PHY 338K Lab Report 3

Ryan J Schlimme
EID: rjs4499

October 29, 2023

Contents

List of Figures

List of Tables

Lab 7: Operational Amplifiers I

Question 1

We are instructed to build the circuit illustrated in Figure 7.1. We used values $V_{CC} = 15\text{V DC} = -V_{EE}$. We also used values $R_G = 10k\Omega = R_{in}$.

We used a sine wave 80 mV pp as V_{in} at 100 Hz. We noticed that the circuit was comparating. $V_{out} \approx V_{CC}$ when $V_{in} > 0$. $V_{out} \approx V_{EE}$ when $V_{in} < 0$. The op amp is trying to make the voltage at the noninverting input the same as that of the inverting input.

As we increase the frequency, we lose the square wave output and instead start to approach closer to a sine wave. We noticed this phenomena begin around 2000 Hz and we observed that it failed to comparate at 10 kHz. This is due to the slowness of the op amp to switch between V_{CC} and V_{EE}, which are spaced roughly 30 V apart, a huge gain compared to the 80 mV pp input signal. As the frequency exceeds the op amp's ability to switch, we start working in the non-ideal linear region where the op amp is between the rail voltages.

Figure 7.2 illustrates this phenomena as we observed.

Question 2

We are instructed to build the inverting amplifier shown in Figure 7.3 using a gain ratio of $R_f/R_{in} = 10$.

We then measured the gain as a function of frequency shown in Table 7.1.

We repeated this measurement for a gain ratio of 100 shown in Table 7.2. The discrepancy in max gain from the design of 100 to the measurement of 36.9 was result of an inability to attenuate the input sufficiently while retaining signal.

We then plotted this data on the same log-log scale to get a rough estimate of the open-loop gain as a function of frequency. This is shown in Figure 7.4.

When analyzing this plot, it appears that the vertical scaling is a function of specified maximum gain whereas the drop off point is consistent regardless of the gain.

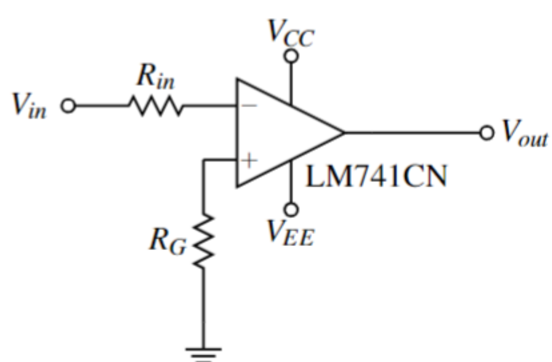

Figure 7.1: Basic operational amplifier circuit

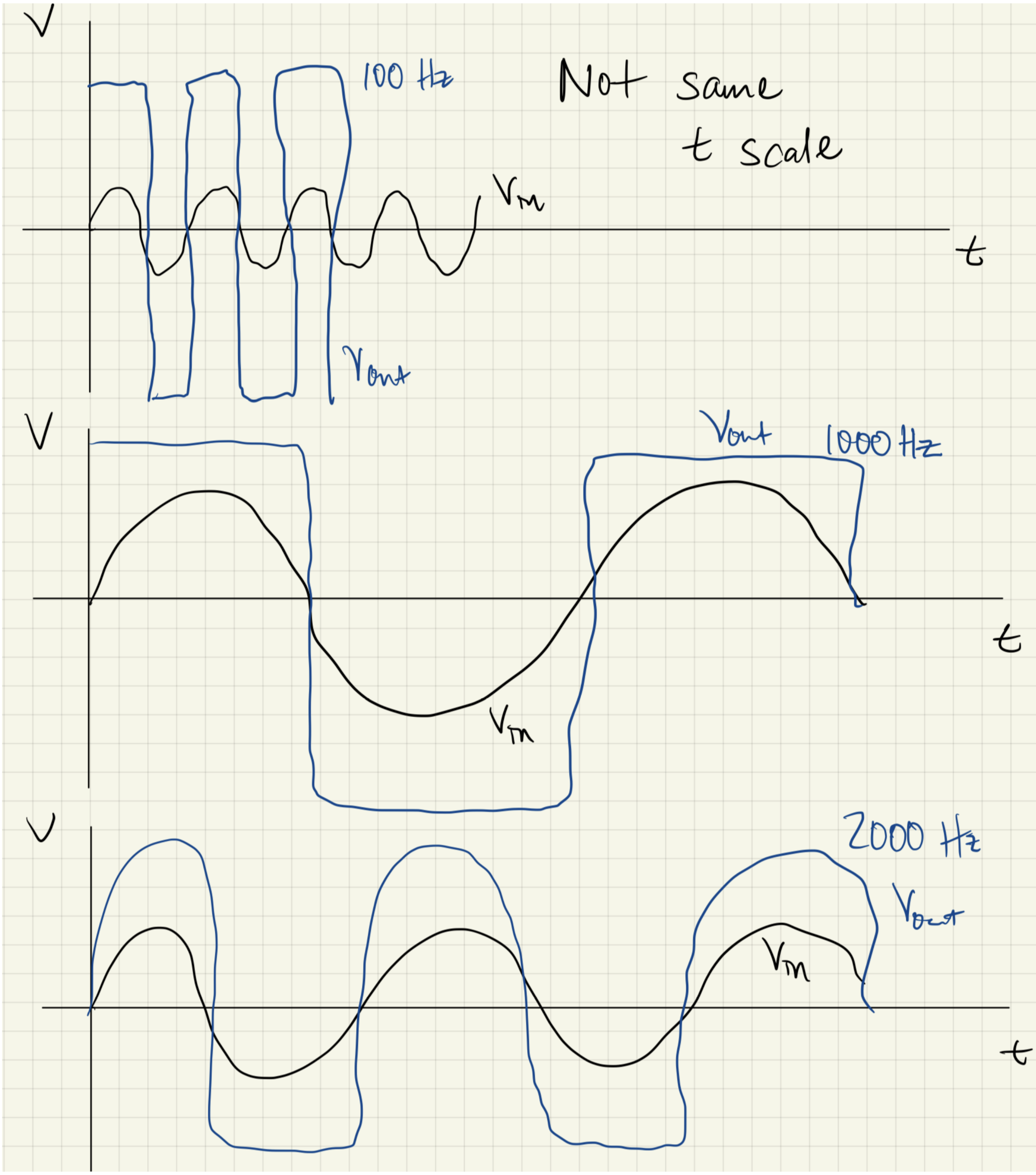

Figure 7.2: Rough sketches illustrating output degeneration of operational amplifier as frequency of input voltage increases

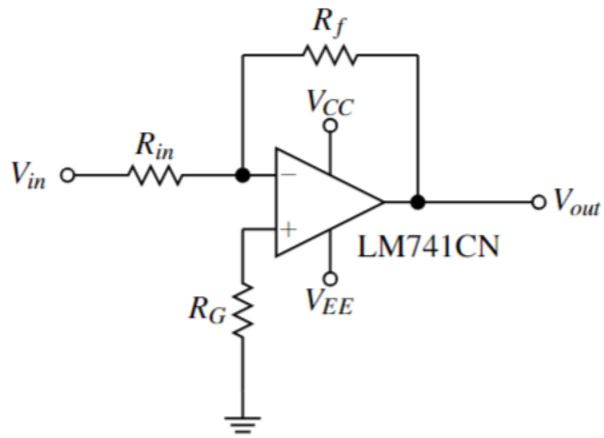

Figure 7.3: Inverting operational amplifier voltage amplifier with gain $= -R_f/R_{in}$

f (Hz)	Gain	f (Hz)	Gain
10	9.5	3000	9.5
20	9.5	10,000	9.4
100	9.5	14,000	9.3
200	9.5	20,000	8.1
400	9.5	24,000	6.9
1000	9.5	30,000	5.8
		35,000	5.0

Table 7.1: Measuring gain of op amp inverting amplifier as a function of input frequency using a designed op amp gain of 10

f (Hz)	Gain	f (Hz)	Gain
100	36.9	10,000	36.7
300	36.9	14,000	26.7
1000	36.9	20,000	19.3
3000	36.9	23,000	16.2
4000	36.9	29,000	13.3

Table 7.2: Measuring gain of op amp inverting amplifier as a function of input frequency using a designed op amp gain of 100

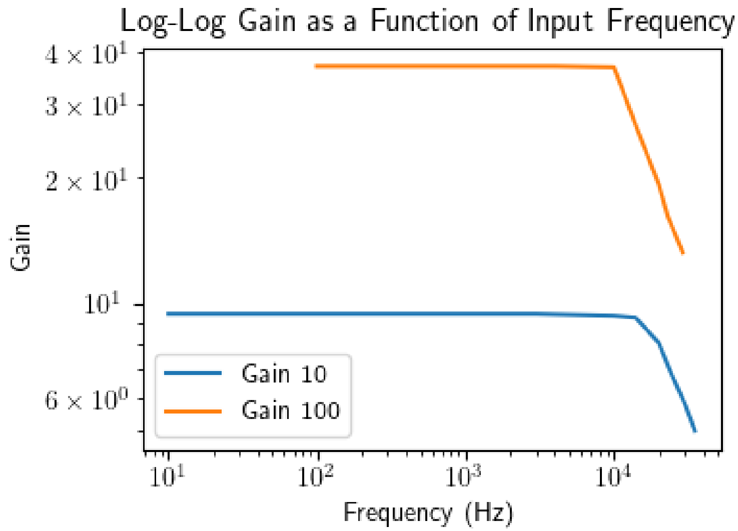

Figure 7.4: Log-log scaled plot of gain as a function of input frequency for two inverting operational amplifiers of gain 10 and 100

Question 3

Two fundamental "rules" of operational amplifiers are:

(1) The output attempts to do whatever is necessary to make the voltage difference between the two inputs zero

(2) The inputs draw no current

When considering an inverting amplifier built using an op amp, we can derive the expression for the gain using the two rules of op amps.

First, we consider that the non-inverting input is at GND. Using **(1)**, we know that the op amp will push the inverting input to GND. Using **(2)**, we know that all current traveling through R_{in} must travel up and through R_f and not into the op amp.

From both of these facts, we can define the current through R_{in} as $i_{in} = V_{in}/R_{in}$. We then consider the current traveling from V_{out} up through R_f which we will call $i_2 = -i_{in}$. Since the voltage at the inverting input is 0, by Ohm's law, $i_2 = -(V_{out} - 0)/R_f = -V_{out}/R_f$.

Now, we have expressions for $V_{in} = i_{in}R_{in}$ and $V_{out} = -i_{in}R_f$. We can express the voltage gain $V_{out}/V_{in} = -R_f/R_{in}$.

Question 4

We then built the circuit shown in Figure 7.5. We first used $V_1 = V_2 = 5$ V DC and $R_1 = R_2 = R_f = 10k\Omega$. We observed $V_{out} = -9.94V$.

Then, we increased V_2 which made V_{out} more negative. Decreasing V_2 made V_{out} more positive. Changing V_1 had the same affect.

Next, we set $V_1 = V_2 = 5$ V DC and started changing values of the resistors as shown in Table 7.3. From these tests, we determined that when $R_1 = R_2 = R_f$, the amplifier sums the two voltages V_1 and V_2 (with a multiplicative factor of -1). However, decreasing either R_1 or R_2 makes the output further negative. Increasing either R_1 or R_2 makes the output less negative, but never positive. Decreasing R_f made the output less negative, but not positive, likely due to lowering the gain of the amplifier.

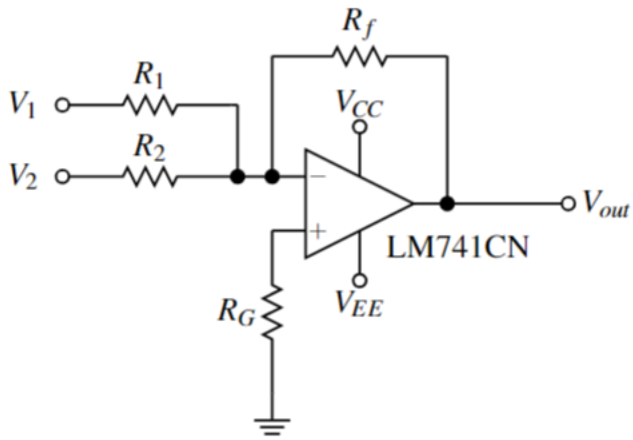

Figure 7.5: Inverting summing operational amplifier circuit

R_1 (kOhm)	10	1	10	10	220
R_2 (kOhm)	10	10	1	10	10
R_f (kOhm)	10	10	10	1	10
V_{out} (V)	-9.9	-15	-15	-1	-5

Table 7.3: Amplifier output dependence on changing R_1, R_2, and R_f one at a time.

Question 5

From Table 7.3, and from my knowledge of amplifiers, I proposed the equation to model the case of a summing inverting amplifier in the case $R_1 = R_2 = R_f$:

$$V_{out} = -(V_1 + V_2)$$

When $R_f \neq R_1 = R_2$, this scales the answer by R_f/R_1 resulting in:

$$V_{out} = -\frac{R_f}{R_1}(V_1 + V_2)$$

Finally, when $R_1 \neq R_2$, the scaling factor is not equal for V_1 and V_2 resulting in:

$$V_{out} = -(\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2)$$

Generalizing this most general equation to N inputs, we yield:

$$V_{out} = -\sum_{i=1}^{N} \frac{R_f}{R_i}V_i$$

It is important to note that if this equation ever yields $|V_{out}| > |V_{EE}|$, $|V_{out}|$ saturates to $|V_{EE}|$.

Bonus 1

Using the summing amplifier in the case $R_1 = R_2 = R_f = 10k\Omega$ for simplicity, we used V_1 and V_2 as 0.2 V pp sine waves.

When these sine waves were identical, we got an output with 0.4 V pp and reflected about the horizontal axis (in reference to $V_1 = V_2$. This is expected as we have made an inverting amplifier which sums the two analog input voltages.

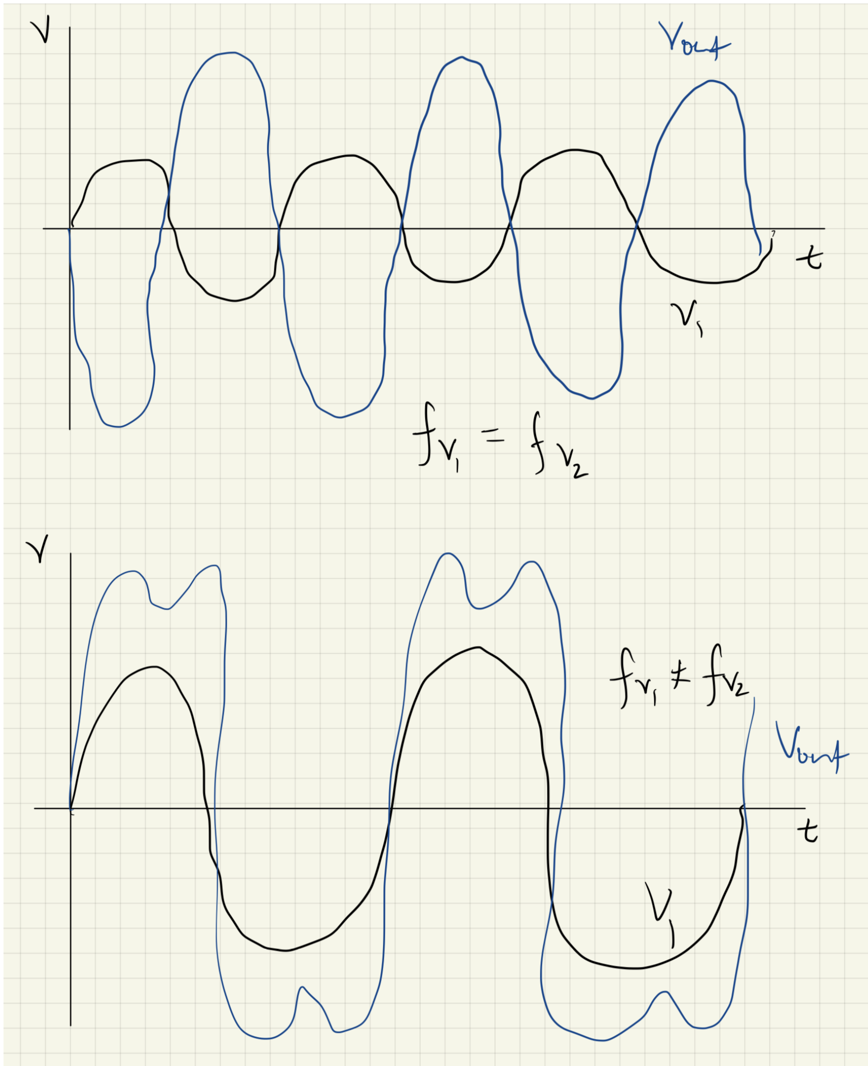

Figure 7.6: Rough sketches illustrating difference in output waveform depending on whether input frequencies match (perfect inverting doubling) or mismatch (beating).

Next, we started to increase the frequency of V_1 while leaving V_2 alone. This quickly creates a beating effect, where $V_{out} \neq -2V_1$. Instead, we produced a complicated signal. This is due to the imbalance in the input voltages for most times, causing both constructive and destructive interference.

A very rough sketch of this phenomena is shown in Figure 7.6.

Lab 8: Operational Amplifiers II

Question 1

We are asked to build the circuit shown in Figure 8.1. We used $V_{CC} = 13$ V and $V_{EE} = -13$ V and $C_f = 18$ nF, $R_G = 10k\Omega$, $Rin = 2k\Omega$. We then drove the input with a 30 Hz square wave and observed the output using an oscilloscope. This produced an inverting amplifier output despite exchanging the feedback resistor with a capacitor.

However, as we increased the frequency, we started to approach the true integral of a square wave, which looks like a "triangle wave". We claimed it operated as an integrator from 3 kHz to about 20 kHz when the response began attenuating. The triangle wave appears in all frequencies above 3 kHz, but the result is no longer the integral of the input function.

Figure 8.2 depicts some rough sketches of our observed outputs.

Question 2

Two fundamental "rules" of operational amplifiers are:

(1) The output attempts to do whatever is necessary to make the voltage difference between the two inputs zero

(2) The inputs draw no current

When considering an inverting amplifier built using an op amp, we can derive the expression for the gain using the two rules of op amps.

First, we consider that the non-inverting input is at GND. Using **(1)**, we know that the op amp will push the inverting input to GND. Using **(2)**, we know that all current traveling through R_{in} must travel up and through C_f and not into the op amp.

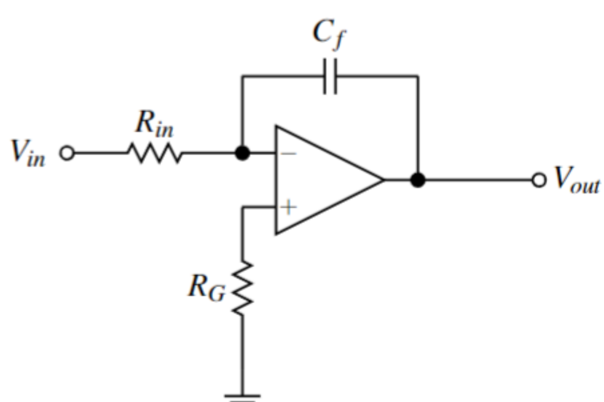

Figure 8.1: Operational amplifier integrator using inverting input capacitor feedback

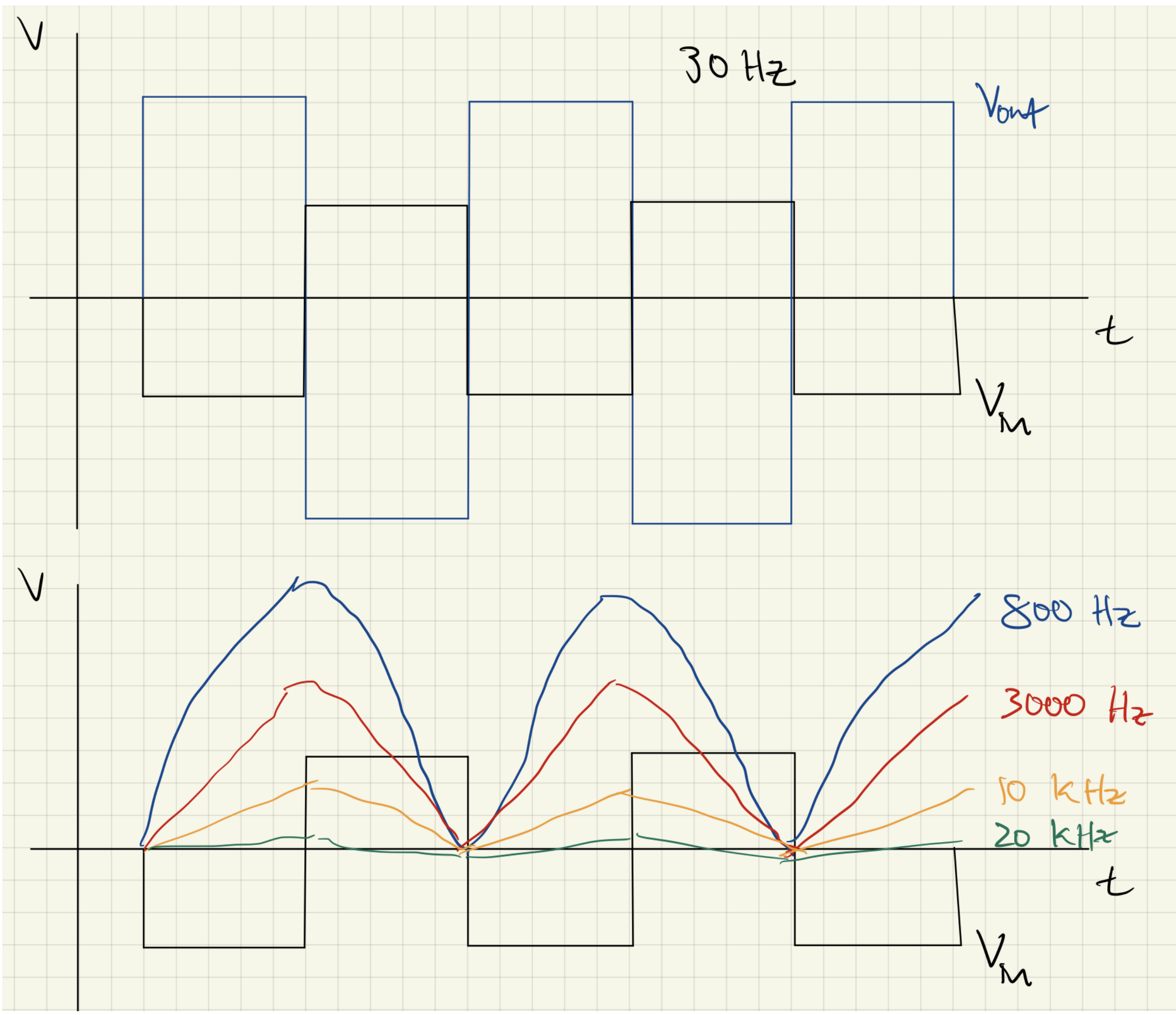

Figure 8.2: Sketches of integration response as a function of driving frequency, transition from inverter to integrator around 3 kHz until attenuation around 20 kHz.

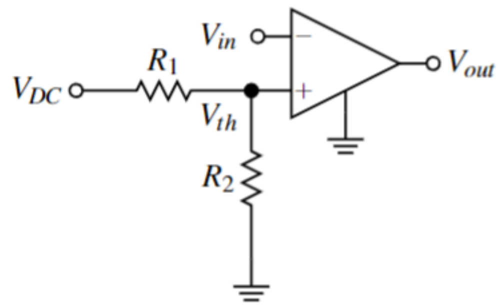

Figure 8.3: Operational amplifier comparator using non-GND threshold voltage via voltage divider on non-inverting input

From both of these facts, we can define the current through R_{in} as $i_{in} = V_{in}/R_{in}$. We then consider the current traveling from V_{out} up through C_f which we will call $i_2 = -i_{in}$. The current through a capacitor is related to the voltage across it by $i = C\frac{dV}{dt}$. Since the voltage at the input terminal is GND, we can express the output voltage merely as the voltage across the capacitor.

Now, we have expressions for $V_{in} = i_{in}R_1$ and $\frac{dV_{out}}{dt} = -i_{in}/C_f$. We can express output voltage as a function of time $V_{out}(t) = \frac{-1}{R_{in}C_f}\int_{t_0}^{t_1} V_{in}(s)\,ds$. (For mathematical rigor, I have performed a dummy change of variables $V_{in}(t) = V_{in}(s), s = t\ \forall t$.

Comparing this to the RC integrator developed in Lab 2, this integrator is stable over a much lower frequency span. The RC integrator only performed integration as we approached 2 MHz where this circuit integrates well at 3 kHz to 20 kHz. This would have better low frequency applications. Additionally, this integrator produces the output multiplied by -1 and shifted negatively.

Question 3

We are then tasked with building the comparator shown in Figure 8.3. Unlike the comparator in Lab 7, this compares V_{in} to $V_{th} \neq$ GND. We used $R_1 = R_2 = 10k\Omega$. Then, we set V_{in} as a sine wave with 4 V pp at 500 Hz. Finally, we supplied several positive DC voltages to the non-inverting input and sketched our observations as shown in Figure 8.4.

As we increased V_{DC}, the graph shifted downward, with less time spent saturated at V_{CC}. This is explainable as the input signal is below the threshold voltage for more total time.

Question 4

Now, we added a 100 kΩ feedback resistor to create a Schmitt trigger as shown in Figure 8.5. We then set V_{in} to be a triangle wave with a 4 V pp at 500 Hz. We then set V_{DC} to 2 V.

With the feedback resistor we saw a switch from low to high voltage output at approximately -1.2 V input and high to low at approximately -1 V input. Without the feedback resistor we saw this switch at -1 V input for both low to high and high to low output switches. Sketches of our observations are shown in Figure 8.6

Question 5

Using the same circuit as Question 4, we now observed the hysteresis curve of the circuit by connecting both input and output of the circuit to the scope in xy mode. Starting with the same triangle wave, we varied the dc offset such that the input got completely above and below the threshold. In both cases, we lost the hysteresis. In the former, the output saturated to V_{CC}, and in the latter, it saturated to GND.

Figure 8.7 depicts sketches of our observations.

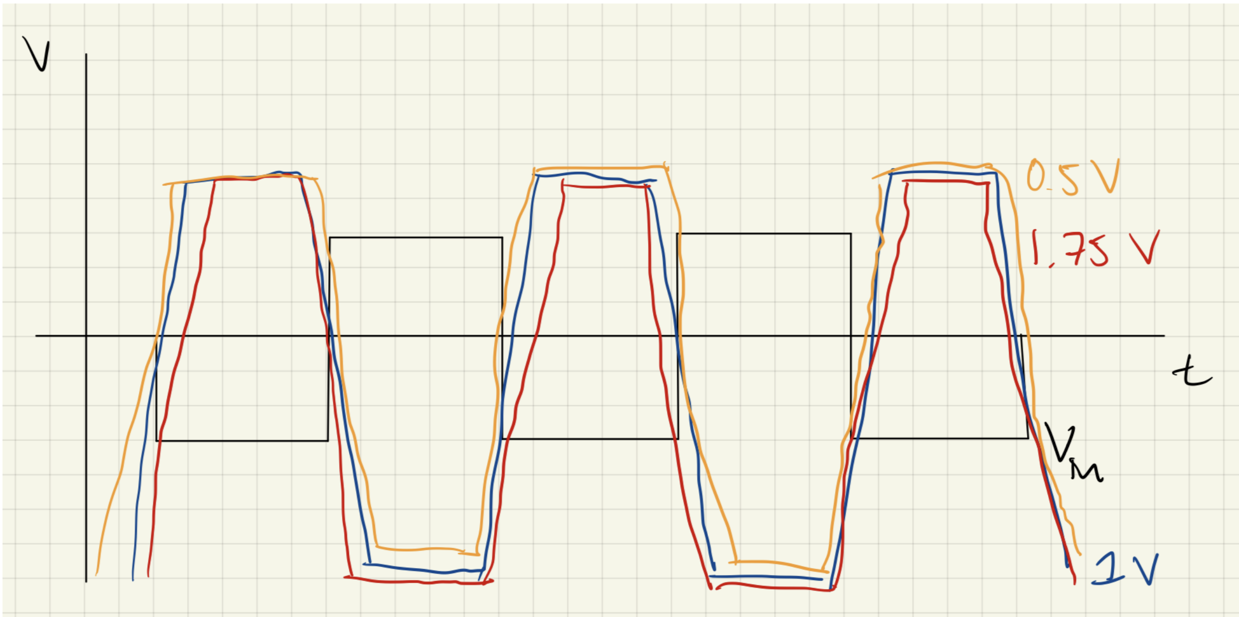

Figure 8.4: Comparator output sketch as a function of DC supply voltage

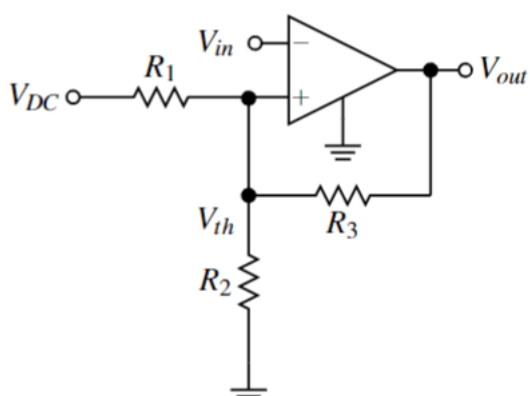

Figure 8.5: Modified operational amplifier comparator using Schmitt trigger feedback resistor

Figure 8.6: Sketch comparing comparator output with and without Schmitt trigger, showing shift in switch voltage in low to high switch (hysteresis)

Figure 8.7: Plot of input function vs output function depicting hysteresis in normal operation, saturation if DC offset applied too heavily

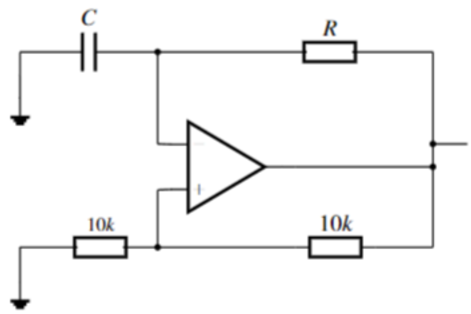

Figure 8.8: Relaxation oscillator built from op amp, capacitor, and resistors

Question 6

For this section, I will use "high" to signify V_{CC} and low to signify GND.

There are two cases of interest in this problem: when V_{in} transitions from low to high and when V_{in} transitions from high to low.

In the former, when V_{in} is low, V_{out} is high. As V_{in} increases, V_{out} transitions to low when V_{in} reaches some threshold voltage V_-. In this region, V_- can be written as a voltage divider across R_1 and R_2 since R_f/ggR_1 and R_2. Therefore, $V_- = \frac{R_1 V_{DC}}{R_1+R_2}$.

In the latter, when V_{in} is high, V_{out} is low. As V_{in} decreases, V_{out} transitions to high when V_{in} reaches some threshold voltage V_+. In this region, we must consider the contribution of R_f. Therefore, $V_+ = \frac{R_1 V_{DC}}{R_1+R_2||R_f} = \frac{R_1 V_{DC}}{R_1+(\frac{1}{R_2}+\frac{1}{R_f})^{-1}}$. This guarantees hysteresis for all values $R_f \neq 0$.

Using these theoretical equations, we can check our observations. $V_+ = 1.05$ V and $V_- = 1.00V$. Our results were slightly more drastic than predicted, but this relied upon our visual inspection of the switching voltage. I am happy with our results.

The entire process of hysteresis relies upon the asymmetry of V_{CC} and V_{EE} along with the asymmetry of R_f in comparison to R_1 and R_2. The asymmetry in power voltages allows for no power to be applied to the op amp when $V_{in} > V_{th}$. Additionally, since R_f is so large, it's contribution in this low current scheme is negligible. However, once the op amp is powered, it plays an effect, creating two different effective resistances by which we determine V_{th}.

Question 7

We are then asked to build the oscillator shown in Figure 8.8 and determine how the frequency relates to the component values used.

First, we used $C = 3$ nF, $R = 50k\Omega$, bottom right $R_r = 50k\Omega$, bottom left $R_l = 10k\Omega$. This produced an oscillating function which saturated for a short duration at both V_{CC} and V_{EE} for a recorded oscillatory frequency of 5.769 kHz.

By reducing $R = 10k\Omega$, we produced a very sharp oscillating triangle wave at 11.84 kHz. Reducing it further to $R = 1k\Omega$ produced an even-higher frequency triangle wave on the order of 100 kHz which we had difficulty measuring exactly.

When we increased to $C = 0.03\mu$F, our frequency decreased to 37.22 kHz while the shape remained a triangle wave.

Sketches of this phenomena is shown in Figure 8.9.

For the first combination, the resultant frequency of 5.769 kHz was below the tolerance of the op amp and it was able to saturate for a short time, as illustrated by the blue sketch in Figure 8.9. Once we changed components and increased the frequency, we always got triangle waves as shown by the red sketch in the same figure. As the frequency increased, the triangle wave became less steep as the op amp struggled to keep up with the "desired" frequency.

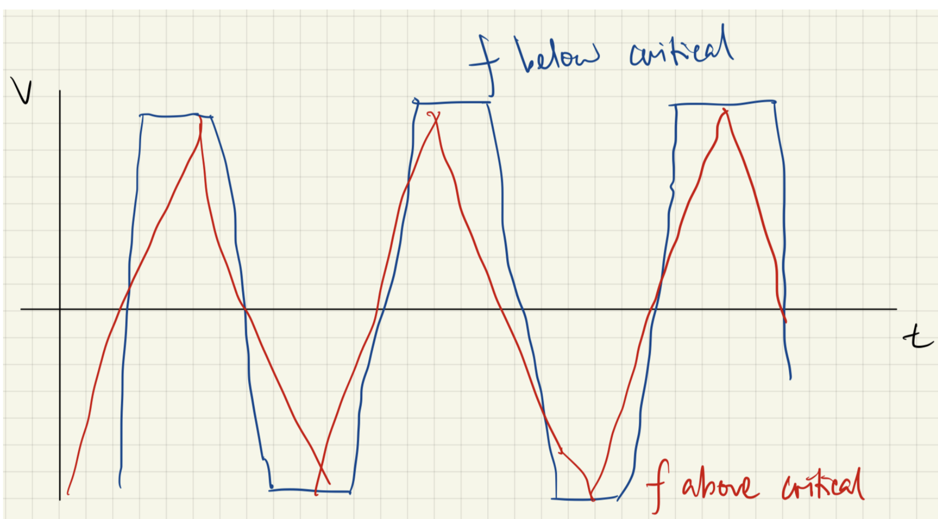

Figure 8.9: Sketches of oscillator when resultant frequency slow enough for op amp to saturate and high enough to create a triangle wave

Lab 9: Johnson Noise

Question 1

First, we are asked to build a simple inverting op amp voltage amplifier with a gain of 10, connecting the non-inverting input to ground with a resistor which we denote as R_N. Later in the lab, this will be our noise resistor. We started with a value of 220 kΩ. Then, we connected the inverting input to ground using a 220 Ω resistor which we denote R_{P1}. Therefore, for a gain of 10, we can use the relationship that gain = R_{f1}/R_{P1}. To reach a gain of 10, we used $R_{f1} = 2.2k\Omega$. We supplied the LM741 operational amplifier with $V_{CC} = 13$ V and $V_{EE} = -13$ V.

We then verified that the amplifier was working properly by driving the inverting amplifier with an attenuated function generator and monitoring the output with an oscilloscope.

Question 2

Next, we are asked to build a simple inverting op amp voltage amplifier with a gain of 100, connecting the non-inverting input to ground with a resistor R_Z much less than R_N; we chose 220 Ω. Then, we connected the inverting input to ground using a 470 Ω resistor which we denote R_{P2}. Therefore, for a gain of 100, we can use the relationship that gain = R_{f2}/R_{P2}. To reach a gain of 10, we used $R_{f2} = 47k\Omega$. We supplied the LM741 operational amplifier with $V_{CC} = 13$ V and $V_{EE} = -13$ V.

We then verified that the amplifier was working properly by driving the inverting amplifier with an attenuated function generator and monitoring the output with an oscilloscope.

Question 3

We then built a two-stage RC bandpass filter with the high pass stage first. We are instructed to build the filter such that the 3 dB points are 100 Hz and 100 kHz.

We struggled to implement the bandpass filter due to high attenuation. Eventually, we settled on values $C_1 = 100$ nF, $R_1 = 16k\Omega$, $C_2 = 30$ pF, $R_2 = 53k\Omega$. These values were found by choosing standard value capacitors and using the relationship that $f_{3dB} = \frac{1}{2\pi RC}$. Using specified capacitor and f_{3dB} values, we can find corresponding $R = \frac{1}{2\pi C f_{3dB}}$.

To try to limit attenuation, we additionally demanded that $R_1 \ll R_2$.

We finally verified our bandpass filter was working by testing its frequency response, driving it with a sine wave, and monitoring it's output attenuation using an oscilloscope. While we had sufficient low pass filtering, our high pass stage was weakly performing for reasons unknown to me. Ultimately, we decided to continue after concluding that the low pass performance was more important as to limit 60 Hz noise from our power source.

Question 4

Finally, we connected our three circuits in order: 10 gain amplifier, bandpass filter, 100 gain amplifier. Figure 9.1 illustrates a diagram of our completed circuit.

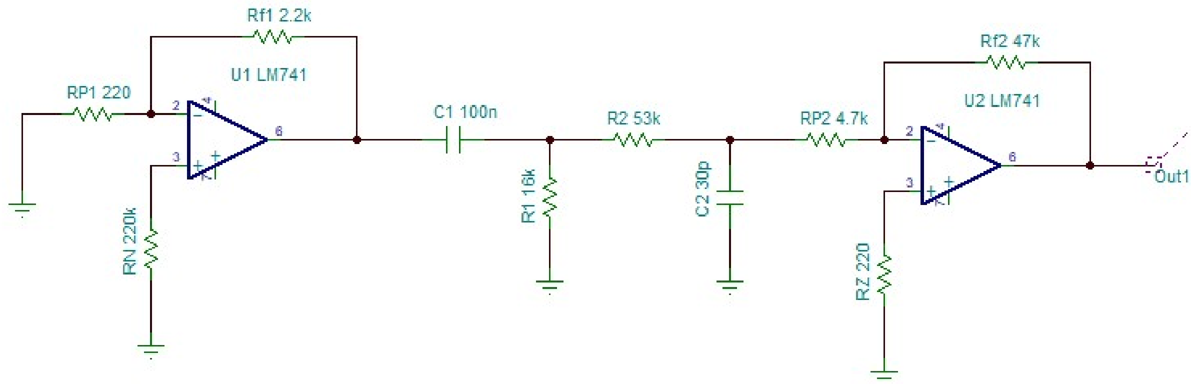

Figure 9.1: A circuit diagram for measuring Johnson noise of a resistor R_N consisting of a pre-amplifier, bandpass filter, and main amplifier. $V_{CC} = 13$ V and $V_{EE} = -13$ V omitted for legibility.

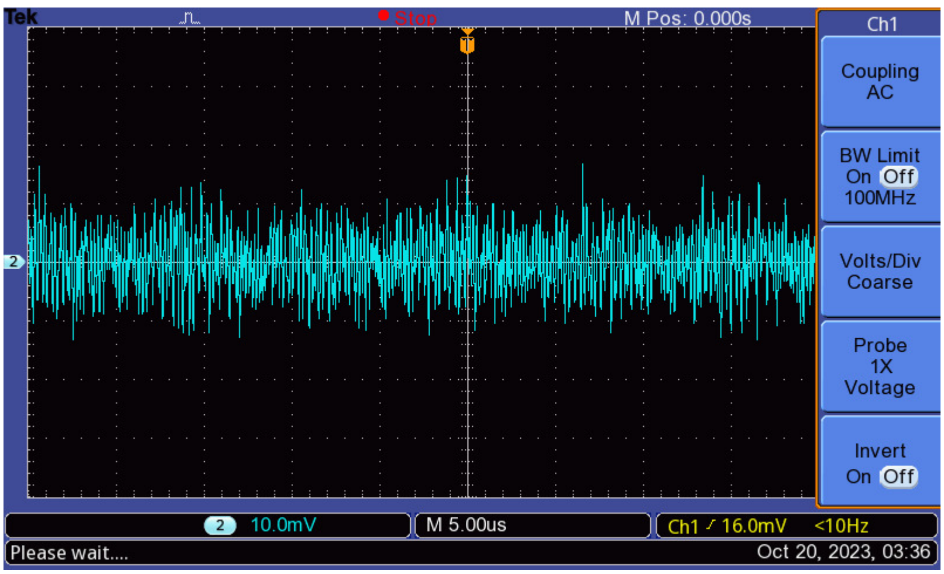

Figure 9.2: A sample trace of the Johnson noise of a resistor as measured in Lab 9

We returned the inverting input of the 10 gain amplifier to ground now that we ensured it's functionality. Then, we connected the output of its op amp to the input of the bandpass filter. We then connected the inverting input of the 100 gain amplifier to the output of the bandpass filter.

Finally, we connected the output of the 100 gain amplifier to our oscilloscope, setting the time base to 5 μs and vertical sensitivity to 10 mV. We observed a noise dominated signal. We then recorded a sweep of the noise using the run/stop function and saved it to a USB for post-processing.

We collected noise data for values of $R_N = 10\Omega, 47k\Omega, 94k\Omega, 220k\Omega, 672k\Omega$. A sample trace is shown in Figure 9.2.

Then, I analyzed our data using an R Script which performed a constant detrend on each raw voltage signals (true voltage - average voltage). This sets the average post-processed signal to 0, meaning all our voltages are now residuals from 0.

Additionally, I subtracted the detrended 10 Ω data from the larger resistor data since the 10 Ω data should theoretically represent the noise of the entire system since R_N is much less than any other resistor in the circuit.

Figure 9.3 illustrates the distributions of the detrended noise signals as a function of resistor value.

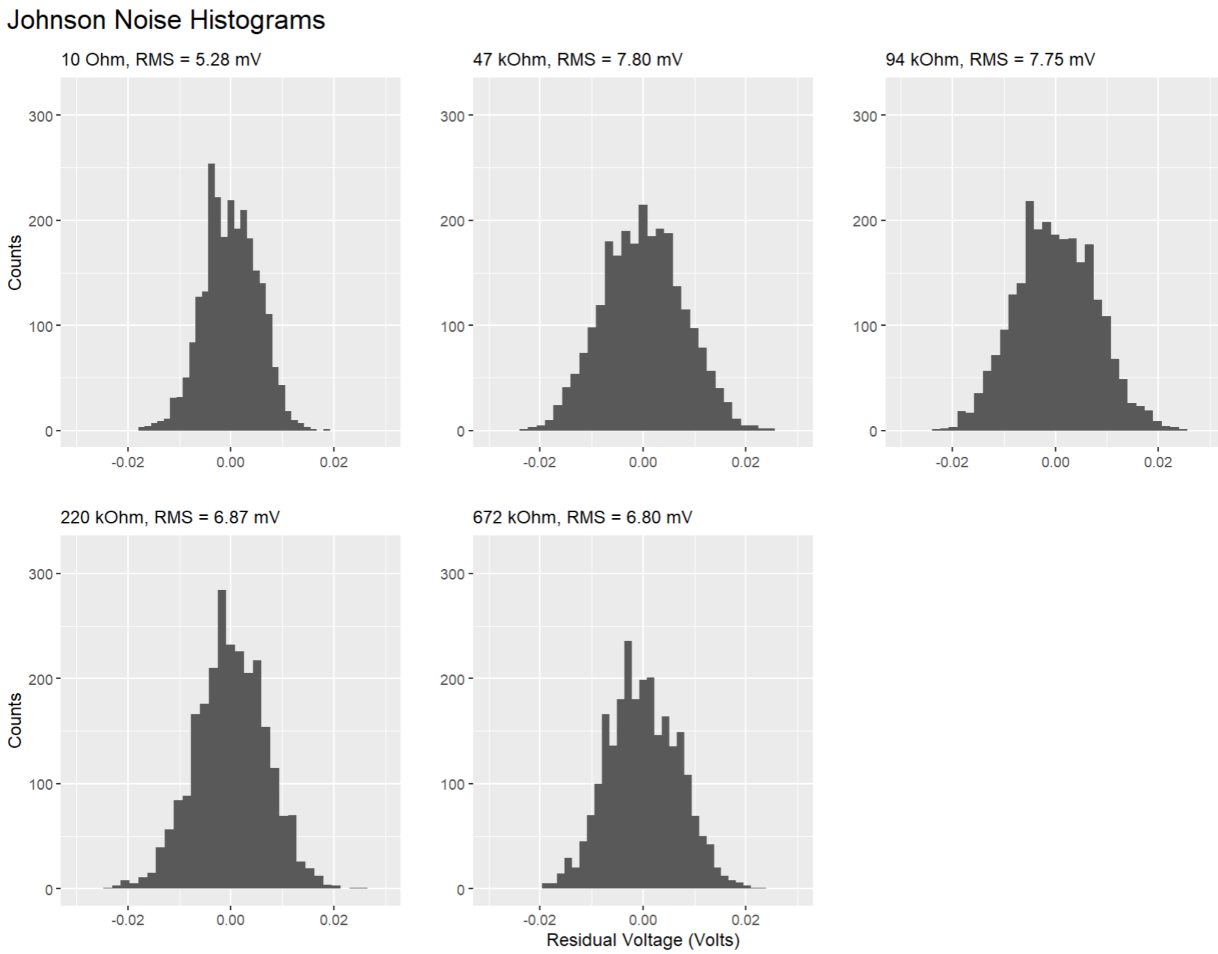

Figure 9.3: Histograms of our noise data along with the corresponding RMS value for each R_N.

Additionally, the RMS of the signal is included. Theoretically, this value should increase linearly with the value of R_N. Unfortunately, we were not able to observe this effect.

Likely, our bandpass filter was attenuating the signal too much to allow for accurate measurement within the uncertainty of the scope.