

1.3 — Preferences

ECON 306 • Microeconomic Analysis • Spring 2022

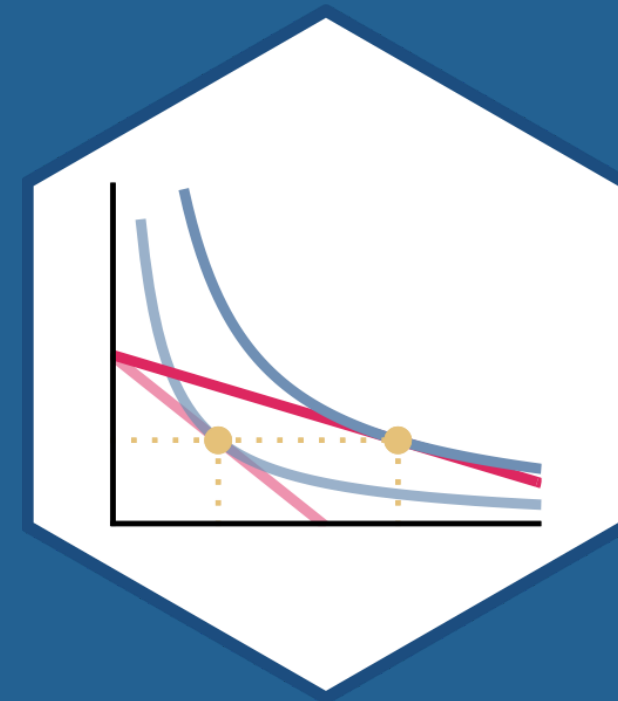
Ryan Safner

Assistant Professor of Economics

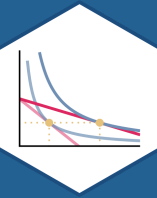
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Outline



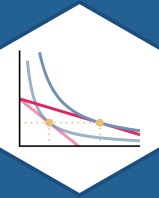
Preferences

Indifference Curves

Marginal Rate of Substitution

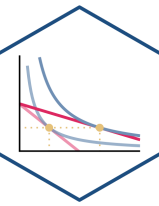
Utility

Marginal Utility



Preferences

Preferences I



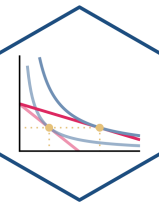
- Which bundles are **preferred** over others?

Example: Between two bundles of (x, y) :

$$a = (4, 12) \text{ or } b = (6, 12)$$



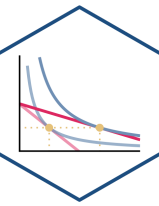
Preferences II



- We will allow **three possible answers**:



Preferences II

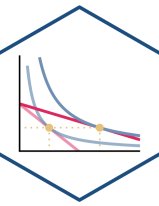


- We will allow **three possible answers**:

1. $a \succ b$: (Strictly) prefer a over b



Preferences II

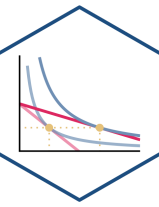


- We will allow **three possible answers**:

1. $a \succ b$: (Strictly) prefer a over b
2. $a \prec b$: (Strictly) prefer b over a



Preferences II

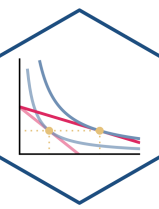


- We will allow **three possible answers**:

1. $a \succ b$: (Strictly) prefer a over b
2. $a \prec b$: (Strictly) prefer b over a
3. $a \sim b$: Indifferent between a and b



Preferences II



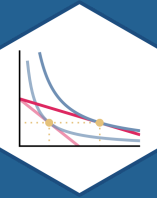
- We will allow **three possible answers**:

1. $a \succ b$: (Strictly) prefer a over b
2. $a \prec b$: (Strictly) prefer b over a
3. $a \sim b$: Indifferent between a and b

- **Preferences** are a list of all such comparisons between all bundles

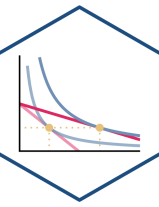
See appendix in [today's class page](#) for more.





Indifference Curves

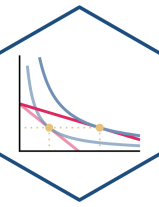
Mapping Preferences Graphically I



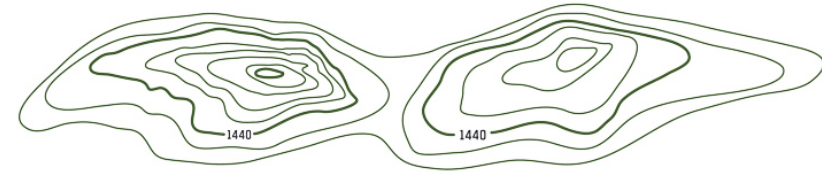
- For each bundle, we now have 3 pieces of information:
 - amount of x
 - amount of y
 - preference compared to other bundles
- How to represent this information graphically?



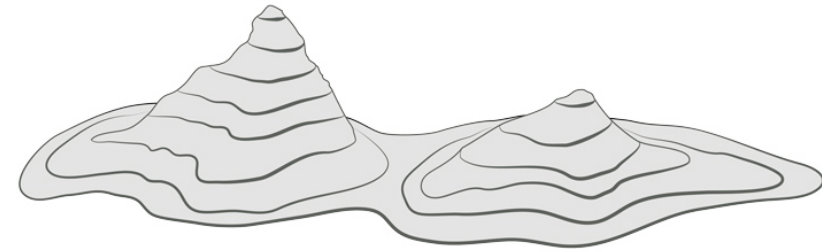
Mapping Preferences Graphically II



- Cartographers have the answer for us
- On a map, **contour lines** link areas of **equal height**
- We will use “**indifference curves**” to link bundles of **equal preference**

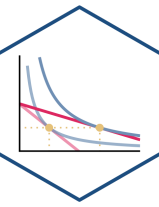


WHAT YOU SEE
ON YOUR MAP

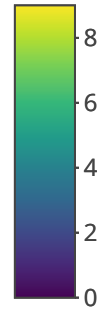


3-D VIEW
OF LANDMARK

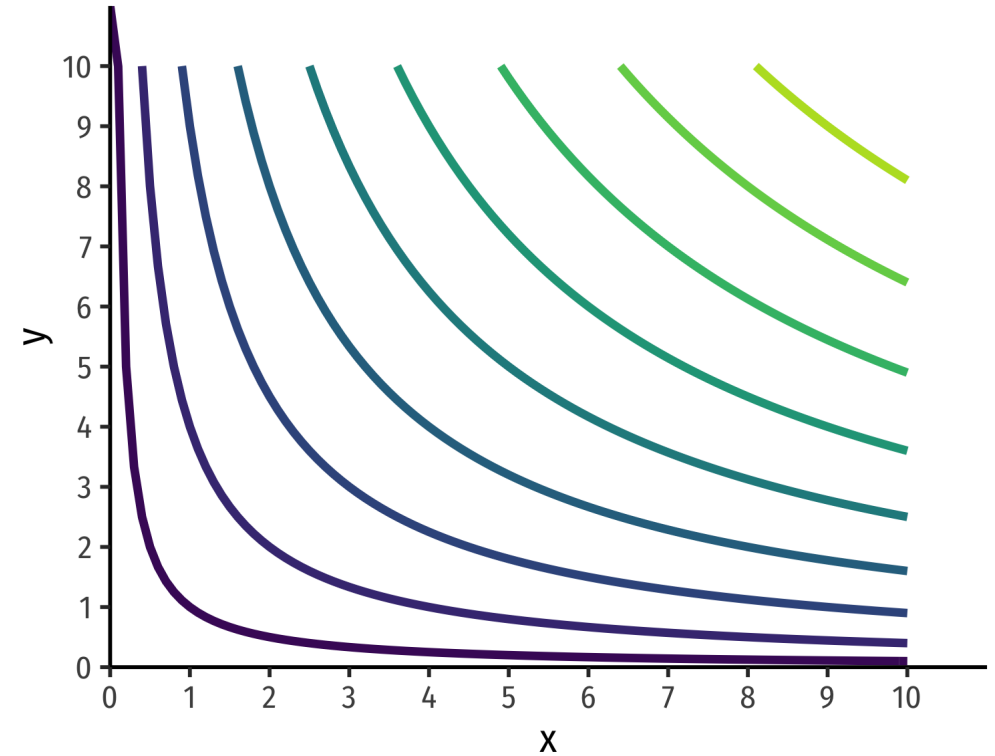
Mapping Preferences Graphically III



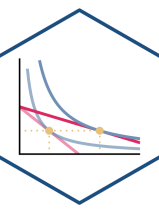
3-D “Mount Utility”



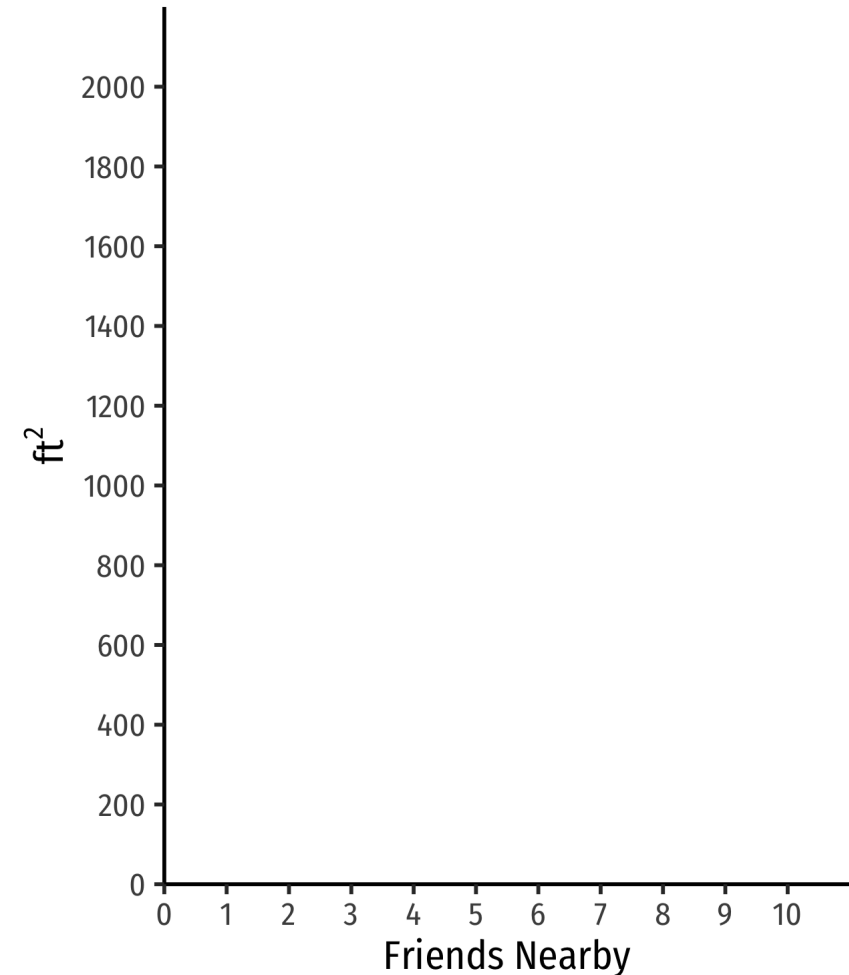
2-D Indifference Curve Contours



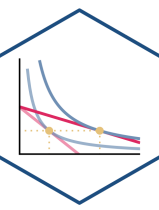
Indifference Curves: Example



Example: Suppose you are hunting for an apartment. You value *both* the size of the apartment and the number of friends that live nearby.

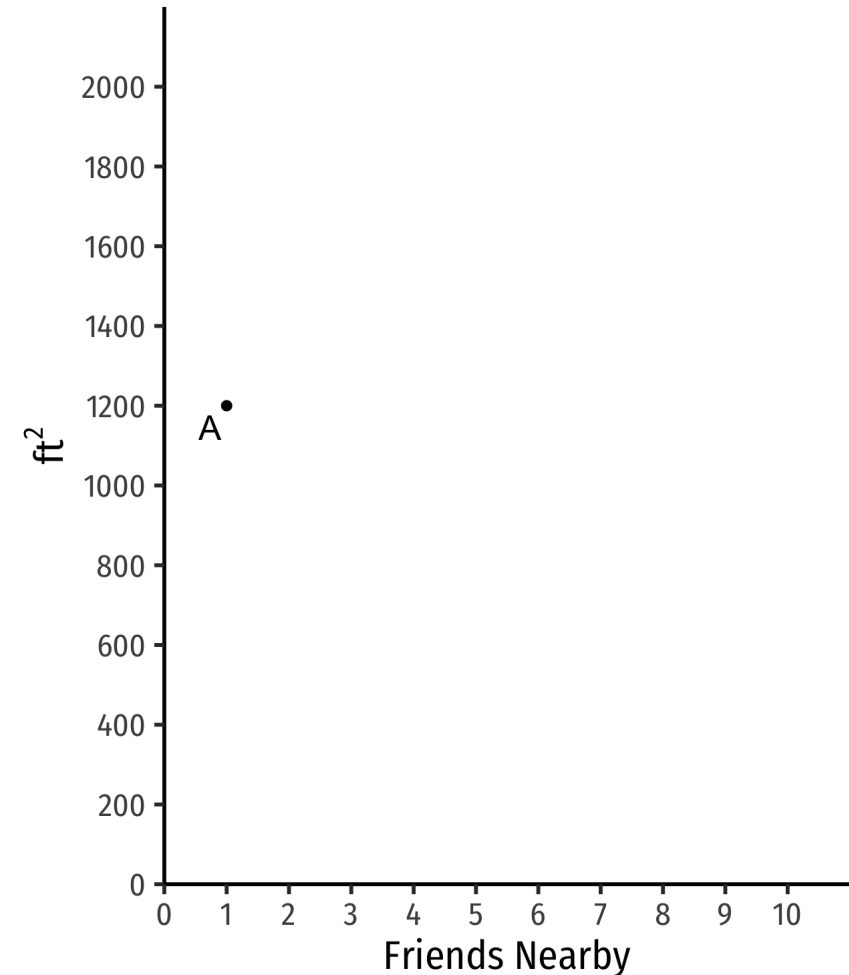


Indifference Curves: Example

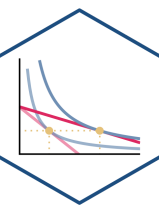


Example: Suppose you are hunting for an apartment. You value *both* the size of the apartment and the number of friends that live nearby.

- Apt. A has 1 friend nearby and is 1,200 ft^2

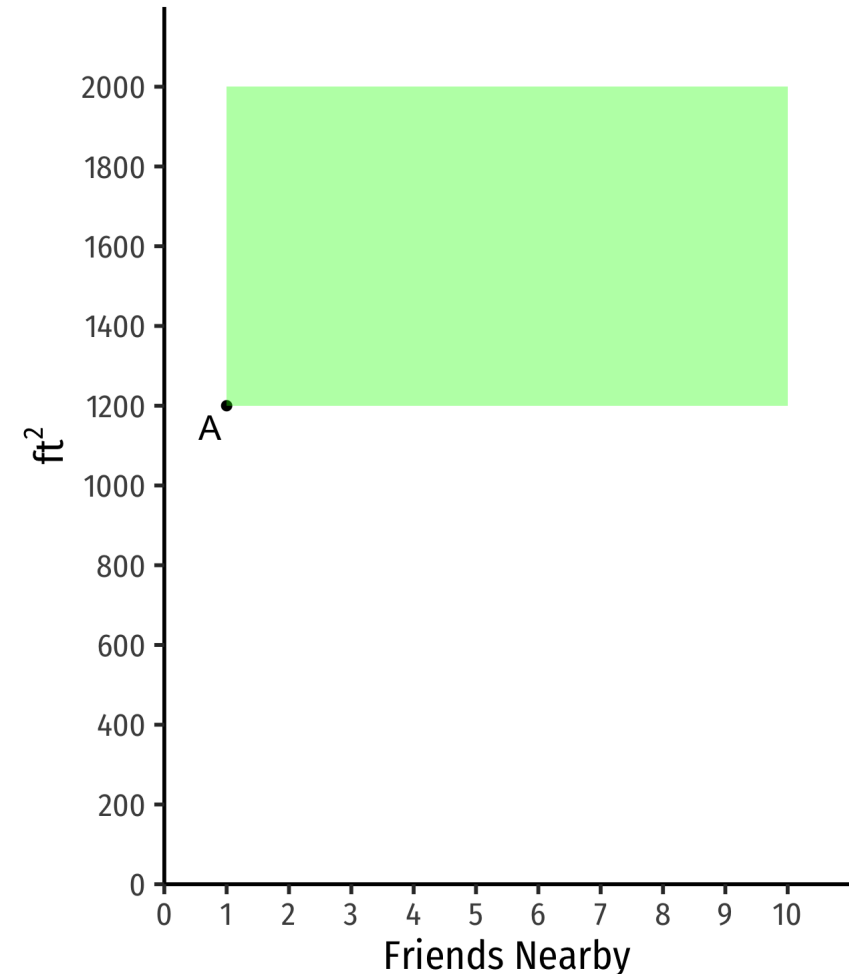


Indifference Curves: Example

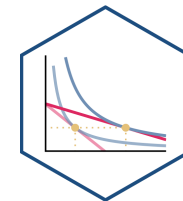


Example: Suppose you are hunting for an apartment. You value *both* the size of the apartment and the number of friends that live nearby.

- Apt. *A* has 1 friend nearby and is 1,200 ft^2
 - Apts that are larger and/or have more friends $\succ A$

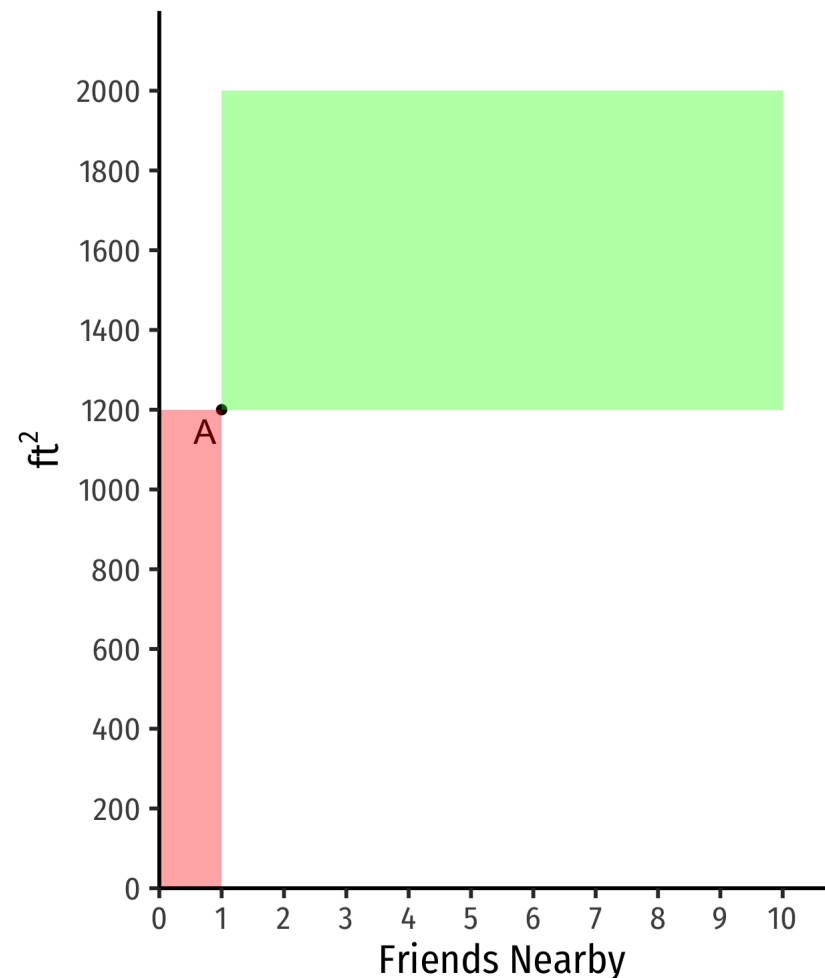


Indifference Curves: Example

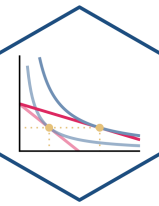


Example: Suppose you are hunting for an apartment. You value *both* the size of the apartment and the number of friends that live nearby.

- Apt. *A* has 1 friend nearby and is 1,200 ft^2
 - Apts that are larger and/or have more friends $\succ A$
 - Apts that are smaller and/or have fewer friends $\prec A$

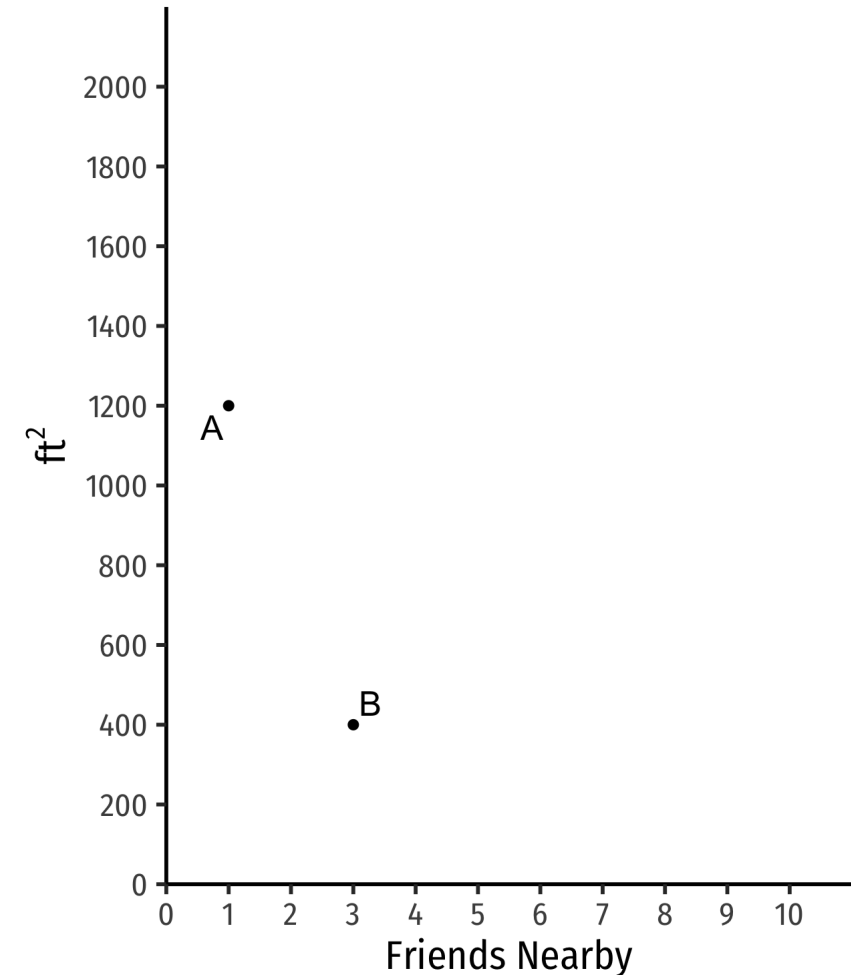


Indifference Curves: Example

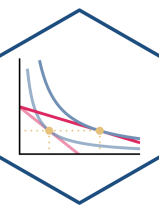


Example:

- Apt. *A* has 1 friend nearby and is 1,200 ft^2
- *B* has *more* friends but *less* ft^2

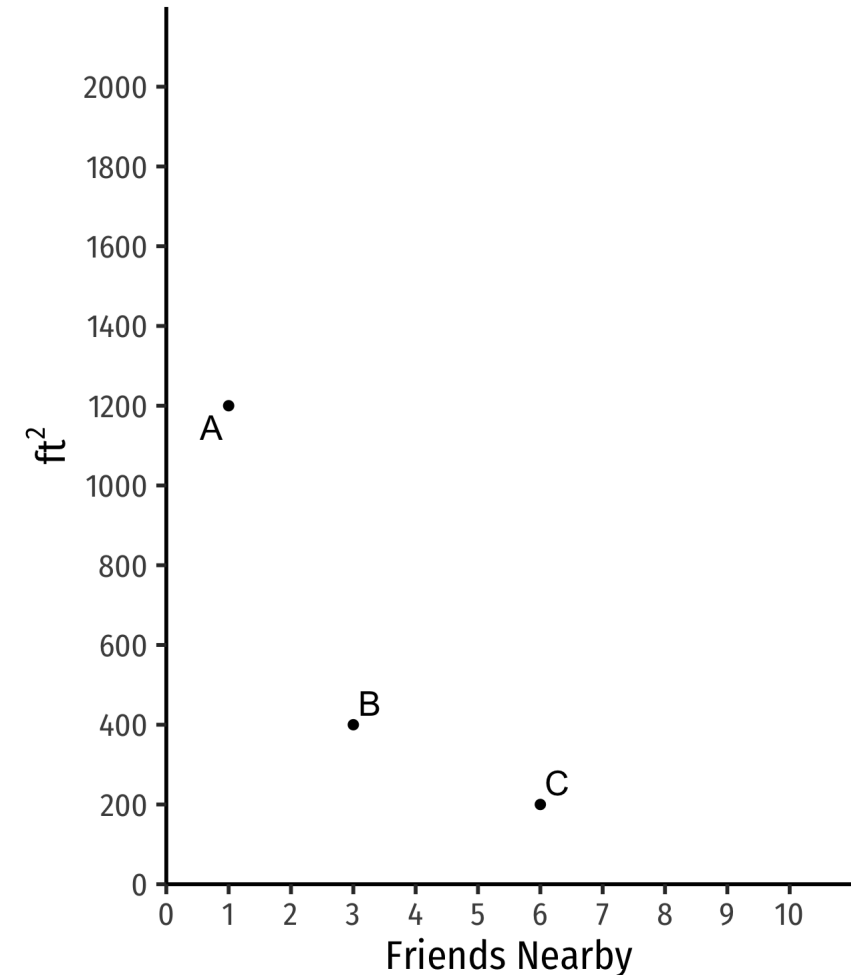


Indifference Curves: Example

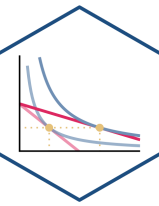


Example:

- Apt. *A* has 1 friend nearby and is 1,200 ft^2
- *B* has *more* friends but *less* ft^2
- *C* has *still more* friends but *less* ft^2

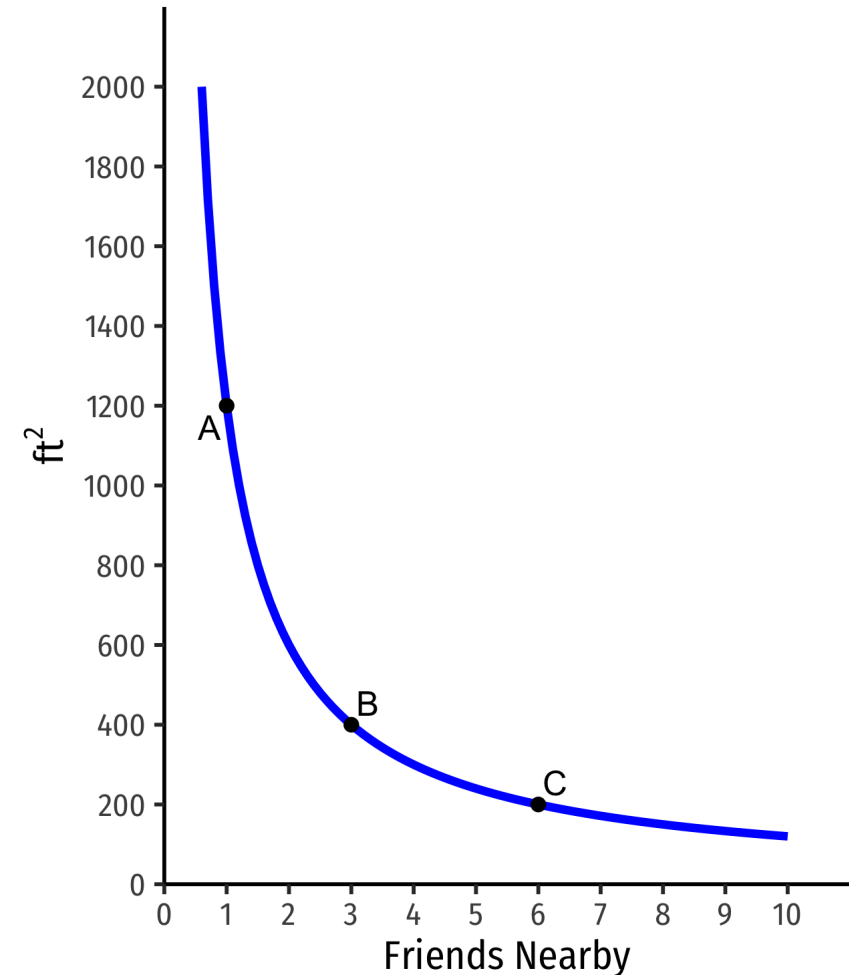


Indifference Curves: Example

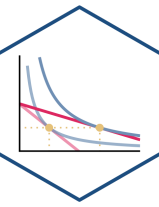


Example:

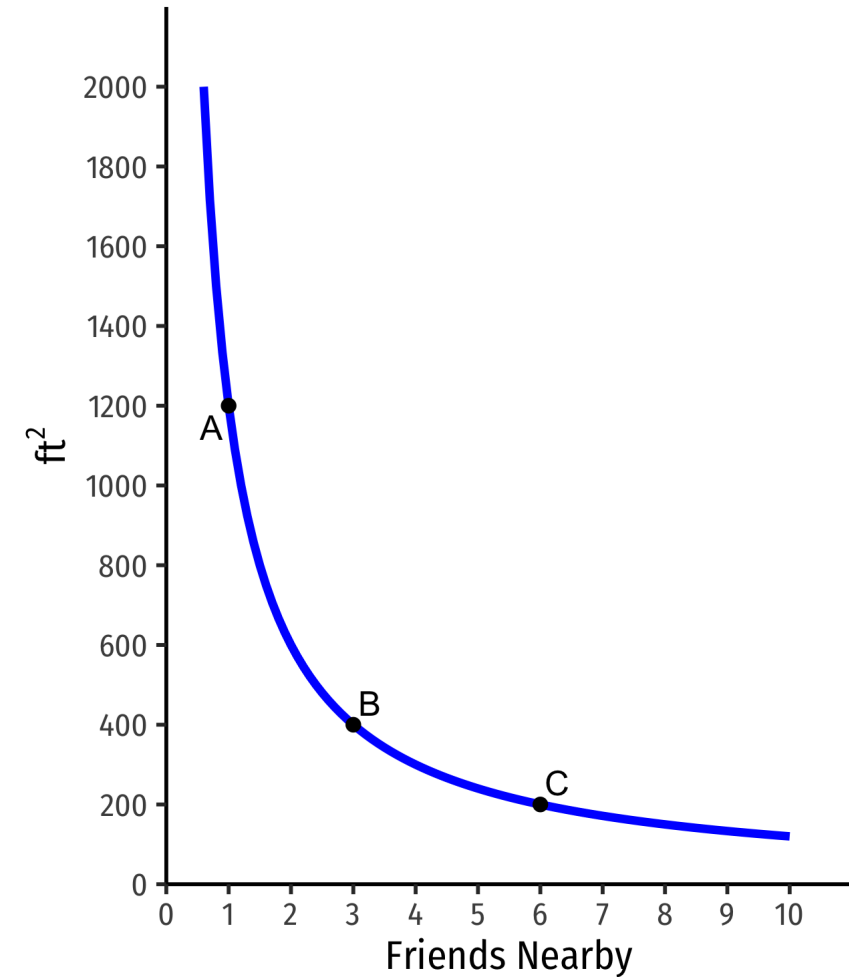
- Apt. *A* has 1 friend nearby and is 1,200 ft^2
- *B* has *more* friends but *less* ft^2
- *C* has *still more* friends but *less* ft^2
- $A \sim B \sim C$: on same **indifference curve**



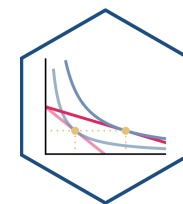
Indifference Curves: Example



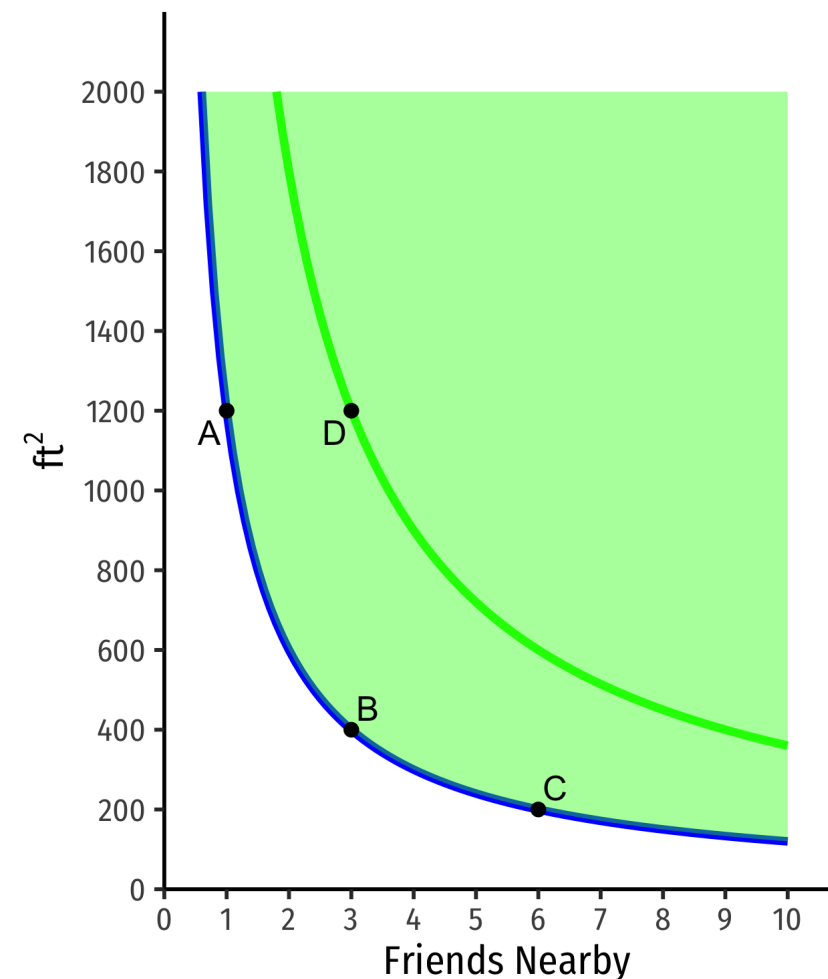
- **Indifferent** between all apartments on the **same** curve



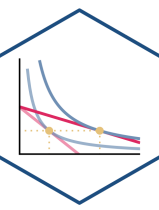
Indifference Curves: Example



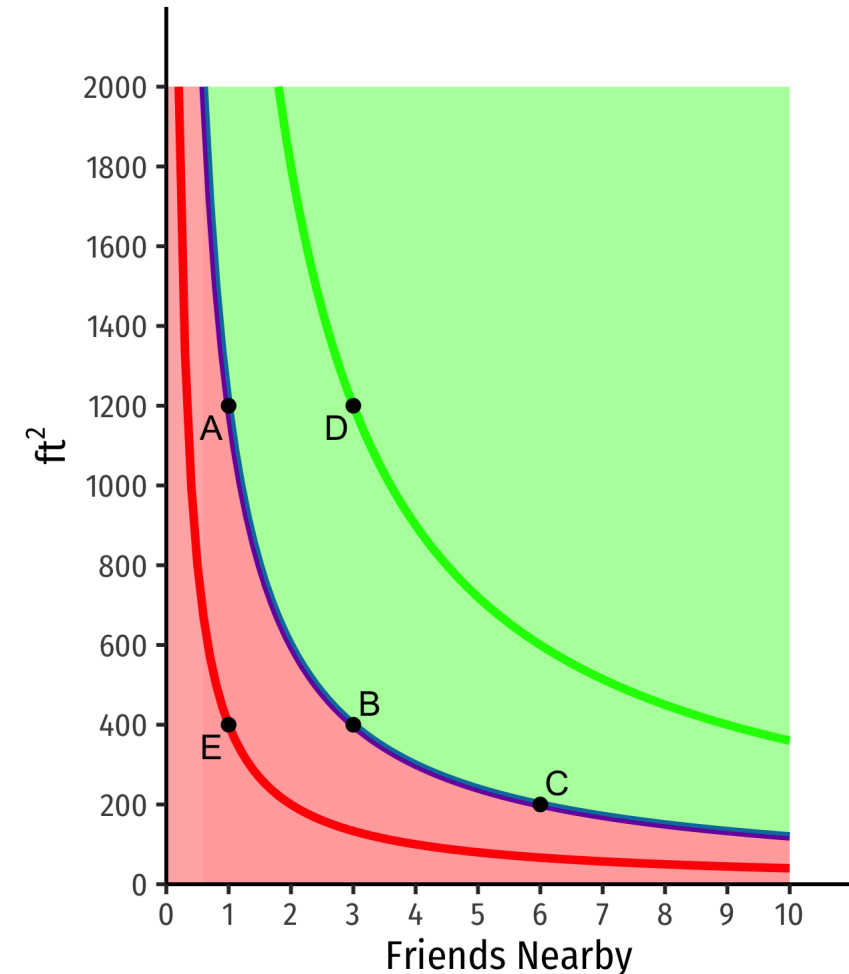
- **Indifferent** between all apartments on the **same** curve
- Apts **above** curve are **preferred over** apts on curve
 - $D \succ A \sim B \sim C$
 - On a **higher curve**



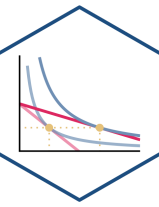
Indifference Curves: Example



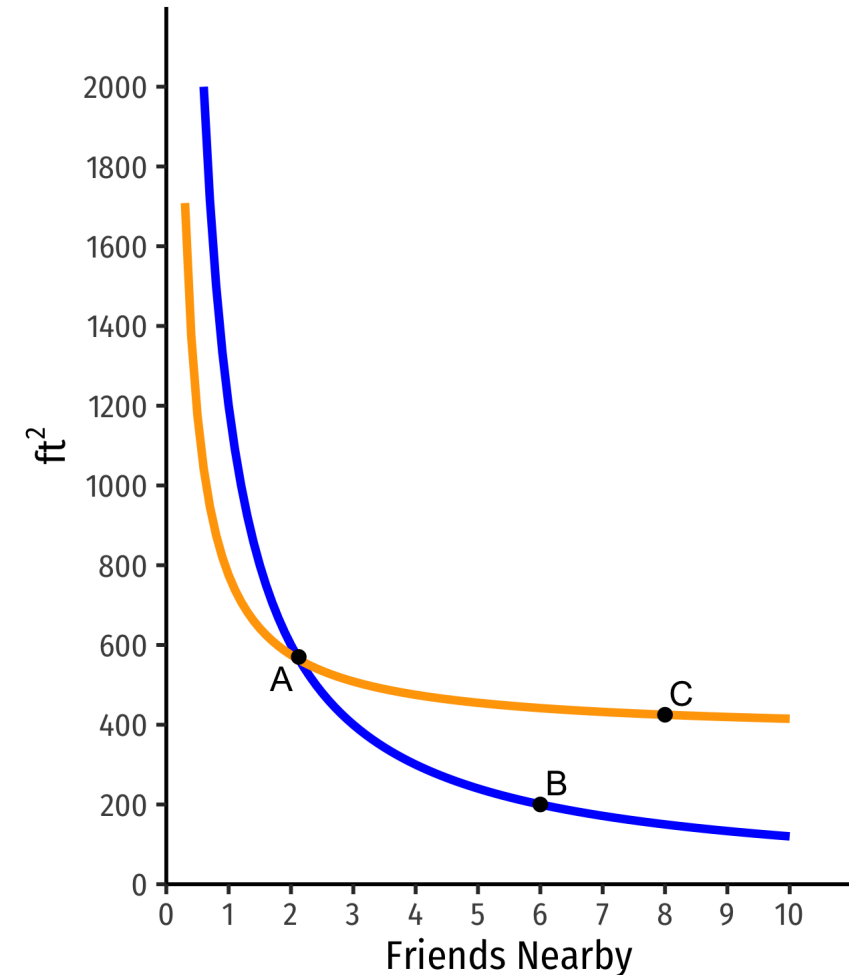
- **Indifferent** between all apartments on the **same** curve
- Apts **above** curve are **preferred over** apts on curve
 - $D \succ A \sim B \sim C$
 - On a **higher curve**
- Apts **below** curve are **less preferred** than apts on curve
 - $E \prec A \sim B \sim C$
 - On a **lower curve**



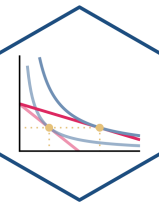
Curves Never Cross!



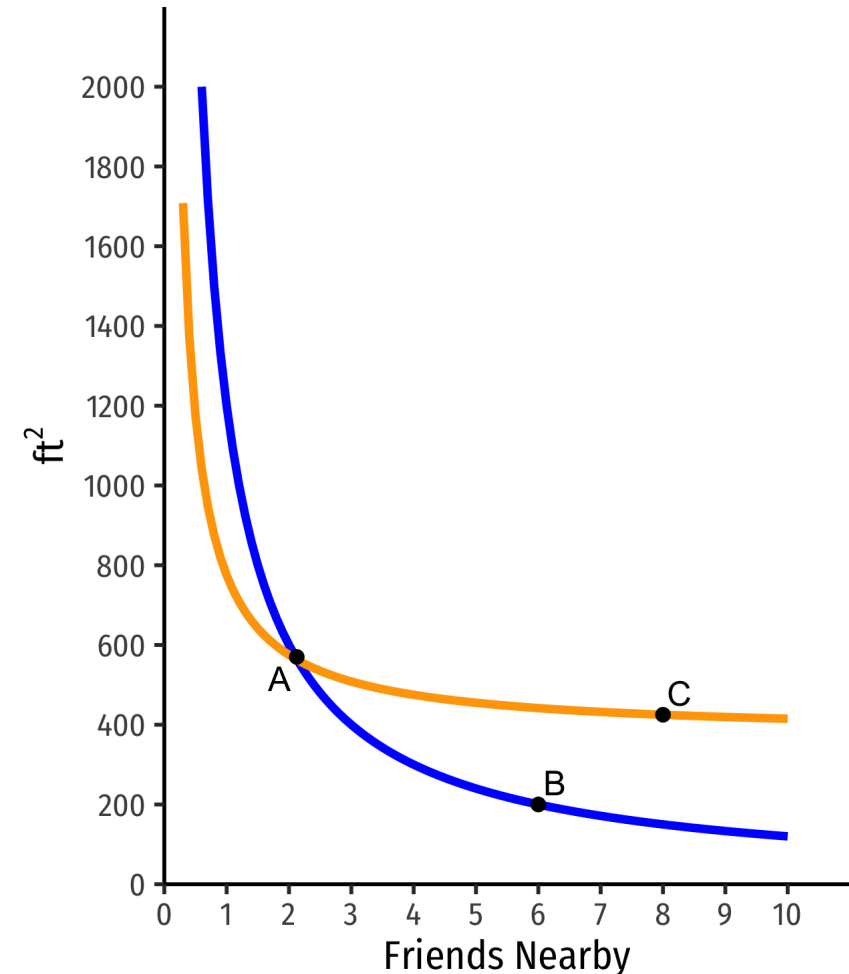
- Indifference curves can never cross:
preferences are **transitive**
 - If I prefer $A \succ B$, and $B \succ C$, I must prefer $A \succ C$

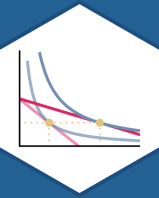


Curves Never Cross!



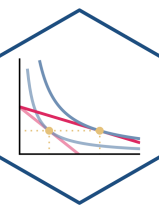
- Indifference curves can never cross:
preferences are **transitive**
 - If I prefer $A \succ B$, and $B \succ C$, I must prefer $A \succ C$
- Suppose two curves crossed:
 - $A \sim B$
 - $B \sim C$
 - But $C \succ B$!
 - Doesn't make sense (not transitive)!





Marginal Rate of Substitution

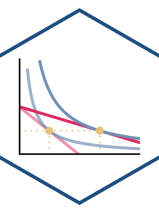
Marginal Rate of Substitution I



- If I find another apt with *1 fewer friend* nearby, how many *more ft^2* would you need to keep you *satisfied*?



Marginal Rate of Substitution I

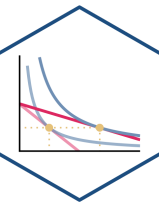


- If I find another apt with *1 fewer friend* nearby, how many *more ft²* would you need to keep you *satisfied*?
- **Marginal Rate of Substitution (MRS)**: rate at which you trade away one good for more of the other and remain *indifferent*
- Think of this as the **relative value** you place on good x :

“I am willing to give up (MRS) units of y to consume 1 more unit of x and stay satisfied.”



Marginal Rate of Substitution II

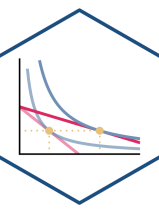


SLOPE



**MARGINAL RATE OF
SUBSTITUTION**

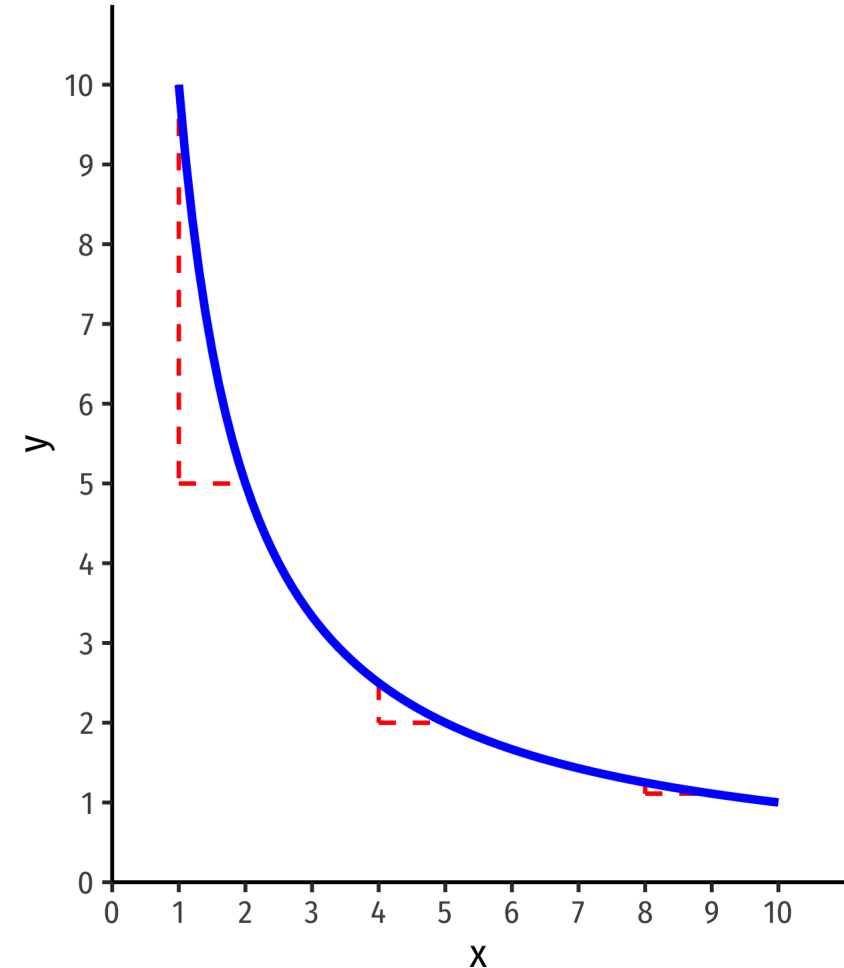
Marginal Rate of Substitution II



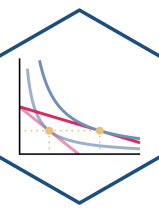
- $MRS =$ **slope of the indifference curve**

$$MRS_{x,y} = -\frac{\Delta y}{\Delta x} = \frac{rise}{run}$$

- Amount of y given up for 1 more x
- Note: slope (MRS) changes along the curve!

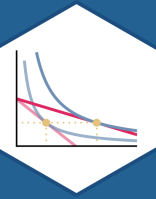


MRS vs. Budget Constraint Slope



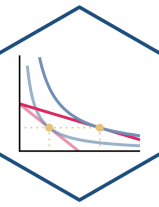
- Budget constraint (slope) from before measured the **market's** tradeoff between x and y based on market prices
- **MRS** here measures your **personal** evaluation of x vs. y based on your preferences
- Foreshadowing: what if these two rates are *different*? Are you truly optimizing?





Utility

So Where are the Numbers?

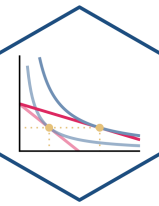


- Long ago (1890s), utility considered a real, measurable, cardinal scale[†]
- Utility thought to be lurking in people's brains
 - Could be understood from first principles: calories, water, warmth, etc
- Obvious problems



[†] “Neuroeconomics” & cognitive scientists are re-attempting a scientific approach to measure utility

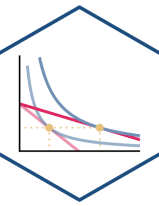
Utility Functions?



- More plausibly **infer people's preferences from their actions!**
 - “Actions speak louder than words”
- **Principle of Revealed Preference:** if a person chooses x over y , and both are affordable, then they must prefer $x \succeq y$
- Flawless? Of course not. But extremely useful approximation!
 - People tend not to leave money on the table



Utility Functions!



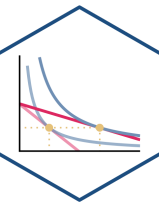
- A **utility function** $u(\cdot)^{\dagger}$ represents preference relations (\succ, \prec, \sim)
- Assign utility numbers to bundles, such that, for any bundles a and b :

$$a \succ b \iff u(a) > u(b)$$



[†] The \cdot is a placeholder for whatever goods we are considering (e.g. x , y , burritos, lattes, etc)

Utility Functions, Pural I



Example: Imagine three alternative bundles of (x, y) :

$$a = (1, 2)$$

$$b = (2, 2)$$

$$c = (4, 3)$$

- Let $u(\cdot)$ assign each bundle a utility of:

$$u(\cdot)$$

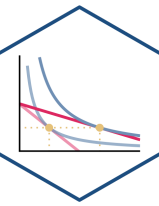
$$u(a) = 1$$

$$u(b) = 2$$

$$u(c) = 3$$

- Does this mean that bundle c is 3 times the utility of a ?

Utility Functions, Pural II



Example: Imagine three alternative bundles of (x, y) :

$$a = (1, 2)$$

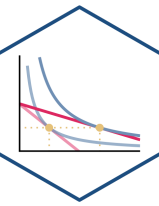
$$b = (2, 2)$$

$$c = (4, 3)$$

- Now consider a 2nd function $v(\cdot)$:

$u(\cdot)$	$v(\cdot)$
$u(a) = 1$	$v(a) = 3$
$u(b) = 2$	$v(b) = 5$
$u(c) = 3$	$v(c) = 7$

Utility Functions, Pural III

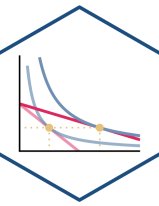


- Utility numbers have an **ordinal** meaning only, **not cardinal**
- Both are valid utility functions:[†]
 - $u(c) > u(b) > u(a)$ ✓
 - $v(c) > v(b) > v(a)$ ✓
 - because $c \succ b \succ a$
- **Only the ranking of utility numbers matters!**



[†] See the Mathematical Appendix in [Today's Class Page](#) for why.

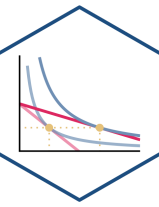
Utility Functions and Indifference Curves I



- Two tools to represent preferences:
indifference curves and **utility functions**
- Indifference curve: all **equally preferred** bundles \iff **same utility level**
- Each indifference curve represents one level (or contour) of utility surface (function)

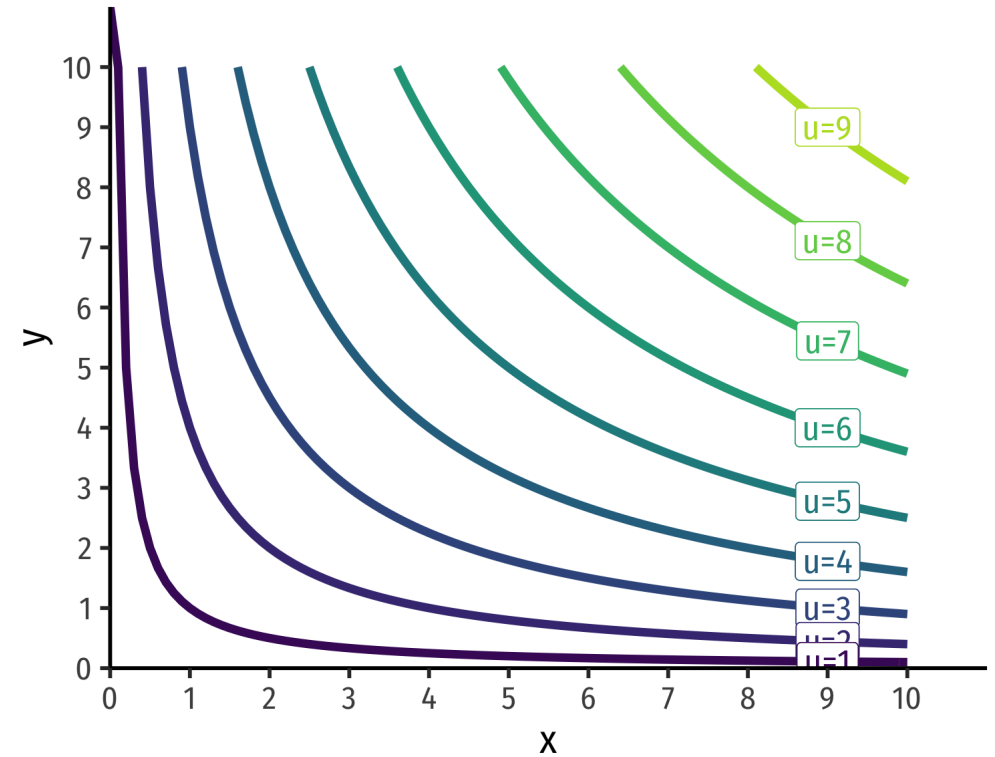
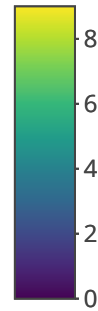


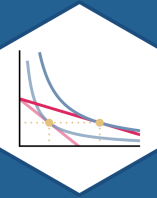
Utility Functions and Indifference Curves II



3-D Utility Function: $u(x, y) = \sqrt{xy}$

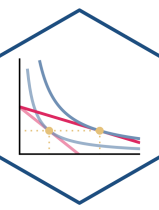
2-D Indifference Curve Contours: $y = \frac{u^2}{x}$



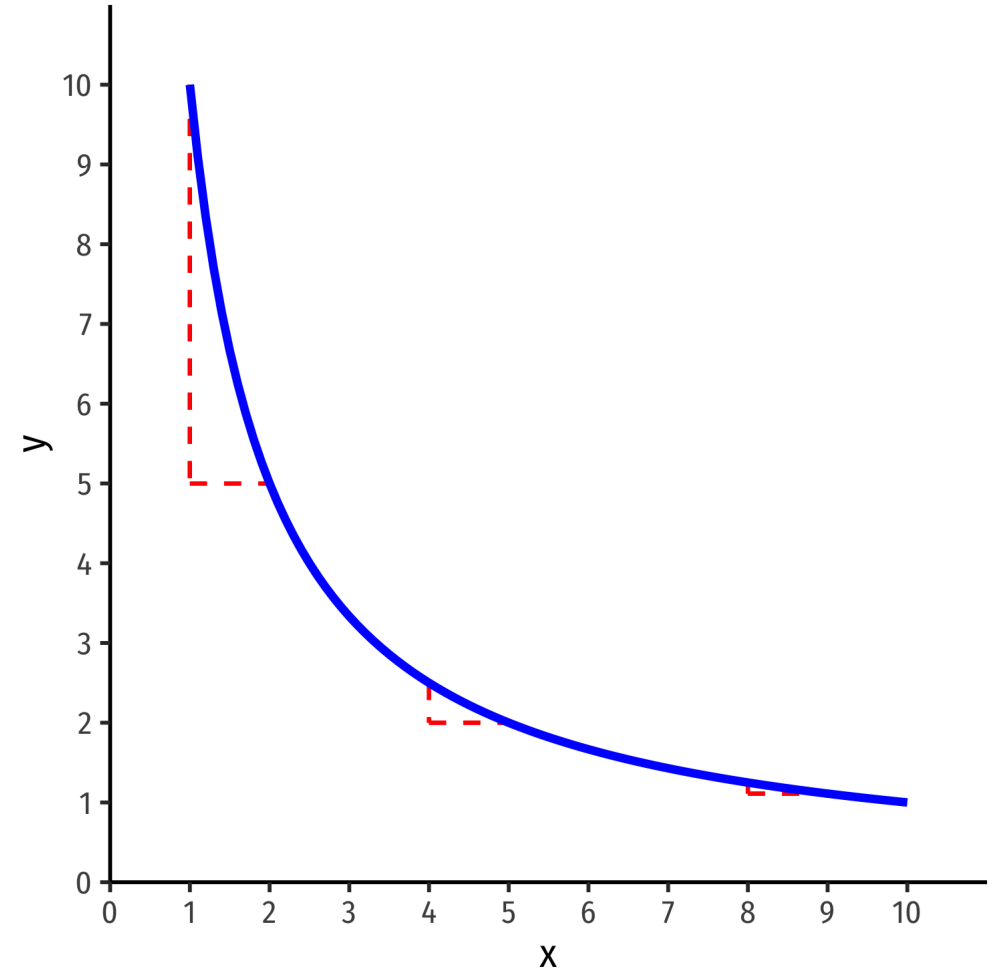


Marginal Utility

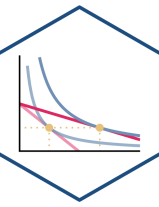
MRS and Marginal Utility I



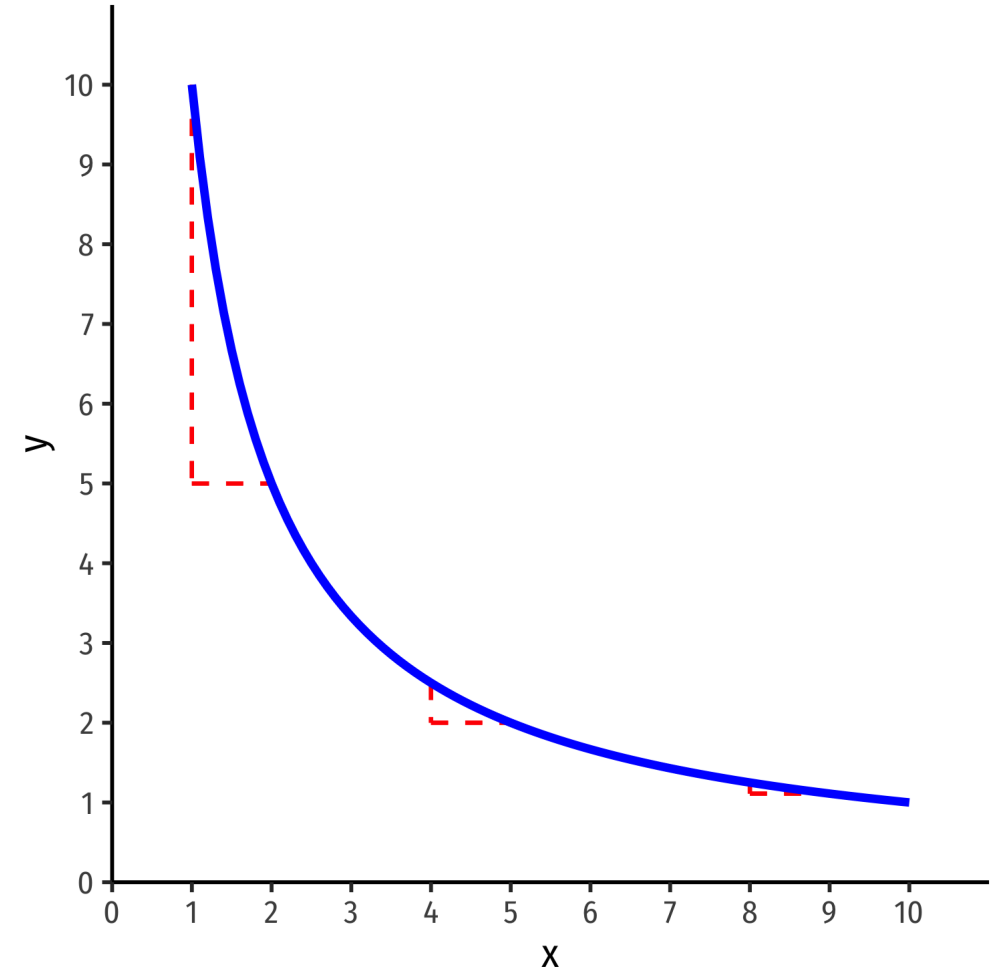
- Recall: **marginal rate of substitution**
 $MRS_{x,y}$ is slope of the indifference curve
 - Amount of y given up for 1 more x
- How to calculate MRS?
 - Recall it changes (not a straight line)!
 - We can calculate it using something from the **utility function**



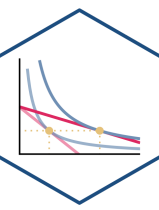
MRS and Marginal Utility II



- **Marginal utility**: change in utility from a marginal increase in consumption



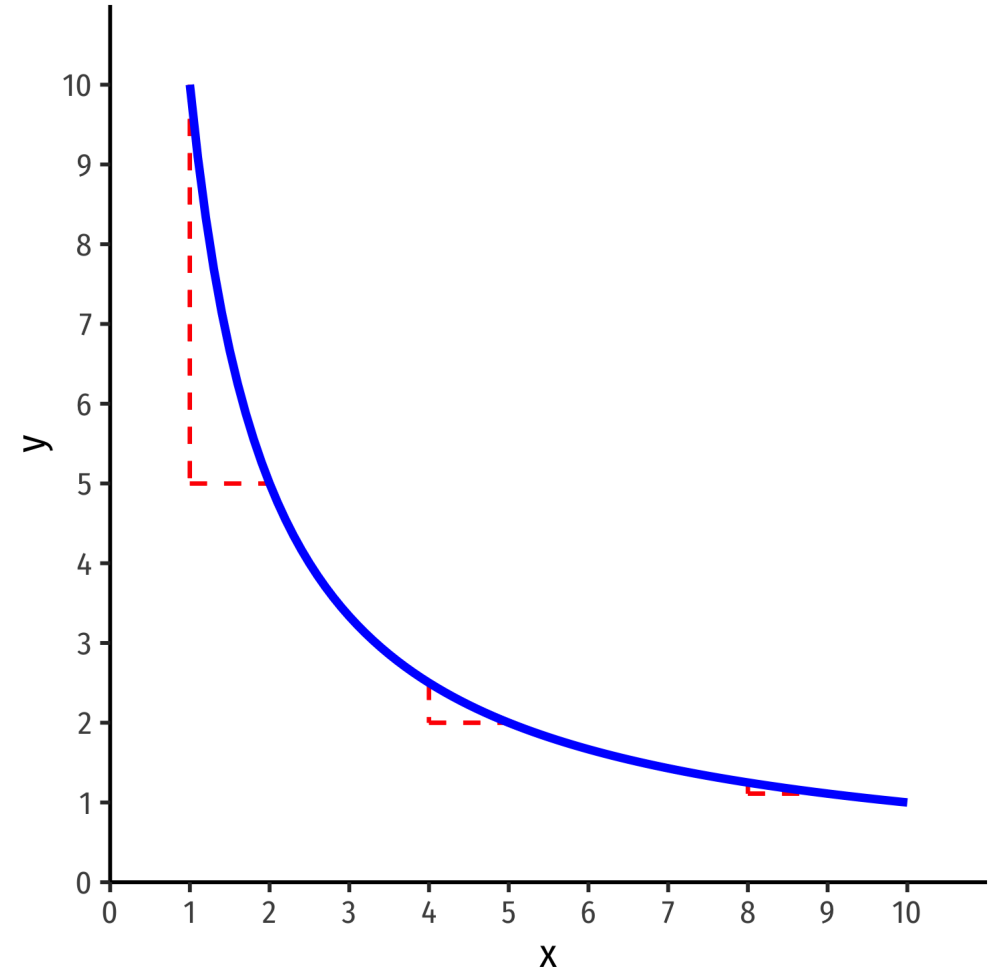
MRS and Marginal Utility II



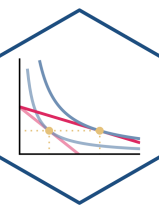
- **Marginal utility**: change in utility from a marginal increase in consumption

Marginal utility of x :

$$MU_x = \frac{\Delta u(x,y)}{\Delta x}$$



MRS and Marginal Utility II



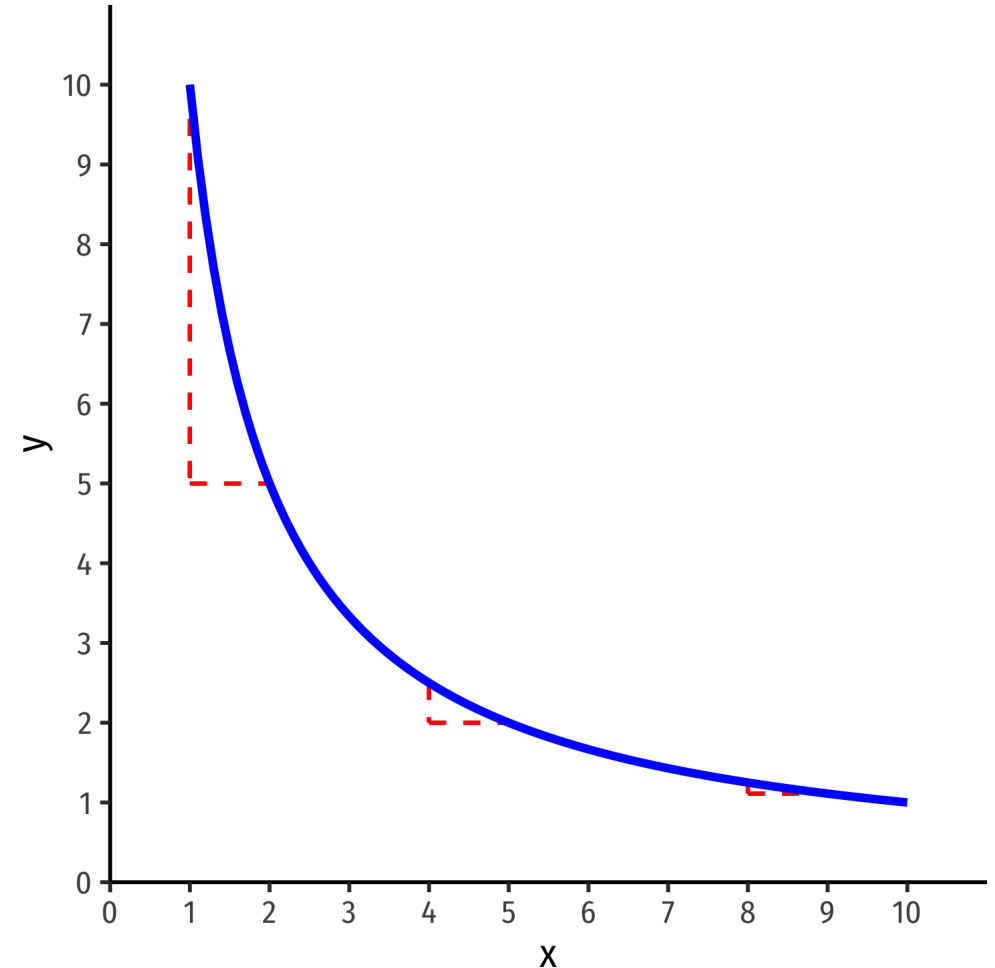
- **Marginal utility**: change in utility from a marginal increase in consumption

Marginal utility of x :

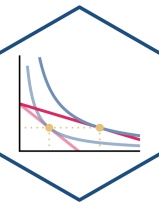
$$MU_x = \frac{\Delta u(x,y)}{\Delta x}$$

Marginal utility of y :

$$MU_y = \frac{\Delta u(x,y)}{\Delta y}$$



MRS and Marginal Utility II

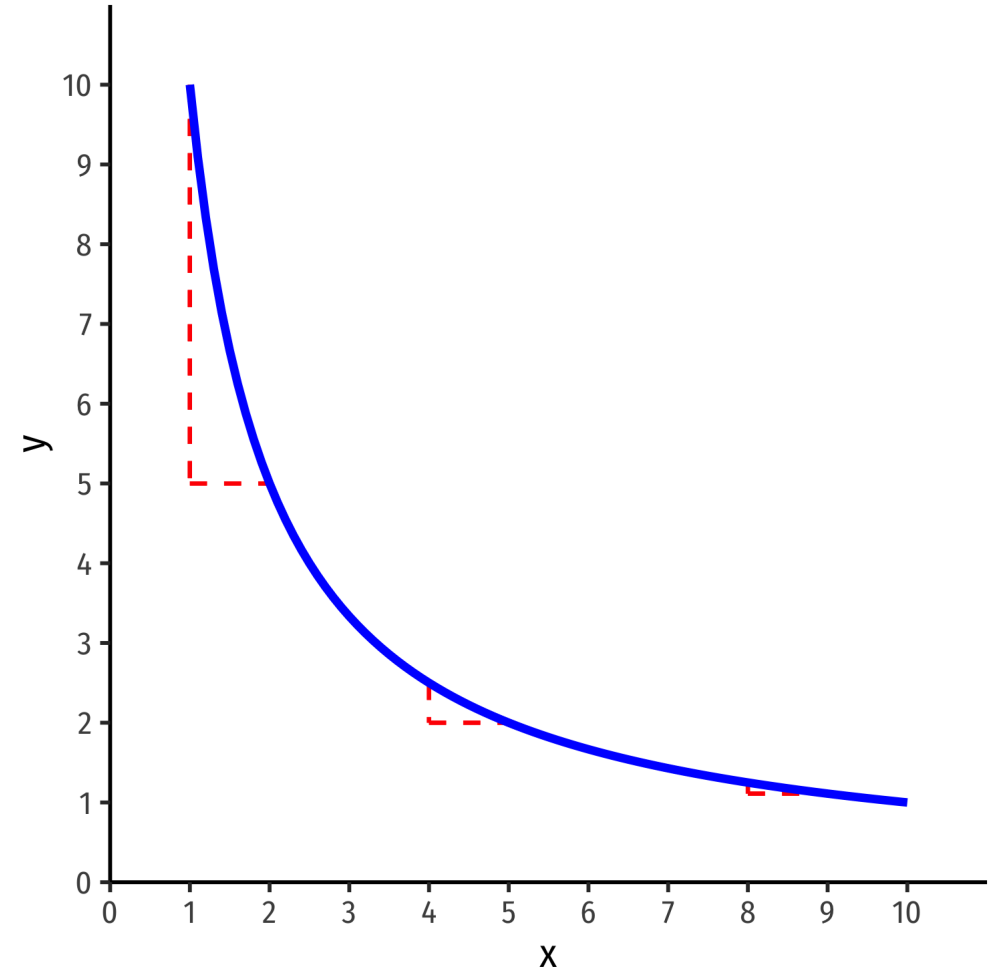


- **Marginal utility**: change in utility from a marginal increase in consumption

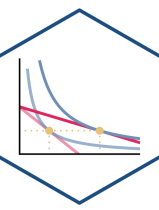
- **Math (calculus)**: “marginal” \iff “derivative with respect to”

$$MU_x = \frac{\partial u(x, y)}{\partial x}$$

- I will always derive marginal utility functions for you



MRS and Marginal Utility: Example

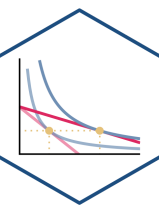


Example: For an example utility function:

$$u(x, y) = x^2 + y^3$$

- Marginal utility of x : $MU_x = 2x$
 - Marginal utility of y : $MU_y = 3y^2$
-
- Again, I will always derive marginal utility functions for you

MRS Equation and Marginal Utility

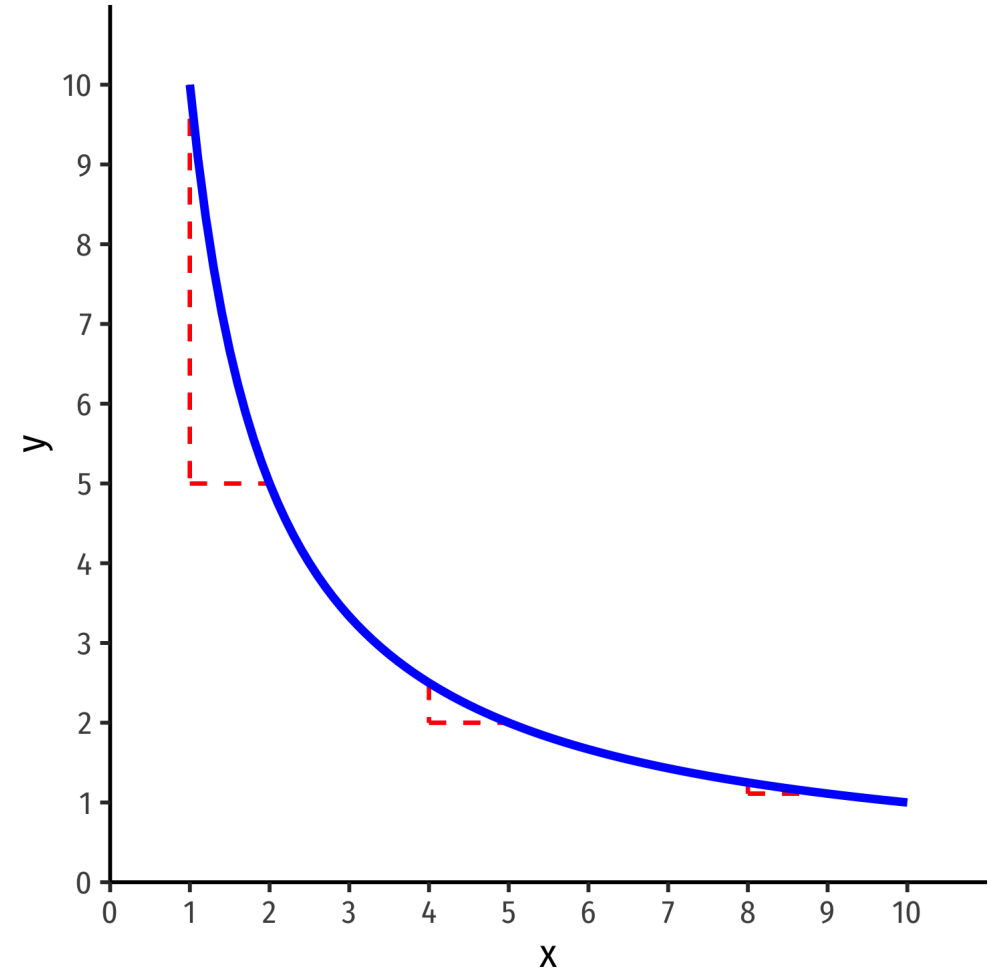


- Relationship between MU and MRS :

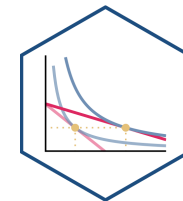
$$\underbrace{\frac{\Delta y}{\Delta x}}_{MRS} = - \frac{MU_x}{MU_y}$$

- See proof in [today's class notes](#)

“I am willing to give up $\frac{MU_x}{MU_y}$ units of y to consume 1 more unit of x and stay satisfied.”

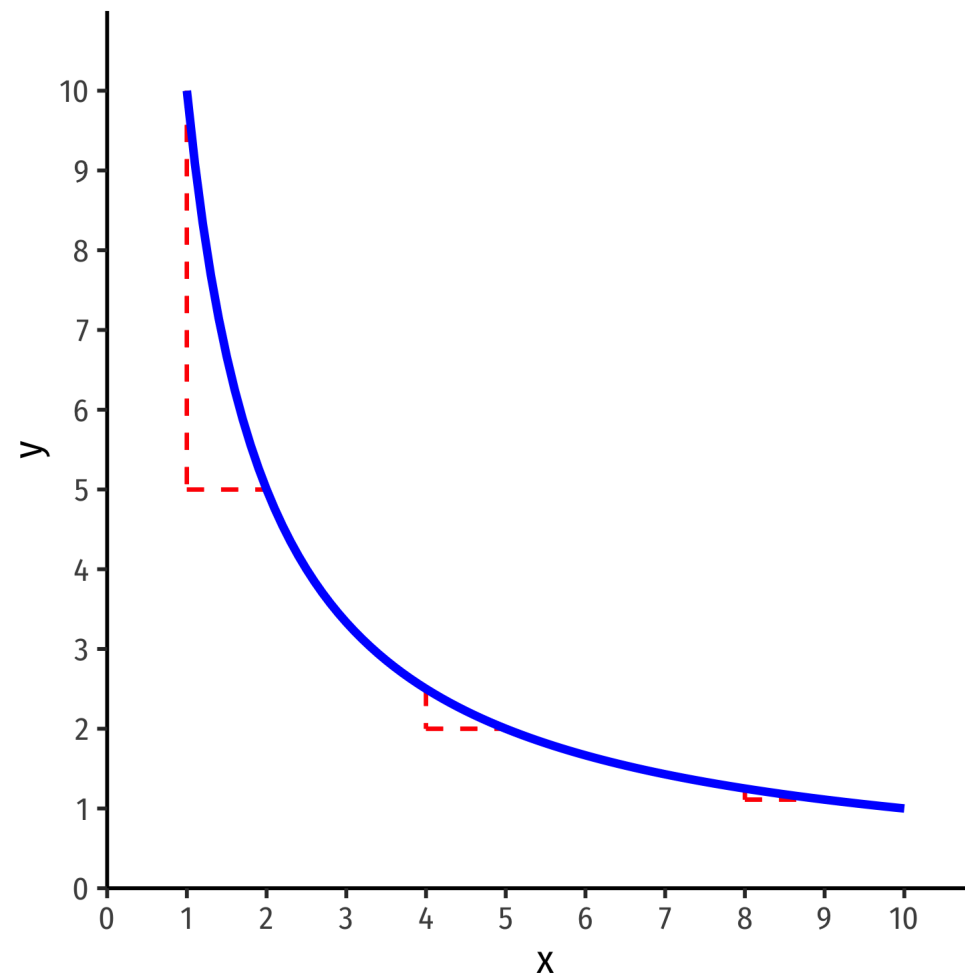


Important Insights About Value

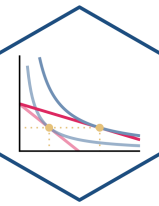


“I am willing to give up $\frac{MU_x}{MU_y}$ units of y to consume 1 more unit of x and stay satisfied.”

- We can't measure MU 's, but we *can* measure $MRS_{x,y}$ and infer the **ratio** of MU 's!
 - **Example:** if $MRS_{x,y} = 5$, a unit of good x gives 5 times the marginal utility of good y at the margin



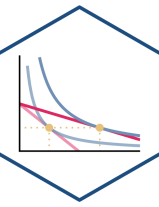
Important Insights About Value



- Value is **subjective**
 - Each of us has our own preferences that determine our ends or objectives
 - Choice is **forward looking**: a comparison of your **expectations** about opportunities
- **Preferences are not comparable across individuals**
 - Only individuals know what they give up at the moment of choice



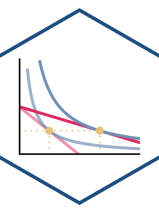
Important Insights About Value



- Value inherently comes from the fact that we must make **tradeoffs**
 - Making one choice means *having to give up* pursuing others!
 - The choice we pursue at the moment must be worth the sacrifice of others! (i.e. highest marginal utility)



Diminishing Marginal Utility



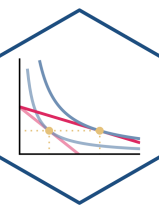
The Law of Diminishing Marginal Utility:

each marginal unit of a good consumed tends to provide less marginal utility than the previous unit, all else equal

- As you consume more x :
 - $\downarrow MU_x$
 - $\downarrow MRS_{x,y}$: willing to give up *fewer* units of y for x

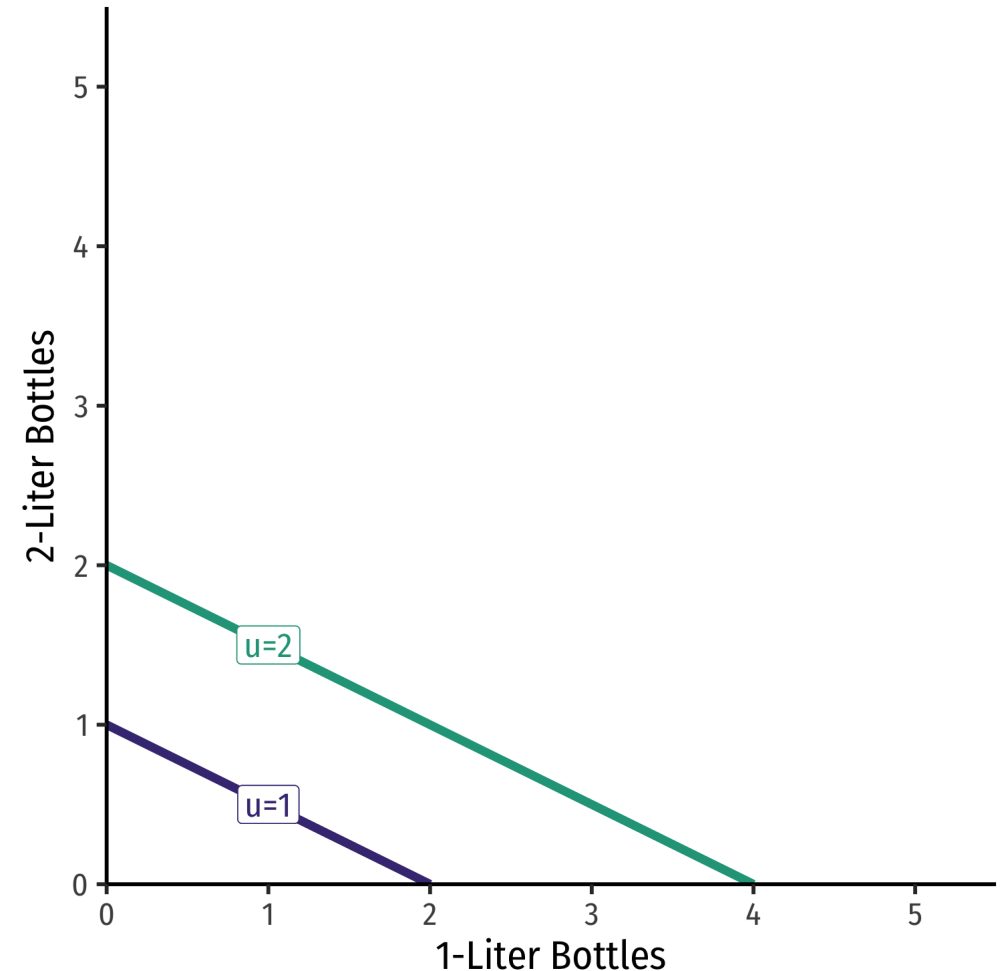


Special Case: Substitutes

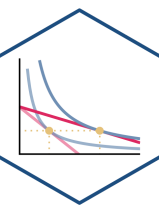


Example: Consider 1-Liter bottles of coke and 2-Liter bottles of coke

- Always willing to substitute between Two 1-L bottles for One 2-L bottle
- **Perfect substitutes:** goods that can be substituted at same fixed rate and yield same utility
- $MRS_{1L,2L} = -0.5$ (a constant!)

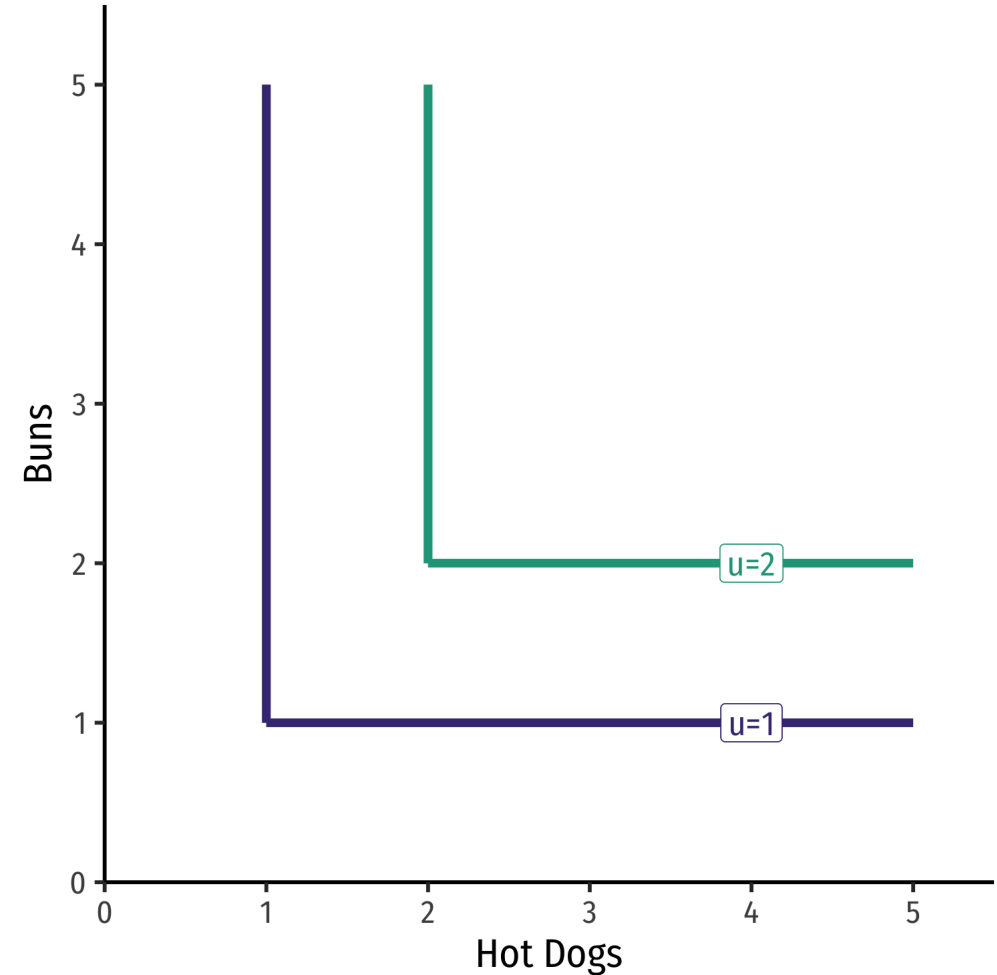


Special Case: Complements

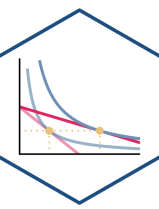


Example: Consider hot dogs and hot dog buns

- Always consume together in fixed proportions (in this case, 1 for 1)
- **Perfect complements:** goods that can be consumed together in same fixed proportion and yield same utility
- $MRS_{H,B} = ?$



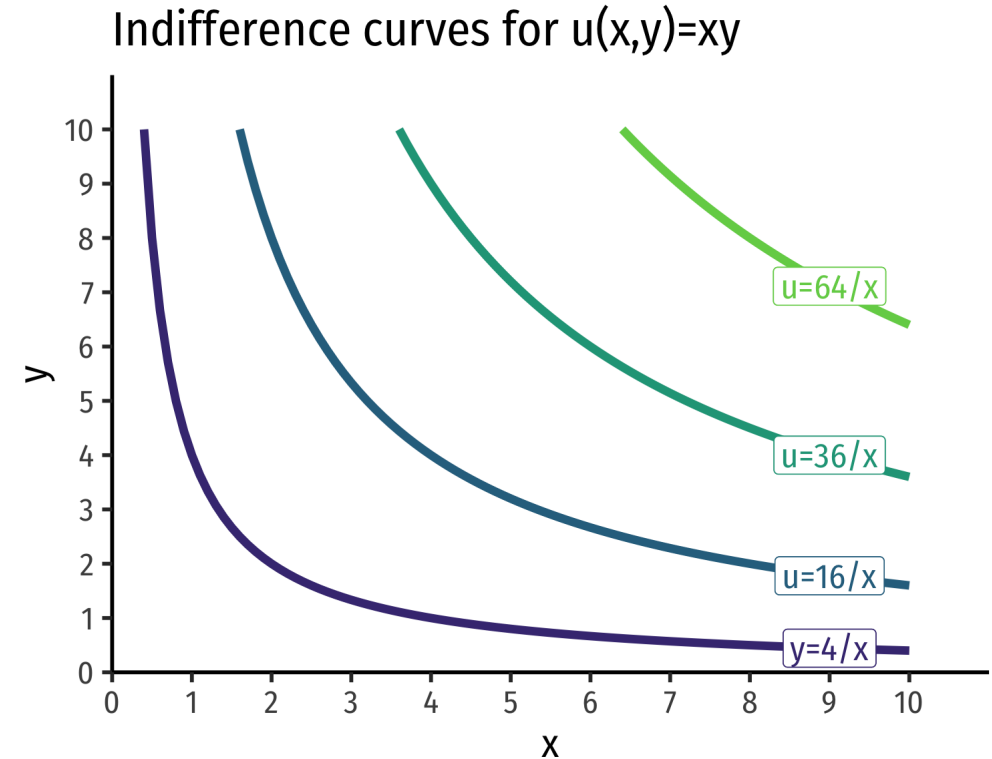
Cobb-Douglas Utility Functions



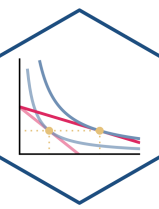
- A very common functional form in economics is **Cobb-Douglas**

$$u(x, y) = x^a y^b$$

- Extremely useful, you will see it often!
 - Lots of nice, useful properties (we'll see later)
 - See the appendix in [today's class page](#)



Practice



Example: Suppose you can consume apples (a) and broccoli (b), and earn utility according to:

$$u(a, b) = 2ab$$

$$MU_a = 2b$$

$$MU_b = 2a$$

1. Put a on the horizontal axis and b on the vertical axis. Write an equation for $MRS_{a,b}$.
2. Would you prefer a bundle of $(1, 4)$ or $(2, 2)$?
3. Suppose you are currently consuming 1 apple and 4 broccoli. a. How many units of broccoli are you willing to give up to eat 1 more apple and remain indifferent? b. How much *more* utility would you get if you were to eat 1 more apple?
4. Repeat question 3, but for when you are consuming 2 of each good.