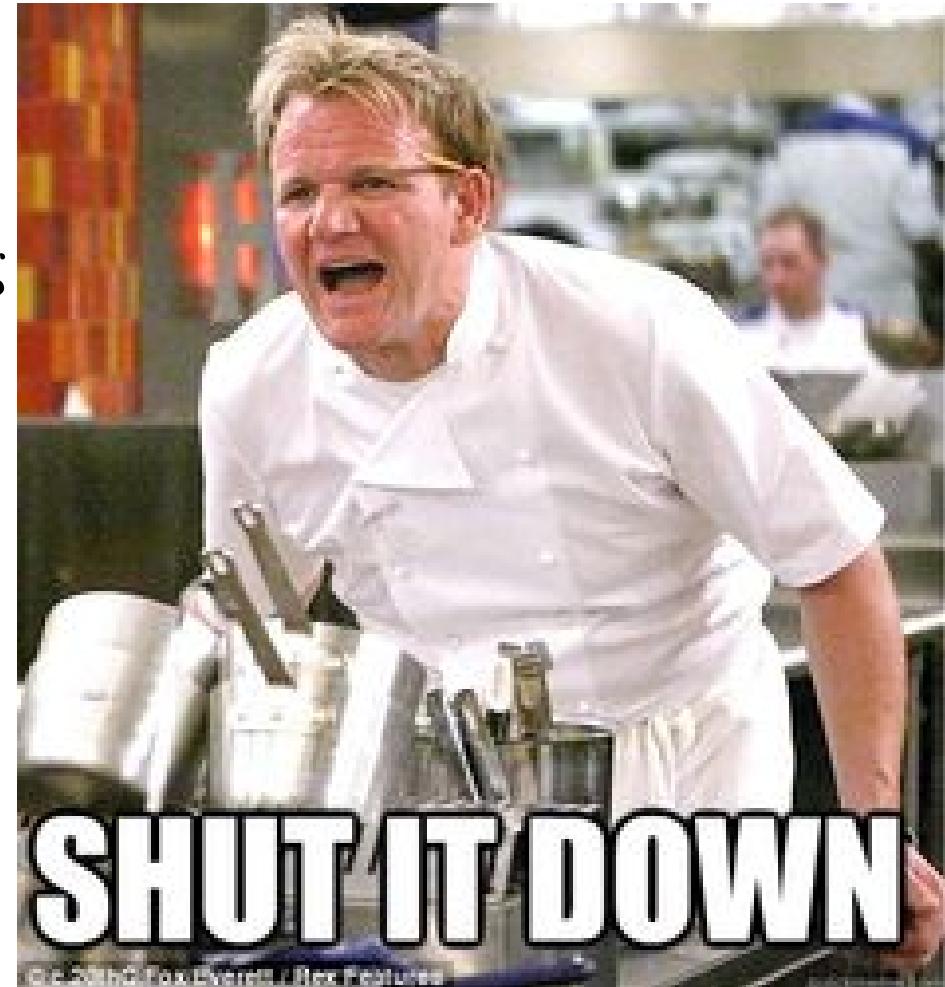
$$pq - VC(q) - f < -f$$

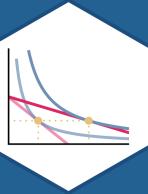
$$pq - VC(q) < 0$$

$$pq < VC(q)$$

$$p < AVC(q)$$

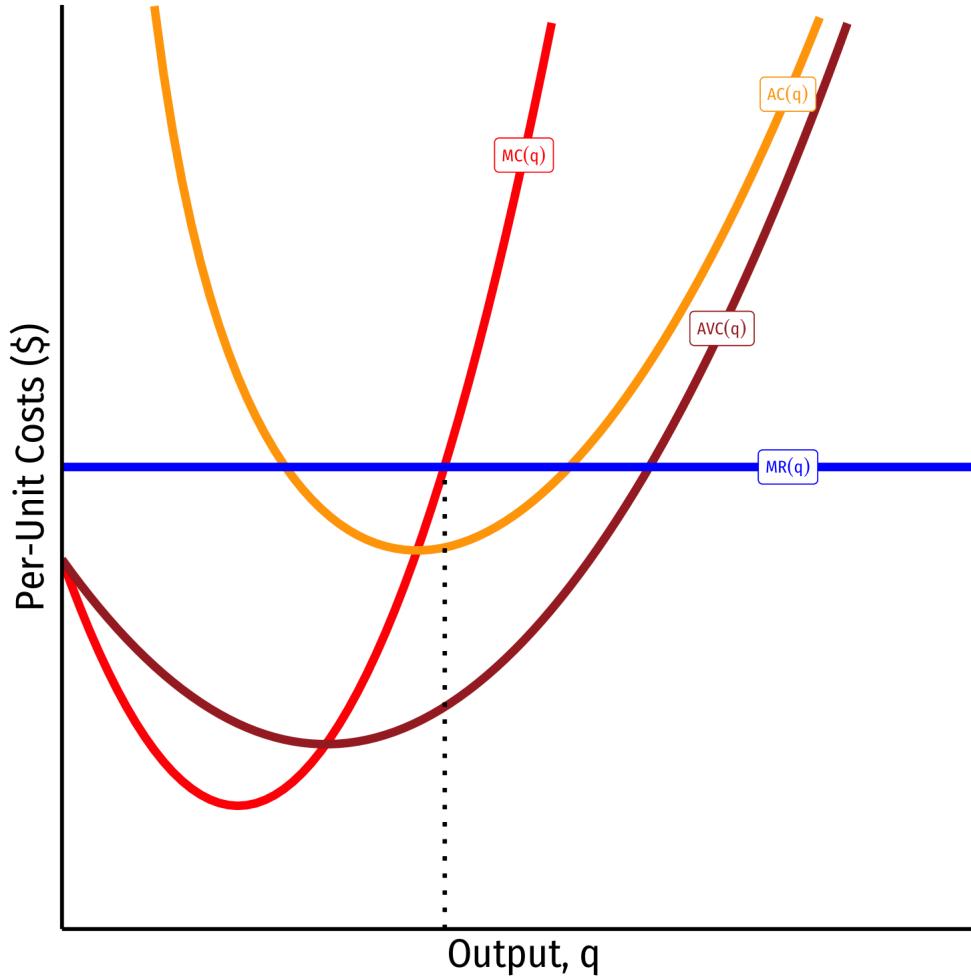
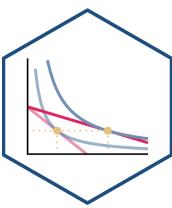
- **Shut down price:** firm will shut down production *in the short run* when $p < AVC(q)$



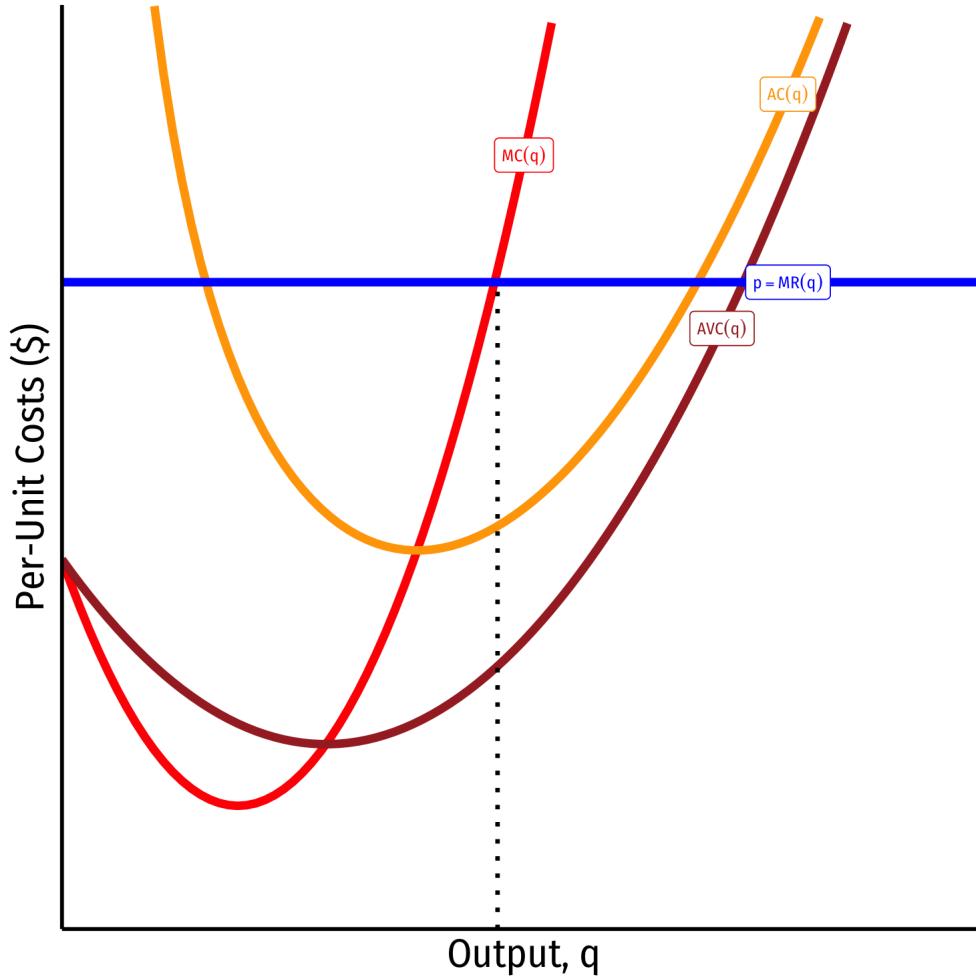
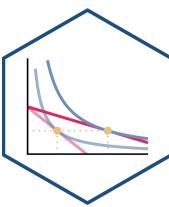


The Firm's Short Run Supply Decision

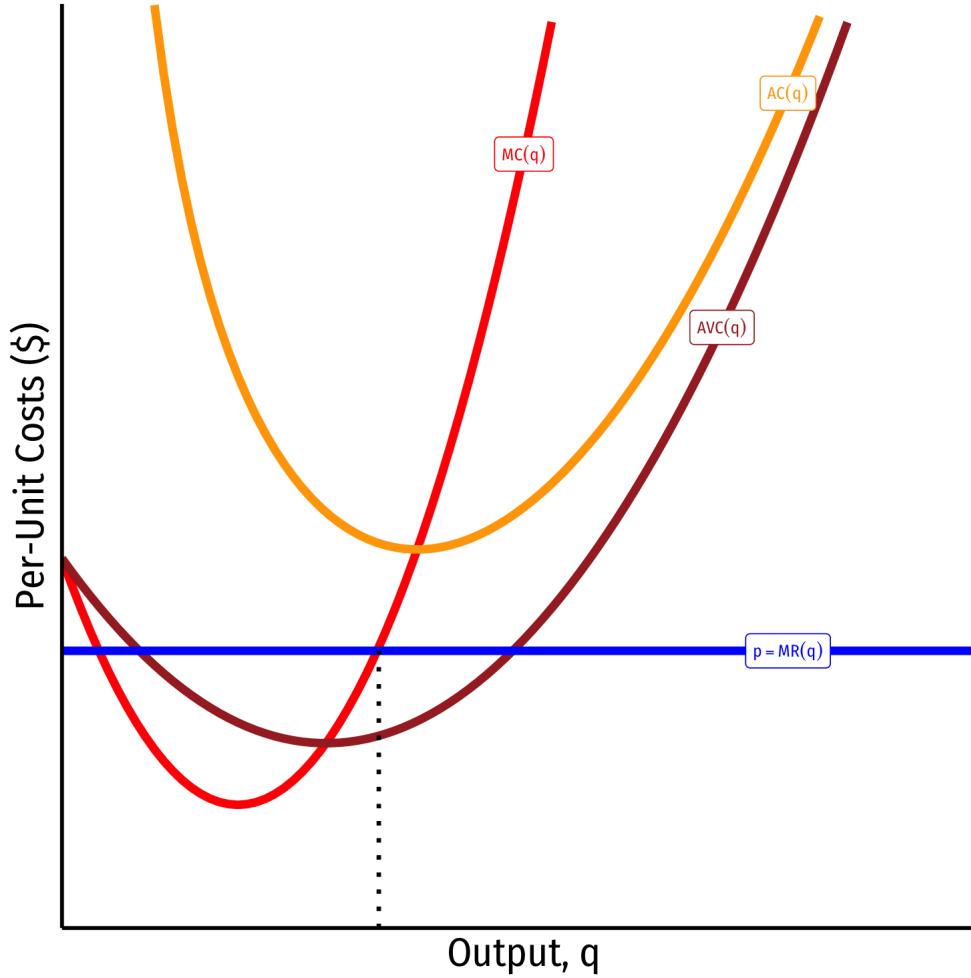
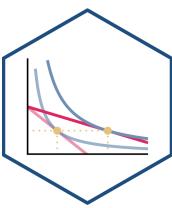
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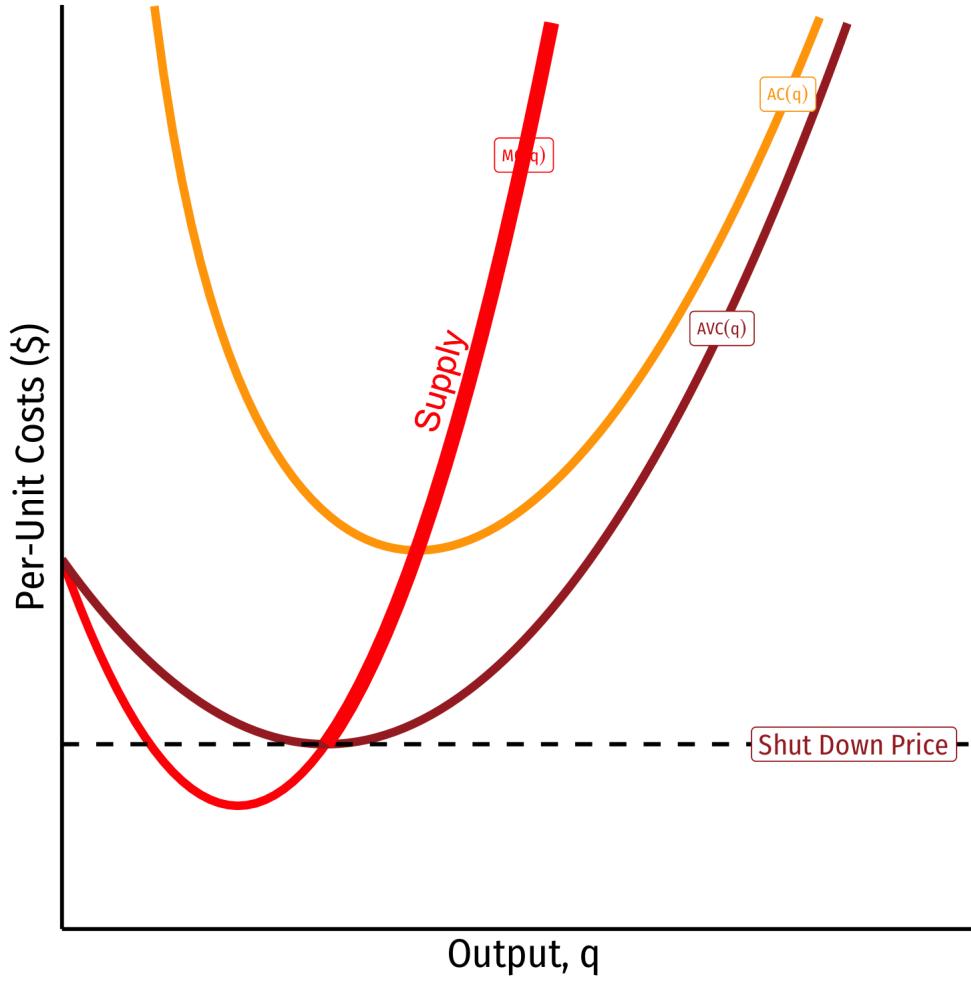
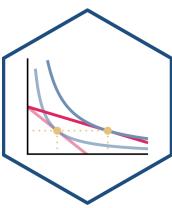
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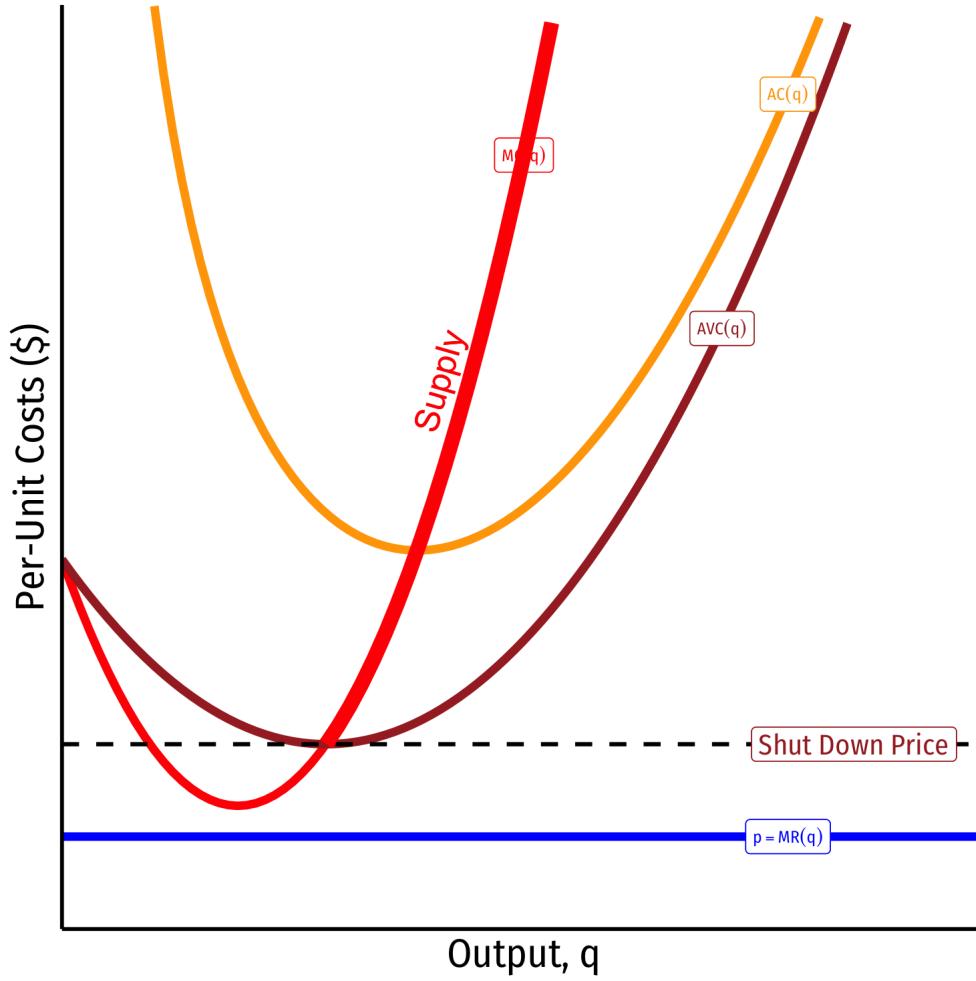
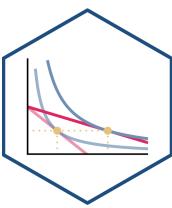
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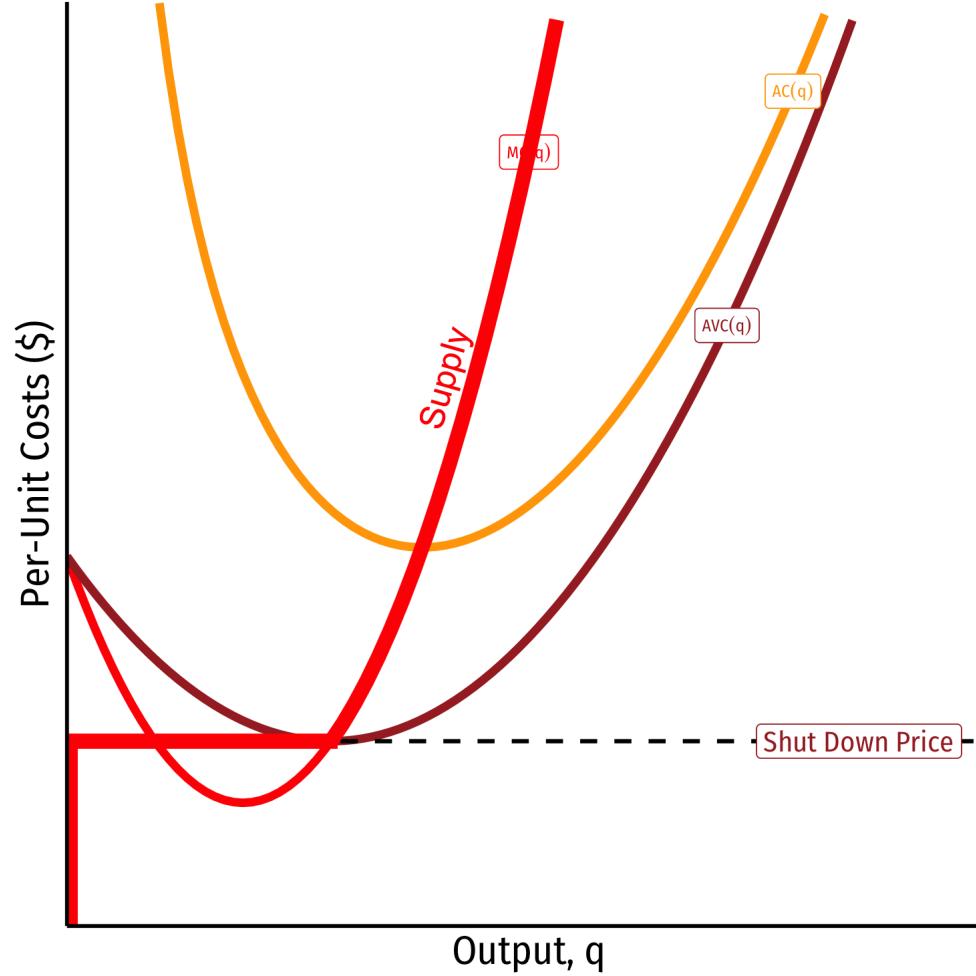
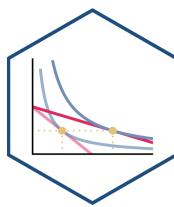
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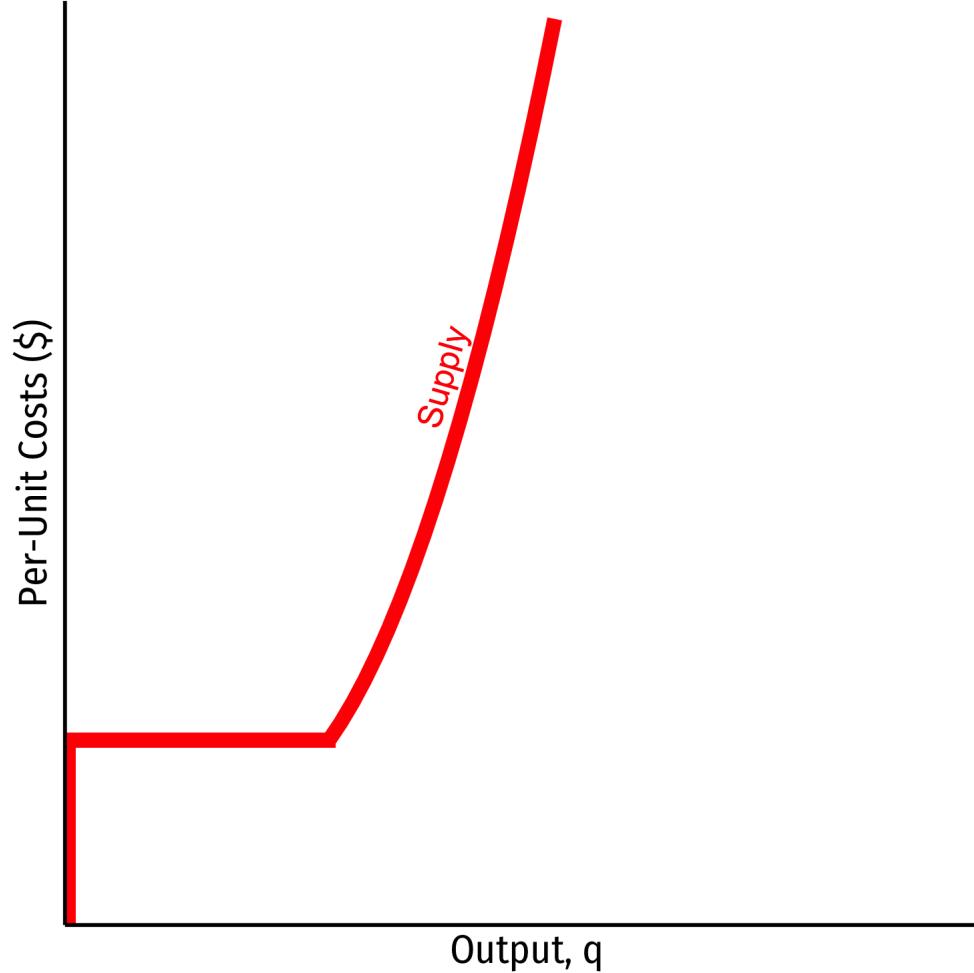
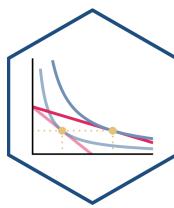
The Firm's Short Run Supply Decision



Firm's short run supply curve:

$$\begin{cases} p = MC(q) & \text{if } p \geq AVC \\ q = 0 & \text{If } p < AVC \end{cases}$$

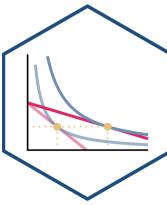
The Firm's Short Run Supply Decision



Firm's short run supply curve:

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Summary:



1. Choose q^* such that $MR(q) = MC(q)$

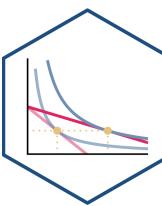
2. Profit $\pi = q[p - AC(q)]$

3. Shut down if $p < AVC(q)$

Firm's short run (inverse) supply:

$$\begin{cases} p = MC(q) & \text{if } p \geq AVC \\ q = 0 & \text{If } p < AVC \end{cases}$$

Choosing the Profit-Maximizing Output q^* : Example



Example: Bob's barbershop gives haircuts in a very competitive market, where barbers cannot differentiate their haircuts. The current market price of a haircut is \$15. Bob's daily short run costs are given by:

$$C(q) = 0.5q^2$$
$$MC(q) = q$$

1. How many haircuts per day would maximize Bob's profits?
2. How much profit will Bob earn per day?
3. Find Bob's shut down price.