

3.2 – Marginal Productivity Theory

ECON 452 • History of Economic Thought • Fall 2020

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 [ryansafner/thoughtF20](https://github.com/ryansafner/thoughtF20)

 thoughtF20.classes.ryansafner.com



Outline



Second Generation Austrian Marginalists

Marginal Productivity Theory.

Product Exhaustion & The Morality of Marginal Productivity.

Second-Generation Marginalists

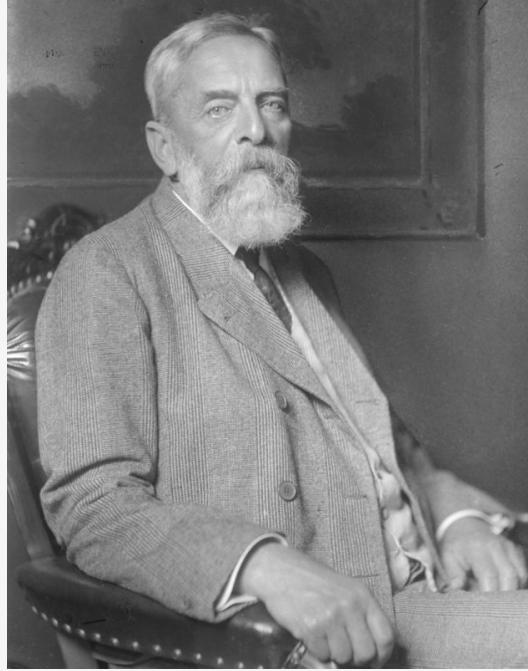


- Primarily extended and applied Jevons, Menger, & Walras' marginalist tendencies to more problems in economics
 - *especially*, the problem of pricing the factors of production
- In England: Alfred Marshall, Phillip Wicksteed, Francis Edgeworth, A.C. Pigou
- In Austria: Friedrich Wieser, Eugen von Böhm-Bawerk
- In Switz./Italy: Enrico Barone, Vilfredo Pareto
- In United States: John Bates Clark, Irving Fisher, Frank Knight, Frank Fetter
- In Sweden: Knut Wicksell



Second-Generation Austrian Marginalists

Friedrich von Wieser

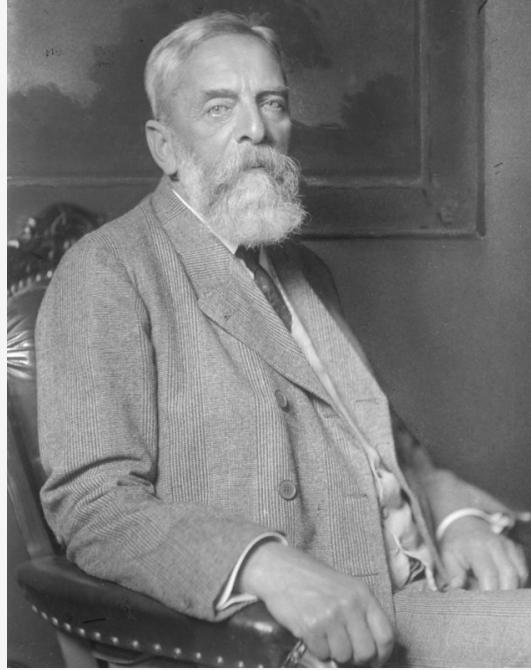


Friedrich von Wieser

1851–1926

- Student of Menger, ultimately replaced Menger as Professor of Political Economy at University of Vienna
- Coined the term “marginal utility” (*Grenznutzen*)
- Teacher to F.A. Hayek
- 1889, *Der natürliche Werth* (*Natural Value*)
- 1914, *Theorie der gesellschaftlichen Wirtschaft* (*Theory of Social Economy*),

Friedrich von Wieser: Alternative Cost Theory

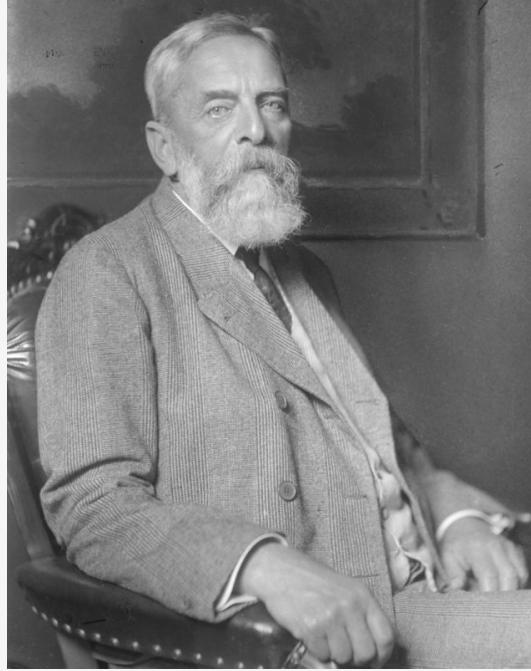


Friedrich von Wieser

1851–1926

- What role do costs of production (payments to factors) play in value of final goods?
- Costs are the values which are forgone in directing resources to a particular production process rather than other production processes
- In this sense, production costs are really a reflection of utilities elsewhere in the economy
- **Alternative cost theory** or **opportunity cost**

Friedrich von Wieser: Alternative Cost Theory

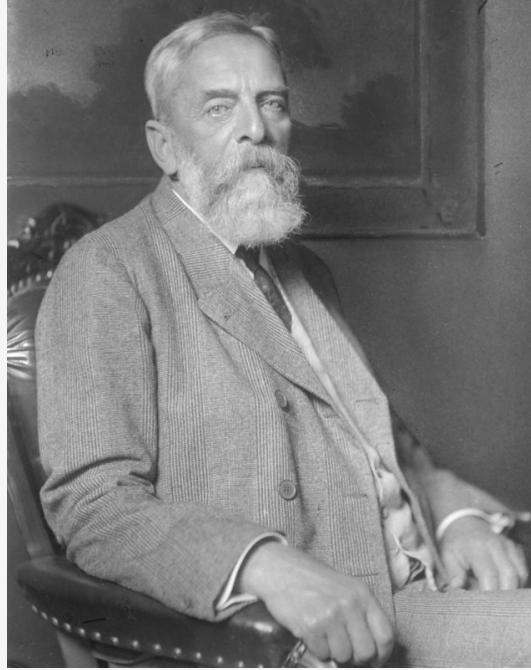


Friedrich von Wieser

1851–1926

- Beginnings of major disagreements:
- Jevons always thought costs were “real” in some sense, e.g. the *disutility* or pain of labor
 - utility of consumption vs. disutility of production; utility & disutility curves
- Marshall & Edgeworth would later argue you can derive an upward-sloping supply/cost curve for non-land factors by disutility of use

Friedrich von Wieser: Imputation Theory

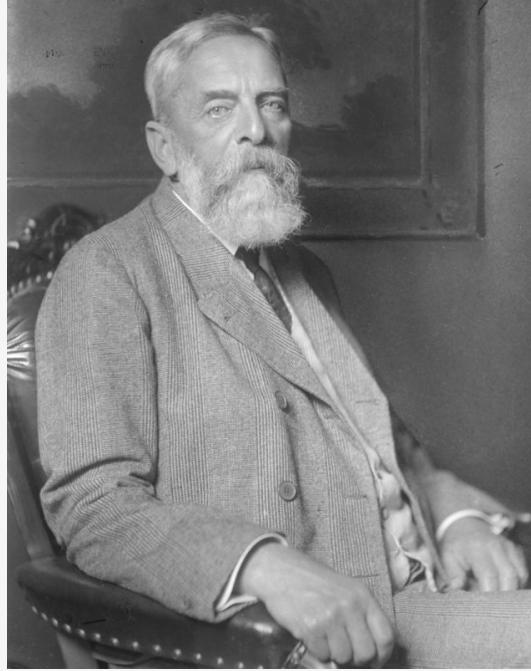


- Menger had clear insights about capital and production: goods of higher order, their complementarity and substitutability, etc.
- If we all agree that prices of final goods reflect their marginal utility, how do we price factor services (land, labor, capital)?
- Wieser, using a legal term, this is a “**problem of imputation**”

Friedrich von Wieser

1851–1926

Friedrich von Wieser: Imputation Theory



Friedrich von Wieser

1851–1926

- Wieser's solution was **linear programming** with simultaneous equations (no calculus)
- **Example:** consider a three-good society, factors in each good's production are x , y , and z , represented by three simultaneous equations:
$$\begin{aligned} x+y &= \$100 \\ 2x+3z &= \$290 \\ 4y+5z &= \$590 \end{aligned}$$
- Solve for x , y , and z (prices of each factor)
- Assumes prices for final goods are given, fixed production coefficients, and no substitution of factors

Eugen von Böhm-Bawerk



Eugen von Böhm-Bawerk

1851–1914

- Studied law at University of Vienna; exposed to Menger but never his direct student
- Friend & brother-in-law to Friedrich Wieser
- Became Minister of Finance of Austria-Hungary; ambassador to Germany
- Later became professor of political economy, teacher to Ludwig von Mises

Eugen von Böhm-Bawerk



Eugen von Böhm-Bawerk

1851–1914

- Direct critique of Marxism: 1896, *Karl Marx and the Close of His System*
- Famously wrote on capital theory and interest theory
 - We will dig into this next week
- *Capital & Interest* 2 volumes:
 - 1884, *History and Critique of Interest Theories*
 - 1889, *Positive Theory of Capital*

Eugen von Böhm-Bawerk



- Took a different approach to the imputation problem (factor pricing) than Wieser:
- Followed a phrase in Menger, “the loss principle” – applying to the price of the final good what would be lost if one of the factor services is withdrawn
- A good start, but in truth, marginal product operates at infinitesimally small changes (derivative)

Eugen von Böhm-Bawerk

1851–1914

Eugen von Böhm-Bawerk: On Opportunity Cost



Eugen von Böhm-Bawerk

1851–1914

“If in any branch of production the price sinks below the cost...men will withdraw from that branch and engage in some better paying branch of production. Conversely, if in one branch of production, the market price of the finished good is considerably higher than the value of the sacrificed or expended means of production, then will men be drawn from less profitable industries. They will press into the better paying branch of production, until through the increased supply, the price is again forced down to cost.

Böhm-Bawerk, Eugen von, 1884, *Capital and Interest*

Eugen von Böhm-Bawerk: On Opportunity Cost



Eugen von Böhm-Bawerk

1851–1914

“What determine the amount of this cost? **The amount of the cost** is identical with the value of the productive power, and, as a rule, is determined by the money marginal utility of this productive power...The price of a definite specie of freely reproducible goods fixes itself in the long run at that point where the money marginal utility, for those who desire to purchase these products, **intersects the money marginal utility of all those who desire to purchase in the other communicating branches of production.** The figure of the two blade of a pair of shears still holds good. One of the two blade, whose coming together determine the height of the price of any species of product, is in truth the marginal utility of this particular product. The other, which we are wont to call "cost," is the marginal utility of the products of other communicating branches of production. Or, according to Wieser, the marginal utility of "production related goods." **It is, therefore, utility and not disutility which, as well on the side of supply as of demand, determine the height of the price.** This, too, even where the so-called law of cost plays its role in giving value to goods. **Jevons, therefore, did not exaggerate the importance of the one side, but came very near the truth when he said "value depends entirely upon utility."** (45)

Price is Determined at the Margin: B-B's Example



- Böhm-Bawerk has a great demonstration of how markets work to set the price at the margin
 - Story of the three “**marginal pairs**”
- Imagine a small public horse market:
 - 3 people, **A**, **B**, and **C** each own 1 horse
 - 3 people, **D**, **E**, and **F** each are potentially interested in buying a horse

Based on Eugen von Böhm-Bawerk's example in *Capital and Interest* (1884)

Price is Determined at the Margin: B-B's Example



Person	Reservation Price
Seller A	Minimum WTA: \$400
Seller B	Minimum WTA: \$500
Seller C	Minimum WTA: \$600
Buyer D	Maximum WTP: \$600
Buyer E	Maximum WTP: \$500
Buyer F	Maximum WTP: \$400

- Suppose Buyer F announces she will pay \$400 for a horse
- Only Seller A is willing to sell at \$400
- Buyers D, E, and F are willing to buy at \$400
 - D and E are willing to pay *more* than F to obtain the 1 horse
 - They raise their bids *above* \$400 to attract sellers

Price is Determined at the Margin: B-B's Example



Person	Reservation Price
Seller A	Minimum WTA: \$400
Seller B	Minimum WTA: \$500
Seller C	Minimum WTA: \$600
Buyer D	Maximum WTP: \$600
Buyer E	Maximum WTP: \$500
Buyer F	Maximum WTP: \$400

- Suppose Seller C announces he will sell his horse for \$600
- Only Buyer D is willing to buy at \$600
- Sellers A, B, and C are willing to sell at \$600
 - A and B are willing to accept *less than* C to sell their horses
 - They lower their asks *below* \$600 to attract buyers

Price is Determined at the Margin: B-B's Example



Person	Reservation Price
Seller A	Minimum WTA: \$400
Seller B	Minimum WTA: \$500
Seller C	Minimum WTA: \$600
Buyer D	Maximum WTP: \$600
Buyer E	Maximum WTP: \$500
Buyer F	Maximum WTP: \$400

Market Price: \$500

- If the market price reaches \$500 (through bids and asks changing)
- Sellers A and B sell their horses for \$500 each
- Buyers D and E buy them at \$500 each

Price is Determined at the Margin: B-B's Example



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Buyer D	Maximum WTP: \$600
Buyer E	Maximum WTP: \$500
Buyer F	Maximum WTP: \$400

Market Price: \$500

- At \$500, **B** and **E** are the "**marginal**" buyer and seller, the "last" ones that *just* got off the fence to exchange in the market
 - **B** has WTA *just* low enough to sell
 - **E** has WTP *just* high enough to buy
- The marginal pair actually are the ones that "set" the market price!

Price is Determined at the Margin: B-B's Example



Person	Reservation Price
Seller A	Minimum WTA: \$400
Seller B	Minimum WTA: \$500
Seller C	Minimum WTA: \$600
Buyer D	Maximum WTP: \$600
Buyer E	Maximum WTP: \$500
Buyer F	Maximum WTP: \$400

Market Price: \$500

- Notice the most possible exchanges take place at a market price of \$500
 - 2 horses get exchanged
- Any price *above or below* \$500, only 1 horse would get exchanged
 - Also, at least one other buyer or seller would raise/lower their bid/ask

Price is Determined at the Margin: B-B's Example



Person	Reservation Price
Seller A	Minimum WTA: \$400
Seller B	Minimum WTA: \$500
Seller C	Minimum WTA: \$600
Buyer D	Maximum WTP: \$600
Buyer E	Maximum WTP: \$500
Buyer F	Maximum WTP: \$400

Market Price: \$500

- At \$500, C and F are the "excluded" buyers and sellers
 - C has WTA *too high* to sell
 - F has WTP *too low* to buy

Price is Determined at the Margin: B-B's Example



Person	Reservation Price
Seller A	Minimum WTA: \$400
Seller B	Minimum WTA: \$500
Seller C	Minimum WTA: \$600
Buyer D	Maximum WTP: \$600
Buyer E	Maximum WTP: \$500
Buyer F	Maximum WTP: \$400

Market Price: \$500

- At \$500, A and D are the "inframarginal" buyers and sellers
 - A has WTA *lower* than market price, earns extra \$100 surplus from exchange
 - D has WTP *higher* than market price, earns extra \$100 surplus from exchange
- These buyers and sellers benefit *the most* from exchange



Marginal Productivity Theory

The Ricardian Roots of Marginal Productivity Theory



- Ricardo's theory of rent applied marginal analysis ("doses" of L+K) to a fixed factor (land), concluding the fixed factor earns a residual surplus (gap between AP>MP) of variable factor (L+K)
- **Marginal productivity theory** takes the other side of the coin: any **variable factor must earn a payment equal to its marginal product** (holding all other factors fixed)

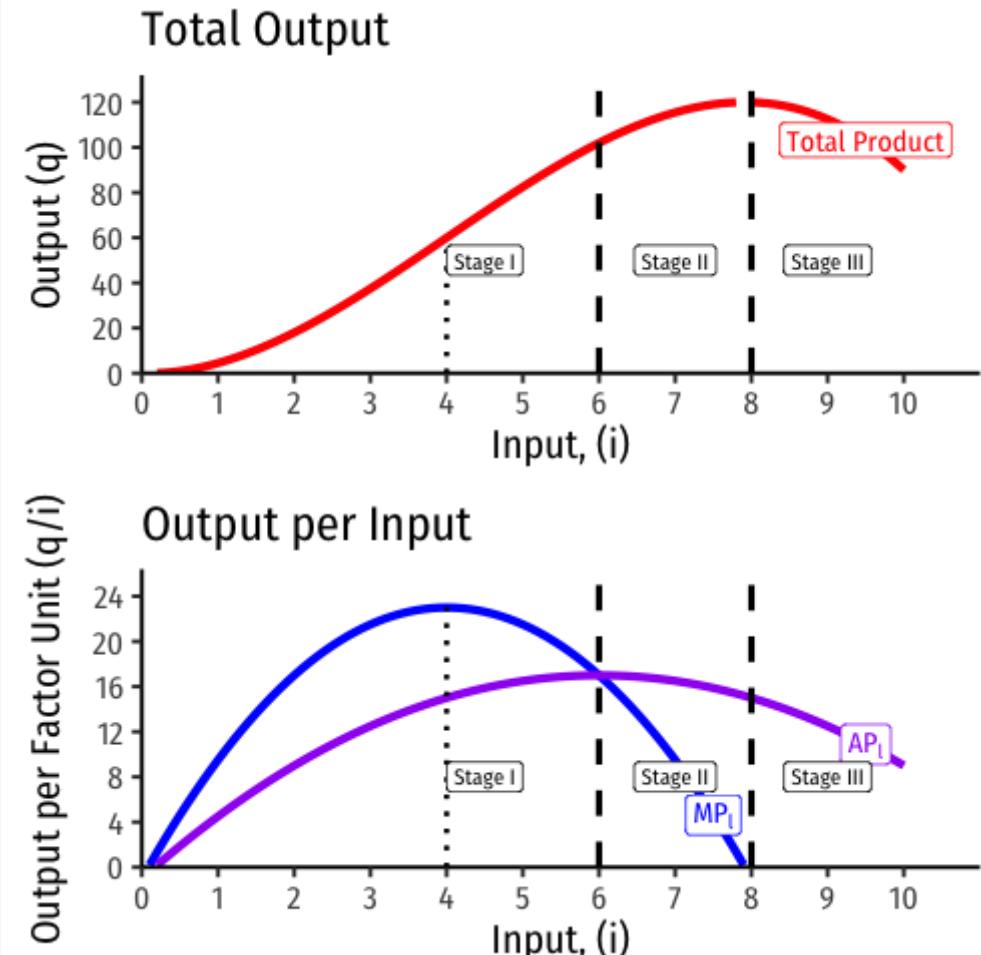
David Ricardo

1772-1823

Marginal Productivity Theory



- Applying Ricardian logic *beyond* agriculture, we arrive at the modern **law of diminishing returns**
 - “**Law of variable proportions**” or “**variation of returns**”
- For *any* one variable factor (holding all others constant), increasing use will eventually yield a diminishing marginal product
 - nothing special about land!



Diminishing Returns



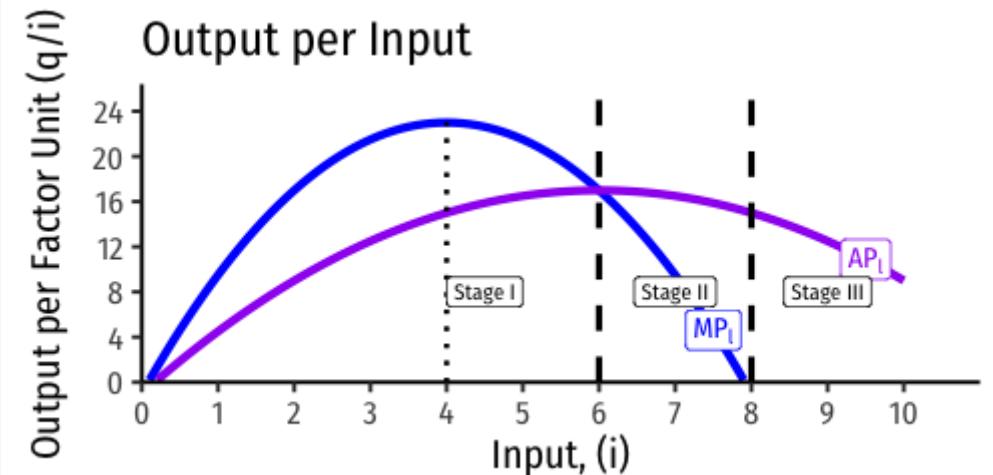
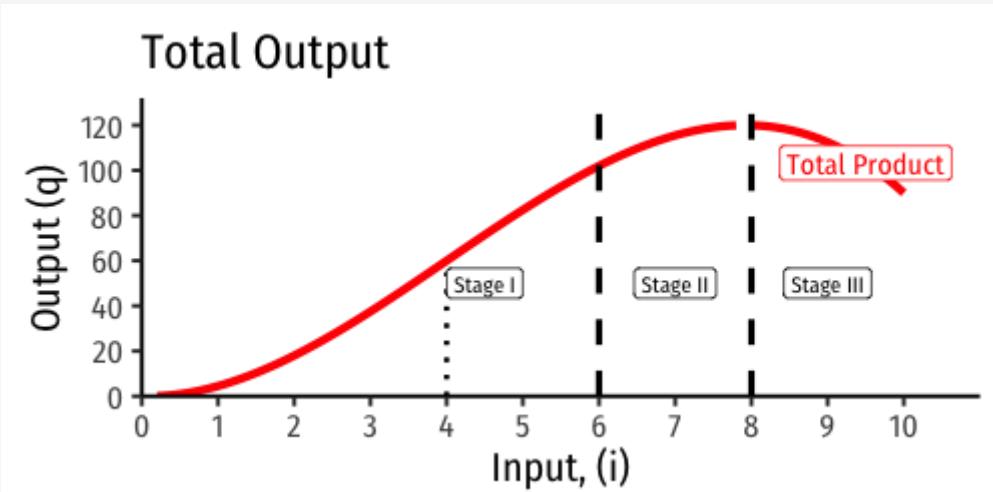
- **Marginal product of factor i, (MP_i)**: additional output produced by adding one more unit of factor i (holding all others constant)

$$MP_i = \frac{\Delta q}{\Delta i}$$

- MP_i is slope of TP at each value of i
- **Average product of factor i (AP_i)**: additional output produced by adding one more unit of factor i (holding all others constant)

$$AP_i = \frac{q}{i}$$

- AP_i is slope of a ray from the origin to the production function at any quantity of i



Derived Demand in Factor Markets



- Demand for factors (e.g. labor) is a “derived demand”:
 - Firm only demands inputs to the extent they **contribute to producing sellable output**
- Firm faces a **tradeoff** when **hiring more labor**, as more labor ΔL creates:
 1. **Marginal Benefit**: Increases output and thus revenue
 2. **Marginal Cost**: Increases costs



Marginal Revenue Product (of Labor)



- Hiring more labor increases output (i.e. labor's MP_L)
 - Recall: $MP_L = \frac{\Delta q}{\Delta L}$, where q is units of output
- Additional output generates (i.e. labor's $MR(q)$)
 - Recall: $MR(q) = \frac{\Delta R(q)}{\Delta q}$, where $R(q)$ is total revenue
- Hiring more labor, on the **margin**, generates a **benefit**, called the **marginal revenue product of labor, MRP_L** :

$$MRP_L = MP_L * MR(q)$$

- i.e. the number of new products a new worker makes times the revenue earned by selling the new products

Marginal Revenue Product for *Competitive Firms*



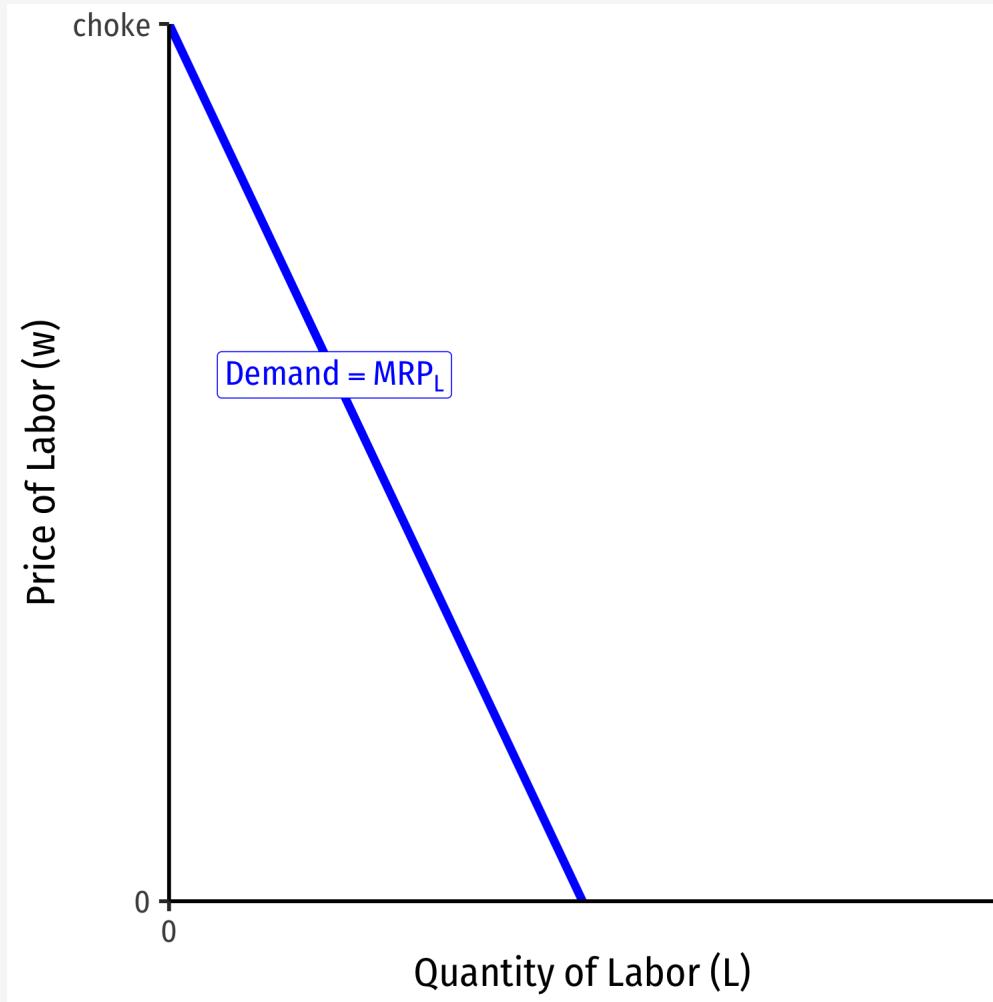
- This is the **Firm's Demand for Labor**:

$$MRP_L = MP_L * MR(q)$$

- For a firm in a **competitive (output) market**, firm's $MR(q) = p$, hence:

$$MRP_L = MP_L * p$$

where p is the price of the firm's *output*



Marginal Revenue Product for *Competitive Firms*

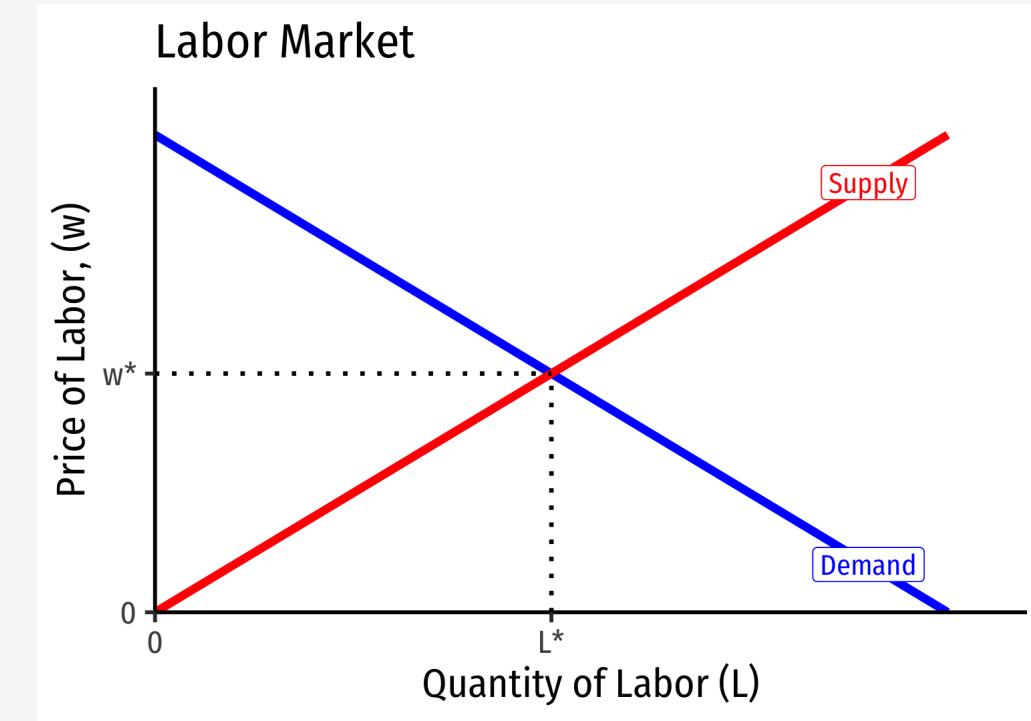
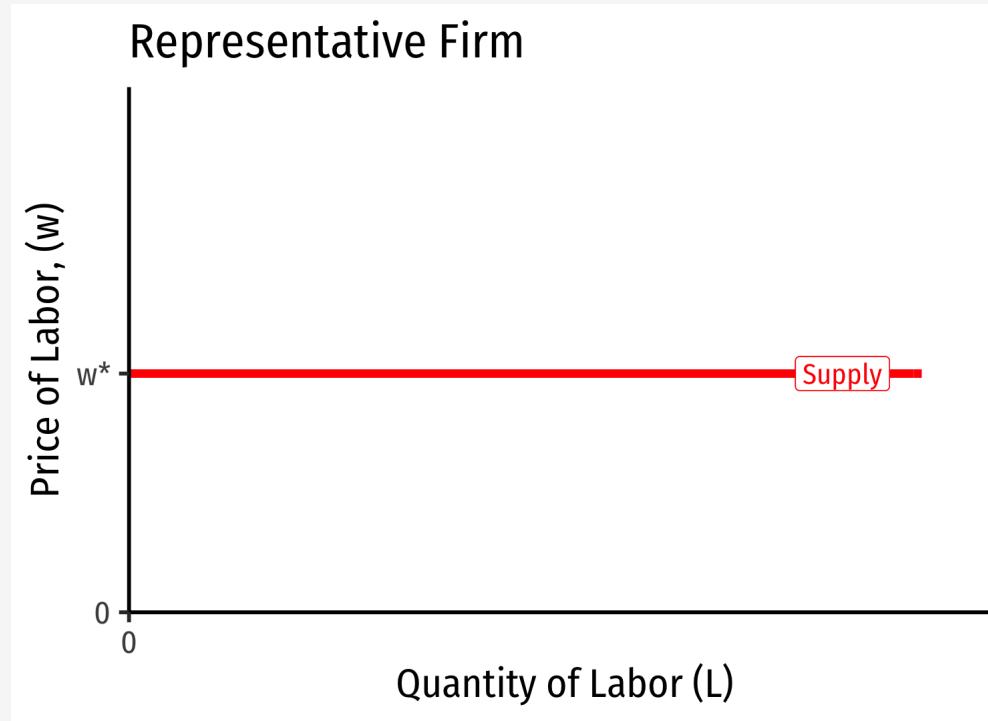


$$MRP_L = MP_L * p$$

- Marginal benefit of hiring labor, MRP_L **falls** with more labor used
 - production exhibits **diminishing marginal returns to labor!**
- **Choke price for labor demand:** price too high for firm to purchase any labor



A Competitive Factor Market

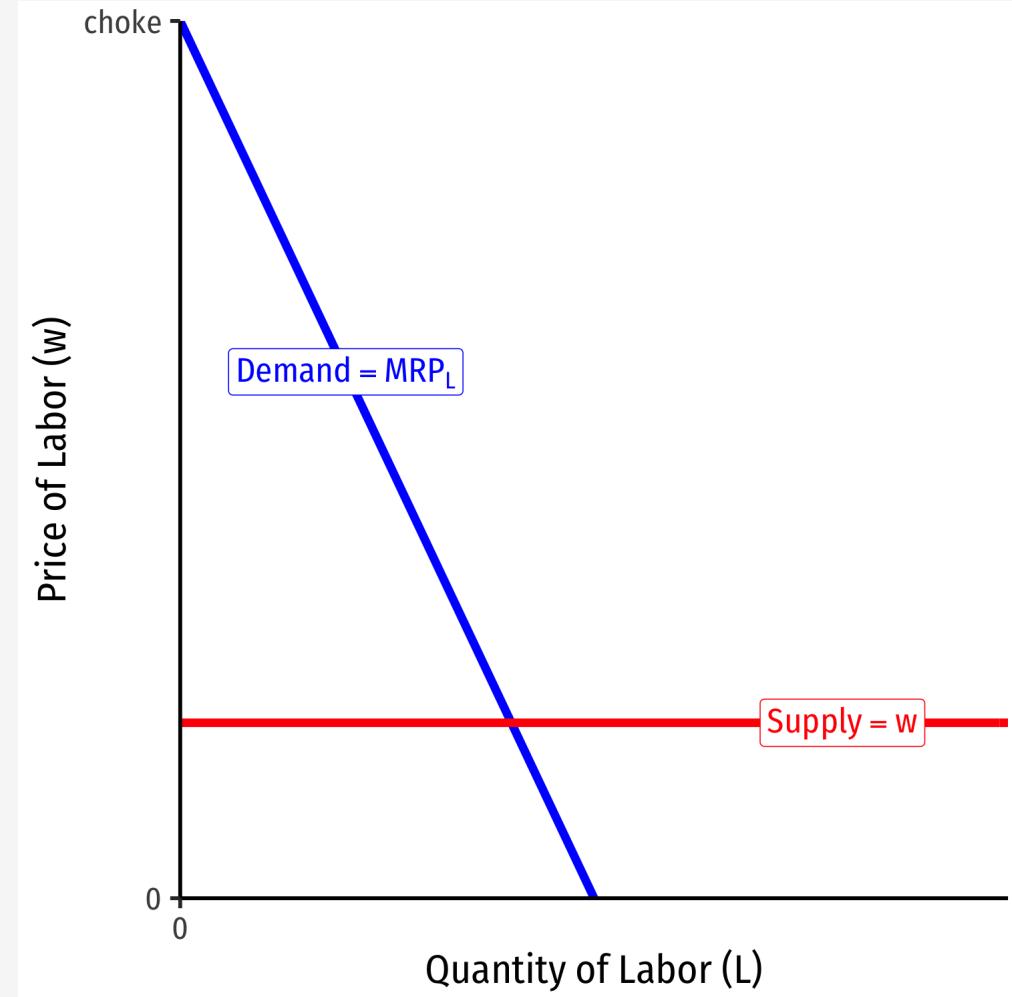


- If the **factor market is competitive**, labor supply for an individual firm is *perfectly elastic* at the market price of labor (w^*)

Labor Supply and Firm's Demand for Labor



- We've seen a falling MRP_L , the marginal benefit of hiring labor
- **Marginal cost of hiring labor**, w , remains constant
 - so long as firm is not a big purchaser (has no market power) in the labor market



Labor Supply and Firm's Demand for Labor



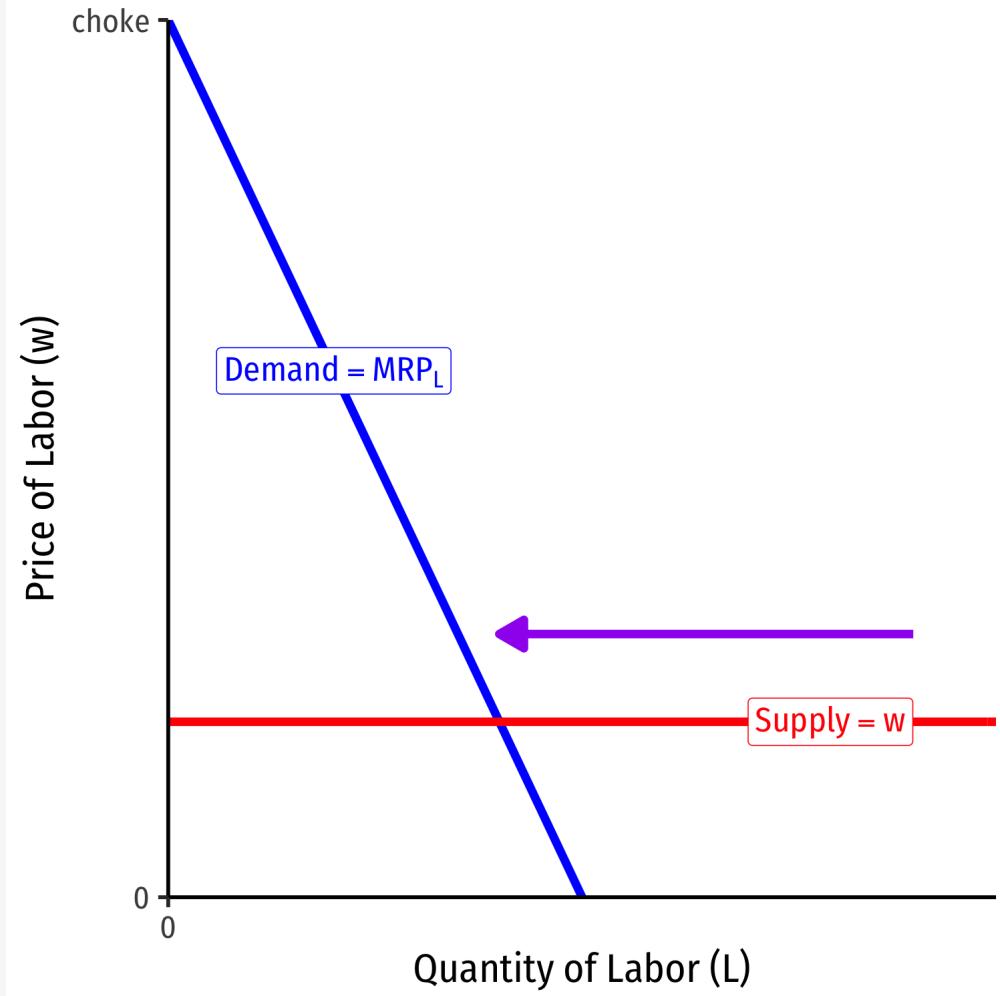
- At low amounts of labor, marginal benefit (MRP_L) < w marginal cost
- Firm will hire more labor



Labor Supply and Firm's Demand for Labor



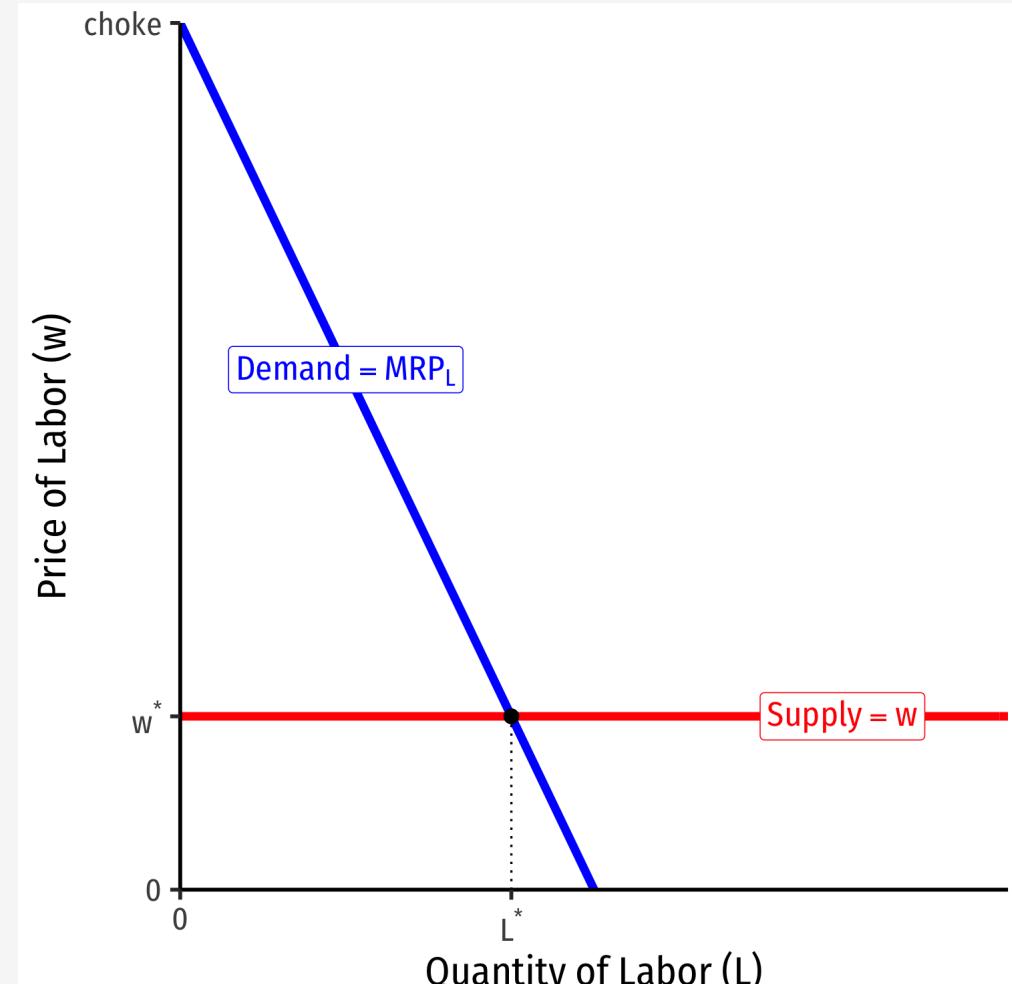
- At high amounts of labor, marginal benefit (MRP_L) < w marginal cost
- Firm will hire less labor



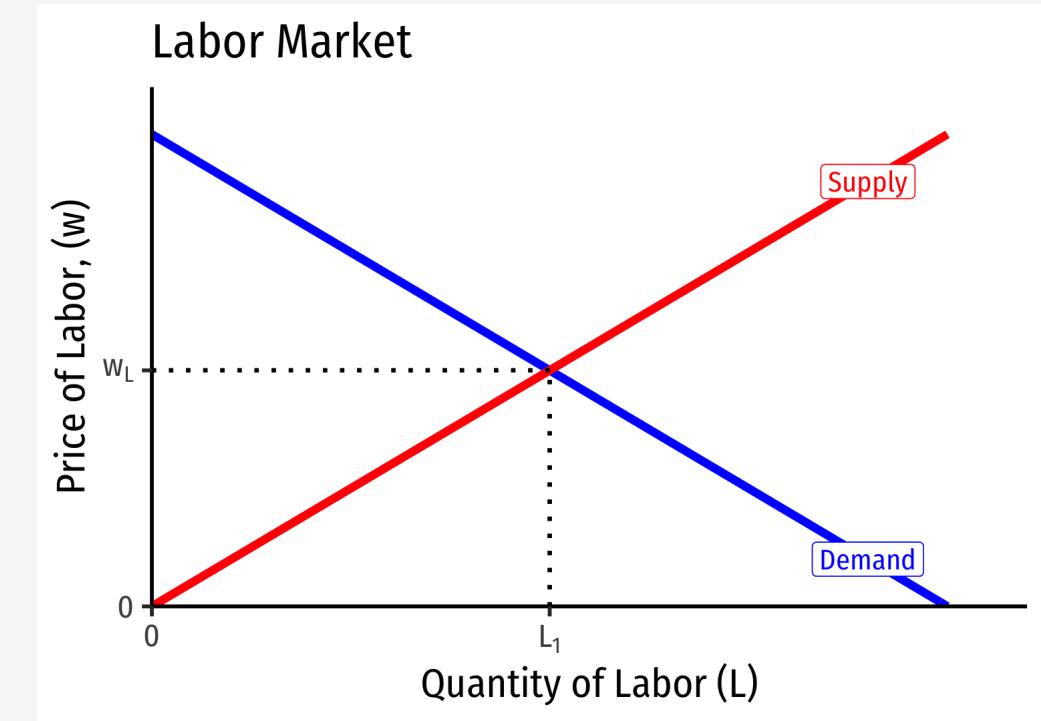
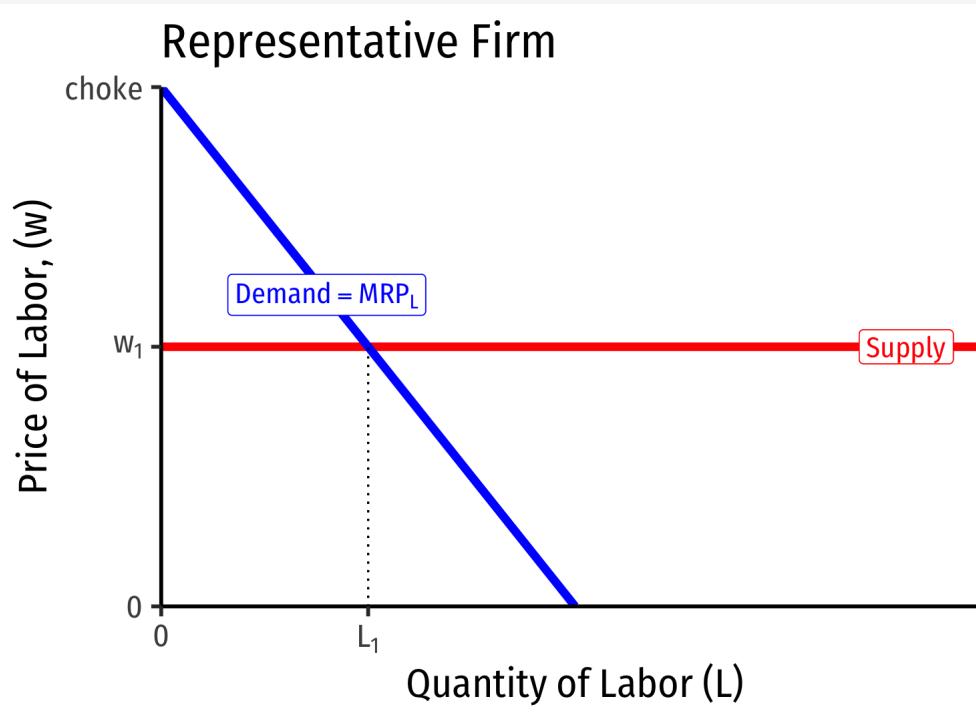
Labor Supply and Firm's Demand for Labor



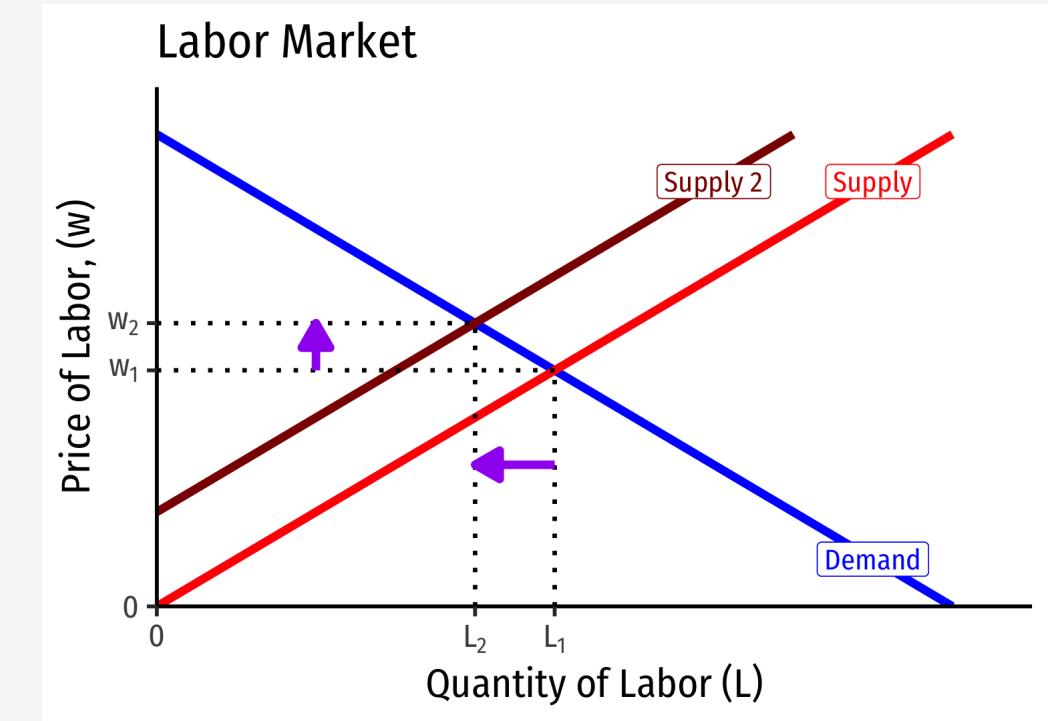
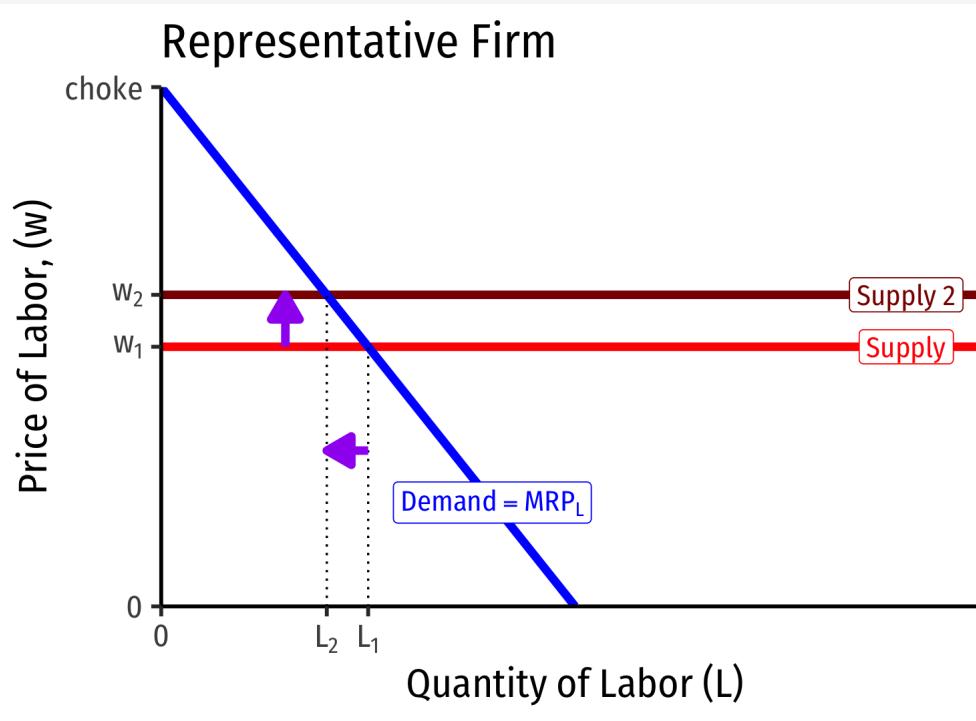
- Firm hires L^* optimal amount of labor where $w = MRP_L$
- i.e. marginal cost of labor = marginal benefit of labor



Labor Supply and Firm's Demand for Labor



Labor Supply and Firm's Demand for Labor



- If market supply of labor decreases, firms hire fewer workers, at higher wages (and vice versa)

Multiple Inputs and Cost Minimization



- But firms produce with many factors, what is the more general rule for hiring the optimal combination of factors?
- Assume three factors: land, labor, capital
- Optimal hiring condition is the **equimarginal rule** (Gossen's Second Law} again:

$$\frac{MP_l}{p_l} = \frac{MP_k}{p_k} = \frac{MP_t}{p_t} = \dots = \frac{MP_n}{p_n}$$

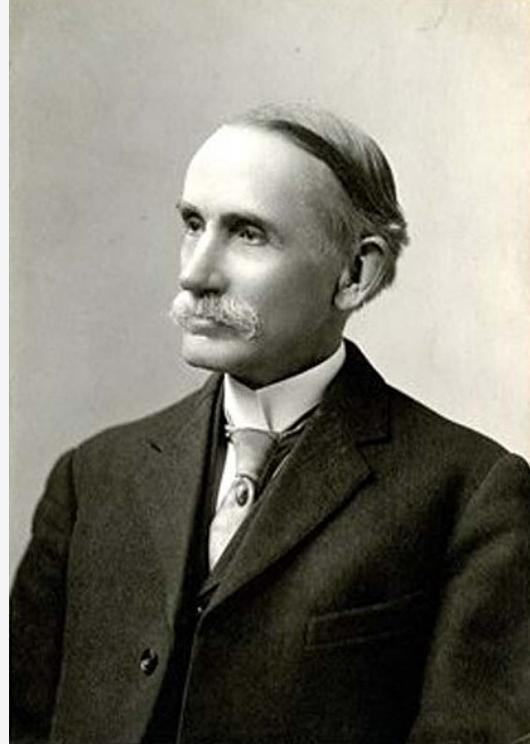
- Cost of production is minimized where the **marginal product per dollar spent** is **equalized** across all n possible inputs
 - the “last dollar spent” on each input provides the same marginal product





Product Exhaustion & The Morality of Marginal Productivity

John Bates Clark



John Bates Clark

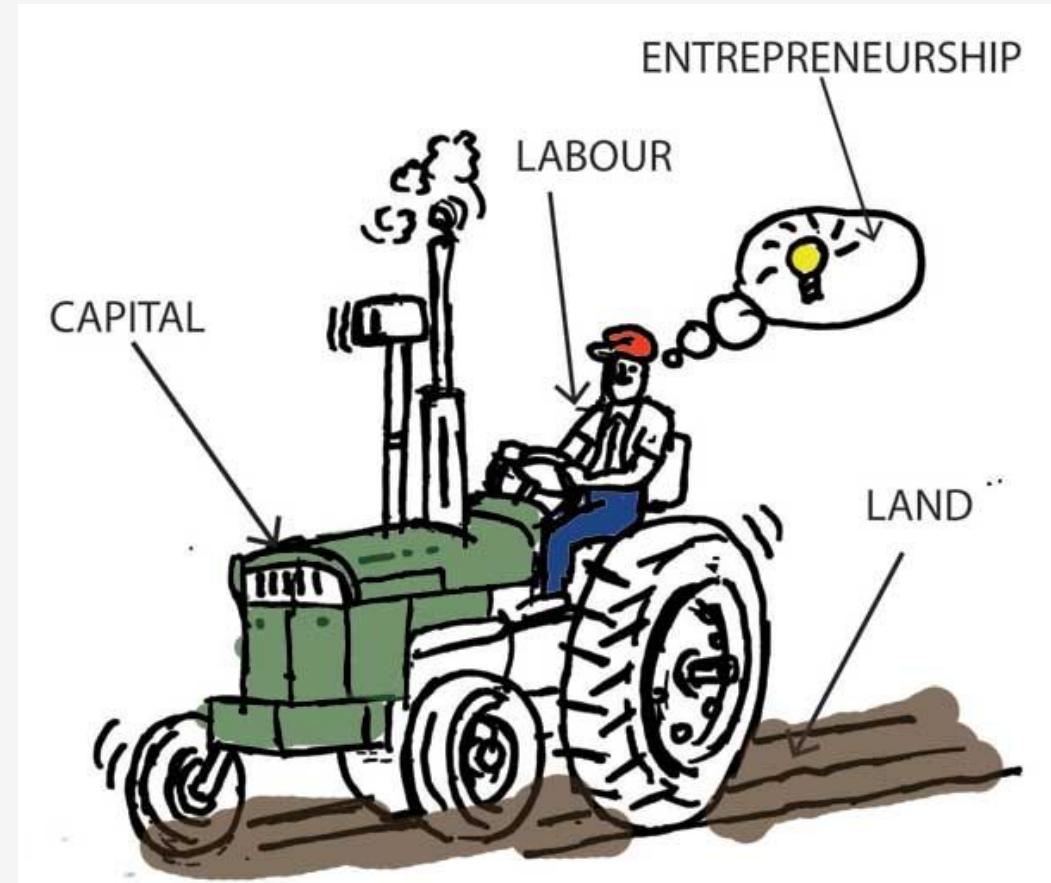
1847–1938

- Initially a German Historicist (studied under Karl Knies) in Germany; a Christian socialist
- Became professor at Columbia, independently derived his own version of marginal utility theory
- Main popularizer of marginal productivity theory, virtues of market competition; opponent to American Institutionalists (see later)
- 1886, *The Philosophy of Wealth*
- 1889, “Possibility of a Scientific Law of Wages” paper at AEA; generalized in 1899 *The Distribution of Wealth*

Product Exhaustion



- Ricardian rent theory defined rent as a residual, will always adjust to fill the gap between output price and wages & profits
 - $\text{output price} = \text{wages \& profits} + \text{rent}$
- Thus, the payments to all factors of production (land, labor, capital) “**fully exhaust the product**”
 - i.e. the sum of factor payments (costs to firm) equals the price

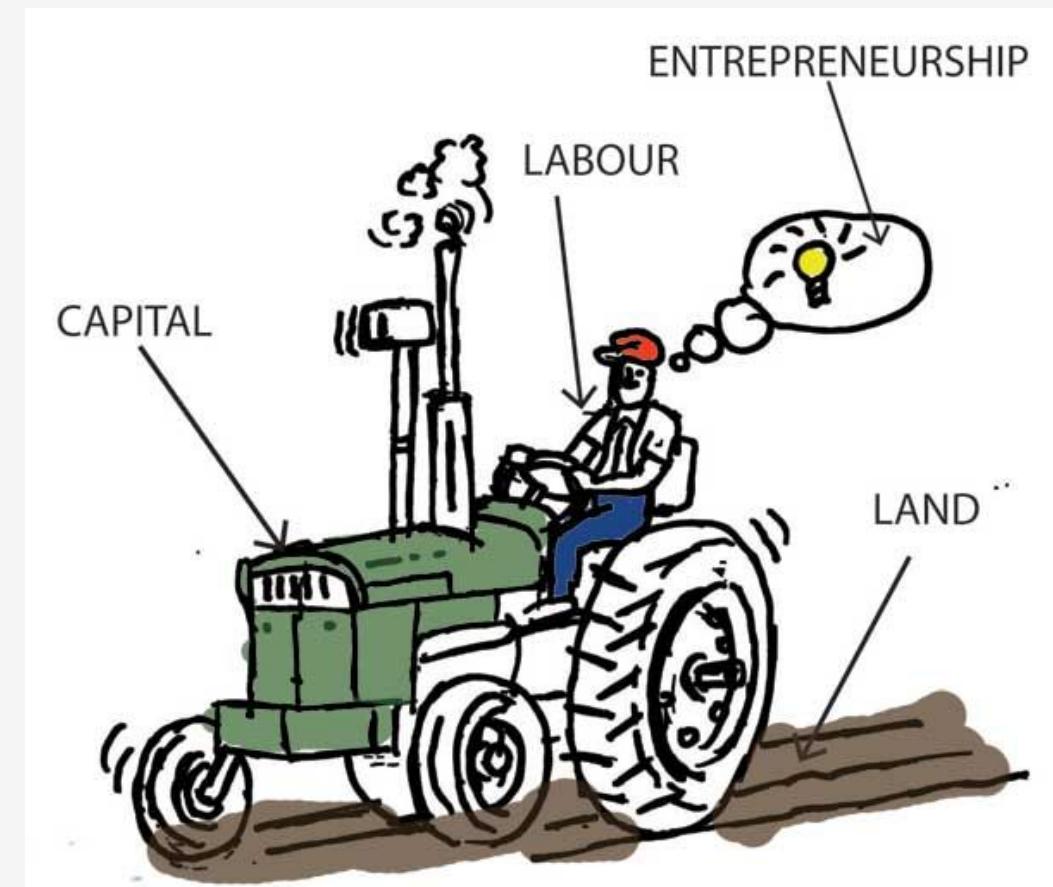


Product Exhaustion

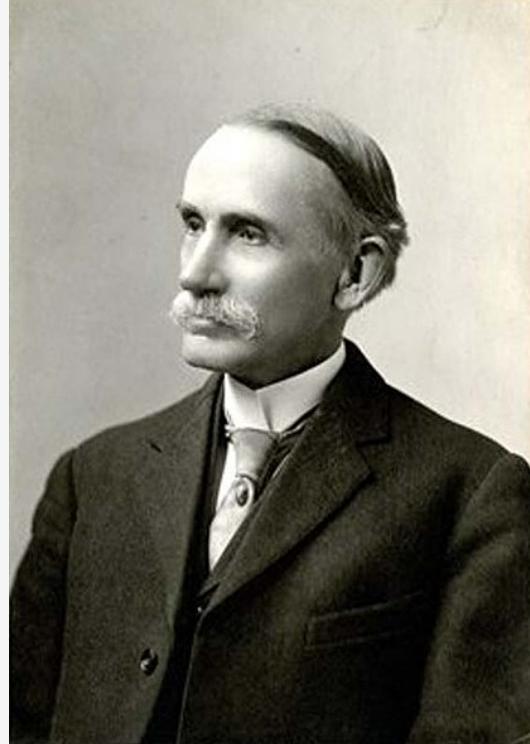


- On a competitive market, each product is paid its marginal (revenue) product
- Does the sum of these marginal products exactly equal the market price of the output?
 - “Product Exhaustion” debate:

$$Q = ? MP_L \times L + MP_K \times K + MP_T \times T$$



John Bates Clark

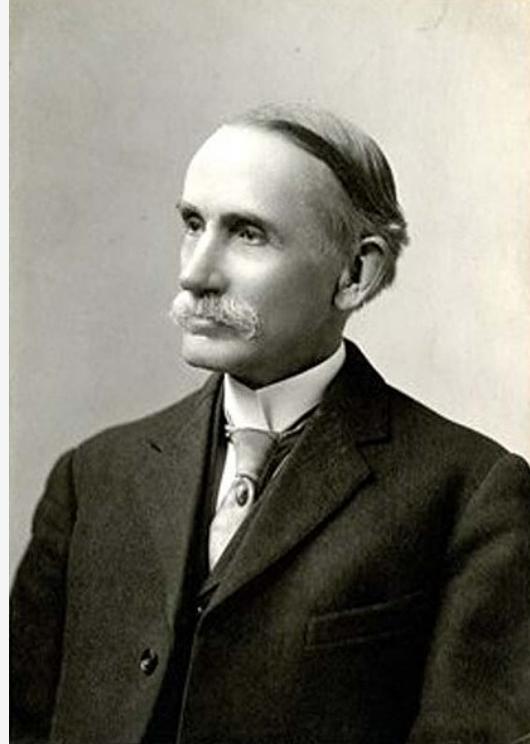


John Bates Clark

1847–1938

- Clark famously argued that on a competitive market, each factor is paid its marginal product, and that this exactly exhausts the product
- Viewed this as a *moral* virtue of markets: each factor is paid for its contribution to society
 - factor prices are not only efficient, they are just
- Offered no *proof* that this is true

John Bates Clark: The Morality of Marginal Productivity

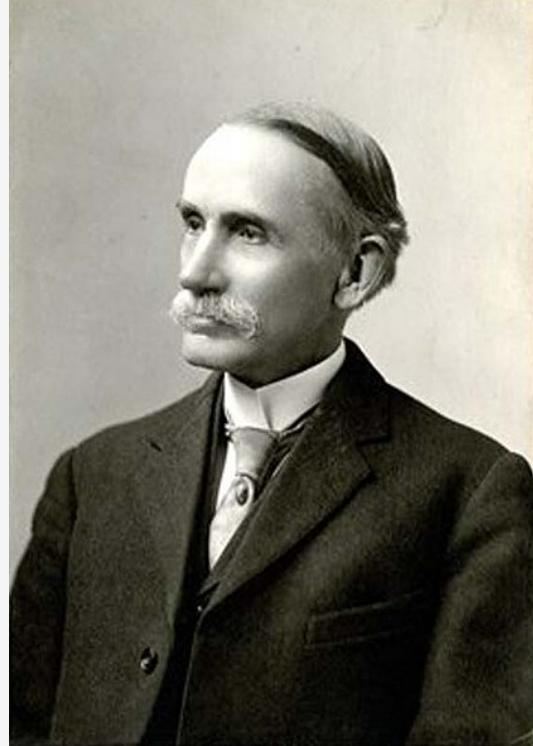


John Bates Clark

1847–1938

- Meant this as a critique of both Karl Marx and Henry George
- Georgists believed rent was undeserved, unearned income of landowners: should go to government
- Marxists believed profit was exploitative and undeserved (surplus value): belonged to workers
- Clark's *Distribution of Wealth* argues that marginal productivity theory shows that under competitive markets, each factor is paid its just due
 - Labor and land and capital are *all* necessary for production, and are paid for their productive contributions

John Bates Clark: The Morality of Marginal Productivity

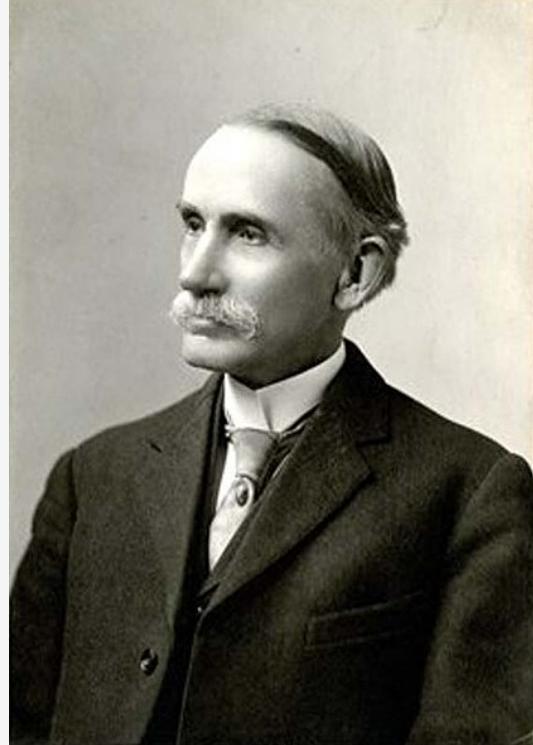


John Bates Clark

1847–1938

- Furthermore, argues that the distribution of income (under competitive markets) is *just* and *deserved!*
- Heavily criticized for this normative theory
 - Problems: not perfectly competitive, monopolies, labor unions, etc.
 - His student, Thorstein Veblen reached the opposite conclusion!
- Hume's is-ought gap

John Bates Clark: The Morality of Marginal Productivity

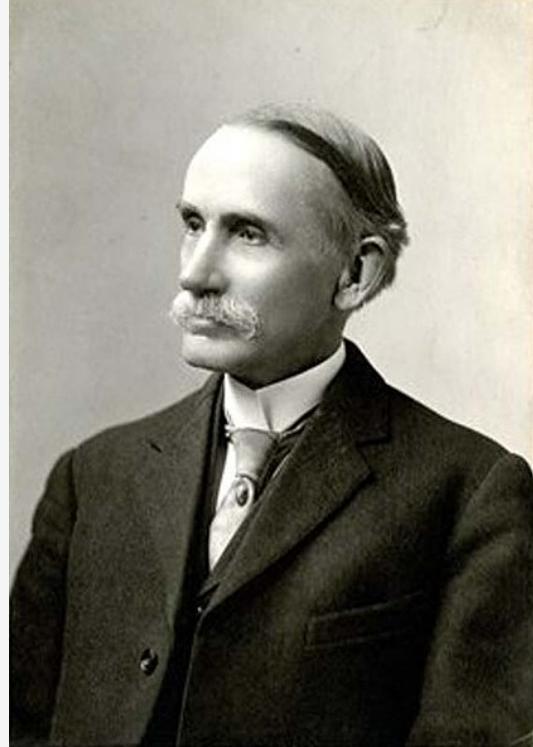


John Bates Clark

1847–1938

- Several flaws with marginal productivity theory as a theory of distribution
- MPT is primarily **a theory of factor pricing**, not about **distribution of relative shares**
- Also an incomplete theory of factor pricing! Considers only demand side of factor market (firms), not the supply side!
- Biggest problem: impossible to observe and measure an individual factor's marginal product!

John Bates Clark: The Morality of Marginal Productivity



- MPT describes the outcome to which we are always *approaching* (i.e. perfect competition), it is an equilibrium state of rest
 - *actual* prices in real world are not equilibrium prices, we are not in perfect competition
- A scarce factor will command higher prices, may be efficient, but is that moral?

John Bates Clark

1847–1938

Phillip Wicksteed



Phillip Wicksteed

1844–1927

- A British economist and unitarian minister
- Learned economics from Jevons, and got inspired to write about political economy after reading Henry George
- 1894, *An Essay on the Co-Ordination of the Laws of Distribution*
 - tries to solve the product exhaustion problem of marginal productivity theory
- 1910, *The Common Sense of Political Economy: Including a Study of the Human Basis of Economic Law*

Phillip Wicksteed



- Uses Euler's Theorem of homogeneous functions to prove product exhaustion under specific conditions:
 - production functions must be linearly homogenous (degree 1)
 - we would say: **constant returns to scale**
 - $cY = f(cL, cK, cT)$

Phillip Wicksteed

1844–1927

Returns to Scale



- The **returns to scale** of production refers to the change in output when all inputs are increased *at the same rate*
- **Constant returns to scale:** output increases at same proportionate rate as inputs increase
 - e.g. if you double all inputs, output doubles
- **Increasing returns to scale:** output increases *more than* proportionately to the change in inputs
 - e.g. if you double all inputs, output *more than* doubles
- **Decreasing returns to scale:** output increases *less than* proportionately to the change in inputs
 - e.g. if you double all inputs, output *less than* doubles

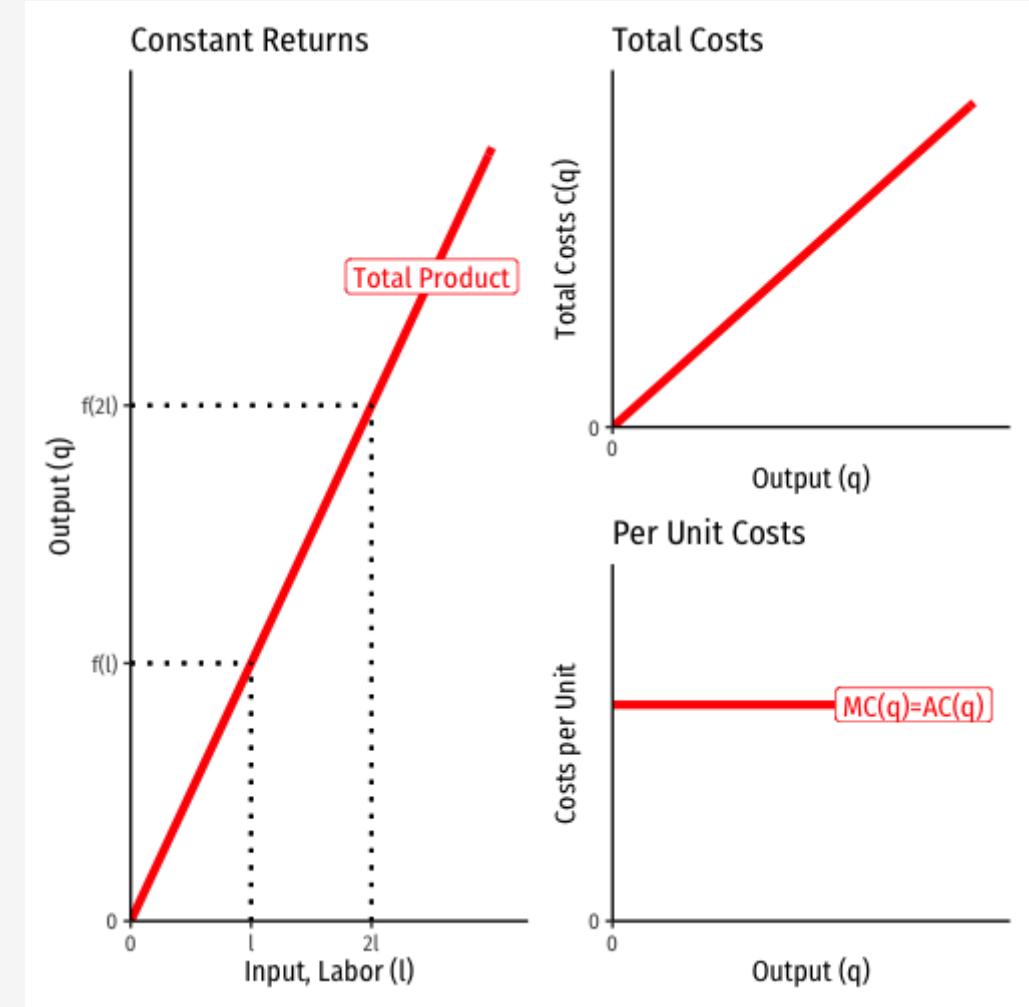
Constant Returns to Scale



- **Constant returns to scale:** doubling all inputs
⇒ double output

$$f(cl, ck, ct) = cf(l, k, t) \quad \forall c > 1$$

- **Constant economies of scale:** average and marginal costs (are equal and) do not vary with output
- Total revenues are completely exhausted by the payments to factors (costs to firm)



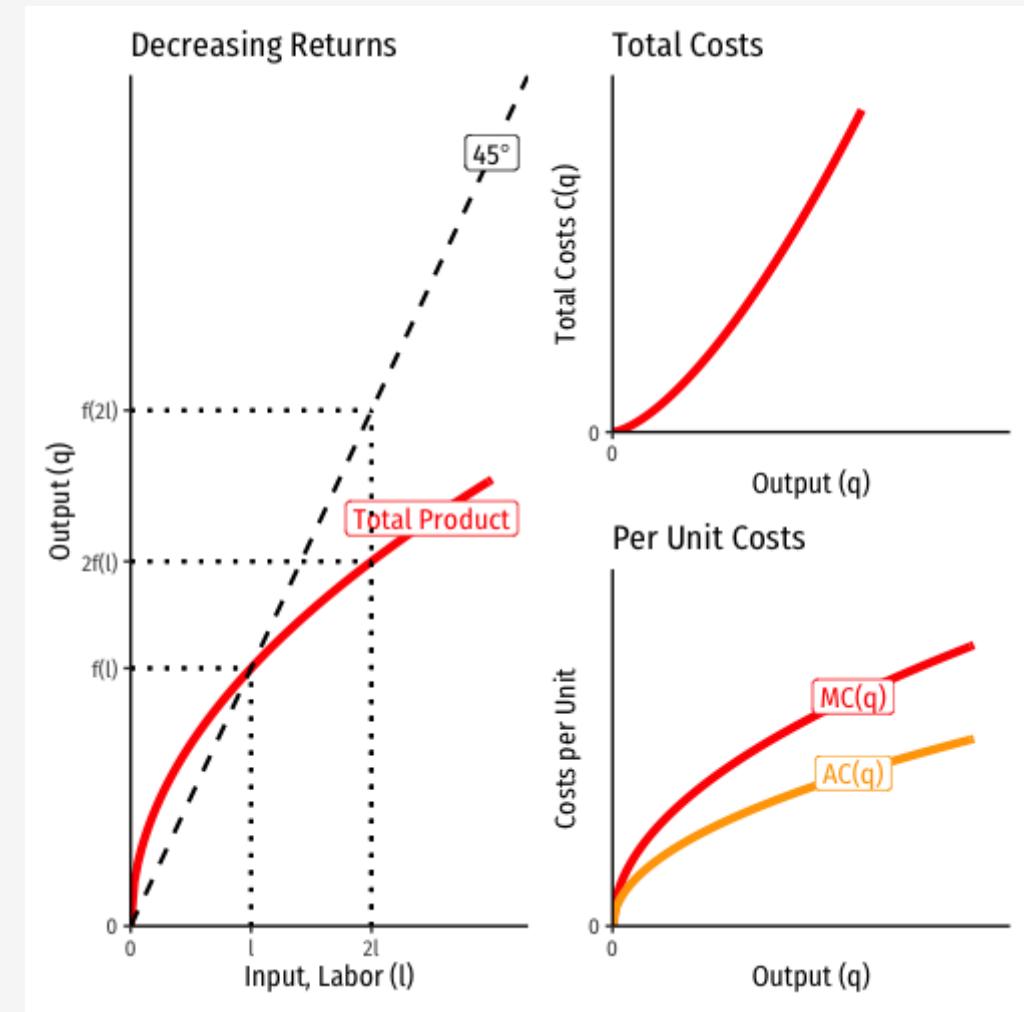
Decreasing Returns to Scale



- **Decreasing returns to scale:** doubling all inputs
 \implies less than double output

$$f(cl, ck, ct) < cf(l, k, t) \quad \forall c > 1$$

- **Diseconomies of scale:** average and marginal costs are increasing with output
 - $AC < MC \implies$ marginal cost pricing is always profitable
 - $Total\ Costs < Total\ Revenues \implies \pi > 0$
- Total revenues are *not* exhausted by the payments to factors (costs to firm); residual leftover!



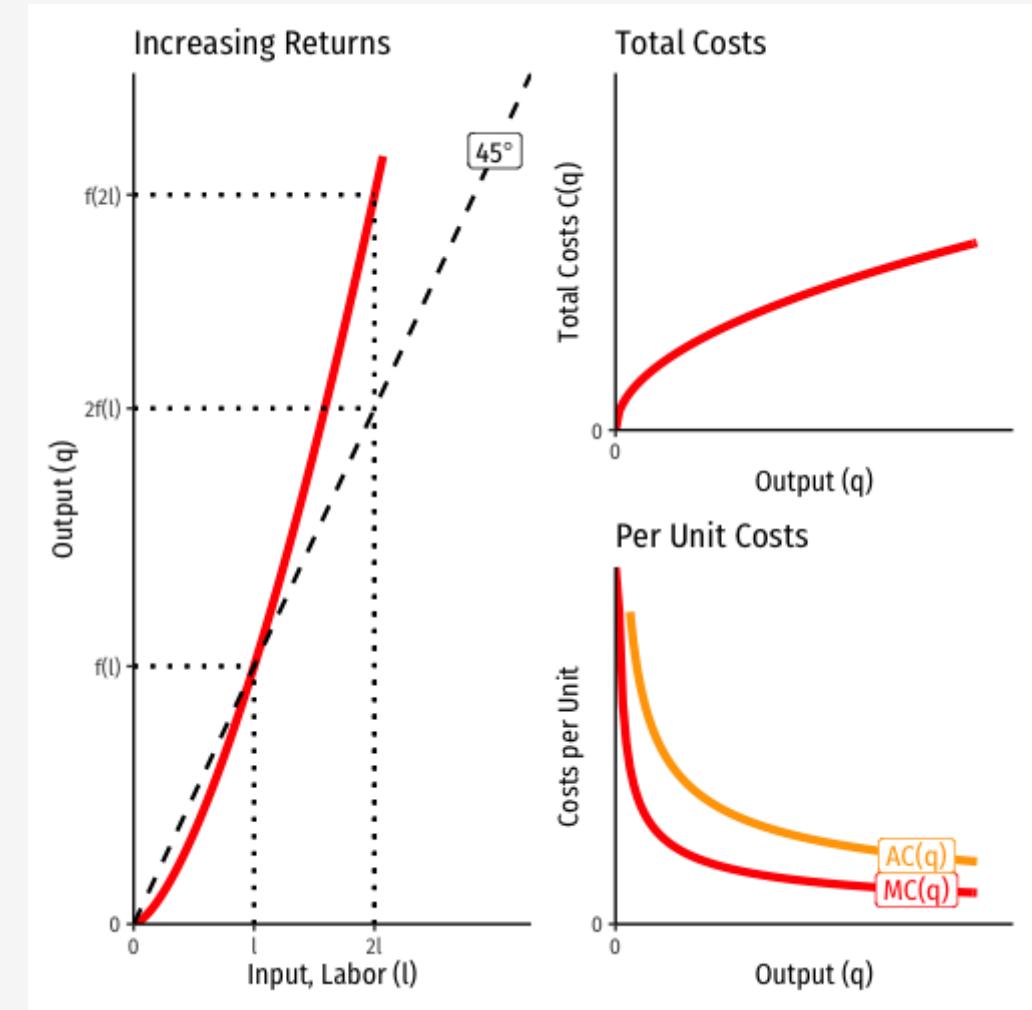
Increasing Returns to Scale



- **Increasing returns to scale:** doubling all inputs
 \Rightarrow more than double output

$$f(cl, ck, ct) > cf(l, k, t) \quad \forall c > 1$$

- **Economies of scale:** average and marginal costs are decreasing with output
 - $AC > MC \Rightarrow$ marginal cost pricing is always loss-inducing
 - $Total\ Costs > Total\ Revenues \Rightarrow \pi < 0$
- Total revenues are *insufficient* to cover the payments to factors (costs to firm); losses!



Knut Wicksell



Knut Wicksell

1851–1926

- Swedish economist at University of Stockholm
- Another supposed independent discoverer of marginal productivity theory
- Made key contributions to capital and interest theory, influence Austrian & Keynesian schools of macroeconomics
 - we'll explore more next week
- 1898, *Interest and Prices*

Wicksell and Product Exhaustion



- Most economists believed that an industry would always be either constant, increasing, or decreasing returns
- Wicksell showed that most **firms actually go through all three phases of returns to scale**
 - developing a **long-run U-shaped average cost curve** for a firm
 - would take a few decades for neoclassical economists to derive and understand shape of AC curve

Knut Wicksell

1851–1926

Wicksell and Product Exhaustion



Knut Wicksell

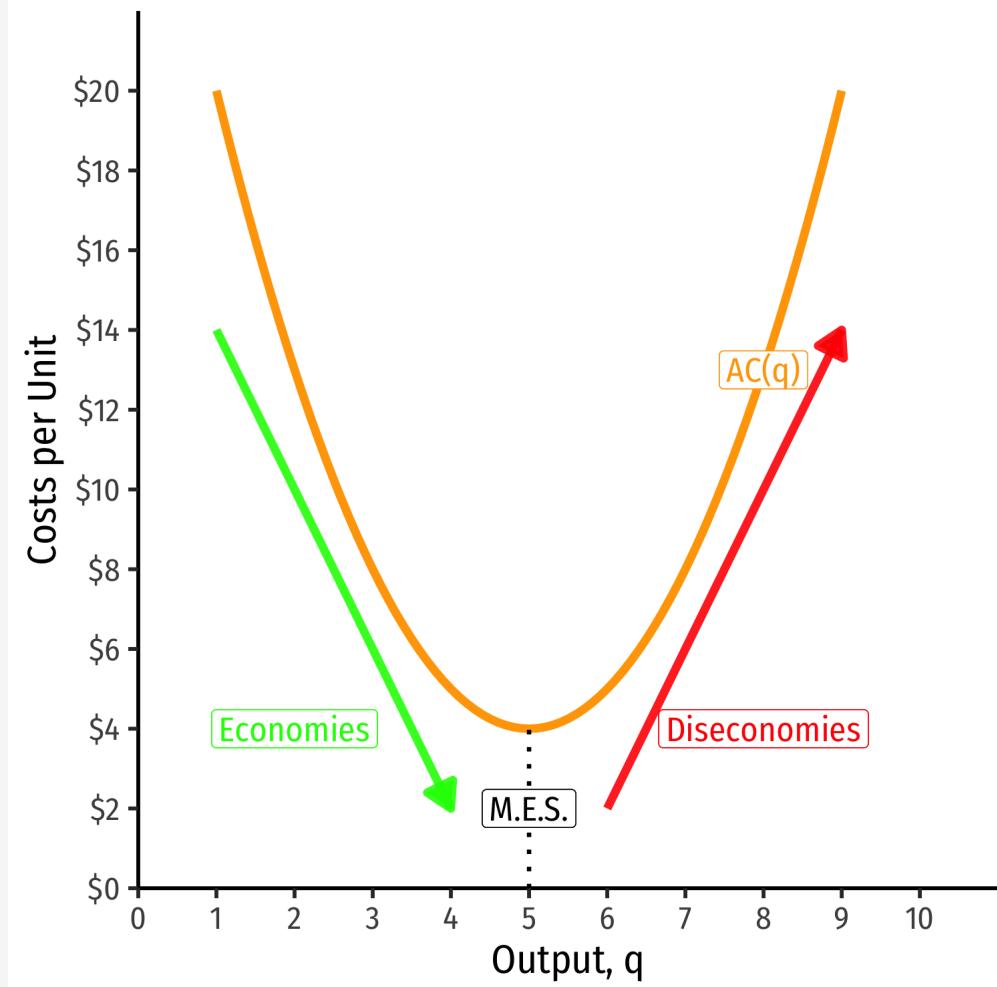
1851–1926

- Thus, it is *not* necessary (as Wicksteed did) to assume constant returns to prove product exhaustion
- Competition would ensure that in the long run, firms are producing at their least-cost combination
 - $p = MC = AC_{min}$
 - $\pi = 0$
 - “product exhaustion”

Wicksell and Product Exhaustion



- **Minimum Efficient Scale:** q with the lowest $AC(q)$
 - constant returns to scale
- **Economies of Scale:** $\uparrow q, \downarrow AC(q)$
 - increasing returns to scale
- **Diseconomies of Scale:** $\uparrow q, \uparrow AC(q)$
 - decreasing returns to scale



Wicksell and Product Exhaustion



- Think about what you learn in microeconomics
- In perfect competition, in the long run, as profits attract entrants and losses force exits, price settles on the break-even point, where profit is 0
 - $p = MC = AC_{min}$
- We still haven't gotten to the famous model of perfect competition, but this is where everything is heading

