PHY 338K Lab Report 1

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Contents

Lab 1: Basic Measurements and Oscilloscope Use **4**
Question 1 . . . 4
Question 2 . . . 4
Part A . . . 4
Part B . . . 4
Part C . . . 4
Question 3 . . . 6
Question 4 . . . 6
Question 5 . . . 6
Question 6 . . . 7
Question 7 . . . 7
Question 8 . . . 10
Question 9 . . . 12
Question 10 . . . 12
Question 11: Bonus . . . 12

Lab 2: RC Circuits **14**
Question 1: Circuit A . . . 14
Part A . . . 14
Part B . . . 14
Part C . . . 16
Question 2: Circuit B . . . 16
Part A . . . 16
Part B . . . 16
Part C . . . 18
Question 3 . . . 18
Question 4 . . . 19

Lab 3: Passive Filters **21**
Question 1 . . . 21
Question 2 . . . 22
Question 3 . . . 22
Question 4 . . . 22
Question 5 . . . 24
Question 6 . . . 24
Part A . . . 24
Part B . . . 26
Part C . . . 26

List of Figures

1.1 Sketch of a basic breadboard depicting vertical and horizontal electronic connections 5
1.2 A simple open circuit consisting of a 10 kΩ resistor 5
1.3 A simple closed circuit consisting of a 10 kΩ resistor and an ammeter 5
1.4 A simple voltage divider consisting of two 10 kΩ resistors in series 6
1.5 A generalized voltage divider consisting of two resistors, R_1 and R_2, in series 7
1.6 Comparing terminated and unterminated function generator circuits 8
1.7 Observed sine waves with and without 50 Ω terminator 8
1.8 Showing the erradic behavior of the sinusoid displayed when the trigger was above the maximum or below the minimum of the signal 9
1.9 Showing the stable behavior of the sinusoid displayed when the trigger was set to TTL or when it was within the vertical limits of the signal 9
1.10 Illustrating the difference between AC and DC coupling on the same signal with a 1V DC offset 10
1.11 A simple circuit illustrating a voltage divider between the output impedance of a function generator, R_{out}, and a load resistance, R_{Load} 11
1.12 A plot of R_{Load} vs V_{out} illustrating a fit between experimental results and theoretical predictions 11
1.13 Sketch illustrating wave form of 110kHz square wave when measuring rise time 12
1.14 Sketch illustrating wave form of 100kHz square wave when measuring fall time 13
2.1 Circuit A: a low pass filter consisting of a 10 kΩ resistor in series with a 0.01 μF capacitor . . 14
2.2 Sketching the time-domain waveform corresponding to Circuit A driven by a square wave without DC offset 15
2.3 A plot illustrating the frequency dependence of the output of Circuit A with outputs denoted by color 15
2.4 Depicting frequency dependence of phase shift and gain of a low pass circuit 16
2.5 Circuit B: a high pass filter consisting of a 10 kΩ resistor in series with a 0.0015 μF capacitor 17
2.6 Sketching the time-domain waveform corresponding to Circuit B driven by a square wave without DC offset 17
2.7 A plot illustrating the frequency dependence of the output of Circuit B with outputs denoted by color 18
2.8 Depicting frequency dependence of phase shift and gain of a high pass circuit 19
3.1 Diagram of a simple RC bandpass circuit 21
3.2 Diagram of a simple RCL circuit measuring V_{out} across the resistor 22
3.3 Diagram of a LRC bandpass circuit 23
3.4 Illustrating the waveform dependence on frequency, V_{low} below resonance, $V_{resonant}$ at resonance, V_{high} above resonance 23
3.5 Illustrating the waveform dependence on the choice of resistor, easier to observe resonance with smaller resistance 24
3.6 Diagram of a different LRC bandpass filter 25
3.7 Illustrating the relationship between circuit gain and frequency of AC input voltage 25

3.8 Plot of the relationship between current through resisitor and frequency of a LRC bandpass circuit 27

List of Tables

1.1 Table illustrating the dependence of V_{out} on R_{Load} corresponding to the circuit illustrated in Figure 1.11 10
1.2 Table illustrating the dependence of pulse frequency and waveform on our voltmeter reading . 12
2.1 Table illustrating the dependence of gain and phase shift on frequency corresponding to the circuit illustrated in Figure 2.1 16
2.2 Table illustrating the dependence of gain and phase shift on frequency corresponding to the circuit illustrated in Figure 2.5 18
3.1 Table illustrating the dependence of gain on frequency corresponding to the circuit illustrated in Figure 3.1 21
3.2 Table illustrating the dependence of gain on frequency corresponding to the circuit illustrated in Figure 3.6 26
3.3 Table illustrating the dependence of I_R on frequency corresponding to the circuit illustrated in Figure 3.6 26

Lab 1: Basic Measurements and Oscilloscope Use

Question 1

Our breadboard consists of several sections of rails which can run horizontally and vertically.Each independent rail is connected beneath the breadboard, serving as a wire to connect circuit components. Figure 1.1 illustrates a simplified version of a breadboard with two vertical rails along the outer edge, one red and one black, and several horizontal rails in blue along the length of the breadboard. In reality, these metal rails are beneath the holes of the breadboard, but for simplicity, this diagram displays the rails as continuous lines. This pattern of vertical rails and horizontal rails continues several times throughout the breadboard.

Question 2

Part A

Using our breadboard, we constructed the circuit shown in Figure 1.2. We then set V_{in} to +5 V and measured the voltage difference between V_{out} and GND to be +5.01 V, which we deemed acceptable to the uncertainty of our multimeter or the electronic within the breadboard.

Part B

Then, we constructed the closed circuit as shown in Figure 1.3 and measured the short circuit output current to be 0.497 mA. Checking against a theoretical prediction using Ohm's Law, this experimental result makes sense.

$$V = IR$$
$$5.01\text{V} = I(10,000\Omega)$$
$$I = 5.01\text{V}/10,000\Omega \; I = 0.501\text{mA}$$

Part C

To calculate the output impedance, we divide the open circuit output voltage which we measured in Part A by the short circuit output current which we measured in Part B.

$$Z = V_{open}/I_{short} = 5.01\text{V}/0.497\text{mA} \approx 10,080\Omega \tag{1.1}$$

An impedance of roughly 10,080Ω makes sense given our simple circuit containing a resistor of 10,000Ω. Due to variability in true resistance intrinsic to the manufacturing process, it is very likely our resistor is not exactly 10 kΩ.

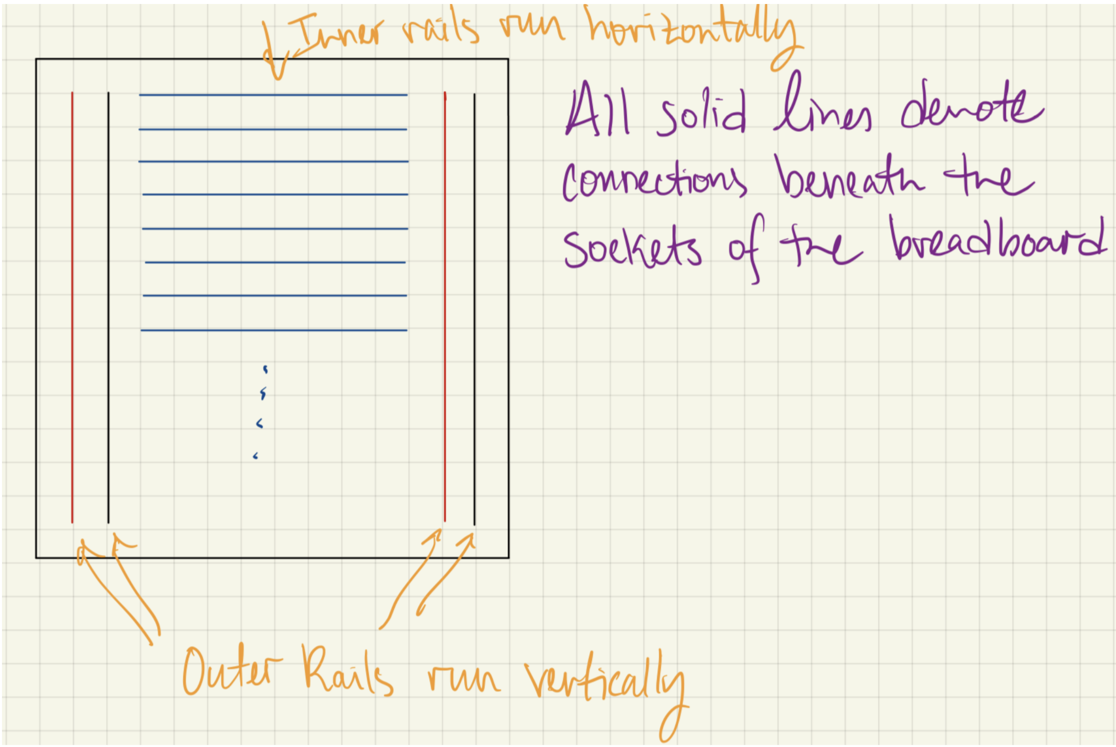


Figure 1.1: Sketch of a basic breadboard depicting vertical and horizontal electronic connections

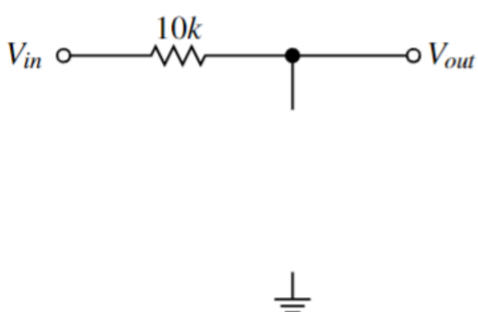


Figure 1.2: A simple open circuit consisting of a 10 kΩ resistor

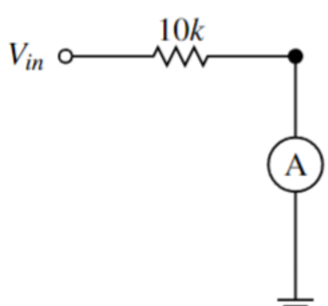


Figure 1.3: A simple closed circuit consisting of a 10 kΩ resistor and an ammeter

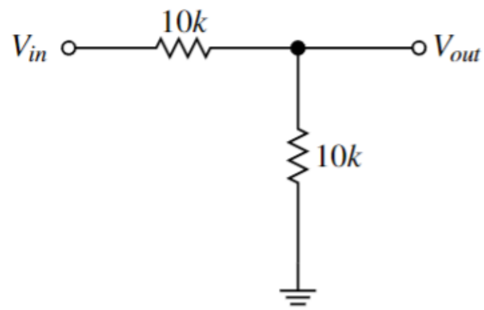


Figure 1.4: A simple voltage divider consisting of two 10 kΩ resistors in series

Question 3

We then constructed a simple voltage divider as shown in Figure 1.4 by adding another 10 kΩ resistor in series. We measured the output voltage V_{out} to be 2.51 V.

Theoretically, a voltage divider should produce an output voltage as follows:

$$V_{out} = V_{in} R_2/(R_1 + R_2) \tag{1.2}$$

where R_1 and R_2 are the two resistors in series and R_2 is the resistor in which the electric potential is measured across. This is often referred to as the voltage divider formula.

To verify the validity of our measurement, we calculated $V_{out} = 5.01\text{V} * 10,000\Omega/(20,000\Omega) = 2.505\text{V}$, matching our result very closely.

Question 4

Equation 1.2 can easily be derived through Ohm's Law, $V = IR$. The following derivation analyzes the circuit depicted in Figure 1.5.

$$V_{in} = IR_{total}$$
$$V_{in} = I(R_1 + R_2)$$
$$I = V_{in}/(R_1 + R_2)$$

$$V_{out} = IR_2$$
$$V_{out} = V_{in} R_2/(R_1 + R_2)$$

There are two limiting cases of interest: $R_2 \gg R_1$ and $R_1 \gg R_2$. In the former, the contribution of R_1 in the denominator becomes negligible, and we can ignore it, leaving $V_{out} = V_{in} R_2/R_2 = V_{in}$. This makes theoretical sense; as R_2 dominates the voltage divider, it "takes" all the input voltage. In the latter case, the contribution of R_1 far surpasses that of R_2, causing the denominator to increase to a value much greater than the numerator. This limiting case results in V_{out} becoming a $1/\infty$ scenario which goes to 0.

Question 5

Moving away from our voltage divider circuit, we now explored our function generator. We set the output waveform to be a sine wave with frequency of 1 kHz. Using the output amperage, we manually adjusted the peak-to-peak (pp) until it read 2 Volts. We observed the waveform, then terminated the BNC cable with a T and a 50Ω terminator. This resulted in the pp dropping to 360 mV. However, there was no period change or phase shift as a result of this change. The difference between the two waveforms, can be seen in Figure 1.7. Since the oscilloscope directly measures voltage, not current, we can think of it as a voltmeter.

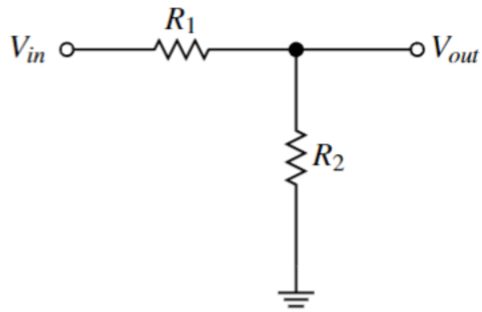


Figure 1.5: A generalized voltage divider consisting of two resistors, R_1 and R_2, in series

To characterize the change in pp, we considered that the terminated system must be acting as a pair of resistors in parallel such that the terminator's effective resistance is much greater than the impedance of the oscilloscope which was set to 1 MΩ. The two circuit diagrams are shown in Figure 1.6.

Question 6

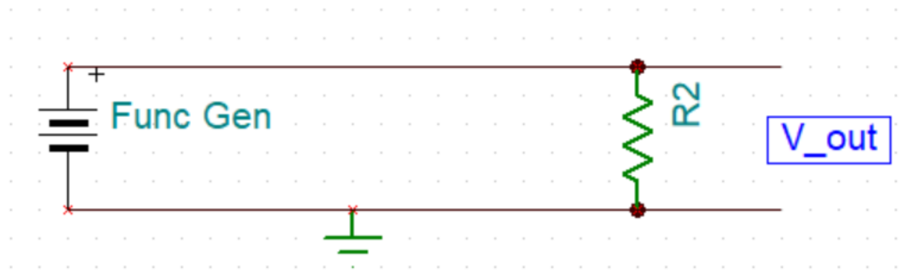
Now, removing the terminator, we investigate the signal characteristics' dependence on the trigger level. When the trigger was well within the sine wave, the oscilloscope displayed a standing wave as if it was taking a picture. However, when the trigger level was beyond the maximum or minimum of the sine wave, the sine wave began moving erradically and it was difficult to see the standing wave. There was no difference in amplitude. This behavior is shown in Figure 1.8.

Now understanding the dependence of waveform stability on the location of the trigger level, we connected another BNC cable to the TTL output of the function generator. Now triggering on this signal, we noticed that we could move the trigger level beyond the sine wave, but within the TTL square wave, and both waves would remain still. This could allow us to quickly trigger a small amplitude wave using a larger amplitude TTL. This behavior is illustrated in Figure 1.9.

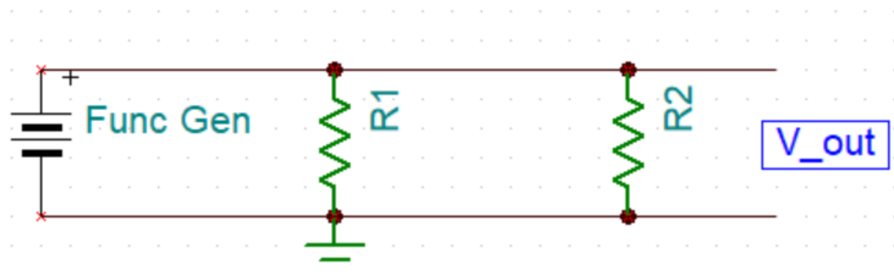
Question 7

Returning to triggering on the output of the function generator, we switched it to a square wave. We then switched between DC and AC coupling while adjusting the DC offset using supplemental voltage from the breadboard. Interestingly, in AC coupling mode, adjusting the DC offset knob resulted in the signal moving vertically on the oscilloscope, but in DC coupling mode, there was no difference.

This is likely since DC coupling removes underlying direct current "noise" to center the oscillating signal around 0 whereas AC coupling allows this underlying direct current "noise." This difference is illustrated in Figure 1.10.



(a) A simple circuit showing R_2 as the impedance of the oscilloscope.



(b) The same circuit, now with a 50 Ω resistor (terminator), denoted as R_1, placed in parallel with the impedance.

Figure 1.6: Comparing terminated and unterminated function generator circuits

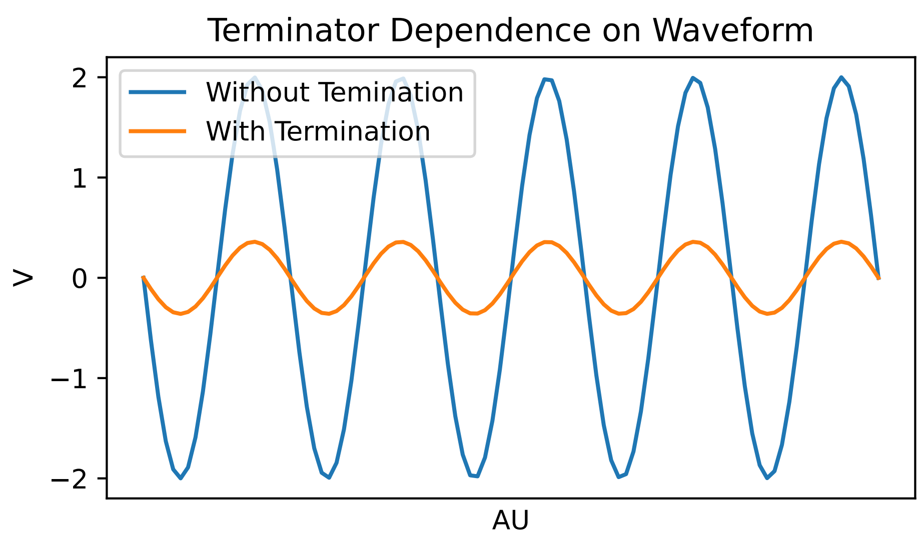


Figure 1.7: Observed sine waves with and without 50 Ω terminator

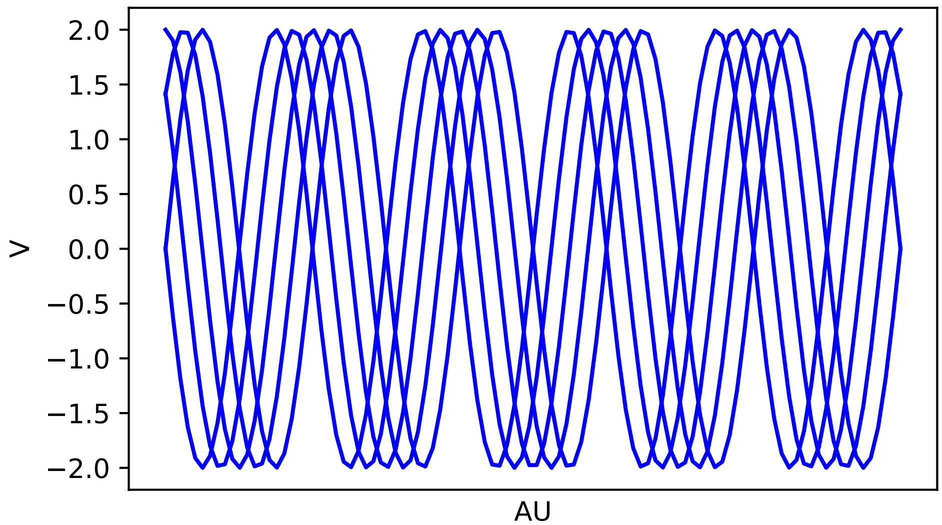


Figure 1.8: Showing the erradic behavior of the sinusoid displayed when the trigger was above the maximum or below the minimum of the signal

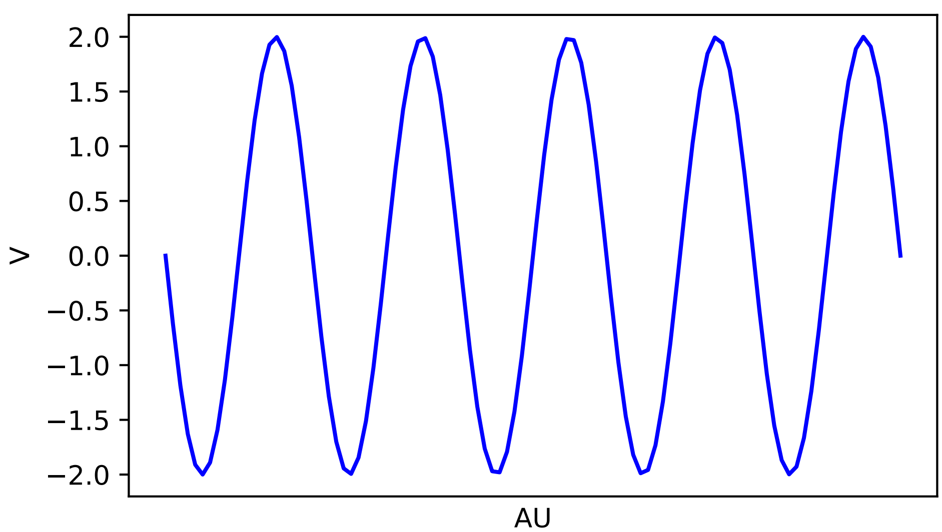


Figure 1.9: Showing the stable behavior of the sinusoid displayed when the trigger was set to TTL or when it was within the vertical limits of the signal

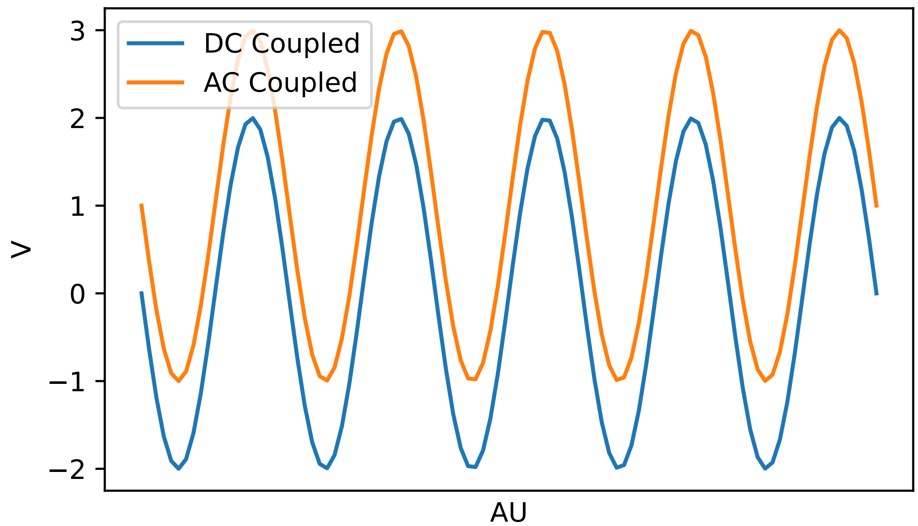


Figure 1.10: Illustrating the difference between AC and DC coupling on the same signal with a 1V DC offset

Question 8

Since the function generator has its own output impedance, we investigated the relationship between V_{out} and R_{Load} by constructing a voltage dividing circuit as shown in Figure 1.11. Using a set value of V = 2.0 V, we used 10 different values for R_{Load} and recorded our data in Table 1.1.

$R_{Load}(\Omega)$	V_{out} (V)	$R_{Load}(\Omega)$	V_{out} (V)
5.3	16e-3	25.0	82e-3
7.1	24e-3	2.3e3	1.6
10.6	32e-3	5.0e3	1.8
12.5	42e-3	5.1e3	2.0
16.7	56e-3	10.0e3	2.0

Table 1.1: Table illustrating the dependence of V_{out} on R_{Load} corresponding to the circuit illustrated in Figure 1.11

To quantify this relationship, we will manipulate the voltage divider formula for the circuit illustrated in Figure 1.11:

$$V_{Out} = V\tfrac{R_{Load}}{R_{out}+R_{Load}}$$

$$V_{Out} = V\tfrac{R_{Load}}{R_{Load}(R_{out}/R_{Load})+1}$$

$$V_{Out} = \tfrac{V}{R_{out}/R_{Load}+1}$$

$$\frac{1}{V_{Out}} = \frac{R_{out}/R_{Load}+1}{V} \tag{1.3}$$

Equation 1.3, when graphed as $1/V_{out}$ vs $1/R_{Load}$ with the realistic restriction that R_{Load} must be positive, produces a roughly linear graph on our closed domain as illustrated Figure 1.12. When fitting to the theoretical graph using a Python script, I found parameters $V = 2.2 \pm 7.1\text{e-}2$ V and $R_{out} = 797 \pm 166\Omega$.

Visually, the fit could be better. A possible source of error is the fluctuations in voltage measurements produced by the oscilloscope at low amplitude signals, likely driven by noise. This led us to reduce significant figures in an effort to read the noisy measurement.

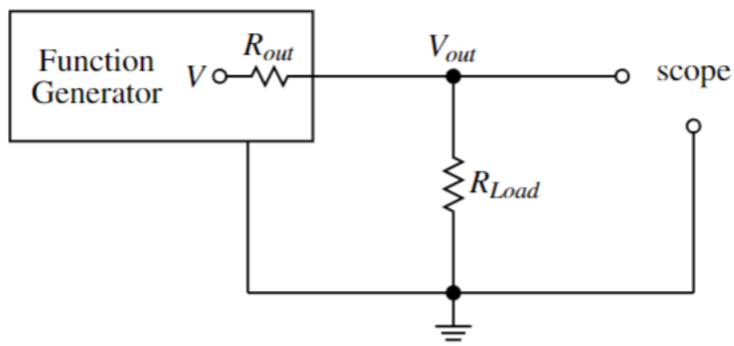


Figure 1.11: A simple circuit illustrating a voltage divider between the output impedance of a function generator, R_{out}, and a load resistance, R_{Load}

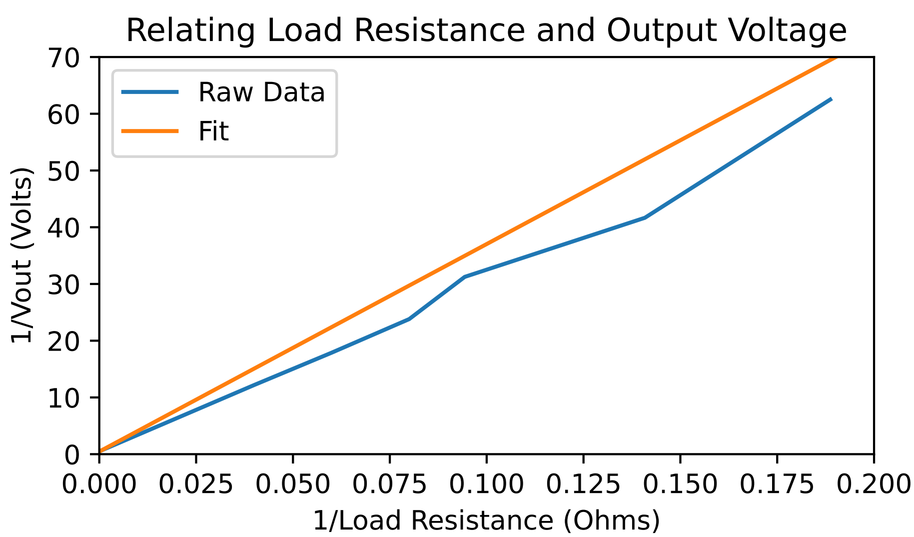


Figure 1.12: A plot of R_{Load} vs V_{out} illustrating a fit between experimental results and theoretical predictions

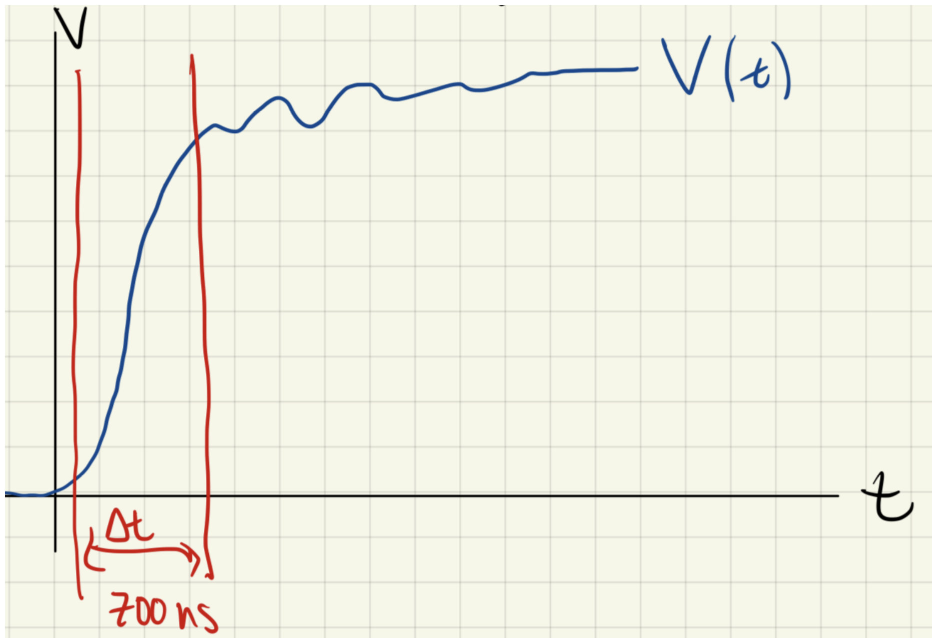


Figure 1.13: Sketch illustrating wave form of 110kHz square wave when measuring rise time

Question 9

Since a voltmeter measures the Root Mean Square (RMS) amplitude of a time varying signal, we investigated our voltmeter's outputs to theory using a 10 Hz, 1 kHz, and 100 kHz sine, square, and triangle wave, each of amplitude 1.00 V. Table 1.2 displays our experimental results with those predicted by the RMS formulas for a sine, square, and triangle wave respectively: $V/\sqrt{2}$, V, and $V/\sqrt{3}$.

	Theory	10 Hz	1 kHz	100 kHz
Sine	0.71 V	0.680 V	0.651 V	0.8 mV
Square	1.00 V	1.10 V	0.98 V	1.0 mV
Triangle	0.58 V	0.54 V	0.52 V	0.7 mV

Table 1.2: Table illustrating the dependence of pulse frequency and waveform on our voltmeter reading

All three waveforms depict major deviations from theoretical at the 100 kHz frequency. This likely indicates that there exists some maximum realizable frequency innate to the voltmeter.

Question 10

To measure the rise time of a square wave, we set the function generator to a square wave at 110 kHz, the maximum frequency we could generate. We measured the rise time as the difference in times between 10% of the rise and 90% of the rise. We observed a 700 ns rise time. Considering each pulse has a period of $1/110$ kHz $= 9.1\mu s$, this seems a reasonable answer. Figure 1.13 displays a sketch of the observed pulse. We noticed that as the pulse reached saturation, the signal jittered slightly.

Question 11: Bonus

To measure the fall time of a square wave, we set the function generator to a square wave at 100 kHz, triggered on the rise of the square wave, then delayed until the fall. We measured the fall time as the difference in times between 90% of the fall and 10% of the fall. We observed a 600 ns rise time. Considering

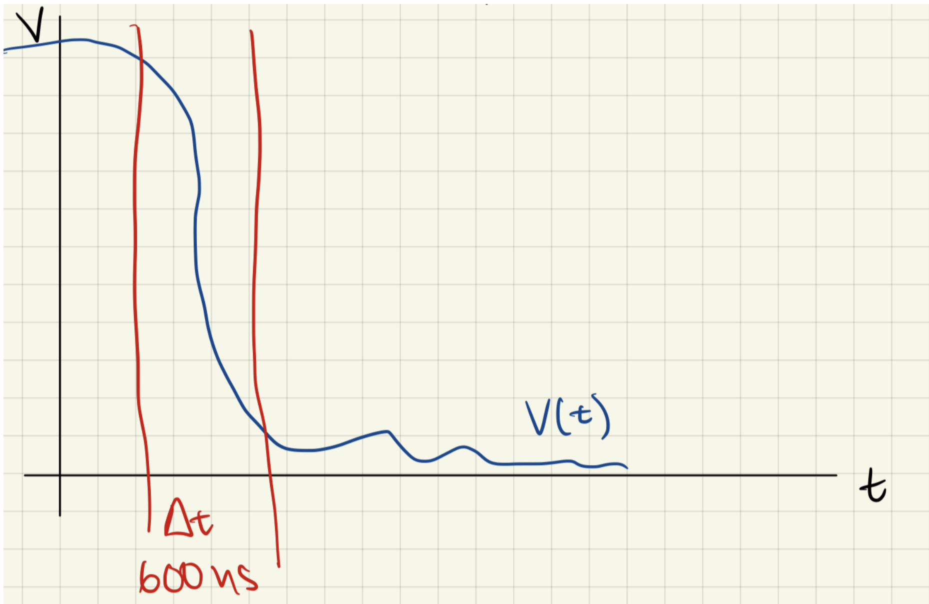


Figure 1.14: Sketch illustrating wave form of 100kHz square wave when measuring fall time

each pulse has a period of 1/100 kHz = 10μs, this seems a reasonable answer. Figure 1.14 displays a sketch of the observed pulse. We noticed that as the pulse reached minimum, the signal jittered slightly, just as had occurred on the rise time pulse.

Lab 2: RC Circuits

Question 1: Circuit A

Part A

We are asked to build circuit A as illustrated in Figure 2.1 with $R = 10k\Omega$ and $C = 0.01\mu F$. Then, we drive it with a square wave with no DC offset, adjusting the frequency and horizontal display until we can see the rise and fall times easily on the screen. We set the oscilloscope to DC coupling mode and observed the output.

Using the built-in rise and fall time measurements, we observed a rise time of $270\mu s$ and a fall time of $276\mu s$ compared to the theoretical rise/fall time of 3 times the time constant $3RC = 300\mu s$. While the fall time was marginally longer than the rise time, both are close to the theoretical prediction. A sketch of our waveform is illustrated in Figure 2.2. It looks almost like a sea wave, with a concave down rising and falling edge, and a corner at the maximum and minimum values.

Part B

To observe the "integrator" nature of the circuit in the regime where $\omega \gg 1/RC$, we drove the circuit with a square wave output, increasing the frequency in relatively small steps. We also adjusted the amplitude of the input to show that the circuit is behaving as expected. An illustration of the frequency dependence of the waveform is shown in Figure 2.3. As the frequency increases, the square wave approaches an oscillatory wave which almost appears to be composed of triangles, with harsh inclines and declines of approximately the same absolute value of their slopes which meet at a corner at the maximum and minimum value.

We confirmed that the circuit was integrating as the output was the sum of the area under the curve. When the input wave went negative, the output wave started to decrease. Conversely, when the input wave jumped to positive, the output wave began to increase.

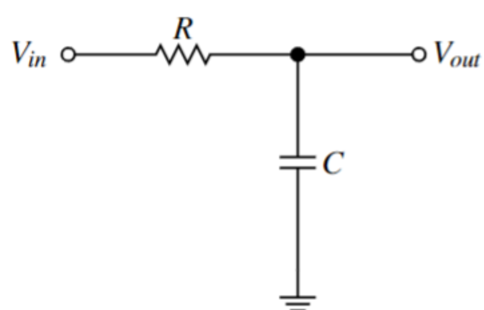


Figure 2.1: Circuit A: a low pass filter consisting of a 10 kΩ resistor in series with a 0.01 μF capacitor

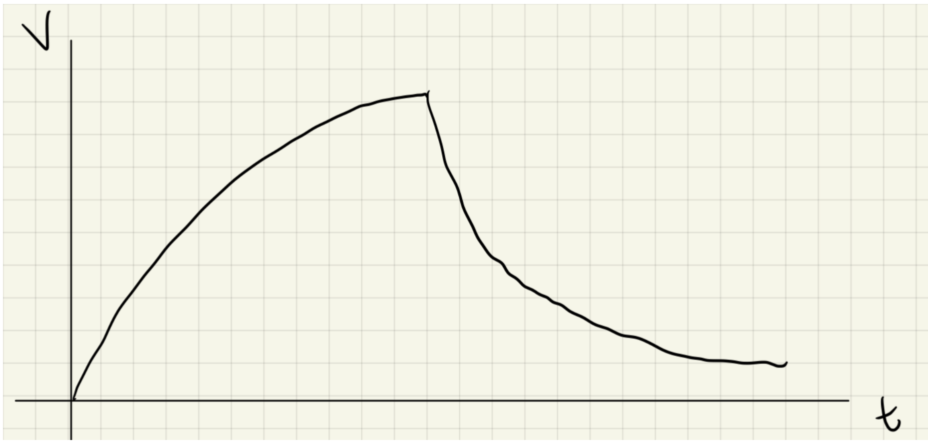
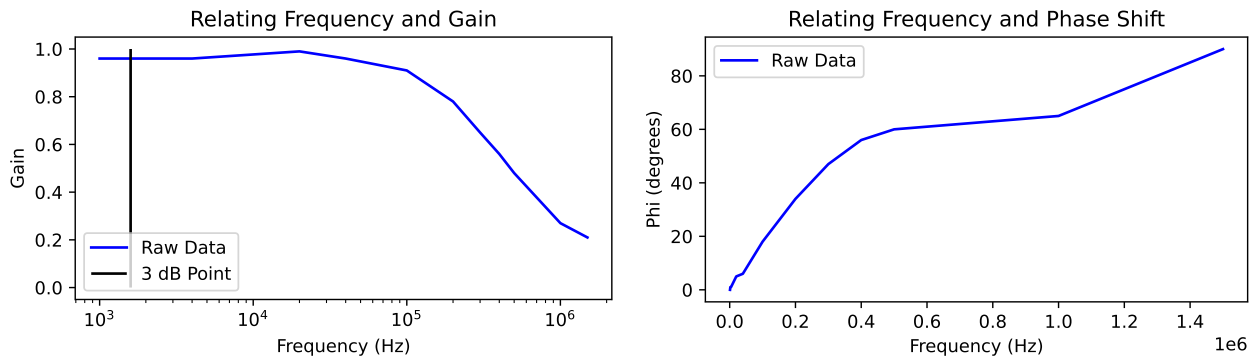


Figure 2.2: Sketching the time-domain waveform corresponding to Circuit A driven by a square wave without DC offset

Figure 2.3: A plot illustrating the frequency dependence of the output of Circuit A with outputs denoted by color



(a) Illustrating exponential decay of gain as frequency increases along with 3 dB point

(b) Illustrating increase of phase shift as frequency increases

Figure 2.4: Depicting frequency dependence of phase shift and gain of a low pass circuit

Part C

To investigate the function dependence of the gain (V_{out}/V_{in}) and phase shift ϕ as a function of input frequency, we drove the circuit with a sine wave and scanned various frequencies. Our data is shown in table 2.1 and plotted in Figure 2.4.

f (kHz)	Gain	ϕ (degrees)	f (kHz)	Gain	ϕ (degrees)
1	0.96	0?	200	0.78	34
2	0.96	1?	300	0.65	47
4	0.96	1?	400	0.56	56
20	0.99	5	500	0.48	60
40	0.96	6	1000	0.27	65
100	0.91	18	1500	0.21	90

Table 2.1: Table illustrating the dependence of gain and phase shift on frequency corresponding to the circuit illustrated in Figure 2.1

Question 2: Circuit B

Part A

We are asked to build circuit B as illustrated in Figure 2.5 with $R = 10k\Omega$ and $C = 0.0015\mu F$. Then, we drive it with a square wave with no DC offset, adjusting the frequency and horizontal display until we can see the fall time easily on the screen. We set the oscilloscope to DC coupling mode and observed the output. We did not observe a rise time.

Using the built-in fall time measurement, we observed a fall time of 53μs compared to the theoretical fall time of 3 times the time constant $3RC = 45\mu$s. While the observed fall time was marginally longer than the theoretical, it is quite close. A sketch of our waveform is illustrated in Figure 2.6. It looks like several jagged peaks extending in the positive and negative vertical directions with near vertical edges as it rises toward an extreme value (either positive or negative) and an exponential "decay" toward 0.

Part B

To observe the "differentiator" nature of the circuit in the regime where $\omega \ll 1/RC$, we drove the circuit with a square wave output, decreasing the frequency in relatively small steps. We also adjusted the amplitude of the input to show that the circuit is behaving as expected. An illustration of the frequency dependence

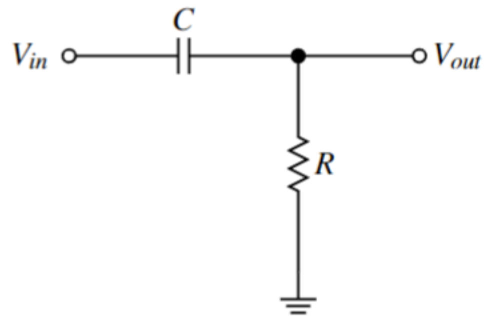


Figure 2.5: Circuit B: a high pass filter consisting of a 10 kΩ resistor in series with a 0.0015 μF capacitor

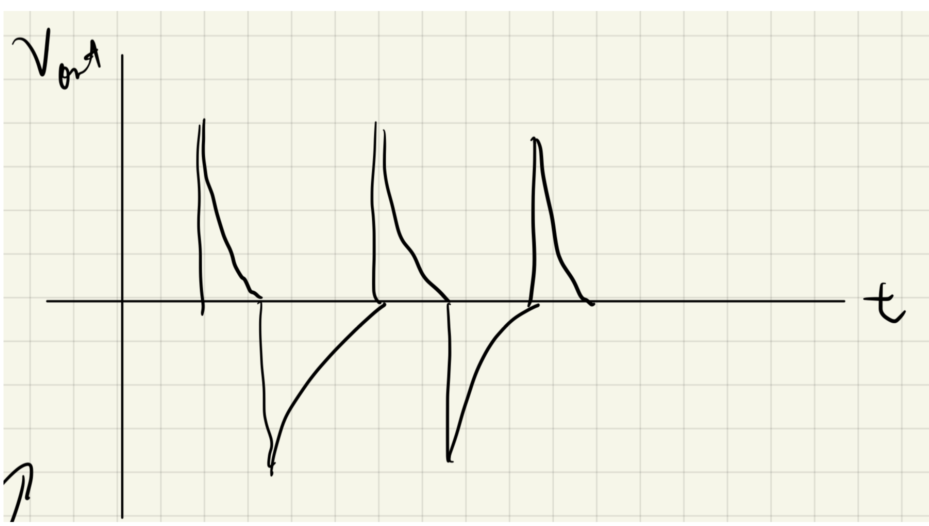


Figure 2.6: Sketching the time-domain waveform corresponding to Circuit B driven by a square wave without DC offset

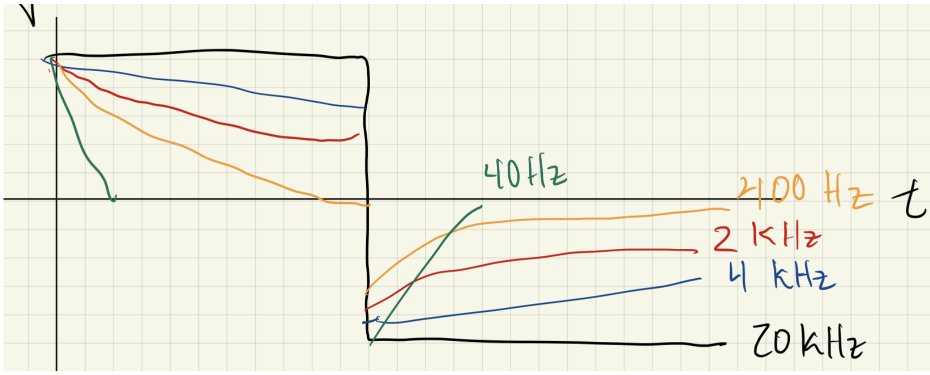


Figure 2.7: A plot illustrating the frequency dependence of the output of Circuit B with outputs denoted by color

of the waveform is shown in Figure 2.7. As the frequency decreases, the square wave approaches a the odd jagged nature referred to in Part a. Additionally, as frequency decreases, the amount of the wave sustained at positive or negative saturation decreases.

We confirmed that the circuit was differentiating as the output was the slope of the input. Since the slope of a vertical line is undefined, the output was not well-defined at the rising or falling edge of the input wave. At lower frequencies, the output displays how the input square wave is in fact not a perfect square wave, yielding a small but non-zero derivative.

Part C

To investigate the function dependence of the gain (V_{out}/V_{in}) and phase shift ϕ as a function of input frequency, we drove the circuit with a sine wave and scanned various frequencies. Our data is shown in table 2.2 and plotted in Figure 2.8.

f (Hz)	Gain	ϕ (degrees)	f (Hz)	Gain	ϕ (degrees)
40	0.16	85	2000	0.89	8
100	0.30	74	3000	0.89	6
200	0.50	58	4000	0.91	4
300	0.63	45	10000	0.92	0?
400	0.70	40	20000	0.92	0?
1000	0.86	16	40000	0.91	0?

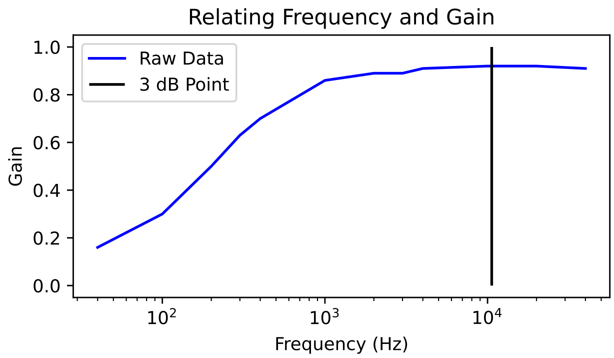
Table 2.2: Table illustrating the dependence of gain and phase shift on frequency corresponding to the circuit illustrated in Figure 2.5

Question 3

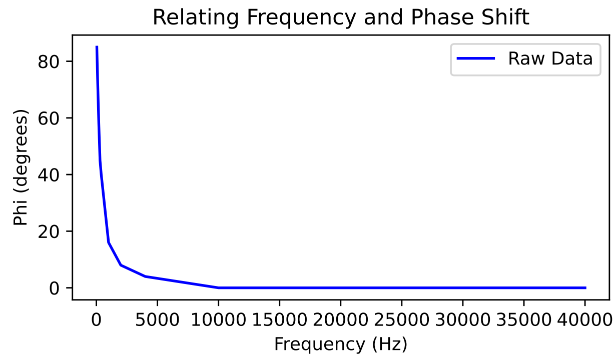
To compare our results from Problem 1 to theory, we will derive the expected gain and phase shift dependencies on frequency.

Using the complex representations of voltage and current, a modified Ohm's law, and the impedance of a capacitor and resistor in series, we can define gain as the ratio of V_{out}/V_{in}.

$$\mathbf{Z} = R - \frac{j}{\omega C}$$
$$\mathbf{I} = \frac{\mathbf{V_{in}}}{\mathbf{Z}} = \frac{V_{in}}{R - \frac{j}{\omega C}}$$
$$\mathbf{V_{out}} = \mathbf{I}Z_c = \frac{\mathbf{I}}{\omega C}$$
$$\mathbf{V_{out}} = \frac{V_{in}}{(R - \frac{j}{\omega C})(\omega C)}$$



(a) Illustrating exponential decay of gain as frequency increases along with 3 dB point



(b) Illustrating decrease of phase shift as frequency increases

Figure 2.8: Depicting frequency dependence of phase shift and gain of a high pass circuit

$$V_{out} = \frac{V_{in}(R+j/\omega C)}{(R-j/\omega C)(\omega C)(R+j/\omega C)}$$
$$V_{out} = V_{in}\frac{1}{\sqrt{1+(\omega RC)^2}}$$
$$\text{Gain} = \frac{1}{\sqrt{1+(\omega RC)^2}} = \frac{1}{\sqrt{1+(2\pi fRC)^2}}$$

This appears to follow our experimental results decently.

Next, we can use this gain, along with some basic assumptions about the waveform of the input and output voltage to derive the phase shift as a function of frequency.

$$\mathbf{V_{in}} = V_{in}e^{j\omega t} \therefore V_{in} = \mathbf{V_{in}}e^{-j\omega t}$$
$$\mathbf{V_{out}} = V_{in}e^{j(\omega t+\phi)} \therefore V_{out} = \mathbf{V_{out}}e^{-j(\omega t+\phi)}$$
$$\frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{1+(\omega RC)^2}} = \Re\left(\frac{\mathbf{V_{out}}e^{-j(\omega t+\phi)}}{\mathbf{V_{in}}e^{-j\omega t}}\right) = \Re(e^{-j\phi}) = \cos\phi$$
$$\phi = \arccos\left(\frac{1}{\sqrt{1+(\omega RC)^2}}\right) = \arccos\left(\frac{1}{\sqrt{1+(2\pi fRC)^2}}\right)$$

This appears to follow our experimental results moderately. Toward higher frequencies, there seems to be something pushing the phase shift to 90°.

Question 4

To compare our results from Problem 2 to theory, we will derive the expected gain and phase shift dependencies on frequency.

Using the complex representations of voltage and current, a modified Ohm's law, and the impedance of a capacitor and resistor in series, we can define gain as the ratio of V_{out}/V_{in}.

$$\mathbf{Z} = R - \frac{j}{\omega C}$$
$$\mathbf{I} = \frac{\mathbf{V_{in}}}{\mathbf{Z}} = \frac{V_{in}}{R-\frac{j}{\omega C}}$$
$$\mathbf{V_{out}} = \mathbf{I}Z_r = \mathbf{I}R$$
$$\mathbf{V_{out}} = \frac{V_{in}R}{(R-\frac{j}{\omega C})}$$
$$V_{out} = \frac{V_{in}R(R+j/\omega C)}{(R-j/\omega C)(R+j/\omega C)}$$
$$V_{out} = V_{in}\frac{R}{\sqrt{R^2+(\omega C)^2}}$$
$$\text{Gain} = \frac{R}{\sqrt{R^2+(\omega C)^2}} = \frac{R}{\sqrt{R^2+(2\pi fC)^2}}$$

This appears to follow our experimental results.

Next, we can use this gain, along with some basic assumptions about the waveform of the input and output voltage to derive the phase shift as a function of frequency.

$$\mathbf{V_{in}} = V_{in}e^{j\omega t} \therefore V_{in} = \mathbf{V_{in}}e^{-j\omega t}$$
$$\mathbf{V_{out}} = V_{in}e^{j(\omega t+\phi)} \therefore V_{out} = \mathbf{V_{out}}e^{-j(\omega t+\phi)}$$
$$\frac{V_{out}}{V_{in}} = \frac{R}{\sqrt{R^2+(\omega C)^2}} = \Re(\frac{\mathbf{V_{out}}e^{-j(\omega t+\phi)}}{\mathbf{V_{in}}e^{-j\omega t}}) = \Re(e^{-j\phi}) = \cos\phi$$
$$\phi = \arccos(\frac{R}{\sqrt{R^2+(\omega C)^2}}) = \arccos(\frac{R}{\sqrt{R^2+(2\pi f C)^2}})$$

This appears to follow our experimental results very well.

Lab 3: Passive Filters

Question 1

First, we are asked to set up a two stage RC bandpass filter as shown in Figure 3.1. We are given the restriction that $1\ \mathrm{M}\Omega \gg R_2 \gg R_1 \gg 50\Omega$ and $1/R_1C_1 \approx 1/R_2C_2 \approx 2\pi * 10$ kHz $= 62.8$ kHz. We chose $R_1 = 1470\Omega, C_1 = 10\mathrm{nF}, R_2 = 15\mathrm{k}\Omega, C_2 = 0.001\mu\mathrm{F}$.

Checking our choices:

$$1/(1470\Omega * 10\ \mathrm{nF}) \approx 68\ \mathrm{kHz}$$
$$1/(15\mathrm{k}\Omega * 0.001\mu\mathrm{F}) \approx 67\ \mathrm{kHz}$$

Therefore, our bandpass filter should have a slightly wider bandwidth than ideal.

To test the frequency response of our circuit, we drove it with a sine wave input without DC offset and measured the gain as a function of frequency, recording our results in Table 3.1.

f (kHz)	Gain	f (kHz)	Gain
0.040	0.77	300	0.77
0.200	0.91	400	0.68
0.400	0.91	600	0.45
4	0.91	800	0.39
8	0.91	1000	0.35
80	0.86	1200	0.29
100	0.86	1600	0.23
200	0.86	2000	0.19
240	0.82		

Table 3.1: Table illustrating the dependence of gain on frequency corresponding to the circuit illustrated in Figure 3.1

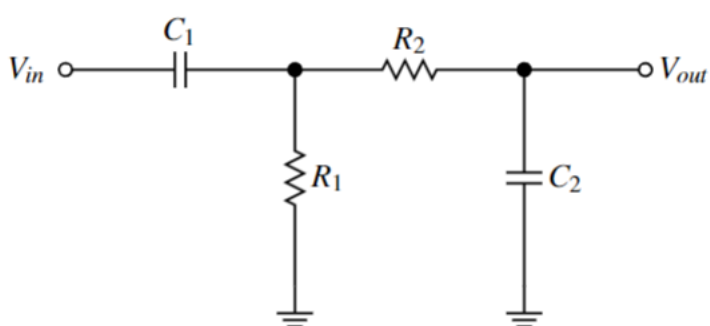


Figure 3.1: Diagram of a simple RC bandpass circuit

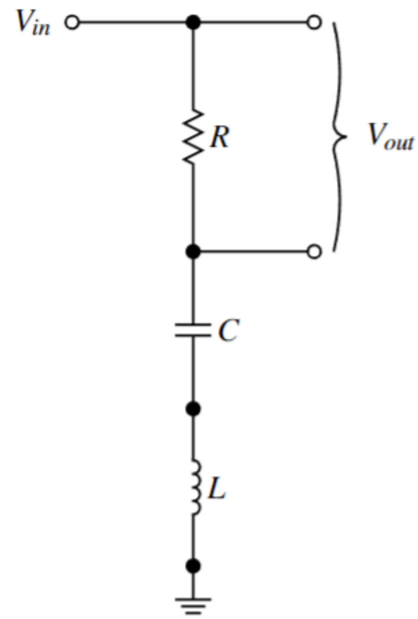


Figure 3.2: Diagram of a simple RCL circuit measuring V_{out} across the resistor

Question 2

From the previous restrictions on R_1 and R_2, we can treat this circuit as a high pass followed by a low pass filter acting independently. Therefore, to derive the theoretical gain as a function of frequency, we merely define an intermediate voltage V_h as the voltage across R_1. Therefore, we can define Gain_h as V_h/V_{in}. Similarly, we can define Gain_l, the gain of the low pass portion, as V_{out}/V_h. By multiplying the two gains together, we get the true gain V_{out}/V_{in}.

Thankfully, we have already derived Gain_h and Gain_l in Lab 2, Question 3 and 4. After multiplication, we get a true gain of $\frac{R_1}{(1+R_1^2+(\omega R_2 C_2)^2+(\omega C_1)^2)^{1/2}}$ or $\frac{R_1}{(1+R_1^2+(2\pi f R_2 C_2)^2+(2\pi f C_1)^2)^{1/2}}$.

In the regime $R_2 \gg R_1$, this equation simplifies to $\frac{1}{sqrt(1+(\omega R_2 C_2}$, acting as if the low pass filter is not even there.

Question 3

We are then asked to construct the LRC circuit shown in Figure 3.2 using a 220 Ω resistor and a capacitor such that $2\pi * 150$ kHz $\approx \omega_0 = 1/sqrt(LC)$. We used a 5 nF capacitor and a 20 μH inductor.

When measuring the output, we attempted to see the resonance peak by continuously varying the frequency near ω_0, but were unable to. The circuit acted as a notch filter such that the gain approached 0 as we approached resonance. It also appeared to shift the signal vertically upwards.

Our failure to observe resonance occurred since measuring the output with an oscilloscope "grounds out" the capacitor and inductor, making the circuit act as a simple voltage divider. Instead, we may want to measure the voltage across LC such that $V_{in} = V_R + V_{LC}$.

Question 4

Next, we built the bandpass circuit shown in Figure 3.3 using the same values for the resistor, capacitor, and inductor. Then, we were able to observe resonance by continuously varying the frequency near ω_0. A sketch of the change in waveform as we approached resonance is illustrated in Figure 3.4.

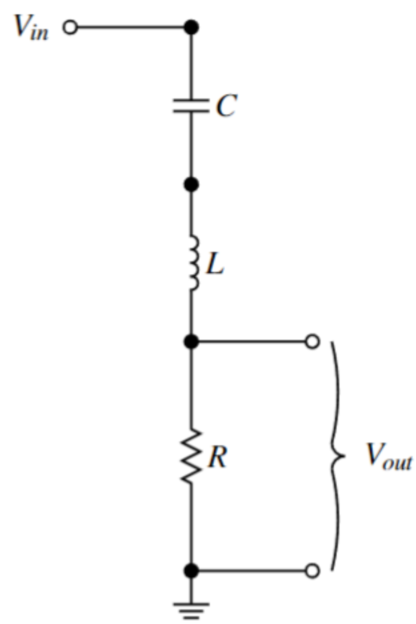


Figure 3.3: Diagram of a LRC bandpass circuit

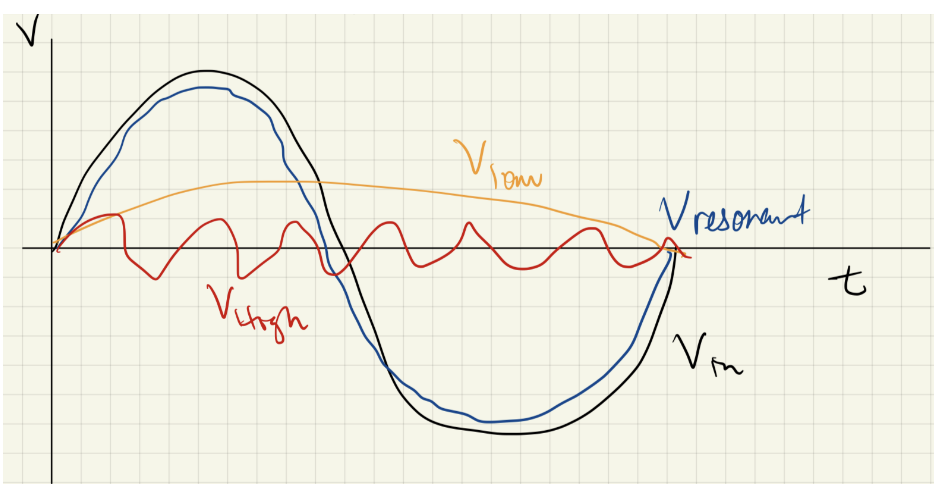


Figure 3.4: Illustrating the waveform dependence on frequency, V_{low} below resonance, $V_{resonant}$ at resonance, V_{high} above resonance

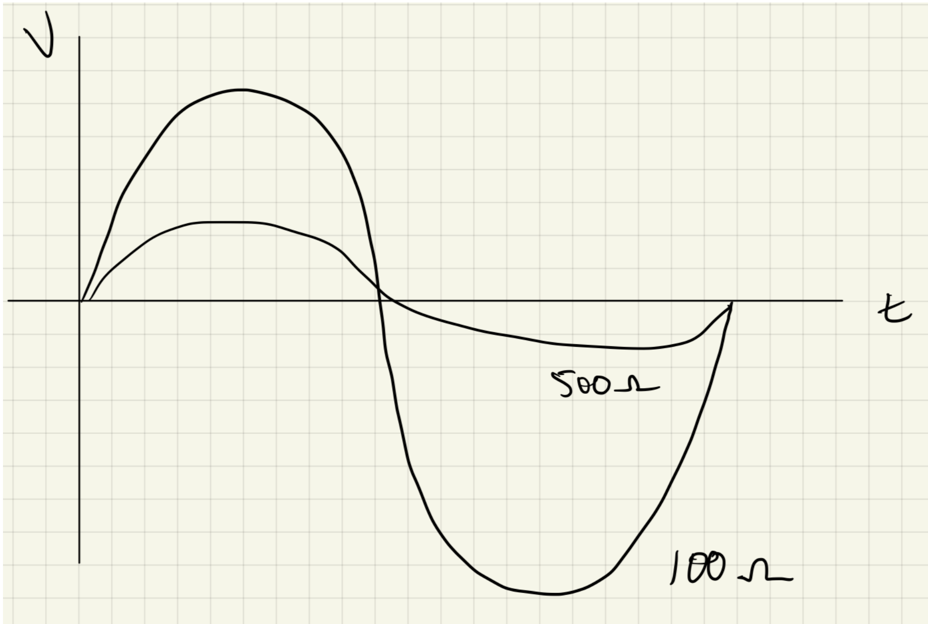


Figure 3.5: Illustrating the waveform dependence on the choice of resistor, easier to observe resonance with smaller resistance

Using our observed resonant frequency of 480 kHz, we can calculate our inductance using the relationship as follows:

$$\omega_0 = 2\pi f = 1/\sqrt{LC}$$
$$L = 1/(C(2\pi f)^2) = 1/(5\text{nF}(2\pi(480\text{kHz}))) = 22.0\mu\text{H}.$$

This matches our stated inductance of 20 μH quite well.

Question 5

Using the same circuit, but replacing the resistor with a 470 Ω resistor, we had a much harder time observing the resonance. Then, we replaced the resistor with a 100 Ω resistor which made it very easy to observe the resonance compared to the other two systems.

Therefore, we concluded that it was best to use a smaller resistor when you want a sharper resonance.

An illustration of our observations is shown in Figure 3.5.

Question 6

Part A

We then built the bandpass circuit illustrated in Figure 3.6 using the same values for L, R, and C as our circuit in Question 5. We then measured the gain as a function of frequency near ω_0 and recorded our data in Table 3.2.

Using this data we can determine $Q = \omega_0/\Delta\omega$ where $\Delta\omega$ is the Full Width at Half Maximum (FWHM) of the resonance.

We found $\omega_0 = 500\text{kHz} * 2\pi = 3142$ krad/s. Using Python, we were able to estimate $\Delta\omega = 600\text{kHz} * 2\pi = 3770$ krad/s. Since $Q = \omega_0/\Delta\omega$, $Q = 3142/3770 = 0.80$ using non-rounded intermediate values for $\omega_0 and \Delta\omega$.

We then plotted the gain as a function of frequency which is illustrated in Figure 3.7.

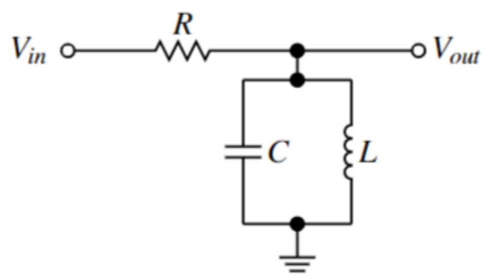


Figure 3.6: Diagram of a different LRC bandpass filter

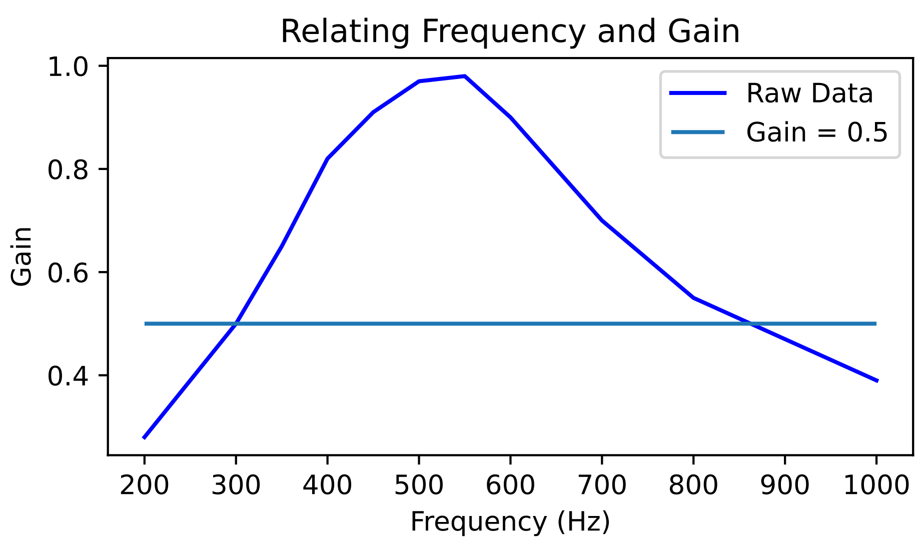


Figure 3.7: Illustrating the relationship between circuit gain and frequency of AC input voltage

f (kHz)	Gain	f (kHz)	Gain
200	0.28	500	0.97
250	0.39	550	0.98
300	0.50	600	0.90
350	0.65	700	0.70
400	0.82	800	0.55
450	0.91	1000	0.39

Table 3.2: Table illustrating the dependence of gain on frequency corresponding to the circuit illustrated in Figure 3.6

Part B

Using the data from Table 3.2, we can also generate a plot of I_R, the current through the resistor as a function of frequency, f, through the relationship between voltage supplied and voltage dissipated through elements in a closed loop. Since V_{out} is measured across the parallel combination of L and C, any voltage not dissipated by those components will be dissipated by the resistor, R. Therefore, $V_R = V_{in} - Vout$. Since $V = IR$, the current through the resistor can be modeled as $I = \frac{V_{in} - V_{out}}{R}$. The voltage supplied $V_{in} = 2.0$ V and $R = 100\Omega$ through the entire frequency scan. Table 3.3 shows the data which generated Figure 3.8.

The plot seems to be a vertical reflection of Figure 3.6 multiplied by some proportionality constant. This makes sense as the more voltage dissipated by the inductor and capacitor, the higher the gain, and the lower the voltage dissipated by the resisitor, therefore the lower the current through the capacitor.

f (kHz)	I_R (mA)	f (kHz)	I_R (mA)
200	14.4	500	0.6
250	12.2	550	0.4
300	10.0	600	2.0
350	7.0	700	6.0
400	3.6	800	9.0
450	1.8	1000	12.2

Table 3.3: Table illustrating the dependence of I_R on frequency corresponding to the circuit illustrated in Figure 3.6

Part C

We can also derive the theoretical gain of the circuit as follows:

$$\mathbf{V_{in}} = V_{in}$$
$$\mathbf{V_{in}} = \mathbf{IZ}$$
$$\mathbf{V_{out}} = \mathbf{IZ_{LC}}$$
$$\mathbf{Z_{LC}} = \frac{j}{1/\omega L - \omega C}$$
$$\mathbf{Z} = R + \mathbf{Z_{LC}}$$
$$\mathbf{I} = V_{in}(R + \frac{j}{1/\omega L - \omega C})$$
$$\mathbf{V_{out}} = V_{in}(R + \frac{j}{1/\omega L - \omega C})(\frac{j}{1/\omega L - \omega C})$$
$$\mathbf{V_{out}} = V_{in}(\frac{Rj}{1/\omega L - \omega C} - \frac{1}{(1/\omega L - \omega C)^2})$$
$$V_{out} = V_{in}(\frac{R^2}{(1/\omega L - \omega C)^2} - \frac{1}{(1/\omega L - \omega C)^4})$$
$$\text{Gain} = \frac{R^2}{(1/\omega L - \omega C)^2} - \frac{1}{(1/\omega L - \omega C)^4}$$

Using our values for R and L, we can compute $Q = \omega_0 L/R = (3142\text{krad/s})(20\mu\text{H})/100\Omega = 0.628$. This does not match our measured value of Q very well as our measured value was too high. There was likely error in calculating the FWHM or in the exact specifications of our circuit components.

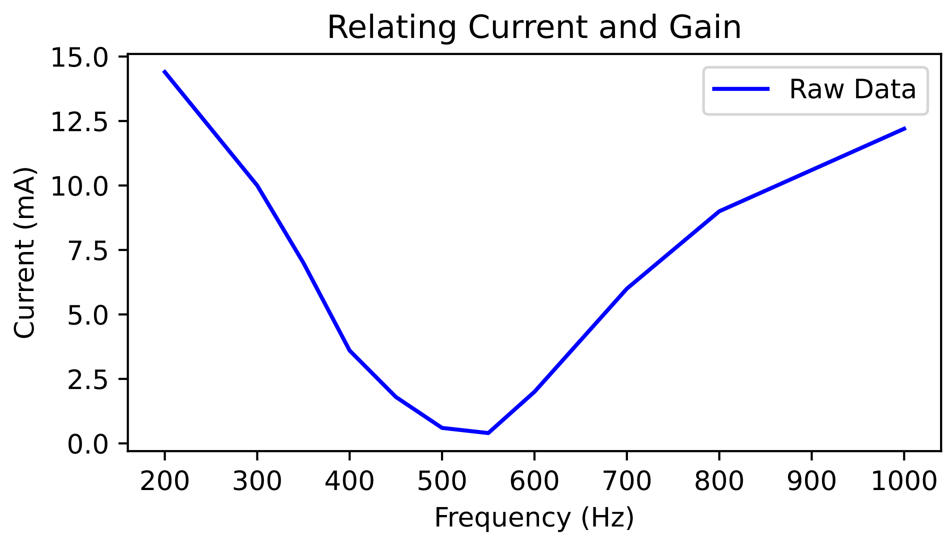


Figure 3.8: Plot of the relationship between current through resisitor and frequency of a LRC bandpass circuit